

United Nations Educational, Scientific and **Cultural Organization**





Primordial Black Holes from Vacuum Bubbles

张君 国际理论物理中心-亚太地区

based on 2303.16810, 1512.01819, 1710.02865, 2006.11907, 2101.11098

in collaboration with 贺吉斌, 邓鹤凌, 朴云松, Jaume Garriga, Alexander Vilenkin

2023.04.09 @ USTC





Masses in the Stellar Graveyard



[Credit:LIGO-Virgo/Aaron Geller/Northwestern University]

• _IGO-Virgo-KAGRA | Aaron Geller | Northwestern







[LVK, PRX (2023)]





Astrophysical origin



[Stevenson+, (2017)]

[LVK, PRX (2023)]



[Tagawa+, (2020)]





Primordial origin



[LVK, PRX (2023)]

 $m_1 \; [\mathrm{M}_\odot]$





Primordial origin

De Luca, Franciolini, Pani, Riotto, Hutsi, Veermae...

GW150914

[Sasaki+, PRL (2016)] [Bird+, PRL (2016)] [Clesse+, Phys. Dark Univ. (2017)]

 10^{-2} ${
m d}eta_i^{
m det}/{
m d}{\cal M}$ 10-10 20

[LVK, PRX (2023)]





Outline



I. Formation mechanism

II. Implications from GWTC-3

III. Outlook



our vacuum



Slow-roll Inflation





our vacuum









 $R(t) \approx H_i^{-1} \left[e^{H_i(t-t_n)} - 1 \right]$



Nucleation rate λ

Size distribution

$$dN = \lambda H_i^4 e^{3H_i t_n} d^3 \mathbf{x} dt_n \qquad dV \equiv e^{3H_i t} d^3 \mathbf{x}$$

$$dn(t) \equiv \frac{dN}{dV} = \lambda \frac{dR}{\left(R + H_i^{-1}\right)^4}$$

$$dn(t_i) \simeq \lambda \frac{dR_i}{R_i^4}$$



 $R(t) \approx H_i^{-1} \left[e^{H_i(t-t_n)} - 1 \right]$









Israel's matching condition

$$M = \frac{4}{3}\pi(\rho_b - \rho_i)R^3 + 4\pi\sigma R^2 [\dot{R}^2 + 1 - H_b^2 R^2]^{1/2} - 8\pi^2 e^{-2\pi i R_b^2} R^2 R_{cont}^2 + 1 - H_b^2 R^2 R_{cont}^2 R_{cont}$$

At the end of inflation

$$\begin{aligned} \frac{4}{3}\pi\rho_m(t_i)R_i^3 &\approx \frac{4}{3}\pi\rho_b R_i^3 + 4\pi\sigma R_i^2 [\dot{R}_i^2 + 1 - H_b^2 R_i^2]^{1/2} \\ \dot{R}_i &\simeq \frac{1}{4}\frac{H_i^2}{H_\sigma}R_i \\ \dot{R}_i &= \frac{H_i R_i + a_i r_i'}{\sqrt{1 - a_i^2 r_i'^2}} \quad ds^2 = -dt^2 + a^2(t)(dt) \end{aligned}$$

Lorentz factor

$$\gamma_i \equiv \frac{1}{\sqrt{1 - a_i^2 r_i'^2}} \sim \frac{H_i}{H_\sigma} \gg 1$$



 \mathcal{X}

Schwarszchild-de Sitter



[Garriga, Vilenkin, JZ, JCAP (2016)]

 $^2 - 8\pi^2 G \sigma^2 R_i^3$

 $(r^2 + r^2 d\Omega^2)$



Initial Condition

$$dn(t_i) \simeq \lambda \frac{dR_i}{R_i^4} \qquad \gamma_i \equiv \frac{1}{\sqrt{1 - a_i^2 r_i'^2}} \sim \frac{H_i}{H_\sigma} \gg$$

