中国科学院大学
University of Chinese Academy of Sciences

Topological modes from non-inertial frames and holography

Ya-Wen Sun, University of Chinese Academy of Sciences,

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Based on arXiv 2004.13380, 2005.02850, 2201.02407 and work in progress

Motivation

- Previous work on topological hydrodynamic modes: topologically trivial hydrodynamic system becomes topologically nontrivial observed in a special non-inertial frame;
- A new observational effect for non-inertial frames in addition to the famous Unruh effect: topologically nontrivial modes observed in non-inertial frames which are trivial in inertial frames.
- An example from relativistic hydrodynamics, also non-relativistic hydrodynamics (Perrot, Delplace, Venaille, 2019, topological modes from inertial forces)

- Further generalizations: holographic realization; other possible systems.
  - Holographic calculation of hydrodynamic modes in non-inertial frames, which is shown to be the same as the hydrodynamic calculation;
  - Topological modes in non-inertial frames for other physical systems: fermions; Weyl semimetal observed in a non-inertial frame, produced due to inertial forces;

Outline

- Topological hydrodynamic modes from non-inertial frames
- Holographic calculation of hydrodynamic modes in non-inertial frames
- Weyl semimetal from non-inertial reference frame
- Summary and open questions

I. Topological hydrodynamic modes from non-inertial frames

- Motivation for the study of topological hydrodynamic modes:
- Classical topological states: sounds/optics
- Classical topological states in gravitational waves?
- Holography: gravitons, hydrodynamic modes
- Possible experimental observational effects?

- Hydrodynamics: small perturbations close to thermal equilibrium, long wave length and long time limit;

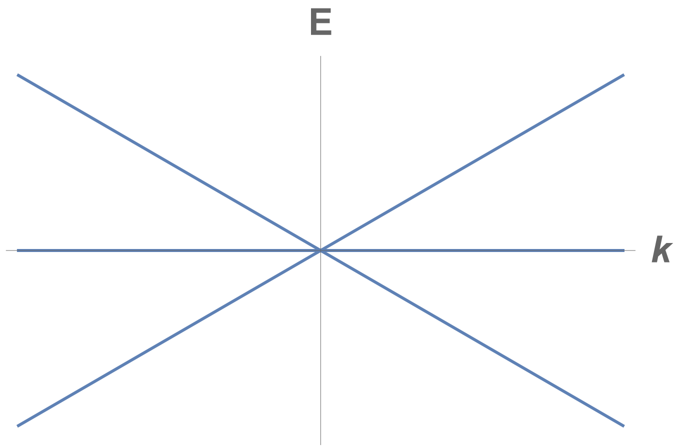
I. Topological hydrodynamic modes from non-inertial frames

Hydrodynamic modes, dynamics determined by the conservation equation

$$\partial_\mu \delta T^{\mu\nu} = 0$$

$$\omega = \pm v_s \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$\omega = 0$$

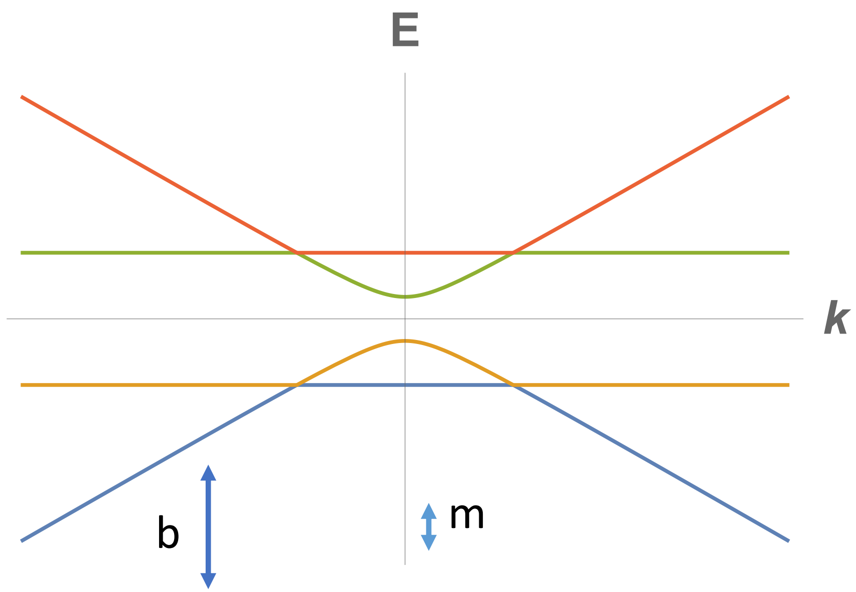


leading order in k

I. Topological hydrodynamic modes from non-inertial frames

- With non-conservation terms

$$\partial_\mu \delta T^{\mu t} = m\delta T^{tx}\,, \quad \partial_\mu \delta T^{\mu x} = -m v_s^2 \delta T^{tt}$$
$$\partial_\mu \delta T^{\mu y} = b v_s \delta T^{tz}\,, \quad \partial_\mu \delta T^{\mu z} = -b v_s \delta T^{ty}$$

