Higher-form symmetries and SymTFT in AdS4/CFT3

2209.xxxxx with M. van Beest, D. Gould and S. Schafer-Nameki

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Concepts of symmetries

• Symmetry is a central concept in physics

• Notions of (global) symmetries:

(1) Ordinary 0-form global symmetry, e. g. flavor symmetries, spacetime symmetries..

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(2) Higher-form symmetries

(3) Higher-group symmetries

(4) Non-invertible symmetries

(5) Sub-system symmetries...

- Lots of recent activities on the subject, applications in high energy and condensed matter physics.

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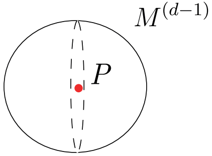
- Let us consider QFTs in d-dimensional space-time.
- The ordinary 0-form symmetry with group G: acts on local operators (0d particles).
- Introduce the topological operator $U(g, M^{(d-1)})$ generating the 0-form symmetry, which corresponds to $g \in G$,

$$U(g_1, M^{(d-1)})U(g_2, M^{(d-1)}) = U(g_1 g_2, M^{(d-1)}) . \quad (1)$$

- $U(g, M^{(d-1)})$ can act non-trivially on a 0-dim. operator $V(\mathcal{P})$ whenever $M^{(d-1)}$ and $\mathcal{P}$ are non-trivially linked.



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Ordinary 0-form global symmetry

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• In differential form language,

$$Q(M^{(d-1)}) = \oint_{M^{(d-1)}} j \,. \tag{2}$$

• e. g. in EM, the $(d-1)$-form conserved current j is given by the Maxwell's equation as

$$d * F = j \,. \tag{3}$$

$$U(g, M^{(d-1)}) = g^{\oint_{M^{(d-1)}} j} \,, \quad g \in U(1) \,. \tag{4}$$

Higher-form symmetry

- Extend the story to a p-form ($p > 0$) global symmetry with abelian group G (Gaiotto, Kapustin, Seiberg, Willett 14')
- A p-form symmetry is generated by a $(d-p-1)$-dimensional topological operator $U(g, M^{(d-p-1)})$:

$$U(g_1, M^{(d-p-1)})U(g_2, M^{(d-p-1)}) = U(g_1 g_2, M^{(d-p-1)}) . \tag{5}$$

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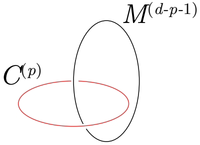
- $U(g, M^{(d-p-1)})$ has non-trivial action on $V(\mathcal{C}^{(p)})$ when $M^{(d-p-1)}$ and $\mathcal{C}^{(p)}$ are non-trivially linked.



- For 1-form symmetry in 3d, both U and V are 1-dim. operators.

3d examples

(1) $U(1)_k$ theory with CS level k

$$S = \frac{k}{4\pi} \int A \wedge dA \tag{6}$$

- The 1-form symmetry $\Gamma^{(1)} = \mathbb{Z}_k^{(1)}$ has the form

$$A \to A + \frac{1}{k}\lambda \tag{7}$$

λ is a properly normalized flat connection ($d\lambda = 0$).

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- The topological generator of the 1-form symmetry:

$$U(e^{2\pi i/k}, M^{(1)}) = \exp\left(i \oint_{M^{(1)}} A \right) \tag{8}$$

- The charged objects under $\mathbb{Z}_k^{(1)}$ are the same Wilson loop operators

$$W_n(\mathcal{C}) = \exp(in \oint_{\mathcal{C}} A) \tag{9}$$

(2) $SU(N)_k$ theory

• The 1-form symmetry $\Gamma^{(1)} = \mathbb{Z}_N^{(1)}$, generated by

$$U(e^{2\pi i n/k}, M^{(1)}) = tr\left[\mathcal{P}\exp\left(in\int_{M^{(1)}} A\right)\right] \;,\; n = \frac{k}{N}\,. \qquad (10)$$

• $\mathbb{Z}_N^{(1)}$ coincides with the center symmetry $\mathbb{Z}_N$ of $SU(N)$.

• Similar to the higher dimensional $SU(N)$ Yang-Mills theories, Wilson loop charged under the $\mathbb{Z}_N$ center symmetry.

3d examples

(3) ABJM theory: $U(N)_k \times U(N)_{-k}$ with bifundamental matter

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- For a single $U(N)_k$ factor, regard it as $(SU(N)_k \times U(1)_{Nk})/\mathbb{Z}_N$.

