Primordial Black Holes from Vacuum Bubbles

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based on 2303.16810, 1512.01819, 1710.02865, 2006.11907, 2101.11098

in collaboration with 贺吉斌, 邓鹤凌, 朴云松, Jaume Garriga, Alexander Vilenkin

United Nations
Educational, Scientific and
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ICTP-AP
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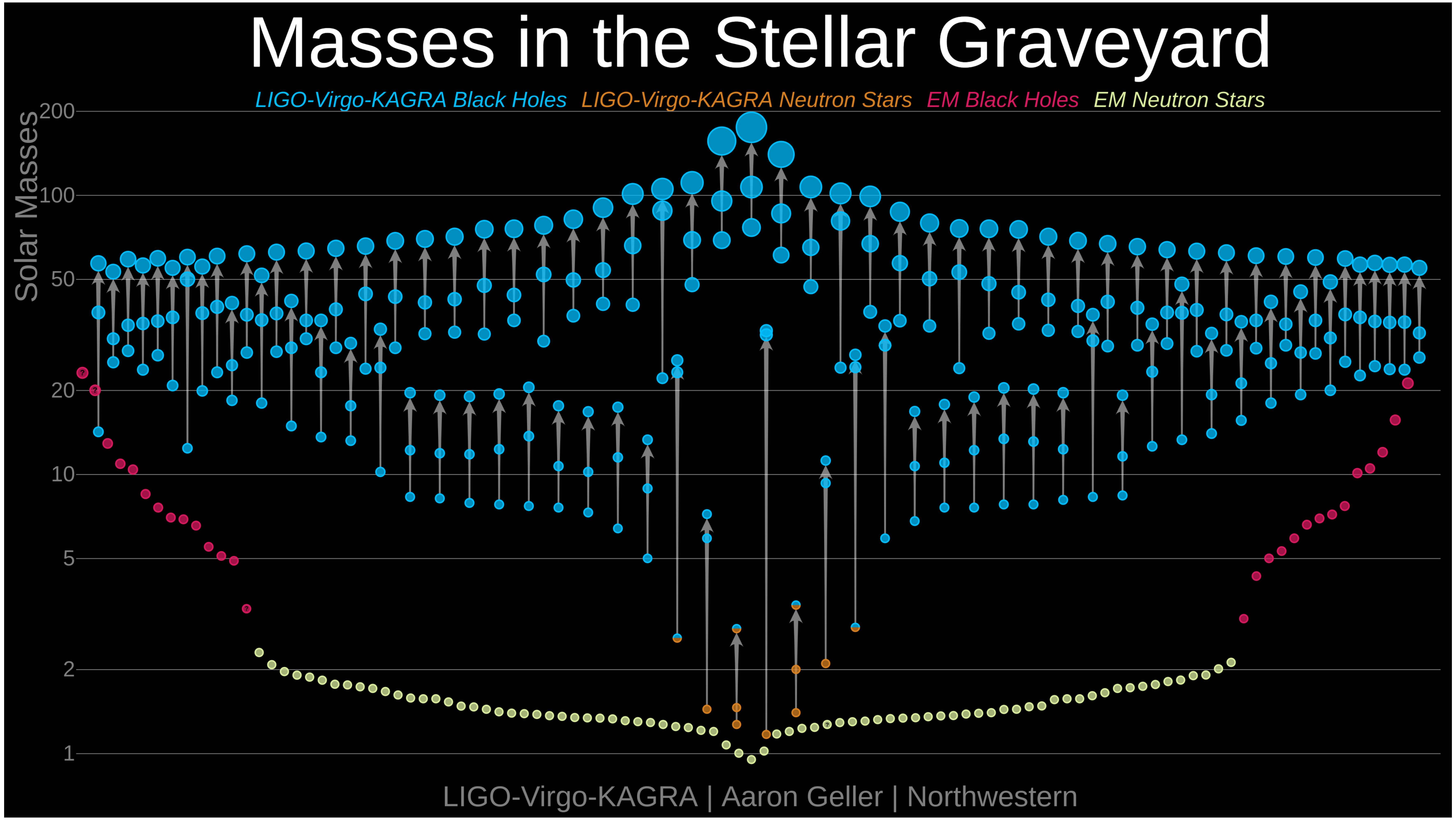
2023.04.09 @ USTC

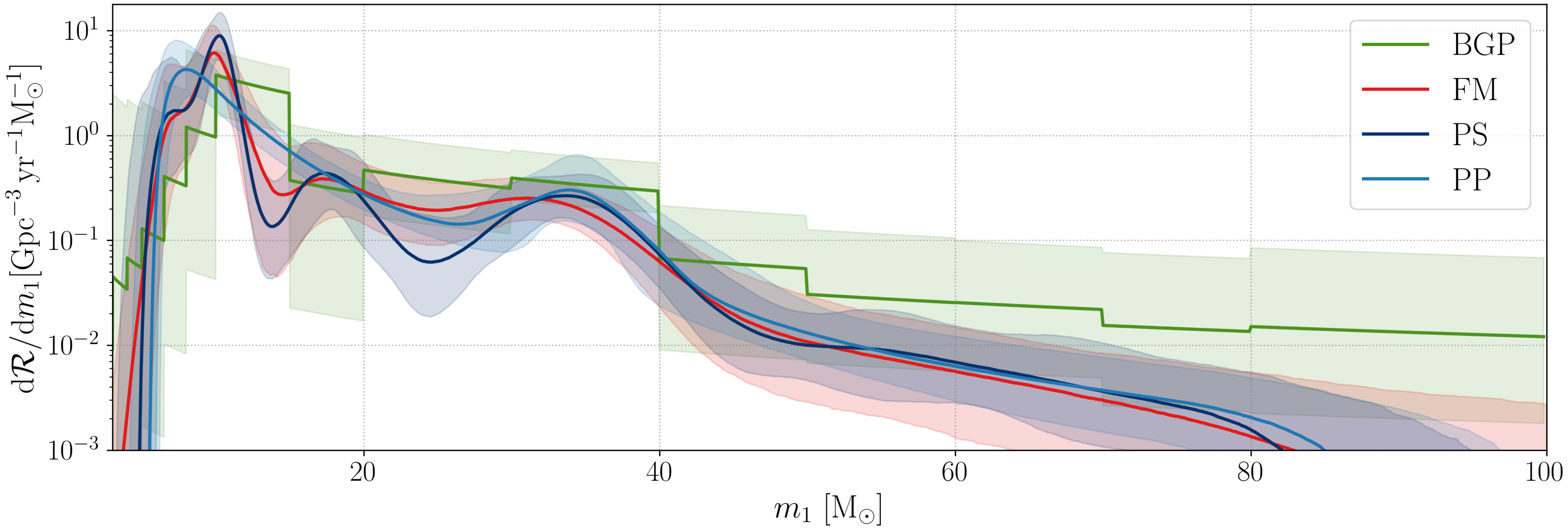
GWTC-3



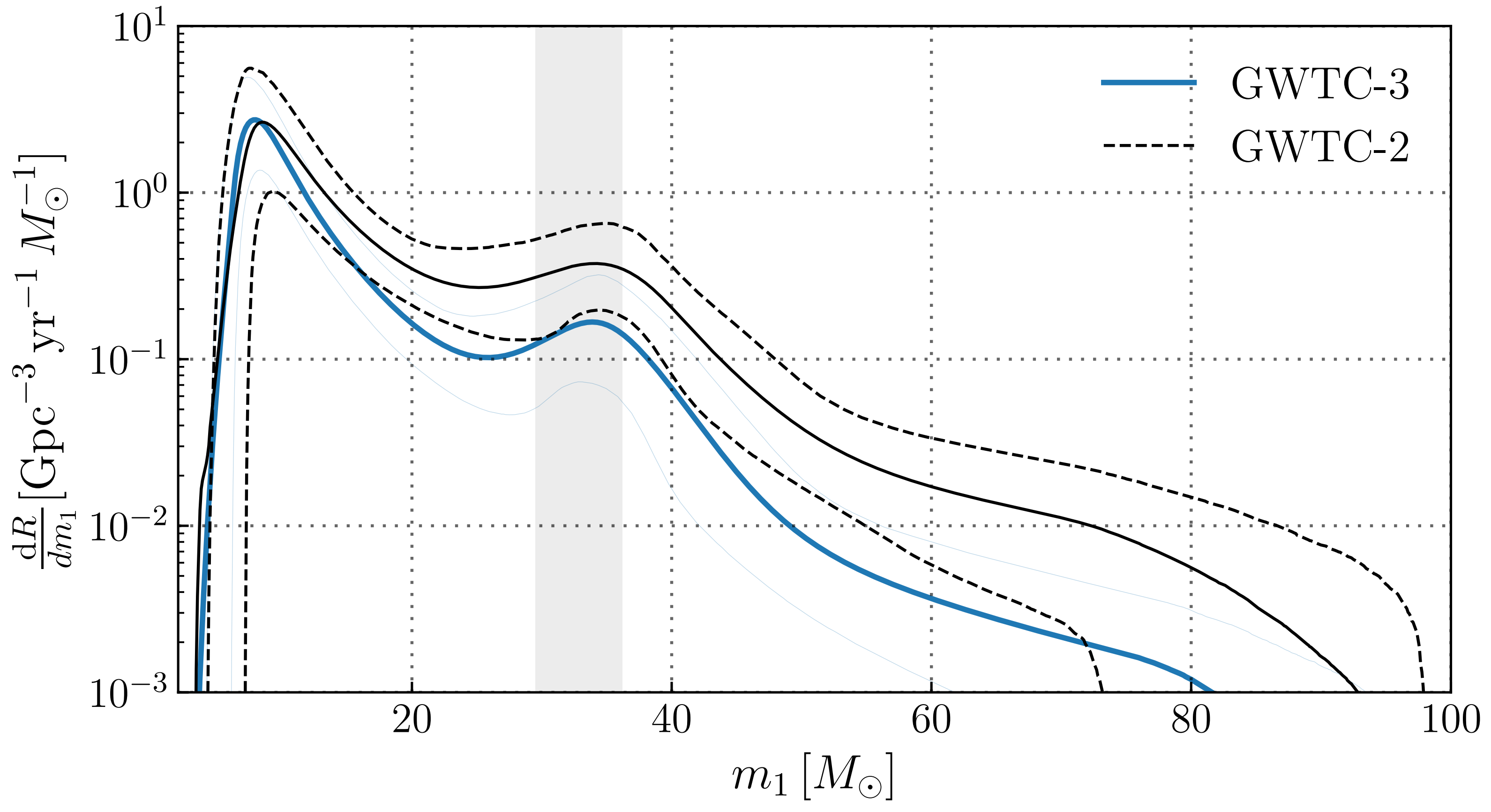
[Credit:LIGO-Virgo/Aaron Geller/Northwestern University]

GWTC-3

[LVK, PRX (2023)]



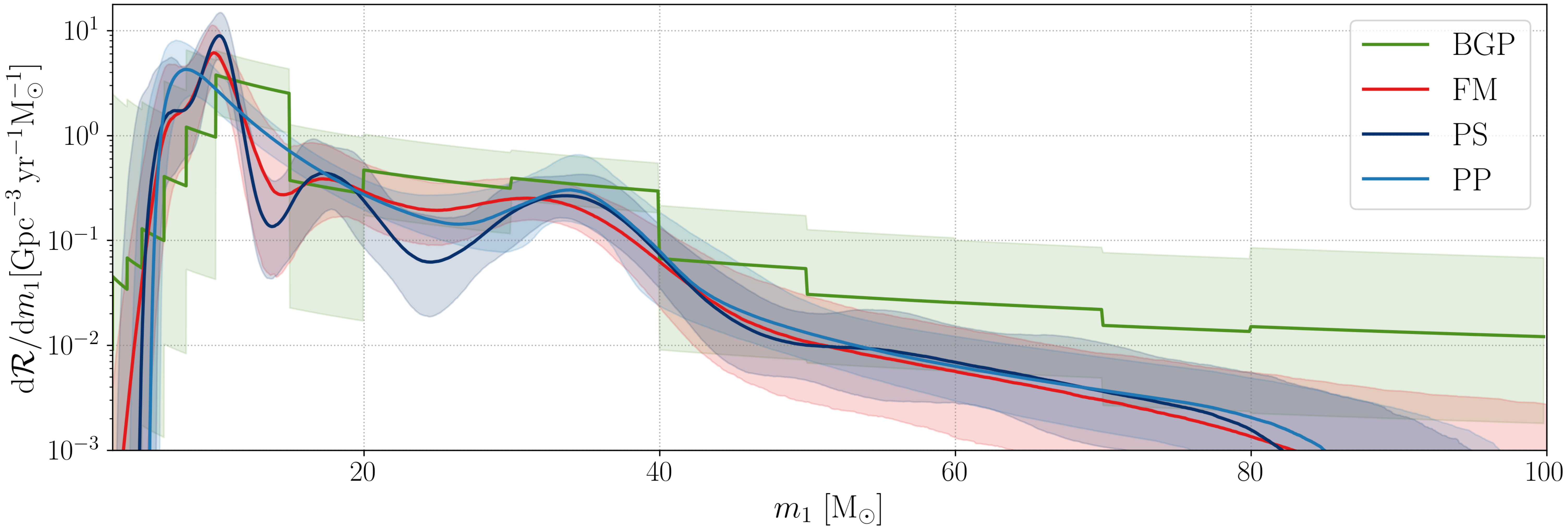
Power Law + Peak



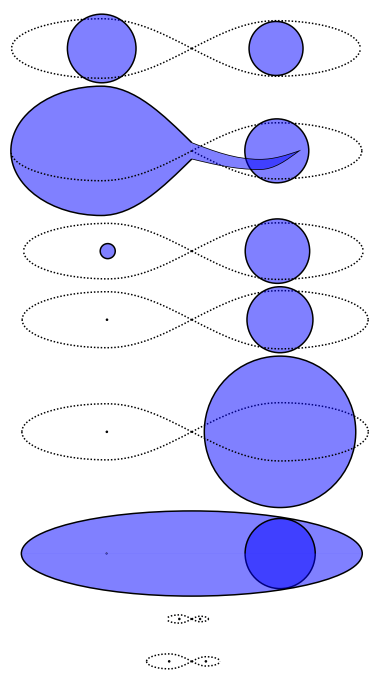
- Two peaks: $\sim 10 M_\odot$ $\sim 35 M_\odot$
- Monotonic decrease for $m > 50 M_\odot$
- Inconclusive evidence for upper mass gap

GWTC-3

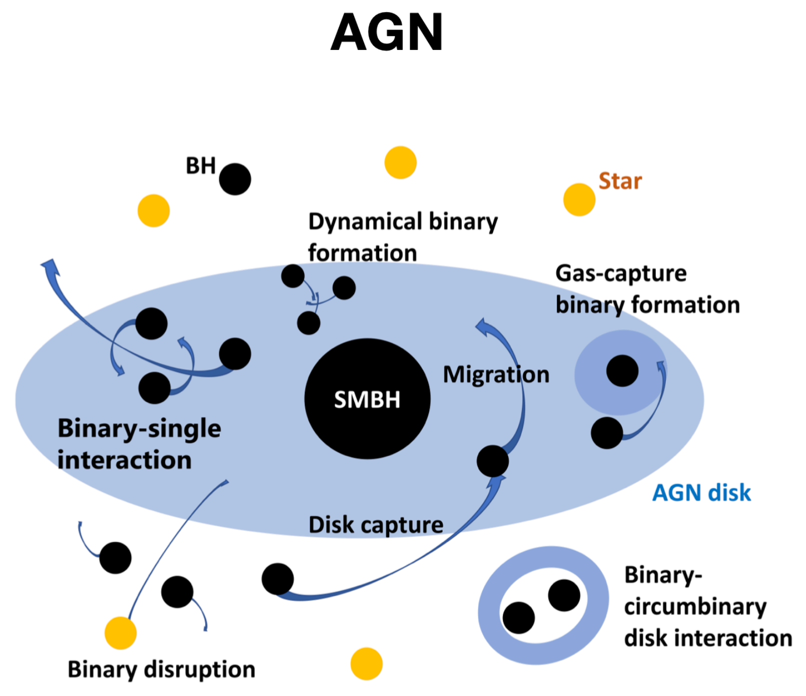
[LVK, PRX (2023)]



Astrophysical origin



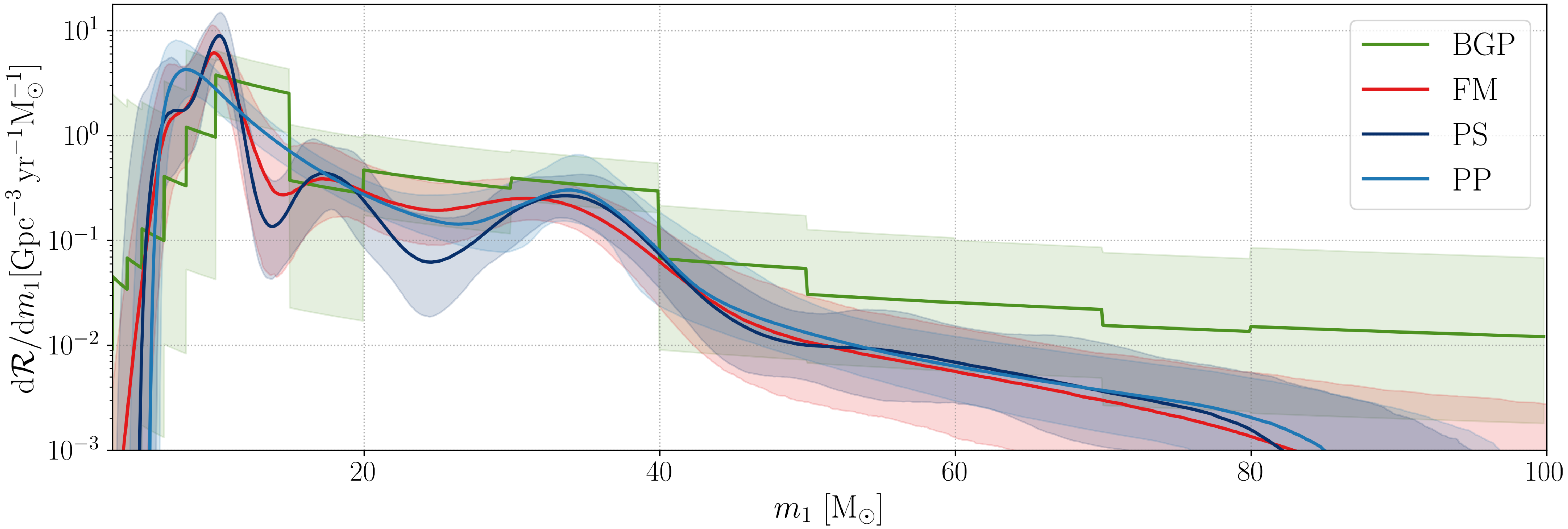
[Stevenson+, (2017)]



