Universality of Phase Transition Dynamics and Holography

曾化碧 扬州大学

With Chuan Yin Xia, Adolfo del Campo, PRL 130, 060402 (2023); e-Print: 2302.11597 [hep-th]

Chuan Yin Xia, PRD 102, 126005 (2020) ;arXiv:2110.07969 [cond-mat.stat-mech]

Hai Qing Zhang, Chuan Yin Xia, JHEP 03 (2021) 136

Peng Huanwu Center for Fundamental Theory USTC April 8, 2023

- * Topological defects formation in phase transition dynamics: Kibble-Zurek mechanism(KZM)
- * Holographic verification of KZM in various dimensions
- * Experiments beyond KZM: fast quench and finite size
- New universal scaling law beyond KZM and numerical verification
- * Summary and outlooks

	Exponent	Definition	Conditions
Specific heat	α	$C \propto t ^{-\alpha}$	$t \rightarrow 0, B \equiv 0$
Order parameter	β	$m \propto (-t)^{\beta}$	$t \to 0$ from below, $B = 0$
Susceptibility	Y	$\chi \propto r ^{-\gamma}$	$t \rightarrow 0, B \equiv 0$
Critical isotherm	8	$B \propto m ^5 \operatorname{sign}(m)$	$B \rightarrow 0, t \equiv 0$
Correlation length	37	$\xi \propto t ^{-v}$	$t \rightarrow 0, B = 0$
Correlation function	η	$G(r) \propto r ^{-d+2-\eta}$	$t \equiv 0, B \equiv 0$
Dynamic	Z	$\tau_c \propto \xi^z$	$t \rightarrow 0, B = 0$

Universality of continuous phase transition

Critical Exponents

Reduced Temperature, t

$$t \equiv \frac{T - T_C}{T_C}$$

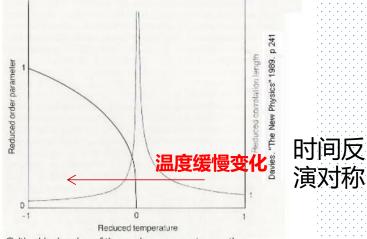
Specific heat $\left|C \propto \left|t\right|^{-\alpha}\right|$

Magnetization $M \propto |t|^{\beta}$

Magnetic susceptibility $~\chi \propto \left|t\right|^{-\gamma}$ Correlation length $~\xi \propto \left|t\right|^{-\nu}$

$$\alpha = 0$$
 (log divergence)

$$\beta = \frac{1}{8} \quad \gamma = \frac{7}{4} \quad \nu = 1$$



Critical behavior of the order parameter an the correlation length. The order parameter vanishes with the power β of the reduced temperature t as the critical point is approached along the line of phase coexistance. The correlation length diverges with the power v of the reduced temperature.

T 600 - T 5 6 7 8 T

发散的弛豫时间

The exponents display critical point universality (don't depend on details of the model). This explains the success of the Ising model in providing a quantitative description of real magnets.

Topology of cosmic domains and strings

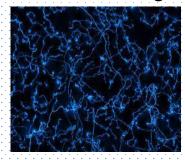
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Journal of Physics A: Mathematical and General, Volume 9, Number 8

Citation T W B Kibble 1976 J. Phys. A: Math. Gen. 9 1387

DOI 10.1088/0305-4470/9/8/029

Cosmic string



Cosmic string

Nature 317, 505–508 (1985) Cite this article

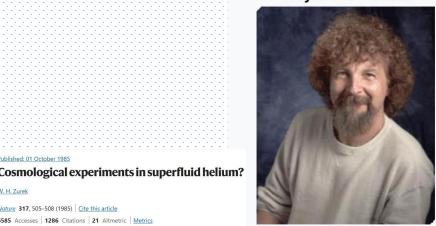
Kibble-Zurek mechanism

Sir **Tom Kibble** CBE FRS MAE



Thomas Walter Bannerman Born Kibble 23 December 1932

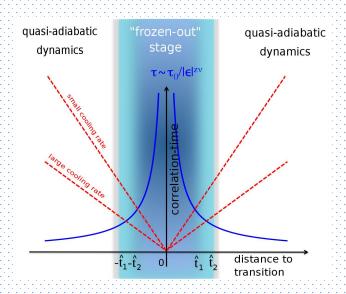
Wojciech H. Zurek

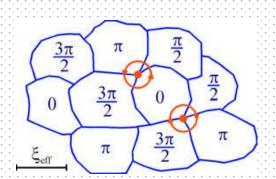


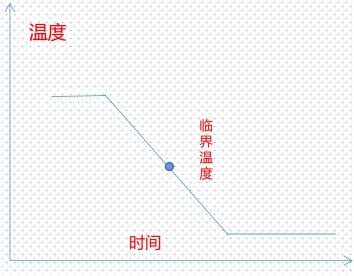
Born

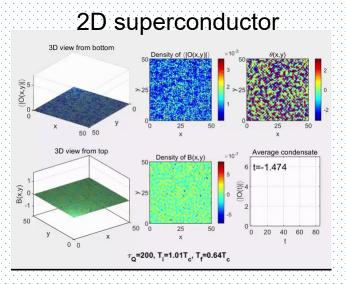
Wojciech H. Żurek 1951 (age 70-71) Bielsko-Biała, Poland

critical slowing down









Inhomogeneous final state with defects inducde by quench

no time reverse symmetry

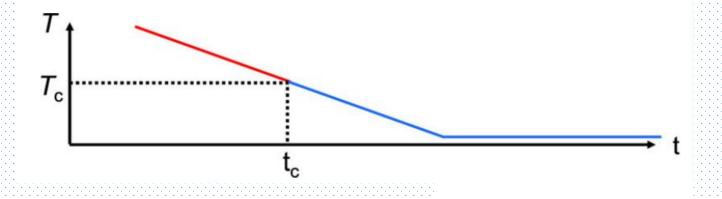
Kibble-Zurek mechanism





Born

Wojciech H. Żurek 1951 (age 70–71) Bielsko-Biała, Poland



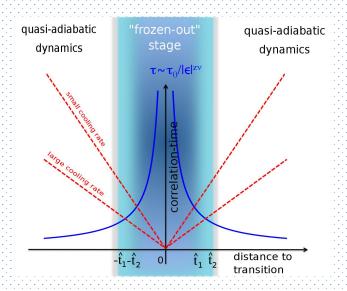
$$T(t) = T_c(1 - \frac{t}{\tau_Q})$$

$$n \sim \frac{L^d}{\xi(\hat{t})^d} \propto \tau_Q^{-\frac{d\nu}{1+z\nu}}$$

Zurek 把拓扑缺陷密度与临界指数 (一个是时间特征尺度,一个是空间特征尺度) 关联起来,尽管看起来过程具有非线性与空间非均匀性质,但Zurek找到了一种普适规律。

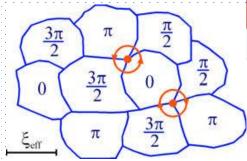
Derivation of Kibble-Zurek scaling

系统何时分块?以什么特征尺度进行分块?



$$T(t) = T_c(1 - \frac{t}{\tau_Q}) \qquad \tau(T) \sim \tau_0(T_c - T)^{-\nu z}$$





arXiv > cond-mat > arXiv:1310.1600

Condensed Matter > Statistical Mechanics

[Submitted on 6 Oct 2013 (v1), last revised 13 Nov 2013 (this version, v3)]

Universality of Phase Transition Dynamics: Topological Defects from Symmet

Adolfo del Campo, Wojciech H. Zurek

In the course of a non-equilibrium continuous phase transition, the dynamics ceases to be adiabatic in the vicinity of the critical point as a reservation time in the neighborhood of the critical point). This enforces a local choice of the broken symmetry and can lead to the formation of

Zurek's verification based on the time dependent Gindzburg-landau equation

1D

OLUME 78, NUMBER 13

PHYSICAL REVIEW LETTERS

31 March 1997

Density of Kinks after a Quench: When Symmetry Breaks, How Big are the Pieces?