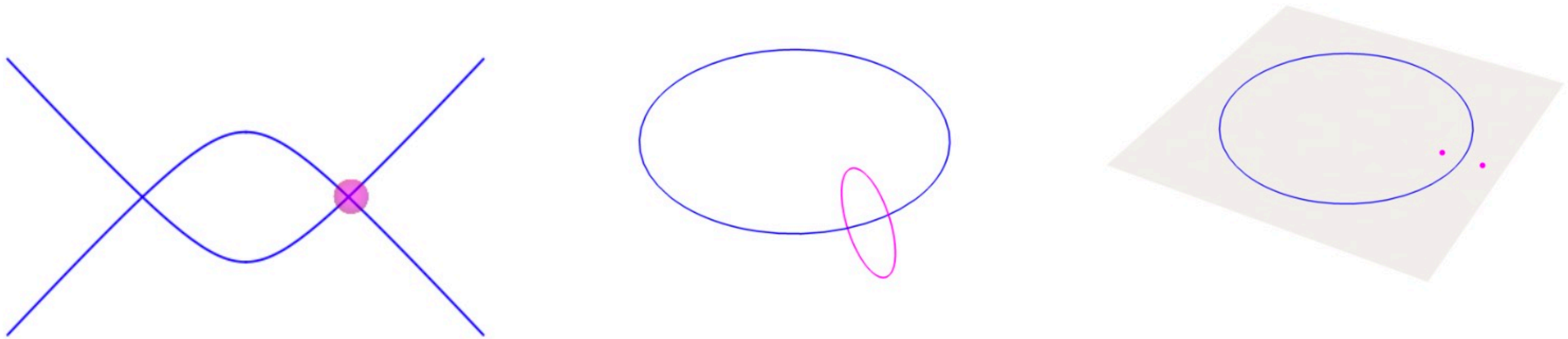


$$\omega = \pm\frac{1}{\sqrt{2}}\sqrt{b^2+k^2+m^2 \pm \sqrt{(k_x^2+m^2-b^2)^2+(k_y^2+k_z^2)^2+2(k_y^2+k_z^2)(k_x^2+m^2+b^2)}}$$

Similar but different spectrum was also found in non-relativistic hydro in e.g. Perrot et.al., Nature Physics, 2019.

I. Topological hydrodynamic modes from non-inertial frames: topological invariants

- Topological invariants



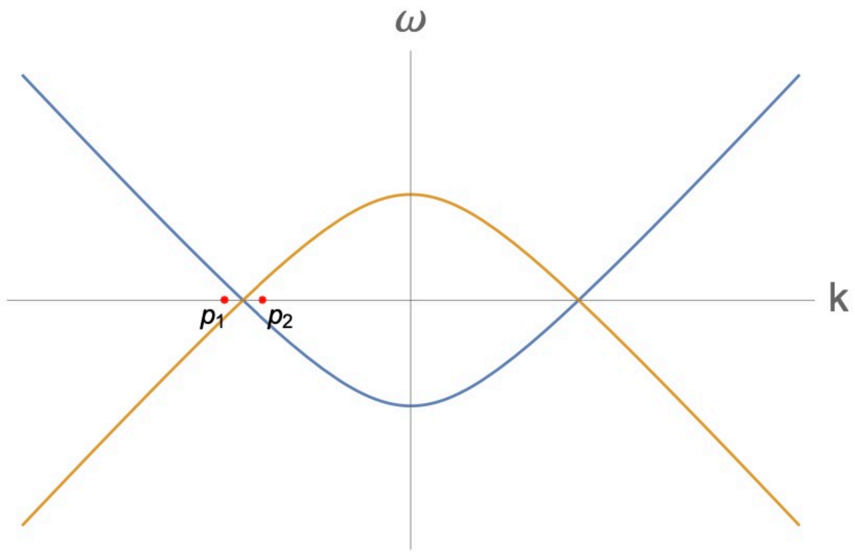
- Symmetry protected topological states: calculate the topological invariants at high symmetric points; reflectional symmetry in two spatial directions