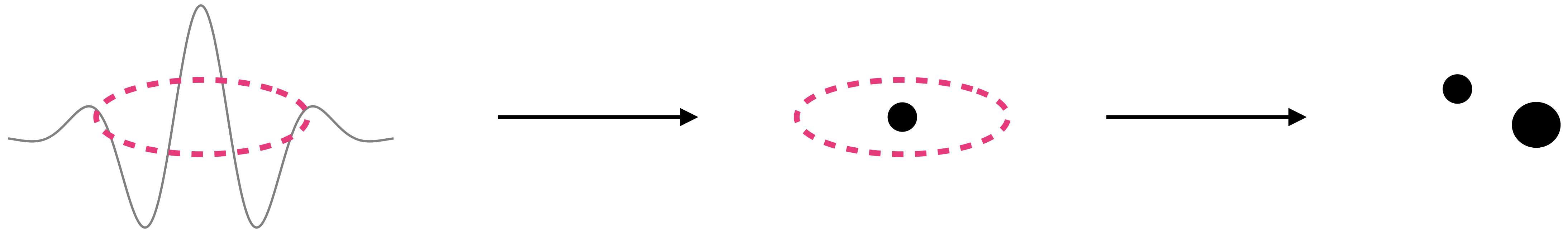
[Tagawa+, (2020)]

GWTC-3

[LVK, PRX (2023)]

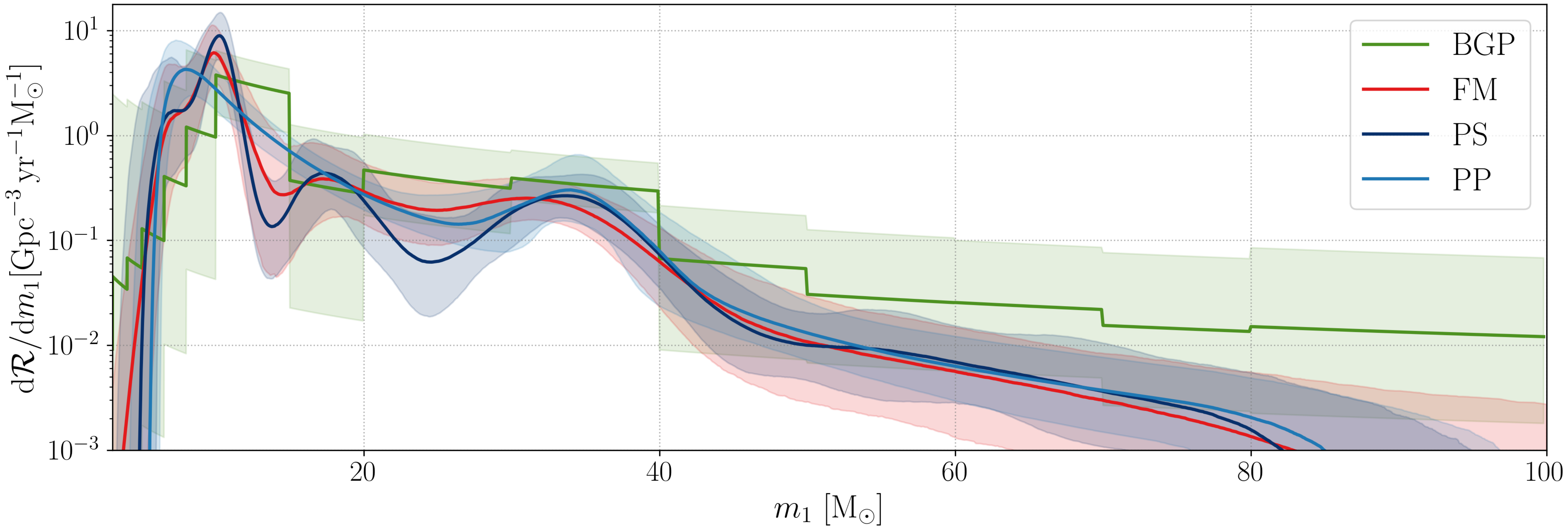


Primordial origin



GWTC-3

[LVK, PRX (2023)]



Primordial origin

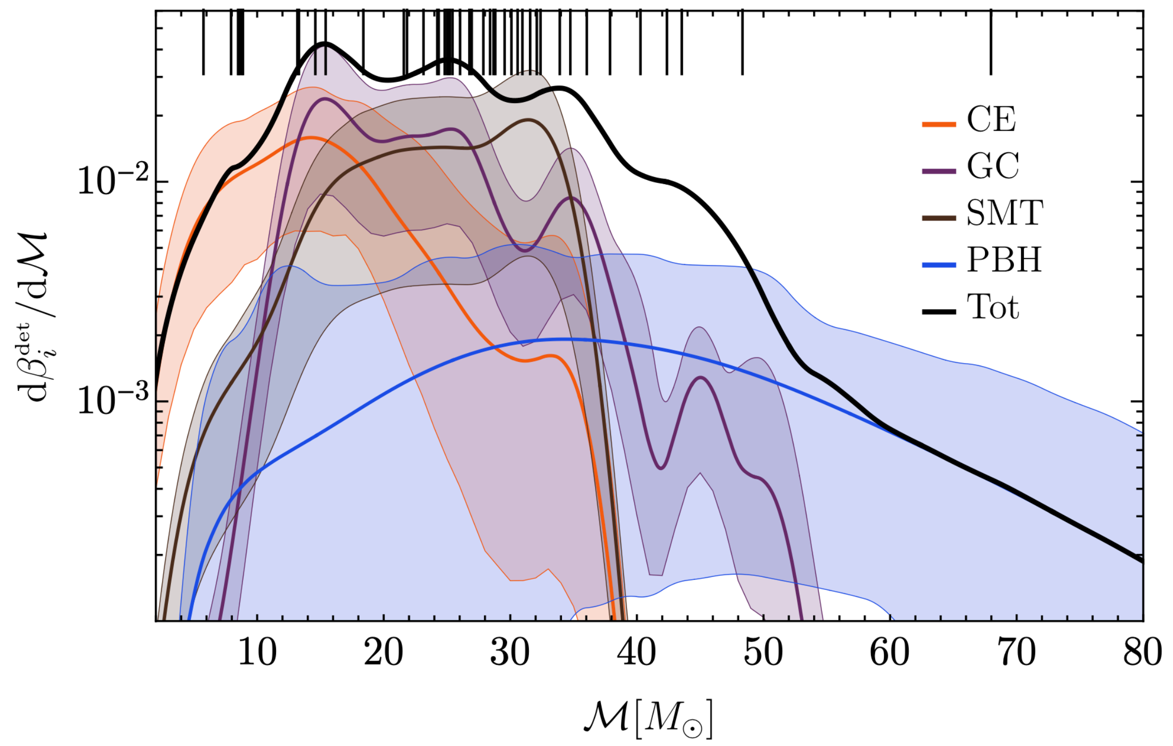
De Luca, Franciolini, Pani, Riotto, Hutsi, Veermae…

GW150914

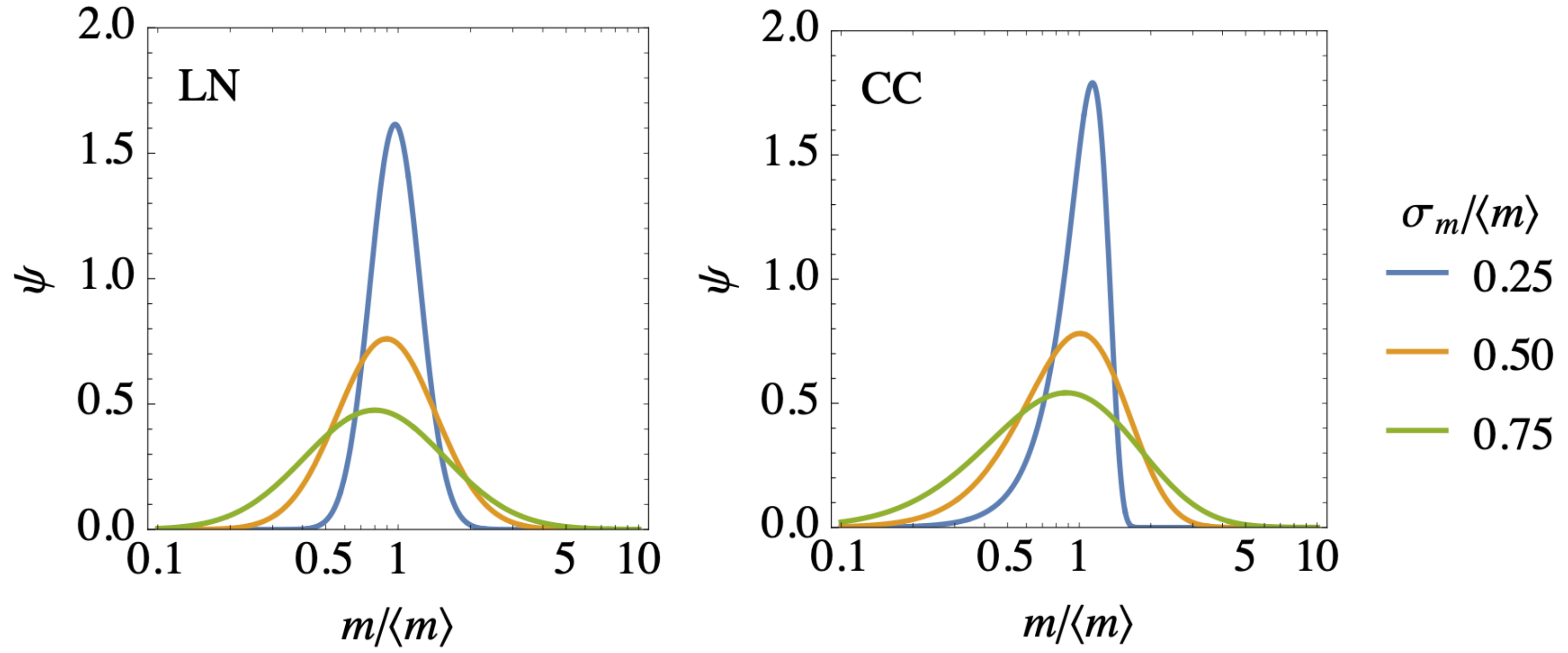
[Sasaki+, PRL (2016)]

[Bird+, PRL (2016)]

[Clesse+, Phys. Dark Univ. (2017)]

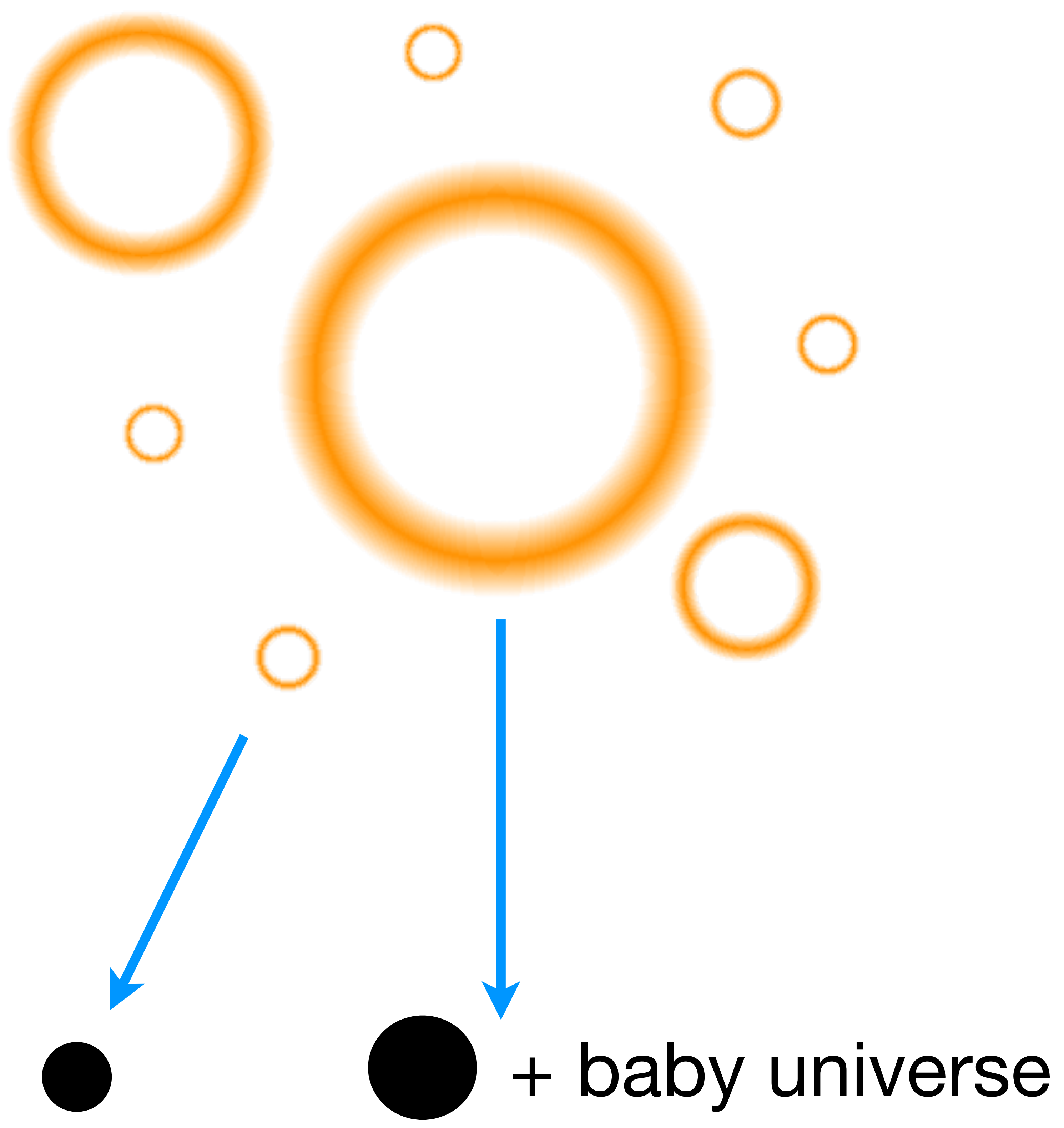


[Franciolini+, PRD (2022)]



[Hutsi+, JCAP (2021)]

