α'-corrections to Near Extremal Dyonic Strings and Weak Gravity Conjecture

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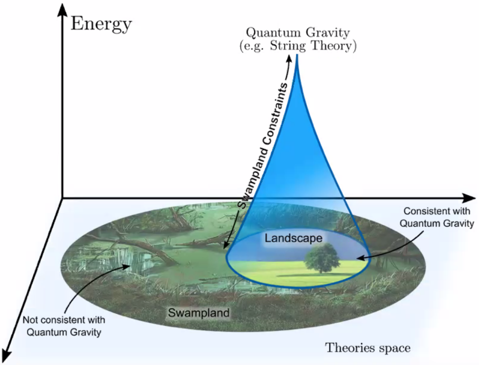
Outline

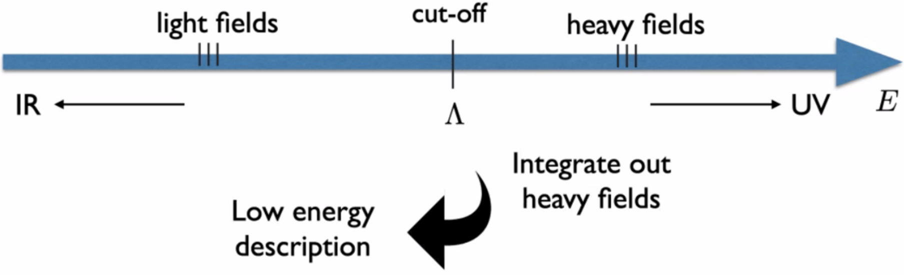
Background & Motivation



Not every EFT can be UV completed in quantum gravity!!!

Swampland: Apparently consistent quantum EFTs that cannot be UV completed in quantum gravity

Effective field theories



Expectation of 'separation of scales':

UV corrections become small in the IR ⟶ Not important to predict experimental results

IR effective theory not very sensitive to UV physics

(Quantum gravitational effects seem irrelevant)

UV/IR mixing

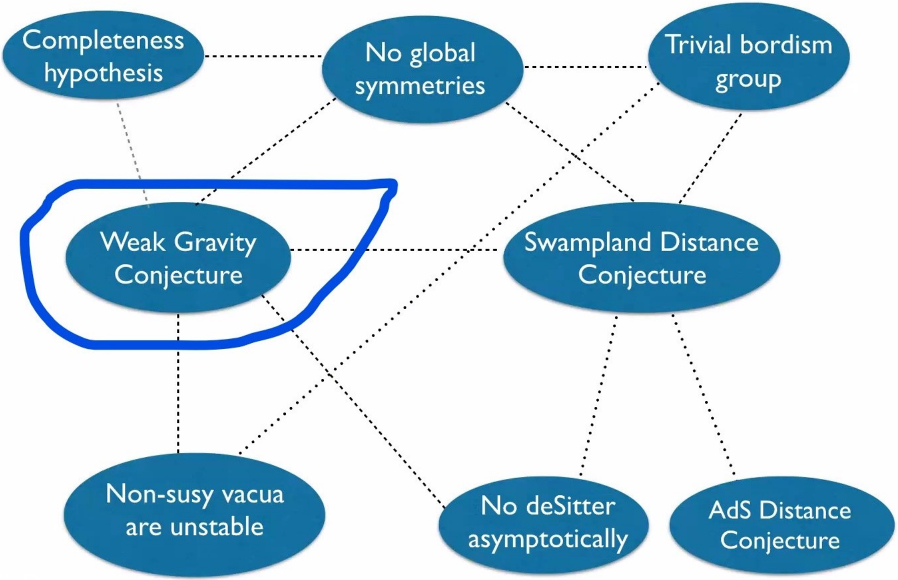
Quantum Gravity breaks with this notion of separation of scales and can impose non-trivial constraints at low energies

- Evidence from String Theory
- Black Hole Physics makes manifest a correlation between high and low energy physics (UV/IR mixing)

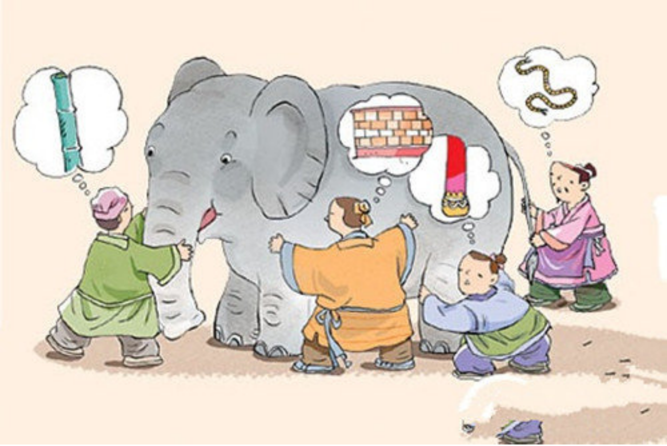
scattering of particles at high energies

large black holes

Swampland Conjectures



These conjectures are based on lots of facts from string theory and intricate interconnections among them without rigorous proofs.