Pablo Laguna^{1,2} and Wojciech Hubert Zurek¹

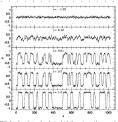
¹Theoretical Astrophysics, MS-B288, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

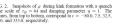
²Department of Astronomy & Astrophysics and CGPG, Penn State University, University Park, Pennsylvania 16802

(Received 19 July 1996)

Numerical study of order parameter evolution in the course of symmetry breaking transitions with Landau-Ginzburg-like dynamics shows that the density of topological defects, kinks which form during the quench, is proportional to the fourth root of its rate. This is a limited (1D) test of the more general theory of domain-size evolution in the course of symmetry breaking transformations proposed by one of us. Using these ideas, it is possible to compute the density of topological defects from the quench time scale and from the equilibrium scaling of the correlation length and relaxation time near the critical point. [S0031-9007(97)02876-7]

PACS numbers: 05.70.Ln, 05.70.Fh, 11.15.Ex, 67.40.Vs





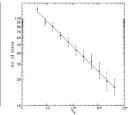
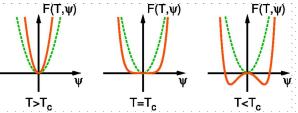


FIG. 3. Number of defects as a function of quench time scale. The plot is obtained at $t\tau_D = 4$ (see Fig. 4). The straight line is the best fit to $N = N_D s^2$ with $a = 0.28 \pm 0.02$ and $N_D = 178 \pm 14$ ($\chi^2 = 1.96$). This exponent composition favorably with the theoretical prediction of 1/4 based on the theory in Ref. 1999.



To investigate this issue, we considered the numerical evolution of a 1D system for a real field φ according to the equation of motion derived from the Landau-Ginzburg potential $V(\varphi) = (\varphi^4 - 2\epsilon \varphi^2 + 1)/8$. The system is in contact with a thermal reservoir. Thus, it obeys the Langevin equation,

$$\ddot{\varphi} + \eta \dot{\varphi} - \partial_{xx} \varphi + \partial_{\varphi} V(\varphi) = \vartheta. \tag{5}$$

t dependent

温度是唯象的参数

Volume 80, Number 25

PHYSICAL REVIEW LETTERS

22 JUNE 1998

Vortex Formation in Two Dimensions: When Symmetry Breaks, How Big Are the Pieces?

Andrew Yates 1,2 and Wojciech H. Zurek1

¹Theoretical Astrophysics T-6, MS B-288, Los Alamos National Laboratory, Los Alamos, New Mexico 87455 ²Département de Physique Théorique, Université de Genève, 24, quai E. Ansermet, CH-1211 Genève 4, Switzerland (Received 12 January 1998)

We investigate the dynamics of second-order phase transitions in two dimensions, breaking a gauged U(1) symmetry. Using numerical simulations, we show that the density of the topological defects formed scales with the quench time scale τ_Q as $n \sim \tau_Q^{-1/2}$ when the dynamics is overdamped at the instant when the freeze-out of thermal fluctuations takes place, and $n \sim \tau_Q^{-2/3}$ in the underdamped case. This is predicted by the scenario proposed by one of us (W. H. Z.). [S0031-9007(98)06423-0]

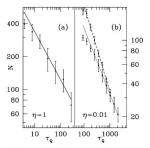


FIG. 2. The variation of defect count N with τ_0 in both (a) overdamped and (b) underdamped regimes. The fitted slopes are (a) -0.44 ± 0.10 ($\chi^2=0.44$, dropping the two leftmost points) and (b) -0.60 ± 0.07 ($\chi^2=0.034$, dropping three points). Predicted values were -1/2 and -2/3. The points fitted by the dashed line are inferred from the fits in Fig. 3. The slope is -0.79 ± 0.04 ($\chi^2=0.27$).

Zurek's verification based on the time dependent Gindzburg-landau equation

3D:

VOLUME 82, NUMBER 14

PHYSICAL REVIEW LETTERS

5 F

Vortex String Formation in a 3D U(1) Temperature Quench

Nuno D. Antunes, ¹ Luís M. A. Bettencourt, ² and Wojciech H. Zurek ²

¹Département de Physique Théorique, Université de Genève, 24 quai E. Ansermet,

CH 1211, Genève 4, Switzerland

²Theoretical Division MS B288, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

(Received 23 November 1998)

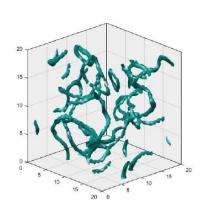


FIG. 1. Vortex string formation under a temperature in AdS₅

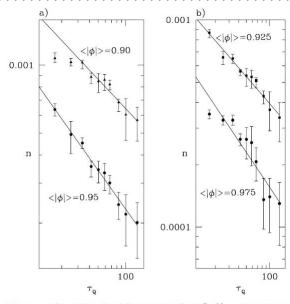


FIG. 3. The string densities measured at $\langle |\phi \rangle| = 0.9, 0.925, 0.95, 0.975$. The fits are to Eq. (8), for $\tau_Q \geq 32$, with $\alpha = 0.4296 \pm 0.043, 0.4378 \pm 0.0289, 0.5692 \pm 0.0256, 0.5600 \pm 0.0797, and <math>f_{\dot{\phi}} = 11.11, 13.99, 12.29, 15.34,$ respectively. The average $\overline{\alpha} = 0.4982 \pm 0.079$ and $\overline{f}_{\dot{\phi}} = 13.18 \pm 1.78$.

Experimental verification

Quantum Simulation of Landau-Zener Model Dynamics Supporting the Kibble-Zurek Mechanism

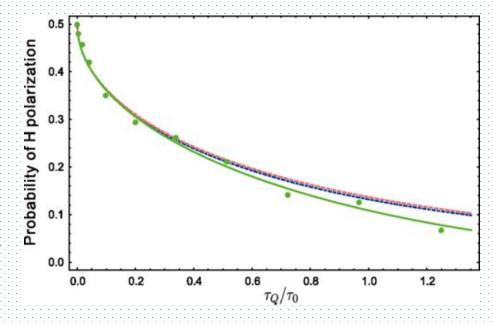
Xiao-Ye Xu, Yong-Jian Han, Kai Sun, Jin-Shi Xu, Jian-Shun Tang, Chuan-Feng Li, and Guang-Can Guo Phys. Rev. Lett. **112**, 035701 – Published 23 January 2014

☑ 中国科大研制出光学量子模拟器用以研究相变机制



中国科学技术大学郭光灿院士领导的中国科学院量子信息重点实验室李传锋教授研究组成功研制出光学量子模拟器,并首次在纯量子模型中证实描述相变过程的Kibble-Zurek(KZ)理论的绝热—脉冲近似成立。此成果发表在1月24日的《物理评论快报》上。

量子计算机拥有经典计算机所无法比拟的巨大优势。受技术所限制,人们尚无法实现通用的普适量子计算机。然而随着量子信息技术的飞速发展,目前人们已经可以利用一些量子系统实现某些特殊的量子计算功能。这类实现特殊任务的量子计算机就是量子模拟器。该研究组研制的量子模拟器就是面向量子相变中的KZ机制这一特殊问题。



nature

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nature > letters > article

Published: 25 July 1996

Laboratory simulation of cosmic string formation in the early Universe using superfluid ³He

C. Bäuerle, Yu. M. Bunkov, S. N. Fisher, H. Godfrin & G. R. Pickett

Nature **382**, 332–334 (1996) Cite this article

Furthermore, the prediction $\beta/\xi_0 = (\tau_Q/\tau_0)^{1/4}$ from the Zurek model is only qualitative.

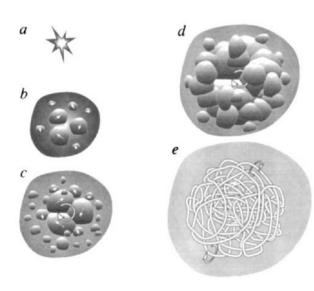


FIG. 1 Schematic view of the creation of a tangle of linear singularities by the nucleation of independent ordered regions as a system passes through a phase transition. In the superfluid ^3He context; at a,a neutron has struck a ^3He nucleus initiating the process n + $^3\text{He} \rightarrow$ $^3\text{H} + p$, liberating 764 keV of energy and creating a small region of very high temperature in the liquid. At a later time b the hot region has expanded and cooled to near the transition temperature. Small regions of superfluid are independently nucleated each having a different value of the order parameter, as shown by the small arrows. At c the three central ordered regions now touch. Although the order parameter may bend to accommodate the boundaries, a full 2π change around the triple contact point remains. This is a vortex. At d, many more regions have been nucleated and overlap, and along the grain boundaries a whole tangle of vortex lines is created. Finally, at e the hot bubble has cooled entirely through the transition and only the tangle of vortices remains.

nature physics

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Letter Published: 16 September 2019