Outline

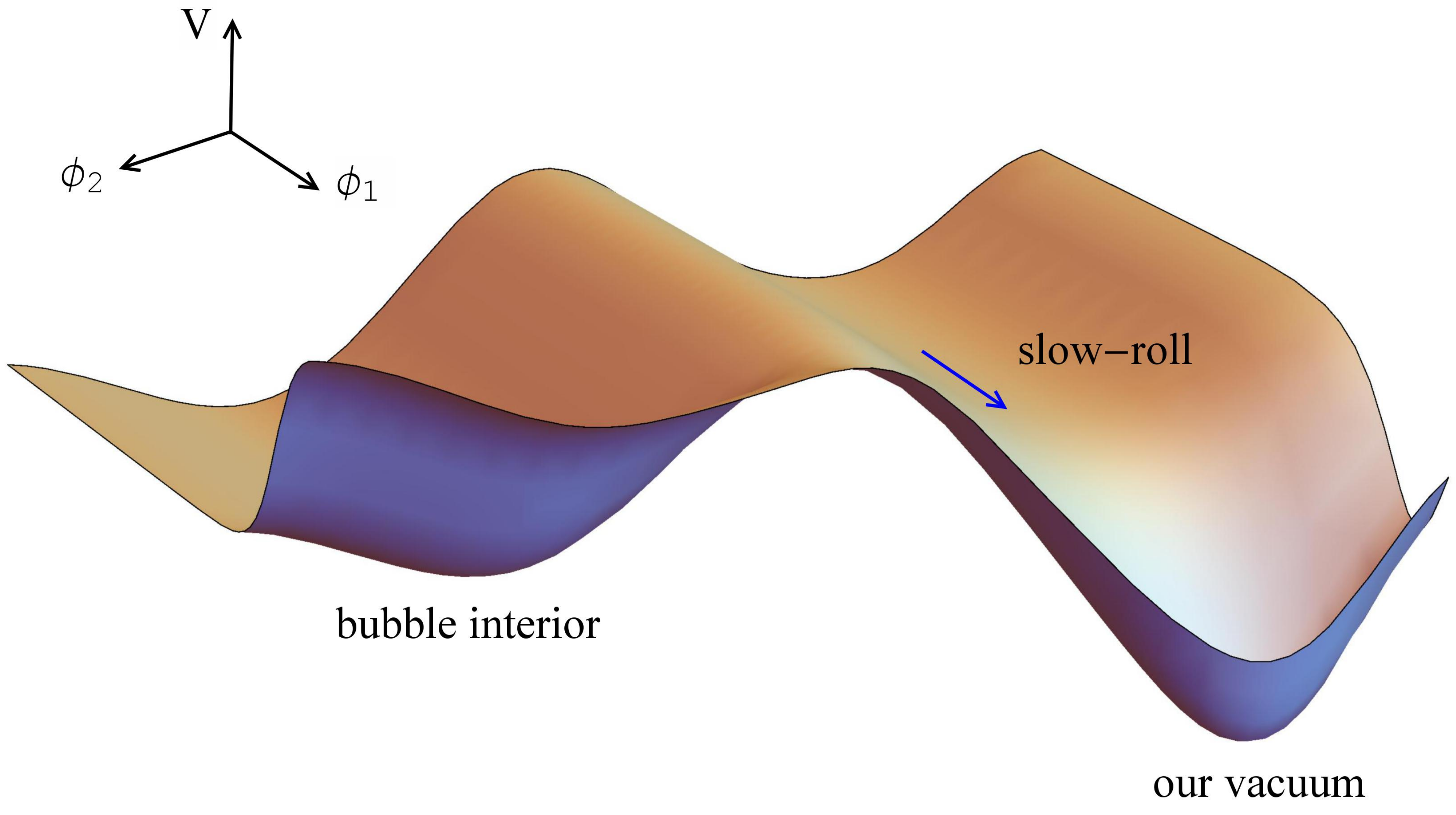


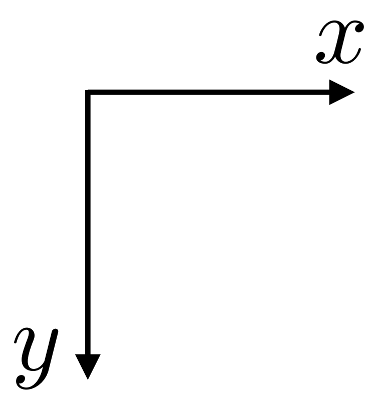
I. Formation mechanism

II. Implications from GWTC-3

III. Outlook

Bubbles from Inflation

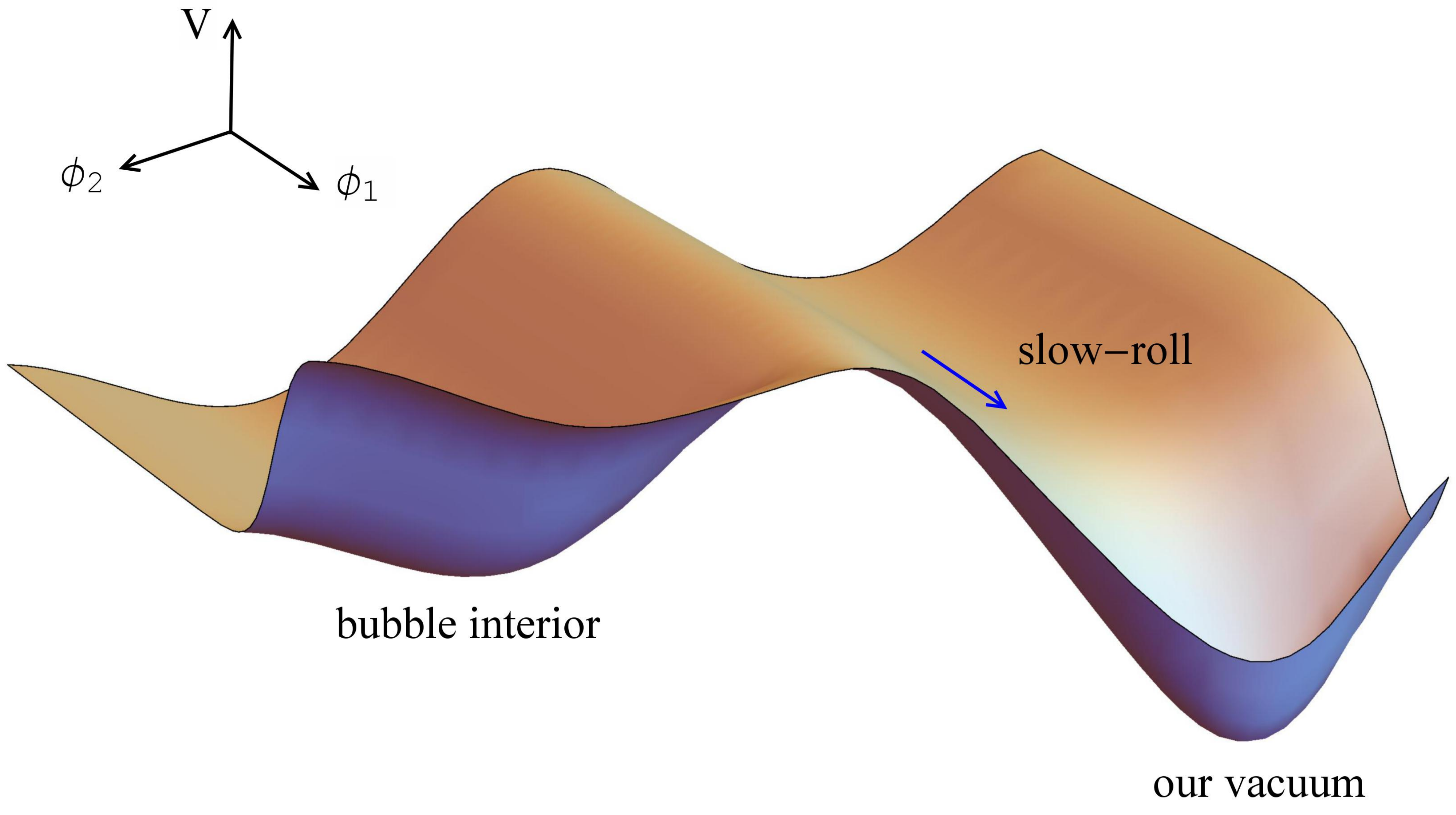




Slow-roll Inflation

[Garriga, Vilenkin, JZ, JCAP (2016)]

Bubbles from Inflation



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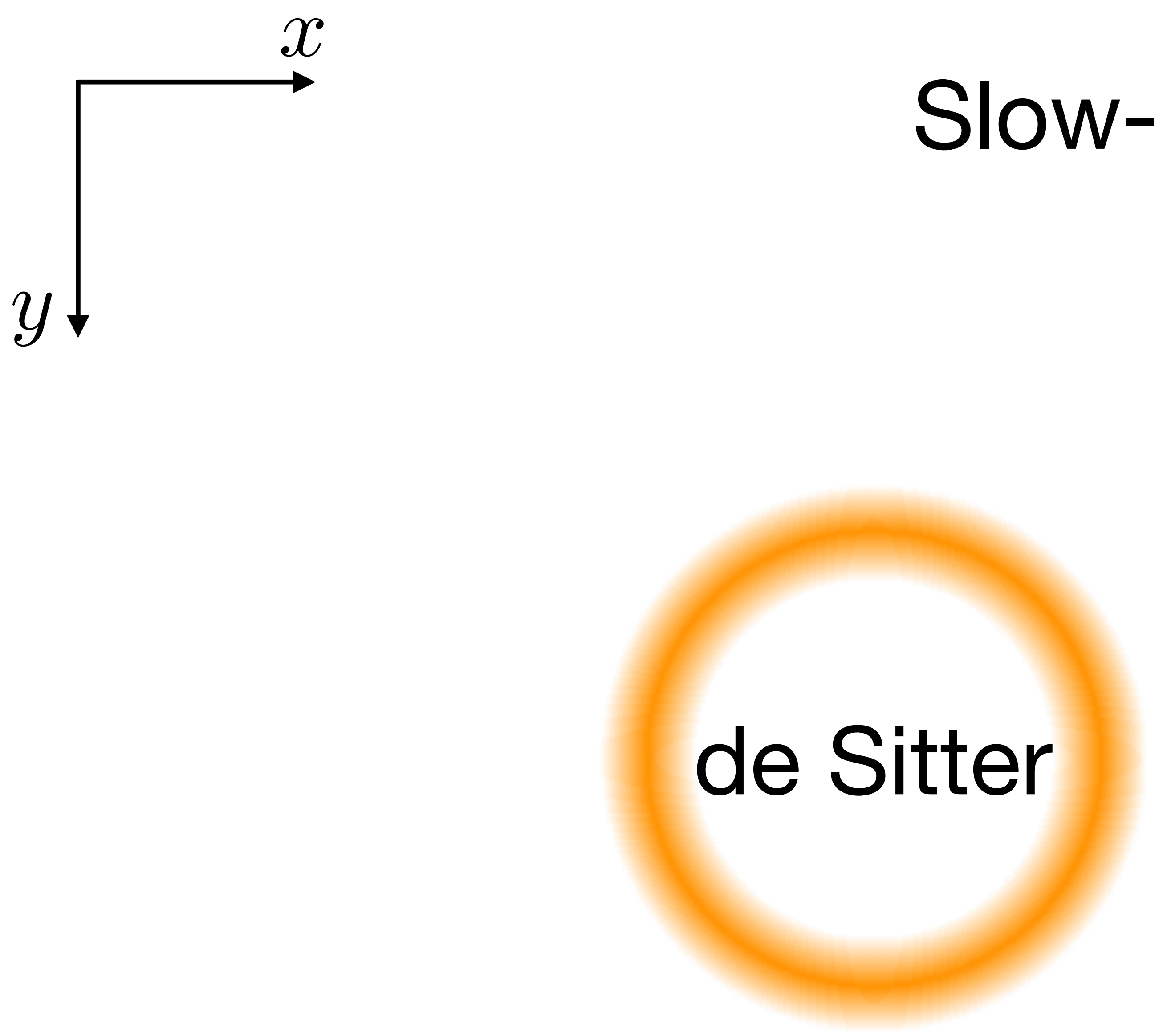

[Garriga, Vilenkin, JZ, JCAP (2016)]

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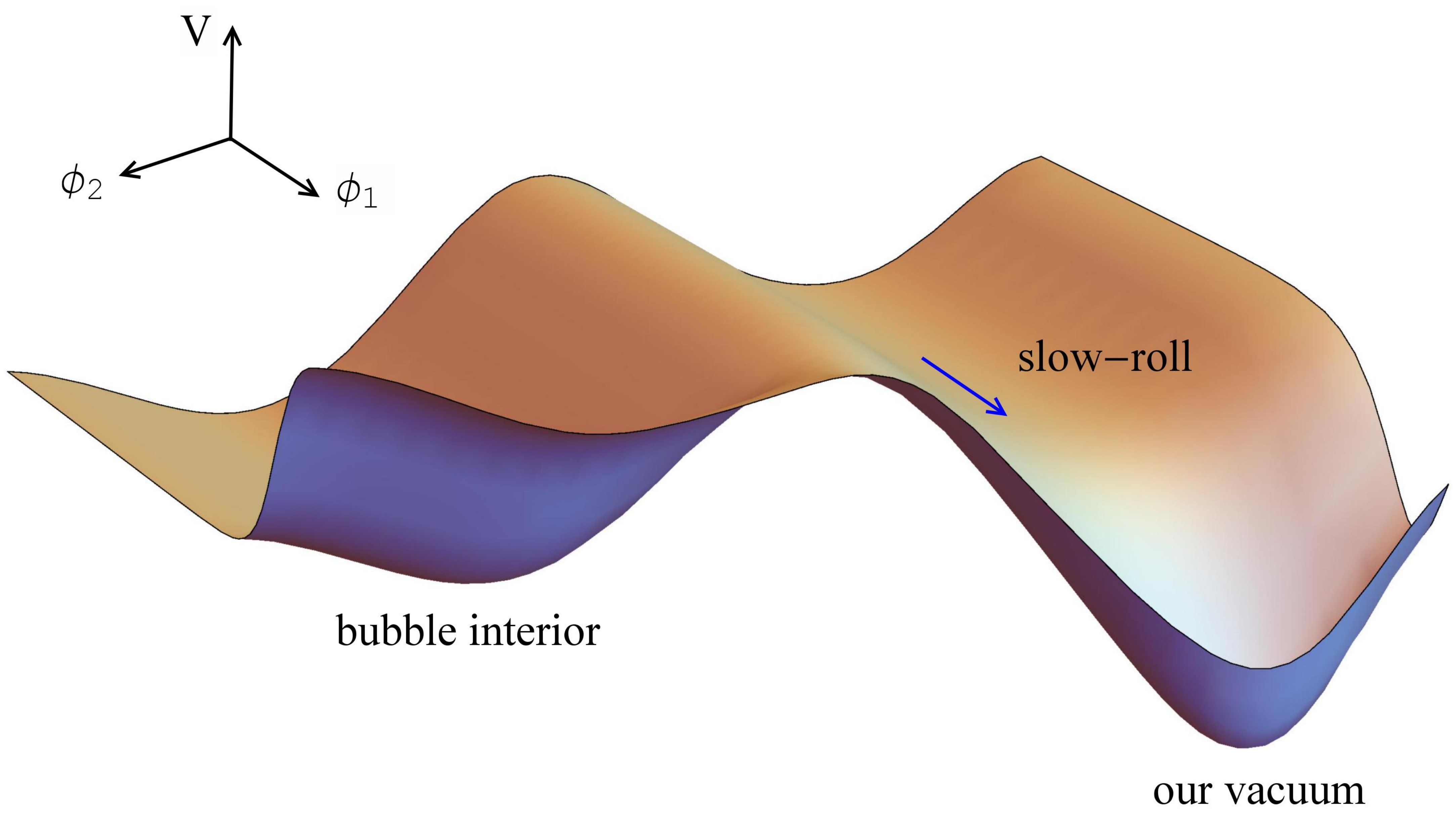

