宇宙背景辐射的引力波记忆效应

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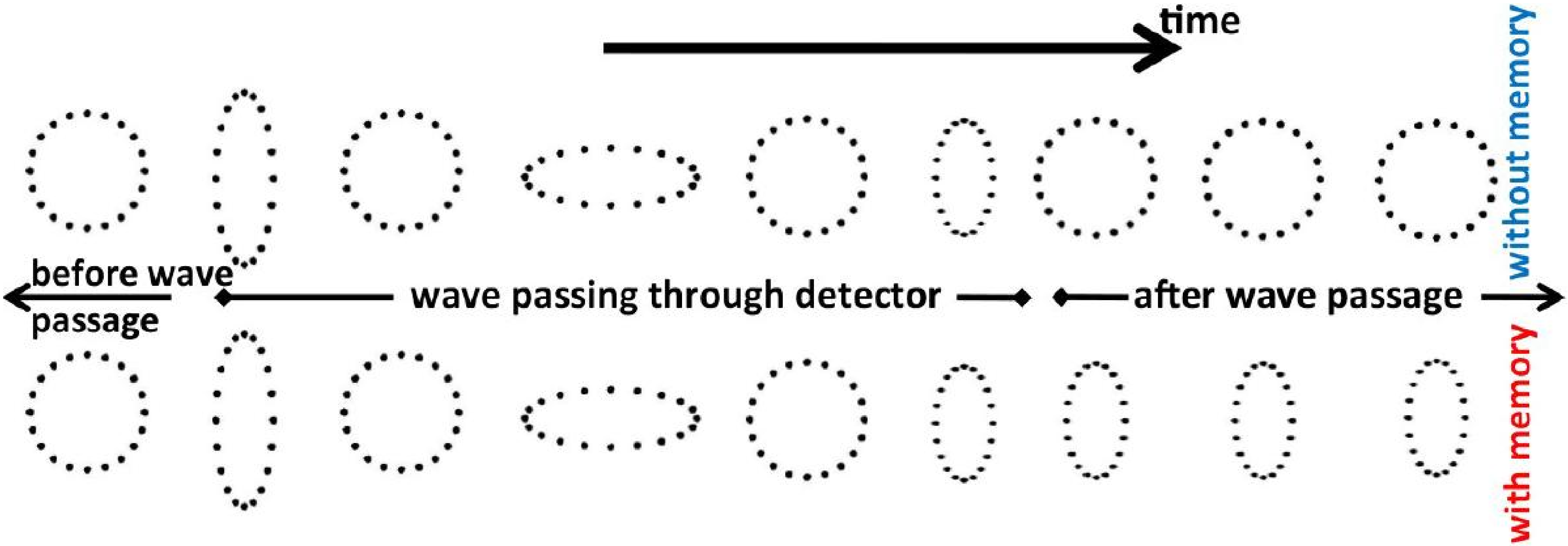
2023-4-7

"2023 引力与宇宙学"专题研讨会 @ 中国科学技术大学

什么是引力波记忆效应



自由运动粒子间距离的永久变化

Linear memory

NATURE VOL. 327 14 MAY 1987 LETTERS TO NATURE 123

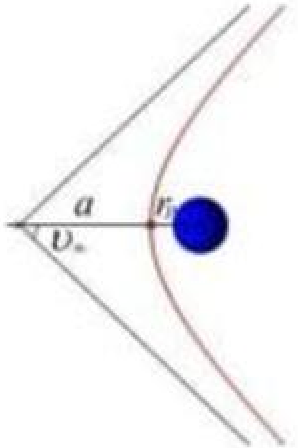
Gravitational-wave bursts with memory and experimental prospects

Vladimir B. Braginsky* & Kip S. Thorne†

* Physics Faculty, Moscow State University, Moscow, USSR
† Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA

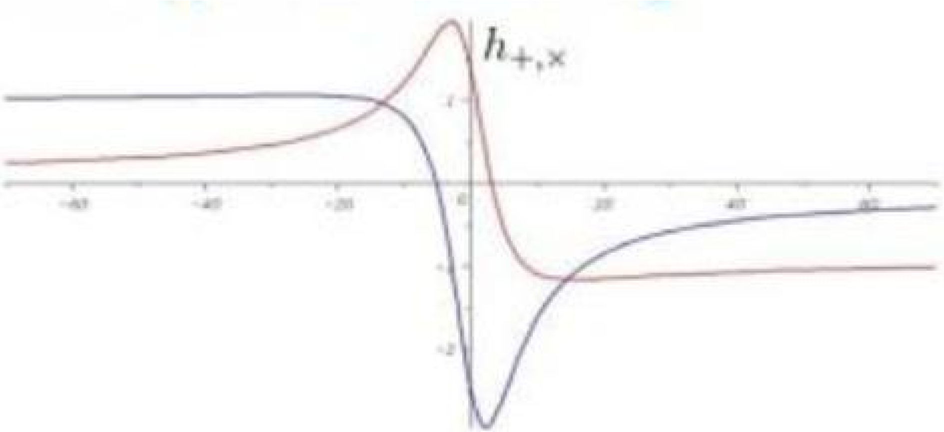
$$h_{ij}(t,r) = \frac{2}{r}\ddot{I}_{ij}(t-r) \quad \Rightarrow \quad h_{ij}(+\infty,r) - h_{ij}(-\infty,r) = \frac{2}{r}(\Delta\ddot{I}_{ij})|_{-\infty}^{+\infty}$$

Linear memory



$$\ddot{I}_{ij} = m(\ddot{x}_i x_j + \dot{x}_i \dot{x}_j + x_i \ddot{x}_j)$$

$$h_{jk}^{\mathrm{TT}} = \sum_{A=1}^{N} \frac{4M_A}{R\sqrt{1 - v_A^2}} \left[\frac{v_A^j v_A^k}{1 - \boldsymbol{v}_A \cdot \boldsymbol{N}} \right]^{\mathrm{TT}} + O(a)$$



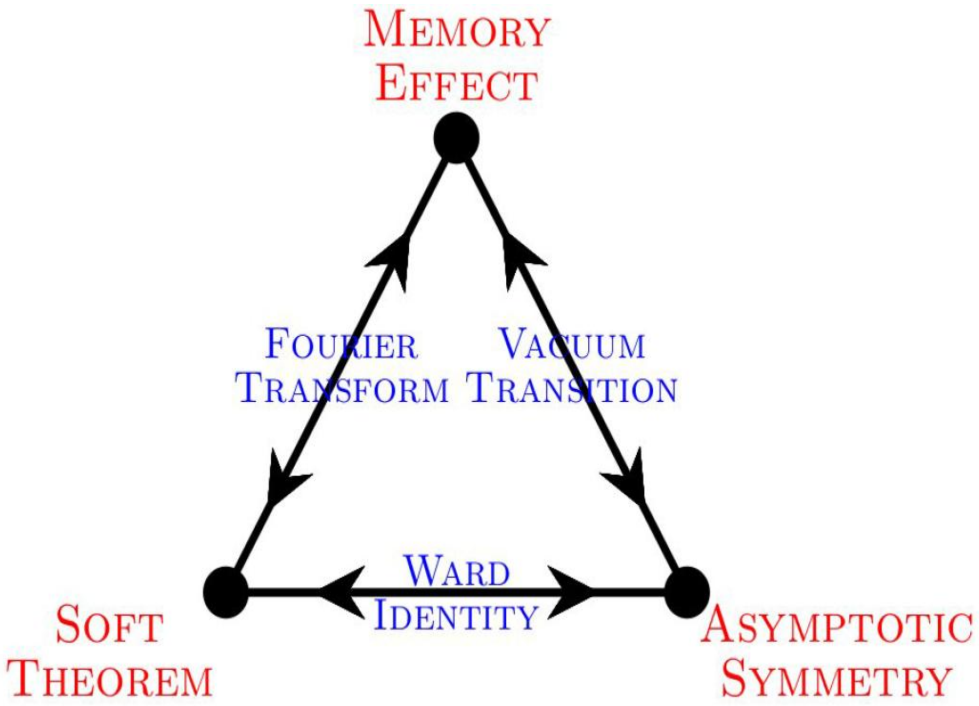
Non-linear memory

Christodoulou
PRL 67, 1486 (1991)

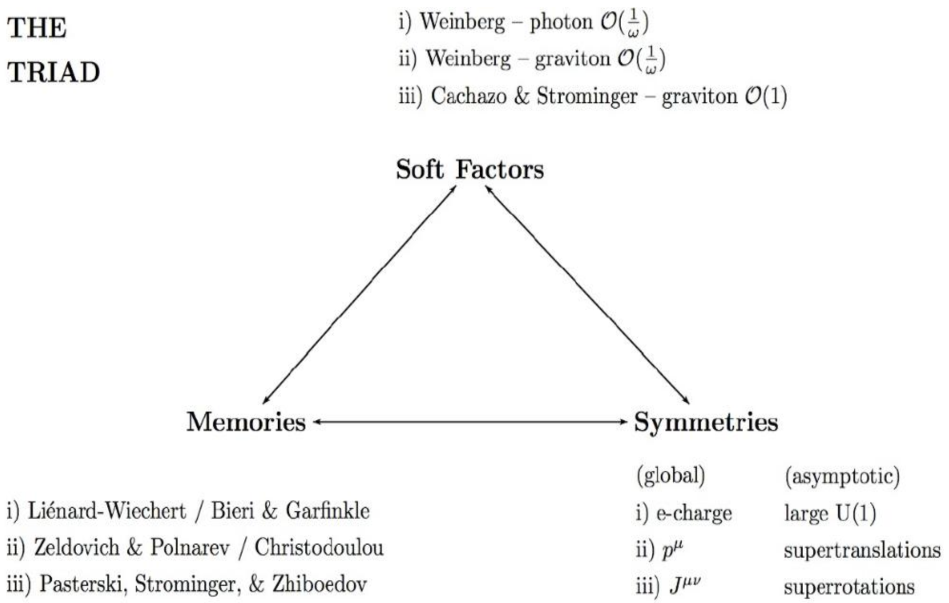
$$\eth^2(\Delta\bar{\sigma}) = \boxed{\underbrace{\int_{-\infty}^{+\infty} |\dot{\sigma}|^2 dt}_{\text{Nonlinear part}}} - \underbrace{\Delta\Psi_2}_{\text{Linear part}}$$

total energy carried by GW along a given direction per solid angle

Memory and soft theorem



S. Weinberg Phys. Rev. 140, B516 (1965)



A. Strominger, arXiv:1703.05448

memory = ‘linear part’ + ‘nonlinear part’