Kibble–Zurek universality in a strongly interacting Fermi superfluid

Bumsuk Ko, Jee Woo Park ≥ & Y. Shin

Nature Physics 15, 1227–1231 (2019) Cite this article

3244 Accesses 21 Citations 2 Altmetric Metrics

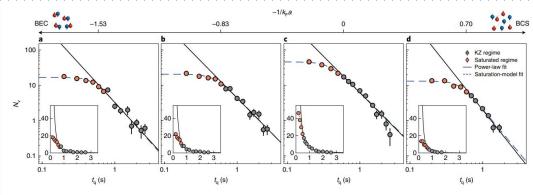
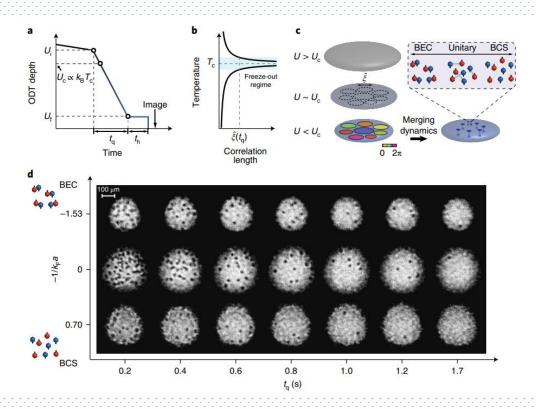


Fig. 2 | Vortex number versus quench time. The average number of detected vortices are plotted as a function of t_q on log-log axes at four different final interaction strengths $-1/k_t a = -1.53$ (757 G) (a), -0.83 (785 G) (b), 0 (832 G) (c) and 0.70 (898 G) (d). The insets show the same data on linear-linear axes. Each data point comprises at least ten realizations of the same experiment, and the error bars are the standard error of the mean. When the error bars are not visible, they are smaller than the marker size.



Observation of Magnetic Flux Generated Spontaneously During a Rapid Quench of Superconducting Films

A. Maniv, E. Polturak, and G. Koren Phys. Rev. Lett. **91**, 197001 – Published 3 November 2003

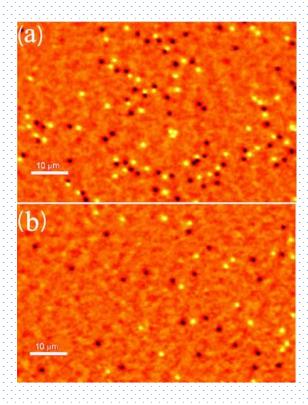


Figure 1 Typical images of spontaneously created vortices in a superconductor cooled at (a) $2\times 10^9~{\rm K/s}$ and (b) $4\times 10^8~{\rm K/s}$. The intensity is proportional to the local magnetic field. Bright and dark spots represent vortices with opposite polarity.

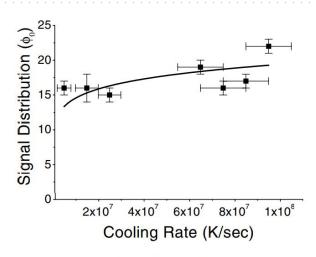


FIG. 4. Dependence of the distribution of spontaneous flux versus the cooling rate. The vertical error bars are those of the fit to a Gaussian distribution. The solid line is the prediction of Refs. [2,17] scaled to fit the data.

Critical exponents of holographic superfluid/superconductor: match G-L theory

Building a Holographic Superconductor

Sean A. Hartnoll (Santa Barbara, KITP), Christopher P. Herzog (Princeton U.), Gary T. Horowitz (UC, Santa Barbara)

Phys.Rev.Lett. 101 (2008) 031601 • e-Print: 0803.3295 • DOI: 10.1103/PhysRevLett.101.031601

Universality class of holographic superconductors

Kengo Maeda, Makoto Natsuume, and Takashi Okamura Phys. Rev. D **79**, 126004 – Published 11 June 2009 Invited Reviews | Published: 09 May 2015

Introduction to holographic superconductor models

RongGen Cai [™], Li Li, LiFang Li & RunQiu Yang

Science China Physics, Mechanics & Astronomy 58, 1–46 (2015) Cite this article

535 Accesses **171** Citations **1** Altmetric Metrics

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Correlation length	ν	ξ ∝ t -*	$t \rightarrow 0, B = 0$
Correlation function	η	$G(r) \propto r ^{-d+2-\eta}$	t = 0, B = 0
Dynamic	Z	$\tau_c \propto \xi^z$	$t \rightarrow 0, B = 0$

$$(\alpha, \beta, \gamma, \delta, \nu, \eta) = (0, \frac{1}{2}, 1, 3, \frac{1}{2}, 0).$$

Dynamical critical exponent z = 2

$$F=rac{lpha}{2}(T-T_c)\phi^2+rac{u}{4!}\phi^4$$
 (Landau-Ginzburg theory),

Near T_c, the effective theory of superconductor is a G-L theory

Published: 02 May 2011

Analytical computation of critical exponents in several holographic superconductors

Hua-Bi Zeng [™], Xin Gao, Yu Jiang & Hong-Shi Zong

Journal of High Energy Physics 2011, Article number: 2 (2011) Cite this article

Ginzburg-Landau theory of a holographic superconductor $F=rac{lpha}{2}(T-T_c)\phi^2+rac{u}{4!}\phi^4$ (Landau-Ginzburg theory),

Lei Yin, Defu Hou, and Hai-cang Ren Phys. Rev. D **91**, 026003 – Published 8 January 2015

and $\omega(\langle O_{\triangle} \rangle)$ takes the Ginzburg-Landau form

$$\omega(\langle O_{\triangle} \rangle) \approx \text{const.} + a \langle O_{\triangle} \rangle^2 + \frac{1}{2} b \langle O_{\triangle} \rangle^4$$

Semiholographic Quantum Criticality

Kristan Jensen

Phys. Rev. Lett. 107, 231601 – Published 28 November 2011

$$S_{\text{EFT}} = \int dt d^d x (\mathcal{L}_{\text{GL}}[\varphi_i, J_i] + \mathcal{L}_{\text{IR}}[\varphi_i; \mathbf{x}]).$$

More than G-L theory

locally quantum critical strange metal

Holographic superconductor away from T_c

quantum critical BCS

Featured in Physics

Editors' Suggestion

Observing the origin of superconductivity in quantum critical metals

J.-H. She, B. J. Overbosch, Y.-W. Sun, Y. Liu, K. E. Schalm, J. A. Mydosh, and J. Zaanen Phys. Rev. B **84**, 144527 – Published 31 October 2011

Physics See Viewpoint: Susceptible to Pairing

Fermion spectra of the BCS/Bogoliubov results

Open Access | Published: 25 March 2010

Photoemission "experiments" on holographic superconductors

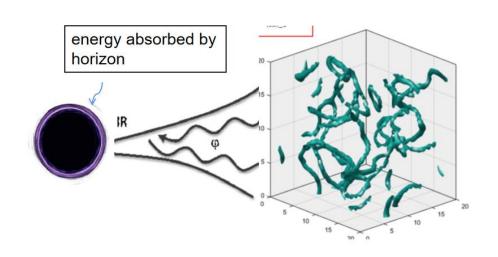
Thomas Faulkner, Gary T. Horowitz, John McGreevy, Matthew M. Roberts [™] & David Vegh

Journal of High Energy Physics 2010, Article number: 121 (2010) Cite this article

KIbble-Zurek机制与AdS/CMT

First principle simulation





Building a Holographic Superconductor

Sean A. Hartnoll (Santa Barbara, KITP), Christopher P. Herzog (Princeton U.), Gary T. Horowitz (UC, Santa Barbara) Phys.Rev.Lett. 101 (2008) 031601 • e-Print: 0803.3295 • DOI: 10.1103/PhysRevLett.101.031601

Einstein-Higgs model in an AdS_{d+1} black hole

The model we used is the usual holographic theory of U(1) symmetry broken, however, in one more spatial dimension, the AdS_5 . the action include a charged scalar and a U(1) gauge field living in a fixed black hole background. The metric of the five dimensional black hole reads

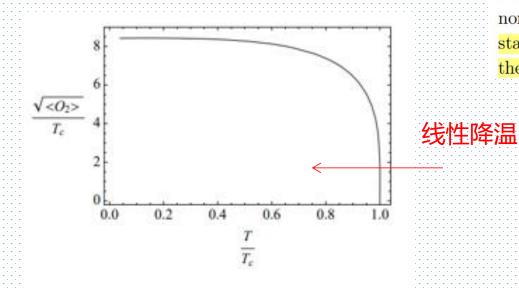
$$ds^{2} = \frac{l^{2}}{u^{2}}(-f(u)dt^{2} - 2dtdu + dx^{2} + dy^{2} + dz^{2}), \quad (1)$$