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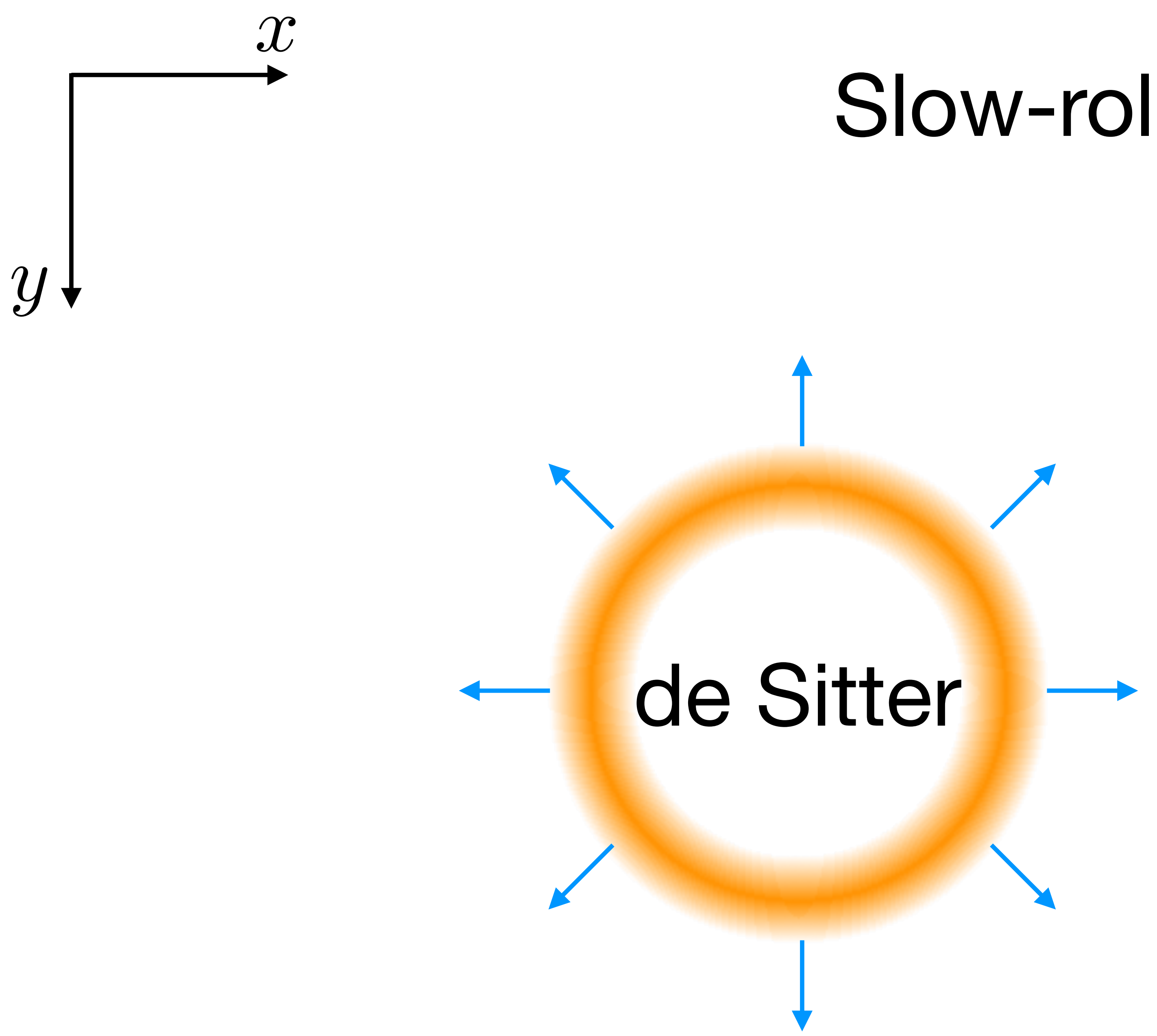
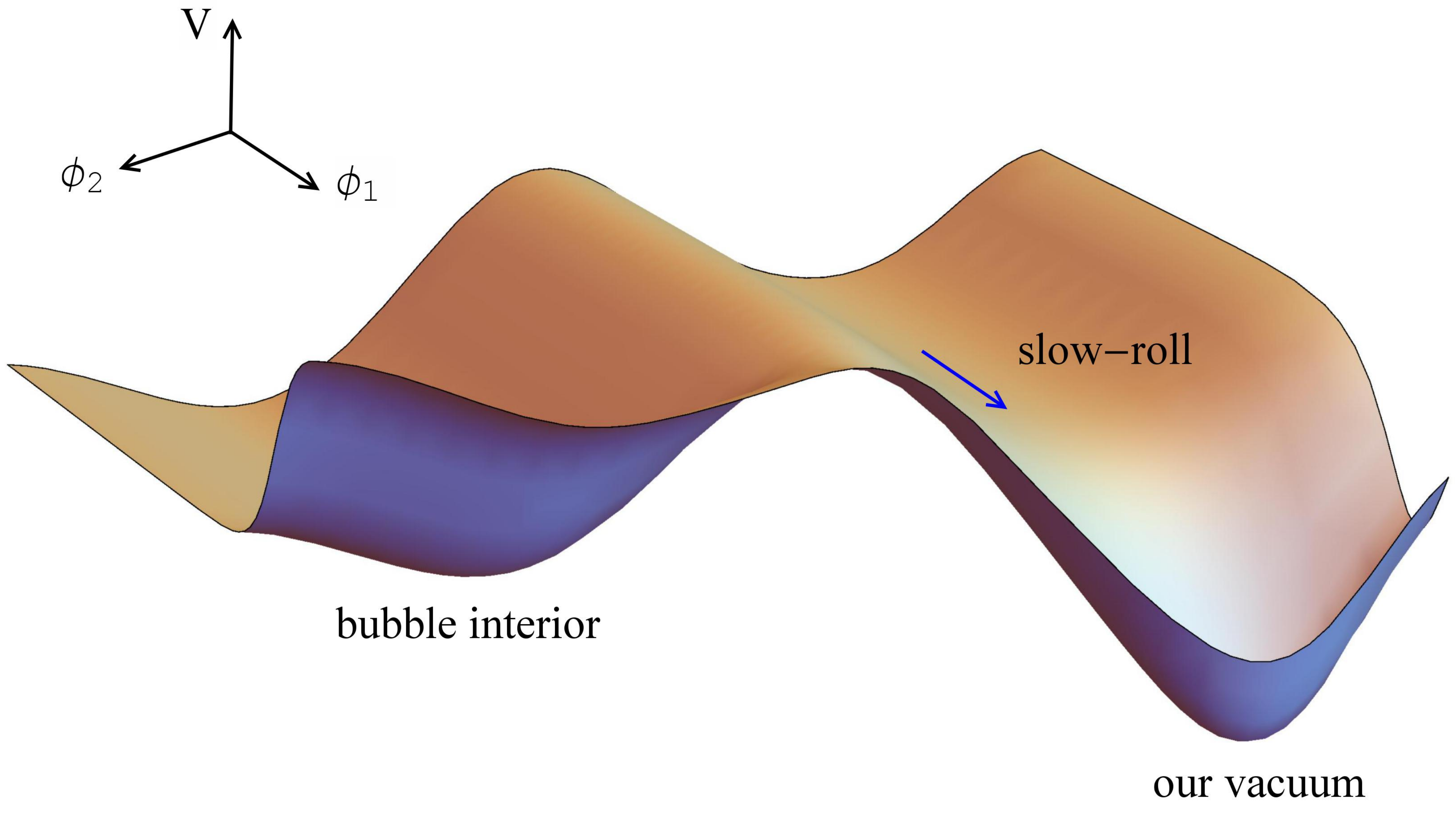
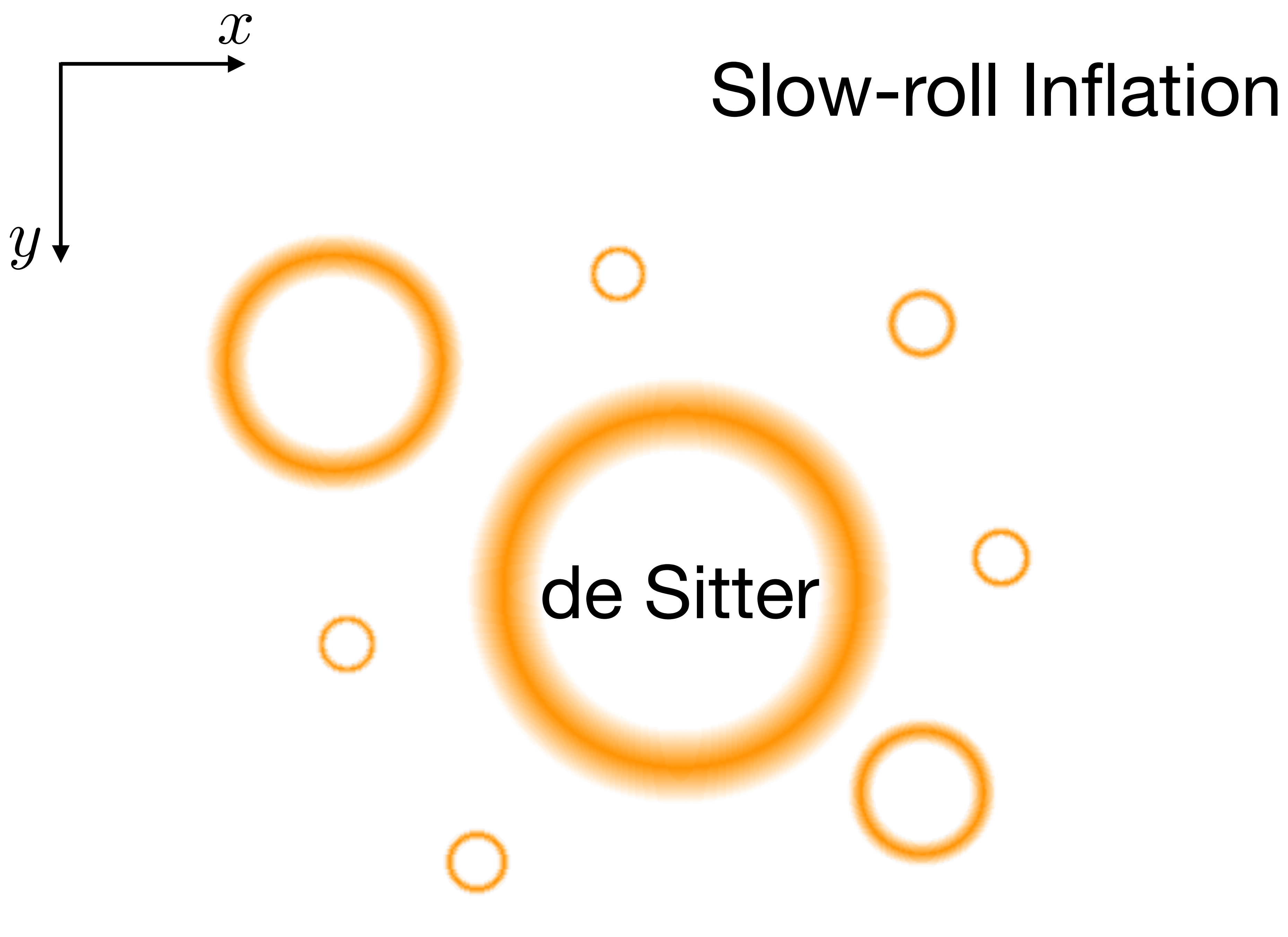


$$R(t) \approx H_i^{-1}\left[e^{H_i(t-t_{\rm n})} - 1\right]$$

[Garriga, Vilenkin, JZ, JCAP (2016)]

Bubbles from Inflation





$$R(t) \approx H_i^{-1}\left[e^{H_i(t-t_{\rm n})} - 1\right]$$

[Garriga, Vilenkin, JZ, JCAP (2016)]

Bubbles from Inflation

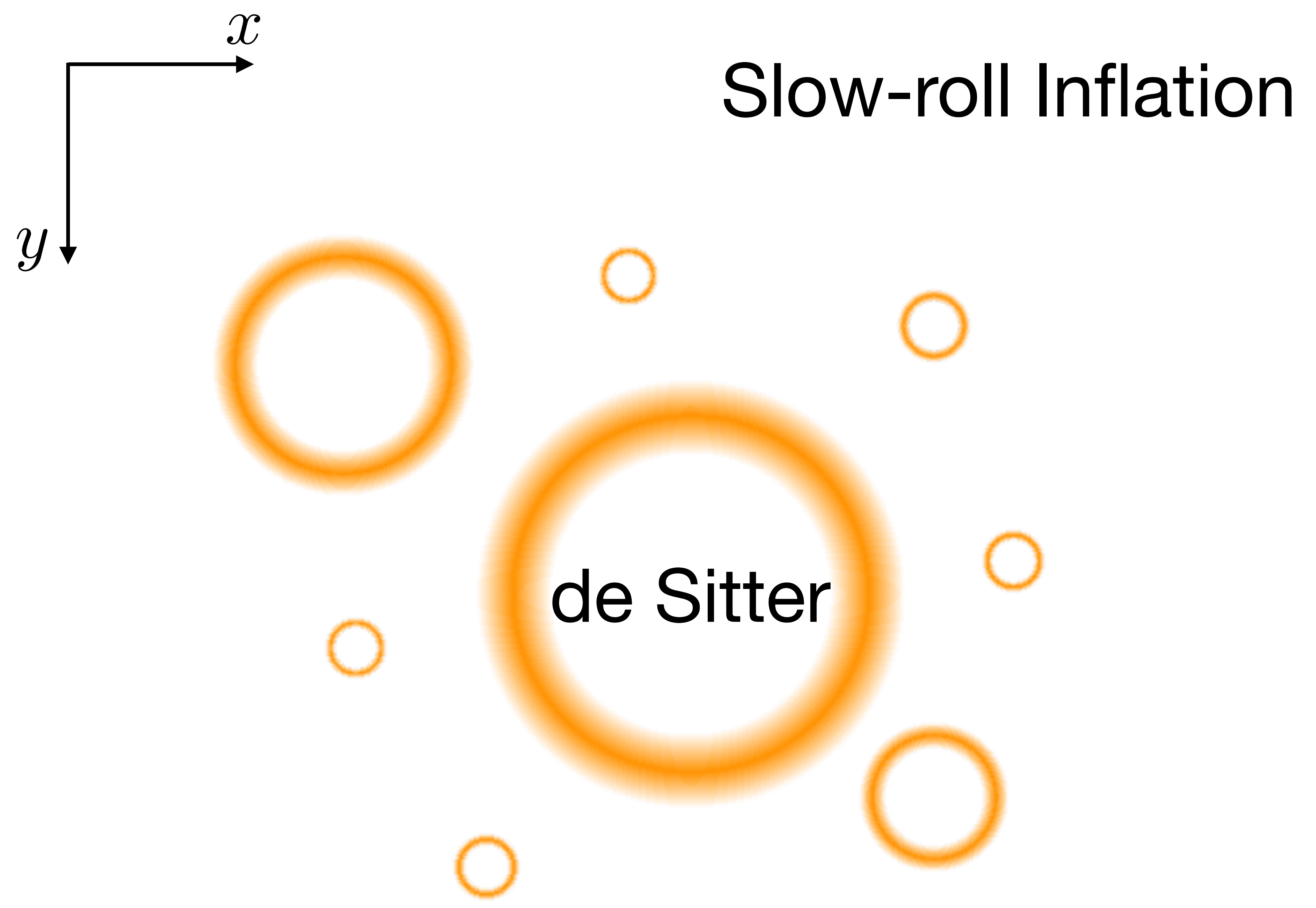
Nucleation rate λ

Size distribution

$$dN = \lambda H_i^4 e^{3H_i t_n} d^3\mathbf{x} dt_n \qquad dV \equiv e^{3H_i t} d^3\mathbf{x}$$

$$dn(t) \equiv \frac{dN}{dV} = \lambda \frac{dR}{\left(R + H_i^{-1}\right)^4}$$

$$dn(t_i) \simeq \lambda \frac{dR_i}{R_i^4}$$



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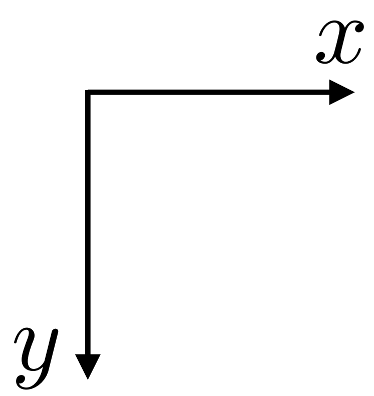
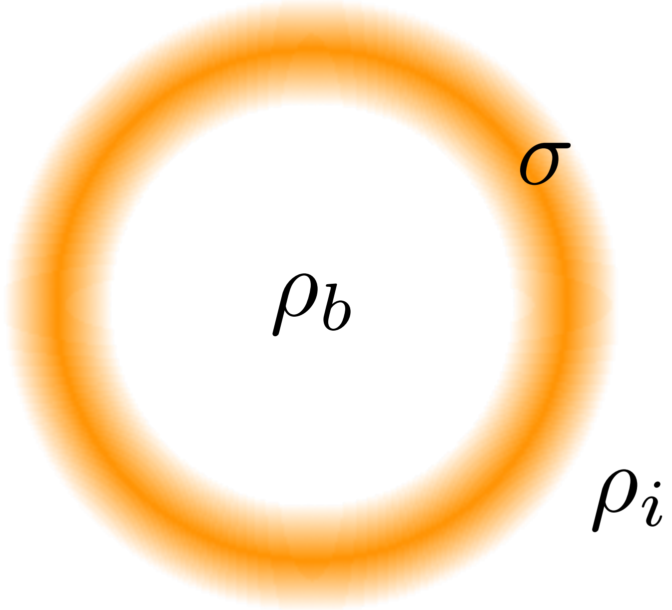


Slow-roll Inflation



$$H_i \equiv \sqrt{3\pi G \rho_i / 3} \sim \eta_i^2 / M_{\mathrm{Pl}}$$

$$H_b \equiv \sqrt{3\pi G \rho_b / 3} \sim \eta_b^2 / M_{\mathrm{Pl}}$$

$$H_\sigma \equiv 2\pi G \sigma \sim \eta_\sigma^3 / M_{\mathrm{Pl}}^2$$

$$\eta_\sigma \sim \eta_b \ll \eta_i \ll M_{\mathrm{Pl}} \quad H_\sigma \ll H_b \ll H_i$$

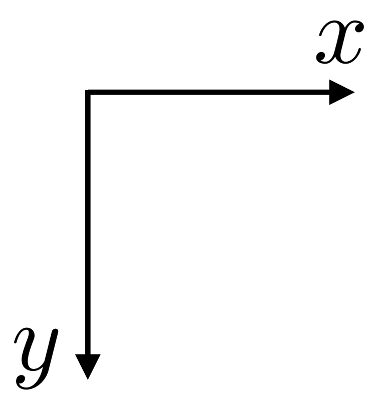
[Garriga, Vilenkin, JZ, JCAP (2016)]

Bubbles from Inflation

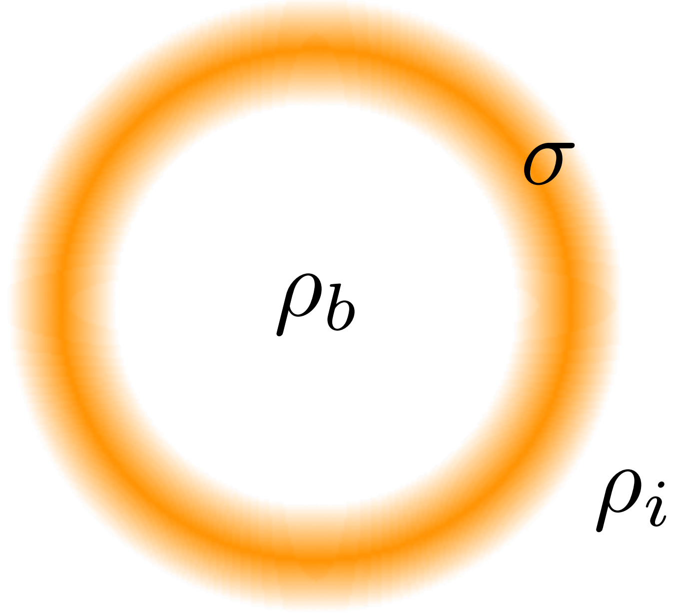
Israel's matching condition

$$\cancel{M} = \frac{4}{3}\pi(\rho_b - \rho_i)R^3 + 4\pi\sigma R^2[\dot{R}^2 + 1 - H_b^2 R^2]^{1/2} - 8\pi^2 G\sigma^2 R^3$$

$\equiv dR/d\tau$



Schwarszchild-de Sitter



At the end of inflation

$$\frac{4}{3}\pi\rho_m(t_i)R_i^3 \approx \frac{4}{3}\pi\rho_b R_i^3 + 4\pi\sigma R_i^2[\dot{R}_i^2 + 1 - H_b^2 R_i^2]^{1/2} - 8\pi^2 G\sigma^2 R_i^3$$

$$\dot{R}_i \simeq \frac{1}{4}\frac{H_i^2}{H_\sigma}R_i$$

$$\dot{R}_i = \frac{H_i R_i + a_i r_i'}{\sqrt{1 - a_i^2 r_i'^2}} \qquad ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega^2)$$

Lorentz factor $\quad \gamma_i \equiv \dfrac{1}{\sqrt{1 - a_i^2 r_i'^2}} \sim \dfrac{H_i}{H_\sigma} \gg 1$

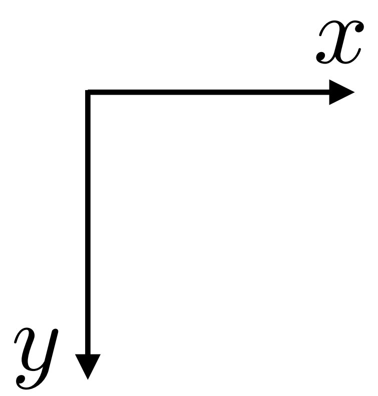
$$H_i \equiv \sqrt{3\pi G\rho_i/3} \sim \eta_i^2/M_{\rm Pl}$$
$$H_b \equiv \sqrt{3\pi G\rho_b/3} \sim \eta_b^2/M_{\rm Pl}$$
$$H_\sigma \equiv 2\pi G\sigma \sim \eta_\sigma^3/M_{\rm Pl}^2$$
$$\eta_\sigma \sim \eta_b \ll \eta_i \ll M_{\rm Pl} \quad H_\sigma \ll H_b \ll H_i$$

[Garriga, Vilenkin, JZ, JCAP (2016)]