	$SU(N)_k$	$U(1)_{Nk}$
$\Gamma^{(1)}$	$\mathbb{Z}_N$	$\mathbb{Z}_{Nk}$

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(4) Gauge the $\mathbb{Z}_k$ 1-form symmetry $\rightarrow (U(N)_k \times U(N)_{-k})/\mathbb{Z}_k$

- Trivial 1-form symmetry

't Hooft anomaly

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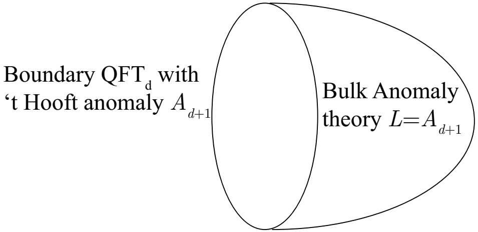
• However, the gauging can be obstructed by 't Hooft anomaly.
• 't Hooft anomaly polynomial A_{d+1} is a $(d+1)$-form
• e. g. 3d ABJ theory $U(N+b)_k \times U(N)_{-k}$ has a $\mathbb{Z}_k^{(1)}$ 1-form symmetry, with background gauge field B_2.

$$A_{d+1} = -\frac{b}{2k} B_2 \wedge B_2 \,, \tag{12}$$

obstruct the gauging of $\mathbb{Z}_k^{(1)}$ 1-form symmetry when $b \nmid k$.
• There can also be mixed 't Hooft anomaly, e. g. BF-terms describing mixed 't Hooft anomalies of 0-form and 1-form symmetry

Anomaly theory

- After we gauge global symmetries with 't Hooft anomaly, one can consider a $(d+1)$-dimensional anomaly theory coupling to the d-dimensional theory on the boundary: (Freed 14')



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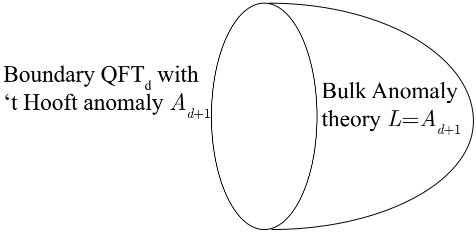
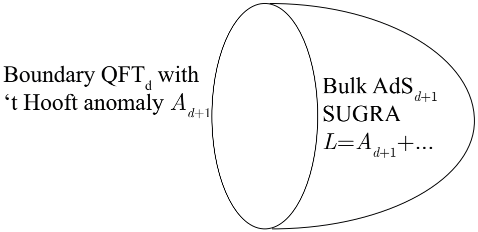


• The whole system is anomaly free

• The bulk anomaly theory is an invertible field theory (a TQFT with 1-dim. Hilbert space), which is the low energy limit of an SPT phase .

AdS/CFT perspective



- 't Hooft anomaly polynomial $A_{d+1} \leftrightarrow$ SUGRA terms in AdS_{d+1}
- Background gauge field C_{p+1} for p-form symmetry $\leftrightarrow$ gauge field in AdS_{d+1}
- Different boundary conditions of $C_{p+1} \rightarrow$ different global form of gauge groups in QFT_d. (Witten 98')

SymTFT

• "Symmetry field theory" (SymTFT): a generalized version of anomaly theory (Apruzzi, Bonetti, Extebarria, Hosseini, Schafer-Nameki 21')

• Encodes the different global structures of QFT (gauge groups, etc..) and 't Hooft anomaly polynomial.

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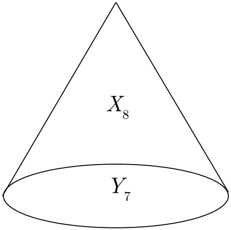
• Can be derived from M-theory action in two setups:

(1) Geometric engineering: M-theory on $\mathbb{R}^{2,1} \times X_8$ (X_8 is non-compact)

(2) AdS/CFT: M-theory on $AdS_4 \times Y_7$, dual to M2 branes probing a singular space X_8. X_8 is a cone over Y_7. (Y_7 is compact)



SymTFT from M-theory

- Consider M-theory on $\mathcal{M}_{11} = \mathcal{M}_4 \times Y_7$
- Starting from topological term

$$S_{11\mathrm{d}} = 2\pi \int_{\mathcal{M}_{11}} \left[-\frac{1}{6} C_3 \wedge G_4 \wedge G_4 - C_3 \wedge \mathcal{X}_8 \right] . \qquad (13)$$