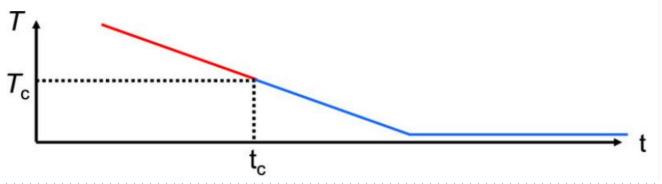
where $f(u) = 1 - (u/u_h)^4$. And the action is

$$S = \int d^5x \sqrt{-g} \left[-\frac{1}{4}F^2 - (|D\Psi|^2 - m^2|\Psi|^2) \right], \quad (2)$$

求解物质场在bulk中的动力学方程得到边界上超导、 超流体的动力学行为

holographic set-up





Before quench, we thermalize the system by adding random seeds into the system in the normal state. The random seeds of the fields are added in the bulk by satisfying the statistical distributions $\langle s(t,x_i)\rangle = 0$ and $\langle s(t,x_i)s(t',x_j)\rangle = h\delta(t-t')\delta(x_i-x_j)$, with the amplitude $h \approx 10^{-3}$. In principle, h cannot be too large since the seeds serve as

t=0, T=T_i, normal state + thermal fluctuations

(Tf-T_i)/v > t >0, linear decrease of temperature

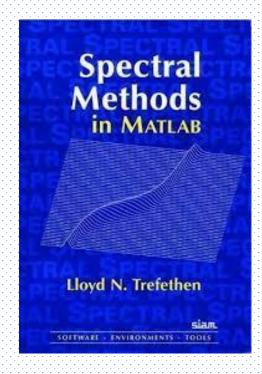
$$t>(Tf-T_i)/v$$
, $T=T_f$

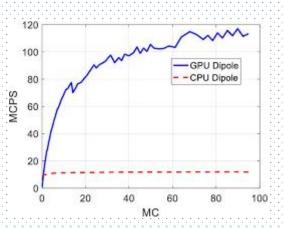
Building a Holographic Superconductor

Numerical scheme

t (Runge-Kutta), x,y,z (Fourier or Chebyshev), u (Chebyshev)

$$\begin{split} u^{-1}\partial_u(u^{-\frac{1}{2}}\partial_t\psi) &= \frac{i}{2}[u^{-1}A_t\psi + \psi(\partial_uA_t - \partial_xA_x - \partial_yA_y - \partial_zA_z) + 2(A_t\partial_t - A_x\partial_x - A_y\partial_y - A_z\partial_z)\psi] \\ &\quad + \frac{1}{2}[(-u^2 - A_x^2 - A_y^2 - A_z^2 + \partial_x\partial_x + \partial_y\partial_y + \partial_z\partial_z - (u^{-1} + 3u^3)\partial_u + (1 - u^4)\partial_u\partial_u)\psi] \\ \\ u^{-1}\partial_u(u^{-\frac{1}{2}}\partial_tA_z) &= -A_z|\psi|^2 + \Im(\psi^*\partial_z\psi) + \frac{1}{2}[(\partial_u - u^4\partial_u - 3u^3 - u^{-1})\partial_uA_z \\ &\quad + \partial_x\partial_xA_z + \partial_y\partial_yA_z + (\partial_u - u^{-1})\partial_zA_t - \partial_y\partial_zA_y - \partial_x\partial_zA_x] \\ \\ u^{-1}\partial_u(u^{-\frac{1}{2}}\partial_tA_y) &= -A_y|\psi|^2 + \Im(\psi^*\partial_y\psi) + \frac{1}{2}[(\partial_u - u^4\partial_u - 3u^3 - u^{-1})\partial_uA_y \\ &\quad + \partial_x\partial_xA_y + \partial_z\partial_zA_y + (\partial_u - u^{-1})\partial_yA_t - \partial_y\partial_zA_z - \partial_x\partial_yA_x] \\ \\ u^{-1}\partial_u(u^{-\frac{1}{2}}\partial_tA_x) &= -A_x|\psi|^2 + \Im(\psi^*\partial_x\psi) + \frac{1}{2}[(\partial_u - u^4\partial_u - 3u^3 - u^{-1})\partial_uA_x \\ &\quad + \partial_y\partial_yA_x + \partial_z\partial_zA_x + (\partial_u - u^{-1})\partial_xA_t - \partial_x\partial_zA_z - \partial_x\partial_yA_y] \\ \\ u\partial_u(u^{-1}A_t) &= -2\Im(\psi^*\partial_u\psi) + \partial_u\partial_zA_z + \partial_u\partial_yA_y + \partial_u\partial_xA_x \\ \\ A_t &= \frac{u^4 - 1}{|\psi|^2}\Im(\psi^*\partial_u\psi) \end{split}$$





Use GPU to save time

1D:

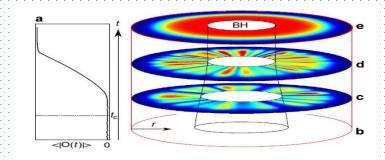
Klbble-Zurek机制与AdS/CMT

Published: 23 June 2015

Universal far-from-equilibrium dynamics of a holographic superconductor

Julian Sonner [™], Adolfo del Campo & Wojciech H. Zurek

Nature Communications 6, Article number: 7406 (2015) Cite this article



Open Access | Published: 12 April 2012

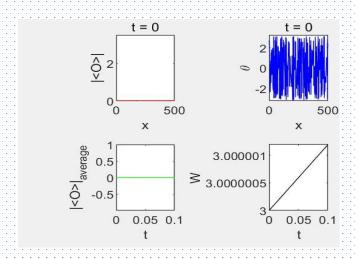
Winding up superfluid in a torus via Bose Einstein condensation

Arnab Das, Jacopo Sabbatini & Wojciech H. Zurek

Scientific Reports 2, Article number: 352 (2012) Cite this article

3355 Accesses | 82 Citations | 1 Altmetric | Metrics

winding numbers $W = \oint_C d\theta/2\pi$



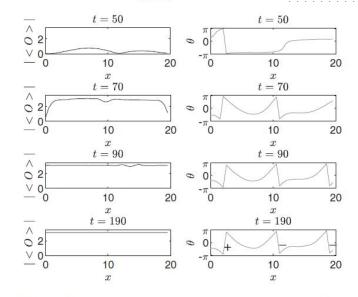
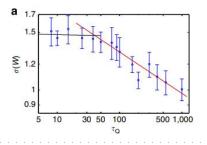


FIG. 1. Winding up a superconducting ring after a temperature quench $\tau_Q = e^3$, the circumference C = 20. In the four rows, we show the magnitude of the order parameter $|\langle O \rangle|$ and its phase θ configuration in the dynamic process. The system finally enters an equilibrium state with constant amplitude of order parameter and a stable configuration of phase field $\theta(x)$.

Universal far-from-equilibrium dynamics of a holographic superconductor

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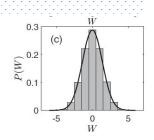
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Winding up a finite size holographic superconducting ring beyor Kibble-Zurek mechanism

Chuan-Yin Xia and Hua-Bi Zeng Phys. Rev. D **102**, 126005 – Published 1 December 2020

winding numbers $W = \oint_C d\theta/2\pi$

方差正比于独立区域的个数 N



the mean field exponents z = 2 and $\nu = 1/2$ in Eq. (1). At large N, $\langle |W| \rangle$ can be computed from the Gaussian distribution, and then we obtain the key prediction at a fixed size [14]

$$\sigma(W) = \sqrt{\langle W \rangle^2} \propto \langle |W| \rangle \propto \tau_Q^{-1/8},$$
 (5)

at a fixed rate $\langle |W| \rangle \propto \sigma(W) \propto \sqrt{N}$.

十万次统计的结果

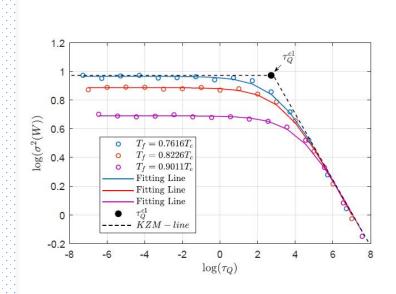


FIG. 2. Universal scalings of invariant of winding number versus quench rate in the quenched 1D holographic superconducting ring for three different quench final temperature. The black dotted line is the line predicted from KZM, the Eq. 9 is used to fit the data.