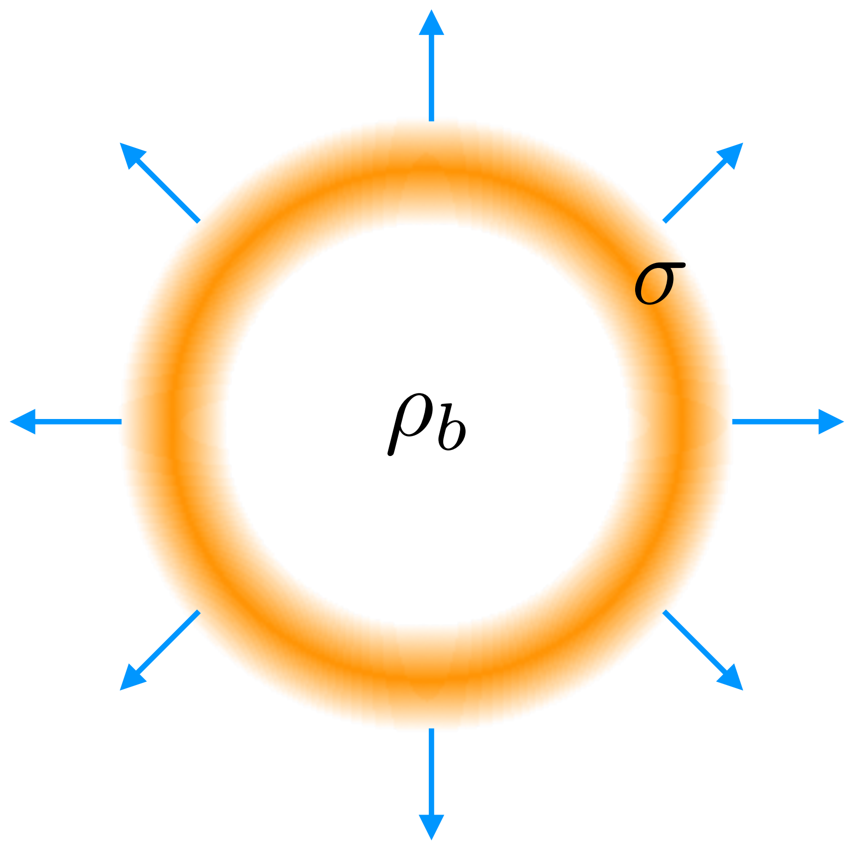
Post-inflation evolution

Initial Condition

$$dn(t_i) \simeq \lambda \frac{dR_i}{R_i^4} \qquad \gamma_i \equiv \frac{1}{\sqrt{1-a_i^2 r_i'^2}} \sim \frac{H_i}{H_\sigma} \gg 1$$



Radiation FRW



[Garriga, Vilenkin, JZ, JCAP (2016)]

Post-inflation evolution

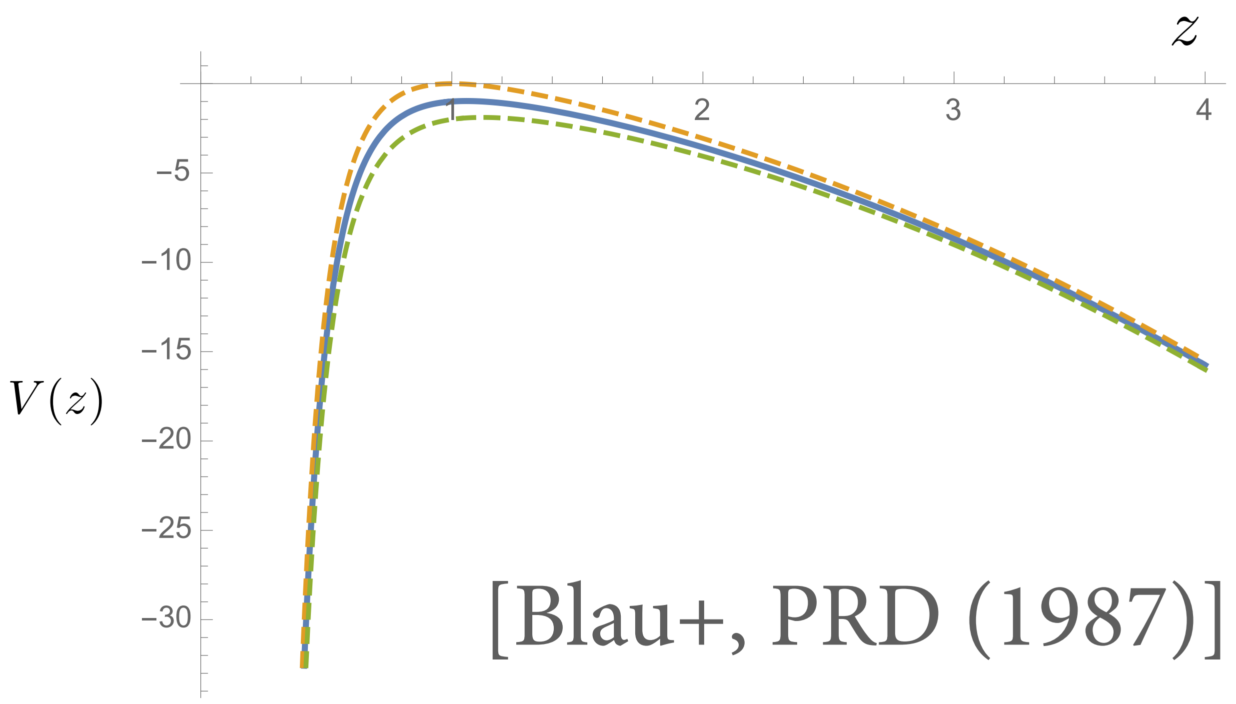
A pedagogical model

$$M_{bh} = \frac{4}{3}\pi\rho_b R^3 + 4\pi\sigma R^2[\dot{R}^2 + 1 - H_b^2 R^2]^{1/2} - 8\pi^2 G\sigma^2 R^3$$

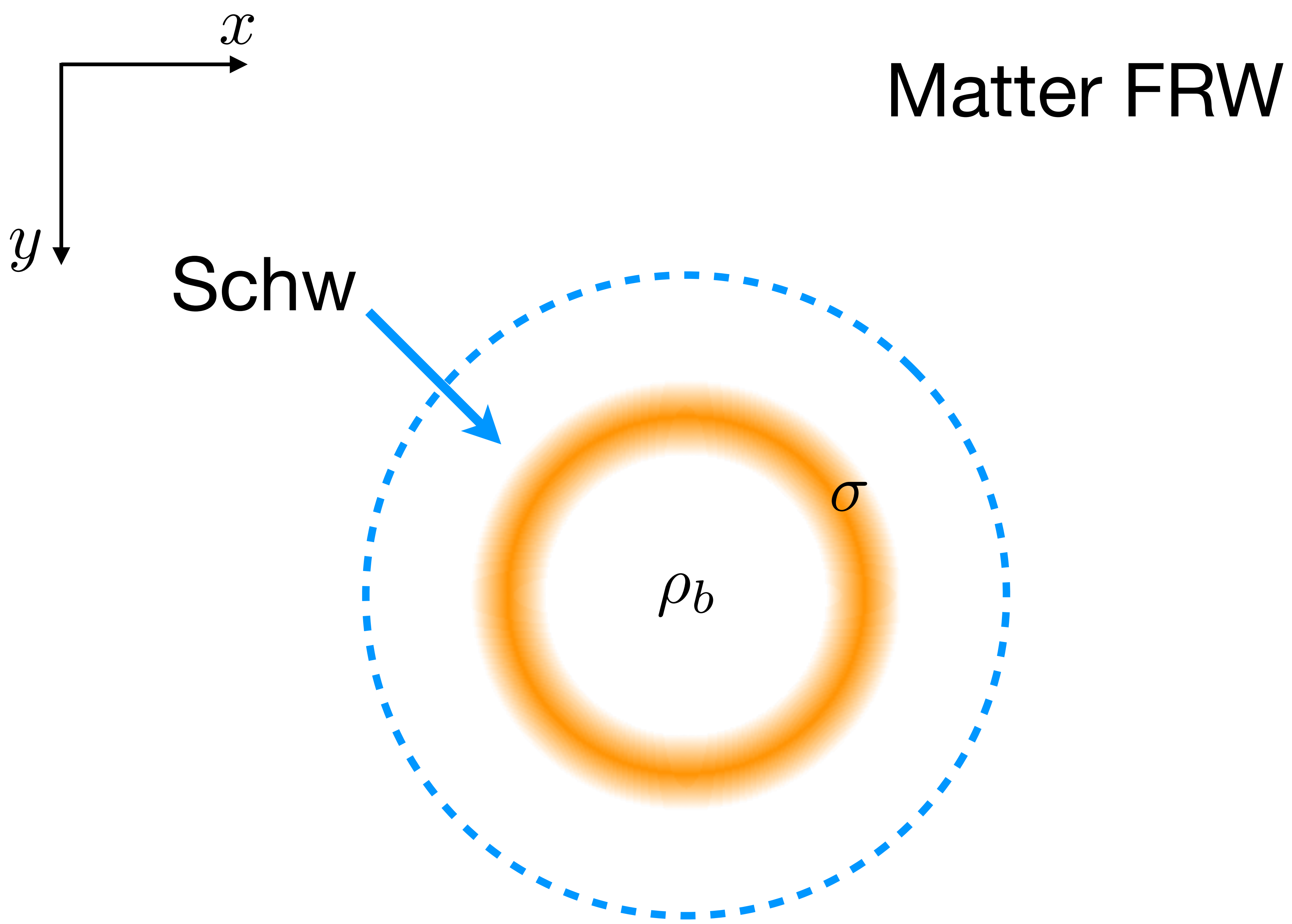
$$\left(\frac{dz}{d\tilde{\tau}}\right)^2 + V(z) = E$$

$$z^3 = \frac{H_+^2}{2GM_{bh}} R^3 \quad E = \frac{-16 H_\sigma^2}{(2GM_{bh})^{2/3} H_+^{8/3}}$$

$$M_* \sim M_{\rm Pl}^3/\eta_b^2$$



[Blau+, PRD (1987)]



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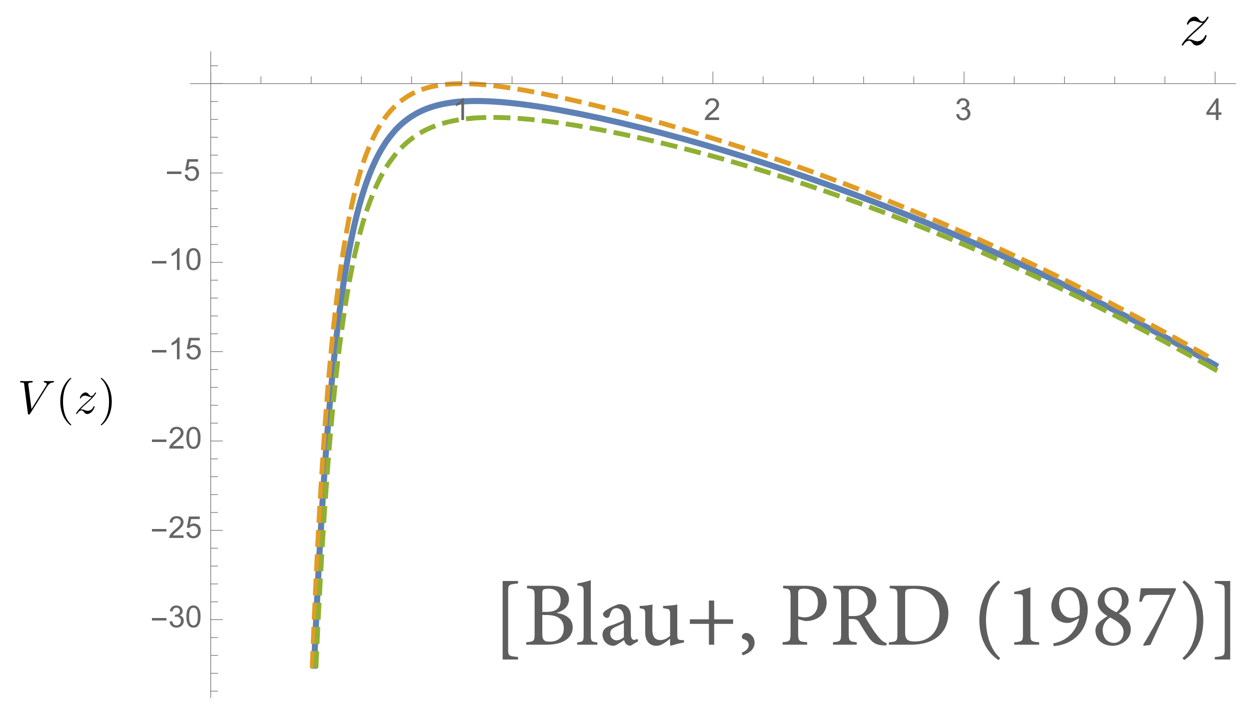
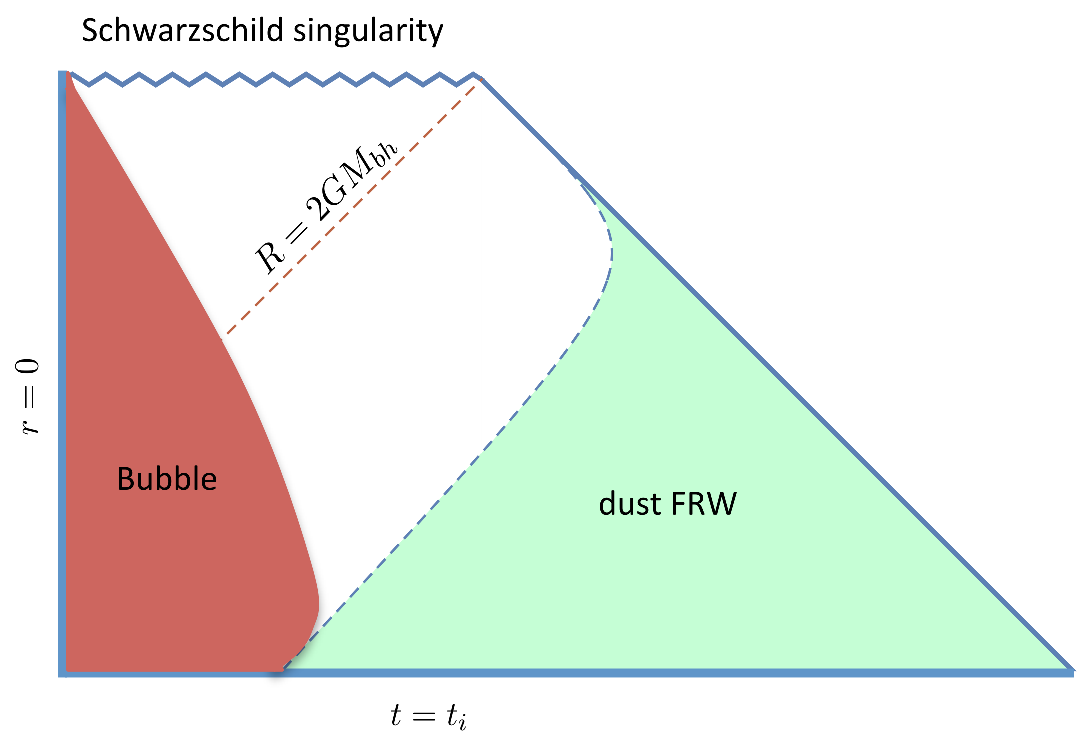
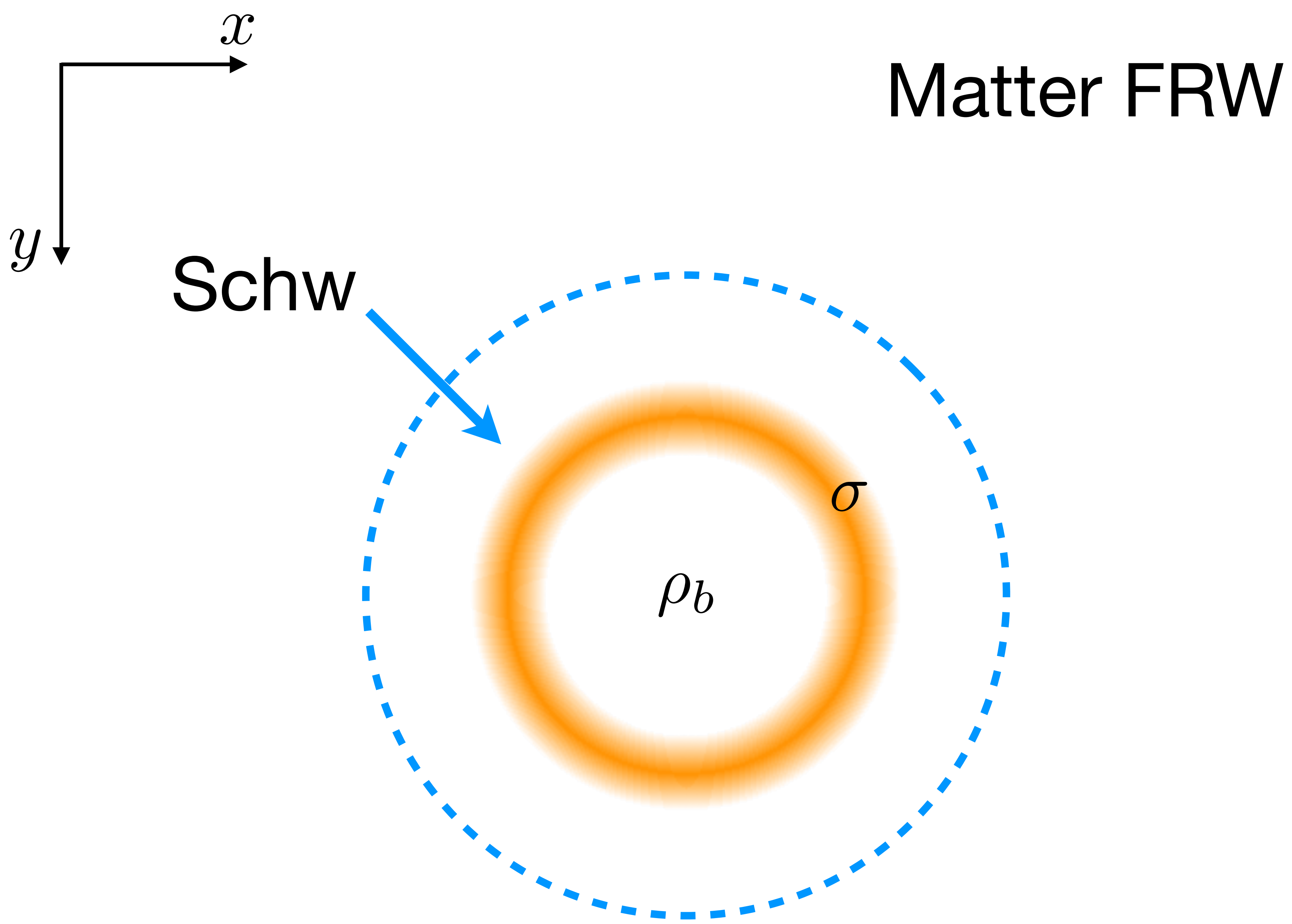


[Blau+, PRD (1987)]

