彭桓武高能基础理论研究中心

Peng Huanwu Center for Fundamental Theory

“2023引力与宇宙学”专题研讨会

玻色星、狄拉克星和虫洞

王永强　兰州大学

2023.4.7

@中国科学技术大学

2022年物理诺贝尔奖：纠缠光子实验、证明违反贝尔不等式和开创量子信息科学

量子纠缠(Einstein-Podolsky-Rosen (EPR) 1935年)

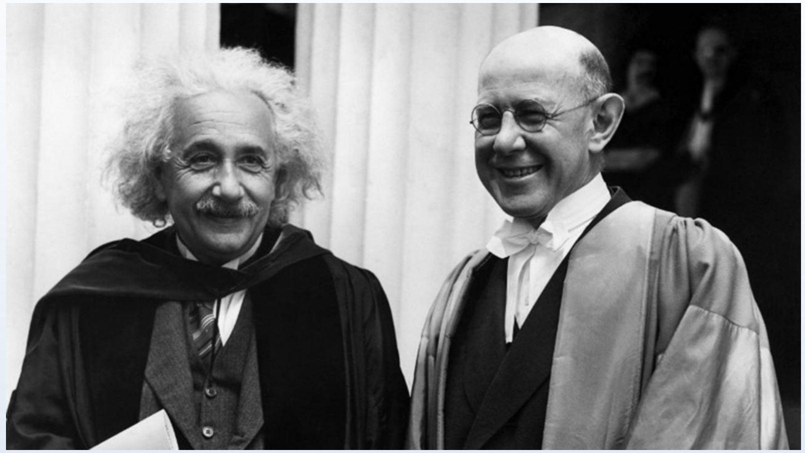
鬼魅般的超距作用

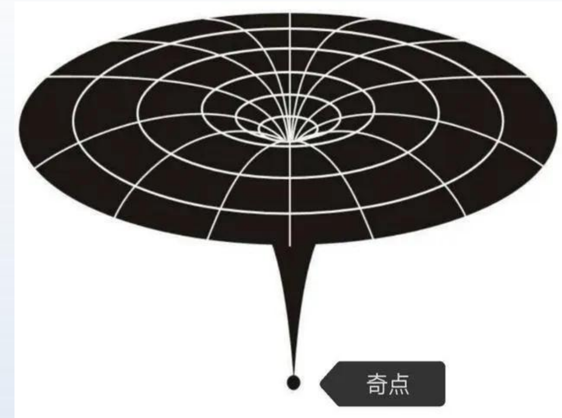
在谷歌量子计算机一个9bit量子位电路上，构造了一个稀疏Sachdev-Ye-Kitaev（SYK）模型，并观察到了虫洞的特征。

1935 三月 EPR

五月

Einstein—Rosen (ER) bridge

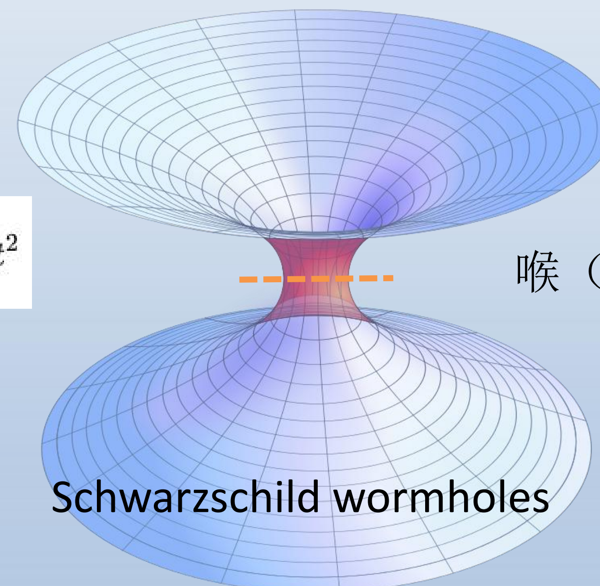




$$ds^2 = -c^2\left(1-\frac{2GM}{rc^2}\right)dt^2 + \frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2(d\theta^2+\sin^2\theta\, d\varphi^2).$$

$$u^2 = r - 2m$$

$$ds^2 = -4(u^2+2m)\,du^2 - (u^2+2m)^2(d\theta^2+\sin^2\theta\, d\varphi^2) + \frac{u^2}{u^2+2m}dt^2$$



Two Schwarzschild solutions joined by a throat

JULY 1, 1935 PHYSICAL REVIEW VOLUME 48

The Particle Problem in the General Theory of Relativity

A. EINSTEIN AND N. ROSEN, *Institute for Advanced Study, Princeton*

(Received May 8, 1935)

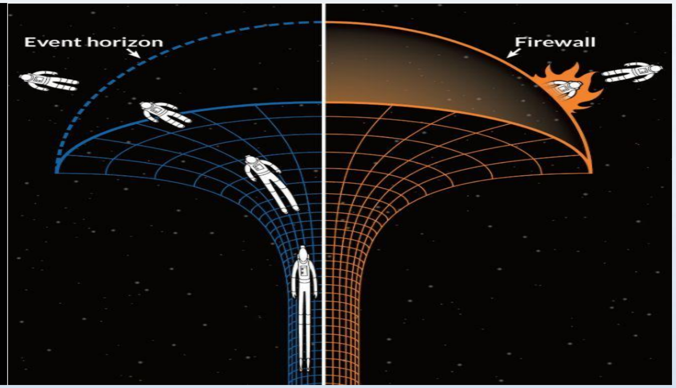
The writers investigate the possibility of an atomistic theory of matter and electricity which, while excluding singularities of the field, makes use of no other variables than the $g_{\mu\nu}$ of the general relativity theory and the φ_μ of the Maxwell theory. By the consideration of a simple example they are led to modify slightly the gravitational equations which then admit regular solutions for the static spherically symmetric case. These solutions involve the mathematical representation of physical space by a space of two identical sheets, a particle being represented by a "bridge" connecting these sheets. One is able to understand why no neutral particles of negative mass are to be found. The combined system of gravitational and electromagnetic equations are treated similarly and lead to a similar interpretation. The most natural elementary charged particle is found to be one of zero mass. The many-particle system is expected to be represented by a regular solution of the field equations corresponding to a space of two identical sheets joined by many bridges. In this case, because of the absence of singularities, the field equations determine both the field and the motion of the particles. The many-particle problem, which would decide the value of the theory, has not yet been treated.

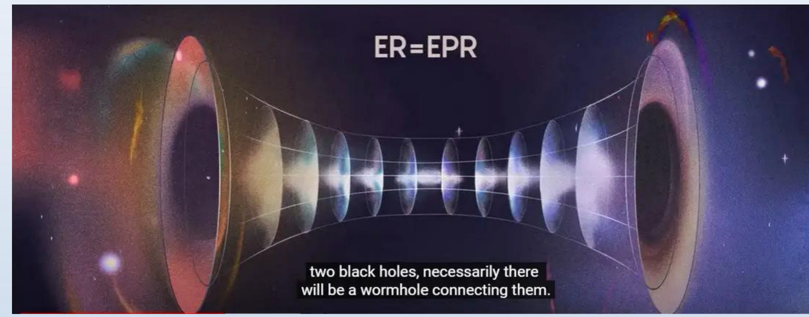
黑洞信息丢失问题

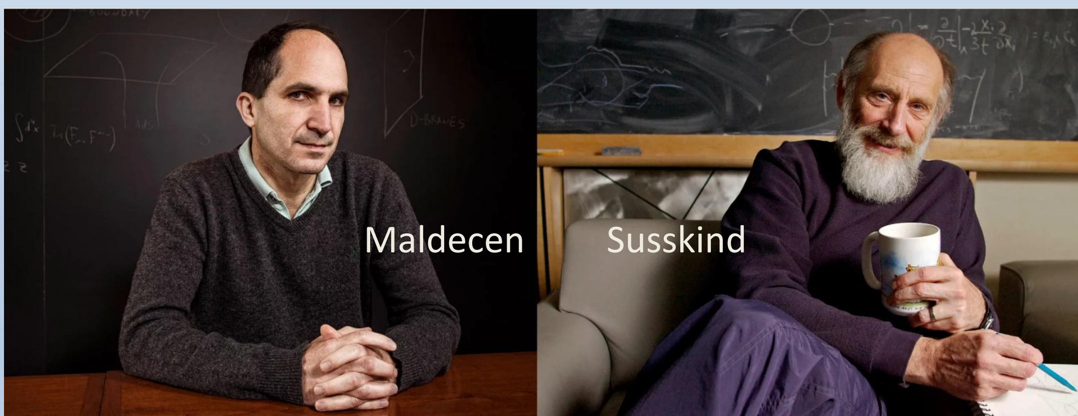
火墙悖论（2012）

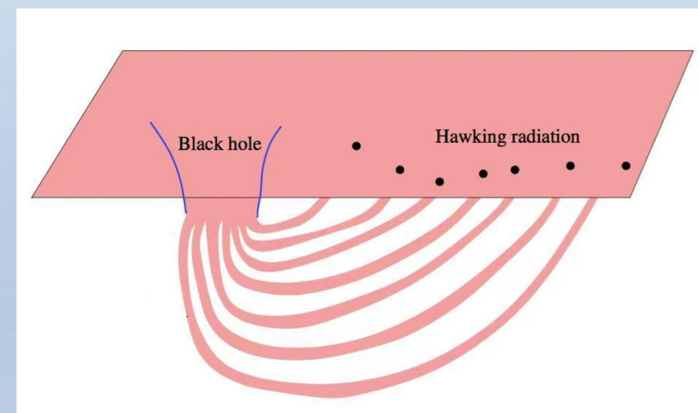
ER=EPR 猜想（2013）

ER bridge between two black holes is created by EPR-like correlations between the microstates of the two black holes.