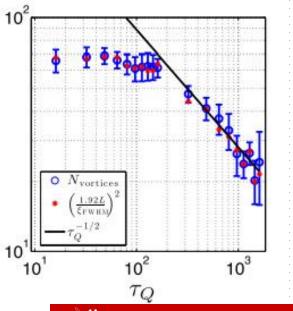
Klbble-Zurek机制与AdS/CMT

2 D:Superfluid

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Defect Formation beyond Kibble-Zurek Mechanism and Holography

Paul M. Chesler, Antonio M. García-García, and Hong Liu Phys. Rev. X **5**, 021015 – Published 14 May 2015



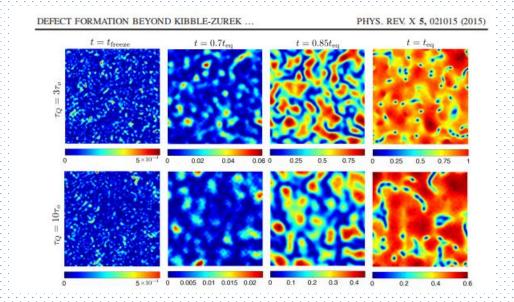
ar 1V > hep-th > arXiv:1005.1776

High Energy Physics - Theory

[Submitted on 11 May 2010 (v1), last revised 27 Aug 2010 (this version, v2)]

Emergent Gauge Fields in Holographic Superconductors

Oriol Domènech, Marc Montull, Alex Pomarol, Alberto Salvio, Pedro J. Silva



$$n \sim \frac{L^d}{\xi(\hat{t})^d} \propto \tau_Q^{-\frac{d\nu}{1+z\nu}} = -1/2$$

$$d=2$$
, $z=2$, $\ln 2/2$

2D: superconductor

Regular Article - Theoretical Physics | Open Access | Published: 12 March 2021

Topological defects as relics of spontaneous symmetry breaking from black hole physics

<u>Hua-Bi Zeng</u>, <u>Chuan-Yin Xia</u> & <u>Hai-Qing Zhang</u> □

Journal of High Energy Physics 2021, Article number: 136 (2021) Cite this article

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ar√i∨ > hep-th > arXiv:1005.1776

High Energy Physics - Theory

[Submitted on 11 May 2010 (v1), last revised 27 Aug 2010 (this version, v2)]

Emergent Gauge Fields in Holographic Superconductors

Oriol Domènech, Marc Montull, Alex Pomarol, Alberto Salvio, Pedro J. Silva

arXiv > hep-th > arXiv:2302.02364

High Energy Physics - Theory

[Submitted on 5 Feb 2023 (v1), last revised 22 Mar 2023 (this version, v3)]

Collective dynamics and the Anderson-Higgs mechanism in a bona fide holographic superconductor

Hyun-Sik Jeong, Matteo Baggioli, Keun-Young Kim, Ya-Wen Sun

T V > hep-th > arXiv:2303.10305

High Energy Physics - Theory

[Submitted on 18 Mar 2023]

How to sit Maxwell and Higgs on the boundary of Anti-de Sitter

Matteo Baggioli

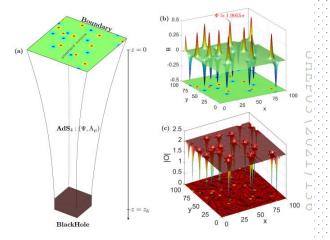
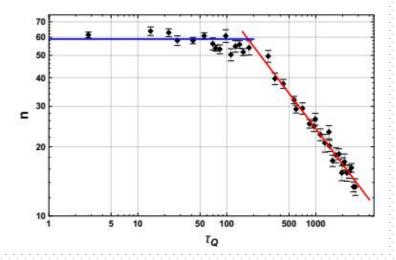
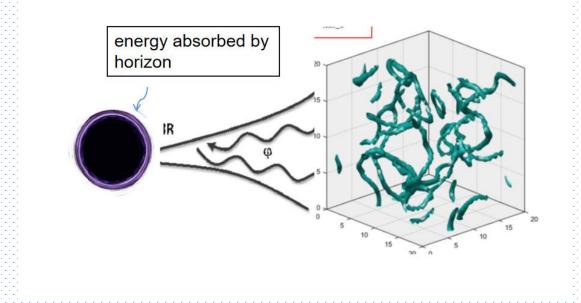


Figure 1. Holographic setup and birth of topological defects. (a) Complex scalar field Ψ and the U(1) gauge field A_{μ} are living in the bulk of Schwarzschild-AdS₄ spacetime. Quenching temperature across the critical point induces quantized vortices on the (2+1)-dimensional boundary, leading to the holographic KZM. (b) Configurations of the magnetic fluxons and their corresponding locations are shown at the bottom. Red arrow points at a positive magnetic fluxon with magnetic flux $\Phi \approx 1.9955\pi$. (c) Configurations of the order parameter vortices and the density plot at the bottom. Streamlines with arrows indicate the directions of phases in the complex plain of the order parameter. Locations of the cores of the vortices and the positions of the magnetic fluxons in panel (b) coincide. For related movie see the supplementary material.



Klbble-Zurek机制与AdS/CMT

3 D: (work in progress) 求解 4+1维非线性偏微分方程组



统计总条数或者总的长度

初步的结果发现和KZM也是吻合的

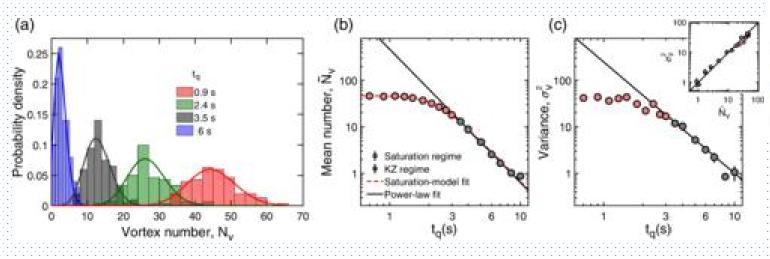
Experiments beyond KZM

fast quench: plateau

Editors' Suggestion

Defect Saturation in a Rapidly Quenched Bose Gas

Junhong Goo, Younghoon Lim, and Y. Shin Phys. Rev. Lett. **127**, 115701 – Published 8 September 2021



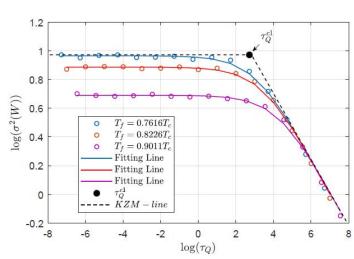


FIG. 2. Universal scalings of invariant of winding number versus quench rate in the quenched 1D holographic superconducting ring for three different quench final temperature. The black dotted line is the line predicted from KZM, the Eq. 9 is used to fit the data.

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Letter Published: 16 September 2019

Kibble–Zurek universality in a strongly interacting Fermi superfluid

Bumsuk Ko, Jee Woo Park ≥ & Y. Shin ≥

Nature Physics 15, 1227–1231 (2019) Cite this article

3244 Accesses 21 Citations 2 Altmetric Metrics

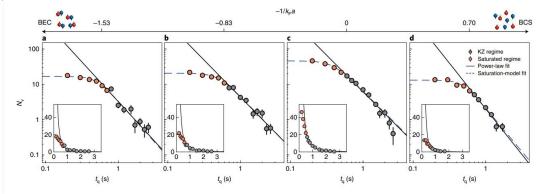
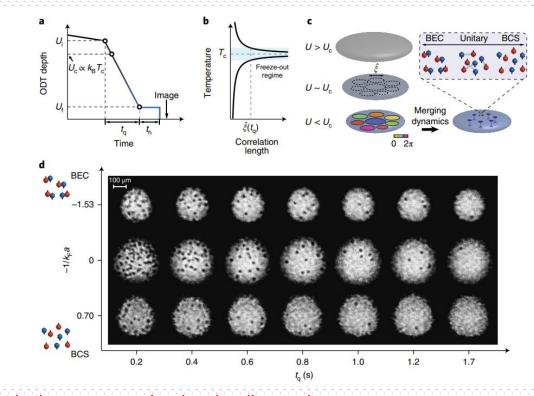


Fig. 2 | Vortex number versus quench time. The average number of detected vortices are plotted as a function of t_q on log-log axes at four different final interaction strengths $-1/k_t a = -1.53$ (757 G) (**a**), -0.83 (785 G) (**b**), 0 (832 G) (**c**) and 0.70 (898 G) (**d**). The insets show the same data on linear-linear axes. Each data point comprises at least ten realizations of the same experiment, and the error bars are the standard error of the mean. When the error bars are not visible, they are smaller than the marker size.

BEC 的实验

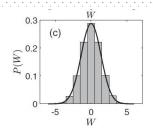


What is the exact mechanism leading to the emergence of the plateau in the defect density?