‘linear part’: changement of charge distribution (ejection of charge to spatial infinity)

‘nonlinear part’: charge flux at null infinity

RAPID COMMUNICATIONS

PHYSICAL REVIEW D

PARTICLES, FIELDS, GRAVITATION, AND COSMOLOGY

THIRD SERIES, VOLUME 44, NUMBER 10 15 NOVEMBER 1991

Christodoulou's nonlinear gravitational-wave memory: Evaluation in the quadrupole approximation

Alan G. Wiseman and Clifford M. Will

PHYSICAL REVIEW D VOLUME 45, NUMBER 2 15 JANUARY 1992

Gravitational-wave bursts with memory: The Christodoulou effect

Kip S. Thorne

Linear memory :

$$\Delta h_{jk}^{\mathrm{TT}} = \Delta \sum_{A=1}^{N} \frac{4M_A}{R\sqrt{1-v_A^2}} \left[\frac{v_A^j v_A^k}{1-\boldsymbol{v}_A \cdot \boldsymbol{N}} \right]^{\mathrm{TT}}$$

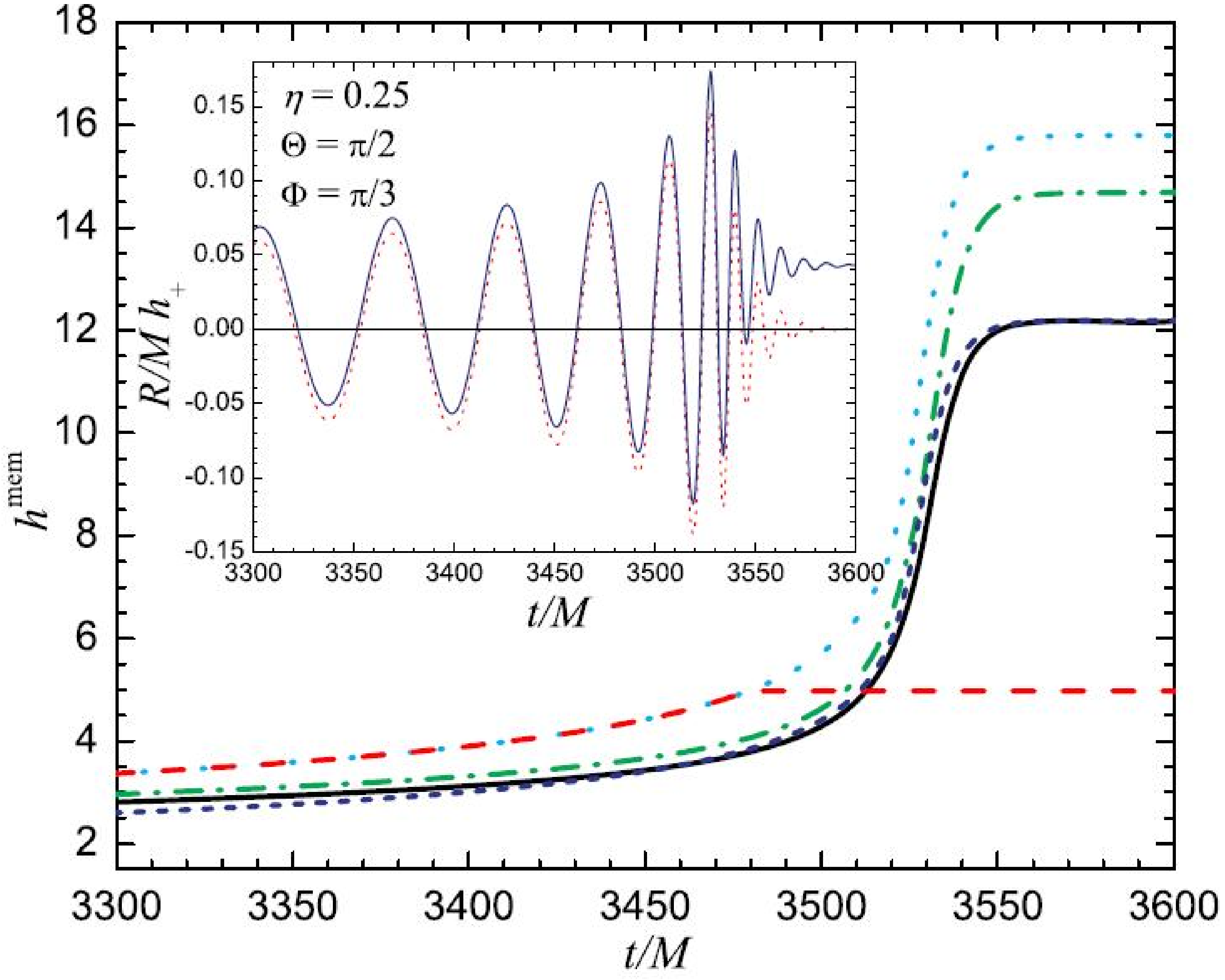
Non-linear memory :

$$\delta h_{jk}^{\mathrm{TT}} = \frac{4}{R} \int_{-\infty}^{T_R} dt' \left[\int \frac{dE^{\mathrm{gw}}}{dt' d\Omega'} \frac{n'_j n'_k}{(1-\boldsymbol{n}' \cdot \boldsymbol{N})} d\Omega' \right]^{\mathrm{TT}}$$

The Astrophysical Journal, 696:L159–L162, 2009 May 10 doi:10.1088/0004-637X/696/2/L159