1





A pedagogical model

$$\begin{split} M_{bh} &= \frac{4}{3} \pi \rho_b R^3 + 4 \pi \sigma R^2 [\dot{R}^2 + 1 - H_b^2 R^2]^{1/2} - 8 \pi^2 G \sigma^2 \\ \left(\frac{dz}{d\tilde{\tau}}\right)^2 + V(z) &= E \\ z^3 &= \frac{H_+^2}{2GM_{bh}} R^3 E = \frac{-16H_\sigma^2}{(2GM_{bh})^{2/3} H_+^{8/3}} V(z) \\ M_* \sim M_{\rm Pl}^3 / \eta_b^2 \end{split}$$
 [Blau+, Plane)







A pedagogical model



• Subcritical







A pedagogical model



Supercritical









Strong interaction with radiation





Radiation FRW



[Deng+, JCAP (2017)]





Strong interaction with radiation





Radiation FRW



[Deng+, JCAP (2017)]





No interaction with radiation









[Deng, JCAP (2020)]



Implications from GWTC-3

Implications from GWTC-3

Mass distribution

$$\psi_{\text{PBH}}(m|m_*,\alpha_1,\alpha_2) = \frac{1}{m_*\left(\alpha_1^{-1} - \alpha_2^{-1}\right)} \begin{cases} (m/m_*)^{\alpha_1 - 1}, & m < m_* \\ (m/m_*)^{\alpha_2 - 1}, & m > m_* \end{cases}$$

Posterior $\mathcal{L}(\Lambda | \mathbf{d})$

Hierarchical Bayesian Inference

Detect fraction

$$\xi(\Lambda) = \int p_{\rm det}(\theta) \, \pi(\theta | \Lambda) d\theta$$



[LVC, Astrophys. J. Lett. (2021)]



Intrinsic parameters Hyperparameters $\mathcal{L}(\boldsymbol{d}|\Lambda) \propto e^{-N(\Lambda)\xi(\Lambda)} [N(\Lambda)]^{N_{\text{det}}} \prod_{i=1}^{N_{\text{det}}} \int \mathcal{L}(d_i|\theta) \, \pi(\theta|\Lambda) \, d\theta$

Merger rate

$$\frac{\mathrm{d}R_{\rm PBH}}{\mathrm{d}m_{1}\mathrm{d}m_{2}} = \frac{1.6 \times 10^{6}}{\mathrm{Gpc}^{3} \,\mathrm{yr}} f_{\rm PBH}^{\frac{53}{37}} \eta^{-\frac{34}{37}} \left(\frac{M}{M_{\odot}}\right)^{-\frac{32}{37}} \left(\frac{t}{t_{0}}\right)^{-\frac{34}{37}} \mathcal{S}\left(M, f_{\rm PBH}, \psi_{\rm PBH}\right) \psi_{\rm PBH}(m_{1}) \psi_{\rm PBH}(m_{2})$$

[He, Deng, Piao, JZ (2023)]