At what quench rates do the KZM scaling laws break down?

How does the plateau value of the defect density depend on the depth of the quench?

Are any of these features universal?



Finite size: suppression of defects formation

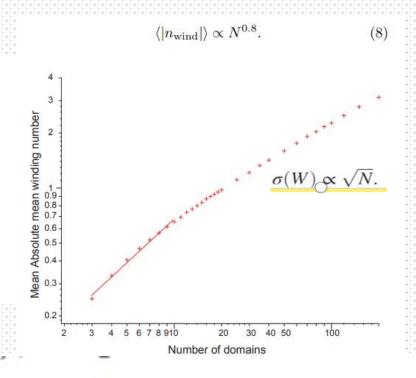
PRL 113,

the mean field exponents z = 2 and $\nu = 1/2$ in Eq. (1). At large N, $\langle |W| \rangle$ can be computed from the Gaussian distribution, and then we obtain the key prediction at a fixed size [14]

1Labore

$$\sigma(W) = \sqrt{\langle W \rangle^2} \propto \langle |W| \rangle \propto \tau_Q^{-1/8},$$
 (5)

at a fixed rate $\langle |W| \rangle \propto \sigma(W) \propto \sqrt{N}$.



$$\langle |n_{\rm wind}| \rangle \propto t_{\rm evap}^{-1/4 \times 0.8} \approx t_{\rm evap}^{-0.2},$$

Beyond KZM: -0.125

Winding up a finite size holographic superconducting ring beyond Kibble-Zurek mechanism

Chuan-Yin Xia and Hua-Bi Zeng

有限尺寸效应

Phys. Rev. D **102**, 126005 – Published 1 December 2020

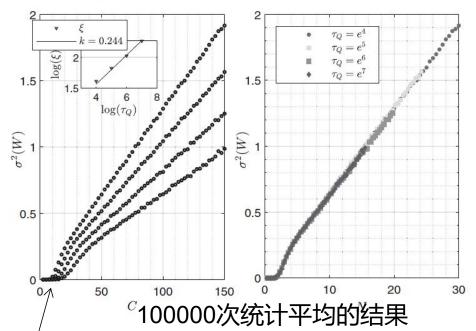


FIG. 2. Size dependence of variance $\sigma^2(W)$. Left: size dependence of $\sigma^2(W)$ for four quench rates, from top to bottom, $\tau_Q = e^4, e^5, e^6, e^7$. The inset shows the scaling of the critical circumference. Right: $\sigma^2(W)$ as a function of pieces number N; all quench rate results are identical to each other.

critical C

 $\xi \propto \tau_O^{\alpha}$,

and the power-law exponent $\alpha = \nu/(1 + \nu z)$

confirm KZM from the critical size

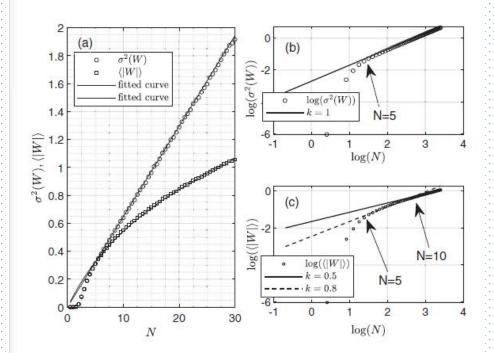


FIG. 3. Size-dependent $\sigma^2(W)$ and $\langle |W| \rangle$. (a) $\sigma^2(W)$ and $\langle |W| \rangle$ as a function of N and their fit from trinomial distribution Eq. (10); when N < 5, the two have the same values. (b) Logarithmic relationship between $\sigma^2(W)$ and N. (c) Logarithmic relationship between $\langle |W| \rangle$, and N. k is the linear fit slop.

Besides the perfectly matched KZM results, from Fig. 2, one also finds that $\sigma^2(W) \propto N$ cannot hold anymore when the ring size is reducing. Figure 3(b) shows that the non-KZM region is $1 < N \le 5$. Also in this region, there is another interesting feature that $\sigma^2(W) = \langle |W| \rangle$, since the winding number can have only three values: -1, 0, and 1. One thing that needs to be emphasized is that in this region, though the KZM pieces N can be larger than 1, W does not take a value larger than 1, and this indicates that the formation of topological defects in a size $N \le 5$ is effectively correlated, which is probably due to the growth of correlation length in the presence of diffusion [13].

Finite size effect dominates when N <10 |W|=0,1,2, real independent region size is 5N.

Density of Kinks after a Quench: When Symmetry Breaks, How Big are the Pieces?

Pablo Laguna and Wojciech Hubert Zurek Phys. Rev. Lett. **78**, 2519 – Published 31 March 1997

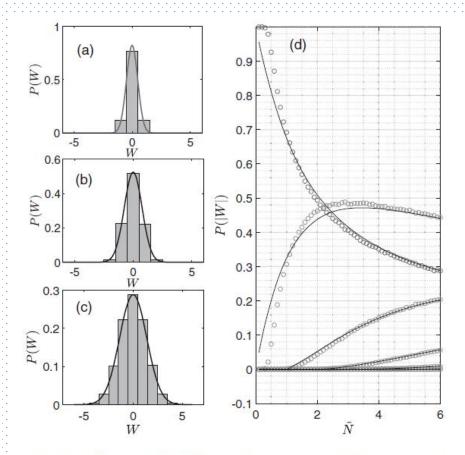


FIG. 4. P(W) and P(|W|). (a–c) Histogram of P(W) for N=8, 18, 30 respectively, when $\tau_Q=e^4$. (d) Numerical results of P(|W|) (circles, from top to bottom are |W|=0, 1, 2, 3, 4, 5, 6, respectively) and their fitted curves derived from Eq. (10) (solid lines).

Because the winding number $W = n^+ - n^-$ and we can label the independent pieces to be "+" "–" or "0," it is natural to expect that the distribution of n^+, n^- can be captured by a trinomial distribution. The probability of the trinomial distribution of trials \tilde{N} reads

$$P(\tilde{N}, n^+, n^-) = \frac{\tilde{N}!}{n^0! n^+! n^-!} \left(\frac{p}{2}\right)^{n^+ + n^-} (1 - p)^{n^0}, \quad (10)$$

where $n^0 = \tilde{N} - (n^+ + n^-)$, n^0 is the number of pieces without either + or -. $(n^+, n^-, n^0) \le \tilde{N}$, and \tilde{N} equals the largest value of $n^+(n^-)$, which can be defined as the number of effectively unrelated pieces. p/2 is the probability for both + and - since the two have the same distribution [58], while 1-p is the 0 probability. From the largest number of $n^+(n^-)$ of a fixed integral N, we find that $\tilde{N} = n_{\max}^{+(-)} = N/5$. Furthermore, we consider the case when the \tilde{N} is not an integral; by increasing from \tilde{N} from a smaller integral number M to M+1, the $\sigma^2(W)$ is

- 1. Number of real independent regions: N= L/(5 \xi).
- 2. Finite size effect dominates when N<10 (Standard Deviation)or N<5(Variance).

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Defect Formation beyond Kibble-Zurek Mechanism and Holography

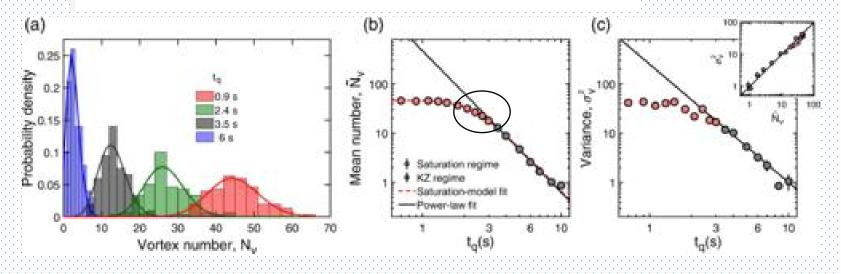
Paul M. Chesler, Antonio M. García-García, and Hong Liu Phys. Rev. X **5**, 021015 – Published 14 May 2015

快速淬火与平台产生的机制

Editors' Suggestion

Defect Saturation in a Rapidly Quenched Bose Gas

Junhong Goo, Younghoon Lim, and Y. Shin Phys. Rev. Lett. **127**, 115701 – Published 8 September 2021



A theory is needed to explain the phenomena

对偏离KZM的理论解释与数值验证

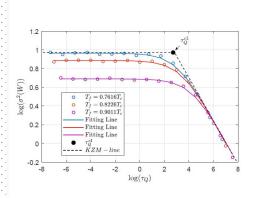


FIG. 2. Universal scalings of invariant of winding number versus quench rate in the quenched 1D holographic superconducting ring for three different quench final temperature. The black dotted line is the line predicted from KZM, the Eq. 9 is used to fit the data.