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- To construct the 4d topological coupling on $\mathcal{M}_4$, we consider a gauge invariant 5-form I_5, such that

$$I_5 = dI_4 \ , \ S_{4\text{d}} = 2\pi \int_{\mathcal{M}_4} I_4 \, . \qquad (14)$$

- I_5 is given by

$$I_5 = \int_{Y_7} I_{12} = \int_{Y_7} \left(-\frac{1}{6} G_4 \wedge G_4 \wedge G_4 - G_4 \wedge \mathcal{X}_8 \right) \qquad (15)$$

SymTFT from M-theory

- From

$$I_5 = \int_{Y_7} I_{12} = \int_{Y_7} \left(-\frac{1}{6} G_4 \wedge G_4 \wedge G_4 - G_4 \wedge \mathcal{X}_8 \right) , \tag{16}$$

we can expand

$$G_4 = \sum_{p=0}^{4} \sum_{i=0}^{b^p(Y_7)} g^i_{4-p} \wedge \omega^i_p \,. \tag{17}$$

$g^i_{4-p} = dc^i_{3-p}$ are field strengths of $(3-p)$-form $U(1)$ gauge fields c^i_{3-p}, ω^i_p are closed differential p-forms of Y_7.

- Integrate over Y_7 to get I_5, and then I_4.

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• We expand $G_4 \in H^4(\mathcal{M}_{11},\mathbb{Z})$ as

$$G_4 = \sum_{p=0}^{4} \sum_{i=0}^{b^p(Y_7)} F^i_{4-p} \smile v^i_p + \sum_{p=0}^{4} \sum_{\alpha} B^\alpha_{4-p} \smile t^\alpha_p \,. \tag{18}$$

(1) $F^i_{4-p} \sim g^i_{4-p}$, $v^i_p \sim \omega^i_p$ are free generators of $H^p(Y_7,\mathbb{Z})$.
(2) B^α_{4-p} are $(4-p)$-form discrete gauge fields, and t^α_p are generators of $\mathrm{Tor}(H^p(Y_7,\mathbb{Z}))$.

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• To write down the topological action using differential forms, use the formulation of differential cohomology

- Uplift G_4 to differential cohomology class

$$\breve{G}_4 = \sum_{p=0}^{4} \sum_{i=0}^{b^p(Y_7)} \breve{F}^i_{4-p} \star \breve{v}^i_p + \sum_{p=0}^{4} \sum_{\alpha} \breve{B}^{\alpha}_{4-p} \star \breve{t}^{\alpha}_p \,, \tag{19}$$

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- Plug into the uplift of topological action

$$\frac{S_{\text{top}}}{2\pi} = \int_{\mathcal{M}_{11}} \breve{I}_{12} \quad (\text{mod } 1) \tag{20}$$

$$\int_{\mathcal{M}_{11}} \breve{I}_{12} = \int_{\mathcal{M}_{11}} -\frac{1}{6} \breve{G}_4 \star \breve{G}_4 \star \breve{G}_4 - \breve{G}_4 \star \breve{X}_8 \,, \tag{21}$$

Here the integration is the "secondary invariant", which is defined in $\mathbb{R}/\mathbb{Z}$.

SymTFT from M-theory

- Using the following assumptions

(1) In the AdS4/CFT3 setups where G_4 background flux is over AdS_4, we can take $F_0^i = 0$

(2) Y_7 is connected, oriented and spin, $H^0(Y_7, \mathbb{Z}) = \mathbb{Z}$, $H^1(Y_7, \mathbb{Z}) = H^3(Y_7, \mathbb{Z}) = 0$

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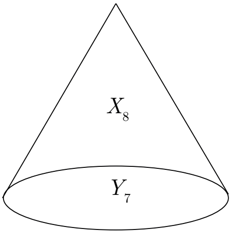
- The SymTFT action is simplified to

$$\frac{S_{\text{top}}}{2\pi} = -\left[\frac{1}{2}\int_{Y_7} \breve{v}_2^i \star \breve{v}_2^j \star \breve{t}_4^\alpha\right]\int_{\mathcal{M}_4} \breve{F}_2^i \star \breve{F}_2^j \star \breve{b}^\alpha - \left[\int_{Y_7} \breve{v}_2^i \star \breve{t}_2^\beta \star \breve{t}_4^\alpha\right]\int_{\mathcal{M}_4} \breve{b}^\alpha \star \breve{B}_2^\beta \star \breve{F}_2^i \\ -\left[\frac{1}{2}\int_{Y_7} \breve{t}_4^\alpha \star \breve{t}_4^\beta\right]\int_{\mathcal{M}_4} \breve{F}_4 \star \breve{b}^\alpha \star \breve{b}^\beta - \left[\frac{1}{2}\int_{Y_7} \breve{t}_2^\alpha \star \breve{t}_2^\beta \star \breve{t}_4^\gamma\right]\int_{\mathcal{M}_4} \breve{b}^\gamma \star \breve{B}_2^\alpha \star \breve{B}_2^\beta\,. \tag{22}$$