Implications from GW

Posterior Mass distribution









/	C.	-3

Parameter	Prior	ABH-PBH	ABH-PBH	ABH	ABH	Description
			f	1		Broken Power Law PBH
M_*/M_{\odot}	[5, 50]	$31.43^{+1.44}_{-1.56}$	$30.54^{+1.37}_{-1.56}$			The critical mass
$\log_{10} f_{\rm PRV}$	[-4, 0]	-2.99+0.07	$-2.95^{+0.05}$			Logarithmic fraction of PBHs in dark matter
10 E 10 J PBH	[,0]	2.99-0.07	2.00-0.05			at the time of formation
$lpha_1$	[0, 10]	$8.80^{+2.15}_{-2.79}$	$8.52^{+2.36}_{-2.82}$			Spectral index of subcritical PBHs
α_2	[0, -10]	$-5.65^{+1.94}_{-3.81}$	$-3.91^{+0.73}_{-0.78}$			Spectral index of supercritical PBHs
				-		TRUNCATED ABH
$\bar{R}_{ m ABH}/ m Gpc^{-3}yr^{-1}$	[0, 50]	$23.80^{+6.20}_{-5.19}$	$24.30^{+6.71}_{-5.62}$	$26.39^{+6.10}_{-4.88}$		Integrated merger rate of ABHs at $z = 0$
β	[-4, 30]	$5.78^{+3.26}_{-2.83}$	$5.21^{+3.65}_{-3.23}$	$10.21^{+4.74}_{-3.27}$		Exponent of the symmetric mass ratio factor
ζ	[0,3]	$1.60^{+0.37}_{-0.22}$	$1.78^{+0.37}_{-0.34}$	$1.10^{+0.10}_{-0.11}$		ABH mass distribution power law scaling
$m_{ m min}/M_{\odot}$	[2, 10]	$6.02^{+0.29}_{-0.27}$	$6.03^{+0.31}_{-0.26}$	$5.94^{+0.30}_{-0.32}$		Minimum mass of the primary mass distribution
100 /M	[30, 100]	$85.28^{+10.6}_{-36.1}$		$75.45^{+9.81}_{-5.18}$		Maximum mass of the primary mass distribution
$m_{\rm max}/M_{\odot}$	[30, 60]	0011	$41.69^{+11.1}_{-8.07}$	0110		Maximum mass of the primary mass distribution
			0101			Broken Power Law ABH
$\bar{R}_{ m ABH}/ m Gpc^{-3}yr^{-1}$	[0, 50]				$23.00^{+6.36}_{-4.72}$	Integrated merger rate of ABHs at $z = 0$
ß.	[_4 12]				2 28+0.42	Power-law slope of the primary mass distribution
$\boldsymbol{\rho}_1$	[, 12]				2.20-0.39	for masses below m_{break}
B	[-4, 12]				$6.73^{+2.08}$	Power-law slope of the primary mass distribution
F -2					-1.43	for masses above m_{break}
$oldsymbol{eta}_q$	[-4, 12]				$0.83^{+0.94}_{-0.72}$	Spectral index for the power-law of the mass ratio distribution
$m_{ m min}/M_{\odot}$	[2, 10]				$5.24^{+0.67}_{-1.39}$	Minimum mass of the primary mass distribution.
$m_{ m max}/M_{\odot}$	[30, 100]				$86.73^{+8.67}_{-9.79}$	Maximum mass of the primary mass distribution.
b	[0, 1]				$0.43^{+0.09}$	The fraction of the way between m_{\min} and m_{\max} at which
	[-, -]				-0.07	the primary mass distribution breaks
δ_m/M_{\odot}	[0, 10]				$4.87^{+3.02}_{-3.11}$	Range of mass tapering on the lower end of the mass distribut

TABLE I. Prior and 68% credible intervals of the hyperparameters. We show the posteriors of the ABH-PBH model with different choices of prior on m_{max} (the third and fourth column), the truncated ABH model (the fifth column), and the broken power law ABH model (the sixth column).

[He, Deng, Piao, JZ (2023)]

stribution

Implications from GWTC-3

- $M_{\rm Pl}^3/\eta_{\rm b}^2 \sim 30 M_\odot \rightarrow \eta_{\rm b} \sim 0.1 \, {\rm GeV}_{\rm c}$
- No subcritical bubble

$$m_{\rm F} \sim \eta_{\rm b}^{-2} \left(\frac{\eta_i^4 M_{\rm Pl}}{\eta_{\sigma}^3} \right)^{3/2} \gtrsim m_* \quad \rightarrow \eta_i^4 > \eta_{\sigma}^3 M_{\rm Pl}$$
$$\eta_i > 10^4 \,\,{\rm GeV}$$