"Geons".
(真子，京子，智子）
Phys Rev 97 (1955) .

"gravitational electromagnetic entity"

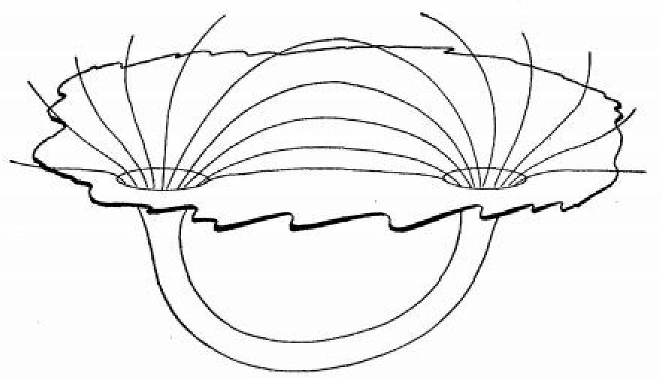
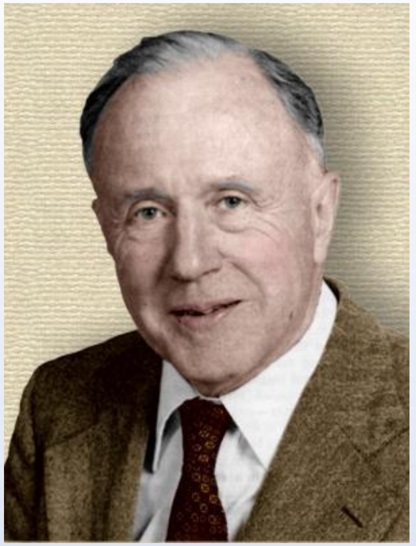
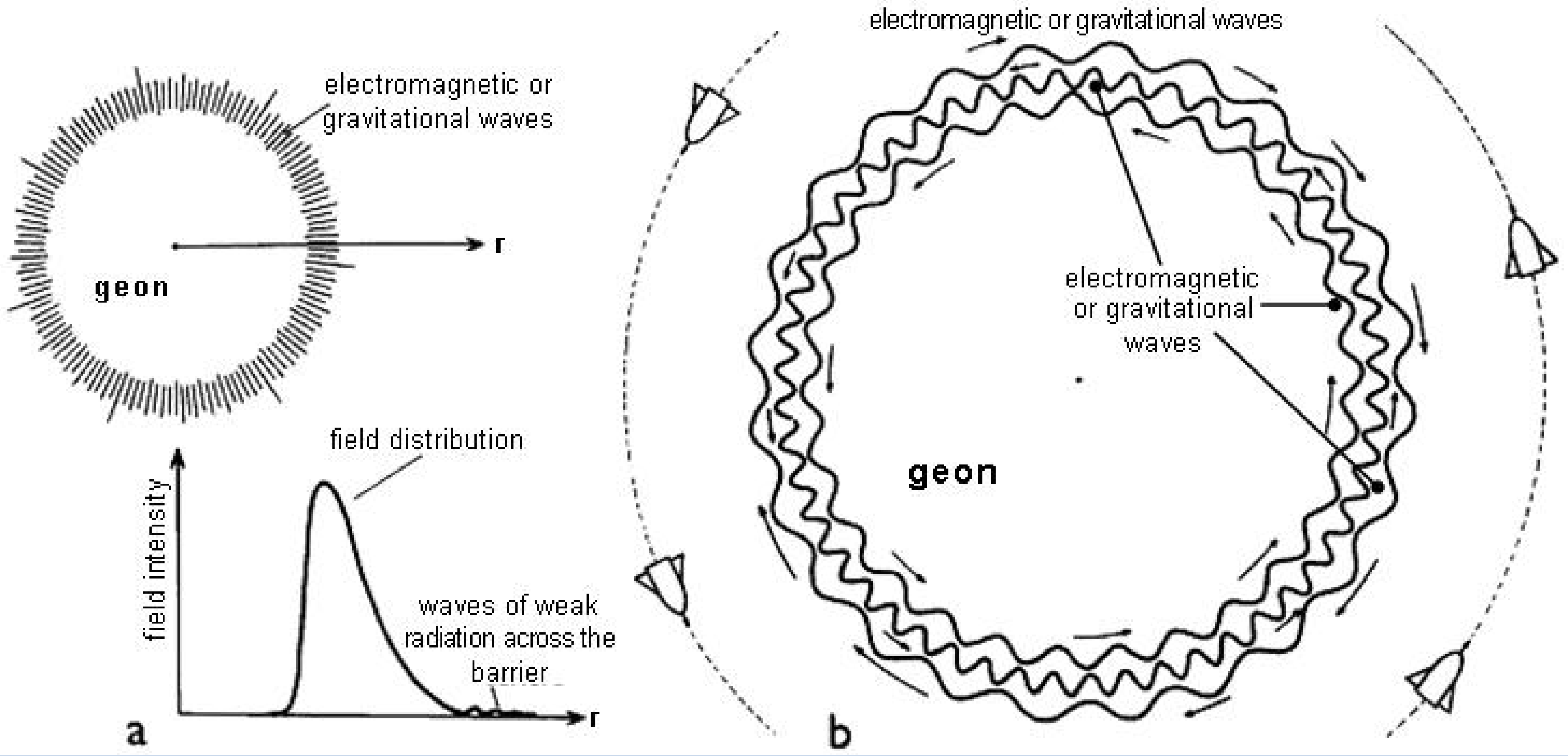


FIG. 7. Schematic representation of lines of force in a doubly-connected space. In the upper continuum the lines of force behave much as if the tunnel mouths were the seats of equal and opposite charges.





主要内容

- 玻色星的介绍
- 狄拉克星的介绍
- 可穿越虫洞的介绍
- Einstein-Dirac-Maxwell中的可穿越虫洞解
- 总结

1. 玻色星的介绍

爱因斯坦方程（1915年）：

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

稳定的particle-like解(3+1)

1.1 有Event Horizon情况

Maxwell field: $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

Schwarzschild **1916**
Reissner–Nordström **(1916-1921)**
Kerr **1962**
Kerr–Newman **1965**

黑洞的无毛定理

Ruffini, Wheeler **(1971)**

Misner, Thorne, Wheeler **(1973)**

Scalar field: $\mathcal{L}_{(0)} = -g^{\alpha\beta} \bar{\Phi}_{,\alpha} \Phi_{,\beta} - \mu^2 \bar{\Phi} \Phi$

Proca field: $\mathcal{L}_{(1)} = -\frac{1}{4} \mathcal{F}_{\alpha\beta} \bar{\mathcal{F}}^{\alpha\beta} - \frac{\mu^2}{2} \mathcal{A}_\alpha \bar{\mathcal{A}}^\alpha$

Dirac field: $\mathcal{L}_{(1/2)} = -i \left[\frac{1}{2} \left(\{\hat{\not{D}} \overline{\Psi}\} \Psi - \overline{\Psi} \hat{\not{D}} \Psi \right) + \mu \overline{\Psi} \Psi \right]$

2014年（近一百年）又发现了新的3+1维黑洞解

1. 带标量毛的旋转黑洞 （标量荷）
C. Herdeiro and E. Radu **PRL 112 (2014)**

2. 带Proca毛的旋转黑洞
C. Herdeiro, E. Radu, H. Rúnarsson **CQG 33 (2016)**

3. 带Dirac毛的旋转黑洞目前未找到

4. 带激发态标量毛的旋转黑洞

YQW, Yu-Xiao Liu, Shao-Wen Wei **PRD 99 (2019)**

Hairy black hole solutions I

Einstein-Yang-Mills theory
Bizón 1990; Kunzle and Masood-ul-Alam, 1990; Volkov and Galtsov, 1990

Einstein-Skyrme,
Einstein-Yang-Mills-Dilaton,
Einstein-Yang-Mills-Higgs,
Einstein-non-Abelian-Proca**,**
etc,
Review by Bizón 1994; Volkov and Gal'tsov (1999)

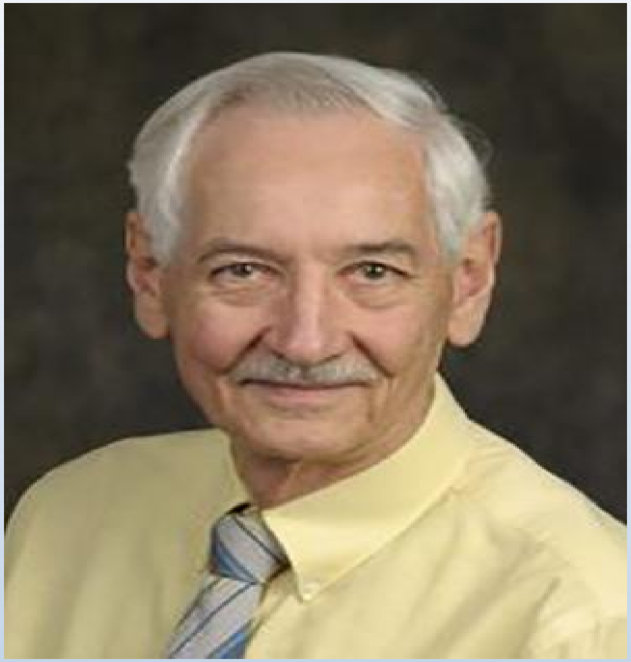
Hairy black hole solutions II
（non-Einstein gravity or general matter field ）

There are lots of papers in this area.