I. Topological hydrodynamic modes from non-inertial frames: topological invariants

- The reflection symmetry: $M : y \to -y, z \to -z.$
- High symmetric point: $k_y = k_z = 0$
- The topological invariant $\xi = N_1 - N_2$

Final result for the topological invariant:

$\xi = 1$ or $\xi = -1.$



N_i: the number of occupied bands at point p_i with eigenvalue of the reflection symmetry M to be 1

$|n_i\rangle$: the occupied state at point p_i

$$|n_1\rangle = \frac{1}{\sqrt{2}}(0, 0, -i, 1)$$

$$|n_2\rangle = \frac{1}{\sqrt{\frac{1}{v_s^2} - 1}}\left(\frac{\sqrt{m^2 + k_x^2}}{v_s(m + ik_x)}, i, 0, 0\right)$$

I. Topological hydrodynamic modes from non-inertial frames

- The most interesting and natural possibility for the symmetric tensor field: the gravitational field

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \mathcal{O}(h_{\mu\nu}) \sim \mathcal{O}(k)$$

- Energy momentum is conserved covariantly $\nabla_\mu T^{\mu\nu} = 0$
- Expanding the covariant conservation equation to first order of $h_{\mu\nu}$

$$\partial_\mu \delta T^{\mu\nu} = -\frac{1}{2}\partial_\alpha h \delta T^{\alpha\nu} - \frac{1}{2}\eta^{\nu\beta}(2\partial_\mu h_{\alpha\beta} - \partial_\beta h_{\mu\alpha})\delta T^{\mu\alpha}$$

I. Topological hydrodynamic modes from non-inertial frames

- With the following nonzero components of $h_{\mu\nu}$

$$h_{tt} = h_{xx} = mx\,, \quad h_{tx} = h_{xt} = \frac{1}{2}mt(v_s^2 + 1)\,,$$
$$h_{ty} = h_{yt} = -\frac{1}{2}bv_s z\,, \quad h_{tz} = h_{zt} = \frac{1}{2}bv_s y\,.$$

infinite many possibilities for $h_{\mu\nu}$, here we pick a simple choice

- The covariant conservation equation gives the non-conservation terms needed

$$\partial_\mu \delta T^{\mu t} = m\delta T^{tx}\,, \quad \partial_\mu \delta T^{\mu x} = -mv_s^2 \delta T^{tt}$$
$$\partial_\mu \delta T^{\mu y} = bv_s \delta T^{tz}\,, \quad \partial_\mu \delta T^{\mu z} = -bv_s \delta T^{ty}$$

- How do we get this gravitational field $h_{\mu\nu}$?
- Surprisingly all Riemann tensors vanish for this metric!
- $h_{\mu\nu}$ could emerge from a coordinate transformation from the flat spacetime

$$\tilde{x}_\mu = x_\mu + \xi_\mu$$

$$\xi_\mu = \left(\frac{mxt}{2}, \quad \frac{mx^2}{4} + \frac{mt^2}{4} v_s^2, \quad -\frac{b}{4} v_s z t, \quad \frac{b}{4} v_s y t \right)$$

- In a specific non-inertial frame, we could observe hydrodynamic modes that are topologically protected even when they are topologically trivial in the original inertial frame.
- Another effect for accelerating frames in addition to the Unruh effect.

I. Topological hydrodynamic modes from non-inertial frames: the non-inertial frame

- A rest observer in the new reference frame

$$d\tilde{x}^i = 0 \text{ for } i = 1,2,3$$

- Solving this equation, we have the movement of the rest observer in the original flat spacetime (at leading order in k)

