Tests of modified gravitational wave propagation

检验修改引力中引力波的传播效应

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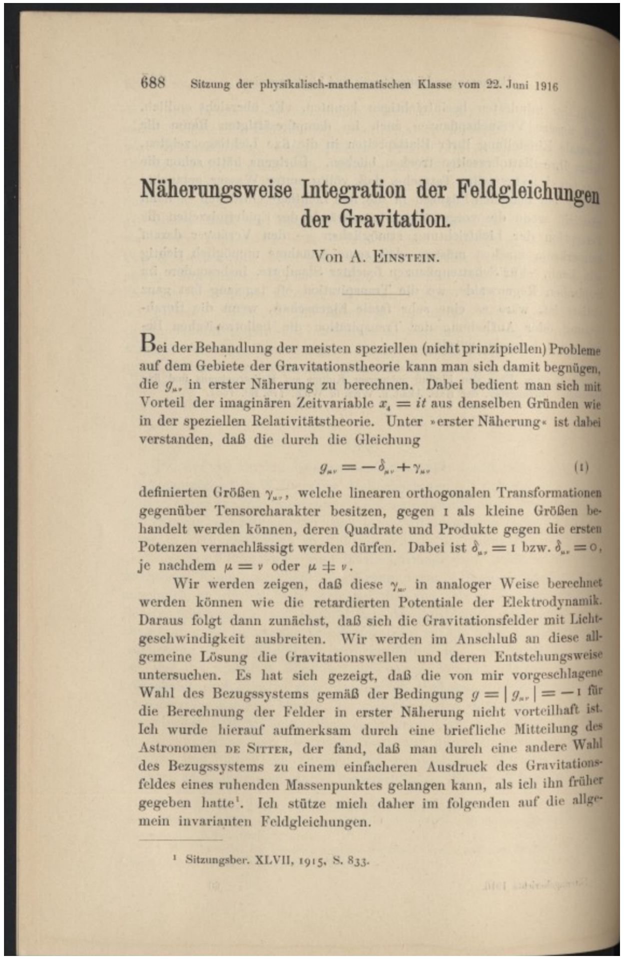
Contents

1.

Tests of gravities with gravitational waves

Gravitational Waves

- ❑ **Predicted by Albert Einstein in 1916, based on general relativity, just proposed a year ago**



688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

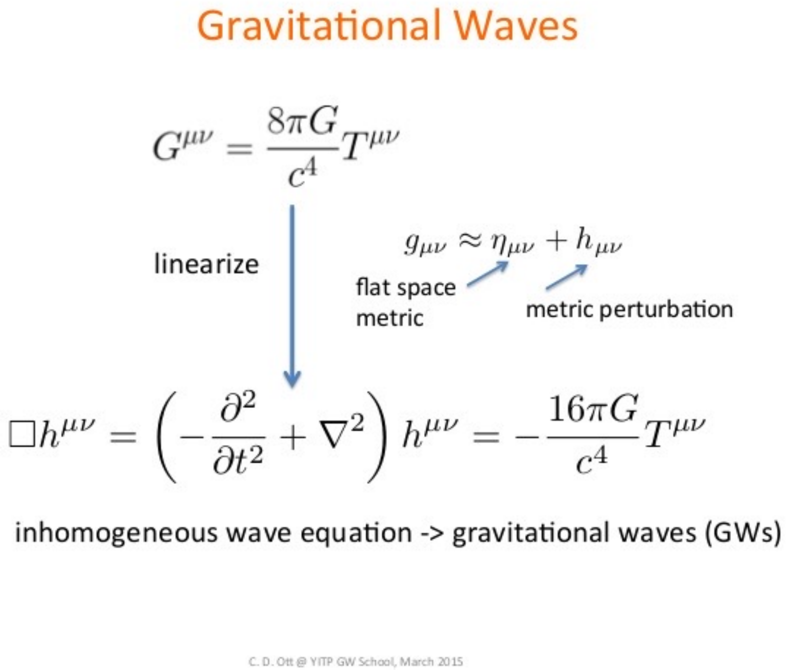
Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

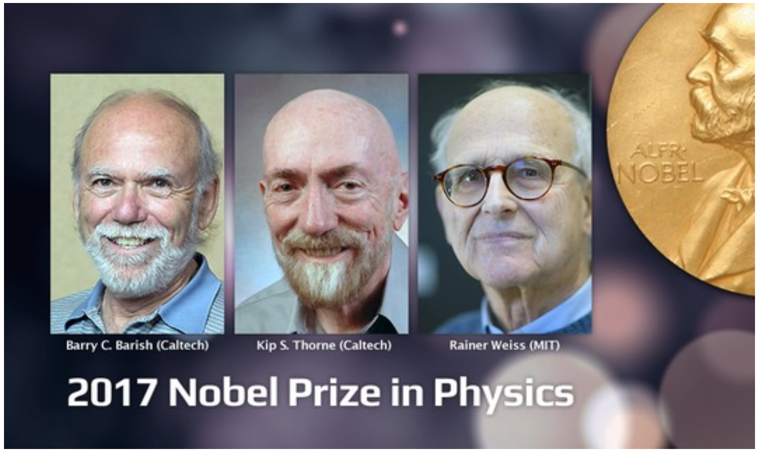
definierten Größen $\gamma_{\mu\nu}$, welche linearen orthogonalen Transformationen gegenüber Tensorcharakter besitzen, gegen 1 als kleine Größen behandelt werden können, deren Quadrate und Produkte gegen die ersten Potenzen vernachlässigt werden dürfen. Dabei ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$.

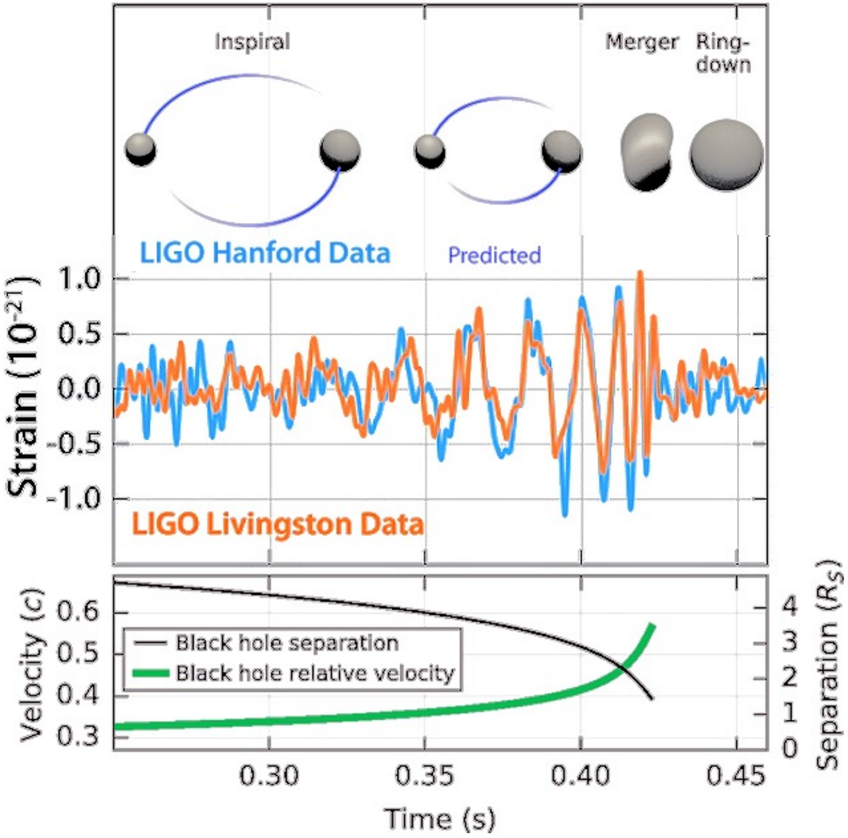
Wir werden zeigen, daß diese $\gamma_{\mu\nu}$ in analoger Weise berechnet werden können wie die retardierten Potentiale der Elektrodynamik. Daraus folgt dann zunächst, daß sich die Gravitationsfelder mit Lichtgeschwindigkeit ausbreiten. Wir werden im Anschluß an diese allgemeine Lösung die Gravitationswellen und deren Entstehungsweise untersuchen. Es hat sich gezeigt, daß die von mir vorgeschlagene Wahl des Bezugssystems gemäß der Bedingung $g = |g_{\mu\nu}| = -1$ für die Berechnung der Felder in erster Näherung nicht vorteilhaft ist. Ich wurde hierauf aufmerksam durch eine briefliche Mitteilung des Astronomen de Sitter, der fand, daß man durch eine andere Wahl des Bezugssystems zu einem einfacheren Ausdruck des Gravitationsfeldes eines ruhenden Massenpunktes gelangen kann, als ich ihn früher gegeben hatte[1]. Ich stütze mich daher im folgenden auf die allgemein invarianten Feldgleichungen.

[1] Sitzungsber. XLVII, 1915, S. 833.