Weak Gravity Conjecture [Arkani-Hamed, Motl, Nocolis, Vafa, 06']

- (Electric WGC) For a U(1) gauge theory coupled to gravity, there exists at least one charged objects with

$$m/q \leq m/q|_{extremal\ BH} \quad \text{in Planck units}$$

so that extremal black holes can decay. Gravity is the *weakest force* for this particle.

$$S = \int d^4X\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{4g^2}F^2 + ...\right]$$

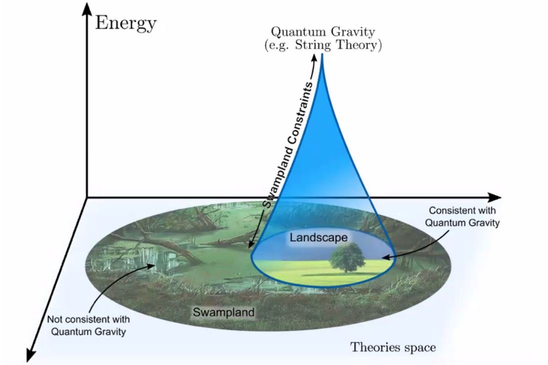
$$m/q|_{extremal\ BH} = \sqrt{2}$$

Moreover, so far no nontrivial version of the conjecture has actually been proven in the sense of being derived from some accepted general principle. A number of promising routes towards a proof of some version of the WGC have been proposed in recent years, but these routes all suffer from at least one of two drawbacks: either they establish some statement which is qualitatively like the WGC, but without the correct $O(1)$ factors included (i.e., “no gauge force can be much weaker than gravity”), or they argue for a precise version of the WGC, but rely on additional, unproven assumptions. In particular in the original

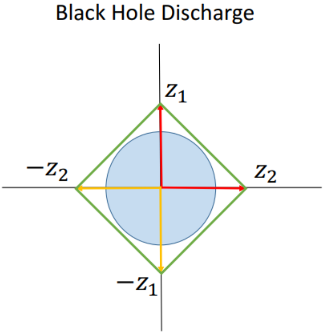
[Harlow, Heidenreich, Reece, Rudelius, 22’]

The usual argument against Global symmetry in QG: “QG must have finite number of degrees of freedom” does not seem to apply to charges associated with a gauge symmetry.

Even though a rigorous proof is still lacking, results from string theory example may well serve as the lamppost in the swampland.



- Generalizations including multiple U(1)s (convex hull condition) and *p*-forms [Cheung Remmen 14', Heidenreich, M. Reece, Rudelius 15']

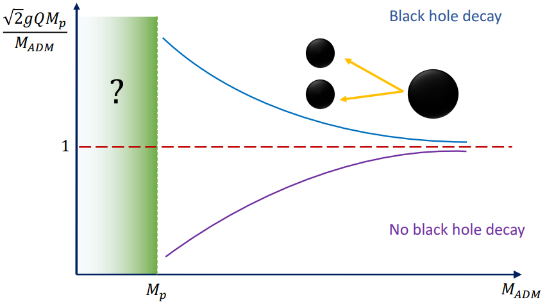


- For Einstein-Maxwell-dilation theory, one needs to distinguish cases where the scalar field mediates force or not. If yes, WGC is replaced by the repulsive force conjecture.

In particular, the charged objects satisfying WGC can be fundamental particles, bound states or even BHs themselves

For an extremal RN black hole to be able to decay to smaller extremal RN black holes [Cheung, Liu, Remmen, 18']

$$\frac{M}{M_p} = \sqrt{2}Q(1 + \frac{c}{Q^2} + \cdots)\,, \quad c < 0$$



The problem

In gravity models preserving supersymmetry, when the extremal limit of charged black holes coincides with BPS limit

$$M = \sum_i Q_i$$

regardless of the detailed structure of the higher derivative couplings. Certainly, not all the higher derivative supergravity models can be UV completed. How can one distinguish those which can be embedded in QG from those which cannot?

Our proposal

We suggest to look at near extremal (near BPS) black holes in supergravity models with higher derivative couplings, in particular, models from string theory.

Clearly, at leading minimal coupling, the attractive force dominates, but one can study whether contributions from the higher derivative couplings is attractive or repulsive.

The setups

1) IIA string compactified on K3

$$\Updownarrow$$

2) Heterotic string compactified on 4-torus

In these two cases, the low energy limit is described by $D = 6$ $\mathcal{N} = (1, 1)$ supergravity coupled to 20 vector multiplets where the scalars parameterize the $O(4, 20)/(O(4) \times O(20))$ coset space.