Here $\breve{b}^\alpha \equiv \breve{B}_0^\alpha \in \mathbb{Z}$ represents the torsional background G_4 flux.

SymTFT from M-theory

- How to compute the numbers in "$\int_{Y_7}$"?
- In this talk, we take Y_7 to be the link Sasaki-Einstein sevenfold of a complex fourfold singularity X_8.



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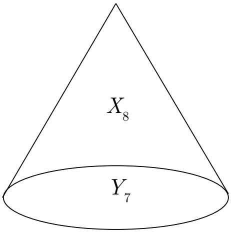

- Consider a resolution $\widetilde{X}_8$ of X_8
- We can identify

(1) Each $\breve{v}_p^i$ with a non-compact $(8-p)$-cycle D_{8-p}^i in $\widetilde{X}_8$

(2) Each $\breve{t}_p^\alpha$ with a compact torsional $(8-p)$-cycle Z_{8-p}^α in $\widetilde{X}_8$, with torsion degree l_α

SymTFT from M-theory

- Hence we have

(1) $\breve{v}_2^i \to$ a non-compact divisor D_i

(2) $\breve{t}_2^\alpha \to$ a compact divisor Z_6^α

(3) $\breve{t}_4^\alpha \to$ a compact 4-cycle Z_4^α

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\begin{aligned}
\frac{1}{2}\int_{Y_7} \breve{v}_2^i \star \breve{v}_2^j \star \breve{t}_4^\alpha &= \left[\frac{1}{2l_\alpha} D_i \cdot D_j \cdot Z_4^\alpha\right] \pmod 1 \\
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\tag{23}
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($l_\alpha, l_\beta, l_\gamma$ are torsion degrees of the corresponding torsional cycles)

SymTFT from M-theory

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- Z_6^α is a linear combination of compact divisors $C_a \subset \widetilde{X}_8$.
- Consider M-theory on $\widetilde{X}_8$, each C_a gives rise to a $U(1)$ gauge field A_a from the decomposition

$$C_3 = \sum_a A_a \wedge \omega_2^a \,, \tag{24}$$

ω_2^a is the Poincaré dual (1,1)-form of C_a.

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• Electrically charged objects are M2-branes wrapping 2-cycles $\mathcal{N}_i$, whose $U(1)_a$-charge is $q_{i,a} = C_a \cdot \mathcal{N}_i$.

• The discrete 1-form symmetry in the setup can be computed by the Smith normal form of $q_{i,a}$.

SymTFT from M-theory

- We decompose the charge matrix

$$q = UDV\,, \tag{25}$$

U and V are square matrices, and D is the Smith normal form

$$D = \begin{pmatrix} l_1 & 0 & \dots & 0 \\ 0 & l_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & l_r \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \tag{26}$$

- Each non-zero element $l_\alpha > 1$ corresponds to a torsional compact divisor $Z_6^\alpha = \sum_a V_{a\alpha} C_a$, with torsion degree l_α.

SymTFT from M-theory

- For compact 4-cycles Z_4^α, compute using the Smith normal decomposition of the intersection matrix between 4-cycles.
- The torsion degree l_α corresponds to a "(-1)-form" symmetry.

Physical applications

(1) ABJ(M) theories: $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$, dual to N M2-branes probing $X_8 = \mathbb{C}^4/\mathbb{Z}_k$ (with torsional G_4 flux).

(2) $Y^{p,k}$ theories: $\mathrm{AdS}_4 \times Y^{p,k}(\mathbb{CP}^2)$, dual to N M2-branes probing a toric fourfold singularity X_8 (with torsional G_4 flux).