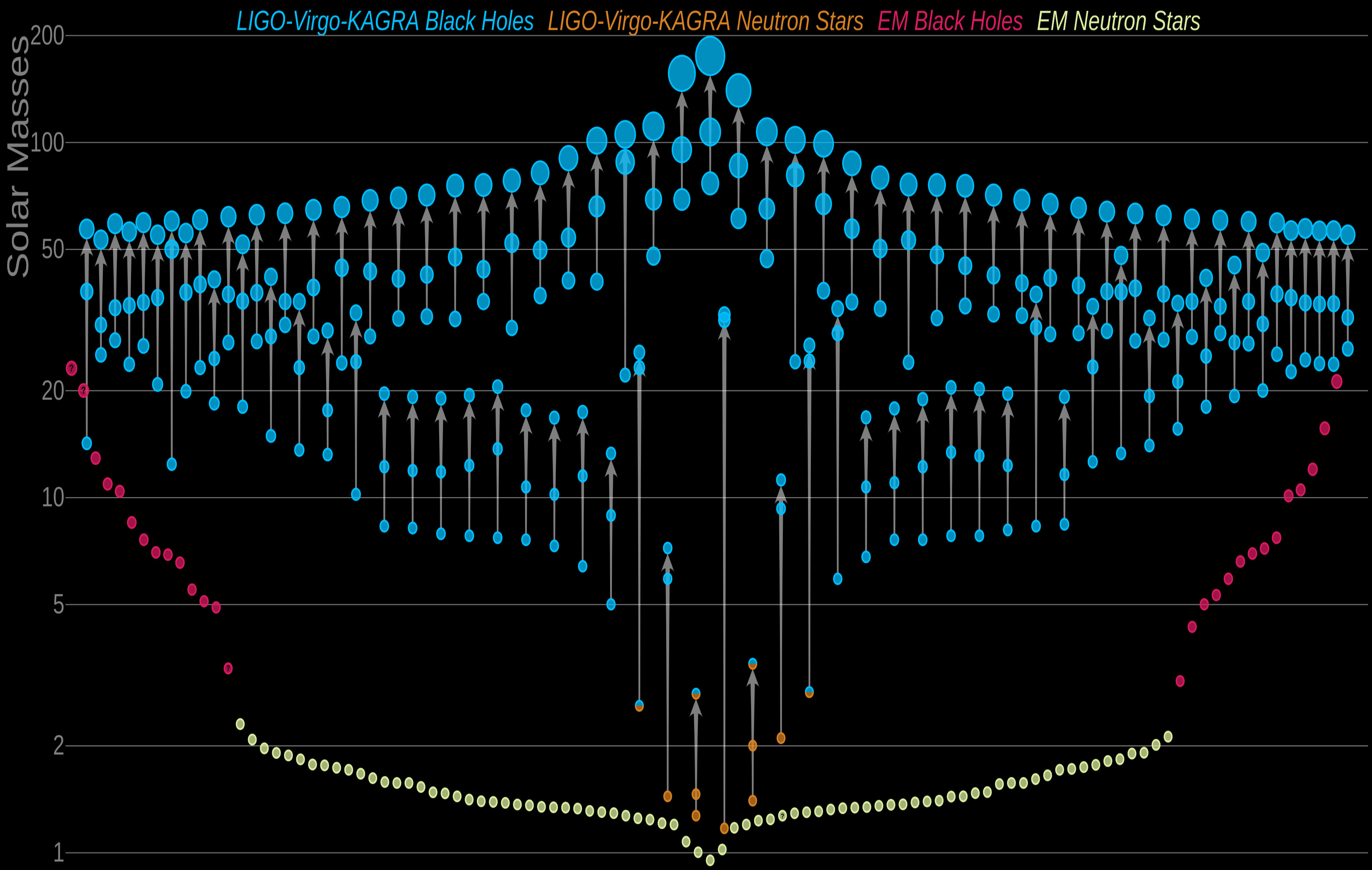
Detection of Gravitational Waves





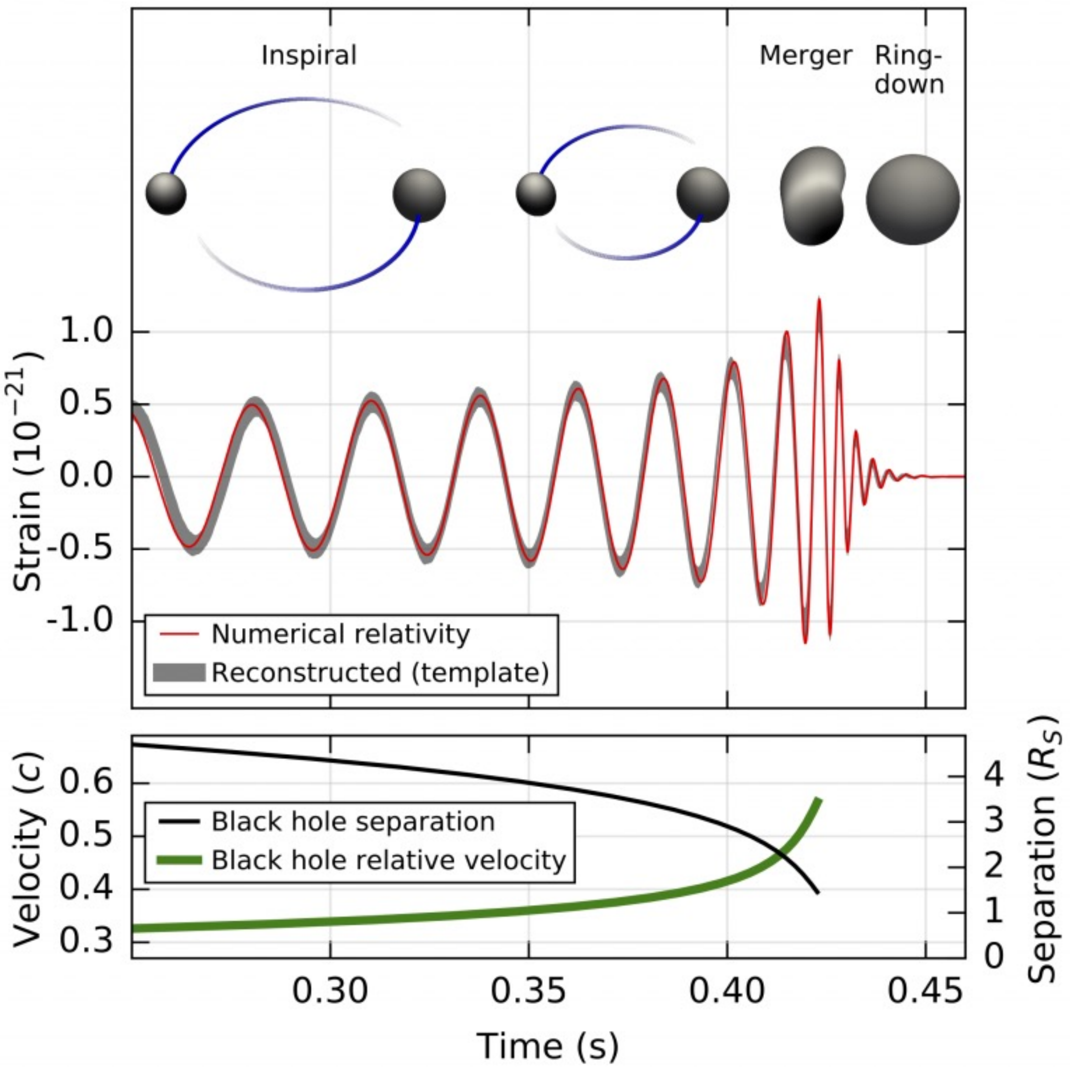
GW150914

Masses in the Stellar Graveyard



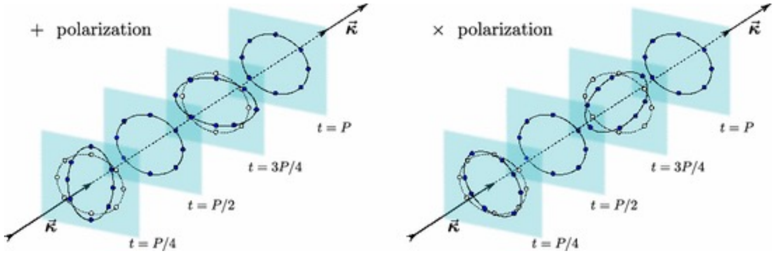
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Signals of GW150914



GWs in general relativity

- ❖ **Two independent polarizations**



- ❖ **Quadrupole radiation**

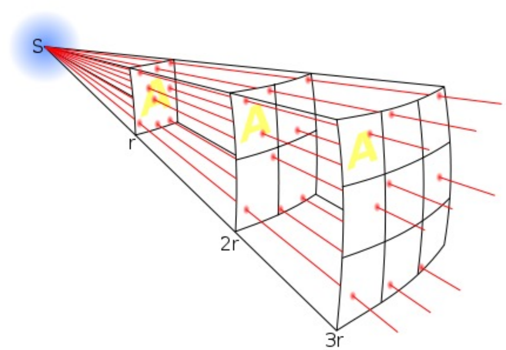
$$\bar{h}_{ij}(t,r) = \frac{2G}{c^4 r}\ddot{I}_{ij}(t - r/c),$$

- ❖ **Propagate at speed of light**
- ❖ **Damping as inverse of d_L**

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)h_{\mu\nu} = 0$$

- ❖ **massless**
- ❖ **No briefringences**

$$h \sim \frac{1}{d_L}$$

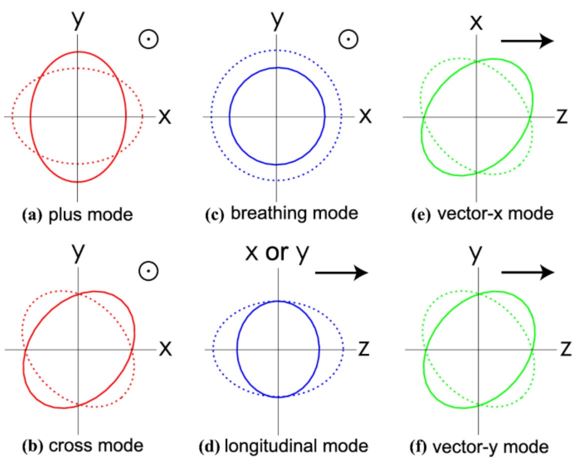


- ❖ **………**

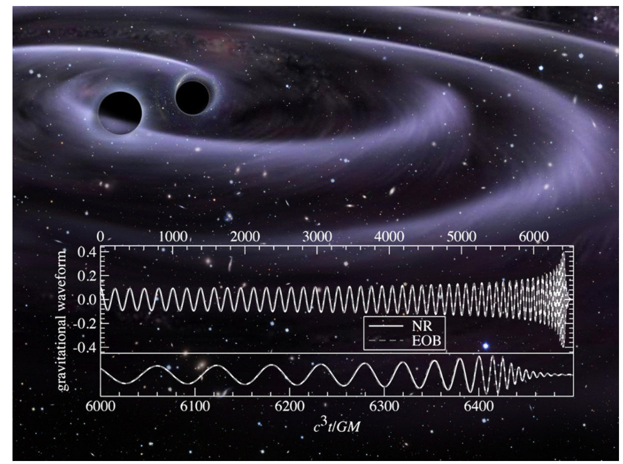
GWs in modified gravities

Modified gravities can affect polarization, generation, propagation, and detection of GWs.

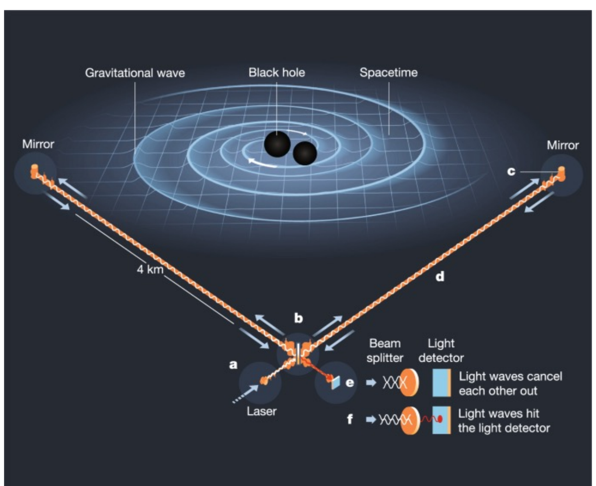
Polarization



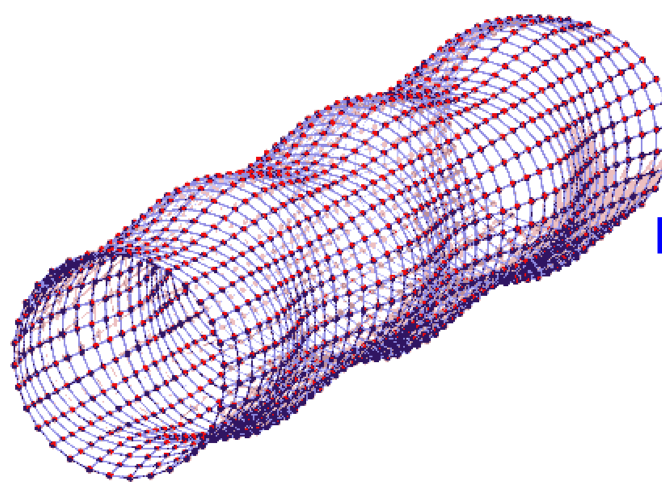
Generation



Detection



Propagation



Tests of gravities with GWs

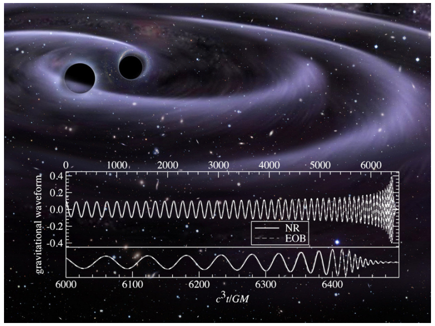
- ❖ Tests of GW generation

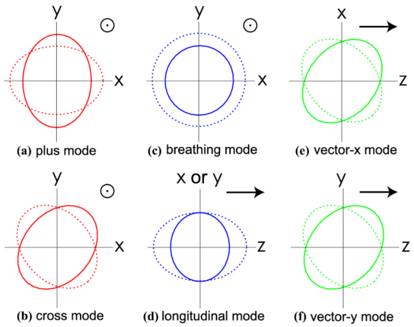
- ❖ Tests of extra polarization

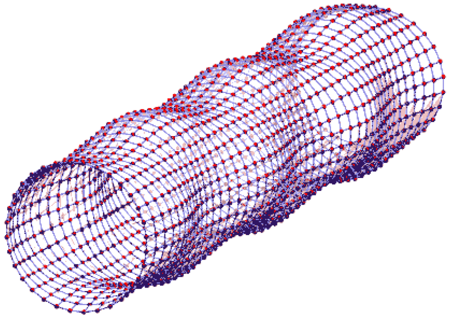
- ❖ Tests of GW propagation

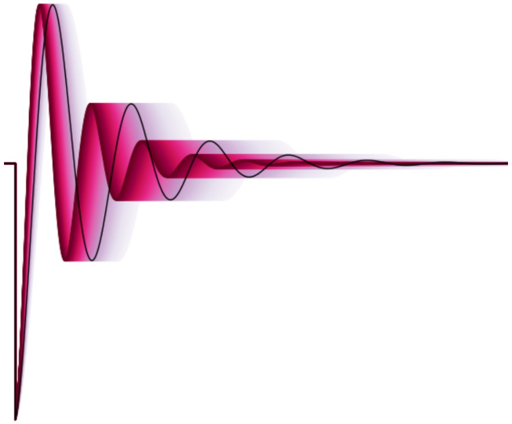
- ❖ Tests of Kerr hypothesis with ringdown signals

