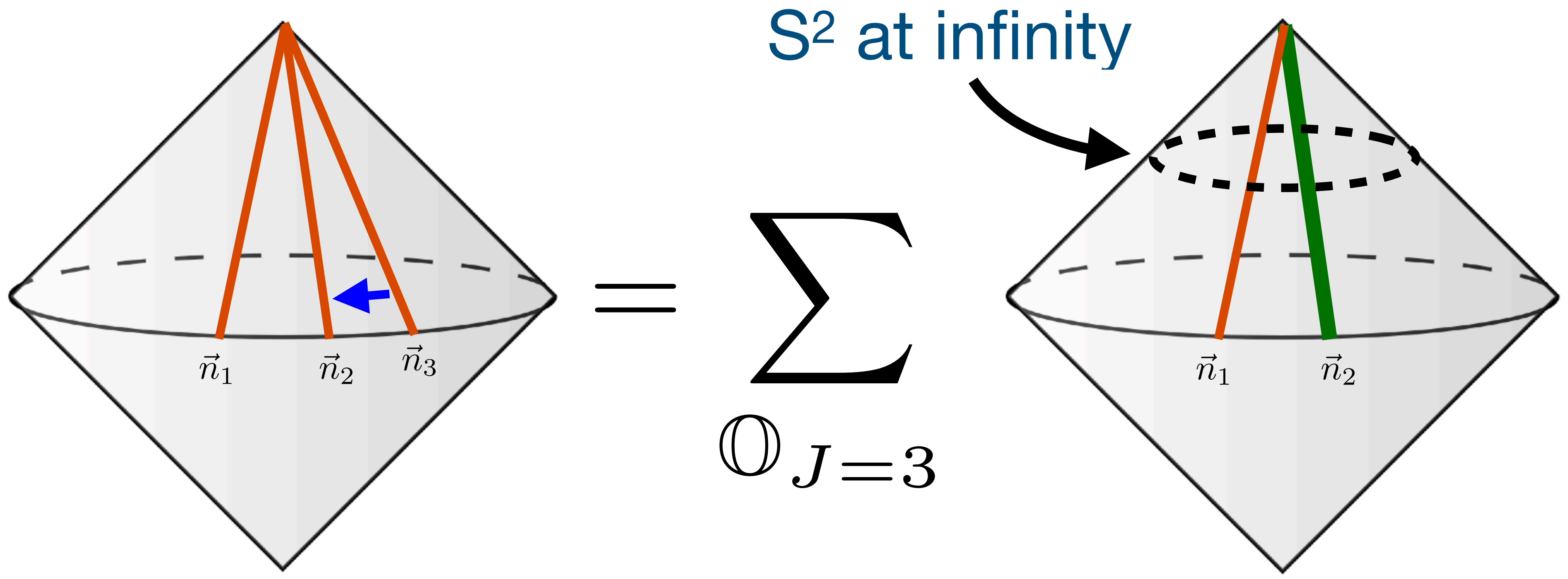