- Subcritical





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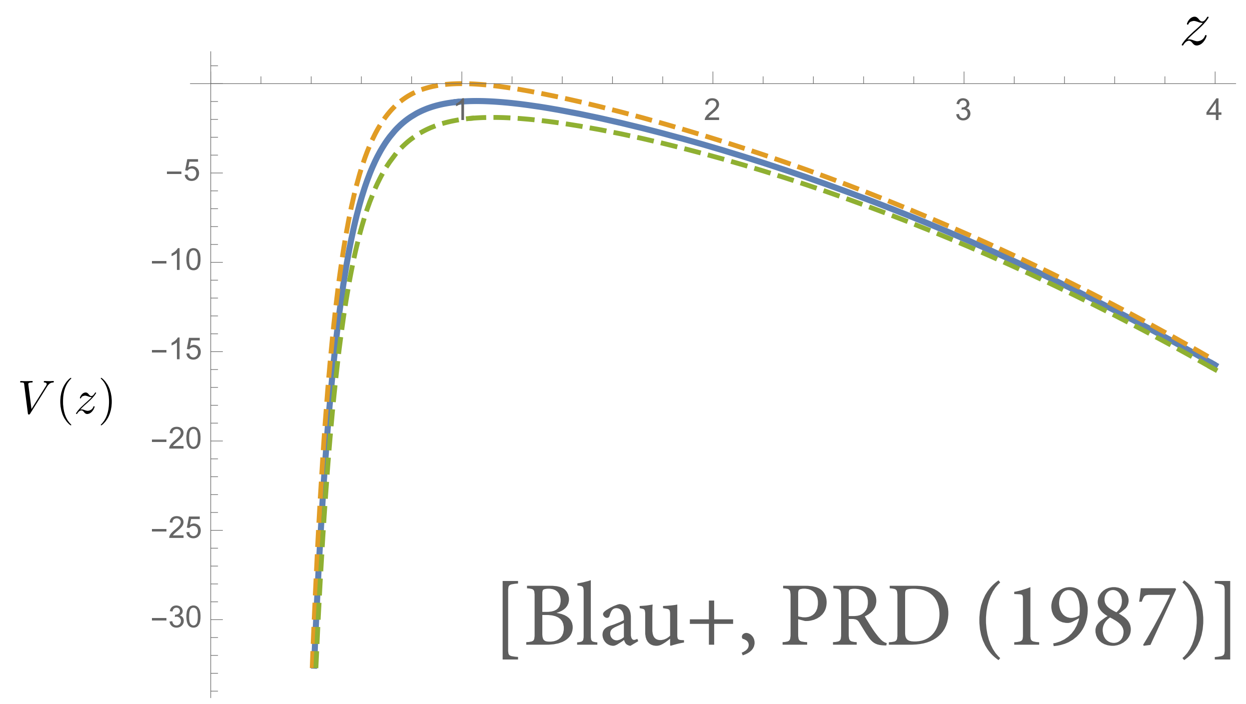

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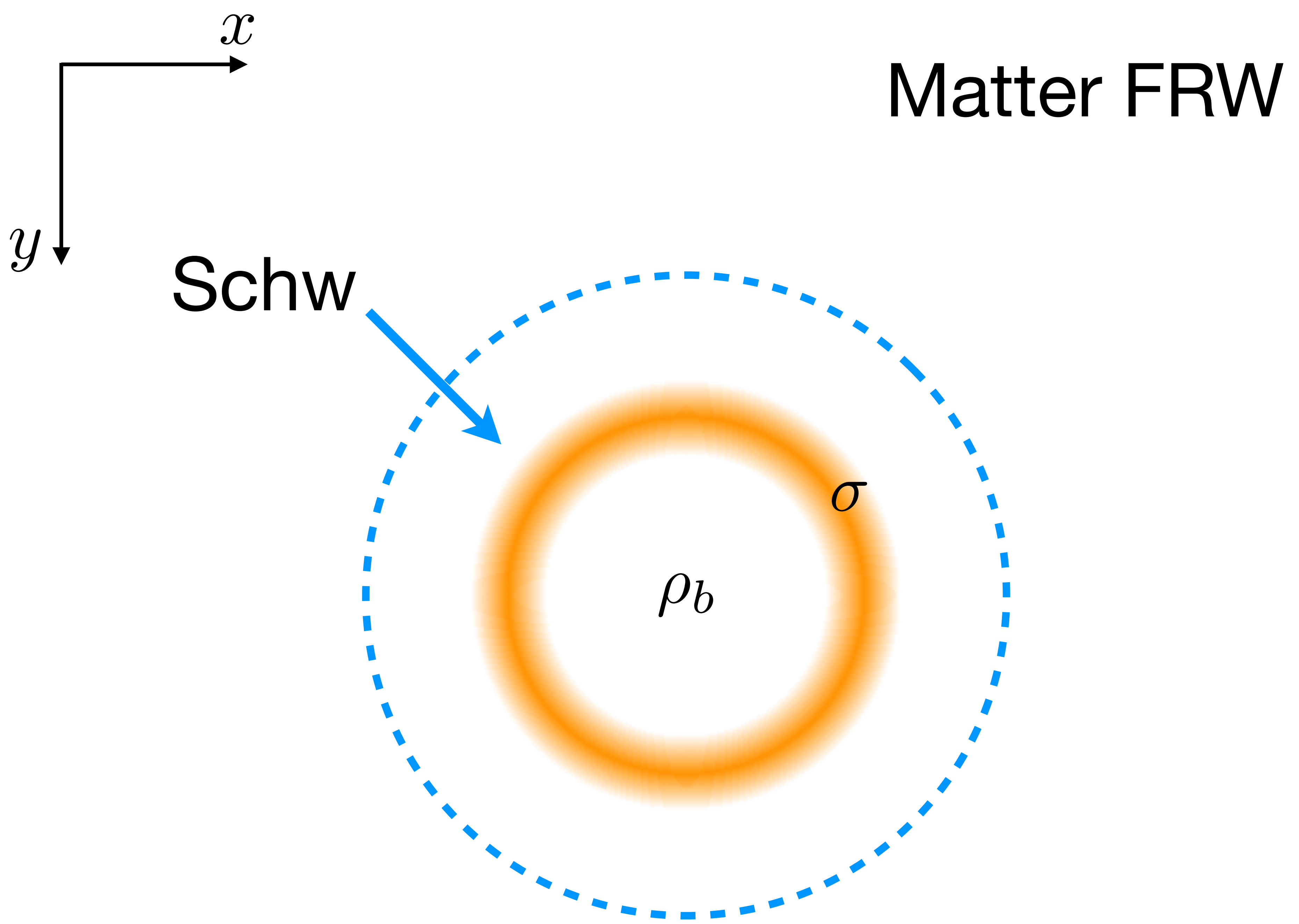
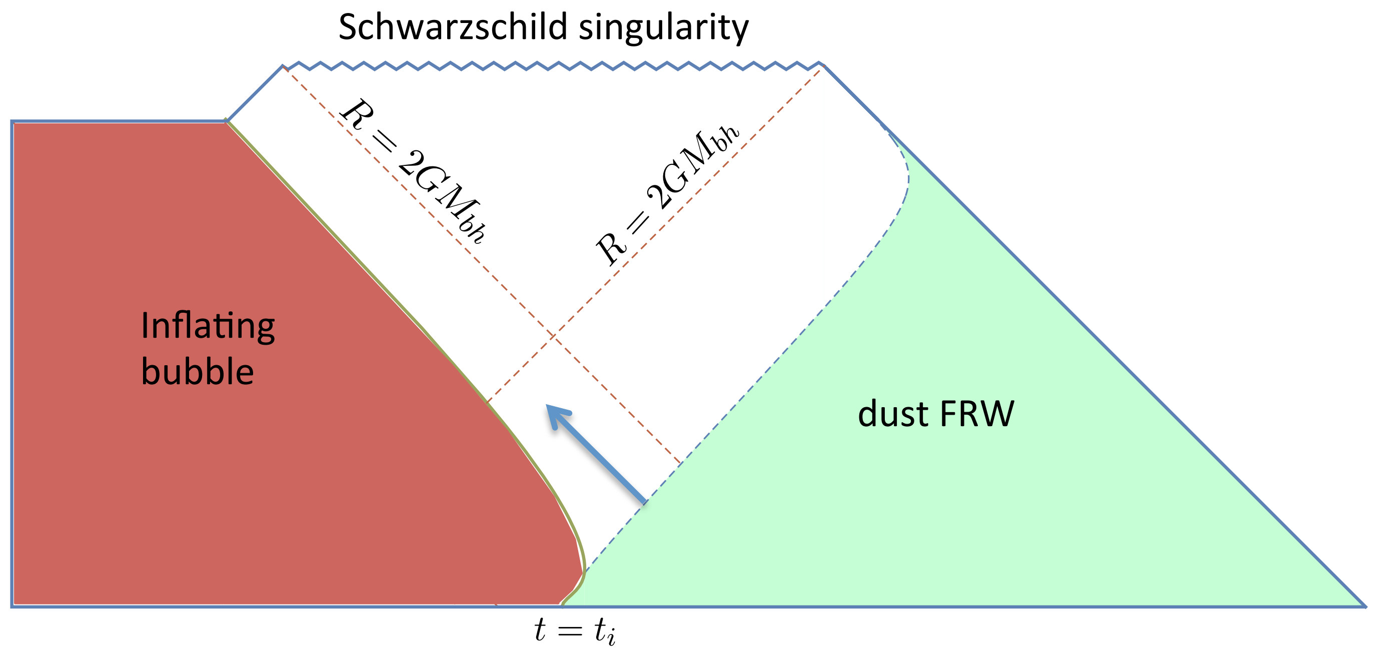
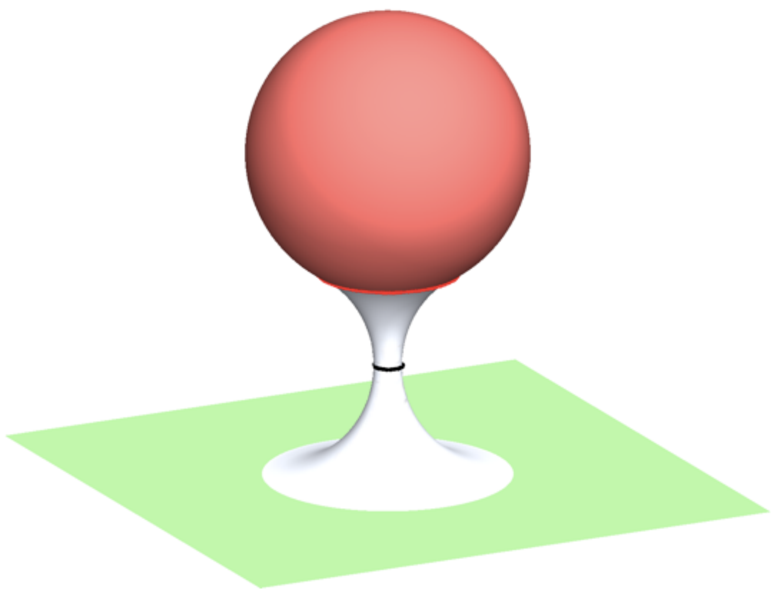


- Supercritical

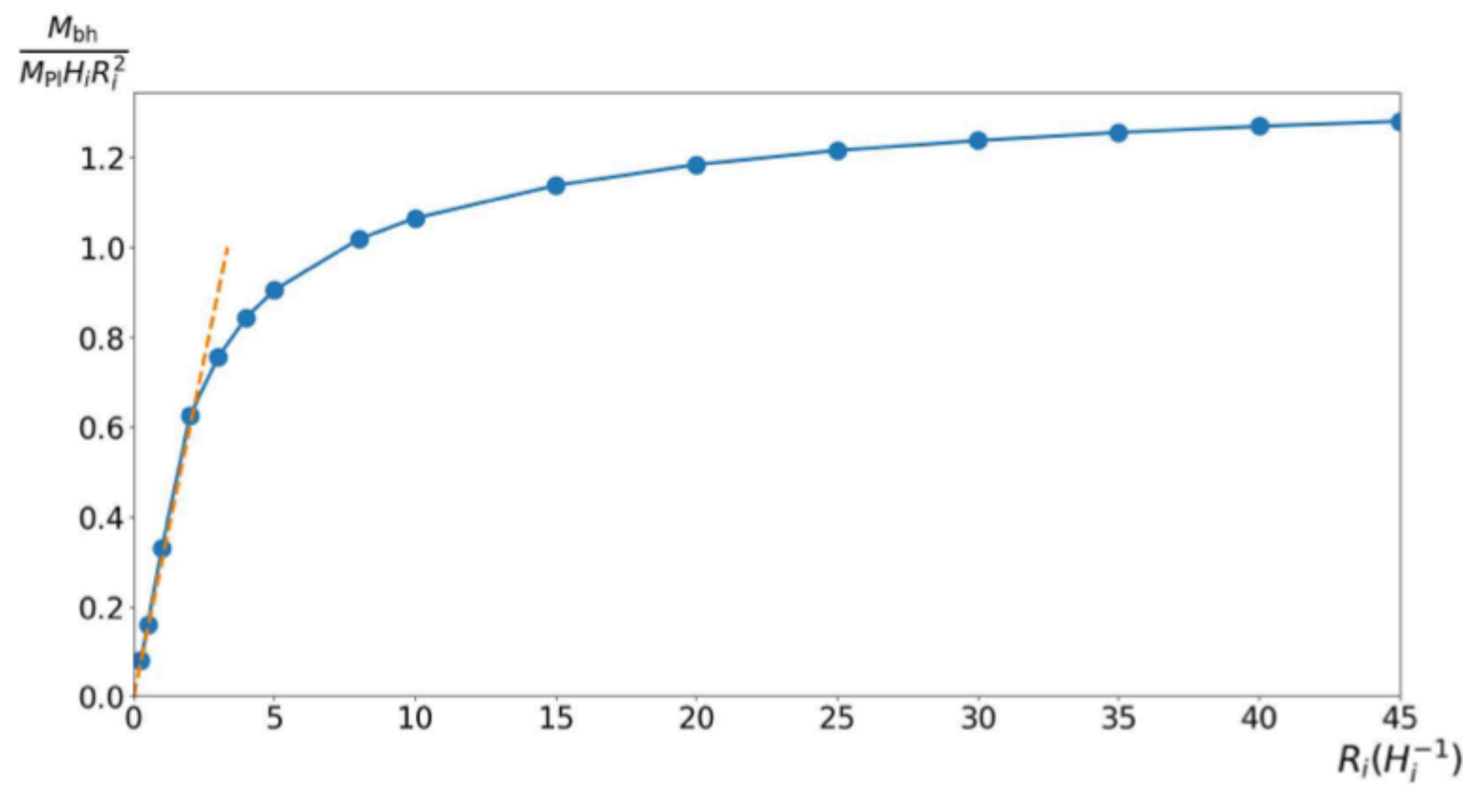


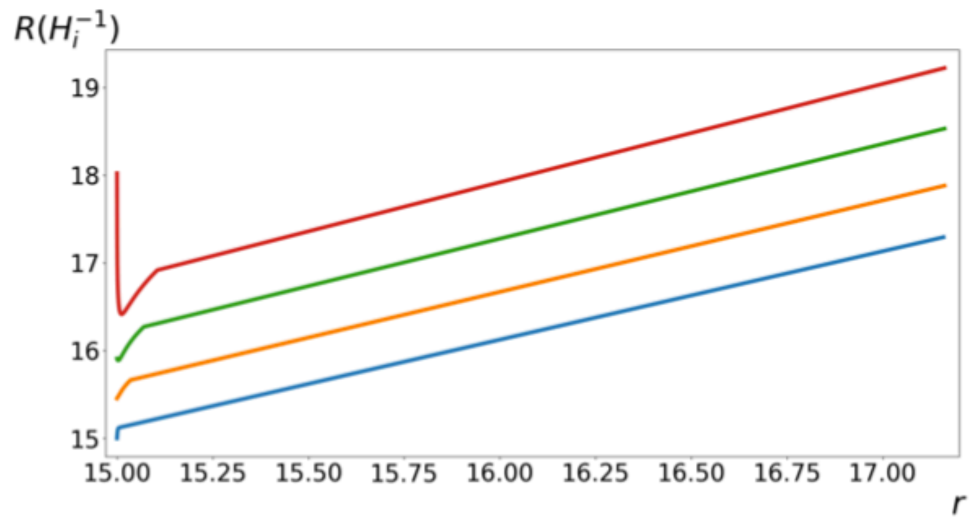


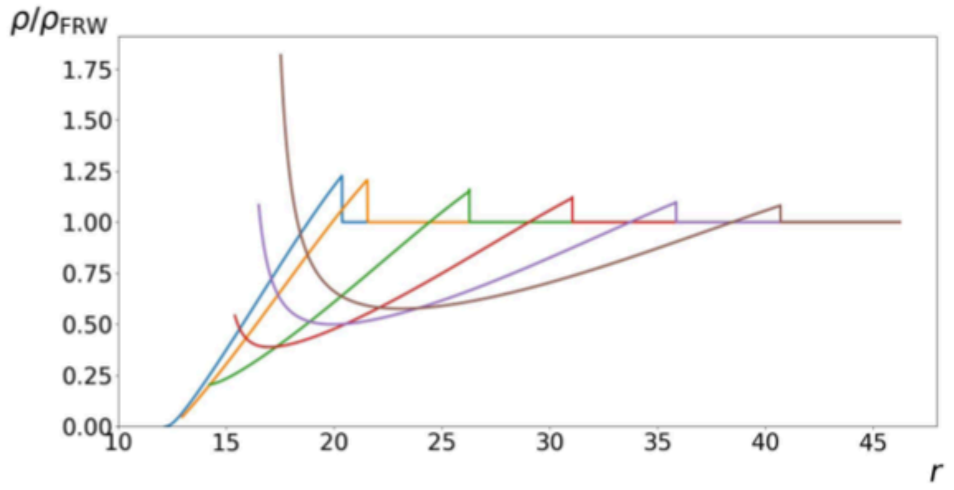
[Garriga, Vilenkin, JZ, JCAP (2016)]