ABJ(M) theories

- Gravity side: M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$, with N units of G_4 background flux and b units of torsional flux

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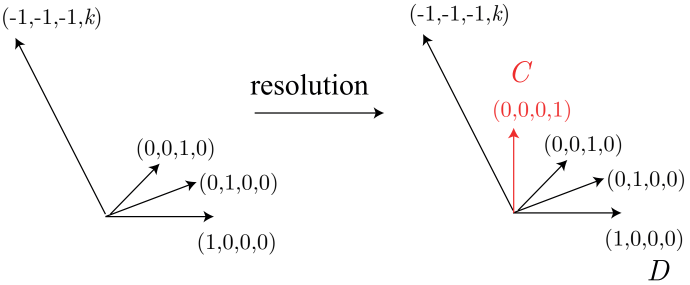
• Compute from string/M-theory:

(1) 't Hooft anomaly BB for 1-form symmetry from SymTFT

(2) topological BF term: mixed 't Hooft anomaly of 0-form and 1-form symmetries

ABJ(M) theories-BB term

- For $Y_7 = S^7/\mathbb{Z}_k$, $X_8 = \mathbb{C}^4/\mathbb{Z}_k$, we choose a toric resolution $\widetilde{X}_8$:



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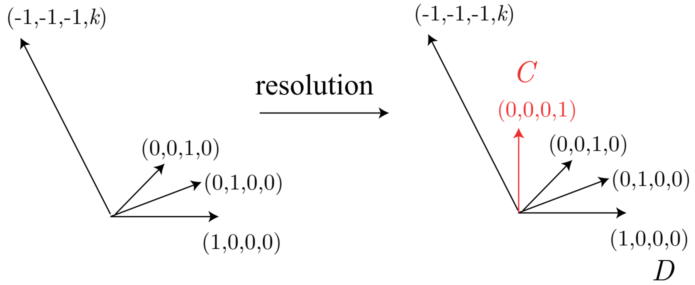

(1) $Z_6 = C$: compact 6-cycle, torsion degree k

(2) $Z_4 = C \cdot D$: compact 4-cycle, torsion degree k

- SymTFT action:

$$\begin{aligned} \frac{S_{\text{top}}}{2\pi} &= -\left[\frac{Z^6 \cdot Z^6 \cdot Z^4}{2k^3}\right]_{\text{mod } 1} \int_{\text{AdS}_4} \breve{B}_2 \star \breve{B}_2 \star \breve{b} \\ &= -\frac{b}{2k} \int_{\text{AdS}_4} \breve{B}_2 \star \breve{B}_2 \pmod{1}. \end{aligned} \tag{27}$$

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- 't Hooft anomaly for $\mathbb{Z}_k^{(1)}$ 1-form symmetry in the $U(N+b)_k \times U(N)_{-k}$ theory:

$$\frac{S_{\text{top}}}{2\pi} = -\frac{b}{2k}\int_{\text{AdS}_4} B_2 \smile B_2 \tag{28}$$

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- For the absence of anomaly after the gauging:

$$\frac{S_{\text{top}}}{2\pi} \in \mathbb{Z} \longrightarrow \frac{b}{2k} \times (2m^2) = \frac{bm^2}{k} = \frac{bk}{(m')^2} \in \mathbb{Z}. \tag{29}$$

ABJ(M) theories-BB term

• $bk/(m')^2 \in \mathbb{Z}$: constrains the possible global form of $(U(N+b)_k \times U(N)_{-k})/\mathbb{Z}_{m'}$.

• For example, consider $k = 4$, $U(N+b)_4 \times U(N)_{-4}$ with $\mathbb{Z}_4^{(1)}$ 1-form symmetry

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(2) If we want to gauge the full $\mathbb{Z}_4^{(1)}$, then $b = 0 \pmod 4$.

- First derivation from geometry!
- Consistent with field theory argument (Tachikawa, Zafrir 19').

ABJ(M) theories-BF term

- BF-coupling term for ABJ(M) theories from IIA derivation (Bergman, Tachikawa, Zafrir 20'):

$$S_{\text{IIA}} = \frac{1}{2\pi} \int_{\text{AdS}_4} B_{NS} \wedge d(kA_{D4} + NA_{D0}) \,. \tag{30}$$

- IIA on $\text{AdS}_4 \times \mathbb{CP}^3$, with N units of F_6 flux over $\mathbb{CP}^3$ and k units of F_2 flux over $\mathbb{CP}^1 \subset \mathbb{CP}^3$.