Universal Breakdown of Kibble-Zurek Scaling in Fast Quenches across a Phase Transition

Hua-Bi Zeng, Chuan-Yin Xia, and Adolfo del Campo Phys. Rev. Lett. **130**, 060402 – Published 9 February 2023

对KZM的直接推广,但Zurek曾认为平台并不是KZM能解释的

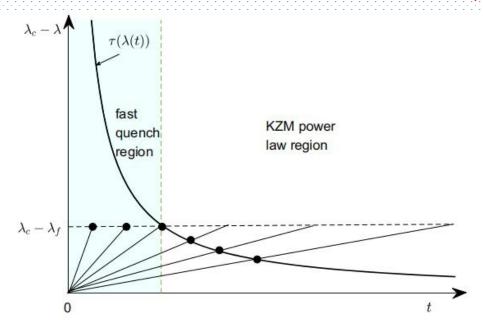


FIG. 1. Universal breakdown of KZM at fast quenches. The KZM relies on the adiabatic impulse approximation to determine the freeze-out time \hat{t} , by equating the time elapsed after crossing the phase transition to relaxation time. For slow quenches, \hat{t} is given by the power law [Eq. (2)]. For fast quenches of finite depth λ_f , it is set by the equilibrium relaxation time with $\hat{t} \sim \tau(\lambda_f)$, and becomes independent of the quench time. Thus, the density of defects saturates at a plateau, leading to the breakdown of the KZM scaling relations at fast quenches.

$$n \sim rac{L^d}{\xi(T_f)^d} \propto L^d (T_c - T_f)^{d
u}.$$

$$\hat{t} \sim \tau(T_f) \propto (T_c - T_f)^{-\nu z}$$
.

$$\frac{\tau_Q^{c1}(T_c - T_f)}{T_c} = \tau_0(T_c - T_f)^{-\nu z},$$

solving the equation we have

$$\tau_Q^{c1} \propto T_c (T_c - T_f)^{-(\nu z + 1)},$$

预测了三个普适规律

快速淬火普适规律在一维全息超导环中的验证



Condensed Matter > Statistical N

[Submitted on 15 Oct 2021]

Kibble Zurek mechani:

Chuan-Yin Xia, Hua-Bi Zeng

We propose a theory to explain the expuench rate au_Q^{c1} above it the KZM sca $n \propto L^d \epsilon_{Tf}^{d\nu}$, the freeze out time \hat{t} ad and z are spatial and dynamical critic quenched superconducting ring via th

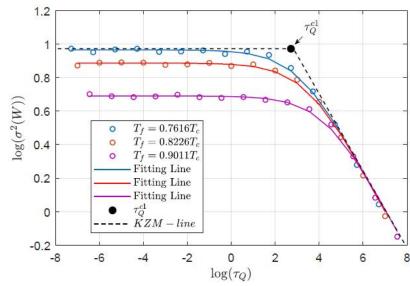


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对偏离KZM的理论解释与数值验证

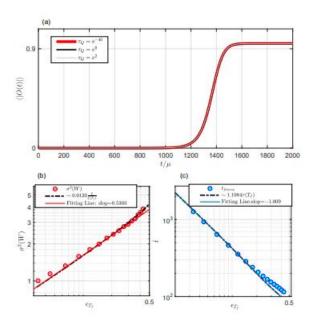


FIG. 3. (a) Dynamics of average absolute value of the order parameter for three sample different quench rates in the fast quench region when $T_f=0.9011T_c$. Before $\hat{t},$ the order parameter will cease to follow or even approximate its equilibrium value, \hat{t} is defined as the moment when the average absolute value reaches $0.01\langle|O_f|\rangle,\,O_f$ is the equilibrium value of the quench ending temperature $T_f.$ (b) Invariance of winding number W versus the reduced quench ending temperature $\epsilon_{T_f}.$ (c) Freeze out time versus $\epsilon_{T_f}.$

Universal Breakdown of Kibble-Zurek Scaling in Fast Quenches across a Phase Transition

Hua-Bi Zeng, Chuan-Yin Xia, and Adolfo del Campo Phys. Rev. Lett. **130**, 060402 – Published 9 February 2023

快速淬火普适规律在一维横场伊幸模型中的验证

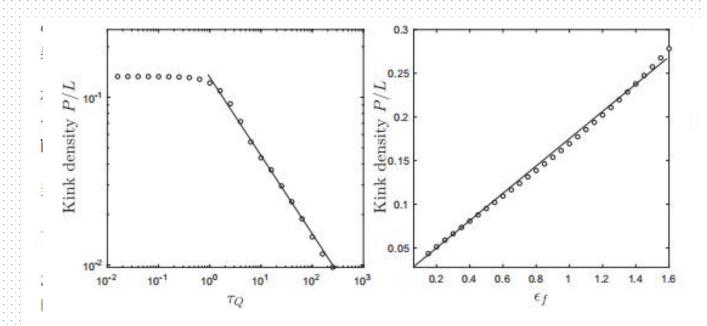


FIG. 3. Universal deviations from the KZM in the 1D transversal field Ising model after a fast thermal quench. (a) The KZM power-law scaling of the kink density is interrupted by the onset of a plateau as the quench time is reduced. A fit to the data reads $P/L = (0.14 \pm 0.03)\tau_Q^{-0.54\pm0.05}$ in the KZM regime. (b) In the fast quench case, the kink density is independent of the quench

对偏离KZM的理论解释与数值验证

nature communications

8

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ottps://doi.org/10.1038/s41467-022-33737-4

Simulating groundstate and dynamical quantum phase transitions on a superconducting quantum computer

Received: 18 May 2022

Accepted: 30 September 2022

Published online: 10 October 2022

Check for updates

James Dborin ®¹, Vinul Wimalaweera¹, F. Barratt², Eric Ostby³, Thomas E. O'Brien³ & A. G. Green ®¹

The phenomena of quantum criticality underlie many novel collective phenomena found in condensed matter systems. They present a challenge for classical and quantum simulation, in part because of diverging correlation lengths and consequently strong finite-size effects. Tensor network techniques that work directly in the thermodynamic limit can negotiate some of these difficulties. Here, we optimise a translationally invariant, sequential quantum circuit on a superconducting quantum device to simulate the groundstate of the quantum Ising model through its quantum critical point. We further demonstrate how the dynamical quantum critical point found in quenches of this model across its quantum critical point can be simulated. Our approach avoids finite-size scaling effects by using sequential quantum circuits inspired by infinite matrix product states. We provide efficient circuits and a variety of error mitigation strategies to implement, optimise and time-evolve these states.

Universal Breakdown of Kibble-Zurek Scaling in Fast Quenches across a Phase Transition

Hua-Bi Zeng, Chuan-Yin Xia, and Adolfo del Campo Phys. Rev. Lett. **130**, 060402 – Published 9 February 202

快速淬火普适规律在ZigZa

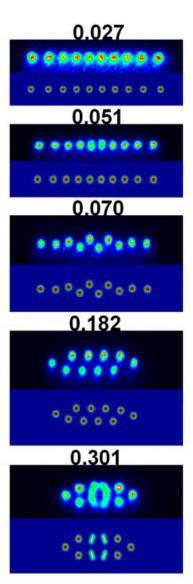
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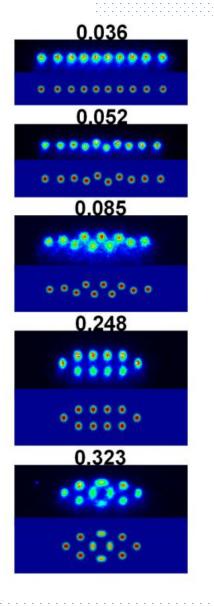
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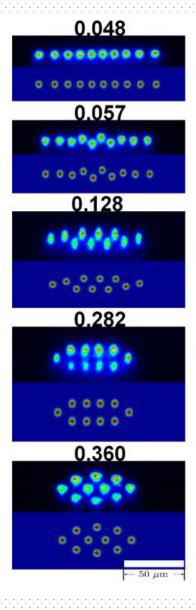
Exploring structure transitions of ic

L. L. Yan1,2, W. Wan1,2, L. Chen1, F. Zhou1

Received: 10 November 2015 Accepted: 27 January 2016 Published: 11 February 2016 Phase transitions have been a research focus under strong Coulomb repulsion, provide a reconfinement by electromagnetic field. We de cooled ⁴⁰Ca⁺ ion crystals in a home-built surf structural phase transition from the linear to complicated two-dimensional configurations with the numerical simulation. Heating due the numerical simulation with the experiment complicated many-body behaviour in the traffer further exploring quantum phase transitit trapped ions.