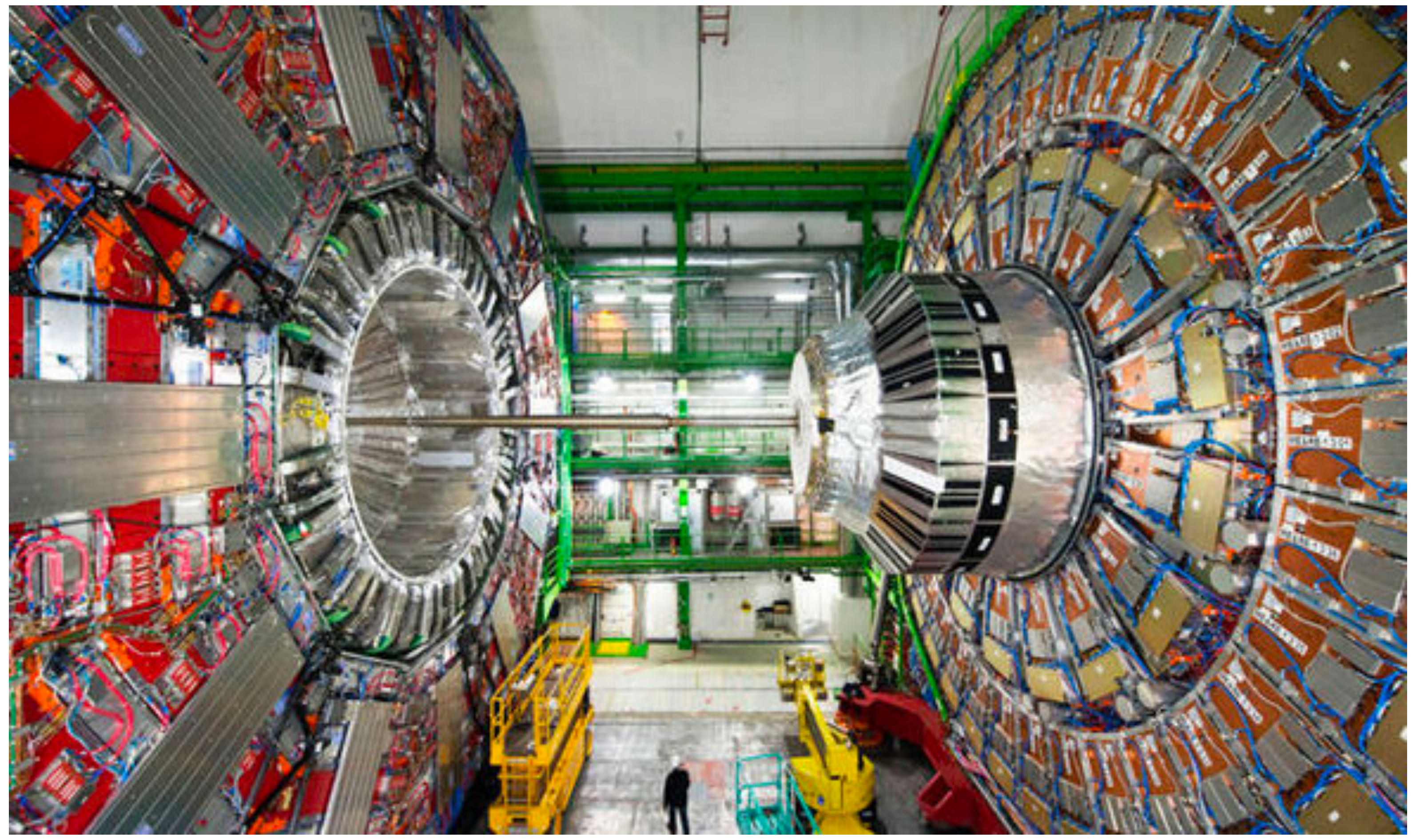
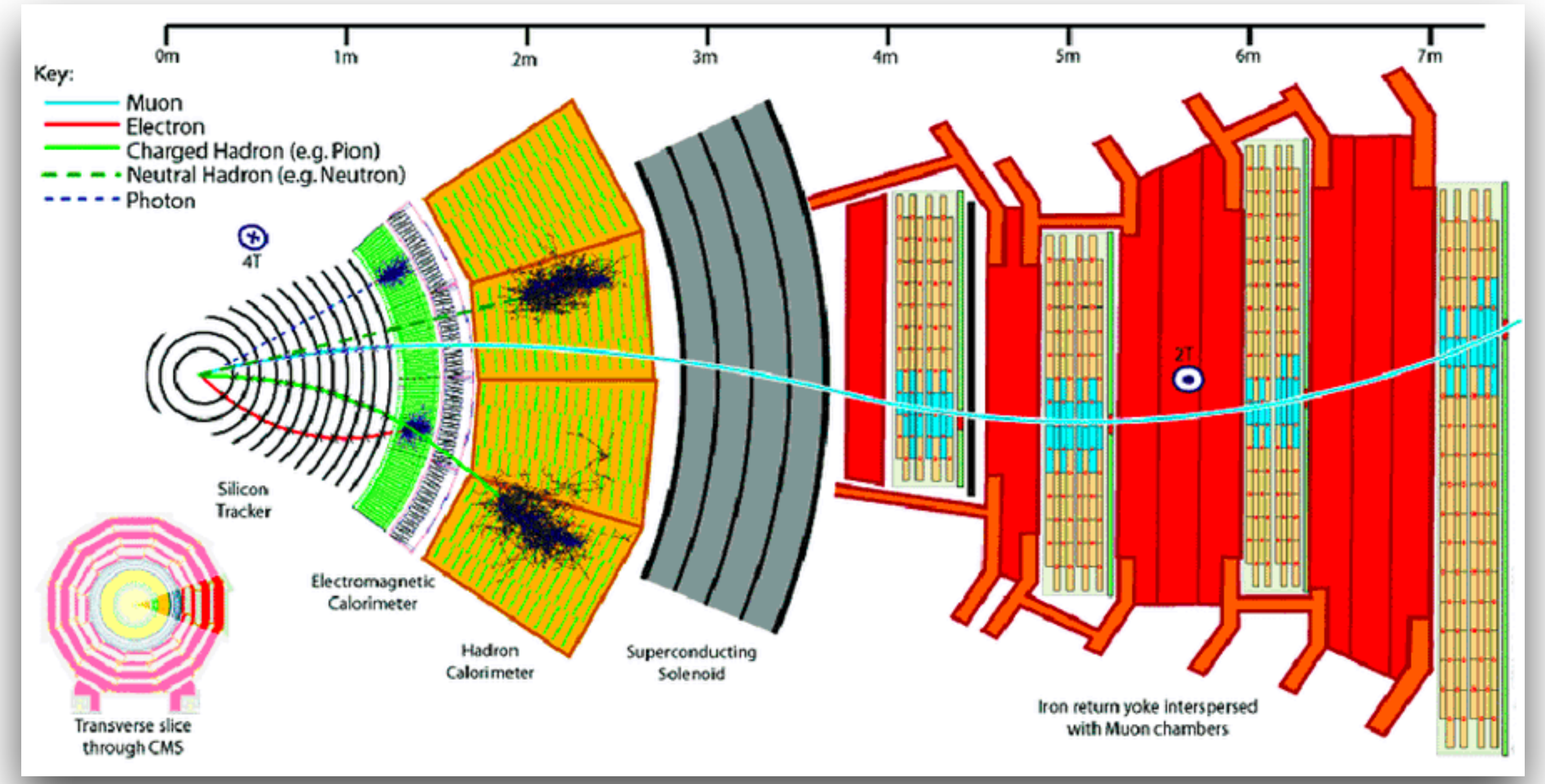