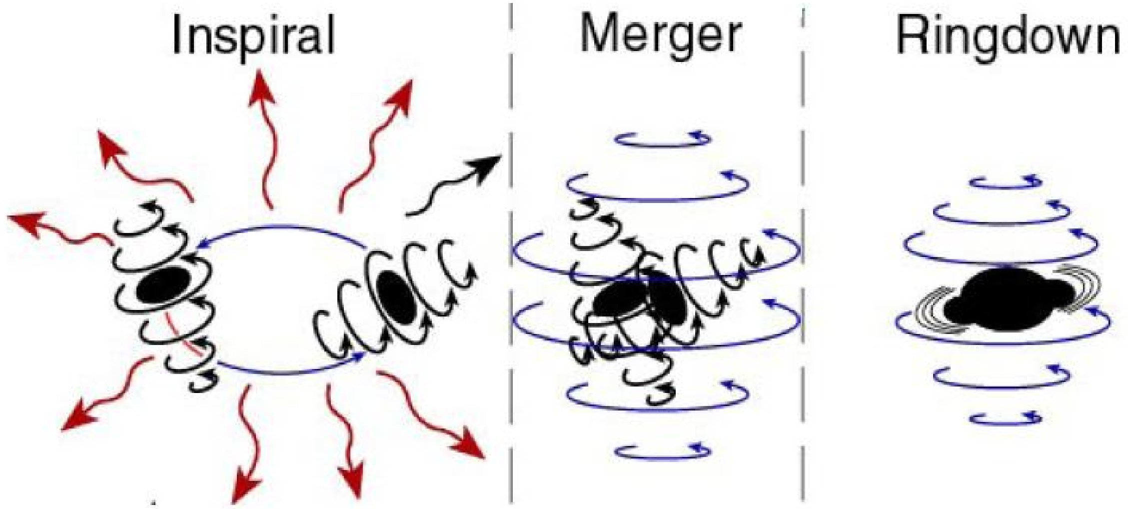
NONLINEAR GRAVITATIONAL-WAVE MEMORY FROM BINARY BLACK HOLE MERGERS

Marc Favata
Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030, USA

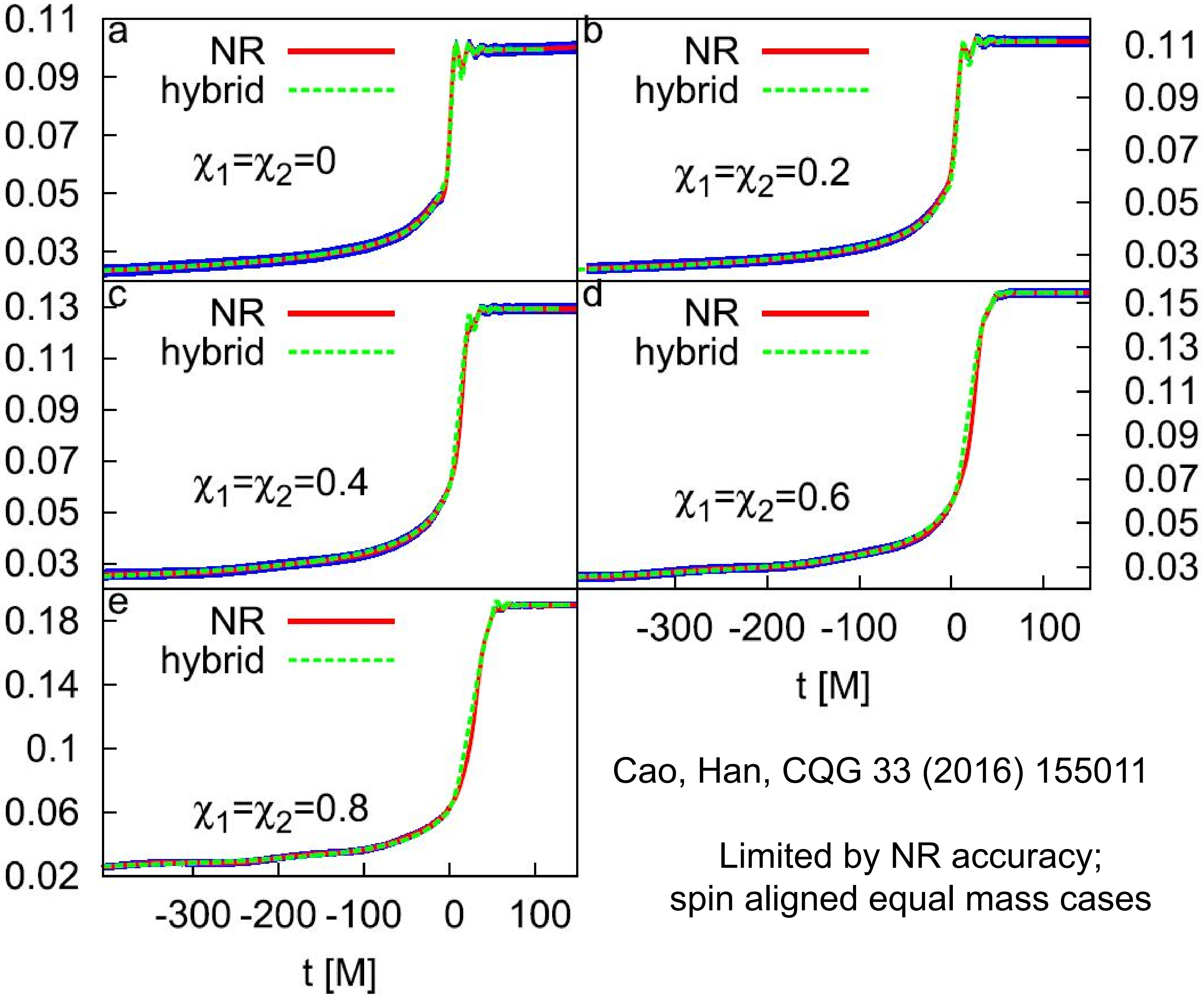


EOBNR waveform model for GW memory

- GW memory mainly happens at merger for BBH
- PN approximation is not valid for merger stage



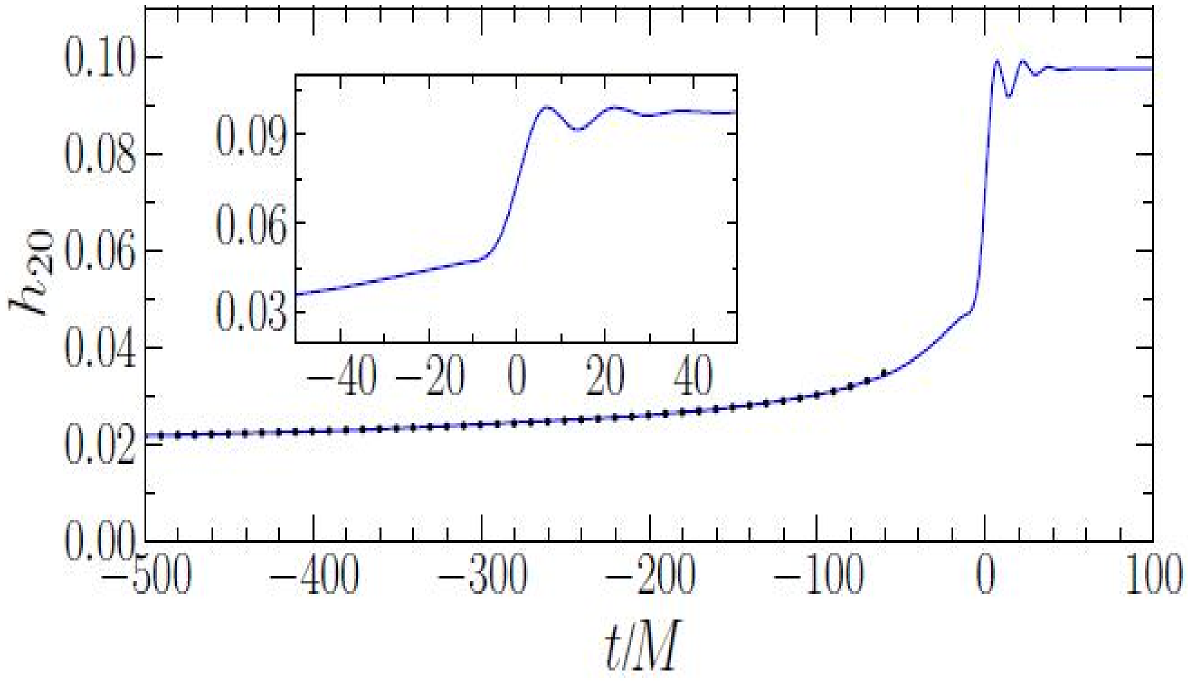
- We need a REAL waveform model for GW memory



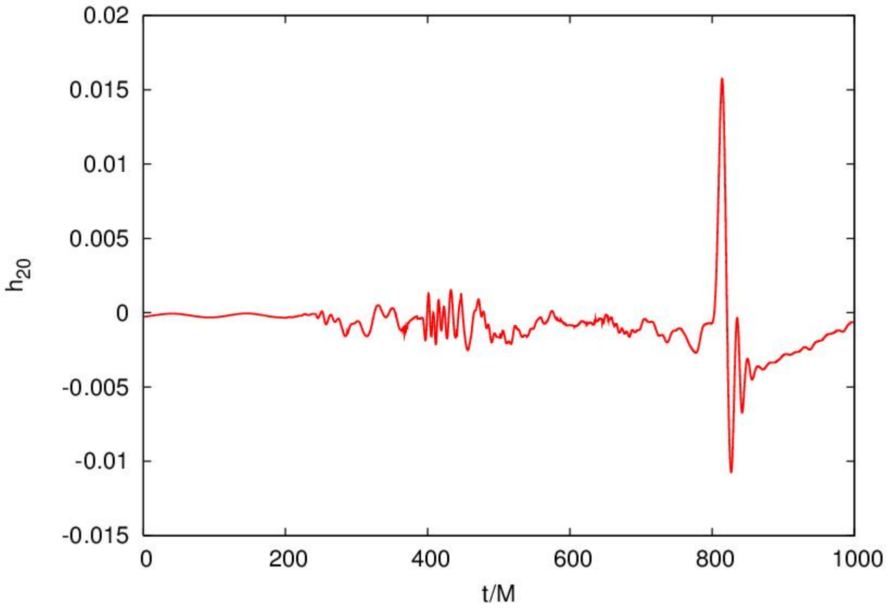
Cao, Han, CQG 33 (2016) 155011

Limited by NR accuracy;
spin aligned equal mass cases

NR results on memory before 2021



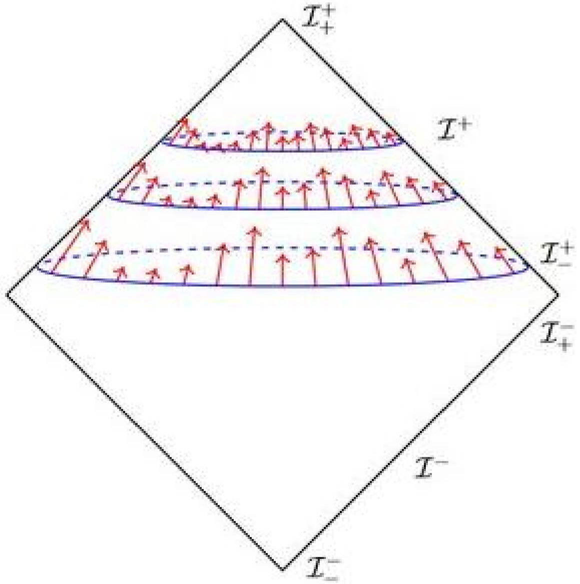
Einstein Toolkit

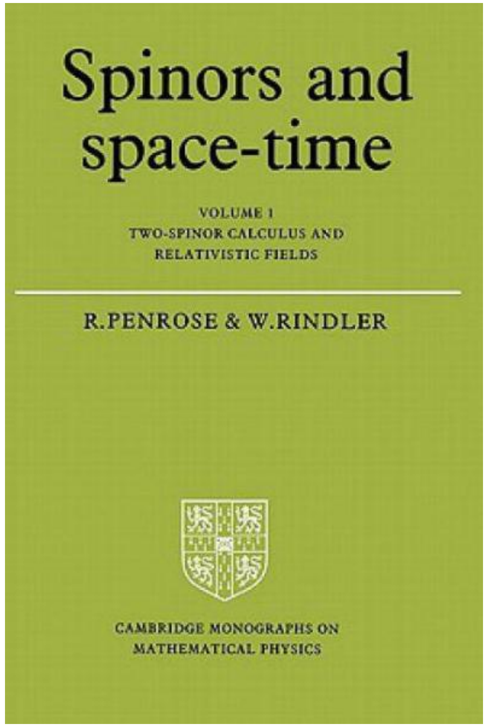


SpEC

Calculate GW memory accurately

Null infinity: mathematical word of radiation region





Balance relation at null infinity

$$\dot{\Psi}_2^\circ = \eth\Psi_3^\circ + \sigma^\circ\Psi_4^\circ, \ \ \Psi_3^\circ = -\eth\dot{\bar{\sigma}}^\circ, \ \ \Psi_4^\circ = -\ddot{\bar{\sigma}}^\circ$$