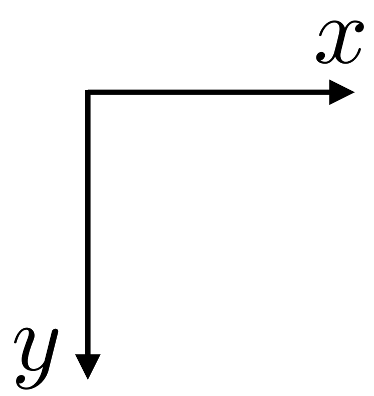
Post-inflation evolution

Strong interaction with radiation









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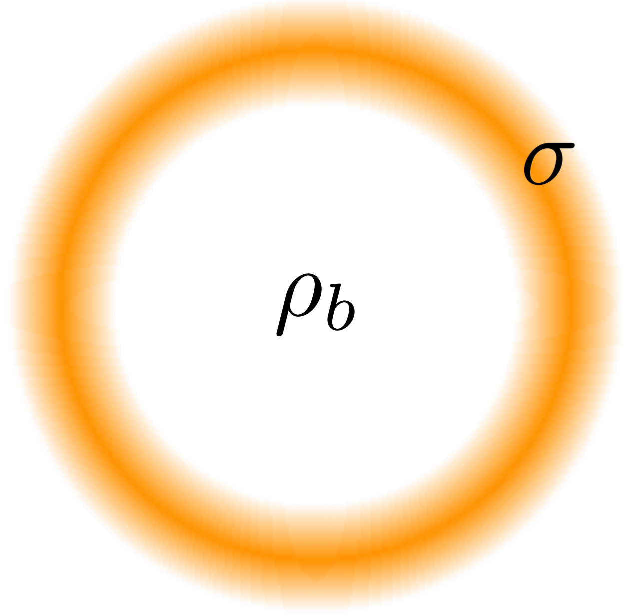

[Deng+, JCAP (2017)]

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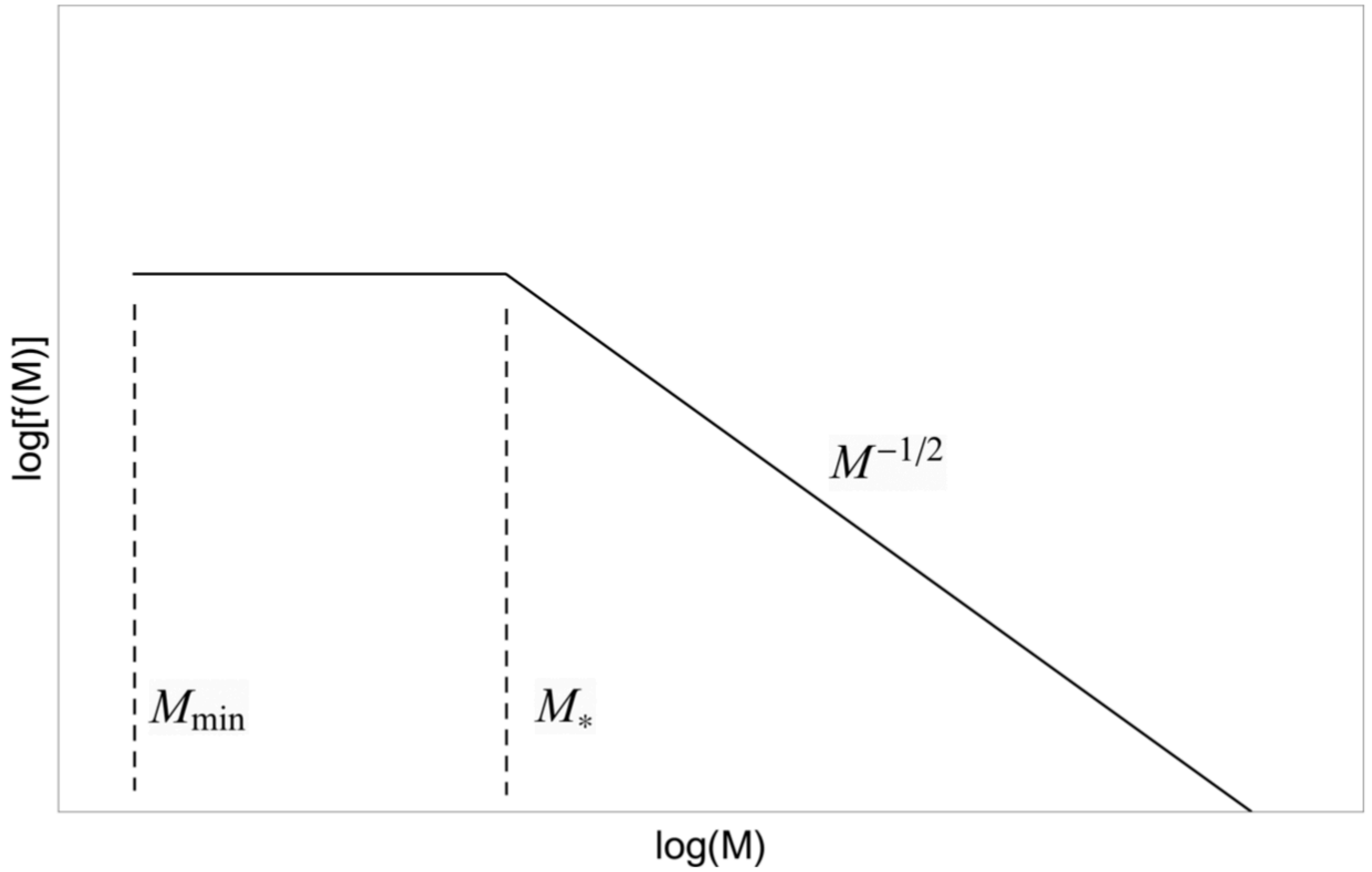
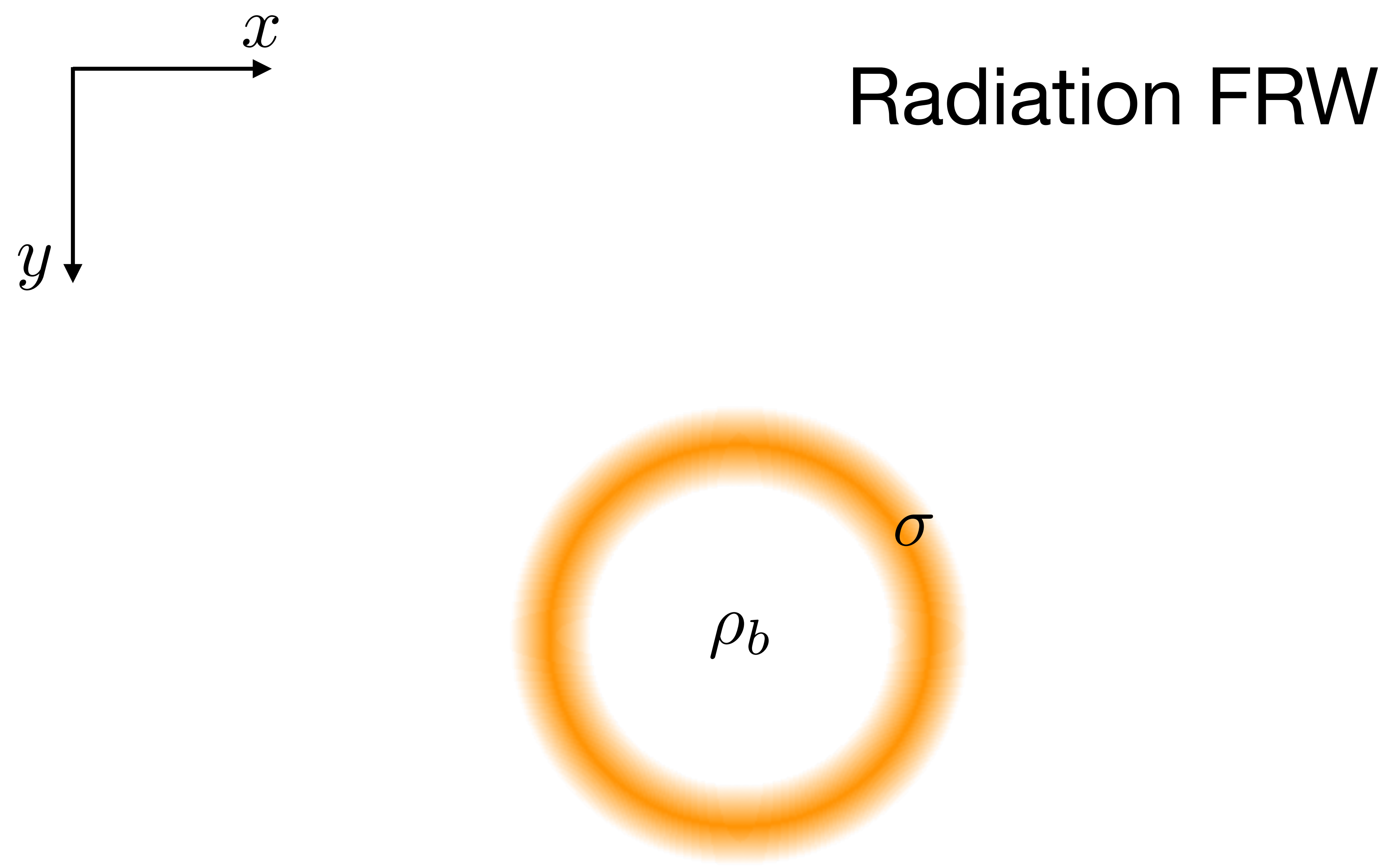


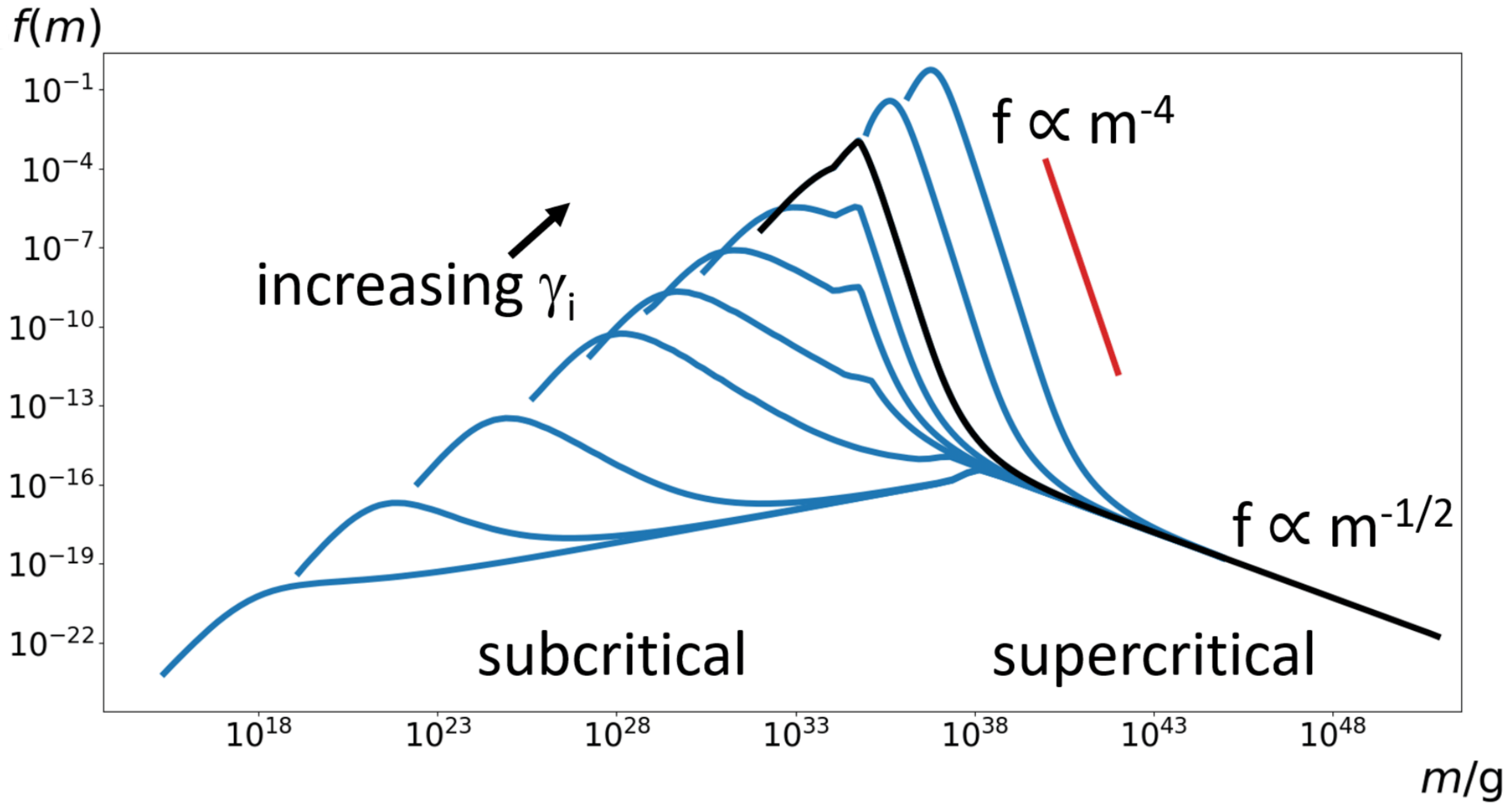
$$\psi_{PBH}(m) \equiv \frac{m}{\rho_{PBH}} \frac{\mathrm{d}n_{PBH}}{\mathrm{d}m} \qquad f(m) \equiv m f_{PBH} \psi_{PBH}(m)$$



[Deng+, JCAP (2017)]

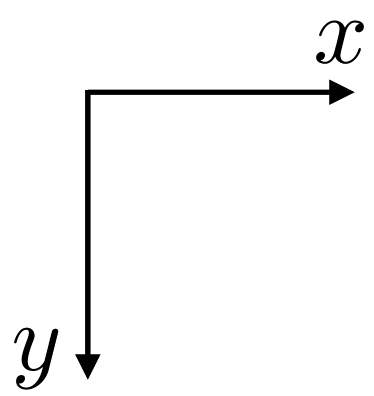
Post-inflation evolution

No interaction with radiation



$$\psi_{\rm PBH}(m) \equiv \frac{m}{\rho_{\rm PBH}} \frac{{\rm d}n_{\rm PBH}}{{\rm d}m}$$

$$f(m) \equiv m f_{\rm PBH} \psi_{\rm PBH}(m)$$



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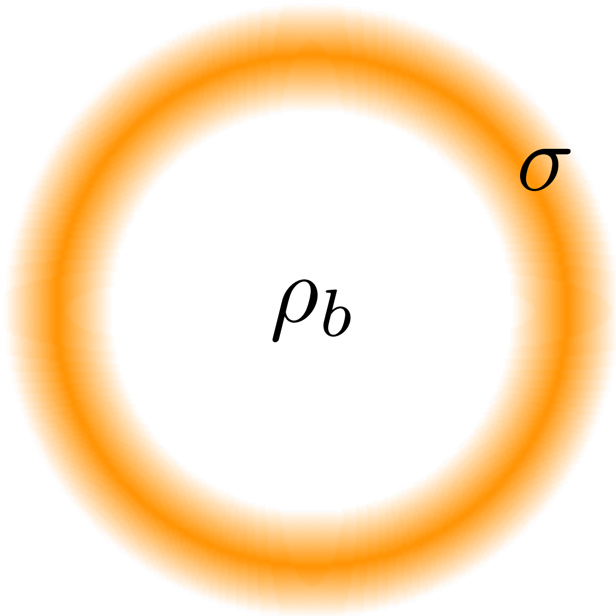


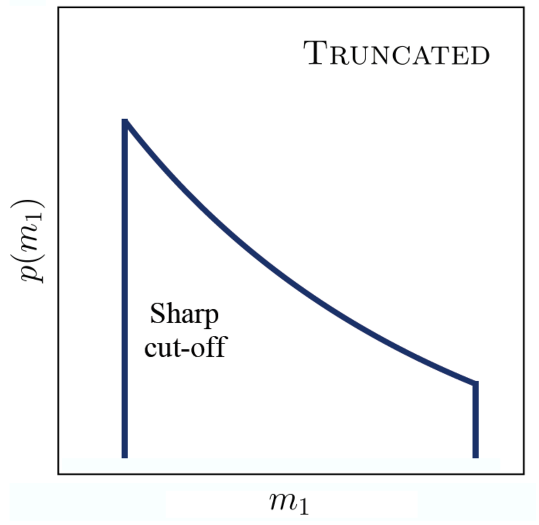
[Deng, JCAP (2020)]

Implications from GWTC-3

Implications from GWTC-3

Mass distribution

[LVC, Astrophys. J. Lett. (2021)]



$$\psi_{\rm PBH}(m|m_*,\alpha_1,\alpha_2)=\frac{1}{m_*\left(\alpha_1^{-1}-\alpha_2^{-1}\right)}\begin{cases}(m/m_*)^{\alpha_1-1}, & m<m_*\\ (m/m_*)^{\alpha_2-1}, & m>m_*\end{cases}$$

Posterior $\mathcal{L}(\Lambda|\mathbf{d})$

Hierarchical Bayesian Inference

Intrinsic parameters

Hyperparameters

$$\mathcal{L}(\boldsymbol{d}|\Lambda)\propto e^{-N(\Lambda)\xi(\Lambda)}[N(\Lambda)]^{N_{\rm det}}\prod_{i=1}^{N_{\rm det}}\int\mathcal{L}(d_i|\theta)\,\pi(\theta|\Lambda)\,d\theta$$