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• In our M-theory analysis:

$$S_{\text{BF}} = 2\pi \int_{\text{AdS}_4} B_2 \wedge d(kB_1 + NA_1)\,. \tag{31}$$

(1) $B_2 \wedge kB_1$ term: from the non-commutativity of G_4 and G_7 flux.
(2) $B_2 \wedge NA_1$ term: from gauging $U(1)$ isometry of Y_7 (equivariant cohomology)

ABJ(M) theories-BF term

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• Let us consider the cases of $b = 0 \to (U(N)_k \times U(N)_{-k})/\mathbb{Z}_{m'}$, BB-term vanishes.

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• B_2 free to fluctuate $\rightarrow kB_1 + NA_1 = 0$

• 0-form symmetry: $U(1) \times \mathbb{Z}_{\gcd(N,k)}$, no 1-form symmetry

$Y^{p,k}$ theories

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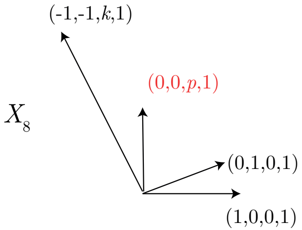
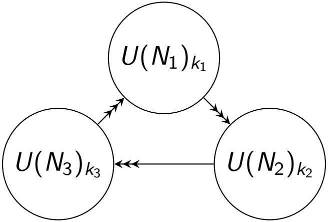


$Y^{p,k}$ theories

- 3d $\mathcal{N} = 2$ quiver gauge theory: (Benini, Closset, Cremonesi 11')



- $n_0, n_1 \in \mathbb{Z}$ parametrizes the discrete flux, in the three windows
  1. $-k \leq n_0 \leq 0\,, \quad 0 \leq 3n_1 - n_0 \leq 3p - k$
  2. $0 \leq n_0 \leq k\,, \quad 0 \leq 3n_1 - n_0 \leq 3p - k$
  3. $k \leq n_0 \leq 2k\,, \quad 0 \leq 3n_1 - n_0 \leq 3p - k$

The field theories are

1. $U(N + n_1 - p - n_0)_{-n_0+\frac{3}{2}n_1} \times U(N)_{\frac{1}{2}n_0-3n_1+\frac{3}{2}p-k} \times U(N - n_1)_{\frac{1}{2}n_0+\frac{3}{2}n_1-\frac{3}{2}p+k}$
2. $U(N + n_1 - p)_{-n_0+\frac{3}{2}n_1} \times U(N)_{2n_0-3n_1+\frac{3}{2}p-k} \times U(N - n_1)_{-n_0+\frac{3}{2}n_1-\frac{3}{2}p+k}$
3. $U(N + n_1 - p)_{\frac{1}{2}n_0+\frac{3}{2}n_1-\frac{3}{2}q} \times U(N)_{\frac{1}{2}n_0-3n_1+\frac{3}{2}p+\frac{1}{2}k} \times U(N - n_1 + n_0 - k)_{-n_0+\frac{3}{2}n_1-\frac{3}{2}p+k}$

- However, it is known that the field theory suffers from a parity anomaly (certain magnetic monopoles have non-integral $U(1)$ charge)
- Remedy: adding off-diagonal Chern-Simons term Λ_{ij} $(i, j = 1, 2, 3)$

$$\delta S = \sum_{ij} \frac{\Lambda_{ij}}{4\pi} \int tr(A_i) \wedge tr(dA_j) \tag{34}$$

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• No explanation from string theory

• Using our SymTFT computation, constrain the possible Λ_{ij}, by comparing with the expected 1-form symmetry from geometry!

$Y^{p,k}$ theories-BF term

- BF term in the $Y^{p,k}(\mathbb{CP}^2)$ theories from M-theory

$$S = \frac{1}{2\pi}\int B_2 \wedge d\left(NA_1 + \mathrm{gcd}(p,k)B_1 + \Omega^{p,k}_{n_0,n_1} c_1\right) , \qquad (35)$$

(1) A_1 term: from gauging isometry of $Y^{p,k}(\mathbb{CP}^2)$

(2) B_1 term: from non-commutativity of G_4 and G_7 flux

(3) c_1 term: from SymTFT

- Different global form of gauge groups, 1-form symmetries . . .

Conclusions

- We developed the SymTFT methods for AdS4/CFT3, from M-theory top-down approach
- For ABJM theories, reproduces correct field theory results from geometry (1-form symmetry, 1-form 't Hooft anomaly BB, mixed 't Hooft anomaly BF)
- For $Y^{p,k}$ theories, put additional constraints on CFT3. Derived the topological action (BF, BB terms . . .) and constrain the possible global form of CFT3.
- Thanks!