- ❖ 。。。。。。



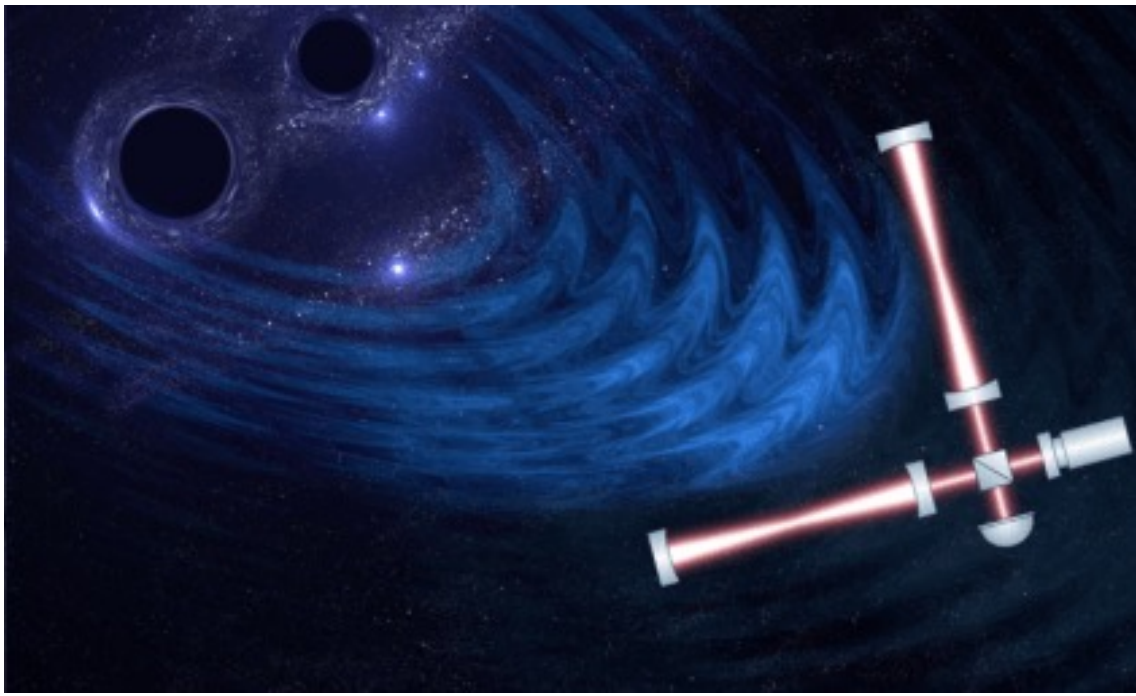


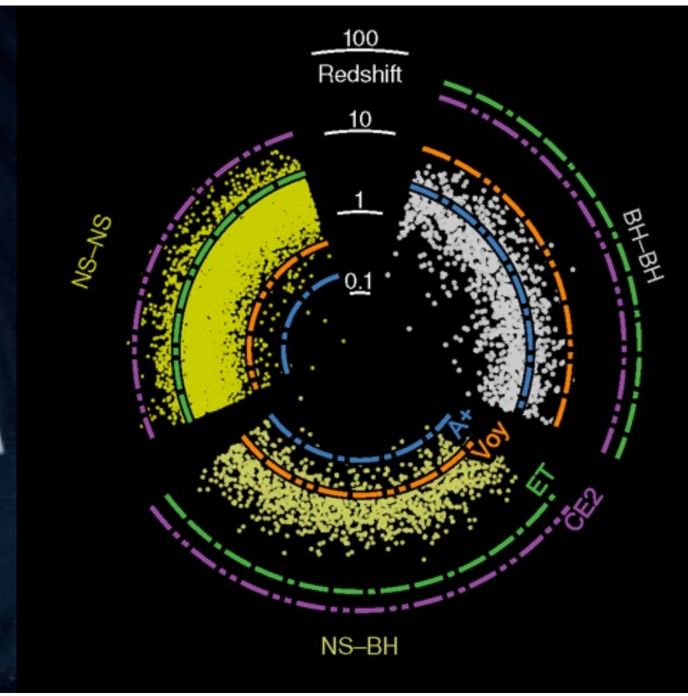




传播效应会随着引力波在宇宙学距离上的传播而不断累积

未来地面和空间引力波探测具有探测宇宙学距离上波源的能力（红移 10 或更远）

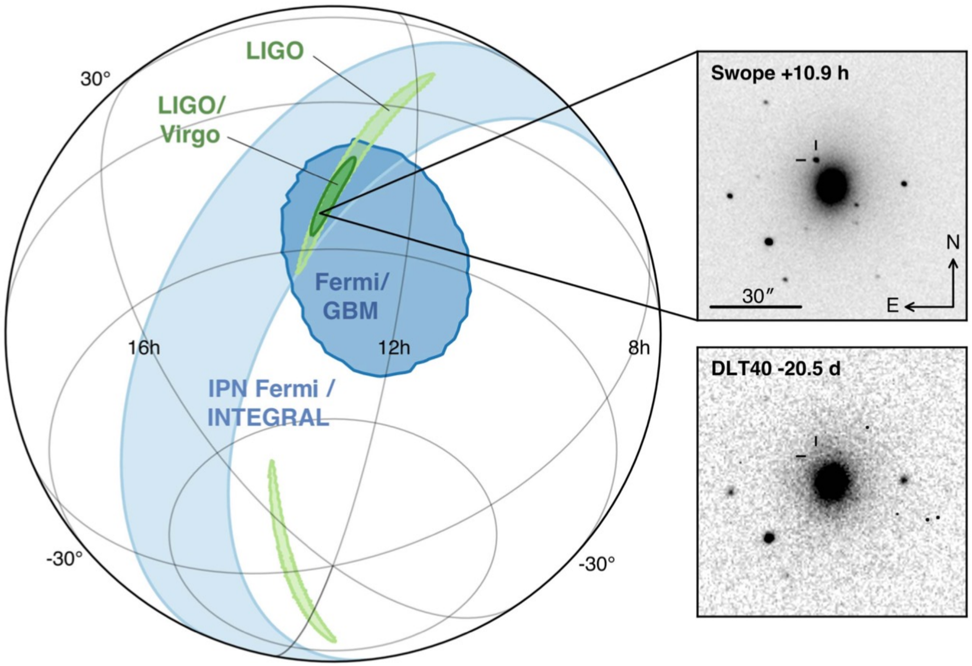




GW170817/GRB170817A 对引力波速度的限制

B. P. Abbott et al.,
ApJL 848, L13 (2017).

$$-3 \times 10^{-15} \leq c_g/c - 1 \leq 7 \times 10^{-16}.$$



- ❖ **极大地压缩了很多引力理论的参数空间**
- ❖ **甚至排除了很多的修改引力**

Tests of GR with LIGO-Virgo-KAGRA events

❖ Tests with GWTC-3

Event	Inst.	Properties					SNR	Tests performed							
		D_L [Gpc]	$(1+z)M$ [$M_\odot$]	$(1+z)\mathcal{M}$ [$M_\odot$]	$(1+z)M_f$ [$M_\odot$]	χ_f		RT	IMR	PAR	SIM	MDR	POL	RD	ECH
GW191109_010717	HL	$1.29^{+1.13}_{-0.65}$	140^{+21}_{-17}	$60.1^{+9.8}_{-9.3}$	135^{+19}_{-15}	$0.61^{+0.18}_{-0.19}$	$17.3^{+0.5}_{-0.5}$	✓	–	–	–	–	✓	✓	✓
GW191129_134029	HL	$0.79^{+0.26}_{-0.33}$	$20.10^{+2.94}_{-0.64}$	$8.49^{+0.06}_{-0.05}$	$19.19^{+3.07}_{-0.67}$	$0.69^{+0.03}_{-0.05}$	$13.1^{+0.2}_{-0.3}$	✓	–	✓	✓	✓	–	–	✓
GW191204_171526	HL	$0.65^{+0.19}_{-0.25}$	$22.74^{+1.94}_{-0.48}$	$9.70^{+0.05}_{-0.05}$	$21.60^{+2.05}_{-0.50}$	$0.73^{+0.03}_{-0.03}$	$17.5^{+0.2}_{-0.2}$	✓	–	✓	✓	✓	✓	–	✓
GW191215_223052	HLV	$1.93^{+0.89}_{-0.86}$	$58.4^{+4.8}_{-3.7}$	$24.9^{+1.5}_{-1.4}$	$55.8^{+4.8}_{-3.3}$	$0.68^{+0.07}_{-0.07}$	$11.2^{+0.3}_{-0.4}$	✓	–	–	–	✓	✓	–	✓
GW191216_213338	HV	$0.34^{+0.12}_{-0.13}$	$21.17^{+2.93}_{-0.66}$	$8.94^{+0.05}_{-0.05}$	$20.18^{+3.06}_{-0.70}$	$0.70^{+0.03}_{-0.04}$	$18.6^{+0.2}_{-0.2}$	✓	–	✓	✓	✓	✓	–	✓
GW191222_033537	HL	$3.0^{+1.7}_{-1.7}$	119^{+16}_{-13}	$51.0^{+7.2}_{-6.5}$	114^{+14}_{-12}	$0.67^{+0.08}_{-0.11}$	$12.5^{+0.2}_{-0.3}$	✓	–	–	–	✓	✓	✓	✓
GW200115_042309	HLV	$0.29^{+0.15}_{-0.10}$	$7.8^{+1.9}_{-1.8}$	$2.58^{+0.01}_{-0.01}$	$7.7^{+1.9}_{-1.8}$	$0.42^{+0.09}_{-0.05}$	$11.3^{+0.3}_{-0.5}$	✓	–	✓	–	–	–	–	✓
GW200129_065458	HLV	$0.90^{+0.29}_{-0.38}$	$74.6^{+4.5}_{-3.8}$	$32.1^{+1.8}_{-2.6}$	$70.9^{+4.2}_{-3.4}$	$0.73^{+0.06}_{-0.05}$	$26.8^{+0.2}_{-0.2}$	✓	✓	✓	✓	✓	✓	✓	✓
GW200202_154313	HLV	$0.41^{+0.15}_{-0.16}$	$19.01^{+1.99}_{-0.34}$	$8.15^{+0.05}_{-0.05}$	$18.12^{+2.09}_{-0.35}$	$0.69^{+0.03}_{-0.04}$	$10.8^{+0.2}_{-0.4}$	✓	–	✓	–	✓	–	–	✓
GW200208_130117	HLV	$2.23^{+1.00}_{-0.85}$	91^{+11}_{-10}	$38.8^{+5.2}_{-4.8}$	$87.5^{+10.3}_{-9.1}$	$0.66^{+0.09}_{-0.13}$	$10.8^{+0.3}_{-0.4}$	✓	✓	–	–	✓	✓	–	✓
GW200219_094415	HLV	$3.4^{+1.7}_{-1.5}$	103^{+14}_{-12}	$43.7^{+6.3}_{-6.2}$	98^{+13}_{-11}	$0.66^{+0.10}_{-0.13}$	$10.7^{+0.3}_{-0.5}$	✓	–	–	–	✓	✓	–	✓
GW200224_222234	HLV	$1.71^{+0.49}_{-0.64}$	$94.9^{+8.3}_{-7.2}$	$40.9^{+3.5}_{-3.8}$	$90.2^{+7.5}_{-6.4}$	$0.73^{+0.07}_{-0.07}$	$20.0^{+0.2}_{-0.2}$	✓	✓	–	–	✓	✓	✓	✓
GW200225_060421	HL	$1.15^{+0.51}_{-0.53}$	$41.2^{+3.0}_{-4.0}$	$17.65^{+0.98}_{-1.97}$	$39.4^{+2.9}_{-3.6}$	$0.66^{+0.07}_{-0.13}$	$12.5^{+0.3}_{-0.4}$	✓	✓	✓	✓	✓	✓	–	✓
GW200311_115853	HLV	$1.17^{+0.28}_{-0.40}$	$75.9^{+6.2}_{-5.7}$	$32.7^{+2.7}_{-2.8}$	$72.4^{+5.6}_{-5.1}$	$0.69^{+0.07}_{-0.08}$	$17.8^{+0.2}_{-0.2}$	✓	✓	✓	–	✓	✓	✓	✓
GW200316_215756	HLV	$1.12^{+0.47}_{-0.44}$	$25.5^{+8.7}_{-1.1}$	$10.68^{+0.12}_{-0.12}$	$24.3^{+9.0}_{-1.1}$	$0.70^{+0.04}_{-0.04}$	$10.3^{+0.4}_{-0.7}$	✓	–	✓	✓	–	–	–	✓

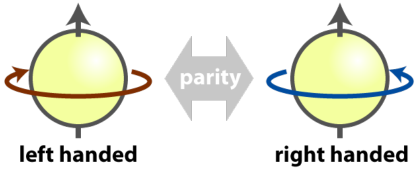
LIGO-Virgo-KAGRA collaboration, arXiv: 2112.06861 [astro-ph].

2.

Modified gravitational wave propagation

GW Propagation in General Relativity

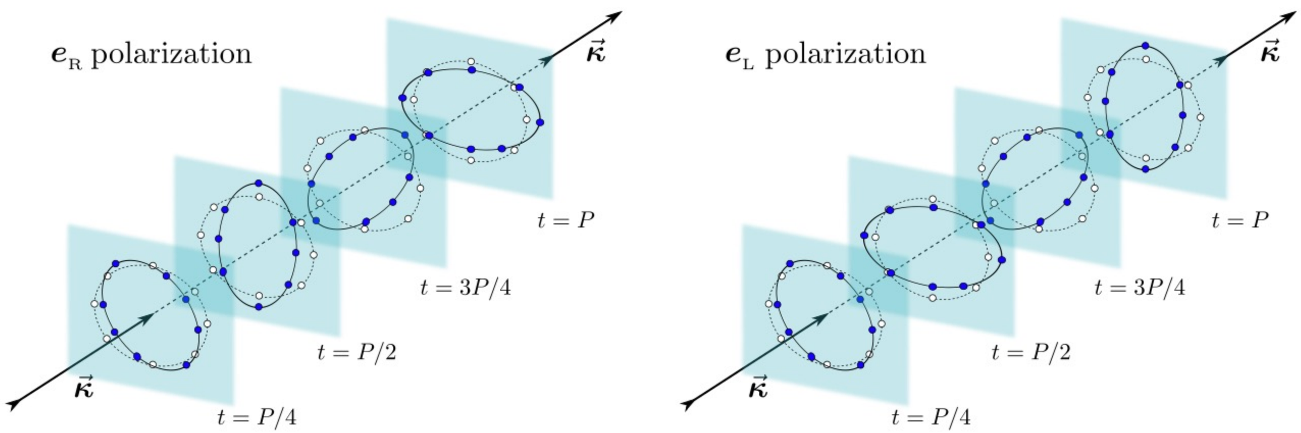
$$h''_A + 2\mathcal{H}h'_A + k^2 h_A = 0, \quad A = R, L$$



$$h_R = \frac{h_+ + ih_\times}{\sqrt{2}}$$

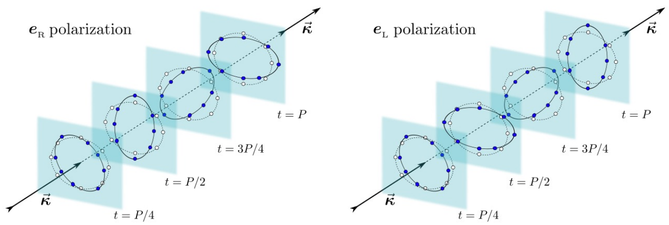
$$h_L = \frac{h_+ - ih_\times}{\sqrt{2}}$$

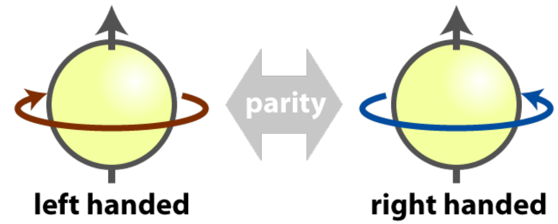
- **Two independent polarization modes: h_R and h_L**
- **Both modes propagate at speed of light**
- **Both modes have the same damping rate**



Credit: 1606.02532

Parametrization of Modified GW propagation





$$h''_A + 2(1+\bar{\nu}+\nu_A)\mathcal{H}h'_A + (1+\bar{\mu}+\mu_A)k^2 h_A = 0, \quad A = R, L$$

GW friction

Amplitude birefringence

Speed or Nonlinear dispersion

Velocity birefringence

- **GW speed and friction (frequency-independent)**
- **Lorentz violation (nonlinear dispersion and frequency-dependent friction)**
- **Parity violation (amplitude and velocity birefringences)**

Case 1: GW speed and friction

frequency-independent $\bar{\mu}, \ \bar{\nu}$

- $\bar{\mu}$ **determine the GW speed：** $c_g = 1 + \bar{\mu}/2$

$$-3 \times 10^{-15} < \bar{\mu}/2 < 7 \times 10^{-16}$$

- $\bar{\nu}$ **modifies GW friction**

$$d_L^{\rm gw}(z) = d_L^{\rm em} \exp\left\{\frac{1}{2}\int_0^z \bar{\nu}\frac{dz'}{1+z'}\right\}$$

GWTC-3 with BBH population gives

$$-3 < \bar{\nu} < 2.5$$

利用电磁对应体检验

利用引力波标准汽笛检验（电磁对应体、宿主星系、质量分布与红移分布、强引力透镜等方法）

Horndeski; Brans-Dicke; f(R); f(T); DHOST; Einstein-Aether; Extra dimension; nonlocal gravity; bigravity; spatial covariant gravity; ……

Case 2: Lorentz violating effects

$\bar{\mu}, \ \bar{\nu}$ are frequency-dependent

$$\mathcal{H}\bar{\nu} = \left[\alpha_{\bar{\nu}}(\tau)\,(k/aM_{\rm LV})^{\beta_{\bar{\nu}}}\right]', \qquad k = 2\pi f$$

$$\bar{\mu} = \alpha_{\bar{\mu}}(\tau)\,(k/aM_{\rm LV})^{\beta_{\bar{\mu}}}, \qquad M_{\rm LV}$$

$M_{\rm LV}$ ：Lorentz 破缺能标

❖ Nonlinear dispersion relation

$$\omega_k^2 = k^2(1+\bar{\mu}) \quad \Longrightarrow \quad c_g = 1 + \frac{1}{2}\bar{\mu}(k)$$

❖ frequency-dependent GW friction

$$|h| \propto a^{-1-\bar{\nu}(k)/2}$$

Multi-fractal spacetime; doubly special relativity; Horava; extra dimension; spatial covariant gravities; Non-Commutative Geometries; consistent 4D Gauss-Bonent; Horava with mixed kinetic terms; standard model extension ; ……..