1.2 无Event Horizon情况 （Geons）

Boson stars

Review: Liebling and Palenzuela (2012)



David J. Kaup

Phys. Rev. 172, 1331 (1968)

Remo Ruffini

Ruffini R and Bonazzola S. Phys. Rev. 1969 (1969)

Einstein-Klein-Gordon theory:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}g^{ab}\left(\Psi^*_{,a}\Psi_{,b} + \Psi^*_{,b}\Psi_{,a}\right) - \mu^2\Psi^*\Psi\right],$$

Spherical boson stars **Kaup (1968); Ruffini, Bonazzola (1969)**

$$ds^2 = -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$\Psi = \phi(r)e^{-iwt}$$

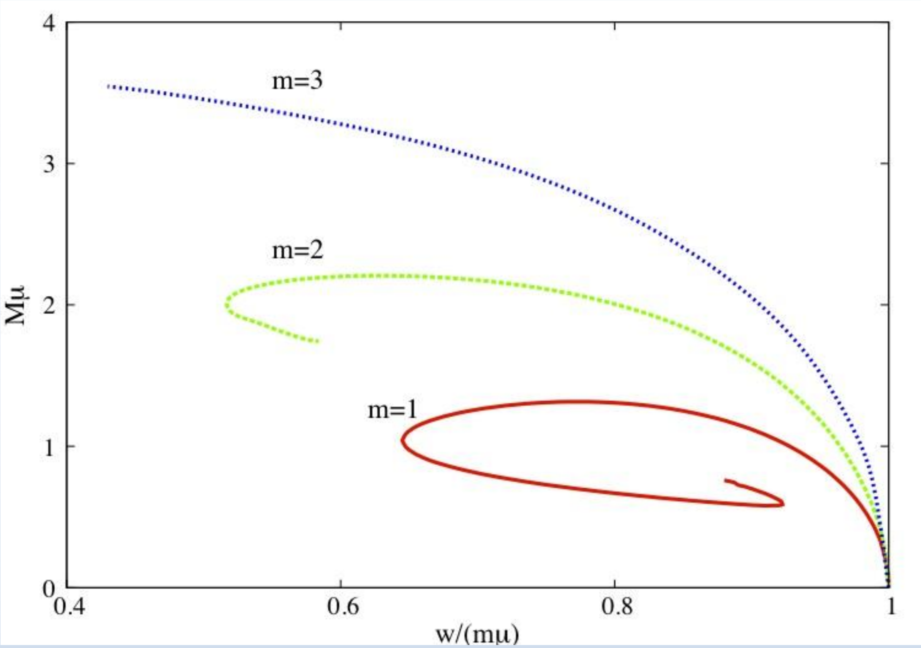
Rotating boson stars **Yoshida, Eriguchi (1997), Schunck, Mielke (1998)**

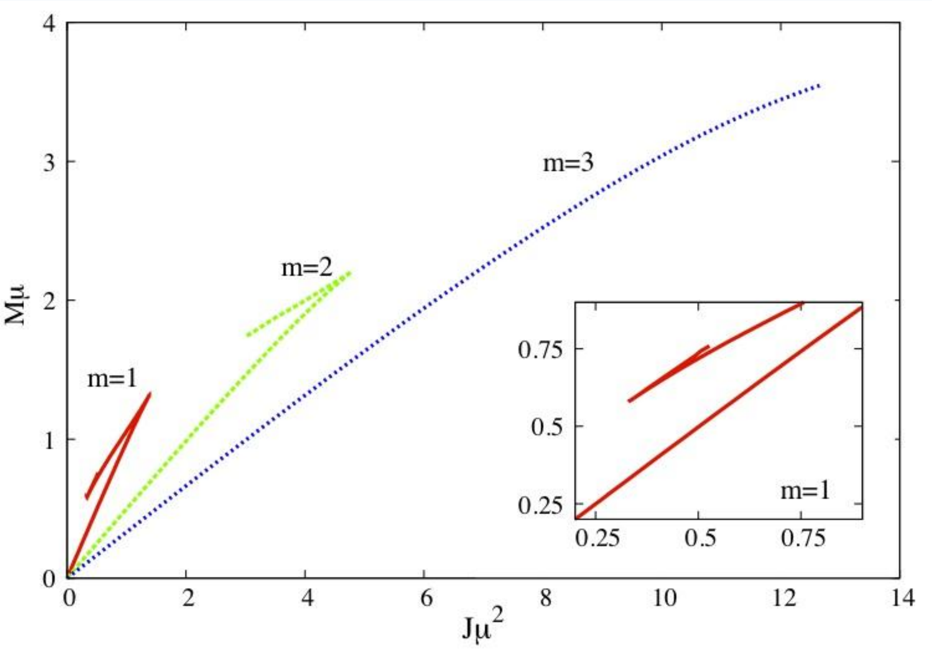
$$ds^2 = -e^{2\nu(r,\theta)}dt^2 + e^{2\alpha(r,\theta)}(dr^2 + r^2 d\theta^2) + e^{2\beta(r,\theta)}r^2\sin^2\theta(d\varphi - \mathrm{w}(r,\theta)dt)^2.$$

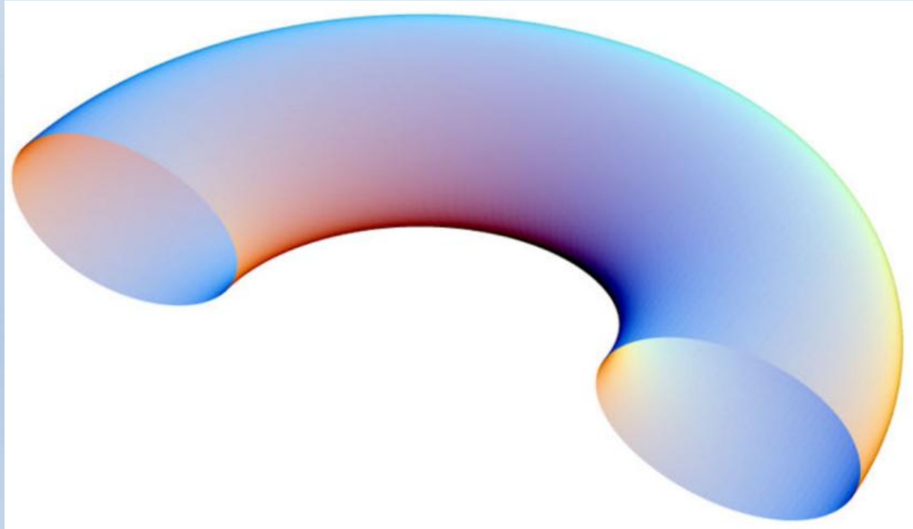
The Complex scalar field $\Psi = \phi(r,\ \theta)\mathrm{e}^{\mathrm{i}(m\varphi - wt)}$

where ω is the scalar field frequency and $m = \pm 1, \pm 2, \ldots$ is the azimuthal harmonic index. Three input parameters: (μ, ω, m)

The boson star mass *M* and the angular momentum *J*







Ergosurfaces

(PRD 89, 124018 (2014))

$$\xi \cdot \xi = 0 \Leftrightarrow 0 = g_{tt} = -e^{2F_0}N + W^2 e^{2F_2} r^2 \sin^2\theta.$$

Varieties of Boson Stars

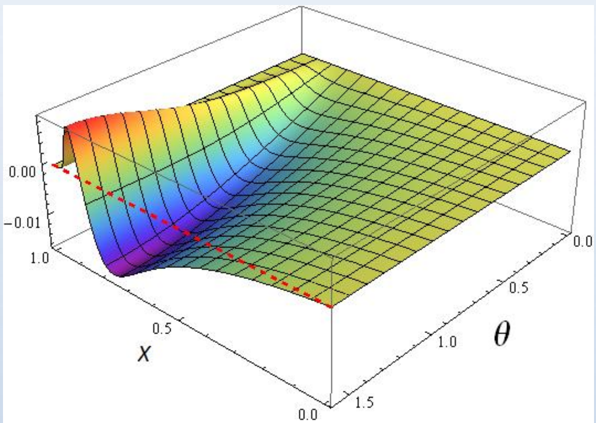
- Self-interaction potential
- Newtonian boson star
- Charged boson star
- Fermionic-bosonic star
- Multi-state boson star
- Alternative theories of gravity
- Gauged boson stars
- AdS boson stars

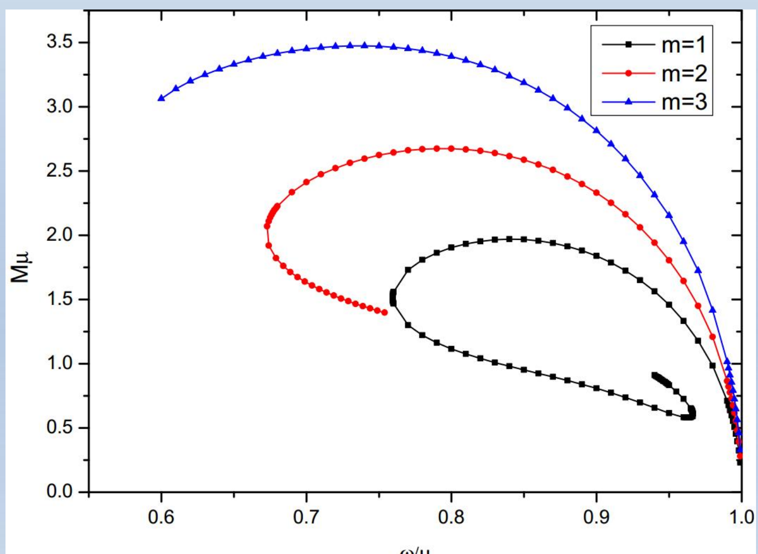
The rotating excited state boson star solutions

YQW, Yu-Xiao Liu and Shao-Wen Wei, PRD 99, 064036 (2019)