The leading α' corrections are fully supersymmetrized in the NS-NS sector $(g_{\mu\nu}, B_{\mu\nu}, \phi)$ [Bergshoeff, de Roo , 89', Liu, Minasian 13', Novak, Ozkan, YP, Tartaglino-Mazzucchelli 17'].

The approach

Step 1) We construct dyonic string solutions with leading α'-corrections.

Step 2) We analyse the force felt by a static probe string (parallel to the macroscopic string).

Step 3) We take the near extremal limit appropriately and extract the contributions from the higher derivative couplings.

Dyonic string solutions in 6D 2-derivative supergravity

The bosonic Lagrangian

$$\mathcal{L}_0 = L(R + L^{-2}\nabla^\mu L \nabla_\mu L - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \cdots), \quad H_{\mu\nu\rho} = 3\partial_{[\mu} B_{\nu\rho]}$$

The static dyonic solution

$$\begin{aligned} ds_6^2 &= D(r)(-h(r)dt^2 + dx^2) + H_p(r)(\frac{dr^2}{f(r)} + r^2 d\Omega_3^2)\,, \\ B_{(2)} &= 2P\sqrt{1+\frac{\mu}{P}}\omega_2 + \sqrt{1+\frac{\mu}{Q}}A(r)dt \wedge dx\,, \qquad L = \frac{1}{D(r)H_p(r)}, \\ D(r) &= A(r) = \frac{r^2}{r^2+Q}, \quad H_p(r) = 1 + \frac{P}{r^2}, \quad h(r) = f(r) = 1 - \frac{\mu}{r^2} \end{aligned}$$

[Cvetic, Tseytlin 95']. $P = 0$, the solution reduces to the fundamental string solution [Duff, Lu 93'].

Thermodynamics

$$
\begin{aligned}
M &= \frac{3\pi}{8}\mu + \frac{\pi}{4}(Q+P), \\
T &= \frac{1}{2\pi}\frac{\sqrt{\mu}}{\sqrt{\mu+P}\sqrt{\mu+Q}}, \\
S &= \frac{1}{2}\pi^2\sqrt{\mu}\sqrt{\mu+P}\sqrt{\mu+Q}, \\
Q_{\rm e} &= \frac{\pi}{4}\sqrt{Q}\sqrt{\mu+Q}, \quad \Phi_{\rm e} = \sqrt{\frac{Q}{\mu+Q}}, \\
Q_{\rm m} &= \frac{\pi}{4}\sqrt{P}\sqrt{\mu+P}, \quad \Phi_{\rm m} = \sqrt{\frac{P}{\mu+P}}
\end{aligned}
$$

- $\mu \to 0,\ T \to 0,\ M \to Q_{\rm e} + Q_{\rm m},\ S \to 0$
- In the canonical ensemble

$$
C_T = \frac{dE}{dT}\bigg|_{Q_{\rm e},Q_{\rm m}} = \frac{\sqrt{\mu}f(\mu,P,Q)}{4PQ-\mu^2}\,, \quad f(\mu,P,Q) > 0
$$

Near extremal, $C_T > 0$ similar to near extremal RN black hole in asymptotically Minkowski.

Probe string

In string frame

$$
\begin{aligned}
S_2 &= -T_2 \int d^2\xi \left(\sqrt{-\det \gamma} + \tfrac{1}{2}\epsilon^{ij}\partial_i X^M \partial_j X^N B_{MN} \right) \\
\gamma_{ij} &= g_{MN}\partial_i X^M \partial_j X^N
\end{aligned}
$$

$$
\xi^0 = t\,, \quad \xi^1 = x
$$

By $x^\perp = \mathrm{constant}$, one obtains the static potential

$$
\mathcal{V} = T_2 \left(\sqrt{-g_{tt}g_{xx}} - B_{tx} \right)\,, \quad \mathcal{F} = -\frac{d\mathcal{V}(r)}{dr}
$$

$\sqrt{-g_{tt}g_{xx}} - B_{tx} = 0$ is one of the off-shell BPS equation [YP 19']. Plugging the solution

$$
\mathcal{V}(r) = \frac{T_2 r^2}{r^2 + Q}\left(\sqrt{1-\mu/r} - \sqrt{1+\mu/Q}\right)
$$

Weak field expansion in the region where $r \gg \max\{\sqrt{\mu},\, \sqrt{Q},\, \sqrt{P}\}$

$$
\mathcal{V}(r)/T_2 = 1 - \sqrt{1+\mu/Q} + \frac{\mu}{2}\frac{1-\sqrt{1+\mu/Q}}{1+\sqrt{1+\mu/Q}} r^{-2} + \mathcal{O}(r^{-4})
$$