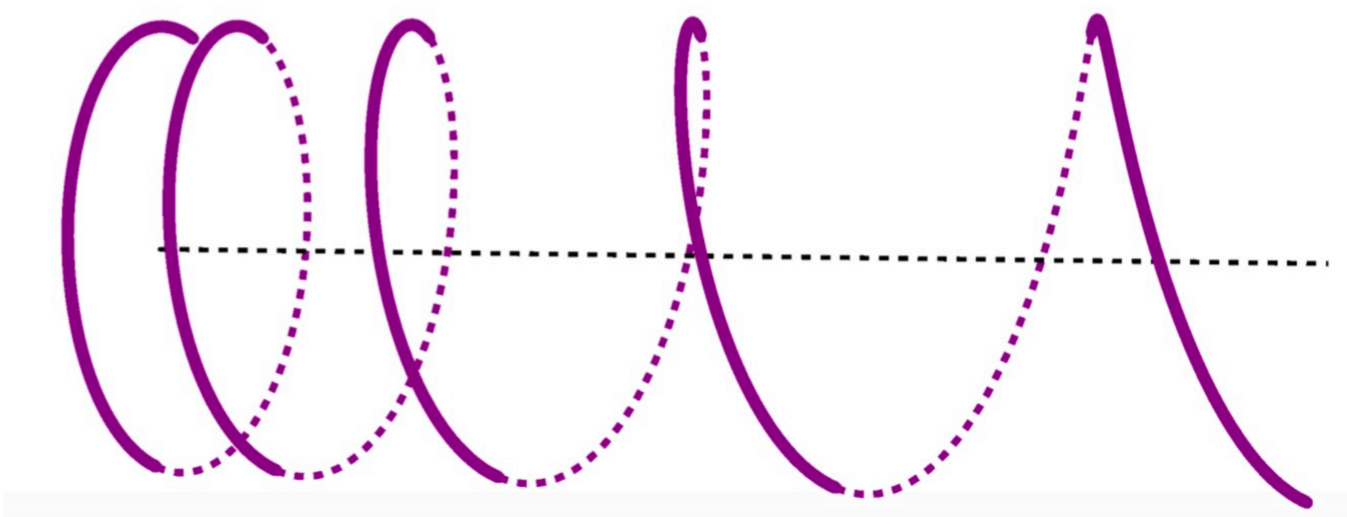
$$d\tilde{t} = dt, \; dx = -\frac{mv_s^2 t dt}{2}, \; dy = \frac{bv_s z dt}{4} \quad dz = -\frac{bv_s y dt}{4}$$

- Integrating these equations with appropriate boundary conditions, we have

$$y = R_0 \cos \frac{bv_s}{4} t \text{ and } z = -R_0 \sin \frac{bv_s}{4} t$$

I. Topological hydrodynamic modes from non-inertial frames: the non-inertial frame

- The rest observer in the new reference frame:
- Rotating with a constant angular velocity $\omega_x = \frac{bv_s}{4}$ in the y-z plane
- Accelerating with a constant acceleration $a = -\frac{mv_s^2}{2}$ in the x direction



x direction

I. Topological hydrodynamic modes from non-inertial frames: the non-inertial frame

- At the same time, the axis of the observer needs to rotate

$$
\begin{aligned}
\frac{\partial}{\partial x'} &= (1 - \frac{1}{2}mx)\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y'} &= \frac{\partial}{\partial y} - bt\frac{\partial}{\partial z} \\
\frac{\partial}{\partial z'} &= bt\frac{\partial}{\partial y} + \frac{\partial}{\partial z}
\end{aligned}
$$

- The best choice is to stay at the origin and the axis rotates at the same time

II. Holographic calculation for hydrodynamic modes in non-inertial frames

- Holographic realization, strongly coupled hydrodynamic systems.
- Hydrodynamic modes-> gravitons
- Non-conservation of energy momentum: massive gravity?
- Another prescription for holographic realization of this system: holographic non-inertial reference frames, coordinate transformation from the original AdS/CFT correspondence
- First step to show that it is indeed the holographic system needed: reproduce the Ward identities due to the energy momentum non-conservation terms

II. Holographic calculation for hydrodynamic modes in non-inertial frames

- Ward identities for the conserved energy momentum tensor

$$k_\mu\left(G^{\mu\nu\lambda\rho} - \eta^{\nu\lambda}\langle T^{\mu\rho}\rangle - \eta^{\nu\rho}\langle T^{\mu\lambda}\rangle - \eta^{\lambda\rho}\langle T^{\mu\nu}\rangle + \eta^{\mu\nu}\langle T^{\lambda\rho}\rangle\right) = 0$$

- With energy momentum non-conservation terms, the Ward identities become

$$k_\mu G^{\mu\nu,\lambda\rho}(k) + i\left[\Gamma^{(1)\mu}{}_{\mu\alpha} G^{\alpha\nu,\lambda\rho}(k) + \Gamma^{(1)\nu}{}_{\mu\alpha} G^{\mu\alpha,\lambda\rho}(k)\right] + \text{contact terms} = 0$$