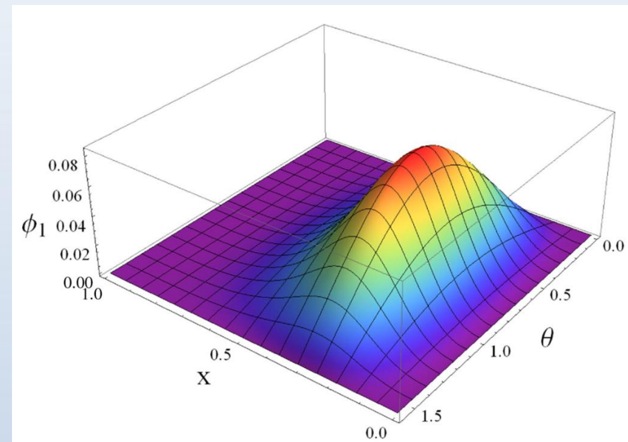
The first-excited states have two types solutions

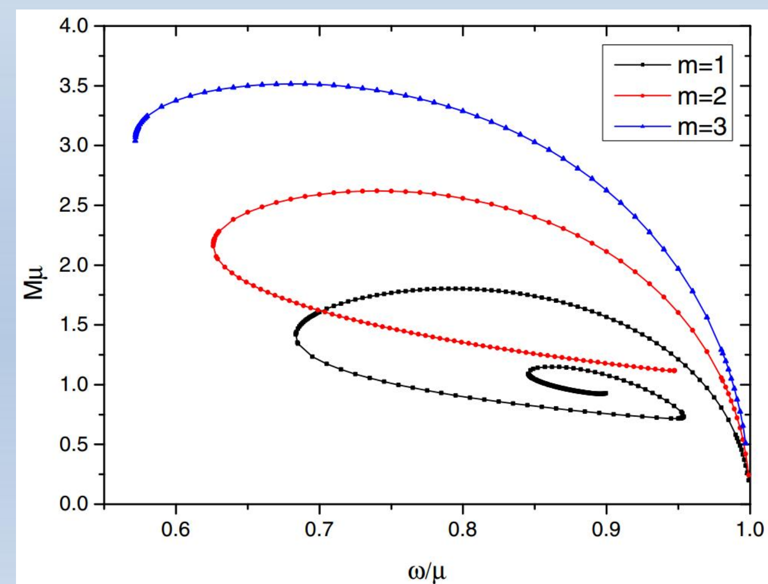
Radial nodes $n_r = 1$





Angular nodes $n_\theta = 1$





Rotating multistate boson stars solutions

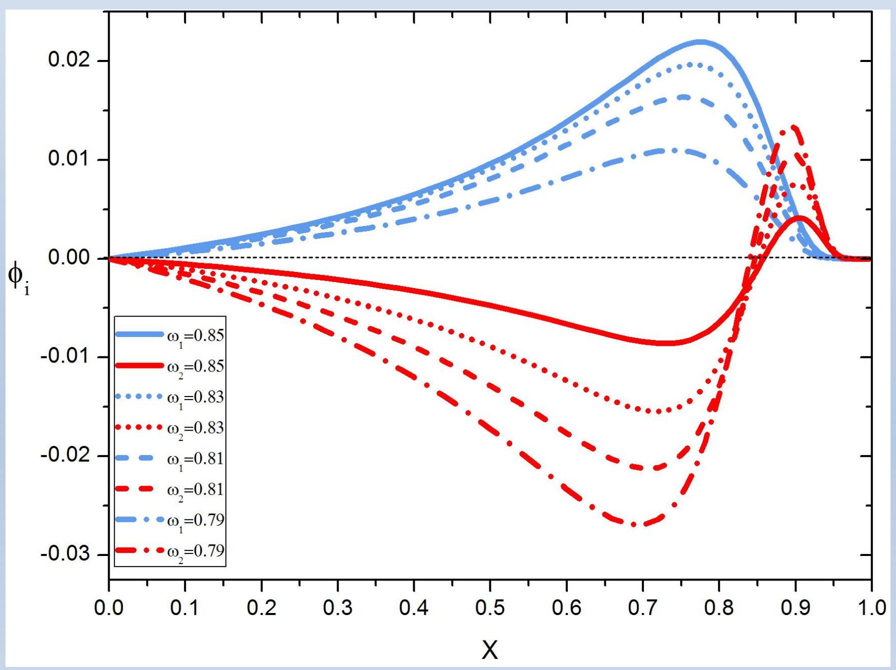
H.B.Li, S.Sun, T.T. Hu, Y.Song, ***YQW***, *Phys.Rev.D 101 (2020)*

The complex multi-scalar fields

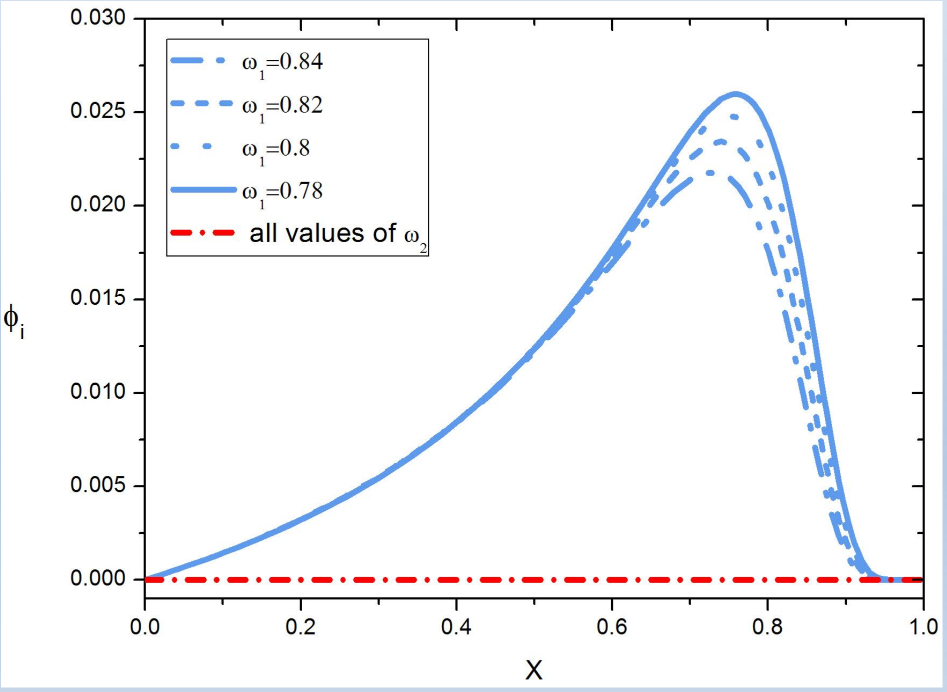
$$\psi_i = \phi_{i(n)}(r,\theta)e^{i(m_i\varphi-\omega_i t)}, \quad i = 1, 2, \quad n = 0, 1, \cdots, \quad m_i = \pm 1, \pm 2, \cdots .$$

The rotating multistate boson stars with same frequency

$^1S\,^2S$ State



$^1S\,^2P$ State



- Proca star （自旋1的有质量矢量场）

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2}\mu^2\mathcal{A}_\alpha\bar{\mathcal{A}}^\alpha\right)$$

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta}, \ \nabla_\alpha\mathcal{F}^{\alpha\beta} = \mu^2\mathcal{A}^\beta$$

$$\nabla_\alpha\mathcal{A}^\alpha = 0$$

Lorentz condition

Spherical N. Rosen, Found. Phys. 24 (1994)

$$ds^2 = -\sigma^2(r)N(r)dt^2 + \frac{dr^2}{N(r)} + r^2 d\Omega_2$$

$$\mathcal{A} = e^{-iwt}\left[f(r)dt + ig(r)dr\right]$$

Rotating R. Brito, V. Cardoso, C. Herdeiro, E. Radu, PLB 752 (2016)