In order to see the contributions from different forces, we switch to Einstein frame

$$g_{MN} = L^{-\frac{1}{2}} g^E_{MN}$$

Then the static potential can be rewritten as

$$\mathcal{V} = T_2 \left(L^{-\frac{1}{2}} \sqrt{-g^E_{tt} g^E_{xx}} - B_{tx} \right)$$

In the asymptotic region where $r \gg \max\{\sqrt{\mu}, \sqrt{Q}, \sqrt{P}\}$, we have

$$\begin{aligned} \sqrt{-g^E_{tt} g^E_{xx}} &= 1 - \frac{P+Q+\mu}{2r^2} + \mathcal{O}(r^{-4}), \\ L^{-\frac{1}{2}}(r) &= 1 + \frac{P-Q}{2r^2} + \mathcal{O}(r^{-4}), \\ B_{tx} &= \sqrt{1+\mu/Q} - \frac{Q\sqrt{1+\mu/Q}}{r^2} + \mathcal{O}(r^{-4}), \end{aligned}$$

Q1:What if $(\phi,\, g_{\mu\nu},\, B_{\mu\nu})$ on the world volume is different from those in the supegravity effective action?

$$g`_{\mu\nu} = g_{\mu\nu} + a g_{\mu\nu} R + b R_{\mu\nu} + \cdots$$

Q2:What about higher derivative corrections to world volume action?

A: They will not affect expansion of potential at leading order in large r.

$$\mathcal{V}(r)/T_2 = 1 - \sqrt{1 + \mu/Q} - \Sigma/r^2 + \mathcal{O}(1/r^3)\,.$$

Leading α'-corrections from IIA string on K3

$$
\begin{aligned}
\Delta S_2\big|_{\rm IIA} \;=\;& \alpha'^3 e^{-2\phi}\Bigg[\frac{\zeta(3)}{3\cdot 2^{11}}\Big(t_8t_8R(\omega_+)^4-\frac{1}{4}\epsilon_8\epsilon_8R(\omega_+)^4-2t_8t_8H^2R(\omega_+)^3 \\
& -\frac{1}{6}\epsilon_9\epsilon_9H^2R(\omega_+)^3+8\cdot 4!\sum_i d_iH^{\mu\nu\lambda}H^{\rho\sigma\tau}\widetilde{Q}^i_{\mu\nu\lambda\rho\sigma\tau}+\cdots\Big)\Bigg] \\
& +\Bigg[\frac{\pi^2}{9\cdot 2^{11}}\alpha'^3 e\Big(t_8t_8R(\omega_+)^4+\frac{1}{4}\epsilon_8\epsilon_8R(\omega_+)^4+\frac{1}{3}\epsilon_9\epsilon_9H^2R(\omega_+)^3-\frac{4}{9}\epsilon_9\epsilon_9H^2(DH)^2R(\omega_+)+\cdots\Big)+ \\
& -\frac{(2\pi)^6}{2}\alpha'^3 B\wedge\Big(X_8(R(\omega_+))+X_8(R(\omega_-))\Big)\Bigg]\,,
\end{aligned}
$$

- The $\zeta(3)$ term that appears at tree level in the 10D effective "vanishes" upon reduction to 6D [Liu, Minasian 19'].

α'-corrections from IIA string on K3

The leading higher derivative correction arises at 1-loop

$$\mathcal{L}_{LR+R^2} = \mathcal{L}_0 + \frac{\alpha'}{16}(\mathcal{L}_{\rm GB} + \mathcal{L}_{\rm Riem^2})$$

$$\begin{aligned}
\mathcal{L}_{\rm GB} &= R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 + \frac{1}{6}RH^2 - R^{\mu\nu}H^2_{\mu\nu} \\
&\quad + \frac{1}{2}R_{\mu\nu\rho\sigma}H^{\mu\nu\lambda}H^{\rho\sigma}{}_{\lambda} + \frac{5}{24}H^4 + \frac{1}{144}\left(H^2\right)^2 - \frac{1}{8}\left(H^2_{\mu\nu}\right)^2 \\
&\quad - \frac{1}{4}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^{\alpha}{}_{\beta}(\omega_+)R_{\lambda\tau}{}^{\beta}{}_{\alpha}(\omega_+)\,, \\
\mathcal{L}_{\rm Riem^2} &= R_{\mu\nu\alpha\beta}(\omega_-)R^{\mu\nu\alpha\beta}(\omega_-) - \frac{1}{4}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^{\alpha}{}_{\beta}(\omega_-)R_{\lambda\tau}{}^{\beta}{}_{\alpha}(\omega_-)
\end{aligned}$$

- $\mathcal{L}_{\rm GB}$ and $\mathcal{L}_{\rm Riem^2}$ preserves 6d (1,0) supersymmetry. The particular combination preserves (1,1) susy.
- They are not related by field redefinition.