II. Holographic calculation for hydrodynamic modes in non-inertial frames

- A prescription to calculate holographic Ward identities without calculating all the components of the Green functions.
- For perturbations of the metric $\delta g_{\mu\nu}(\vec{k})$, deriving equations of motion for this system and substituting the solutions into the action, we could get the on-shell action.
- The action has to be composed of gauge invariant combinations; holographic Ward identities--- diffeomorphism;

II. Holographic calculation for hydrodynamic modes in non-inertial frames

- For asymptotic AdS systems, all possible gauge invariant combinations:

$$Z_1 = \frac{\delta g_{xx}}{2k_x^2} + \frac{\delta g_{tx}}{\omega k_x} + \frac{\delta g_{tt}}{2\omega^2}, \quad Z_2 = \frac{\delta g_{yy}}{2k_y^2} + \frac{\delta g_{ty}}{\omega k_y} + \frac{\delta g_{tt}}{2\omega^2},$$

$$Z_3 = \frac{\delta g_{zz}}{2k_z^2} + \frac{\delta g_{tz}}{\omega k_z} + \frac{\delta g_{tt}}{2\omega^2}, \quad Z_4 = \frac{\delta g_{xx}}{2k_x^2} - \frac{\delta g_{xy}}{k_x k_y} + \frac{\delta g_{yy}}{2k_y^2},$$

$$Z_5 = \frac{\delta g_{xx}}{2k_x^2} - \frac{\delta g_{xz}}{k_x k_z} + \frac{\delta g_{zz}}{2k_z^2}, \quad Z_6 = \frac{\delta g_{yy}}{2k_y^2} - \frac{\delta g_{yz}}{k_y k_x} + \frac{\delta g_{zz}}{2k_z^2}$$

II. Holographic calculation for hydrodynamic modes in non-inertial frames

- The on-shell action should be

$$S \supset \int \frac{d^4k}{(2\pi)^4} G_{ij}(r) Z_i'(-\vec{k}) Z_j(\vec{k}) \Big|_{r_h}^{r_b}$$

- All 55 components of Green functions should be expressed using the 21 independent Gij functions.
- Eliminating all Gij's, we obtain 34 identities for holographic Green functions.
- 40 Ward identities need to be reproduced, 6 of which could be derived from the other 34 identities
- They match to each other.

II. Holographic calculation for hydrodynamic modes in non-inertial frames

- The metric for the coordinate transformed AdS spacetime:

$$g_{\mu\nu}^{\text{bulk}} = g_{\mu\nu}^{AdS} + h_{\mu\nu}^{\text{bulk}}$$

$$h_{\mu\nu}^{\text{bulk}} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

- With the new metric, the form of the on-shell action would be different from the AdS one, nevertheless, it can still be written as sums of gauge invariant terms.

II. Holographic calculation for hydrodynamic modes in non-inertial frames

- New gauge invariant combinations

$$Z_1 = \frac{\delta g_{xx}}{2k_x^2} + \frac{\delta g_{tx}}{\omega k_x} + \frac{\delta g_{tt}}{2\omega^2} - \frac{im\delta g_{tt}}{4k_x\omega^2} + \frac{im\delta g_{tx}}{2k_x^2\omega} - \frac{imv_s^2\delta g_{xx}}{4k_x\omega^2}$$

$$Z_2 = \frac{\delta g_{yy}}{2k_y^2} + \frac{\delta g_{ty}}{\omega k_y} + \frac{\delta g_{tt}}{2\omega^2} - \frac{imv_s^2\delta g_{xx}}{4k_x\omega^2} - \frac{ibv_s\delta g_{zz}}{4k_yk_z\omega},$$

$$Z_3 = \frac{\delta g_{zz}}{2k_z^2} + \frac{\delta g_{tz}}{\omega k_z} + \frac{\delta g_{tt}}{2\omega^2} - \frac{imv_s^2\delta g_{xx}}{4k_x\omega^2} + \frac{ibv_s\delta g_{yy}}{4k_yk_z\omega},$$

$$Z_4 = \frac{\delta g_{xx}}{2k_x^2} - \frac{\delta g_{xy}}{k_xk_y} + \frac{\delta g_{yy}}{2k_y^2} - \frac{im\delta g_{xx}}{4k_x^3},$$

$$Z_5 = \frac{\delta g_{xx}}{2k_x^2} - \frac{\delta g_{xz}}{k_xk_z} + \frac{\delta g_{zz}}{2k_z^2},$$

$$Z_6 = \frac{\delta g_{yy}}{2k_y^2} - \frac{\delta g_{yz}}{k_yk_z} + \frac{\delta g_{zz}}{2k_z^2} - \frac{im\delta g_{xx}}{4k_x^3}.$$

Using the same method as the asymptotic AdS case, we could match the Ward identities from both sides

II. Holographic calculation for hydrodynamic modes in non-inertial frames

- Remarks:
  - This method for calculating holographic Ward identities could also be generalized to massive gravities.
  - Other holographic realizations, massive gravity? External fields?