PN approximation → adiabatic approximation (otherwise exact)

Weak field + slow velocity

$$h = h^n + h^m$$

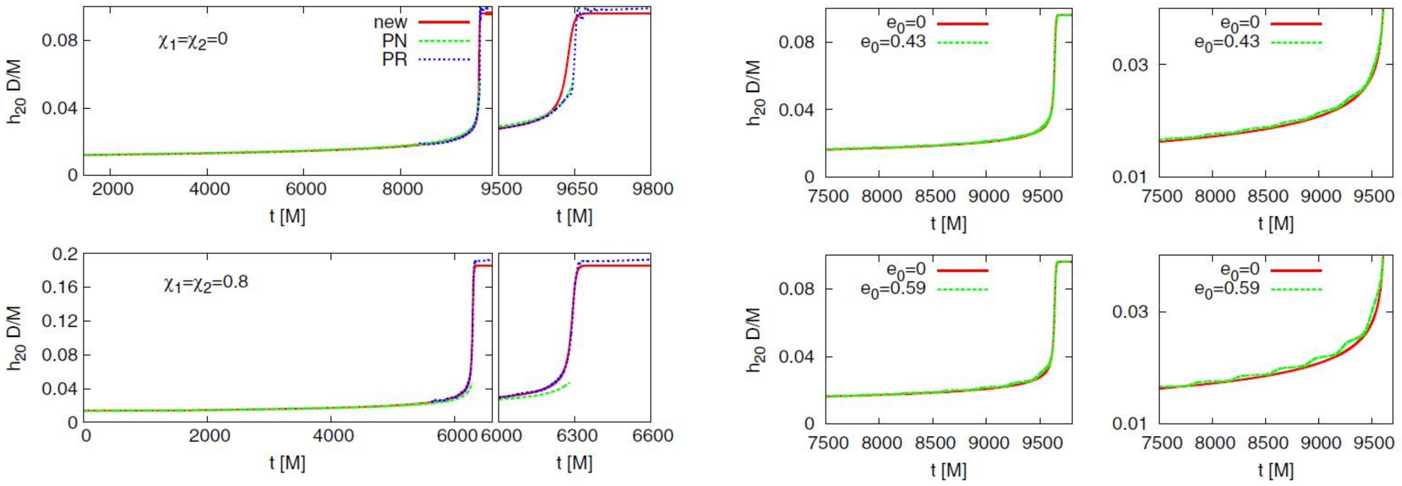
$$\dot{h}^n \gg \dot{h}^m$$

$$h^m(t) = f[h^n(t)]$$

X. Liu, X. He, and Z. Cao, Phys. Rev. D 103, 043005 (2021)

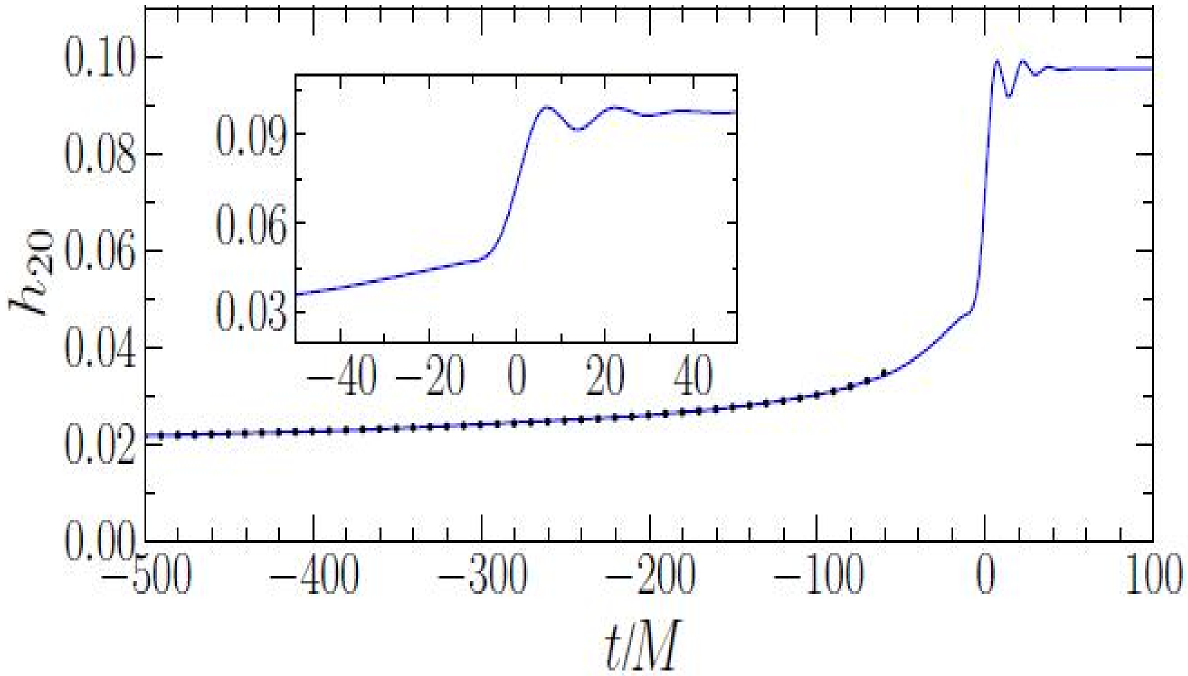
引力波记忆效应恒等式

$$
h_{lm}\Big|_{t_1}^{t_2} = -\sqrt{\frac{(l-2)!}{(l+2)!}}\Bigg[\frac{4}{D}\int \Psi_2^{\circ}[{}^0Y_{lm}]\sin\theta d\theta d\phi\Big|_{t_1}^{t_2} - D\sum_{l'=2}^{\infty}\sum_{l''=2}^{\infty}\sum_{m'=-l'}^{l'}\sum_{m''=-l''}^{l''}\Gamma_{l'l''lm'-m''-m}\times \left(\int_{t_1}^{t_2}\dot{h}_{l'm'}\dot{\bar{h}}_{l''m''}dt - \dot{h}_{l'm'}(t_2)\bar{h}_{l''m''}(t_2) + \dot{h}_{l'm'}(t_1)\bar{h}_{l''m''}(t_1)\right)\Bigg]
$$

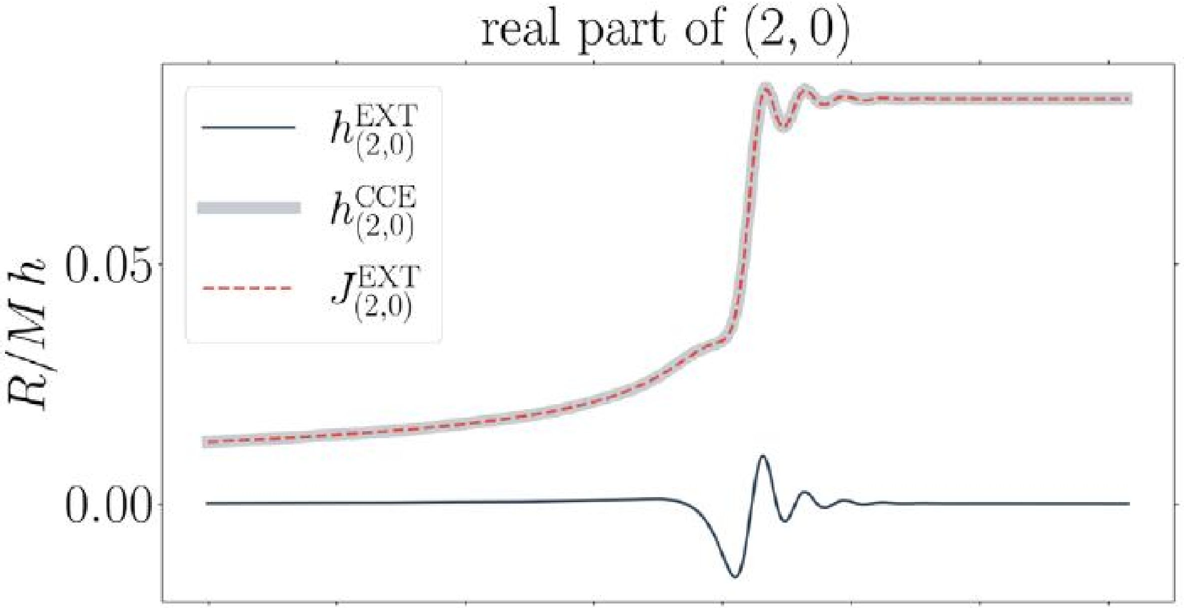


X. Liu, X. He, and Z. Cao, Phys. Rev. D 103, 043005 (2021)

NR results on memory after 2021



Einstein Toolkit



SpEC

Memory on detector

$$h = \Re[(F^+ + iF^\times) \cdot (h^n + h^m)]$$

$$h(t = \infty) = \Re[(F^+ + iF^\times)h^m]$$

$$\text{At } t = \infty,\ h^n = \dot{h}^m = 0$$

So, our previous GW memory calculation result is exact, no approximation is needed