$$R_{\mu\nu}{}^{\alpha}{}_{\beta}(\omega_\pm) = \partial_\mu\omega^{\alpha}_{\pm\nu\beta} + \omega^{\alpha}_{\pm\mu\gamma}\omega^{\gamma}_{\pm\nu\beta} - (\mu\leftrightarrow\nu)\,, \quad \omega^{\alpha}_{\pm\mu\beta} = \omega^{\alpha}_{\mu\beta} \pm \frac{1}{2}H_{\mu}{}^{\alpha}{}_{\beta}$$

Up to cubic fermions, the off-shell transformation rule is given by

$$\begin{aligned}
\delta e_\mu{}^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu \ , \\
\delta\psi_\mu{}^i &= (\partial_\mu + \frac{1}{4}\omega_{\mu ab}\gamma^{ab})\epsilon^i + V_\mu{}^i{}_j\epsilon^j + \frac{1}{8}H_{\mu\nu\rho}\gamma^{\nu\rho}\epsilon^i \ , \\
\delta B_{\mu\nu} &= -\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]} \ , \\
\delta\chi^i &= \frac{1}{2\sqrt{2}}\gamma^\mu\delta^{ij}\partial_\mu L\epsilon_j - \frac{1}{4}\gamma^\mu E_\mu\epsilon^i + \frac{1}{\sqrt{2}}\gamma^\mu V'^{(i}_{\mu\,k}\delta^{j)k}L\epsilon_j - \frac{1}{12\sqrt{2}}L\delta^{ij}\gamma\cdot H\epsilon_j \ , \\
\delta L &= \frac{1}{\sqrt{2}}\bar{\epsilon}^i\chi^j\delta_{ij} \ , \\
\delta C_{\mu\nu\rho\sigma} &= L\bar{\epsilon}^i\gamma_{[\mu\nu\rho}\psi^j_{\sigma]}\delta_{ij} - \frac{1}{2\sqrt{2}}\bar{\epsilon}\gamma_{\mu\nu\rho\sigma}\chi \ , \\
\delta V_\mu{}^{ij} &= \frac{1}{2}\bar{\epsilon}^{(i}\gamma^\rho\psi^{j)}_{\mu\rho} + \frac{1}{12}\bar{\epsilon}^{(i}\gamma\cdot H\psi^{j)}_\mu + \frac{1}{8}\sigma^{-1}\bar{\epsilon}^{(i}\gamma^\rho\Big(H_{[\mu}{}^{ab}\gamma_{ab}\psi^{j)}_{\rho]}\Big)
\end{aligned}$$

Thus the full ansatz takes the form

$$ds_6^2 = D(r)\left(-h(r)dt^2 + dx^2 + 2\omega dt dx\right) + H_p(r)\left(\frac{dr^2}{f(r)} + r^2 d\Omega_3^2\right),$$
$$B_{(2)} = 2P\sqrt{1+\frac{\mu}{P}}\omega_2 + \sqrt{1+\frac{\mu}{Q}}A(r)dt\wedge dx\,, \quad L = L(r)$$

$$L = L_0 + \delta L, \quad D = D_0 + \delta D, \quad A = D_0 + \delta A$$
$$f = f_0 + \delta f, \quad h = f_0 + \delta h, \quad \omega = \delta\omega$$

- α'-correction generates a velocity in the x-direction, however, the

solution carries 0 linear momentum.

Thermodynamic quantities with α' corrections

$$
\begin{aligned}
M &= \frac{3\pi}{8}\mu + \frac{\pi}{4}(Q+P) - \frac{3\pi\mu^2(3\mu+2Q)}{32(\mu+Q)^2(\mu+2Q)}\alpha', \\
T &= \frac{1}{2\pi}\frac{\sqrt{\mu}}{\sqrt{\mu+P}\sqrt{\mu+Q}} - \frac{\sqrt{\mu}Q(5\mu+4Q)}{4\pi\sqrt{\mu+P}(\mu+Q)^{5/2}(\mu+2Q)}\alpha', \\
S &= \frac{1}{2}\pi^2\sqrt{\mu}\sqrt{\mu+P}\sqrt{\mu+Q} + \frac{\pi^2\sqrt{\mu}\sqrt{\mu+P}Q(5\mu+4Q)}{4(\mu+Q)^{3/2}(\mu+2Q)}\alpha', \\
Q_{\rm e} &= \frac{\pi}{4}\sqrt{Q}\sqrt{\mu+Q}, \quad \Phi_{\rm e} = \sqrt{\frac{Q}{\mu+Q}} + \frac{\mu\sqrt{Q}(5\mu+4Q)}{2(\mu+Q)^{5/2}(\mu+2Q)}\alpha', \\
Q_{\rm m} &= \frac{\pi}{4}\sqrt{P}\sqrt{\mu+P}, \quad \Phi_{\rm m} = \sqrt{\frac{P}{\mu+P}}
\end{aligned}
$$