- Next step: hydrodynamics modes reproduced in holography;

II. Holographic calculation for hydrodynamic modes in non-inertial frames

- Hydrodynamic modes in holography: perturbations of the metric $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$ (Policastro, Son, Starinets, 2002) ;
- Solve the equations of motion order by order in w and q; obtain the Green functions; poles in the Green functions
- The poles could be directly seen from the coefficients in the solutions of perturbations; assume k to be in the z direction;
- For the vector modes:

$$G_R^{txtx} \propto \frac{1}{H^{(0)}_{tx}}(\frac{1}{u}H'_{tx})\,|_{u=\epsilon}= \frac{q^2}{iw-\frac{q^2}{2}}$$

$$\begin{cases} H^{(0)}_{tx} = C_1 H^{inc}_{tx}\,|_{u=0} + C_2 H^{I}_{tx} \\ H^{(0)}_{xz} = C_1 H^{inc}_{xz}\,|_{u=0} + C_2 H^{I}_{xz} \end{cases} \Longrightarrow \begin{cases} C_1 = \frac{q^2 H^{(0)}_{tx} + wq H^{(0)}_{xz}}{iw-\frac{q^2}{2}} \\ C_2 = \frac{iw H^{(0)}_{tx} + \frac{wq}{2} H^{(0)}_{xz}}{iw-\frac{q^2}{2}} \end{cases}$$

$$G_R^{txxz} \propto \frac{1}{H^{(0)}_{xz}}(\frac{1}{u}H'_{tx})\,|_{u=\epsilon}= \frac{wq}{iw-\frac{q^2}{2}}$$

$$G_R^{xzxz} \propto \frac{1}{H^{(0)}_{xz}}(\frac{f}{u}H'_{xz})\,|_{u=\epsilon}= \frac{w^2}{iw-\frac{q^2}{2}}$$

II. Holographic calculation for hydrodynamic modes in non-inertial frames

- For the scalar modes

$$H(u) = C_0 H^{inc}(u) + C_1 H^{I}(u) + C_2 H^{II}(u) + C_3 H^{III}(u)$$

$$H_{tt}, H_{tz}, H_{zz} \quad H_{xx} + H_{yy}$$

- Coefficients C's are determined by the boundary values of H fields
- Solve for the coefficients and the determinant from the linear equation gives the poles

$$24(3w^2 - q^2) + O(w^3, q^3)$$

II. Holographic calculation for hydrodynamic modes in non-inertial frames

- A simple way to calculate the hydrodynamic modes in non-inertial frames without solving the equations in the new background geometry:
    - Start from the inertial frame results and perform a rotation coordinate transformation to put k in arbitrary direction;
    - Perform a coordinate transformation (change of reference frame) in the fields;
    - Find the poles of the transformed system from the determinant of the equation for the coefficients;
- Incoming boundary condition at the horizon: does not change

II. Holographic calculation for hydrodynamic modes in non-inertial frames

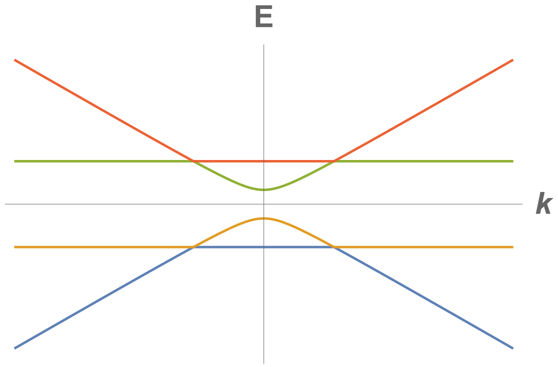
$$\xi^\mu = (-\frac{1}{2}mxt, \frac{1}{4}mx^2 + \frac{1}{4}mv_s^2t^2, -\frac{1}{2}bv_szt, \frac{1}{2}bv_syt, 0)$$

$$h'_{\mu\nu}(x) = h_{\mu\nu}(x) - h_{\alpha\nu}(x)\partial_\mu\xi^\alpha - h_{\mu\beta}(x)\partial_\nu\xi^\beta - \xi^\lambda\partial_\lambda h_{\mu\nu}(x)$$