Case 3: Parity violating effects

$\nu_A, \ \mu_A$ **are frequency-dependent**

$$\mathcal{H}\nu_{\rm A} = \left[\rho_{\rm A}\alpha_\nu(\tau)\,(k/aM_{\rm PV})^{\beta_\nu}\right]',$$
$$\mu_{\rm A} = \rho_{\rm A}\alpha_\mu(\tau)\,(k/aM_{\rm PV})^{\beta_\mu},$$

$k = 2\pi f$

$M_{\rm PV}$ ：宇称破缺能标

- **Amplitude birefringence**

$$|h_A| \propto a^{-1-\nu_A(k)/2}$$

$A = R$ or L

$\nu_R = -\nu_L$

$\mu_R = -\mu_L$

- **Velocity birefringence**

$$\omega_A^2(k) = k^2(1+\mu_A) \Longrightarrow c_A = 1 + \frac{1}{2}\mu_A$$

Chern-Simons; chiral scalar-tensor; Horava with parity violation; spatial covariant gravities; Nieh-Yan; symmetric teleparallel gravities; Host action; standard model extension; ……

TABLE I. Corresponding parameters $\mathcal{H}\bar{\nu}$, $\bar{\mu}$, $\mathcal{H}\nu_A$, and μ_A in specific modified theories of gravity. The numbers in the brackets are the values of $\beta_{\bar{\nu}}$, $\beta_{\bar{\mu}}$, β_{ν}, and β_{μ} for each theory, which represent the frequency dependences of $\mathcal{H}\bar{\nu}$, $\bar{\mu}$, $\mathcal{H}\nu_A$, and μ_A.

	Friction and speed		Birefringences		Damping and dispersion	
Theories of gravity	$\mathcal{H}\bar{\nu}$	$\bar{\mu}$	$\mathcal{H}\nu_A$ (β_ν)	μ_A (β_μ)	$\mathcal{H}\bar{\nu}$ ($\beta_{\bar{\nu}}$)	$\bar{\mu}$ ($\beta_{\bar{\mu}}$)
Nonlocal gravity [15, 18, 19]	✓	—	—	—	—	—
Time-dependent Planck mass gravity [20]	✓	—	—	—	—	—
Extra dimension (DGP) [21, 22]	✓	—	—	—	—	—
$f(R)$ gravity [23]	✓	—	—	—	—	—
$f(T)$ gravity [24]	✓	—	—	—	—	—
$f(T, B)$ gravity [25]	✓	—	—	—	—	—
$f(Q)$ gravity [28]	✓	—	—	—	—	—
Galileon Cosmology [29]	✓	—	—	—	—	—
Horndeski [30, 31]	✓	✓	—	—	—	—
beyond Horndeski GLPV [32]	✓	✓	—	—	—	—
DHOST [33]	✓	✓	—	—	—	—
SME gravity sector [34, 35]	✓	✓	—	—	—	—
generalized scalar-torsion gravity [37]	✓	✓	—	—	—	—
teleparallel Horndeski [25]	—	✓	—	—	—	—
generalized TeVeS theory [26, 27]	—	✓	—	—	—	—
effective field theory of inflation [38]	—	✓	—	—	—	—
Scalar-Gauss-Bonnet [36]	—	✓	—	—	—	—
Einstein-Æether [39, 40]	—	✓	—	—	—	—
bumblebee gravity [41]	—	✓	—	—	—	—
Chern-Simons gravity [42, 49–51]	—	—	✓(1)	—	—	—
Palatini Chern-Simons [43]	—	—	✓(1)	✓(1)	—	—
Chiral-scalar-tensor [44–46]	—	—	✓(1)	✓(1)	—	—
Parity-violating scalar-nonmetricity [52–54]	—	—	✓(1)	✓(-1, 1)	—	—
Metric-affine Chern-Simons [47, 48]	—	—	—	✓(-1)	—	—
Nieh-Yan teleparallel [55–57]	—	—	—	✓(-1)	—	—
New general relativity [58]	—	—	—	✓(-1)	—	—
Chiral Weyl gravity [85]	—	—	—	✓(1)	—	✓(2)
Spatial covariant gravities [61–63]	✓	✓	✓ (1)	✓ (1, 3)	✓ (2)	✓ (2, 4)
Havara with parity violation [64–66]	—	✓	—	✓(1, 3)	—	✓(2, 4)
linear gravity with Lorentz violation [77]	—	✓	—	✓($d-4 \geq 1$)	—	✓($d-4 \geq 2$)
diffeomorphism/Lorentz violating linear gravity [78]	—	✓	—	✓($d-4 \geq -1$)	—	✓($d-4 \geq -2$)
Horava with mixed derivative coupling [67]	—	✓	—	—	✓(2)	✓(2, 4)
Horava gravity [68–72]	—	✓	—	—	—	✓(2, 4)
modified dispersion in extra dimension [79]	—	—	—	—	—	✓(2)
Noncommutative Geometry [80, 81]	—	—	—	—	—	✓(-2, 2)
Double special relativity theory [82–84]	—	—	—	—	—	✓(-2, 1)
consistent 4D Einstein-Gauss-Bonnet [73–75]	—	—	—	—	—	✓(2)
Lorentz violating Weyl gravity [76]	—	—	—	—	—	✓(2)
Massive gravity [59, 60]	—	—	—	—	—	✓(-2)

Amplitude and phase modifications to waveform

$$h''_{\rm A} + (2+\bar{\nu}+\nu_{\rm A})\mathcal{H}h'_{\rm A} + (1+\bar{\mu}+\mu_{\rm A})k^2 h_{\rm A} = 0, \qquad {\rm A} = {\rm R \ or \ L}$$

摩擦项修正　振幅双折射　色散关系修正　速度双折射

$$\mathcal{H}\bar{\nu} = \left[\alpha_{\bar{\nu}}(\tau)\,(k/aM_{\rm LV})^{\beta_{\bar{\nu}}}\right]', \quad \mathcal{H}\nu_{\rm A} = \left[\rho_{\rm A}\alpha_{\nu}(\tau)\,(k/aM_{\rm PV})^{\beta_{\nu}}\right]',$$

$$\bar{\mu} = \alpha_{\bar{\mu}}(\tau)\,(k/aM_{\rm LV})^{\beta_{\bar{\mu}}}, \qquad \mu_{\rm A} = \rho_{\rm A}\alpha_{\mu}(\tau)\,(k/aM_{\rm PV})^{\beta_{\mu}},$$

Amplitude modifications

$$h_A = h_A^{\rm GR}(1 + \rho_A \delta h_1 + \delta h_2)$$

Amplitude birefringence ($\rho_A \delta h_1$)　GW friction (δh_2)

Phase modifications

$$h_A = h_A^{\rm GR} e^{i(\rho_A \delta\Psi_1 + \delta\Psi_2)}$$

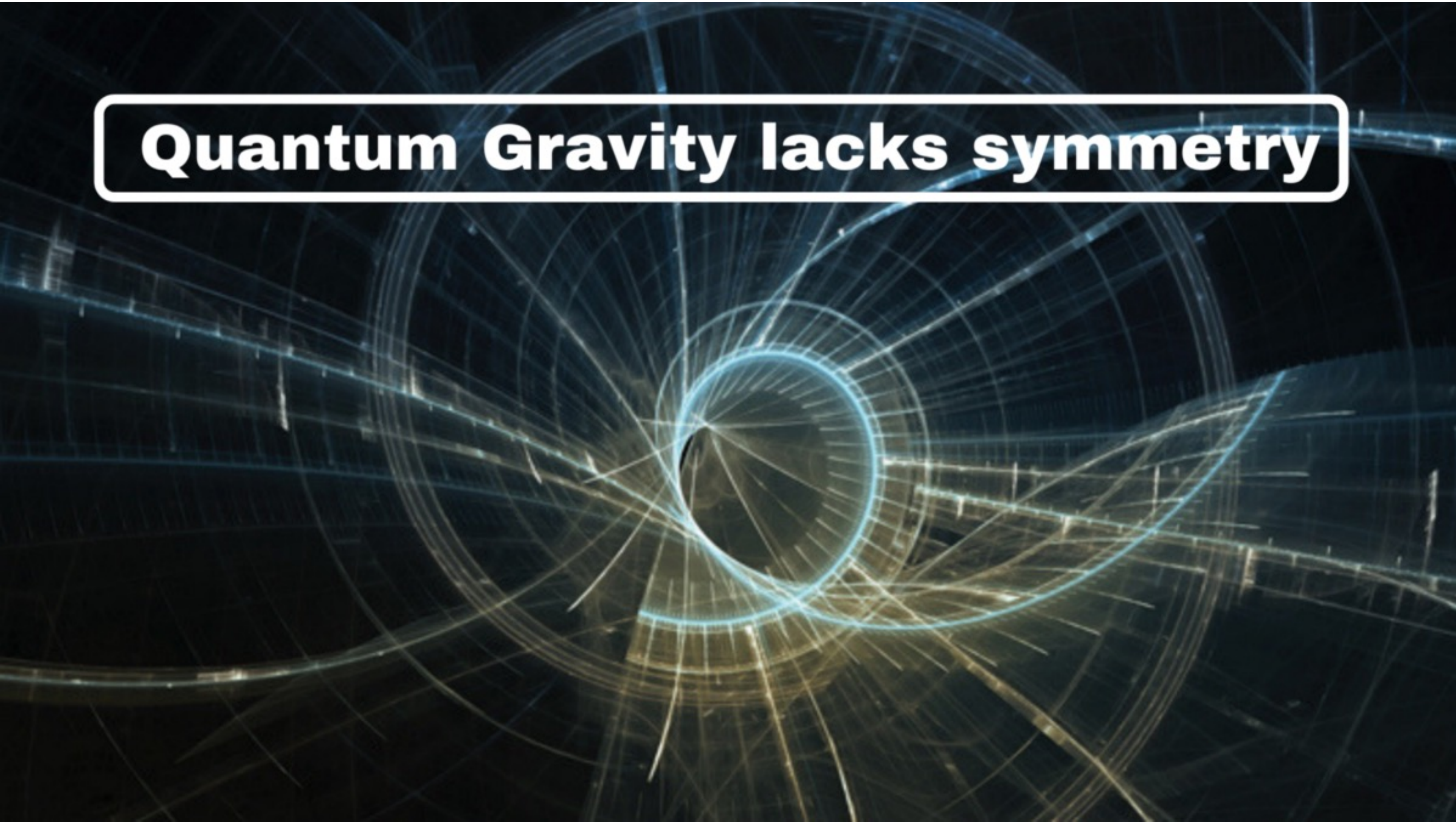
Velocity birefringence ($\rho_A \delta\Psi_1$)　Nonlinear dispersion ($\delta\Psi_2$)

W.Zhao, T. Liu, L. Wen, TZ, A. Wang, EPJC 80 (2020) 630

Y.F. Wang, R. Niu, TZ, W.Zhao, ApJ 908, 58 (2021)

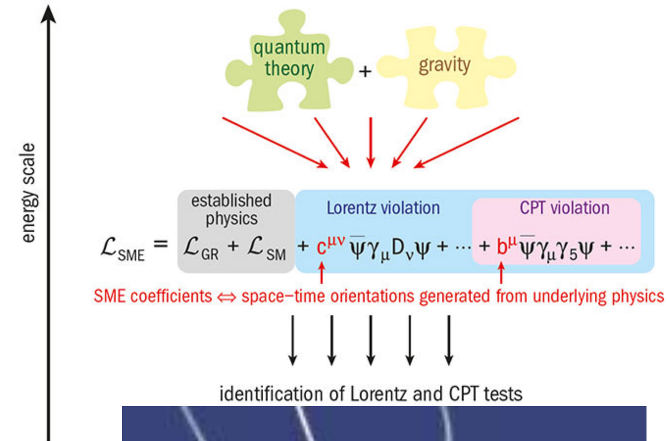
3.