Instead of measure the waveform,
we concern the overall GW memory on the detector

$$h^{\text{mem}} = \frac{M}{D}\Re[(F^+(\theta,\phi,\psi) + iF^\times(\theta,\phi,\psi))\times \sum_{l=2}^{\infty}\sum_{m=-l}^{l} h^t_{lm} Y_{-2lm}(\iota,\beta)]$$

$$
\begin{aligned}
h^{\mathrm{mem}} &= \frac{M}{D}\Re[(F^{+}(\theta,\phi,\psi)+iF^{\times}(\theta,\phi,\psi))\times \\
&\quad \sum_{l=2}^{\infty}\sum_{m=-l}^{l} h^{t}_{lm}Y_{-2lm}(\iota,\beta)] \\
&\approx \frac{M}{D}F^{+}(\theta,\phi,\psi)h^{t}_{20}Y_{-220}(\iota),
\end{aligned}
$$

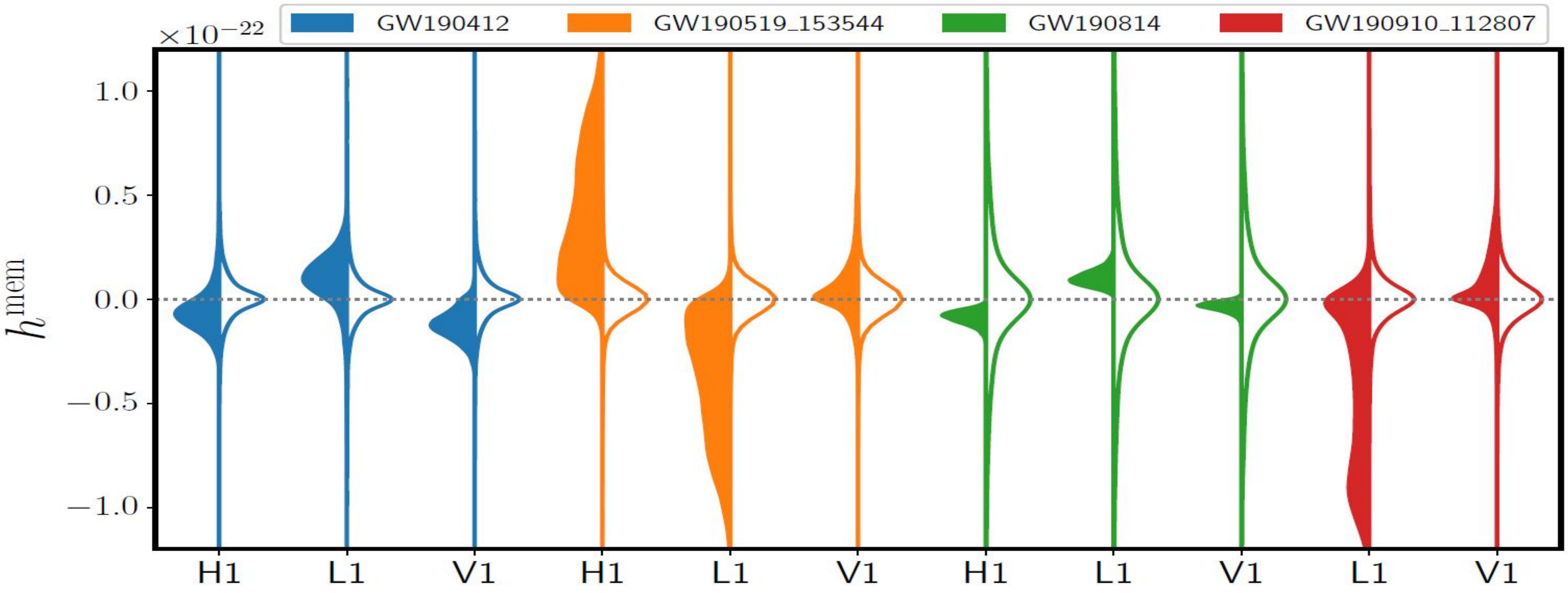
$$
\begin{aligned}
F^{+}(\theta,\phi,\psi) &\equiv -\frac{1}{2}(1+\cos^{2}\theta)\cos 2\phi\cos 2\psi \\
&\quad -\cos\theta\sin 2\phi\sin 2\psi, \\
F^{\times}(\theta,\phi,\psi) &\equiv +\frac{1}{2}(1+\cos^{2}\theta)\cos 2\phi\sin 2\psi \\
&\quad -\cos\theta\sin 2\phi\cos 2\psi,
\end{aligned}
$$

The overall GW memory depends on parameters

$$(M, q, \vec{\chi}_1, \vec{\chi}_2, D, \iota, \theta, \phi, \psi)$$

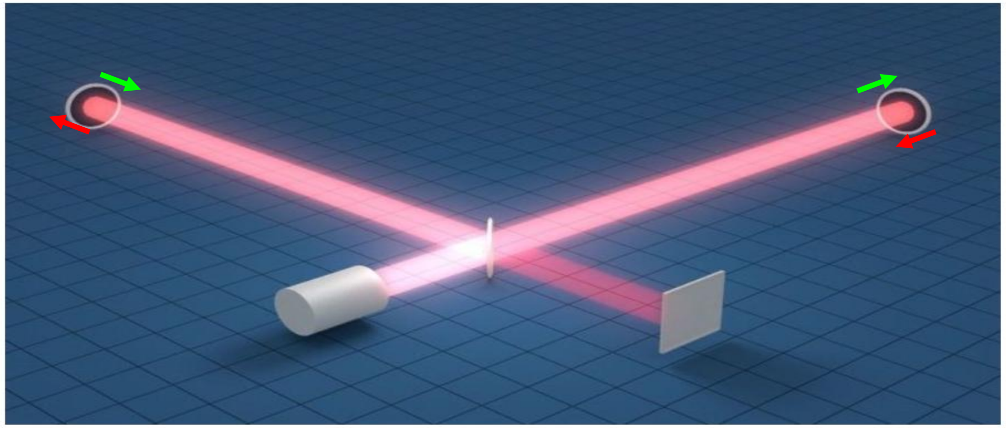
Where h^{t}_{lm} has been calculated by our previous EXACT calculation, is determined by $(M, q, \vec{\chi}_1, \vec{\chi}_2)$

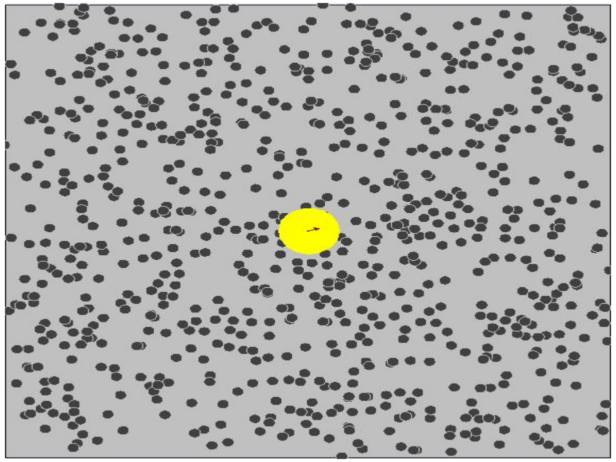
Golden events for GW memory



Stochastic background of GW memory

Multiple successive GW memory events



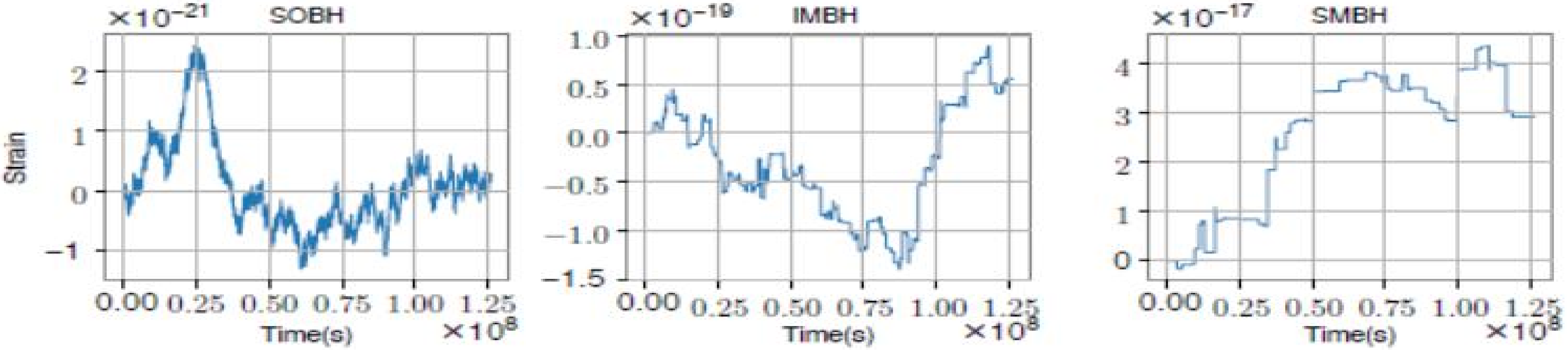


behave as one dimensional Brownian motion

$$\mathfrak{M} = \sum_{j=1}^{\infty} \Re[(F^{+}(\theta_j, \phi_j, \psi_j) + iF^{\times}(\theta_j, \phi_j, \psi_j))\times h(q_j, M_j, \vec{\chi}_{1j}, \vec{\chi}_{2j}, d_L, \iota_j, \phi_{cj})]$$

$$\langle \mathfrak{M}^2(t) \rangle = 2Dt.$$

SGWMB for BBH mergers



D: 3.16×10^{-50} 8.42×10^{-47} 1.73×10^{-42}

For Gauss type Brownian motion:

$$D = \frac{\sigma^2}{2\Delta t}$$

σ : variance of the Gauss distribution

$\Delta t.$: averaged time between two successive GW memory events

$$\mathcal{A} = \frac{M}{D_L} F^{+}(\theta, \phi, \psi) Y_{-220}(\iota)[0.0969 + 0.0562\chi_{\rm up} +$$
$$0.0340\chi_{\rm up}^2 + 0.0296\chi_{\rm up}^3 + 0.0206\chi_{\rm up}^4](4\eta)^{1.65},$$
$$\chi_{\rm up} \equiv \chi_{\rm eff} + \frac{3}{8}\sqrt{1-4\eta}\chi_{\rm A},$$
$$\chi_{\rm eff} \equiv (m_1\vec{\chi}_1 + m_2\vec{\chi}_2) \cdot \hat{N}/M,$$
$$\chi_{\rm A} \equiv (m_1\vec{\chi}_1 - m_2\vec{\chi}_2) \cdot \hat{N}/M,$$

parameters $m_{1,2}$, $\vec{\chi}_{1,2}$, D_L, ι, θ, ϕ, and ψ are random variables.