Universal Breakdown of Kibble-Zurek Scaling in Fast Quenches across a Phase Transition

Hua-Bi Zeng, Chuan-Yin Xia, and Adolfo del Campo Phys. Rev. Lett. **130**, 060402 – Published 9 February 2023

$$\ddot{\phi}_l + \eta \dot{\phi}_l + \partial_{\phi_l} V(\{\phi_i\}, t) + \zeta_l = 0.$$

$$V(\{\phi_l\},t) = \sum_{l=1}^{L} \frac{1}{2} [\lambda(t)\phi_l^2 + \phi_l^4] + c \sum_{l=1}^{L-1} \phi_l \phi_{l+1},$$

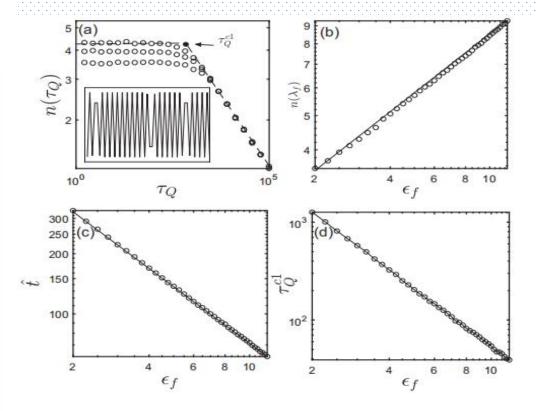


FIG. 2. Universal breakdown of KZM at fast quenches in a one-dimensional ϕ^4 model in Log-Log plots. (a) The dependence of the density on the quench time is shown for three different values of $\lambda_f = -2, -1.5, -1$, from top to bottom. The inset shows a single realization of the scalar field ϕ_l , in the zigzag phase with excitations in the form kinks (at the interface of adjacent zigzag domains) after crossing the phase transition. In the KZM scaling regime, $n = (21.1 \pm 0.4)\tau_Q^{-0.245\pm0.007}$, while the onset and value of the plateau depend on λ_f . The value of the density (b) and the

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Structural phase transition and its critical dynamics from holography

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We introduce a gravitational lattice theory defined in an AdS_3 black hole background that provides a holographic dual description of the linear-to-zigzag structural phase transition, characterized by the spontaneous breaking of parity symmetry observed in, e.g., confined Coulomb crystals. The transition from the high-symmetry linear phase to the broken-symmetry doubly-degenerate zigzag phase can be driven by quenching the coupling between adjacent sites through the critical point. An analysis of the equilibrium correlation length and relaxation time reveals mean-field critical exponents. We explore the nonequilibrium phase transition dynamics leading to kink formation. The kink density obeys universal scaling laws in the limit of slow quenches, described by the Kibble-Zurek mechanism (KZM), and at fast quenches, characterized by a universal breakdown of the KZM.

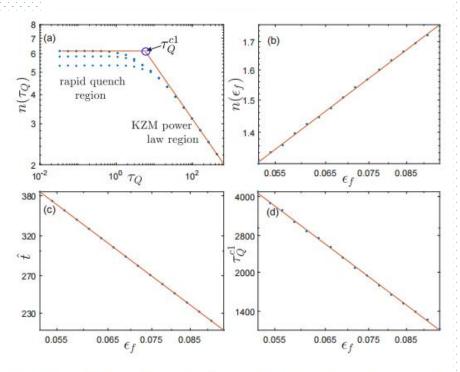


Figure 5. Universal scaling laws at fast and slow quenches in a log-log representation. (a) The mean number n of kinks as function of the quench time τ_Q (N=100). The behavior at slow quenches exhibits a universal power-law scaling with the quench time, in agreement with KZM. As the quench time is reduced, there is a crossover to a plateau region with a value of the kink average number that is independent of the quench time. The crossover occurs for τ_Q shorter than τ_Q^{c1} . For rapid quenches, panels (b)-(d) show that the mean number n of kinks, the freeze-out time $\hat{\xi}$, and the critical quenching time τ_Q^{c1} scale universally as a function of the quench depth ϵ_f . The corresponding data is fitted by the following power-laws $n=(5.59\pm0.09)(C_f-C_c)^{0.491\pm0.007}$, $\hat{t}=(20.96\pm0.09)(C_f-C_c)^{-0.986\pm0.001}$, and $\tau_Q^{c1}=(8.2\pm0.7)(C_f-C_c)^{-2.11\pm0.04}$.

Universal Breakdown of Kibble-Zurek Scaling in Fast Quenches across a Phase Transition

Hua-Bi Zeng, Chuan-Yin Xia, and Adolfo del Campo Phys. Rev. Lett. **130**, 060402 – Published 9 February 2023

快速淬火普适规律在2维BEC中的验证

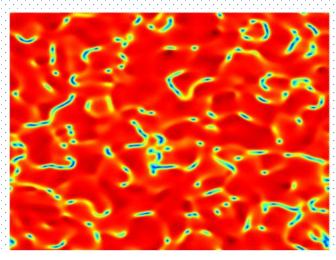
we consider the stochastic Gross-Pitaevskii equation

$$(i-\gamma)\frac{\partial\phi}{\partial t} = -\frac{1}{2}\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) + \lambda(t)\phi + \tilde{g}|\phi|^2\phi + \eta(\vec{x},t).$$

$$\lambda(t) = -\frac{t}{\tau_Q}$$

在二维全息超导/超流模型中也得到了一致的结果

对偏离KZM的理论解释与数值验证



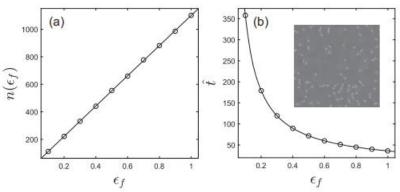


FIG. 4. Universal deviations from the KZM in a 2D superfluid after a fast thermal quench. (a) The vortex number is independent of the quench rate and scales universally with the quench depth ϵ_f . A fit to the data reads $n_{\lambda_f} = (1100 \pm 5) \epsilon_f^{0.99 \pm 0.01}$. (b) Corresponding universal scaling of the freeze-out time as function of ϵ_f , with data fitted to $\hat{t} = (35.9 \pm 0.2) \epsilon_f^{-0.99 \pm 0.03}$. The inset shows the distribution of vortices in a single realization.

Kibble-Zurek Mechanism is of great significance for understanding the universal laws in non equilibrium phase transition processes.

New experiments have discovered phenomena that the original KZM cannot explain.

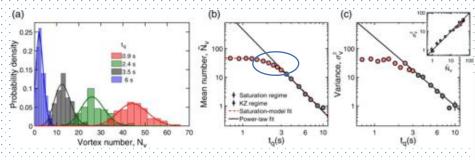
The universal laws of finite size effect and rapid quenching can still be understood within the basic framework of KZM. The existence of new universal laws and numerical calculations of holography (etc.) perfectly verify the universality.

COMMUNICATIONS

Experiments in progress, with USTC



PHYSICS



How to understand the smooth transition from the platform to the KZM region

KZM and Nonequilibrium Steady State Phase Transition Process by an External Field



KZM and first order phase transition

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Published: 05 July 2013

Periodically driven holographic superconductor

<u>Wei-Jia Li, Yu Tian</u> & <u>Hongbao Zhang</u> [™]

Journal of High Energy Physics 2013, Article number: 30 (2013) Cite this article

112 Accesses 37 Citations Metrics

Universal critical exponents of nonequilibrium phase transitions from holography

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We study the critical exponents in the universal scaling laws of a holographic nonequilibrium steady state near its critical point of phase transition, which is driven by an ac electric field sitting in the boundary of the bulk. The applied electric field drives the initial superconducting state into a nonequilibrium steady state with vanishing condensate as its amplitude is greater than a critical value. In the vicinity of the nonequilibrium critical point, we numerically calculate the six static and one dynamical critical exponents, and find that they have similar values to those in equilibrium systems within numerical errors.

Quench Dynamics in Holographic First-Order Phase Transition

Qian Chen (Beijing, GUCAS), Yuxuan Liu (Beijing, GUCAS), Yu Tian (Beijing, GUCAS and E

Wu (Beijing, Inst. Math.), Hongbao Zhang (Beijing Normal U.) (Nov 21, 2022)

e-Print: 2211.11291 [hep-th]

THANKS!