These satisfy the first law of thermodynamics up to $\mathcal{O}(\alpha'^2)$

$$
dM - TdS - \Phi_{\rm e}dQ_{\rm e} - \Phi_{\rm m}dQ_{\rm m} = \mathcal{O}(\alpha'^2)
$$

- The BPS limit is still defined at $\mu = 0$

$$T = 0\,, \quad M = Q_{\rm e} + Q_{\rm m}\,, \quad S = 0$$

- Reparametrization degree of freedom at $\mathcal{O}(\alpha')$

$$\mu \to \mu + \alpha' f_1(\mu, P, Q)\,, \quad Q \to Q + \alpha' f_2(\mu, P, Q)\,, \quad P \to P\alpha' f_3(\mu, P, Q)$$

However, the relation among physical quantities are unaffected

- The leading order solution suffices for deriving the thermodynamic quantities at $\mathcal{O}(\alpha')$ [Reall, Santos 19']

$$I_E = I_{E0}(T, \Phi_{\rm e}, Q_{\rm m}) + I_{\rm hd}(T, \Phi_{\rm e}, Q_{\rm m})$$

based on which one can also show

$$\delta S(M,\ Q_{\rm e},\ Q_{\rm m}) = -I_{\rm hd}$$

Here

$$\delta S(M,\ Q_{\rm e},\ Q_{\rm m}) = \frac{\pi^2\sqrt{\mu}\sqrt{\mu + P}(9\mu + 8Q)}{16(\mu + Q)^{3/2}}\alpha'$$

Static potential felt by the probe string

$$\begin{aligned} \mathcal{V}(r)/T_2 &= 1-\sqrt{1+\mu/Q}-\Sigma/r^2+\mathcal{O}(1/r^3)\,, \\ \Sigma &= \frac{\mu}{2}+Q\left(1-\sqrt{\frac{\mu+Q}{Q}}\right)-\frac{3\alpha'\,(3\mu+2Q)\,\mu^2}{8(\mu+Q)^2(\mu+2Q)} \end{aligned}$$

The large r expansions of the fields in Einstein frame are given by

$$\begin{aligned} \sqrt{-g^E_{tt}g^E_{xx}} &= 1-\frac{\sigma_g}{r^2}+\mathcal{O}(r^{-4})\,, \quad \sigma_g=\frac{1}{2}\left(P+Q+\mu-\frac{3\alpha'(3\mu+2Q)\mu^2}{8(Q+\mu)^2(2Q+\mu)}\right) \\ L^{-\frac{1}{2}}(r) &= 1-\frac{\sigma_L}{r^2}+\mathcal{O}(r^{-4})\,, \quad \sigma_L=\frac{1}{2}\left(Q-P-\frac{3\alpha'(3\mu+2Q)\mu^2}{8(Q+\mu)^2(2Q+\mu)}\right)\,, \\ B_{tx} &= \sqrt{1+\mu/Q}-\frac{\sigma_B}{r^2}+\mathcal{O}(r^{-4})\,, \quad \sigma_B=Q\sqrt{1+\mu/Q} \end{aligned}$$

$$\Sigma=\sigma_g+\sigma_L-\sigma_B$$

It is clear that if the α' correction $\delta\Sigma$ is positive, it enhances the attractive force, while a negative $\delta\Sigma$ is repulsive.

- α' correction to the static potential expressed in terms of $(\mu,\ P,\ Q)$ which are not physical quantities and can be redefined with α' dependent terms
- However, the relation among physical quantities should not depend on the reparameterization. We thus need to recast Σ in terms of physical quantities in order to discuss the α'-correction to the static force

  We can first solve for parameters Q and P from

$$Q = \frac{\sqrt{\pi^2\mu^2 + 64Q_{\rm e}^2} - \pi\mu}{2\pi}, \quad P = \frac{\sqrt{\pi^2\mu^2 + 64Q_{\rm m}^2} - \pi\mu}{2\pi}$$

- Express μ in terms of $(T,\ Q_{\rm e},\ Q_{\rm m})$, $(M,\ Q_{\rm e},\ Q_{\rm m})$ or $(S,\ Q_{\rm e},\ Q_{\rm m})$. They correspond to the inclusion of the higher derivative interactions in 3 different physical processes: 1) isothermal process; 2) isoenergetic process; 3) isoentropic process