$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	Parameter	Prior	ABH-PBH	ABH-PBH	ABH	ABH	Description
$\begin{array}{c c c c c c c c c c c c c c c c c c c $							Broken Power Law PBH
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	M_*/M_{\odot}	[5, 50]	$31.43^{+1.44}_{-1.56}$	$30.54^{+1.37}_{-1.56}$			The critical mass
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$\log_{10} f_{\mathrm{PBH}}$	[-4,0]	$-2.99^{+0.07}_{-0.07}$	$-2.95^{+0.05}_{-0.05}$			Logarithmic fraction of PBHs in dark matter at the time of formation
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$lpha_1$	[0, 10]	$8.80^{+2.15}_{-2.79}$	$8.52^{+2.36}_{-2.82}$			Spectral index of subcritical PBHs
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$lpha_2$	[0, -10]	$-5.65^{+1.94}_{-3.81}$	$-3.91^{+0.73}_{-0.78}$			Spectral index of supercritical PBHs
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				- Diff. and an			TRUNCATED ABH
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$\bar{R}_{\rm ABH}/{ m Gpc^{-3}yr^{-1}}$	[0, 50]	$23.80^{+6.20}_{-5.19}$	$24.30^{+6.71}_{-5.62}$	$26.39^{+6.10}_{-4.88}$		Integrated merger rate of ABHs at $z = 0$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	β	[-4, 30]	$5.78^{+3.26}_{-2.83}$	$5.21^{+3.65}_{-3.23}$	$10.21^{+4.74}_{-3.27}$		Exponent of the symmetric mass ratio factor
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	ζ	[0,3]	$1.60^{+0.37}_{-0.22}$	$1.78^{+0.37}_{-0.34}$	$1.10^{+0.10}_{-0.11}$		ABH mass distribution power law scaling
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$m_{ m min}/M_{\odot}$	[2, 10]	$6.02^{+0.29}_{-0.27}$	$6.03^{+0.31}_{-0.26}$	$5.94^{+0.30}_{-0.32}$		Minimum mass of the primary mass distribution
m_{max}/M_{\odot} [30, 60] $41.69_{-8.07}^{+11.1}$ Maximum mass of the primary mass distribution $\bar{R}_{ABH}/Gpc^{-3}yr^{-1}$ [0, 50] $23.00_{-4.72}^{+6.36}$ Integrated merger rate of ABHs at $z = 0$ β_1 $[-4, 12]$ $2.28_{-0.39}^{+0.42}$ Power-law slope of the primary mass distribution β_2 $[-4, 12]$ $6.73_{-1.43}^{+2.08}$ Power-law slope of the primary mass distribution β_q $[-4, 12]$ $0.83_{-0.72}^{+0.72}$ Spectral index for the power-law of the mass ratio distribution m_{min}/M_{\odot} $[2, 10]$ $5.24_{-1.39}^{+0.67}$ Maximum mass of the primary mass distribution. m_{max}/M_{\odot} $[30, 100]$ $86.73_{-9.79}^{+8.67}$ Maximum mass of the primary mass distribution. b $[0, 1]$ $0.43_{-0.09}^{+0.09}$ The fraction of the way between m_{min} and m_{max} at which the primary mass distribution breaks δ_m/M_{\odot} $[0, 10]$ $4.87_{-3.11}^{+3.02}$ Range of mass tapering on the lower end of the mass distribution	m/M_{-}	[30, 100]	$85.28^{+10.6}_{-36.1}$		$75.45^{+9.81}_{-5.18}$		Maximum mass of the primary mass distribution
$\bar{R}_{ABH}/Gpc^{-3}yr^{-1}$ [0,50] $23.00^{+6.36}_{-4.72}$ Integrated merger rate of ABHs at $z = 0$ β_1 $[-4, 12]$ $2.28^{+0.42}_{-0.39}$ Power-law slope of the primary mass distribution for masses below m_{break} β_2 $[-4, 12]$ $6.73^{+2.08}_{-1.43}$ Power-law slope of the primary mass distribution for masses above m_{break} β_q $[-4, 12]$ $0.83^{+0.94}_{-0.72}$ Spectral index for the power-law of the mass ratio distribution for masses above m_{break} β_q $[-4, 12]$ $0.83^{+0.94}_{-0.72}$ Spectral index for the power-law of the mass ratio distribution 	$m_{\rm max}/m_{\odot}$	[30, 60]		$41.69^{+11.1}_{-8.07}$			Maximum mass of the primary mass distribution