- The determinant in the coefficient matrix gives (in the final result, k chosen to be in the k_x direction; note that it cannot be chosen to be in the k_x direction at the beginning as derivatives in k are needed in the process)

$$-\frac{1}{12}(b^2 - 12\omega^2)(k_1^2 + m^2 - 3\omega^2)$$

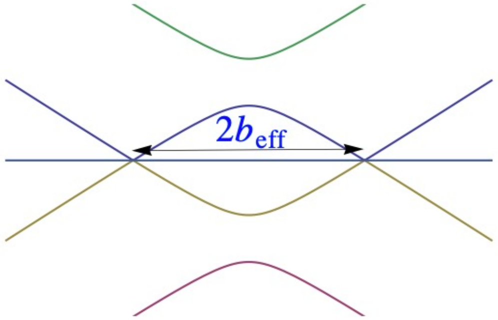
Thus the poles are at $(b^2 - 12\omega^2)(k_1^2 + m^2 - 3\omega^2) = 0$, which agrees with the hydrodynamic results.



III. Topological modes in non-inertial frames for other physical systems: fermions

- Motivation: generalization to other systems, i.e. could other trivial states be seen as topologically nontrivial by a non-inertial observer?
- Fermions, gapless topological systems: semimetals
- The Lagrangian for the Weyl semimetal

$$\mathcal{L} = \bar{\Psi}\left(i\partial\!\!\!/ - eA\!\!\!/ - \gamma_5\vec{\gamma}\cdot\vec{b} + M\right)\Psi$$



- Could a Dirac fermion (without the *b* term above) in an inertial frame be seen as a Weyl semimetal in a non-inertial frame?

III. Topological modes in non-inertial frames for other physical systems: fermions

- Weyl semimetals in non-inertial frames from inertial frame Dirac fermions
- Start from the equation of motion for the fermion in the non-inertial frame

$$i\gamma^a e_a^\nu D_\nu \psi - m\psi = 0$$

$$D_\nu = \partial_\nu - \frac{i}{4}\omega_{\nu bc}\sigma^{bc}$$

$$\sigma^{bc} = \frac{i}{2}\left[\gamma^b, \gamma^c\right]$$

$$\omega_{\nu bc} = e_{b\lambda}\nabla_\nu e_c^\lambda = e_{b\lambda}(\partial_\nu e_c^\lambda + \Gamma^\lambda_{\nu\rho})e_c^\rho$$

- With a background metric different from the flat one
- Try to reproduce the Weyl Lagrangian from a specific metric

III. Topological modes in non-inertial frames for other physical systems: fermions

- Assume that $e_a^\mu = \delta_a^\mu + \epsilon f_a^\mu$
- We have an extra term in the equation of motion

$$i\gamma^a \partial_a \psi - m\psi + \Sigma\psi = 0$$

- If we choose

$$f_a^\mu = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & bt & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Then we have $\Sigma = \begin{pmatrix} 0 & 0 & b & 0 \\ 0 & 0 & 0 & -b \\ b & 0 & 0 & 0 \\ 0 & -b & 0 & 0 \end{pmatrix}$, which gives the Weyl Lagrangian

III. Topological modes in non-inertial frames for other physical systems: fermions

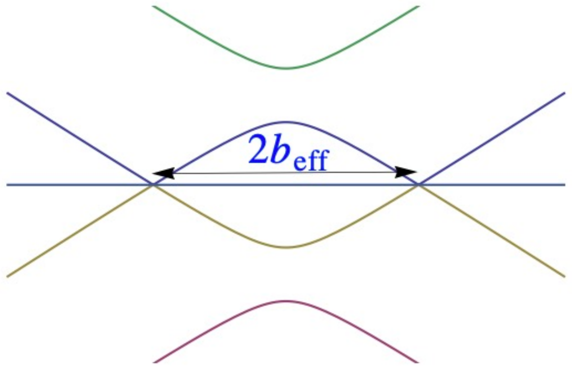
- We have $i\gamma^\mu \partial_\mu \psi - m\psi - b\gamma^5\gamma^3\psi = 0$
- With the spectrum

$$\omega = -\sqrt{\frac{b^2}{16} + k_1^2 + k_2^2 + k_3^2 + m^2 - \frac{1}{2}\sqrt{b^2(k_3^2 + m^2)}}$$

$$\omega = \sqrt{\frac{b^2}{16} + k_1^2 + k_2^2 + k_3^2 + m^2 - \frac{1}{2}\sqrt{b^2(k_3^2 + m^2)}}$$

$$\omega = -\sqrt{\frac{b^2}{16} + k_1^2 + k_2^2 + k_3^2 + m^2 + \frac{1}{2}\sqrt{b^2 k_3^2 + b^2 m^2}}$$

$$\omega = \sqrt{\frac{b^2}{16} + k_1^2 + k_2^2 + k_3^2 + m^2 + \frac{1}{2}\sqrt{b^2 k_3^2 + b^2 m^2}}$$