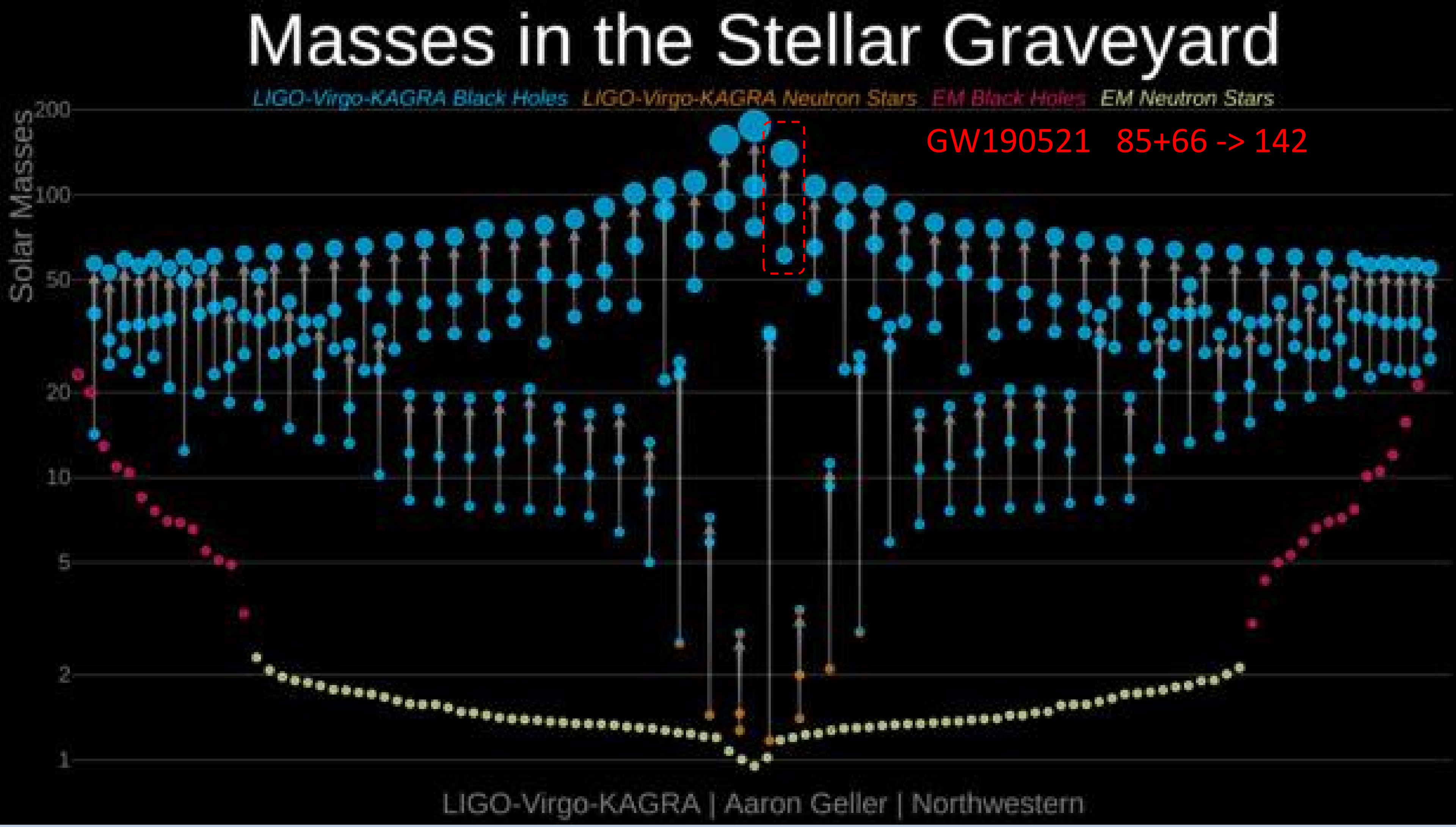
$$ds^2 = -e^{2F_0(r,\theta)}dt^2 + e^{2F_1(r,\theta)}(dr^2 + r^2 d\theta^2) + e^{2F_2(r,\theta)}r^2\sin^2\theta\left(d\varphi - \frac{W(r,\theta)}{r}dt\right)^2$$

$$A = \left(\frac{H_1}{r}dr + H_2 d\theta + iH_3\sin\theta d\varphi + iV dt\right)e^{i(m\varphi - wt)}$$

在天文物理研究中的玻色星

1. 黑洞的mimic

2. 暗物质存在形式的候选（标量暗物质，暗光子）



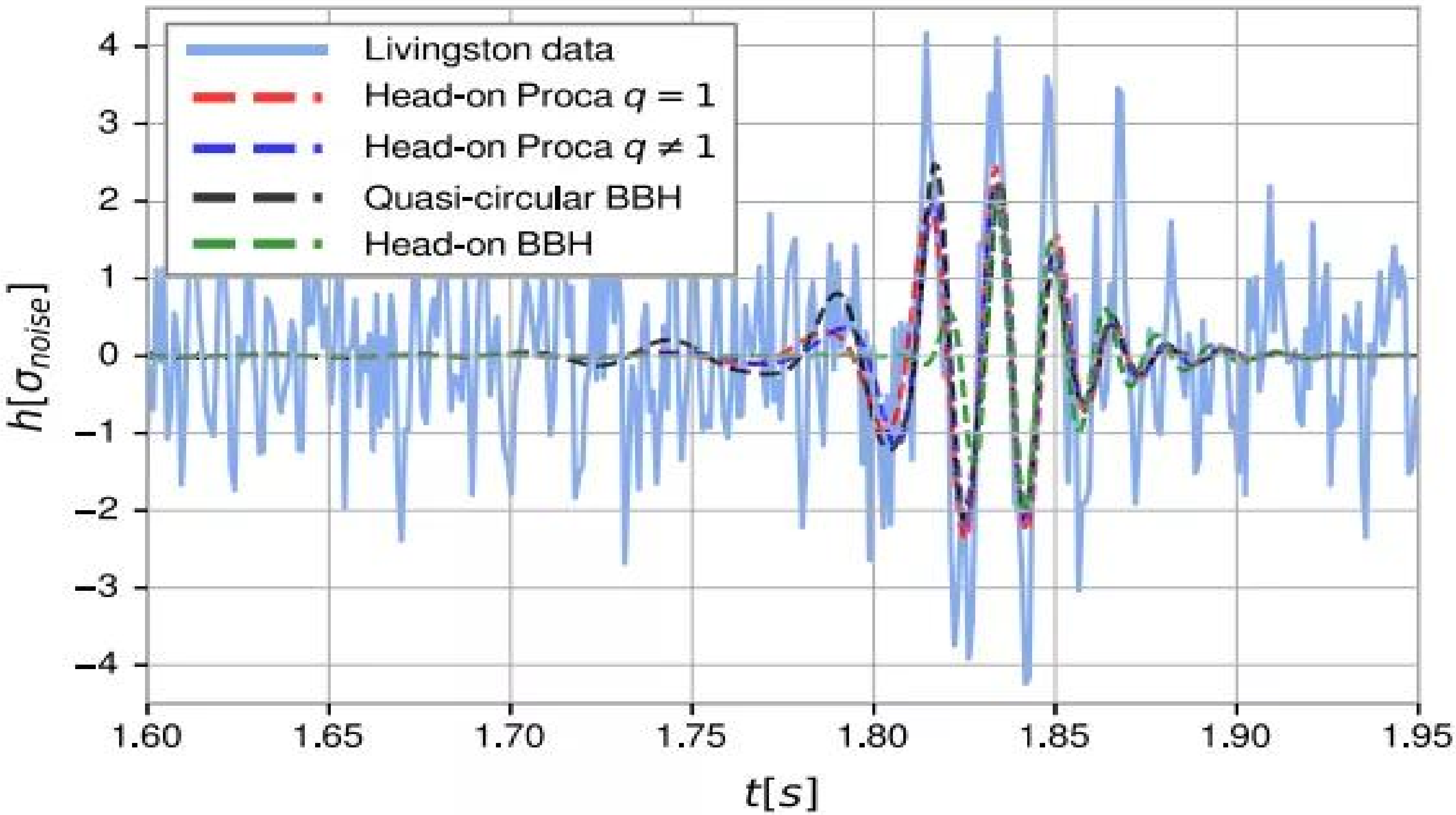
中等质量黑洞第一次被探测到

- GW190521。 两个**85倍**太阳质量和**66倍**太阳质量的黑洞碰撞相一致，碰撞结果产生了一个具有**142倍**太阳质量的黑洞。 0.1S 旋转 4周 频率 30~80HZ

PHYSICAL REVIEW LETTERS

GW190521 as a Merger of Proca Stars: A Potential New Vector Boson of 8.7×10^{-13} eV

Juan Calderón Bustillo, Nicolas Sanchis-Gual, Alejandro Torres-Forné, José A. Font, Avi Vajpeyi, Rory Smith, Carlos Herdeiro, Eugen Radu, and Samson H. W. Leong



2. 狄拉克星解

$$\mathcal{L}_{(1/2)} = -i\left[\frac{1}{2}\left(\{\hat{\not{D}}\overline{\Psi}\}\Psi - \overline{\Psi}\hat{\not{D}}\Psi\right) + \mu\overline{\Psi}\Psi\right]$$

Dirac 场的类粒子解

平直时空

- D. Ivanenko, Sov. Phys. 13, 141 (1938)
- H. Weyl, Phys. Rev. 77, 699 (1950)
- W. Heisenberg, Physica 19, 897 (1953)
- R. Finkelstein, R. LeLevier, M. Ruderman, Phys. Rev. 83, 326(1951)
- R. Finkelstein, C.F. Fronsdal, P. Kaus, Phys. Rev. 103, 1571 (1956)
- **M. Soler, Phys. Rev. D 1, 2766 (1970)** **a numerical study**
- **T. Cazenave, L. Vazquez, Commun. Math. Phys. 105, 35–47 (1986)** **a proof of existence**

考虑引力

D.R. Brill, J.A. Wheeler, Rev. Mod. Phys. 29, 465–479 (1957)

T.D. Lee, Y. Pang, Phys. Rev. D 35, 3678 (1987)

F. Finster, J. Smoller, S.T. Yau, Phys. Rev. D 59, 104020 (1999).

Dirac 星

引力与旋量场（spin-1/2)

单个费米子

球对称构型

两个费米子（自旋相反）

两电子自旋波函数： 总自旋为1的三重态
自旋为零的单态 (纠缠态)

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R + \mathcal{L}_{(s)}\right]$$

$$\mathcal{L}^{[A]}_{(1/2)} = -i\left[\frac{1}{2}\left(\{\hat{\not{D}}\overline{\Psi}^{[A]}\}\Psi^{[A]} - \overline{\Psi}^{[A]}\hat{\not{D}}\Psi^{[A]}\right) + \mu\overline{\Psi}^{[A]}\Psi^{[A]}\right]$$

$$A = 1, 2$$

$$\Psi^{[1]} = \begin{pmatrix} \cos(\frac{\theta}{2})z(r) \\ i\sin(\frac{\theta}{2})\bar{z}(r) \\ -i\cos(\frac{\theta}{2})\bar{z}(r) \\ -\sin(\frac{\theta}{2})z(r) \end{pmatrix} e^{i(\frac{1}{2}\varphi - wt)}\ , \qquad \Psi^{[2]} = \begin{pmatrix} i\sin(\frac{\theta}{2})z(r) \\ \cos(\frac{\theta}{2})\bar{z}(r) \\ \sin(\frac{\theta}{2})\bar{z}(r) \\ i\cos(\frac{\theta}{2})z(r) \end{pmatrix} e^{i(-\frac{1}{2}\varphi - wt)}$$

$$z(r) \equiv (1+i)f(r) + (1-i)g(r)$$

Dirac-boson stars Chen Liang, Ji-Rong Ren, Shi-Xian Sun, and WYQ，JHEP 02 (2023) 249