In the isothermal process,

$$\begin{aligned}\sigma_g &= \frac{2(Q_\mathrm{e}+Q_\mathrm{m})}{\pi} + 64\pi Q_\mathrm{e} T^4 \left(Q_\mathrm{e} Q_\mathrm{m} + Q_\mathrm{m}^2\right) \\ &\quad +16\pi^2 T^4 \left(4 Q_\mathrm{e} Q_\mathrm{m} + Q_\mathrm{m}^2\right)\alpha' + \mathcal{O}(T^6)\,, \\ \sigma_L &= \frac{2(Q_\mathrm{e}-Q_\mathrm{m})}{\pi} - 64\pi Q_\mathrm{e} T^4 \left(Q_\mathrm{e} Q_\mathrm{m} - Q_\mathrm{m}^2\right) \\ &\quad +16\pi^2 T^4 \left(Q_\mathrm{m}^2 - 4 Q_\mathrm{e} Q_\mathrm{m}\right)\alpha' + \mathcal{O}(T^6)\,, \\ \sigma_B &= \frac{4Q_\mathrm{e}}{\pi} + \mathcal{O}(T^6)\end{aligned}$$

$$\Sigma = \sigma_g + \sigma_L - \sigma_B = 128\pi Q_\mathrm{e} Q_\mathrm{m}^2 T^4 + 32\pi^2 Q_\mathrm{m}^2 T^4 \alpha' + \mathcal{O}(T^6)$$

In the isoenergetic process, up to first order in α' we have

$$\sigma_g = \frac{2(Q_\mathrm{e}+Q_\mathrm{m})}{\pi} + \frac{Q_\mathrm{e}+Q_\mathrm{m}}{Q_\mathrm{e}Q_\mathrm{m}}\frac{\delta m^2}{\pi} - \frac{3\delta m^2}{4Q_\mathrm{e}^2}\alpha' + \mathcal{O}(\delta m^3)\,,$$

$$\sigma_L = \frac{2(Q_\mathrm{e}-Q_\mathrm{m})}{\pi} + \frac{Q_\mathrm{m}-Q_\mathrm{e}}{Q_\mathrm{e}Q_\mathrm{m}}\frac{\delta m^2}{\pi} - \frac{3\delta m^2}{4Q_\mathrm{e}^2}\alpha' + \mathcal{O}(\delta m^3)\,,$$

$$\sigma_B = \frac{4Q_\mathrm{e}}{\pi} + \mathcal{O}(\delta m^3)$$

$$\Sigma = \sigma_g + \sigma_L - \sigma_B = \frac{2\delta m^2}{\pi Q_\mathrm{e}} - \frac{3\delta m^2\alpha'}{2Q_\mathrm{e}^2} + \mathcal{O}(\delta m^3)$$

In the isoentropic process, the leading falloff coefficients of various fields in Einstein frame, up to first order in α' we have

$$
\begin{aligned}
\sigma_g &= \frac{2(Q_{\rm e}+Q_{\rm m})}{\pi}+\frac{Q_{\rm e}+Q_{\rm m}}{1024\pi^3 Q_{\rm e}^3 Q_{\rm m}^3}s^4-\frac{4Q_{\rm e}+7Q_{\rm m}}{4096\pi^2 Q_{\rm e}^4 Q_{\rm m}^3}s^4\alpha'+\mathcal{O}(s^6)\,,\\
\sigma_L &= \frac{2(Q_{\rm e}-Q_{\rm m})}{\pi}+\frac{Q_{\rm m}-Q_{\rm e}}{1024\pi^3 Q_{\rm e}^3 Q_{\rm m}^3}s^4-\frac{7Q_{\rm m}-4Q_{\rm e}}{4096\pi^2 Q_{\rm e}^4 Q_{\rm m}^3}s^4\alpha'+\mathcal{O}(s^6)\,,\\
\sigma_B &= \frac{4Q_{\rm e}}{\pi}+\mathcal{O}(s^6)
\end{aligned}
$$

$$
\Sigma=\sigma_g+\sigma_L-\sigma_B=\frac{s^4}{512\pi^3 Q_{\rm e}^3 Q_{\rm m}^2}-\frac{7s^4}{2048\pi^2 Q_{\rm e}^4 Q_{\rm m}^2}\alpha'+\mathcal{O}(s^6)
$$

α'-corrections from heterotic string on 4-torus

The leading higher derivative correction arises at tree-level

$$\mathcal{L}_{LR+LR^2} = \sqrt{-g}L\left(R + L^{-2}\nabla^\mu L\nabla_\mu L - \frac{1}{12}\widetilde{H}_{\mu\nu\rho}\widetilde{H}^{\mu\nu\rho} + \frac{\alpha'}{8}R_{\mu\nu\alpha\beta}(\omega_+)R^{\mu\nu\alpha\beta}(\omega_+)\right)$$

where the 3-form field strength $\tilde{H}_{\mu\nu\rho}$ includes the Lorentz Chern-Simons term

$$\widetilde{H}_{(3)} = dB_{(2)} + \frac{1}{4}\alpha'\mathrm{CS}_{(3)}(\omega_+)$$

such that it obeys a deformed Bianchi identity

$$d\widetilde{H}_{(3)} = \frac{1}{4}\alpha' R^a{}_b(\omega_+) \wedge R^b{}_a(\omega_+)$$