Tests of Lorentz symmetry of gravity

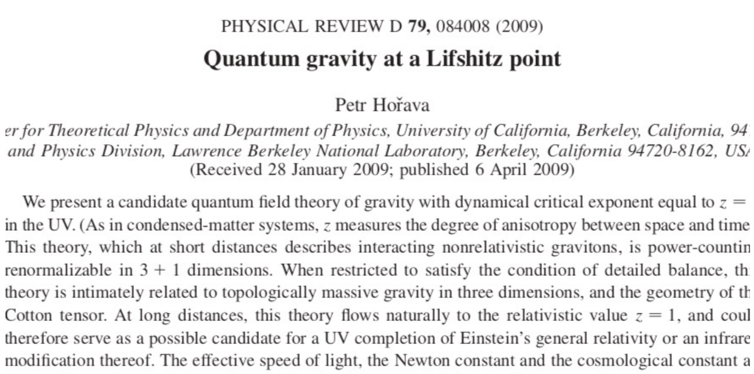

Quantum Gravity lacks symmetry

Lorentz violation in gravity

Standard Model Extension



Horava-Lifshitz Gravity


PHYSICAL REVIEW D **79,** 084008 (2009)

Quantum gravity at a Lifshitz point

Petr Hořava

er for Theoretical Physics and Department of Physics, University of California, Berkeley, California, 94
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(Received 28 January 2009; published 6 April 2009)

We present a candidate quantum field theory of gravity with dynamical critical exponent equal to $z =$ in the UV. (As in condensed-matter systems, z measures the degree of anisotropy between space and time This theory, which at short distances describes interacting nonrelativistic gravitons, is power-countin renormalizable in $3 + 1$ dimensions. When restricted to satisfy the condition of detailed balance, th theory is intimately related to topologically massive gravity in three dimensions, and the geometry of th Cotton tensor. At long distances, this theory flows naturally to the relativistic value $z = 1$, and coul therefore serve as a possible candidate for a UV completion of Einstein's general relativity or an infrare modification thereof. The effective speed of light, the Newton constant and the cosmological constant a

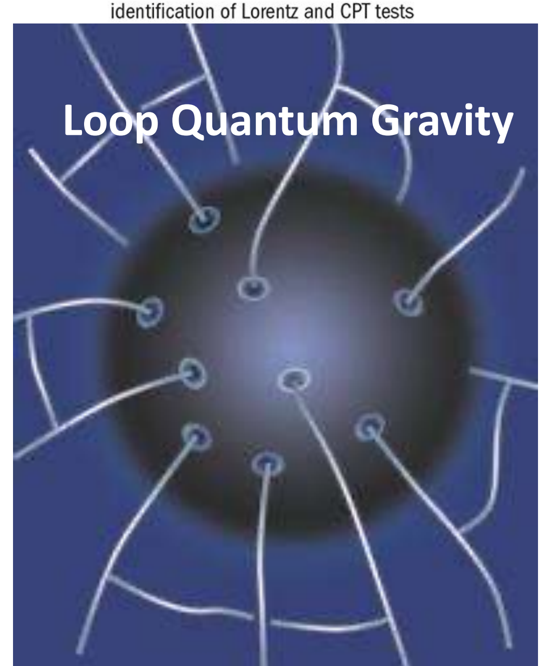


TABLE I. Corresponding parameters $\mathcal{H}\bar{\nu}$, $\bar{\mu}$, $\mathcal{H}\nu_A$, and μ_A in specific modified theories of gravity. The numbers in the brackets are the values of $\beta_{\bar{\nu}}$, $\beta_{\bar{\mu}}$, β_{ν}, and β_{μ} for each theory, which represent the frequency dependences of $\mathcal{H}\bar{\nu}$, $\bar{\mu}$, $\mathcal{H}\nu_A$, and μ_A.

Theories of gravity	Friction and speed		Birefringences		Damping and dispersion	
	$\mathcal{H}\bar{\nu}$	$\bar{\mu}$	$\mathcal{H}\nu_A$ (β_ν)	μ_A (β_μ)	$\mathcal{H}\bar{\nu}$ ($\beta_{\bar{\nu}}$)	$\bar{\mu}$ ($\beta_{\bar{\mu}}$)
Nonlocal gravity [15, 18, 19]	✓	—	—	—	—	—
Time-dependent Planck mass gravity [20]	✓	—	—	—	—	—
Extra dimension (DGP) [21, 22]	✓	—	—	—	—	—
$f(R)$ gravity [23]	✓	—	—	—	—	—
$f(T)$ gravity [24]	✓	—	—	—	—	—
$f(T, B)$ gravity [25]	✓	—	—	—	—	—
$f(Q)$ gravity [28]	✓	—	—	—	—	—
Galileon Cosmology [29]	✓	—	—	—	—	—
Horndeski [30, 31]	✓	✓	—	—	—	—
beyond Horndeski GLPV [32]	✓	✓	—	—	—	—
DHOST [33]	✓	✓	—	—	—	—
SME gravity sector [34, 35]	✓	✓	—	—	—	—
generalized scalar-torsion gravity [37]	✓	✓	—	—	—	—
teleparallel Horndeski [25]	—	✓	—	—	—	—
generalized TeVeS theory [26, 27]	—	✓	—	—	—	—
effective field theory of inflation [38]	—	✓	—	—	—	—
Scalar-Gauss-Bonnet [36]	—	✓	—	—	—	—
Einstein-Æther [39, 40]	—	✓	—	—	—	—
bumblebee gravity [41]	—	✓	—	—	—	—
Chern-Simons gravity [42, 49–51]	—	—	✓(1)	—	—	—
Palatini Chern-Simons [43]	—	—	✓(1)	✓(1)	—	—
Chiral-scalar-tensor [44–46]	—	—	✓(1)	✓(1)	—	—
Parity-violating scalar-nonmetricity [52–54]	—	—	✓(1)	✓(-1, 1)	—	—
Metric-affine Chern-Simons [47, 48]	—	—	—	✓(-1)	—	—
Nieh-Yan teleparallel [55–57]	—	—	—	✓(-1)	—	—
New general relativity [58]	—	—	—	✓(-1)	—	—
Chiral Weyl gravity [85]	—	—	—	✓(1)	—	✓(2)
Spatial covariant gravities [61–63]	✓	✓	✓ (1)	✓ (1, 3)	✓ (2)	✓ (2, 4)
Havara with parity violation [64–66]	—	✓	—	✓(1, 3)	—	✓(2, 4)
linear gravity with Lorentz violation [77]	—	✓	—	✓($d-4\geq 1$)	—	✓($d-4\geq 2$)
diffeomorphism/Lorentz violating linear gravity [78]	—	✓	—	✓($d-4\geq -1$)	—	✓($d-4\geq -2$)
Horava with mixed derivative coupling [67]	—	✓	—	—	✓(2)	✓(2, 4)
Horava gravity [68–72]	—	✓	—	—	—	✓(2, 4)
modified dispersion in extra dimension [79]	—	—	—	—	—	✓(2)
Noncommutative Geometry [80, 81]	—	—	—	—	—	✓(-2, 2)
Double special relativity theory [82–84]	—	—	—	—	—	✓(-2, 1)
consistent 4D Einstein-Gauss-Bonnet [73–75]	—	—	—	—	—	✓(2)
Lorentz violating Weyl gravity [76]	—	—	—	—	—	✓(2)
Massive gravity [59, 60]	—	—	—	—	—	✓(-2)

GW waveform with Lorentz violation

$\bar{\mu}$，$\bar{\nu}$ 和引力波频率相关

$$\mathcal{H}\bar{\nu} = \left[\alpha_{\bar{\nu}}(\tau)\left(k/aM_{\rm LV}\right)^{\beta_{\bar{\nu}}}\right]',$$

$$\bar{\mu} = \alpha_{\bar{\mu}}(\tau)\left(k/aM_{\rm LV}\right)^{\beta_{\bar{\mu}}},$$

$k = 2\pi f$

$M_{\rm LV}$ ： Lorentz 破缺能标

❖ **Lorentz 破缺的引力波波形**

$$\tilde{h}_A(f) = \tilde{h}_A^{\rm GR}(1 + \rho_A \delta h_1 + \delta h_2)e^{i(\rho_A \delta\Psi_1 + \delta\Psi_2)},$$

$$\delta h_2 = -\frac{1}{2}\left(\frac{2\pi f}{M_{\rm LV}}\right)^{\beta_{\bar{\nu}}}\left[\alpha_{\bar{\nu}}(\tau_0) - \alpha_{\bar{\nu}}(\tau_e)(1+z)^{\beta_{\bar{\nu}}}\right]$$

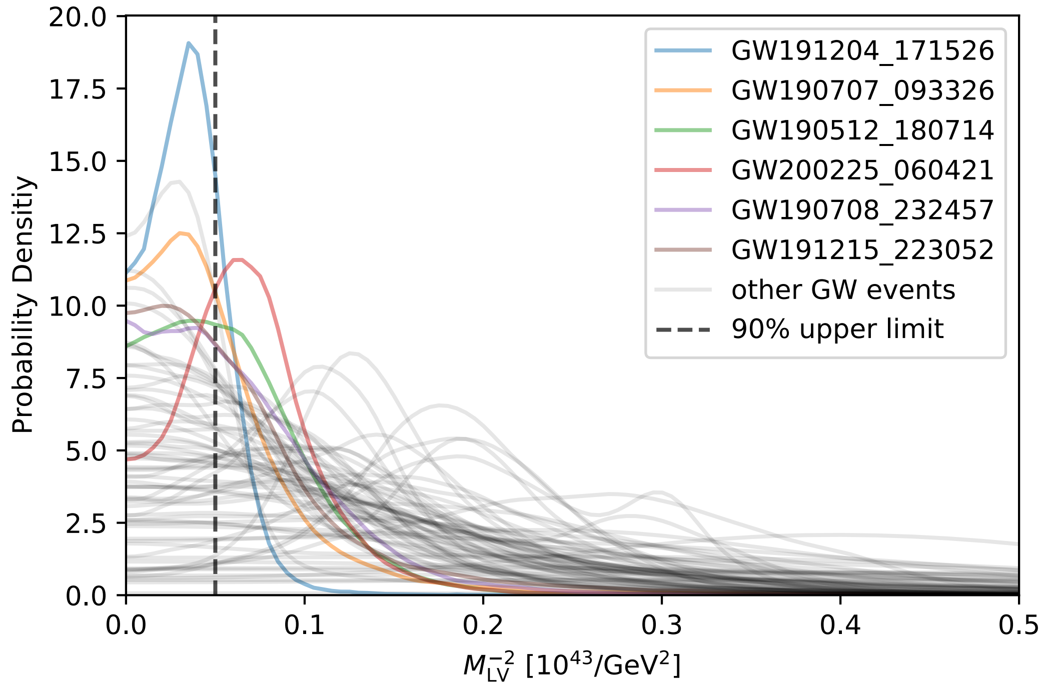
$$\delta\Psi_2 = \begin{cases} \frac{(2/M_{\rm LV})^{\beta_{\bar{\mu}}}}{\beta_{\bar{\mu}}+1}(\pi f)^{\beta_{\bar{\mu}}+1}\int_{t_e}^{t_0}\frac{\alpha_{\bar{\mu}}}{a^{\beta_{\bar{\mu}}+1}}dt, & \beta_{\bar{\mu}} \neq -1, \\ \frac{M_{\rm LV}}{2}\ln u \int_{t_e}^{t_0}\alpha_{\mu}dt, & \beta_{\bar{\mu}} = -1, \end{cases}$$

Multi-fractal spacetime; doubly special relativity; Horava; extra dimension; spatial covariant gravities; Non-Commutative Geometries; consistent 4D Gauss-Bonent; Horava with mixed kinetic terms; standard model extension； ……..

Test of frequency-dependent damping rate

$\bar{\nu}$ 和引力波频率相关

$$\delta h_2 = -\frac{1}{2}\left(\frac{2\pi f}{M_{\rm LV}}\right)^{\beta_{\bar{\nu}}}\left[\alpha_{\bar{\nu}}(\tau_0) - \alpha_{\bar{\nu}}(\tau_e)(1+z)^{\beta_{\bar{\nu}}}\right] \qquad \beta_{\bar{\nu}} = 2$$



$$M_{\rm LV} \gtrsim 1.4 \times 10^{-21}\ \mathrm{GeV}$$

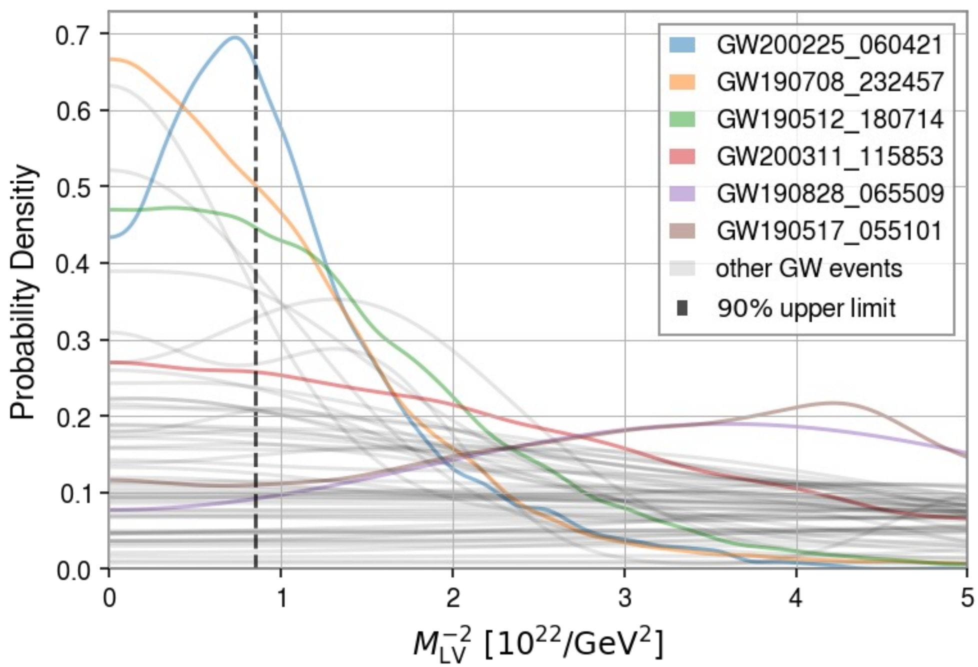
T. Zhu, W. Zhao, A. Wang, et. al., in preparation (2023)