系统作用量：

$$S=\int\sqrt{-g}d^4x\left(\frac{R}{16\pi G}+\mathcal{L}_S+\mathcal{L}_D\right),$$

场方程：

$$R_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R=8\pi G\left(T^S_{\alpha\beta}+T^D_{\alpha\beta}\right),$$

$$\nabla^2\psi-\mu_S^2\psi=0,$$

$$\gamma^\mu\hat{D}_\mu\Psi^{(k)}-\mu_D\Psi^{(k)}=0,$$

$$ds^2=-N(r)\sigma^2(r)dt^2+\frac{dr^2}{N(r)}+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right).$$

$$\psi=\phi(r)e^{-i\omega_S t},$$

$$\Psi^{(1)}=\begin{pmatrix}\cos(\frac{\theta}{2})z(r)\\ i\sin(\frac{\theta}{2})\overline{z}(r)\\ -i\cos(\frac{\theta}{2})\overline{z}(r)\\ -\sin(\frac{\theta}{2})z(r)\end{pmatrix}e^{i\frac{\varphi}{2}-i\omega_D t},\quad \Psi^{(2)}=\begin{pmatrix}i\sin(\frac{\theta}{2})z(r)\\ \cos(\frac{\theta}{2})\overline{z}_n(r)\\ \sin(\frac{\theta}{2})\overline{z}(r)\\ i\cos(\frac{\theta}{2})z(r)\end{pmatrix}e^{-i\frac{\varphi}{2}-i\omega_D t},$$

3 可穿越虫洞

PHYSICAL REVIEW VOLUME 97, NUMBER 2 JANUARY 15, 1955

Geons*

JOHN ARCHIBALD WHEELER
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received September 8, 1954)

Associated with an electromagnetic disturbance is a mass, the gravitational attraction of which under appropriate circumstances is capable of holding the disturbance together for a time long in comparison with the characteristic periods of the system. Such gravitational-electromagnetic entities, or "geons"; are analyzed via classical relativity theory. They furnish for the first time a completely classical, divergence-free, self-consistent picture of the Newtonian concept of body over the range of masses from $\sim 10^{39}$ g to $\sim 10^{57}$ g. Smaller geons are quantum objects whose analysis would call for the treatment of characteristic new effects. Topics covered in the discussion include: 1. Need for a self-consistent of equations of self-consistent geon; mass and radius values. 4. Transformations and interactions of electromagnetic geons; evaluation of refractive index barrier penetration integral for spherical geon; photon-photon collision processes as additional mechanism for escape of energy from system; restatement in language of coupling of characteristic modes; the thermal geon; comparison of gravitation and virtual electron pair phenomena as sources of coupling between modes; gravitational coupling and collective vibrations of geon; fission of a geon; interaction between two geons simple at large distances; orientation dependence and exponential term at intermediate distances; violent transmutation

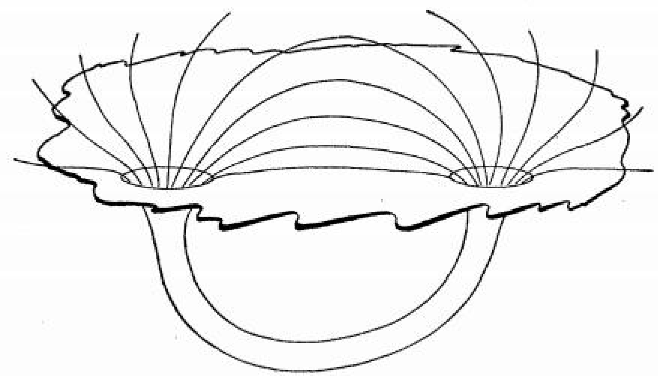


FIG. 7. Schematic representation of lines of force in a doubly-connected space. In the upper continuum the lines of force behave much as if the tunnel mouths were the seats of equal and opposite charges.

ANNALS OF PHYSICS: **2**, 525–603 (1957)

Classical Physics as Geometry

Gravitation, Electromagnetism, Unquantized Charge, and Mass as Properties of Curved Empty Space*

CHARLES W. MISNER† AND JOHN A. WHEELER‡

Lorentz Institute, University of Leiden, Leiden, Netherlands, and Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

ditions for Maxwell's equations in a closed sp
consider situations—such as described by one
is a net flux of lines of force through what t
the multiply-connected space and what phys
for more vividly terming a "wormhole". The
from the mouth of a small wormhole appears t
resolving power to come from an elementary ele

PHYSICAL REVIEW VOLUME 128, NUMBER 2 OCTOBER 15, 1962

Causality and Multiply Connected Space-Time

ROBERT W. FULLER*
Pupin Physical Laboratories, Columbia University, New York, New York

AND

JOHN A. WHEELER†
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received May 16, 1962)

to exist in the example considered in the paper. It is shown that no signal can ever be sent from one to the other. The key point in preventing any violation of causality is simple: The (Schwarzschild) throat of the wormhole pinches off in a finite time and traps the signal in a region of infinite curvature. This investigation also displays some of the unusual geometric features of the Schwarzschild solution of Einstein's equations for

Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity

Michael S. Morris and Kip S. Thorne
Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125

(Received 16 March 1987; accepted for publication 17 July 1987)

Rapid interstellar travel by means of spacetime wormholes is described in a way that is useful for teaching elementary general relativity. The description touches base with Carl Sagan's novel *Contact*, which, unlike most science fiction novels, treats such travel in a manner that accords with the best 1986 knowledge of the laws of physics. Many objections are given against the use of black holes or Schwarzschild wormholes for rapid interstellar travel. A new class of solutions of the Einstein field equations is presented, which describe wormholes that, in principle, could be traversed by human beings. It is essential in these solutions that the wormhole possess a throat at which there is no horizon; and this property, together with the Einstein field equations, places an extreme constraint on the material that generates the wormhole's spacetime curvature: In the wormhole's throat that material must possess a radial tension τ_0 with the enormous magnitude $\tau_0 \sim$ (pressure at the center of the most massive of neutron stars) $\times$ (20 km)^2/(circumference of throat)^2. Moreover, this tension must exceed the material's density of mass-energy, $\rho_0 c^2$. No known material has this $\tau_0 > \rho_0 c^2$ property, and such material would violate all the "energy conditions" that underlie some deeply cherished theorems in general relativity. However, it is not possible today to rule out firmly the existence of such material; and quantum field theory gives tantalizing hints that such material might, in fact, be possible.

exotic matter

a stress-energy tensor (that violates the null energy condition (NEC))

$$T_{\mu\nu}k^{\mu}k^{\nu} < 0$$

where k^{μ} is *any* null vector

phantom field

$$\frac{1}{4\pi\alpha}\mathcal{R} + \mathcal{L}_{\rm ph}$$

$$\mathcal{L}_{\rm ph} = \frac{1}{2}\partial_{\mu}\Psi\partial^{\mu}\Psi$$

Thin shell wormhole

M. Visser, Phys. Rev. D 39, 3182 (1989); Nucl. Phys. H 328, 203 (1989).

Schwarzschild surgery

"cutting and pasting"

$$ds^2 = -(1 - \tfrac{2M}{r})dt^2 + \frac{dr^2}{(1 - \tfrac{2M}{r})} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)$$

cutting

$$\Omega_{1,2} \equiv \{r_{1,2} \le a \mid a > 2M\}$$

$$\partial\Omega_{1,2} \equiv \{r_{1,2} = a \mid a > 2M\}$$

pasting

$$\partial\Omega_1 \equiv \partial\Omega_2$$

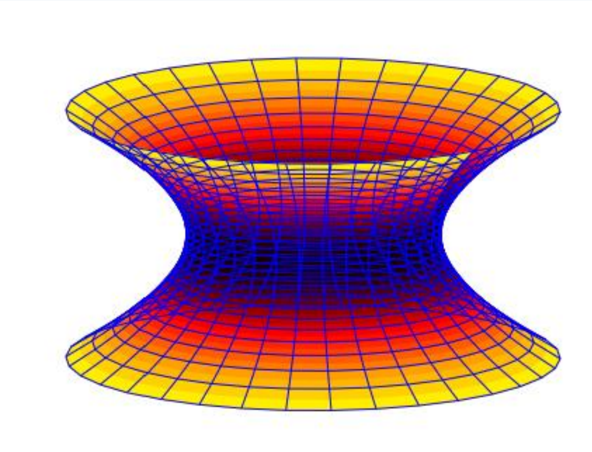
The throat of the thin wormhole

Modified-gravity WHs without exotic matter

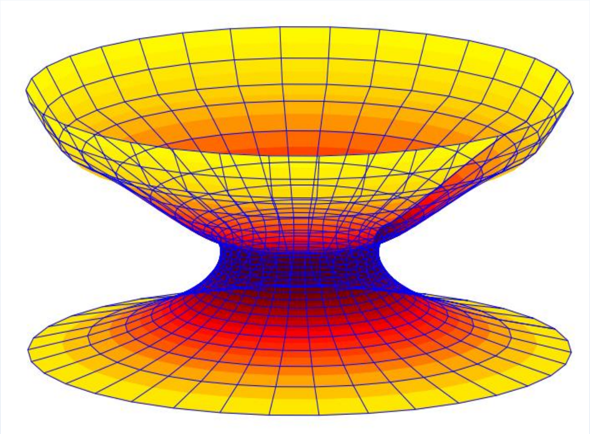
$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}^{\text{eff}}$$