$$\sigma^2 = \langle \mathcal{A}^2 \rangle - \langle \mathcal{A} \rangle^2.$$

$$\mathcal{A} = \mathcal{A}_{\rm bbh}\mathcal{A}_{\rm ang},$$
$$\mathcal{A}_{\rm bbh} \equiv \frac{M}{D_L}[0.0969 + 0.0562\chi_{\rm up} +$$
$$0.0340\chi_{\rm up}^2 + 0.0296\chi_{\rm up}^3 + 0.0206\chi_{\rm up}^4](4\eta)^{1.65},$$
$$\mathcal{A}_{\rm ang} \equiv F^{+}(\theta, \phi, \psi) Y_{-220}(\iota).$$

parameters $m_{1,2}$, $\vec{\chi}_{1,2}$, D_L, ι, θ, ϕ, and ψ are independent

$\Rightarrow$ $\mathcal{A}_{\rm bbh}$ and $\mathcal{A}_{\rm ang}$ are independent

uniform distribution of ι, θ, ϕ, and ψ

$\Rightarrow$
$$\langle \mathcal{A}_{\rm ang} \rangle = 0$$

$$\langle \mathcal{A} \rangle = 0$$

$$\langle \mathcal{A}_{\rm ang}^2 \rangle - \langle \mathcal{A}_{\rm ang} \rangle^2 \equiv \sigma_{\rm ang}^2 = \frac{1}{20\pi}.$$

$$\sigma_{\rm bbh}^2 \equiv \langle \mathcal{A}_{\rm bbh}^2 \rangle - \langle \mathcal{A}_{\rm bbh} \rangle^2, \mu_{\rm bbh} \equiv \langle \mathcal{A}_{\rm bbh} \rangle,$$
$$\sigma = \sigma_{\rm ang}\sqrt{\sigma_{\rm bbh}^2 + \mu_{\rm bbh}^2} = \frac{1}{\sqrt{20\pi}}\sqrt{\sigma_{\rm bbh}^2 + \mu_{\rm bbh}^2}.$$

$\mu_{\rm bbh}$, $\sigma_{\rm bbh}$ and Δt. are determined by and only by event rates of BBH merger

Corresponding theoretical D: 3.16×10^{-50}, 8.41×10^{-47} and 1.73×10^{-42}

Power spectrum of SGWMB

$$S^{\mathfrak{M}}(f) \equiv \lim_{T\to\infty} \frac{1}{T} \left| \int_0^T e^{-2\pi i f t} \mathfrak{M}(t) dt \right|^2$$

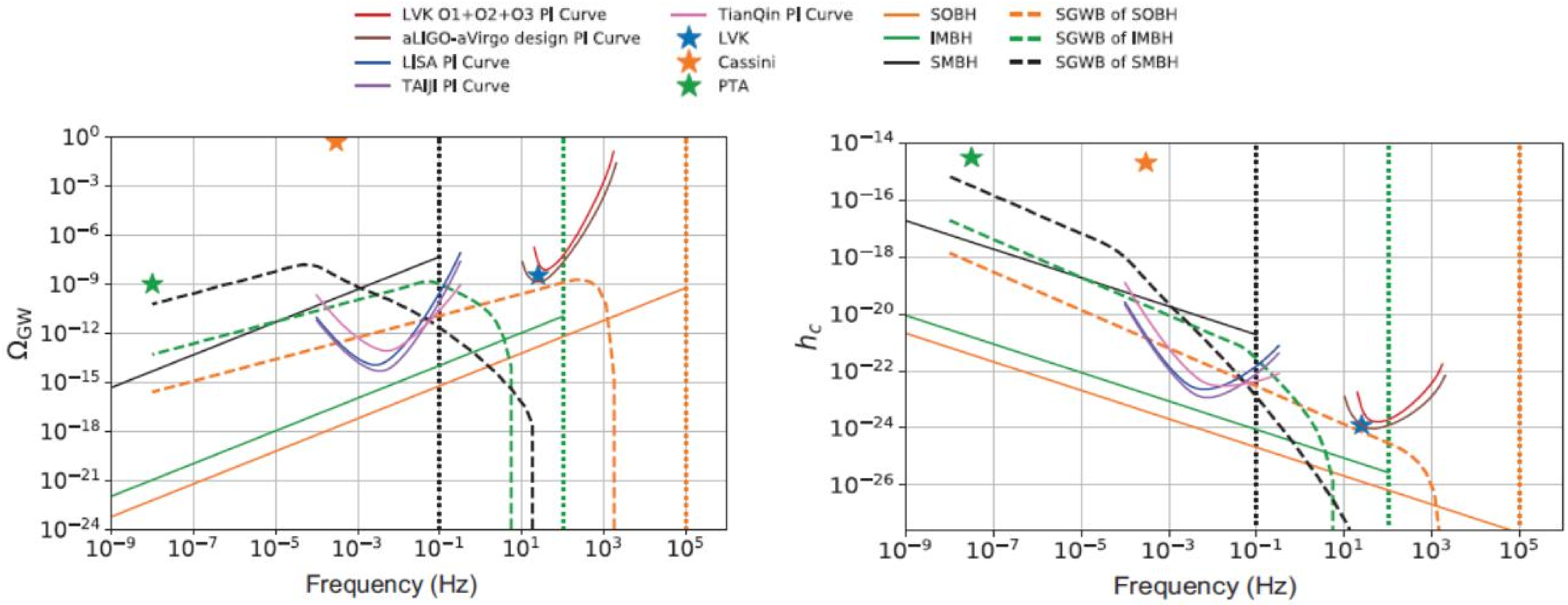
$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T \int_0^T dt_1 dt_2 \cos(2\pi f(t_1 - t_2)) \langle \mathfrak{M}(t_1) \mathfrak{M}(t_2) \rangle$$

$$= \lim_{T\to\infty} \frac{D}{\pi^2 f^2} \left[1 - \frac{\sin(2\pi f T)}{2\pi f T} \right]$$

$$= \frac{D}{\pi^2 f^2}.$$

$$h_c^{\mathfrak{M}}(f) = \sqrt{2 f S^{\mathfrak{M}}} = \frac{1}{\pi} \sqrt{\frac{\sigma_{\mathrm{bbh}}^2 + \mu_{\mathrm{bbh}}^2}{20\pi f \Delta t}}.$$

Detectability of SGWMB



Energy flux and GW memory

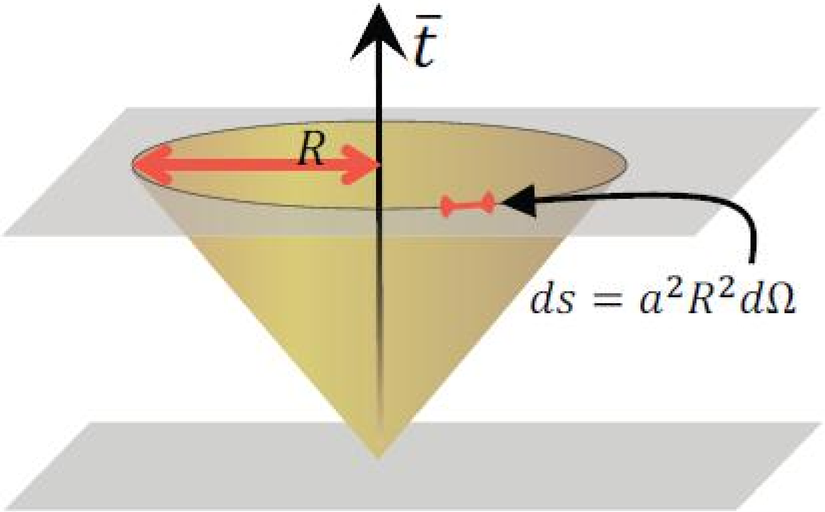
$$h_{ij}^{\mathrm{m}} = \frac{4}{D_L}\int_{-\infty}^{t} dt' \left[\int D_L^2 F^{\circ} \frac{n_i' n_j'}{1 - \mathbf{n}' \cdot \mathbf{N}} d\Omega'\right]^{\mathrm{TT}}$$

$$h_{ij}^{\mathrm{m}} = h_{+} e_{ij}^{+} + h_{\times} e_{ij}^{\times} \qquad h^{\mathrm{m}} \equiv h_{+}^{\mathrm{m}} - i h_{\times}^{\mathrm{m}}$$

$$h^{\mathrm{m}} = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} h_{lm}^{\mathrm{m}} [{}^{-2}Y_{lm}]$$

$$\eth^2 h \Leftrightarrow F^{\circ} \quad \Rightarrow \quad h_{lm}^{\mathrm{m}} = \frac{32\pi}{D_L}\sqrt{\frac{(l-2)!}{(l+2)!}} \int_{-\infty}^{t} \int D_L^2 F^{\circ}(t') dt' \overline{[{}^{0}Y_{lm}]} d\Omega', l \geq 2$$