$\bar{R}_{ABH}/Gpc^{-3}yr^{-1}$ [0, 50] $23.00^{+6.36}_{-4.72}$ Integrated merger rate of ABHs at $z = 0$ β_1 $[-4, 12]$ $2.28^{+0.42}_{-0.39}$ Power-law slope of the primary mass distribution for masses below m_{break} β_2 $[-4, 12]$ $6.73^{+2.08}_{-1.43}$ Power-law slope of the primary mass distribution for masses above m_{break} β_q $[-4, 12]$ $0.83^{+0.94}_{-0.72}$ Spectral index for the power-law of the mass ratio distribution for masses of the primary mass distribution. m_{min}/M_{\odot} $[2, 10]$ $5.24^{+0.67}_{-1.39}$ Minimum mass of the primary mass distribution. m_{max}/M_{\odot} $[30, 100]$ $86.73^{+8.67}_{-9.79}$ Maximum mass of the primary mass distribution.b $[0, 1]$ $0.43^{+0.09}_{-0.07}$ Maximum mass distribution breaks δ_m/M_{\odot} $[0, 10]$ $4.87^{+3.02}_{-3.11}$ Range of mass tapering on the lower end of the mass distribution							Broken Power Law ABH
$ \begin{array}{c c} \beta_1 & [-4,12] \\ & 2.28^{+0.42}_{-0.39} \end{array} \begin{array}{c} \text{Power-law slope of the primary mass distribution} \\ & \text{for masses below } m_{\text{break}} \\ \text{Power-law slope of the primary mass distribution} \\ & \text{for masses above } m_{\text{break}} \\ & \text{Power-law slope of the primary mass distribution} \\ & \text{for masses above } m_{\text{break}} \\ & \beta_q & [-4,12] \\ & \beta_q & [-4,12] \\ & M_{\min}/M_{\odot} & [2,10] \\ & m_{\max}/M_{\odot} & [30,100] \\ & b & [0,1] \\ & b & [0,1] \\ & 0.43^{+0.09}_{-0.07} \\ & \delta_m/M_{\odot} & [0,10] \\ \end{array} \begin{array}{c} 2.28^{+0.42}_{-0.39} \\ & \text{Power-law slope of the primary mass distribution} \\ & \text{for masses above } m_{\text{break}} \\ & \text{Power-law slope of the primary mass distribution} \\ & \text{for masses above } m_{\text{break}} \\ & \text{Minimum mass of the primary mass distribution.} \\ & \text{Maximum mass of the primary mass distribution.} \\ & \text{The fraction of the way between } m_{\text{min}} \text{ and } m_{\text{max}} \text{ at which} \\ & \text{the primary mass distribution breaks} \\ & \delta_m/M_{\odot} & [0,10] \\ & 4.87^{+3.02}_{-3.11} \end{array} \end{array}$	$\bar{R}_{\rm ABH}/{ m Gpc^{-3}yr^{-1}}$	[0, 50]				$23.00^{+6.36}_{-4.72}$	Integrated merger rate of ABHs at $z = 0$
$\beta_{1} \qquad (1 + 3) = 1 \qquad (1 + 3$	β_1	[-4, 12]				$2.28^{+0.42}$	Power-law slope of the primary mass distribution
β_2 $[-4, 12]$ $6.73^{+2.08}_{-1.43}$ Power-law slope of the primary mass distribution for masses above m_{break} β_q $[-4, 12]$ $0.83^{+0.94}_{-0.72}$ Spectral index for the power-law of the mass ratio distribution m_{min}/M_{\odot} $[2, 10]$ $5.24^{+0.67}_{-1.39}$ Minimum mass of the primary mass distribution. m_{max}/M_{\odot} $[30, 100]$ $86.73^{+8.67}_{-9.79}$ Maximum mass of the primary mass distribution. b $[0, 1]$ $0.43^{+0.09}_{-0.07}$ The fraction of the way between m_{min} and m_{max} at which the primary mass distribution breaks δ_m/M_{\odot} $[0, 10]$ $4.87^{+3.02}_{-3.11}$ Range of mass tapering on the lower end of the mass distribution	<i>P</i> 1	[.,]					for masses below m_{break}