Tests of nonlinear dispersion relation

$\bar{\mu}$ 和引力波频率相关

$$\delta\Psi_2 = \begin{cases} \frac{(2/M_{\rm LV})^{\beta_{\bar{\mu}}}}{\beta_{\bar{\mu}}+1}(\pi f)^{\beta_{\bar{\mu}}+1}\int_{t_e}^{t_0}\frac{\alpha_{\bar{\mu}}}{a^{\beta_{\bar{\mu}}+1}}dt, & \beta_{\bar{\mu}} \neq -1, \\ \frac{M_{\rm LV}}{2}\ln u\int_{t_e}^{t_0}\alpha_{\mu}dt, & \beta_{\bar{\mu}} = -1, \end{cases}$$

$\beta_{\bar{\mu}} = 2$ and 4



$\beta_{\bar{\mu}} = 2$

$$M_{\rm LV} \gtrsim 1.1 \times 10^{-11}\ {\rm GeV}$$

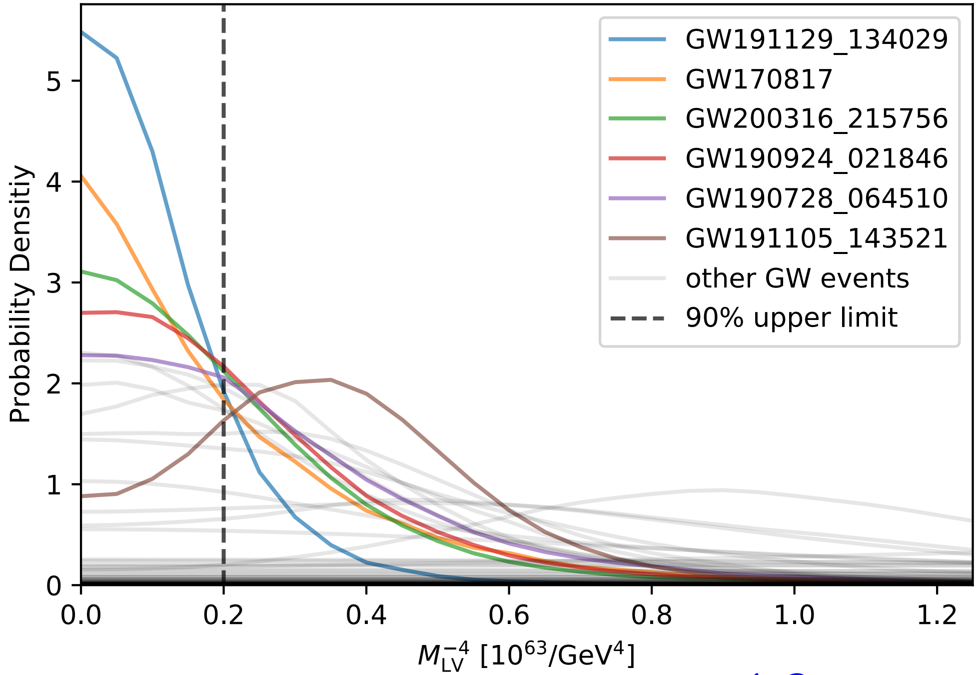
T. Zhu, W. Zhao, A. Wang, et. al., in preparation (2023)

Tests of nonlinear dispersion relation： high-order spatial derivative case

$\bar{\mu}$ 和引力波频率相关

$$\delta\Psi_2 = \begin{cases} \frac{(2/M_{\rm LV})^{\beta_{\bar{\mu}}}}{\beta_{\bar{\mu}}+1}(\pi f)^{\beta_{\bar{\mu}}+1}\int_{t_e}^{t_0}\frac{\alpha_{\bar{\mu}}}{a^{\beta_{\bar{\mu}}+1}}dt, & \beta_{\bar{\mu}} \neq -1, \\ \frac{M_{\rm LV}}{2}\ln u \int_{t_e}^{t_0}\alpha_\mu dt, & \beta_{\bar{\mu}} = -1, \end{cases}$$

$\beta_{\bar{\mu}} = 2$ and 4



$\beta_{\bar{\mu}} = 4$

$$M_{\rm LV} \gtrsim 2.7 \times 10^{-16}\ {\rm GeV}$$

C. Gong, T. Zhu, R. Niu, Q. Wu, J.L. Cui, X. Zhang, W. Zhao, A. Wang, PRD 105, 033034 (2022).

T. Zhu, W. Zhao, A. Wang, et. al., in preparation (2023)

4.

Tests of parity symmetry of gravity

Nature is parity violating

The End of Parity Symmetry in Weak Interaction

(T.D. Lee and C.N. Yang 1956; C.S. Wu 1957)

$$^{60}_{27}\mathrm{Co} \rightarrow \, ^{60}_{28}\mathrm{Ni} + e^{-} + \bar{\nu}_e \, .$$

TABLE I. Corresponding parameters $\mathcal{H}\bar{\nu}$, $\bar{\mu}$, $\mathcal{H}\nu_A$, and μ_A in specific modified theories of gravity. The numbers in the brackets are the values of $\beta_{\bar{\nu}}$, $\beta_{\bar{\mu}}$, β_{ν}, and β_{μ} for each theory, which represent the frequency dependences of $\mathcal{H}\bar{\nu}$, $\bar{\mu}$, $\mathcal{H}\nu_A$, and μ_A.

Theories of gravity	Friction and speed		Birefringences		Damping and dispersion	
	$\mathcal{H}\bar{\nu}$	$\bar{\mu}$	$\mathcal{H}\nu_A$ (β_ν)	μ_A (β_μ)	$\mathcal{H}\bar{\nu}$ ($\beta_{\bar{\nu}}$)	$\bar{\mu}$ ($\beta_{\bar{\mu}}$)
Nonlocal gravity [15, 18, 19]	✓	—	—	—	—	—
Time-dependent Planck mass gravity [20]	✓	—	—	—	—	—
Extra dimension (DGP) [21, 22]	✓	—	—	—	—	—
$f(R)$ gravity [23]	✓	—	—	—	—	—
$f(T)$ gravity [24]	✓	—	—	—	—	—
$f(T, B)$ gravity [25]	✓	—	—	—	—	—
$f(Q)$ gravity [28]	✓	—	—	—	—	—
Galileon Cosmology [29]	✓	—	—	—	—	—
Horndeski [30, 31]	✓	✓	—	—	—	—
beyond Horndeski GLPV [32]	✓	✓	—	—	—	—
DHOST [33]	✓	✓	—	—	—	—
SME gravity sector [34, 35]	✓	✓	—	—	—	—
generalized scalar-torsion gravity [37]	✓	✓	—	—	—	—
teleparallel Horndeski [25]	—	✓	—	—	—	—
generalized TeVeS theory [26, 27]	—	✓	—	—	—	—
effective field theory of inflation [38]	—	✓	—	—	—	—
Scalar-Gauss-Bonnet [36]	—	✓	—	—	—	—
Einstein-Æether [39, 40]	—	✓	—	—	—	—
bumblebee gravity [41]	—	✓	—	—	—	—
Chern-Simons gravity [42, 49–51]	—	—	✓(1)	—	—	—
Palatini Chern-Simons [43]	—	—	✓(1)	✓(1)	—	—
Chiral-scalar-tensor [44–46]	—	—	✓(1)	✓(1)	—	—
Parity-violating scalar-nonmetricity [52–54]	—	—	✓(1)	✓(-1, 1)	—	—
Metric-affine Chern-Simons [47, 48]	—	—	—	✓(-1)	—	—
Nieh-Yan teleparallel [55–57]	—	—	—	✓(-1)	—	—
New general relativity [58]	—	—	—	✓(-1)	—	—
Chiral Weyl gravity [85]	—	—	—	✓(1)	—	✓(2)
Spatial covariant gravities [61–63]	✓	✓	✓ (1)	✓ (1, 3)	✓ (2)	✓ (2, 4)
Havara with parity violation [64–66]	—	✓	—	✓(1, 3)	—	✓(2, 4)
linear gravity with Lorentz violation [77]	—	✓	—	✓($d-4 \geq 1$)	—	✓($d-4 \geq 2$)
diffeomorphism/Lorentz violating linear gravity [78]	—	✓	—	✓($d-4 \geq -1$)	—	✓($d-4 \geq -2$)
Horava with mixed derivative coupling [67]	—	✓	—	—	✓(2)	✓(2, 4)
Horava gravity [68–72]	—	✓	—	—	—	✓(2, 4)
modified dispersion in extra dimension [79]	—	—	—	—	—	✓(2)
Noncommutative Geometry [80, 81]	—	—	—	—	—	✓(-2, 2)
Double special relativity theory [82–84]	—	—	—	—	—	✓(-2, 1)
consistent 4D Einstein-Gauss-Bonnet [73–75]	—	—	—	—	—	✓(2)
Lorentz violating Weyl gravity [76]	—	—	—	—	—	✓(2)
Massive gravity [59, 60]	—	—	—	—	—	✓(-2)

Amplitude Birefringence of GWs

$$h_A'' + (2 + \bar{\nu} + \nu_A)\mathcal{H}h_A' + (1 + \bar{\mu} + \mu_A)k^2 h_A = 0, \quad \mathrm{A = R \text{ or } L}$$

⇓

It determines amplitude evolution of GWs. Different values for left and right-hand modes follows different dampings of circular polarization modes. This is the amplitude birefringence.

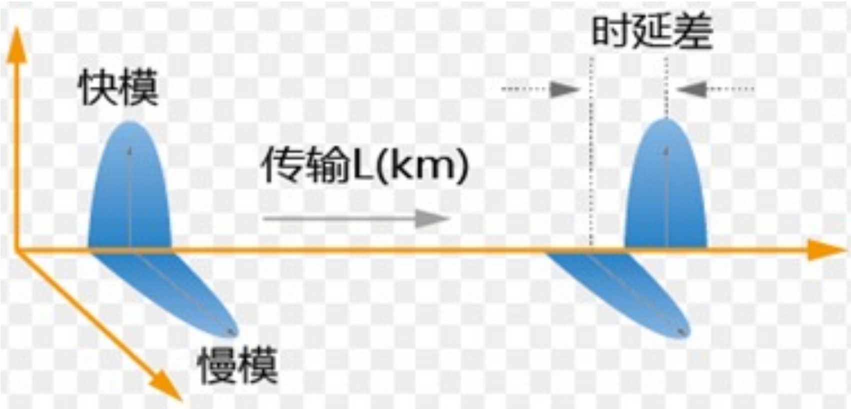
If ignoring the time-dependence of ν_A, the evolution of GWs follows the relation

$$|\tilde{h}_A| \propto a^{-1-\nu_A/2},$$

Velocity Birefringence of GWs

$$h''_{\rm A} + (2+\bar{\nu}+\nu_{\rm A})\mathcal{H}h'_{\rm A} + (1+\bar{\mu}+\mu_{\rm A})k^2 h_{\rm A} = 0$$

It determines the velocity of graviton. Different values for left and right-hand polarization modes follow that the velocities of two modes are different, which is the so-called velocity birefringence.



In general, $\mu_{\rm A}$ can be parameterized as $\mu_{\rm A} = \alpha\rho_{\rm A}(k/a\Lambda)^\beta$, $\rho_{\rm R} = 1$ and $\rho_{\rm L} = -1$, which equals to the modified dispersion relation

$$\omega_{\rm A}^2(k) = k^2[1+{\rm sgn}(\alpha)\rho_{\rm A}(k/a\Lambda)^\beta],$$

which follows the group velocity of GWs, *i.e.*

$$v_{\rm A} = 1 - {\rm sgn}(\alpha)(1/2)\rho_{\rm A}(k/a\Lambda)^\beta,$$

* which follows that if one polarization mode is superluminal, then the other is subluminal.

GW waveform with parity violation

ν_A，μ_A 和引力波频率相关

$A = R \text{ or } L$

$$\mathcal{H}\nu_{\rm A} = \left[\rho_{\rm A}\alpha_\nu(\tau)\,(k/aM_{\rm PV})^{\beta_\nu}\right]',$$

$$\mu_{\rm A} = \rho_{\rm A}\alpha_\mu(\tau)\,(k/aM_{\rm PV})^{\beta_\mu},$$

$k = 2\pi f$

$M_{\rm PV}$：宇称破缺能标

❖ 宇称破缺的引力波波形

$$\tilde{h}_A(f) = \tilde{h}_A^{\rm GR}(1 + \rho_A\delta h_1 + \delta h_2)e^{i(\rho_A\delta\Psi_1 + \delta\Psi_2)},$$

$$\delta h_1 = -\frac{1}{2}\left(\frac{2\pi f}{M_{\rm PV}}\right)^{\beta_\nu}\left[\alpha_\nu(\tau_0) - \alpha_\nu(\tau_e)(1+z)^{\beta_\nu}\right]$$

$$\delta\Psi_1 = \begin{cases} \frac{(2/M_{\rm PV})^{\beta_\mu}}{\beta_\mu+1}(\pi f)^{\beta_\mu+1}\int_{t_e}^{t_0}\frac{\alpha_\mu}{a^{\beta_\mu+1}}dt, & \beta_\mu \neq -1, \\ \frac{M_{\rm PV}}{2}\ln u\int_{t_e}^{t_0}\alpha_{\bar\mu}dt, & \beta_\mu = -1, \end{cases}$$