GW memory from cosmic radiation

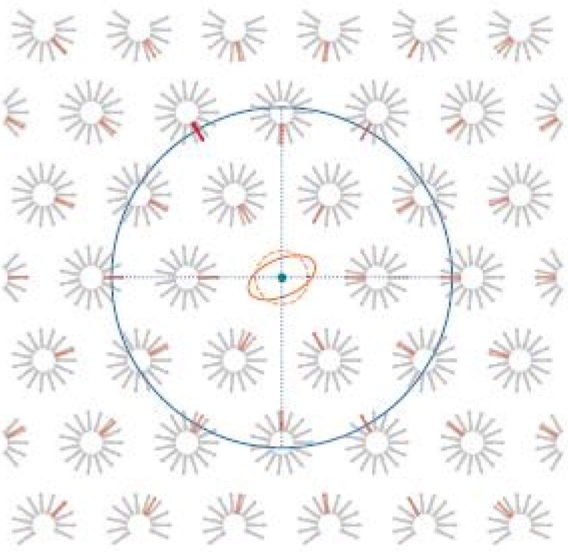


$$h \propto \frac{1}{R}, F^{\circ} \propto \frac{1}{D_L^2} = \frac{1}{a^2R^2}$$

$$h^{\mathrm{m}}_{lm} = 32\pi R\sqrt{\frac{(l-2)!}{(l+2)!}}\int_{-\infty}^{t}\int a^2(t')F^{\circ}dt'\overline{[{}^0Y_{lm}]}d\Omega', l \geq 2.$$

GW memory from cosmic radiation

From source frame
to detector frame



$$h^{\mathrm{m}}_{ij} = \int h^{\mathrm{m}}_{ij}(\theta,\phi)d\Omega.$$

$$h^{\mathrm{m}}_{ij}(\theta,\phi) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l}$$
$$\{\Re[h^{m}_{lm}\,{}^{-2}Y_{lm}(\pi-\theta,2\pi-\phi)]e^{+}_{ij}(\pi-\theta,2\pi-\phi)$$
$$-\Im[h^{m}_{lm}\,{}^{-2}Y_{lm}(\pi-\theta,2\pi-\phi)]e^{\times}_{ij}(\pi-\theta,2\pi-\phi)\}$$

$$\dot{h}^{\mathrm{m}}_{ij} = \int \dot{h}^{\mathrm{m}}_{ij}(\theta,\phi)d\Omega$$

$$\dot{h}^{\mathrm{m}}_{ij}(\theta,\phi) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l}$$
$$\{\Re[\dot{h}^{m}_{lm}\,{}^{-2}Y_{lm}(\pi-\theta,2\pi-\phi)]e^{+}_{ij}(\pi-\theta,2\pi-\phi)$$
$$-\Im[\dot{h}^{m}_{lm}\,{}^{-2}Y_{lm}(\pi-\theta,2\pi-\phi)]e^{\times}_{ij}(\pi-\theta,2\pi-\phi)\},$$

$$\dot{h}^{\mathrm{m}}_{lm} = 32\pi R\sqrt{\frac{(l-2)!}{(l+2)!}}\int F^{\circ}\overline{[{}^{0}Y_{lm}]}d\Omega', l\geq 2,$$

Anisotropic cosmic radiation produce GW memory

$$F^\circ = F_0 \rho(\theta', \phi')$$

$$\int \rho(\theta', \phi') d\Omega' = 1$$

$$h^{\mathrm{m}}_{lm} = 32\pi F_0 R \sqrt{\frac{(l-2)!}{(l+2)!}} a_{lm}, l \geq 2.$$

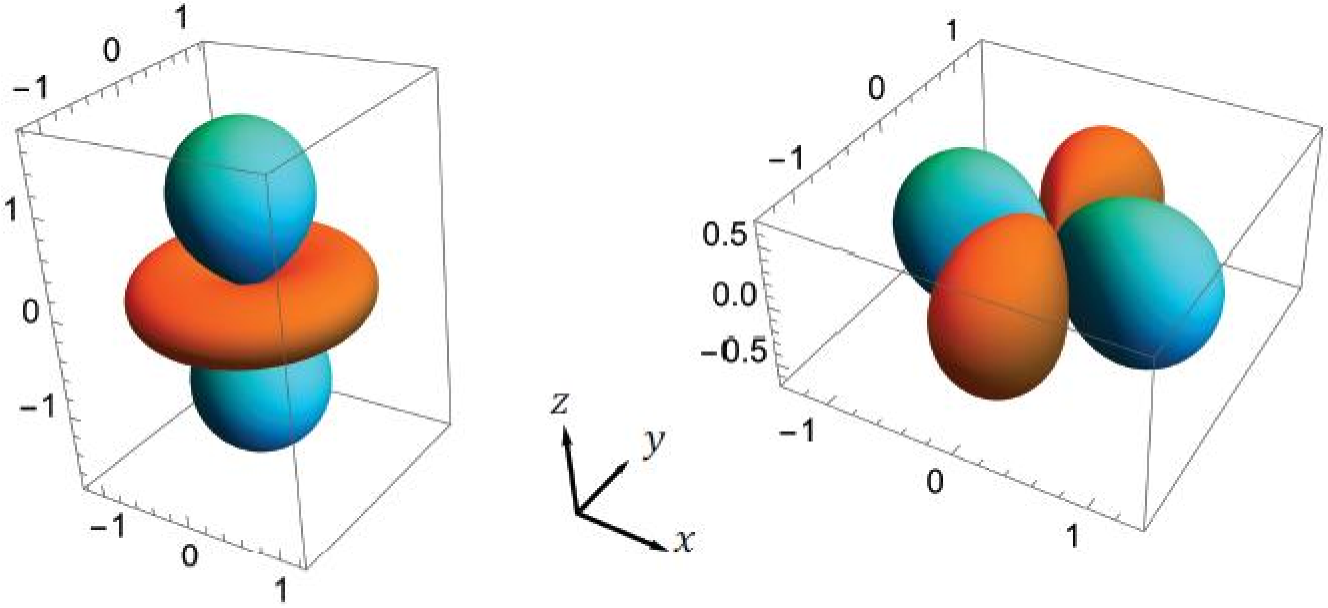
$$a_{lm} \equiv \int \rho \overline{[{}^0 Y_{lm}]} d\Omega'.$$

$$h^{\mathrm{m}}_{ij} = F_0 R \Re \left[\sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm} W_{lmij} \right],$$

$$W_{lmij} \equiv 32\pi \sqrt{\frac{(l-2)!}{(l+2)!}} \int {}^{-2}Y_{lm}(\pi - \theta, 2\pi - \phi) \times$$
$$\left[e^{+}_{ij}(\pi - \theta, 2\pi - \phi) + i e^{\times}_{ij}(\pi - \theta, 2\pi - \phi) \right] d\Omega.$$

$$W_{lmij} = 0, l \neq 2$$

$$\dot{h}^{\mathrm{m}}_{ij} = F_0 R \Re\left[\sum_{m=-2}^{2} a_{2m} W_{2mij}\right]$$



$$\Re[W_{2-2ij}] \xrightarrow{\phi\to\frac{\pi}{4}+\phi} \Im[W_{2-2ij}],$$

$$\Re[W_{2-1ij}] \xrightarrow{\phi\to\frac{\pi}{2}+\phi} \Im[W_{2-1ij}],$$

$$\Re[W_{21ij}] \xrightarrow{\phi\to\frac{\pi}{2}-\phi} \Im[W_{2-1ij}],$$

$$\Re[W_{22ij}] \xrightarrow{\phi\to\frac{\pi}{4}-\phi} \Im[W_{22ij}],$$

$$\Re[W_{2-2ij}] = \Re[W_{22ij}],$$

$$\Re[W_{2-1ij}] \xrightarrow{\phi\to\pi-\phi} \Re[W_{21ij}],$$

$$\Im[W_{2-2ij}] \xrightarrow{x\to z, z\to -x} \Im[W_{2-1ij}],$$

GW memory effect on FRW metric

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j$$

$$\dot{a}\delta_{ij} \Rightarrow \dot{a}\delta_{ij} + a\dot{h}_{ij}$$

$$H_0 \Rightarrow H_0\delta_{ij} + \dot{h}_{ij}$$

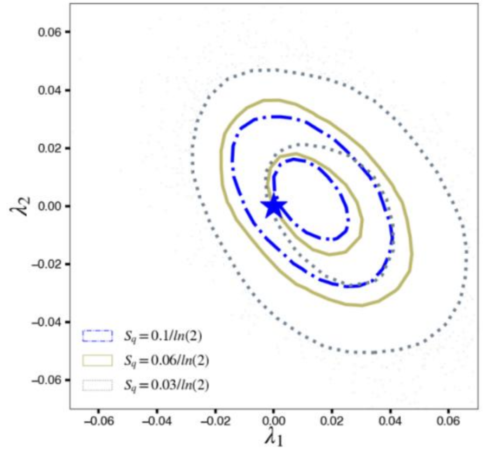
MNRAS **519,** 4841–4855 (2023)
Advance Access publication 2022 December 30