β_q $[-4, 12]$ $0.83^{+0.94}_{-0.72}$ Spectral index for the power-law of the mass ratio distribution m_{\min}/M_{\odot} $[2, 10]$ $5.24^{+0.67}_{-1.39}$ Minimum mass of the primary mass distribution. m_{\max}/M_{\odot} $[30, 100]$ $86.73^{+8.67}_{-9.79}$ Maximum mass of the primary mass distribution. b $[0, 1]$ $0.43^{+0.09}_{-0.07}$ The fraction of the way between m_{\min} and m_{\max} at which the primary mass distribution breaks δ_m/M_{\odot} $[0, 10]$ $4.87^{+3.02}_{-3.11}$ Range of mass tapering on the lower end of the mass distribut	$oldsymbol{eta}_2$	[-4, 12]				$6.73^{+2.08}_{-1.43}$	for masses above m
p_q $[-4, 12]$ $0.83_{-0.72}$ Spectral index for the power-law of the mass ratio distribution m_{\min}/M_{\odot} $[2, 10]$ $5.24_{-1.39}^{+0.67}$ Minimum mass of the primary mass distribution. m_{\max}/M_{\odot} $[30, 100]$ $86.73_{-9.79}^{+8.67}$ Maximum mass of the primary mass distribution. b $[0, 1]$ $0.43_{-0.07}^{+0.09}$ The fraction of the way between m_{\min} and m_{\max} at which the primary mass distribution breaks δ_m/M_{\odot} $[0, 10]$ $4.87_{-3.11}^{+3.02}$ Range of mass tapering on the lower end of the mass distribution	ß	[_/ 12]				0.83+0.94	Spectral index for the power law of the mass ratio distribution
m_{\min}/M_{\odot} [2, 10] $5.24^{-1.39}_{-1.39}$ Minimum mass of the primary mass distribution. m_{\max}/M_{\odot} [30, 100] $86.73^{+8.67}_{-9.79}$ Maximum mass of the primary mass distribution. b [0, 1] $0.43^{+0.09}_{-0.07}$ The fraction of the way between m_{\min} and m_{\max} at which the primary mass distribution breaks δ_m/M_{\odot} [0, 10] $4.87^{+3.02}_{-3.11}$ Range of mass tapering on the lower end of the mass distribution	P_q	[-4, 12]				$5.03_{-0.72}$	Minimum mass of the primory mass distribution
m_{max}/M_{\odot} [30, 100] $86.73^{+8.07}_{-9.79}$ Maximum mass of the primary mass distribution. b $[0, 1]$ $0.43^{+0.09}_{-0.07}$ The fraction of the way between m_{min} and m_{max} at which the primary mass distribution breaks δ_m/M_{\odot} $[0, 10]$ $4.87^{+3.02}_{-3.11}$ Range of mass tapering on the lower end of the mass distribution	$m_{\rm min}/M_{\odot}$	[2, 10]				$5.24_{-1.39}$	Minimum mass of the primary mass distribution.
b $[0,1]$ $0.43^{+0.09}_{-0.07}$ The fraction of the way between m_{\min} and m_{\max} at which the primary mass distribution breaks δ_m/M_{\odot} $[0,10]$ $4.87^{+3.02}_{-3.11}$ Range of mass tapering on the lower end of the mass distribut	$m_{ m max}/M_{\odot}$	[30, 100]				$86.73_{-9.79}^{+8.67}$	Maximum mass of the primary mass distribution.
$\frac{\delta_m/M_{\odot}}{M_{\odot}} \qquad [0,10] \qquad \qquad 4.87^{+3.02}_{-3.11} \text{ Range of mass tapering on the lower end of the mass distribution}$	b	[0,1]				$0.43^{+0.09}_{-0.07}$	The fraction of the way between m_{\min} and m_{\max} at which the primary mass distribution breaks
	δ_m/M_\odot	[0, 10]				$4.87^{+3.02}_{-3.11}$	Range of mass tapering on the lower end of the mass distribut