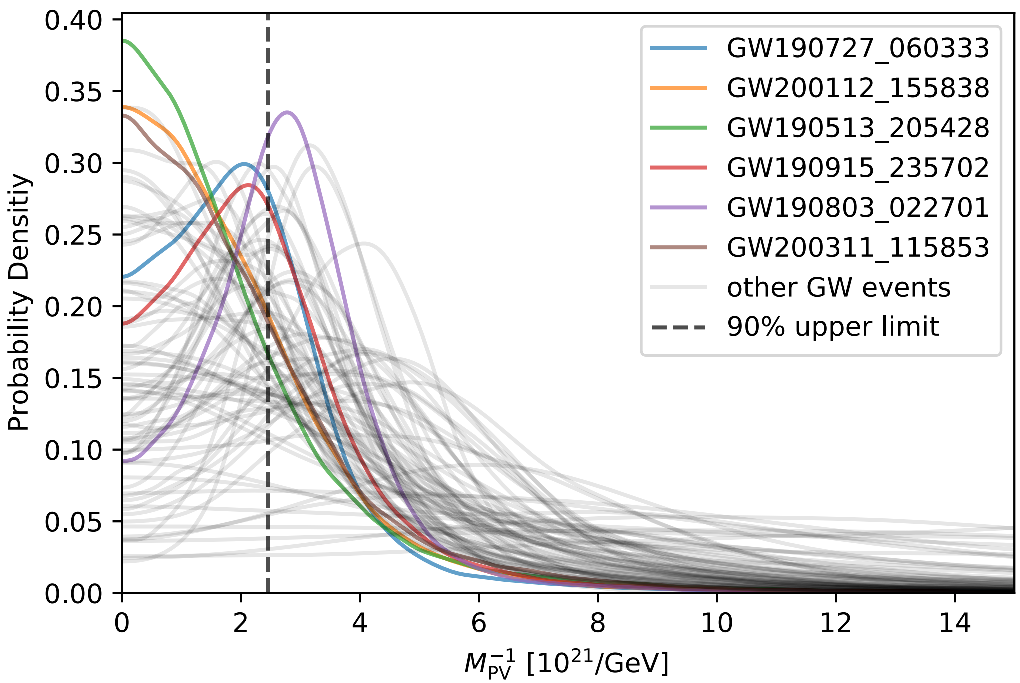
Chern-Simons; chiral scalar-tensor; Horava with parity violation; spatial covariant gravities; Nieh-Yan; symmetric teleparallel gravities; Host action; standard model extension; ……

Tests of amplitude birefringence

ν_A 和引力波频率相关

$$\delta h_1 = -\frac{1}{2}\left(\frac{2\pi f}{M_{\rm PV}}\right)^{\beta_\nu}\left[\alpha_\nu(\tau_0) - \alpha_\nu(\tau_e)(1+z)^{\beta_\nu}\right]$$

$\beta_\nu = 1$



$\beta_\nu = 1$

$$M_{\rm PV} \gtrsim 4.1 \times 10^{-22}\ {\rm GeV}$$

Y.F. Wang, R. Niu, T. Zhu, W. Zhao, APJ 908, 58 (2021).

T. Zhu, W. Zhao, A. Wang, et. al., in preparation (2023)

Phase corrections due to velocity birefringence

$$h_+ = h_+^{\rm GR} - h_\times^{GR}\delta\Psi_1$$
$$h_\times = h_\times^{\rm GR} + h_+^{\rm GR}\delta\Psi_1$$

$$\delta\Psi_1 \sim f^{\beta_\mu+1}$$

$$\delta\Psi_1 = \begin{cases} \frac{(2/M_{\rm PV})^{\beta_\mu}}{\beta_\mu+1} \frac{u^{\beta_\mu+1}}{\mathcal{M}^{\beta_\mu+1}} \int_{t_e}^{t_0} \frac{\alpha_\mu}{a^{\beta_\mu+1}}, & \beta_\mu \neq -1, \\ \frac{M_{\rm PV}}{2} \ln u \int_{t_e}^{t_0} \alpha_\mu dt, & \beta_\mu = -1. \end{cases}$$

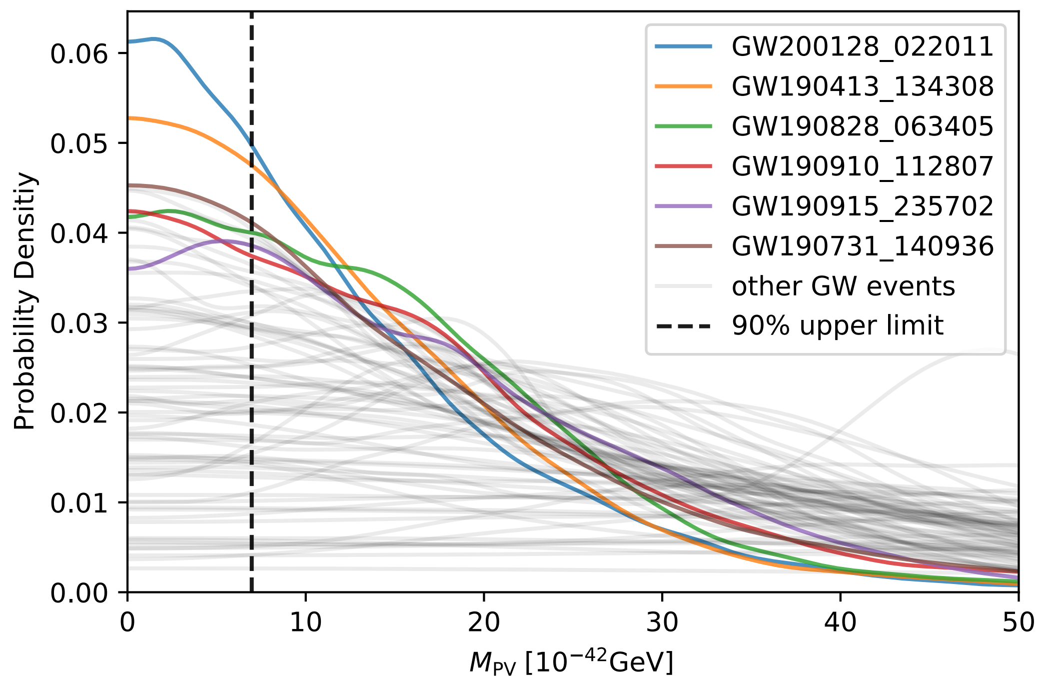
$\beta_\mu = -1$	**Nieh-Yan modified teleparallel gravity** **Symmetric teleparallel equivalence of GR theory; …**
$\beta_\mu = 1$	Ghost-free PV gravities, Chrial scalar-tensor gravity Horava-Lifshitz gravity + parity violation term; Symmetric teleparallel equivalence of GR theory; Spatial covariant gravities; ….
$\beta_\mu = 3$	Horava-Lifshitz gravity + parity violation term; Spatial covariant gravities; ….

Tests of velocity birefringence: $\beta_\mu = -1$

μ_A 和引力波频率相关

$$\delta\Psi_1 = \begin{cases} \frac{(2/M_{\rm PV})^{\beta_\mu}}{\beta_\mu+1}(\pi f)^{\beta_\mu+1}\int_{t_e}^{t_0}\frac{\alpha_\mu}{a^{\beta_\mu+1}}dt, & \beta_\mu \neq -1, \\ \frac{M_{\rm PV}}{2}\ln u\int_{t_e}^{t_0}\alpha_{\bar\mu}dt, & \beta_\mu = -1, \end{cases}$$

$\beta_\mu = -1$



$\beta_\mu = -1$

$$M_{\rm PV} \lesssim 6.9 \times 10^{-42}\ {\rm GeV}$$

Q. Wu, **T. Zhu**, R. Niu, W. Zhao and A. Wang, PRD105, 024035 (2022)

T. Zhu, W. Zhao, A. Wang, et. al., in preparation (2023)

Tests of velocity birefringence: $\beta_\mu = 1$

μ_A 和引力波频率相关

$$\delta\Psi_1 = \begin{cases} \frac{(2/M_{\rm PV})^{\beta_\mu}}{\beta_\mu+1}(\pi f)^{\beta_\mu+1}\int_{t_e}^{t_0}\frac{\alpha_\mu}{a^{\beta_\mu+1}}dt, & \beta_\mu \neq -1, \\ \frac{M_{\rm PV}}{2}\ln u \int_{t_e}^{t_0}\alpha_{\bar{\mu}}dt, & \beta_\mu = -1, \end{cases} \qquad \beta_\mu = 1$$

$$M_{\rm PV} \gtrsim 10\ {\rm GeV}$$

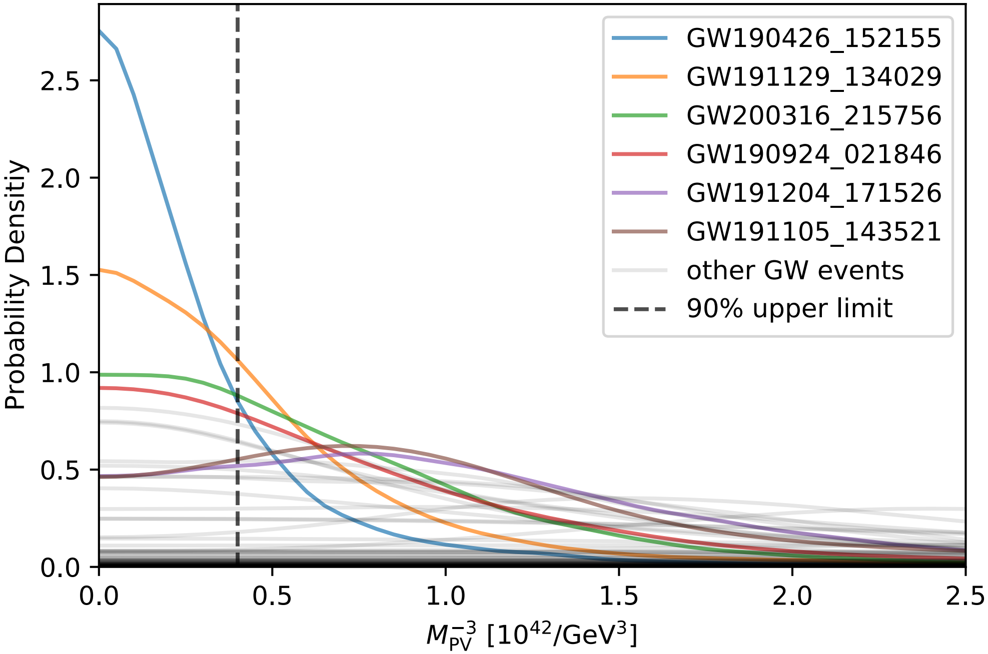
- S. Wang, EPJC80, 342 (2020) ;
- S. Wang and Z.C.Zhao, EPJC80,1032 (2020);
- Y.F.Wang, R.Niu, T.Zhu and W.Zhao, APJ908, 58 (2021) ;
- Y.F.Wang, S.M.Brown, L.Shao and W.Zhao, PRD106, 084005 (2022) ;
- Z.C.Zhao, Z.Cao and S.Wang, APJ 930, 139 (2022).

Tests of velocity birefringence: $\beta_\mu = 3$

μ_A 和引力波频率相关

$$\delta\Psi_1 = \begin{cases} \frac{(2/M_{\rm PV})^{\beta_\mu}}{\beta_\mu+1}(\pi f)^{\beta_\mu+1}\int_{t_e}^{t_0}\frac{\alpha_\mu}{a^{\beta_\mu+1}}dt, & \beta_\mu \neq -1, \\ \frac{M_{\rm PV}}{2}\ln u \int_{t_e}^{t_0}\alpha_{\bar{\mu}}dt, & \beta_\mu = -1, \end{cases}$$

$\beta_\mu = 3$



$\beta_\mu = 3$

$$M_{\rm PV} \gtrsim 1.2 \times 10^{-14}\ {\rm GeV}$$

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Summary

- A parametrized GW propagations

$$h''_A + 2(1+\bar{\nu}+\nu_A)\mathcal{H}h'_A + (1+\bar{\mu}+\mu_A)k^2 h_A = 0, \quad A = R, L$$

- GW waveform with Lorentz/parity violation

$$\tilde{h}_{\rm A}(f) = \tilde{h}^{\rm GR}_{\rm A}(f)(1+\rho_{\rm A}\delta h_1 + \delta h_2)e^{i(\rho_{\rm A}\delta\Psi_1 + \delta\Psi_2)},$$

- GW constraints on M_LV and M_PV

TABLE III. Results from the Bayesian analysis of the parity- and Lorentz-violating waveforms with GW events in GWTC-3. The table shows 90%-credible upper bounds on $M_{\rm PV}$ for $\beta_\nu = -1$ (for velocity birefringence) and lower bounds on $M_{\rm PV}$ and $M_{\rm LV}$ for other cases. We also include bounds for several cases derived from existing tests with GWTC-1/GWTC-2/GWTC-3 in Refs. [6–8, 57, 63, 89] for comparison.

	$M_{\rm PV}$ [GeV]			$M_{\rm LV}$ [GeV]		
	$\beta_\nu = 1$	$\beta_\mu = -1$	$\beta_\mu = 3$	$\beta_{\bar\nu} = 2$	$\beta_{\bar\mu} = 2$	$\beta_{\bar\mu} = 4$
GWTC-1	1.0×10^{-22} [89]	—		—	0.8×10^{-11} [6]	—
GWTC-2	—	6.5×10^{-42} [57]	1.0×10^{-14} [63]	—	1.3×10^{-11} [7]	2.4×10^{-16} [63]
GWTC-3	—	—	—	—	1.8×10^{-11} [8]	—
This work	4.1×10^{-22}	6.9×10^{-42}	1.2×10^{-14}	1.4×10^{-21}	1.1×10^{-11}	2.7×10^{-16}

Summary

$\bar{\mu}$, $\bar{\nu}$ 与 ν_A, μ_A 和引力波频率相关

TABLE III. Results from the Bayesian analysis of the parity- and Lorentz-violating waveforms with GW events in GWTC-3. The table shows 90%-credible upper bounds on $M_{\rm PV}$ for $\beta_\nu = -1$ (for velocity birefringence) and lower bounds on $M_{\rm PV}$ and $M_{\rm LV}$ for other cases. We also include bounds for several cases derived from existing tests with GWTC-1/GWTC-2/GWTC-3 in Refs. [6–8, 57, 63, 89] for comparison.