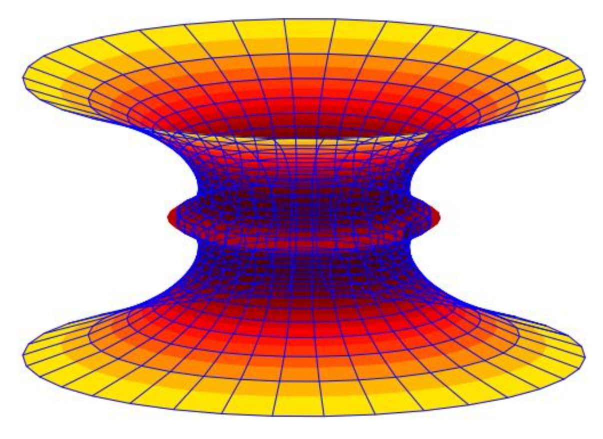
$$T_{\mu\nu}^{\text{eff}} k^\mu k^\nu < 0 \qquad T_{\mu\nu} k^\mu k^\nu \geq 0$$



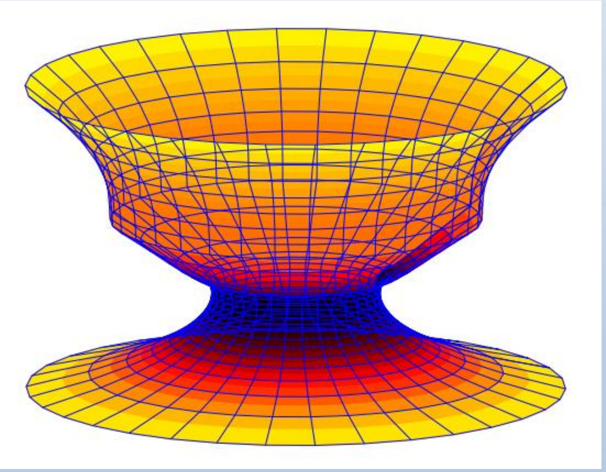
symmetric、 a single throat



asymmetric 、a single throat



symmetric 、 a double throat



asymmetric 、 a double throat

4. Einstein-Dirac-Maxwell中的可穿越虫洞解

Einstein-Dirac-Maxwell model （BSKR Wormhole）

J. L. Blazquez-Salcedo, C. Knoll and E. Radu, Phys. Rev. Lett. 126 101102 (2021)

$$S = \frac{1}{4\pi}\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{4}R + \mathcal{L}_D - \frac{1}{4}F^2\right]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L}_D = \sum_{\epsilon=1,2}\left[\frac{\mathrm{i}}{2}\overline{\Psi}_\epsilon\gamma^\nu\hat{D}_\nu\Psi_\epsilon - \frac{\mathrm{i}}{2}\hat{D}_\nu\overline{\Psi}_\epsilon\gamma^\nu\Psi_\epsilon - \mu\overline{\Psi}_\epsilon\Psi_\epsilon\right]$$

$$\hat{D}_\mu = \partial_\mu + \Gamma_\mu - iqA_\mu$$

$$\Psi_{\epsilon=1,2}$$

$$A = V(r)dt$$

$$\mathcal{R}_1 = -i\mathcal{R}_2 = \begin{bmatrix}\phi(r)\\ -i\bar{\phi}(r)\end{bmatrix},$$

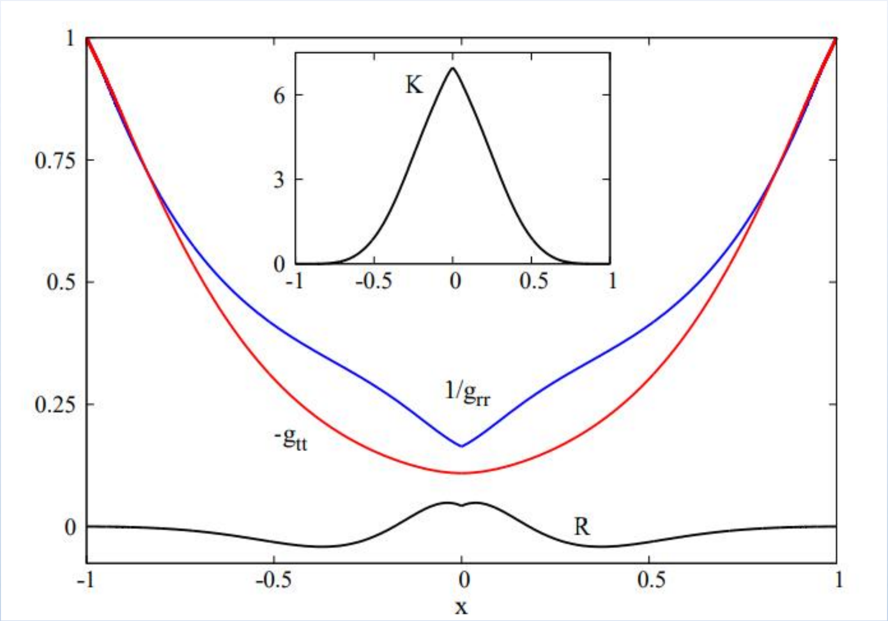
$$\Theta_1 = \begin{bmatrix}-\kappa\sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\end{bmatrix}e^{i\frac{\varphi}{2}},\quad \Theta_2 = \begin{bmatrix}\kappa\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\end{bmatrix}e^{-i\frac{\varphi}{2}}$$

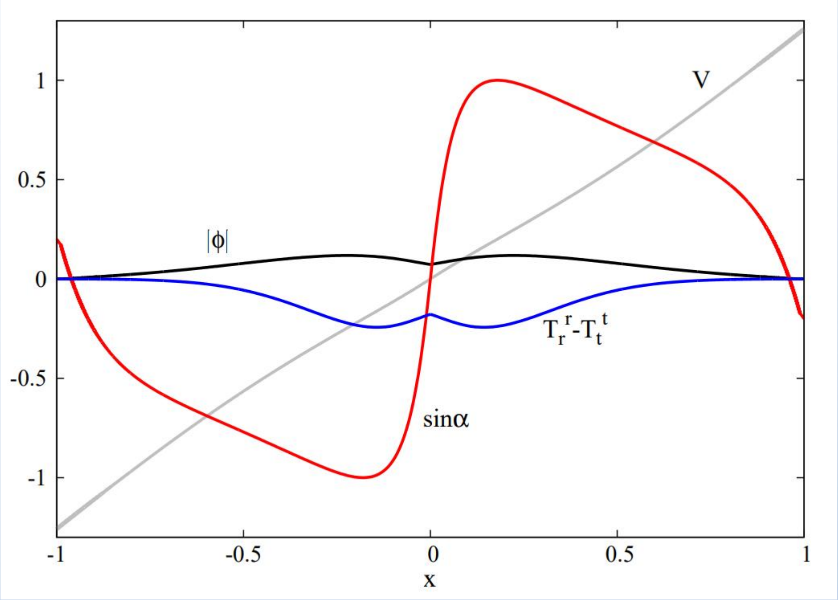
$$\Psi_\epsilon = \mathrm{e}^{-\mathrm{i}wt}\mathcal{R}_\epsilon(r)\otimes\Theta_\epsilon(\theta,\varphi)$$

$$\phi = e^{i\pi/4}F - e^{-i\pi/4}G$$

$$ds^2 = -e^{2\nu(r)}dt^2 + f(r)dr^2 + (r^2 + r_0^2)d\Omega^2.$$

对称的虫洞解





负能量 violates the null energy condition $T_{\mu\nu}k^{\mu}k^{\nu} < 0$

$$T_r^r - T_t^t < 0$$

PHYSICAL REVIEW D **104,** 124055 (2021)

Blázquez-Salcedo–Knoll–Radu wormholes are not solutions to the Einstein-Dirac-Maxwell equations

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(Received 22 September 2021; accepted 15 November 2021; published 17 December 2021)

Recently, Blázquez-Salcedo, Knoll, and Radu (BSKR) have given a class of static, spherically symmetric traversable wormhole spacetimes with Dirac and Maxwell fields. The BSKR wormholes are obtained by joining a classical solution to the Einstein-Dirac-Maxwell (EDM) equations on the "up" side of the wormhole ($r \geq 0$) to a corresponding solution on the "down" side of the wormhole ($r \leq 0$). However, it can be seen that the BSKR metric fails to be C^3 on the wormhole throat at $r = 0$. We prove that if the matching were done in such a way that the resulting spacetime metric, Dirac field, and Maxwell field

1. For all of the BSKR wormholes, all of these fields are not smooth (C^∞).

2. BSKR wormholes all must contain spurious sources spurious distributional source for the Dirac field
at Σ $(r = 0)$.

PHYSICAL REVIEW LETTERS **128,** 091104 (2022)