This theory is related to IIA on K3 by S-duality

$$L^{\mathrm{IIA}} \star H^{\mathrm{IIA}} = \widetilde{H}^{\mathrm{het}}, \quad L^{\mathrm{IIA}} g^{\mathrm{IIA}}_{\mu\nu} = g^{\mathrm{het}}_{\mu\nu}, \quad L^{\mathrm{IIA}} = 1/L^{\mathrm{het}}$$

- Using duality and applying suitable change of coordinates, we obtain the dyonic string solution with α' corrections in heterotic string on 4-torus.

$$M^{(\mathrm{het})} = \frac{3\pi}{8}\mu + \frac{\pi}{4}(Q+P) - \frac{3\pi\mu^2(3\mu+2P)}{32(\mu+P)^2(\mu+2P)}\alpha',$$

$$T^{(\mathrm{het})} = \frac{1}{2\pi}\frac{\sqrt{\mu}}{\sqrt{\mu+P}\sqrt{\mu+Q}} - \frac{\sqrt{\mu}P(5\mu+4P)}{4\pi\sqrt{\mu+Q}(\mu+P)^{5/2}(\mu+2P)}\alpha',$$

$$S^{(\mathrm{het})} = \frac{1}{2}\pi^2\sqrt{\mu}\sqrt{\mu+P}\sqrt{\mu+Q} + \frac{\pi^2\sqrt{\mu}\sqrt{\mu+Q}P(5\mu+4P)}{4(\mu+P)^{3/2}(\mu+2P)}\alpha',$$

$$Q_{\mathrm{e}}^{(\mathrm{het})} = \frac{1}{4}\pi\sqrt{Q}\sqrt{\mu+Q}, \quad \Phi_{\mathrm{e}}^{(\mathrm{het})} = \sqrt{\frac{Q}{\mu+Q}},$$

$$Q_{\mathrm{m}}^{(\mathrm{het})} = \frac{\pi}{4}\sqrt{P}\sqrt{\mu+P}, \quad \Phi_{\mathrm{m}}^{(\mathrm{het})} = \sqrt{\frac{P}{\mu+P}} + \frac{\mu\sqrt{P}(5\mu+4P)}{2(\mu+P)^{5/2}(\mu+2P)}\alpha'$$

which are related to those in the IIA case by interchanging parameters P, Q. At fixed conserved charges $(M,\ Q_{\mathrm{e}},\ Q_{\mathrm{m}})$

$$\delta S^{\mathrm{het}} = \frac{\pi^2\sqrt{\mu}\sqrt{\mu+Q}(9\mu+8P)}{16(\mu+P)^{3/2}}\alpha' > 0$$

Static potential felt by the probe string

The probe string in heterotic side is electrically charged, dual to the magnetically charged one in the IIA side we find that the static potential takes the form

$$\mathcal{V}(r)/T_2 = 1 - \sqrt{1+\mu/Q} - \Sigma/r^2 + \mathcal{O}(1/r^3)\,, \quad \Sigma = \frac{\mu}{2} + Q\left(1 - \sqrt{\frac{\mu+Q}{Q}}\right)$$

1) Isothermal

$$\Sigma = 128\pi Q_{\rm e} Q_{\rm m}^2 T^4 + 128\pi^2 Q_{\rm e} Q_{\rm m} T^4 \alpha'$$

2) Isoenergetic

$$\Sigma = \frac{2\delta m^2}{\pi Q_{\rm e}} + \left(\frac{3}{2' Q_{\rm e} Q_{\rm m}^2}\alpha' - \frac{2}{\pi Q_{\rm e}^2} - \frac{2}{\pi Q_{\rm e} Q_{\rm m}}\right)\delta m^3$$

3) Isoentropic

$$\Sigma = \frac{s^4}{512\pi^3 Q_{\rm e}^3 Q_{\rm m}^2} - \frac{s^4}{512\pi^2 Q_{\rm e}^3 Q_{\rm m}^3}\alpha' + \mathcal{O}(s^6)$$

Summary

- We obtained static dyonic string in both IIA on K3 and heterotic on 4-torus with leading α' corrections
- We worked out α' corrected thermodynamics $\Delta S > 0$
- If the inclusion of the correction is an isoentropic process, the higher derivative corrections always tend to reduce the attractive force when the system slightly deviates away from extremality

Future direction

- Most importantly, we would like to understand the special role played by entropy and how general the statement can be in other string theory setup?
- If yes, it can be used to constrain gravitational EFT.

Thank you for your attention