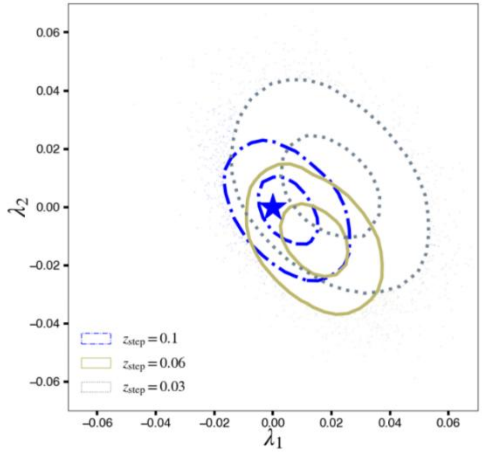
https://doi.org/10.1093/mnras/stac3812

The quadrupole in the local Hubble parameter: first constraints using Type Ia supernova data and forecasts for future surveys

Suhail Dhawan,[1★] Antonin Borderies,[2] Hayley J. Macpherson[3★] and Asta Heinesen[2★]

$$H(\boldsymbol{e}) = H_{\mathrm{m}} + \boldsymbol{H}_{\mathrm{q}} \cdot \boldsymbol{ee}\,\mathcal{F}_{\mathrm{quad}}(z, S)$$





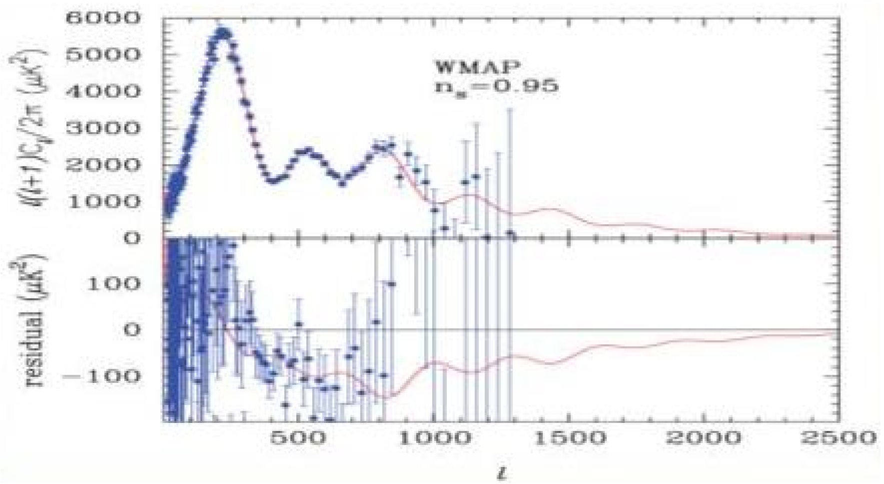
GW memory due to CMB

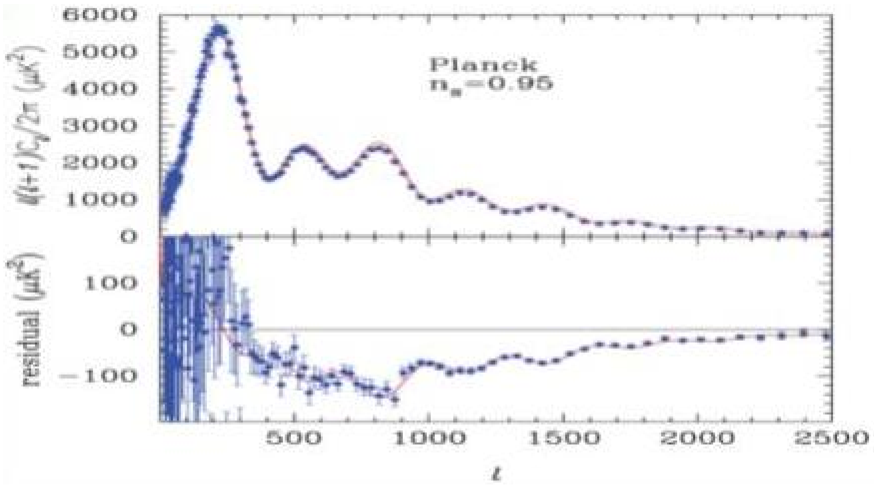
$$F_0 \approx 10^{-42}\mathrm{s}^{-2}$$

$$R \sim 10^{17}\mathrm{s}$$

$$a_{2m} \sim 10^{-5}$$

$$\dot{h}_{ij}^{\mathrm{mCMB}} \approx 10^{-35}\mathrm{s}^{-1} \qquad (H_0 \sim 10^{-17}\mathrm{s}^{-1})$$





GW memory due to CnuB

$$F_0 = \frac{3H_0^2}{8\pi}\Omega_\nu \qquad \Omega_\nu = \frac{1}{h^2}\frac{\sum_i m_{\nu_i}}{93.2\mathrm{eV}}$$

$$0.06\mathrm{eV} \lesssim \sum_i m_{\nu_i} \lesssim 6\mathrm{eV}$$

$$\dot{h}_{ij}^{\mathrm{mC\nu B}} \approx \Omega_\nu \times 10^{-18}\mathrm{s}^{-1}$$

$$\boxed{\dot{h}_{ij}^{\mathrm{mC\nu B}} > 10^{-22}\mathrm{s}^{-1}}$$

$$\sum_i m_{\nu_i} < 93.2 \times 10^{18} \times h^2 \mathcal{I}\mathrm{eV}$$

$\mathcal{I} \sim 10^{-19}\mathrm{s}^{-1}$ [MNRAS 519, 4841 (2023)]

$$\sum_i m_{\nu_i} < 5\mathrm{eV}$$

GW memory due to SGWB

Standard inflation theory predict: $\rho_{\rm GW} \approx 10^{-15}$ for above frequency $10^{-17}{\rm Hz}$

$$R \sim 10^{17}{\rm s} \qquad F_0 \approx 10^{-50}{\rm s}^{-2}$$

$$\dot{h}^m_{ij} < 10^{-33}{\rm s}^{-1}$$

GW memory due to SGWB

CMB + BAO + BBN: $F_0 < 3.8 \times 10^6 \times \underbrace{\frac{3H_0^2}{8\pi} \sim 10^{-42} \mathrm{s}^{-2} \qquad R \sim}\ 10^{17} \mathrm{s}$

$$\Downarrow$$

$$\dot{h}_{ij}^{\mathrm{m}_{\mathrm{relicSGWB}}} \lesssim 10^{-23} \mathrm{s}^{-1}$$

GW memory due to SGWB

From CBC:

$$\Omega_{\rm GW}(f) = A_{\rm ref}\left(\frac{f}{f_{\rm ref}}\right)^{\frac{2}{3}}$$

$$F_0 = \frac{9H_0^2}{16\pi}\frac{A_{\rm ref}}{f_{\rm ref}^{\frac{2}{3}}}\left(f_{\rm merg}^{\frac{2}{3}} - f_{\rm form}^{\frac{2}{3}}\right)$$

Since $f_{\rm merg} \gg f_{\rm form}$ we have

$$F_0 \approx \frac{9H_0^2}{16\pi}A_{\rm ref}\left(\frac{f_{\rm merg}}{f_{\rm ref}}\right)^{\frac{2}{3}}.$$

LIGO: $A_{\rm ref} < 10^{-9}$ at $f_{\rm ref} = 25{\rm Hz}$ $f_{\rm merg} \approx 10^2{\rm Hz}$ $\Rightarrow$ $F_0 \lesssim 10^{-44}{\rm s}^{-2}$

PTA: $A_{\rm ref} < 10^{-6}$ at $f_{\rm ref} = 10^{-8}{\rm Hz}$ $f_{\rm merg} \approx 10^{-3}{\rm Hz}$ $\Rightarrow$ $F_0 \lesssim 10^{-39}{\rm s}^{-2}$

$$\boxed{\begin{aligned} \dot{h}_{ij}^{\rm m_{stelarCBCSGWB}} &\approx 10^{-28}{\rm s}^{-1}, \\ \dot{h}_{ij}^{\rm m_{superCBCSGWB}} &\approx 10^{-23}{\rm s}^{-1}. \end{aligned}}$$

GW memory due to SGWF

From GW foreground of binary white dwarfs:

$$S_h(f) \simeq 1.9 \times 10^{-44} (f/\mathrm{Hz})^{-7/3}\,\mathrm{Hz}^{-1} \times \left(\frac{\mathcal{D}_{\rm char}}{6.4\ \mathrm{kpc}}\right)^{-2} \left(\frac{\mathcal{R}_{\rm gal}}{0.015/\mathrm{yr}}\right) \left(\frac{\mathcal{M}_{z,\rm char}}{0.35\,M_\odot}\right)^{5/3}$$

$$F_0 \simeq 4.5 \times 10^{-44} \times (f_{\rm up}^{\frac{2}{3}} - f_{\rm low}^{\frac{2}{3}})$$

$$f_{\rm up}^{\frac{2}{3}} - f_{\rm low}^{\frac{2}{3}} \sim 1 \qquad R \sim 10^{11}\mathrm{s}$$

$$\dot{h}_{ij}^{\rm mBWDSGWB} \lesssim 10^{-34}\mathrm{s}^{-1}$$

Summary

- GW memory is an outstanding character of GR
- Waveform model of GW memory has been constructed and detection is possible
- Overall GW memory has been estimated, and golden events have been shown
- SGWMB of BBH mergers is promising for LISA/Taiji/Tianqin
- GW memory of CnuB may be detected or be used to constraint mass of nu

$$0.06\mathrm{eV} \lesssim \sum_i m_{\nu_i} \lesssim 6\mathrm{eV} \quad \Rightarrow \quad \boxed{\dot{h}_{ij}^{\mathrm{m_{C\nu B}}} > 10^{-22}\mathrm{s}^{-1}}$$