Traversable Wormholes in General Relativity

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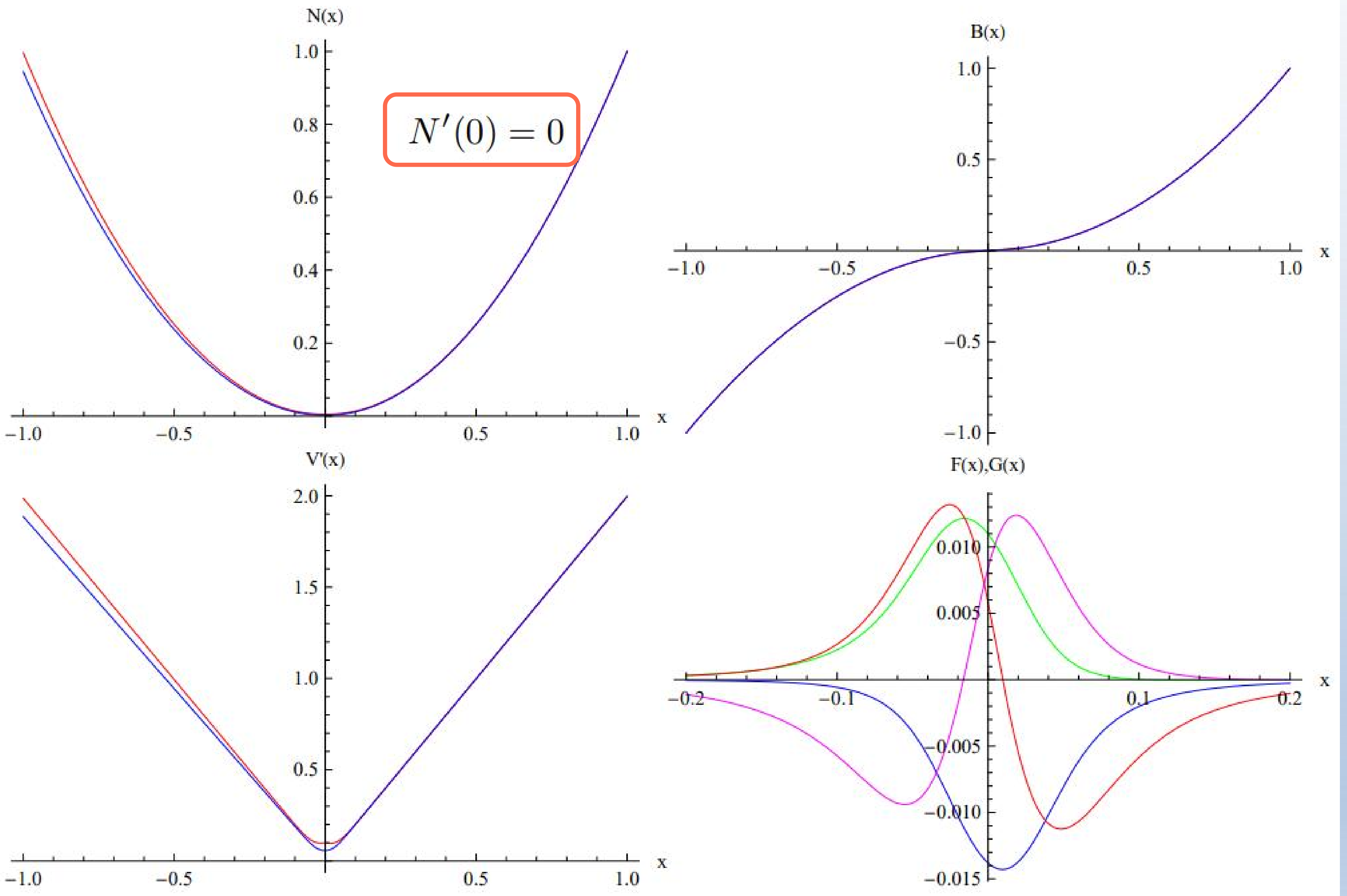
Blázquez-Salcedo *et al.* [Phys. Rev. Lett. **126**, 101102 (2021)] obtained asymptotically flat traversable wormhole solutions in the Einstein-Dirac-Maxwell theory without using phantom matter. The normalizable numerical solutions found therein require a peculiar behavior at the throat: the mirror symmetry relatively the throat leads to the nonsmoothness of gravitational and matter fields. In particular, one must postulate changing of the sign of the fermionic charge density at the throat, requiring coexistence of particle and antiparticles without annihilation and posing a membrane of matter at the throat with specific properties. Apparently this kind of configuration could not exist in nature. We show that there are wormhole solutions, which are asymmetric relative the throat and endowed by smooth gravitational and matter fields, thereby being free from all the above problems. This indicates that such wormhole configurations could also be supported in a realistic scenario.

非对称的虫洞解（**KZ wormhole**）



N(x)
1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5
1.0
x
$N'(0) = 0$
B(x)
1.0
0.5
−0.5
−1.0
−1.0
−0.5
0.5
1.0
x
V'(x)
2.0
1.5
1.0
0.5
−1.0
−0.5
0.5
1.0
x
F(x),G(x)
0.010
0.005
−0.005
−0.010
−0.015
−0.2
−0.1
0.1
0.2
x

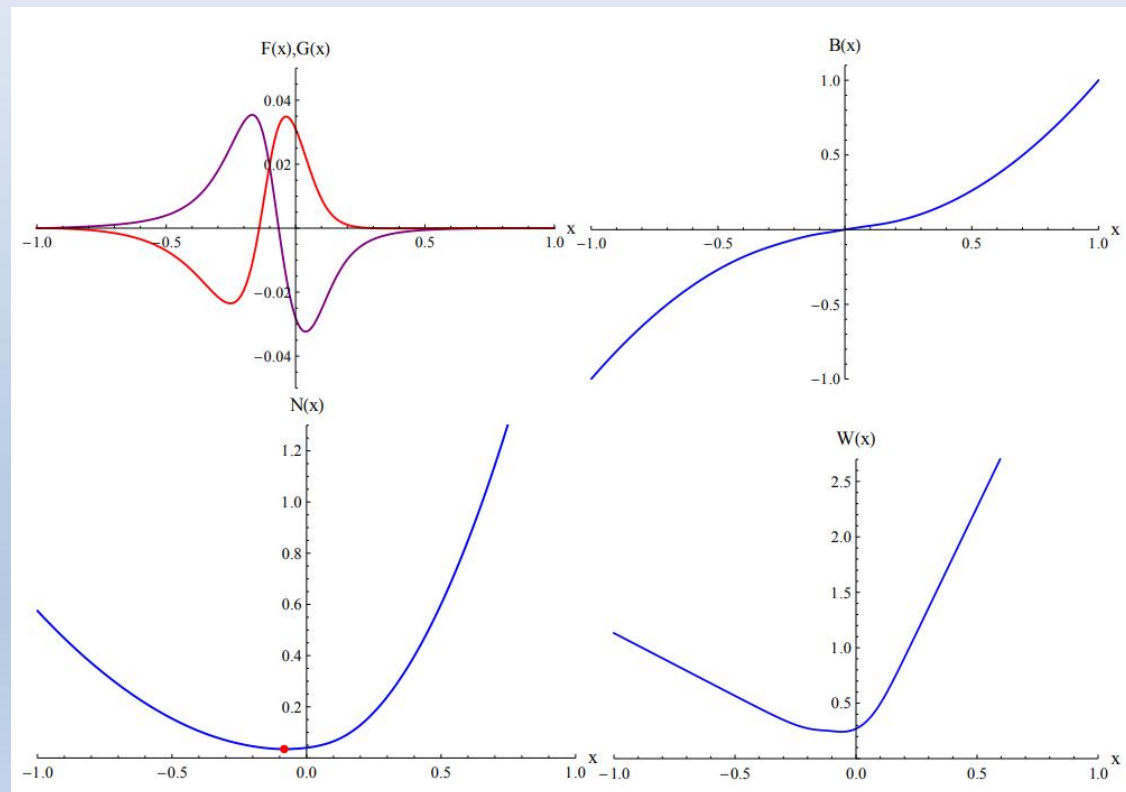
$N'(0) = 0$ → $N'(0) = \text{Constant}$

b_i	g_i	f_i	n_i/σ_+	σ_-/σ_+	Q_+/r_0	Q_-/r_0	M_+/r_0	M_-/r_0	$\omega r_0/\sigma_+$	$N'(0)$
0.289865	0.0049816	0.033000	0.0521623	1.62879	0.978917	0.977803	0.977428	0.977230	−0.120251	−0.0622616
0.289043	0.0177181	0.025000	0.0491175	1.21799	0.976157	0.974928	0.974764	0.973952	−0.115032	−0.0263646
0.288946	0.0256779	0.016095	0.0448804	1.00440	0.975806	0.974565	0.974637	0.973344	−0.105347	0
0.289748	0.0329166	0	0.0345718	0.72252	0.978405	0.977286	0.977742	0.975932	−0.080254	0.0421234
0.291968	0.0310424	−0.028000	0.0173830	0.24981	0.985702	0.984627	0.985574	0.982341	−0.034964	0.1299027

TABLE I: The families of wormhole solutions with the parameters $qr_0 = 0.03$ and $\mu r_0 = 0.2$.

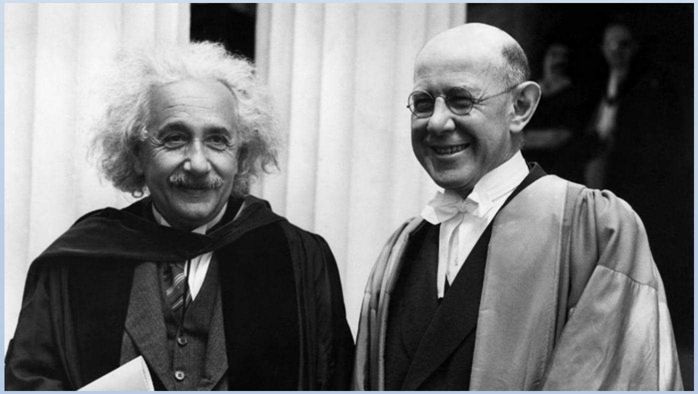
3. Einstein-Dirac-Maxwell
中的一般可穿越虫洞解

WYQ， Shao-Wen Wei, Yu-Xiao Liu, arXiv:2206.12250



4 总结

1. 当扩展狄拉克星（Einstein-Dirac-Maxwell）到虫洞结构可以构造一般可穿越虫洞解
2. 该虫洞解为非对称虫洞
3. 依然破坏能量条件, 但不依赖奇异物质。



ER 、EPR

玻色星 、 虫洞

谢谢！