III. Topological modes in non-inertial frames for other physical systems: fermions

- The metric again has all zero Riemann tensors: a non-inertial reference frame
- Solve for the reference frame: $x^\mu \to x'^\mu = x^\mu + \xi^\mu$

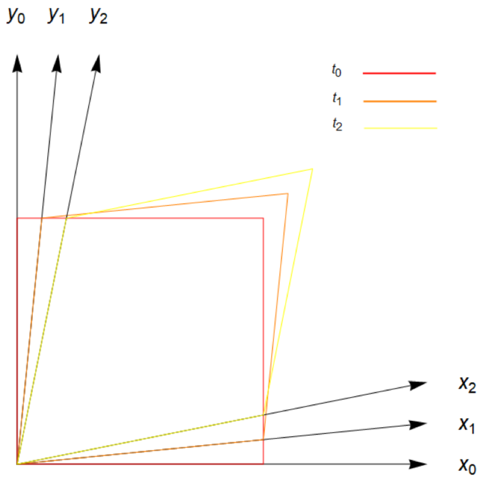
$$\xi^\mu = (\frac{b}{2}xy, \frac{b}{2}ty, \frac{b}{2}tx, 0,)$$

- The non-inertial frame: elastic observers, i.e. the observer at rest in the new frame is not a rigid body in the original inertial frame
- The formula for the movement of the accelerating observer in the inertial frame

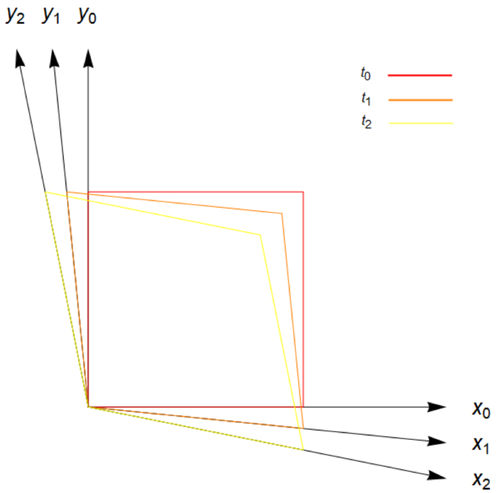
$$t = t' - \frac{b}{2}x'y' \quad x = x' - \frac{b}{2}t'y' \quad y = y' - \frac{b}{2}x't'$$

III. Topological modes in non-inertial frames for other physical systems: fermions

The movement of the accelerating observer in the inertial frame



The movement of the material (rest in the inertial frame) seen by the observer



Several remarks

- The only scale of the system: b
- $t << 1/b$, only at an instant, any use in real systems?
- b, m are both small enough in real systems?

III. Topological modes in non-inertial frames for other physical systems: fermions

- The underlying mechanism that topologically trivial systems could become topologically nontrivial

  ✯How could a single band crossing point become two or four crossing nodes just viewed by a different observer: real modes of w and k could become complex after a reference frame change and complex modes become real, i.e. we are observing the topological structure in the complex spectrum.

  ✯Viewed from the physical perspective: inertial forces introduce interactions that change the topological structure of the system;

Summary

- Based on our previous work of topologically nontrivial hydrodynamic modes observed in a non-inertial reference frame, we have shown that
  - In the non-inertial frame holographic system, the Ward identities are shown to be the same and the same hydrodynamic modes are obtained;
  - This property could be generalized to fermionic systems, for which we have shown that normal Dirac fermions could become a Weyl semimetal observed in a non-inertial frame, which, however, requires an elastic observer.

Open questions

- Topological states from non-inertial frames in other systems, e.g. photons?
- Possible other fermionic topological states from non-inertial frames?
- Gapped ones?
- Any realistic realization that could be observed in laboratories?
- More explicit explanation for the underlying mechanism?
- More transport properties?

Thank you!