Detect fraction

$$\xi(\Lambda)=\int p_{\rm det}(\theta)\,\pi(\theta|\Lambda)\mathrm{d}\theta$$

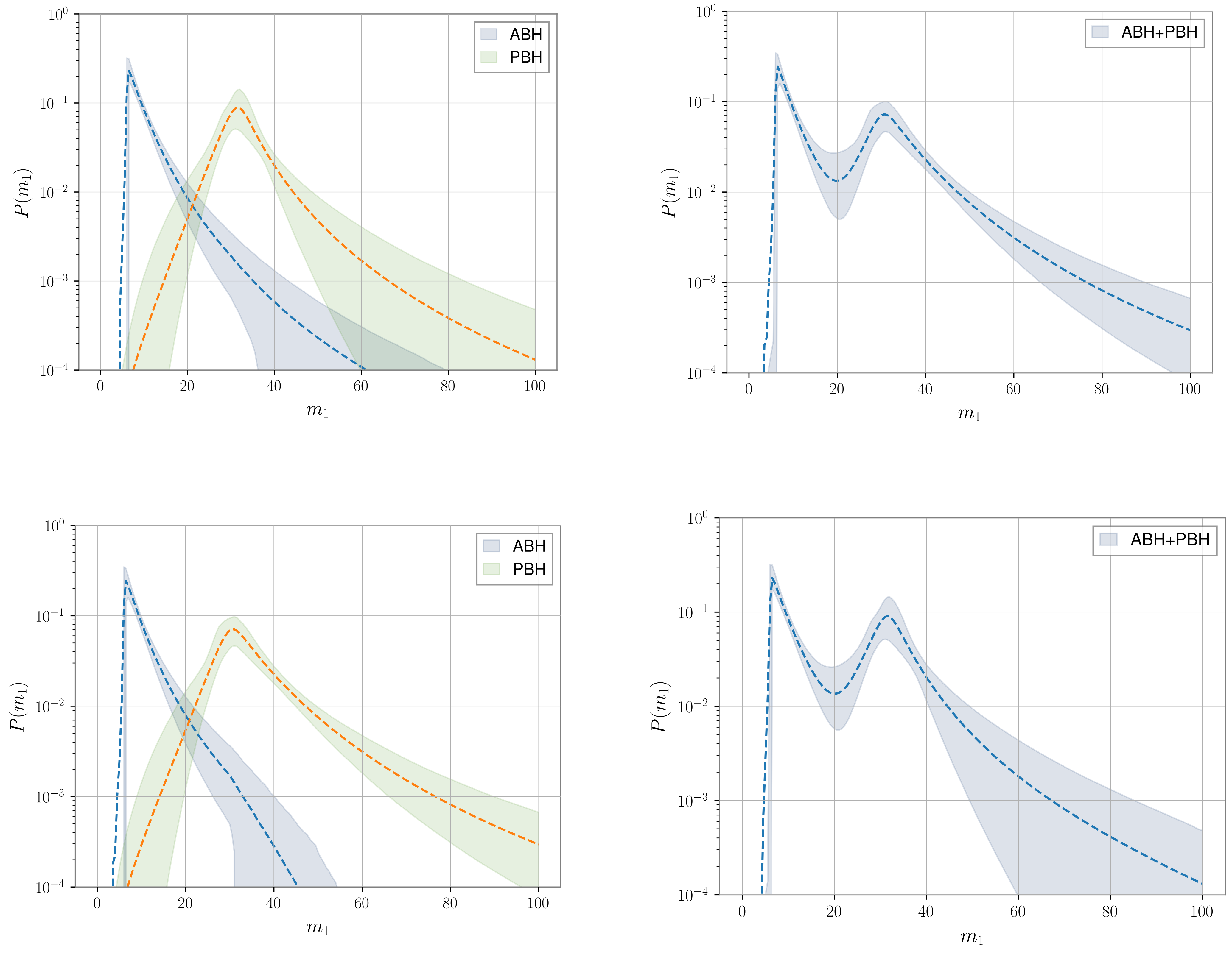
Merger rate

$$\frac{\mathrm{d}R_{\rm PBH}}{\mathrm{d}m_1\mathrm{d}m_2}=\frac{1.6\times10^6}{\rm Gpc^3\,yr}f_{\rm PBH}^{\frac{53}{37}}\eta^{-\frac{34}{37}}\left(\frac{M}{M_\odot}\right)^{-\frac{32}{37}}\left(\frac{t}{t_0}\right)^{-\frac{34}{37}}S(M,f_{\rm PBH},\psi_{\rm PBH})\,\psi_{\rm PBH}(m_1)\,\psi_{\rm PBH}(m_2)$$

[He, Deng, Piao, JZ (2023)]

Implications from GWTC-3

Posterior Mass distribution



Parameter	Prior	ABH-PBH	ABH-PBH	ABH	ABH	Description
						BROKEN POWER LAW PBH
$M_*/M_\odot$	$[5, 50]$	$31.43^{+1.44}_{-1.56}$	$30.54^{+1.37}_{-1.56}$			The critical mass
$\log_{10} f_{\rm PBH}$	$[-4, 0]$	$-2.99^{+0.07}_{-0.07}$	$-2.95^{+0.05}_{-0.05}$			Logarithmic fraction of PBHs in dark matter at the time of formation
α_1	$[0, 10]$	$8.80^{+2.15}_{-2.79}$	$8.52^{+2.36}_{-2.82}$			Spectral index of subcritical PBHs
α_2	$[0, -10]$	$-5.65^{+1.94}_{-3.81}$	$-3.91^{+0.73}_{-0.78}$			Spectral index of supercritical PBHs
						TRUNCATED ABH
$\bar{R}_{\rm ABH}/{\rm Gpc}^{-3}{\rm yr}^{-1}$	$[0, 50]$	$23.80^{+6.20}_{-5.19}$	$24.30^{+6.71}_{-5.62}$	$26.39^{+6.10}_{-4.88}$		Integrated merger rate of ABHs at $z = 0$
β	$[-4, 30]$	$5.78^{+3.26}_{-2.83}$	$5.21^{+3.65}_{-3.23}$	$10.21^{+4.74}_{-3.27}$		Exponent of the symmetric mass ratio factor
ζ	$[0, 3]$	$1.60^{+0.37}_{-0.22}$	$1.78^{+0.37}_{-0.34}$	$1.10^{+0.10}_{-0.11}$		ABH mass distribution power law scaling
$m_{\rm min}/M_\odot$	$[2, 10]$	$6.02^{+0.29}_{-0.27}$	$6.03^{+0.31}_{-0.26}$	$5.94^{+0.30}_{-0.32}$		Minimum mass of the primary mass distribution
$m_{\rm max}/M_\odot$	$[30, 100]$	$85.28^{+10.6}_{-36.1}$		$75.45^{+9.81}_{-5.18}$		Maximum mass of the primary mass distribution
	$[30, 60]$		$41.69^{+11.1}_{-8.07}$			
						BROKEN POWER LAW ABH
$\bar{R}_{\rm ABH}/{\rm Gpc}^{-3}{\rm yr}^{-1}$	$[0, 50]$				$23.00^{+6.36}_{-4.72}$	Integrated merger rate of ABHs at $z = 0$
β_1	$[-4, 12]$				$2.28^{+0.42}_{-0.39}$	Power-law slope of the primary mass distribution for masses below $m_{\rm break}$
β_2	$[-4, 12]$				$6.73^{+2.08}_{-1.43}$	Power-law slope of the primary mass distribution for masses above $m_{\rm break}$
β_q	$[-4, 12]$				$0.83^{+0.94}_{-0.72}$	Spectral index for the power-law of the mass ratio distribution
$m_{\rm min}/M_\odot$	$[2, 10]$				$5.24^{+0.67}_{-1.39}$	Minimum mass of the primary mass distribution.
$m_{\rm max}/M_\odot$	$[30, 100]$				$86.73^{+8.67}_{-9.79}$	Maximum mass of the primary mass distribution.
b	$[0, 1]$				$0.43^{+0.09}_{-0.07}$	The fraction of the way between $m_{\rm min}$ and $m_{\rm max}$ at which the primary mass distribution breaks
$\delta_m/M_\odot$	$[0, 10]$				$4.87^{+3.02}_{-3.11}$	Range of mass tapering on the lower end of the mass distribution

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[He, Deng, Piao, JZ (2023)]

Implications from GWTC-3

- $M_{\rm Pl}^3/\eta_{\rm b}^2 \sim 30 M_\odot \rightarrow \eta_{\rm b} \sim 0.1\ {\rm GeV}$
- No subcritical bubble

$$m_{\rm F} \sim \eta_{\rm b}^{-2}\left(\frac{\eta_i^4 M_{\rm Pl}}{\eta_\sigma^3}\right)^{3/2} \gtrsim m_* \quad \rightarrow \eta_i^4 > \eta_\sigma^3 M_{\rm Pl}$$

$$\eta_i > 10^4\ {\rm GeV}$$

Parameter	Prior	ABH-PBH	ABH-PBH	ABH	ABH	Description
						Broken Power Law PBH
$M_*/M_\odot$	[5, 50]	$31.43^{+1.44}_{-1.56}$	$30.54^{+1.37}_{-1.56}$			The critical mass
$\log_{10} f_{\rm PBH}$	[−4, 0]	$-2.99^{+0.07}_{-0.07}$	$-2.95^{+0.05}_{-0.05}$			Logarithmic fraction of PBHs in dark matter at the time of formation
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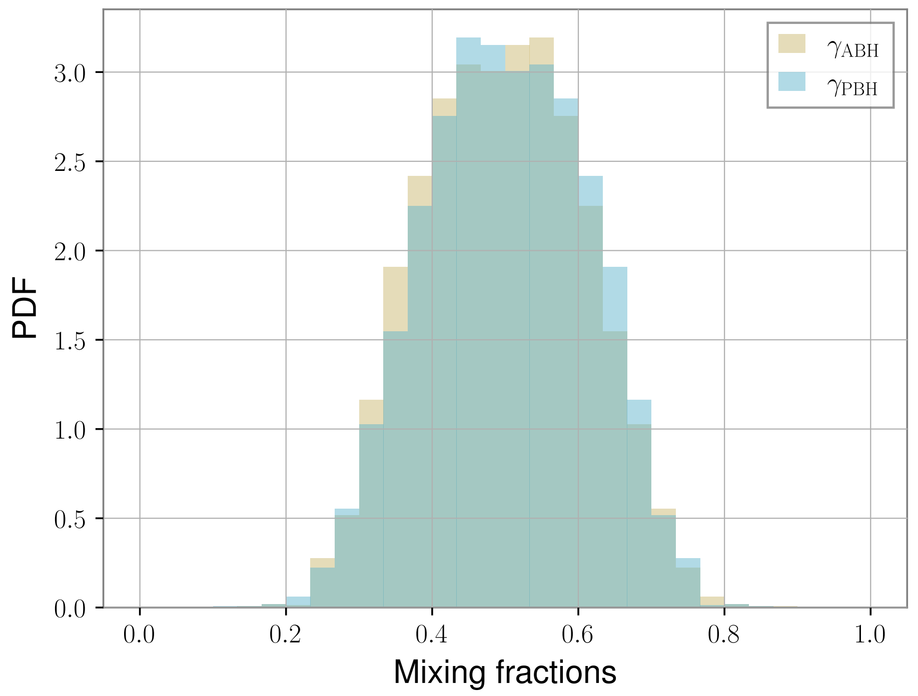
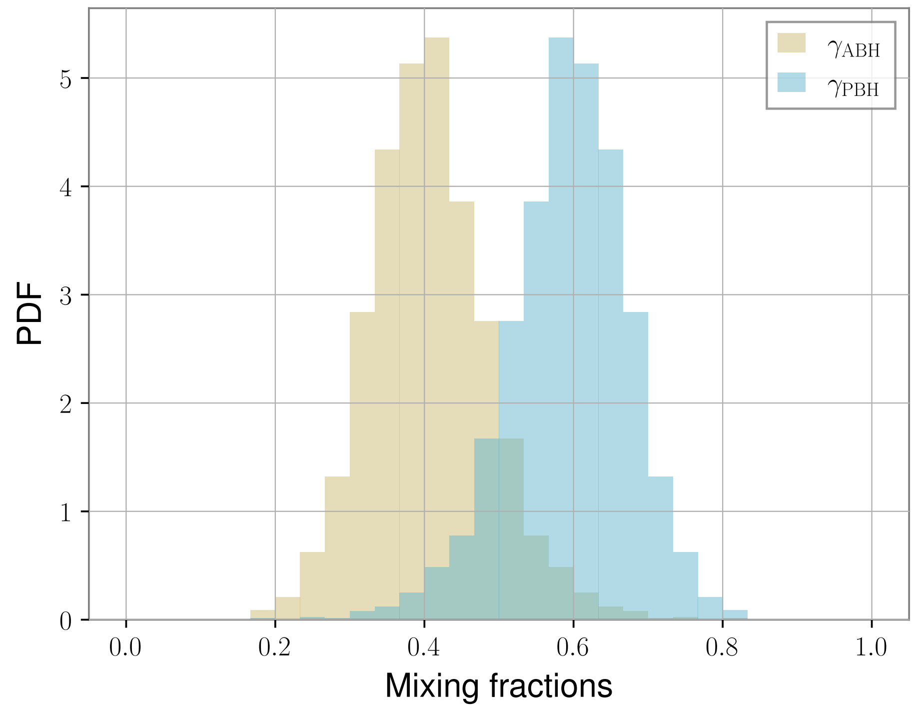
Implications from GWTC-3

Posterior Mass distribution

$$\gamma_{\rm PBH} \equiv N^{\rm det}_{\rm PBH}/(N^{\rm det}_{\rm ABH} + N^{\rm det}_{\rm PBH})\,,$$
$$\gamma_{\rm ABH} \equiv 1 - \gamma_{\rm PBH}.$$





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  [in the next version]

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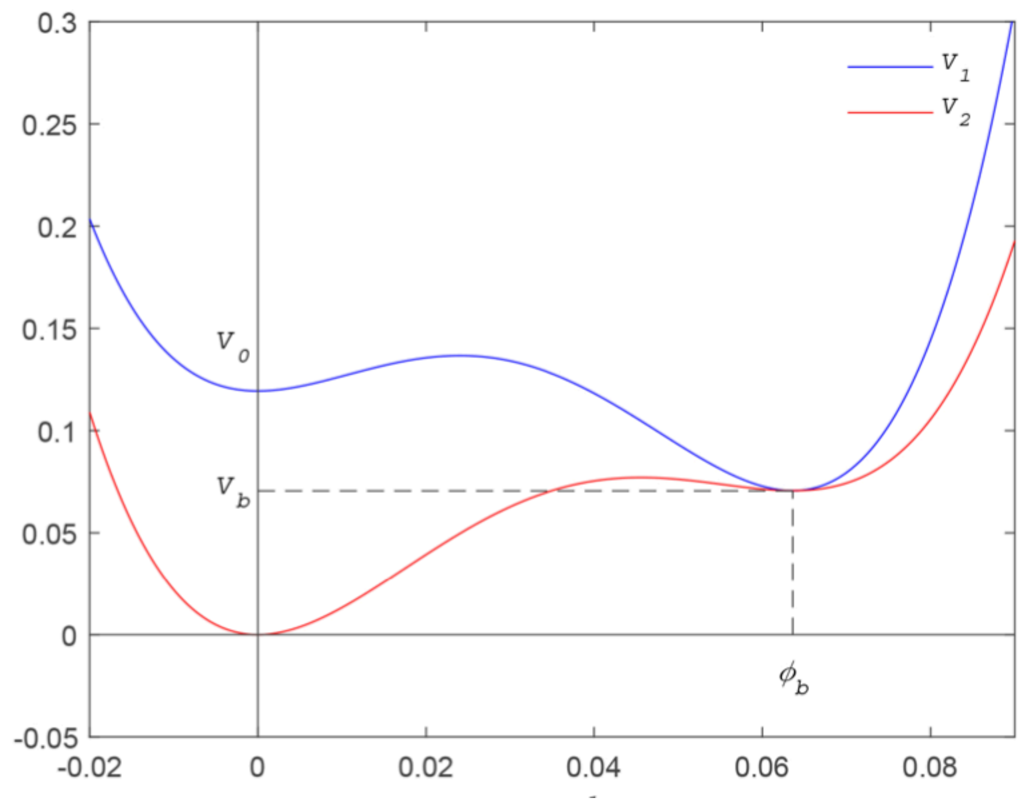

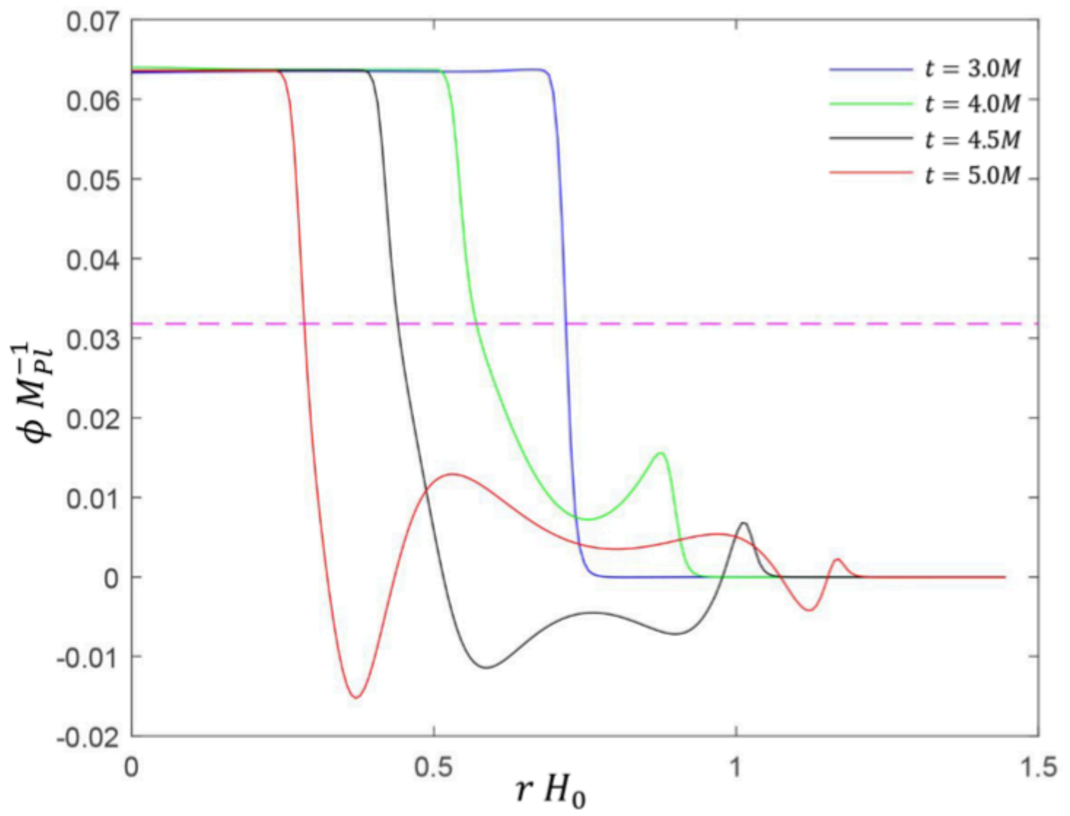

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- Possibility of seeding supermassive black holes

Thanks!