TABLE I. Prior and 68% credible intervals of the hyperparameters. We show the posteriors of the ABH-PBH model with different choices of prior on m_{max} (the third and fourth column), the truncated ABH model (the fifth column), and the broken power law ABH model (the sixth column).

[He, Deng, Piao, JZ (2023)]

stribution

Implications from GWTC-3

Posterior Mass distribution



$$egin{aligned} &\gamma_{ ext{PBH}} \equiv N_{ ext{PBH}}^{ ext{det}} / (N_{ ext{ABH}}^{ ext{det}} + N_{ ext{PBH}}^{ ext{det}}) \,, \ &\gamma_{ ext{ABH}} \equiv 1 - \gamma_{ ext{PBH}}. \end{aligned}$$

Parameter	Prior	ABH-PBH	ABH-PBH	ABH	ABH	Description
						Broken Power Law PBH
M_*/M_{\odot}	[5, 50]	$31.43^{+1.44}_{-1.56}$	$30.54^{+1.37}_{-1.56}$			The critical mass
$\log_{10} f_{\mathrm{PBH}}$	[-4,0]	$-2.99\substack{+0.07\\-0.07}$	$-2.95^{+0.05}_{-0.05}$			Logarithmic fraction of PBHs in dark matter at the time of formation
$lpha_1$	[0, 10]	$8.80^{+2.15}_{-2.79}$	$8.52^{+2.36}_{-2.82}$			Spectral index of subcritical PBHs
$lpha_2$	[0, -10]	$-5.65^{+1.94}_{-3.81}$	$-3.91^{+0.73}_{-0.78}$			Spectral index of supercritical PBHs
						TRUNCATED ABH
$\bar{R}_{\rm ABH}/{ m Gpc^{-3}yr^{-1}}$	[0, 50]	$23.80^{+6.20}_{-5.19}$	$24.30^{+6.71}_{-5.62}$	$26.39^{+6.10}_{-4.88}$		Integrated merger rate of ABHs at $z = 0$
β	[-4, 30]	$5.78^{+3.26}_{-2.83}$	$5.21^{+3.65}_{-3.23}$	$10.21^{+4.74}_{-3.27}$		Exponent of the symmetric mass ratio factor
ζ	[0,3]	$1.60^{+0.37}_{-0.22}$	$1.78^{+0.37}_{-0.34}$	$1.10^{+0.10}_{-0.11}$		ABH mass distribution power law scaling
$m_{ m min}/M_{\odot}$	[2, 10]	$6.02^{+0.29}_{-0.27}$	$6.03^{+0.31}_{-0.26}$	$5.94_{-0.32}^{+0.30}$		Minimum mass of the primary mass distribution
$m_{ m max}/M_{\odot}$	[30, 100]	$85.28^{+10.6}_{-36.1}$	41 60+11.1	$75.45^{+9.81}_{-5.18}$		Maximum mass of the primary mass distribution
	[30, 00]		41.09_8.07			De event Devent L ADU
$\bar{\mathbf{p}}$ / \mathbf{C} = -3 = -1	[0, 50]				22 00+6.36	BROKEN POWER LAW ABH
K_{ABH}/Gpc yr	[0, 50]				23.00-4.72	Integrated merger rate of ABHs at $z = 0$ Power-law slope of the primary mass distribution
$oldsymbol{eta}_1$	[-4, 12]				$2.28^{+0.42}_{-0.39}$	for masses below <i>m</i> _{break}
0	r (101				(-72+2)	Power-law slope of the primary mass distribution
β_2	[-4, 12]				$6.73_{-1.43}$	for masses above m_{break}
eta_q	[-4, 12]				$0.83^{+0.94}_{-0.72}$	Spectral index for the power-law of the mass ratio distribution
$m_{ m min}/M_{\odot}$	[2, 10]				$5.24^{+0.67}_{-1.39}$	Minimum mass of the primary mass distribution.
$m_{ m max}/M_{\odot}$	[30, 100]				$86.73_{-9.79}^{+8.67}$	Maximum mass of the primary mass distribution.
b	[0,1]				$0.43^{+0.09}_{-0.07}$	The fraction of the way between m_{\min} and m_{\max} at which the primary mass distribution breaks
δ_m/M_\odot	[0, 10]				$4.87^{+3.02}_{-3.11}$	Range of mass tapering on the lower end of the mass distribut

TABLE I. Prior and 68% credible intervals of the hyperparameters. We show the posteriors of the ABH-PBH model with different choices of prior on m_{max} (the third and fourth column), the truncated ABH model (the fifth column), and the broken power law ABH model (the sixth column).

[He, Deng, Piao, JZ (2023)]

stribution

- Analyzing with more sophisticated ABH model
 - [in the next version]

- Analyzing with more sophisticated ABH model [in the next version]
- Bubble dynamics in FRW universe

[Huang, JZ, Piao, in progress]





- Analyzing with more sophisticated ABH model [in the next version]
- Bubble dynamics in FRW universe [Huang, JZ, Piao, in progress]
- Digging more information from GWTC-3

- Analyzing with more sophisticated ABH model [in the next version]
- Bubble dynamics in FRW universe [Huang, JZ, Piao, in progress]
- Digging more information from GWTC-3
- Possibility of seeding supermassive black holes

Thanks!