	$M_{\rm PV}$ [GeV]			$M_{\rm LV}$ [GeV]		
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GWTC-3	—	—	—	—	1.8×10^{-11} [8]	—
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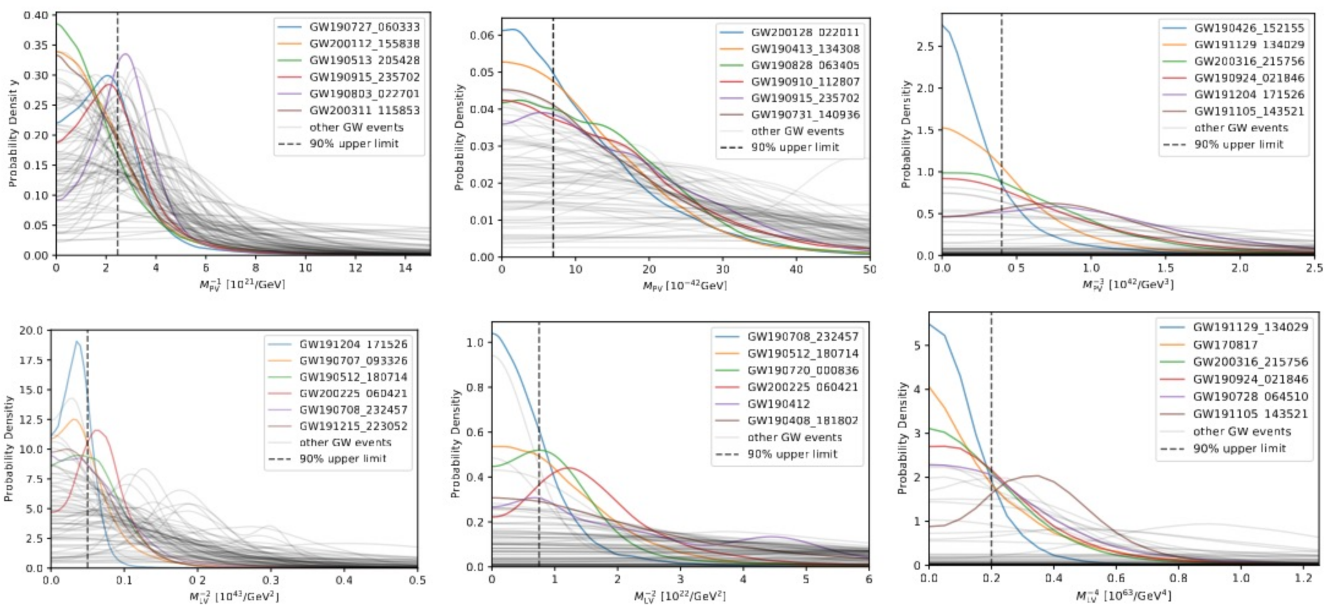

FIG. 1. The posterior distributions for $M_{\rm PV}^{-\beta_\nu}$ with $\beta_\nu = 1$, $M_{\rm PV}^{-\beta_\mu}$ with $\beta_\mu = -1, 3$, $M_{\rm LV}^{-\beta_{\bar{\nu}}}$ with $\beta_\mu = 2$, and $M_{\rm LV}^{-\beta_{\bar{\mu}}}$ with $\beta_{\bar{\mu}} = 2, 4$ from selected GW events in the GWTC-3. The legend indicates the events that give the tightest constraints. The vertical dash line in each figure denotes the 90% upper limits from the combined result.

TABLE I. Corresponding parameters $\mathcal{H}\bar{\nu}$, $\bar{\mu}$, $\mathcal{H}\nu_A$, and μ_A in specific modified theories of gravity. The numbers in the brackets are the values of $\beta_{\bar{\nu}}$, $\beta_{\bar{\mu}}$, β_{ν}, and β_{μ} for each theory, which represent the frequency dependences of $\mathcal{H}\bar{\nu}$, $\bar{\mu}$, $\mathcal{H}\nu_A$, and μ_A.

Theories of gravity	Friction and speed		Birefringences		Damping and dispersion	
	$\mathcal{H}\bar{\nu}$	$\bar{\mu}$	$\mathcal{H}\nu_A$ (β_ν)	μ_A (β_μ)	$\mathcal{H}\bar{\nu}$ ($\beta_{\bar{\nu}}$)	$\bar{\mu}$ ($\beta_{\bar{\mu}}$)
Nonlocal gravity [15, 18, 19]	✓	—	—	—	—	—
Time-dependent Planck mass gravity [20]	✓	—	—	—	—	—
Extra dimension (DGP) [21, 22]	✓	—	—	—	—	—
$f(R)$ gravity [23]	✓	—	—	—	—	—
$f(T)$ gravity [24]	✓	—	—	—	—	—
$f(T,B)$ gravity [25]	✓	—	—	—	—	—
$f(Q)$ gravity [28]	✓	—	—	—	—	—
Galileon Cosmology [29]	✓	—	—	—	—	—
Horndeski [30, 31]	✓	✓	—	—	—	—
beyond Horndeski GLPV [32]	✓	✓	—	—	—	—
DHOST [33]	✓	✓	—	—	—	—
SME gravity sector [34, 35]	✓	✓	—	—	—	—
generalized scalar-torsion gravity [37]	✓	✓	—	—	—	—
teleparallel Horndeski [25]	—	✓	—	—	—	—
generalized TeVeS theory [26, 27]	—	✓	—	—	—	—
effective field theory of inflation [38]	—	✓	—	—	—	—
Scalar-Gauss-Bonnet [36]	—	✓	—	—	—	—
Einstein-Æther [39, 40]	—	✓	—	—	—	—
bumblebee gravity [41]	—	✓	—	—	—	—
Chern-Simons gravity [42, 49–51]	—	—	✓(1)	—	—	—
Palatini Chern-Simons [43]	—	—	✓(1)	✓(1)	—	—
Chiral-scalar-tensor [44–46]	—	—	✓(1)	✓(1)	—	—
Parity-violating scalar-nonmetricity [52–54]	—	—	✓(1)	✓(-1, 1)	—	—
Metric-affine Chern-Simons [47, 48]	—	—	—	✓(-1)	—	—
Nieh-Yan teleparallel [55–57]	—	—	—	✓(-1)	—	—
New general relativity [58]	—	—	—	✓(-1)	—	—
Chiral Weyl gravity [85]	—	—	—	✓(1)	—	✓(2)
Spatial covariant gravities [61–63]	✓	✓	✓ (1)	✓ (1, 3)	✓ (2)	✓ (2, 4)
Havara with parity violation [64–66]	—	✓	—	✓(1, 3)	—	✓(2, 4)
linear gravity with Lorentz violation [77]	—	✓	—	✓($d-4\geq 1$)	—	✓($d-4\geq 2$)
diffeomorphism/Lorentz violating linear gravity [78]	—	✓	—	✓($d-4\geq -1$)	—	✓($d-4\geq -2$)
Horava with mixed derivative coupling [67]	—	✓	—	—	✓(2)	✓(2, 4)
Horava gravity [68–72]	—	✓	—	—	—	✓(2, 4)
modified dispersion in extra dimension [79]	—	—	—	—	—	✓(2)
Noncommutative Geometry [80, 81]	—	—	—	—	—	✓(-2, 2)
Double special relativity theory [82–84]	—	—	—	—	—	✓(-2, 1)
consistent 4D Einstein-Gauss-Bonnet [73–75]	—	—	—	—	—	✓(2)
Lorentz violating Weyl gravity [76]	—	—	—	—	—	✓(2)
Massive gravity [59, 60]	—	—	—	—	—	✓(-2)

Spatial covariant gravities

- ❖ **Time diffeomorphism violation but keep spatial covariance**
- ❖ **A lot of modified gravities can map to spatial covariant gravities**
- ❖ **A general framework for exploring different modified gravities**

d	(d_t, d_s)	operators
0	(0, 0)	1
1	(1,0)	K
	(0, 1)	-
2	(2, 0)	$K_{ij},\ K^2$
	(1, 1)	-
	(0, 2)	R
3	(3, 0)	$K_{ij}K^{jk}K^i_k,\ K_{ij}K^{ij}K,\ K^3$
	(2, 1)	$\varepsilon_{ijk}K^i_l\nabla^j K^{kl}$
	(1, 2)	$\nabla^i\nabla^j K_{ij},\ \nabla^2 K,\ R^{ij}K_{ij},\ RK$
	(0, 3)	$\omega_3(\Gamma)$
4	(4, 0)	$K_{ij}K^{jk}K^i_k K,\ \left(K_{ij}K^{ij}\right)^2,\ K_{ij}K^{ij}K^2,\ K^4$
	(3, 1)	$\varepsilon_{ijk}\nabla_m K^i_n K^{jm}K^{kn},\quad \varepsilon_{ijk}\nabla^i K^j_m K^k_n K^{mn},\quad \varepsilon_{ijk}\nabla^i K^j_l K^{kl}K$
	(2, 2)	$\nabla_k K_{ij}\nabla^k K^{ij},\ \nabla_i K_{jk}\nabla^k K^{ij},\ \nabla_i K^{ij}\nabla_k K^k_j,\ \nabla_i K^{ij}\nabla_j K,\ \nabla_i K\nabla^i K,\ R_{ij}K^i_k K^{jk},\ RK_{ij}K^{ij},\ R_{ij}K^{ij}K,\ RK^2$
	(1, 3)	$\varepsilon_{ijk}R^{il}\nabla^j K^k_l,\quad \varepsilon_{ijk}\nabla^i R^j_l K^{kl},\ \omega_3(\Gamma)K$
	(0, 4)	$\nabla^i\nabla^j R_{ij},\quad \nabla^2 R,\quad R_{ij}R^{ij},\quad R^2$

Tests of spatial covariant gravities

TABLE II. The corresponding values of the parameters $(\alpha_{\bar{\nu}}, \beta_{\bar{\nu}}, \alpha_{\bar{\mu}}, \beta_{\bar{\mu}}, \alpha_{\nu}, \beta_{\nu}, \alpha_{\mu}, \beta_{\mu})$ for different terms in defined in (3.16) and (3.17).

	$\mathcal{H}\bar{\nu}$		$\bar{\mu}$		$\mathcal{H}\nu_A$		μ_A		
	$\alpha_{\bar{\nu}}$	$\beta_{\bar{\nu}}$	$\alpha_{\bar{\mu}}$	$\beta_{\bar{\mu}}$	α_{ν}	β_{ν}	α_{μ}	β_{μ}	related coefficients
$\mathcal{G}_0$	$\ln \mathcal{G}_0$	0	$1/\mathcal{G}_0$	0	—	—	—	—	$c_1^{(2,0)}, c_1^{(3,0)}, c_2^{(3,0)}, c_1^{(4,0)}, c_2^{(4,0)}, c_3^{(4,0)}$
$\mathcal{G}_1$	—	—	—	—	$\mathcal{G}_1 M_{\rm PV}$	1	$-\mathcal{G}_1 M_{\rm PV}$	1	$c_1^{(2,1)}, c_1^{(3,1)}, c_2^{(3,1)}, c_3^{(3,1)}$
$\mathcal{G}_2$	$\mathcal{G}_2 M_{\rm LV}^2$	2	$-\mathcal{G}_2 M_{\rm LV}^2$	2	—	—	—	—	$c_1^{(2,2)}$
$\mathcal{W}_0$	—	—	$\mathcal{W}_0$	0	—	—	—	—	$c_1^{(0,2)}, c_3^{(1,2)}, c_4^{(1,2)}, c_6^{(2,2)}, c_7^{(2,2)}, c_8^{(2,2)}, c_9^{(2,2)}$
$\mathcal{W}_1$	—	—	—	—	—	—	$\mathcal{W}_1 M_{\rm PV}$	1	$c_1^{(0,3)}, c_1^{(1,3)}, c_2^{(1,3)}, c_3^{(1,3)}$
$\mathcal{W}_2$	—	—	$\mathcal{G}_2 M_{\rm LV}^2$	2	—	—	—	—	$c_3^{(0,4)}$

TABLE IV. Summary of estimations for bounds of the coupling coefficients in spatial covariant gravities. Note that all the coefficients are estimated approximately at present time, i..e, $z = 0$. Here $[a_{\rm min}, b_{\rm max}]$ represents constraints with $a_{\rm bin}$ and $b_{\rm max}$ being the lower the upper bounds, respectively.

Coefficients	bounds	related coefficients	datasets used
$\sqrt{\frac{\mathcal{G}_0}{\mathcal{W}_0}} - 1$	$[-30, 7] \times 10^{-16}$	$c_1^{(2,0)}, c_1^{(3,0)}, c_2^{(3,0)}, c_1^{(4,0)}, c_2^{(4,0)}, c_3^{(4,0)}$, $c_1^{(0,2)}, c_3^{(1,2)}, c_4^{(1,2)}, c_6^{(2,2)}, c_7^{(2,2)}, c_8^{(2,2)}, c_9^{(2,2)}$	mult-messenge observations of GW170817 [37, 38]
$\frac{(\ln \mathcal{G}_0)'}{\mathcal{H}}$	$[-3.0,\ 2.5]$	$c_1^{(2,0)}, c_1^{(3,0)}, c_2^{(3,0)}, c_1^{(4,0)}, c_2^{(4,0)}, c_3^{(4,0)}$	dark sirens in GWTC-3 with BBH mass distributions [47]
$\lvert\mathcal{G}_1\rvert$	$\lesssim 2065$ km	$c_1^{(2,1)}, c_1^{(3,1)}, c_2^{(3,1)}, c_3^{(3,1)}$	tests of amplitude birefringence with LIGO-Virgo O1/O2 [48]
	$\lesssim 1000$ km		from statistic distribution of $\cos\iota$ in GWTC-2 [50]
$\lvert\mathcal{W}_1 - \mathcal{G}_1\rvert$	$\lesssim 4.4 \times 10^{-18}$ km	$c_1^{(0,3)}, c_1^{(1,3)}, c_2^{(1,3)}, c_3^{(1,3)}$, $c_1^{(2,1)}, c_1^{(3,1)}, c_2^{(3,1)}, c_3^{(3,1)}$	tests of velocity birefringence with 4-OGC [58]
$\lvert\mathcal{W}_2 - \mathcal{G}_2\rvert$	$\lesssim 1.2 \times 10^{-10}$ m^2	$c_1^{(2,2)},\ c_3^{(0,4)}$	tests of Lorentz-violating dispersion with GWTC-3 [13]

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Thanks!