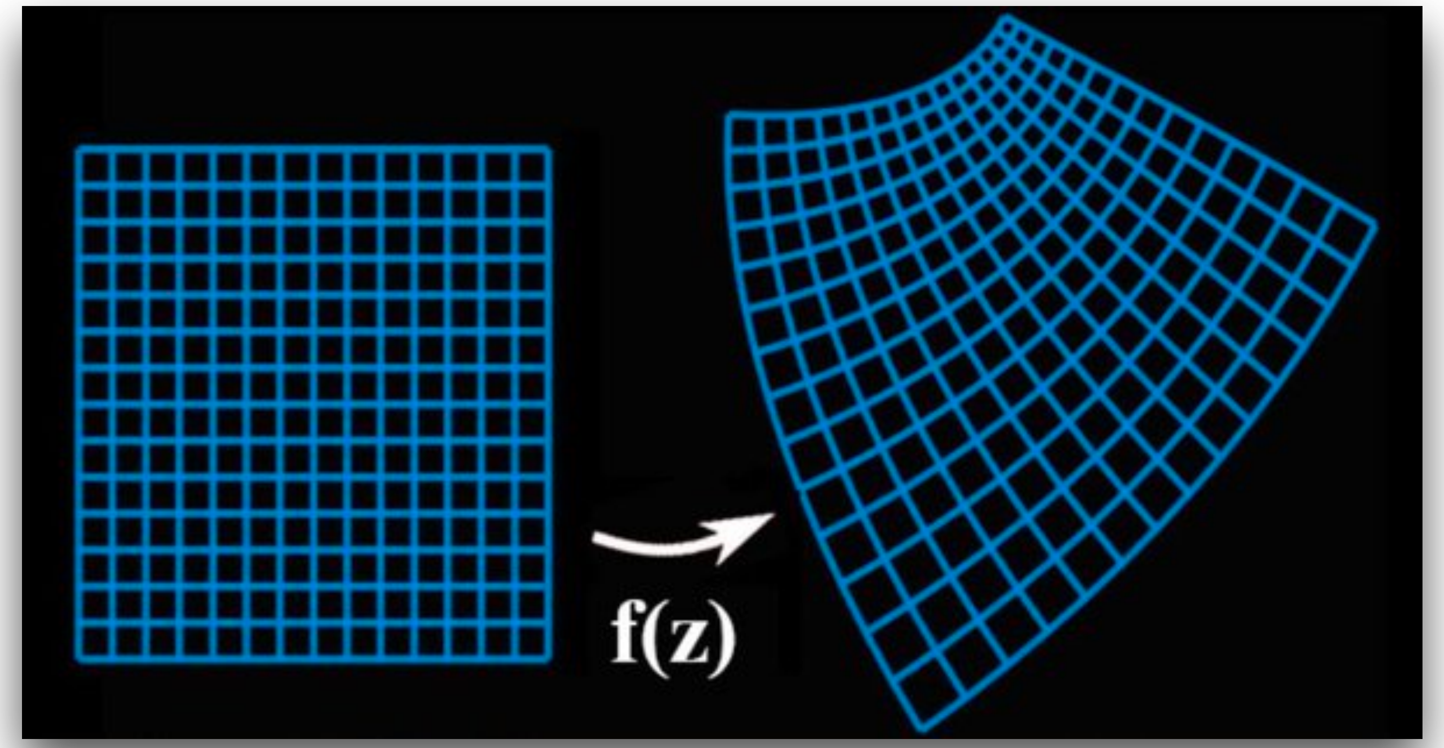


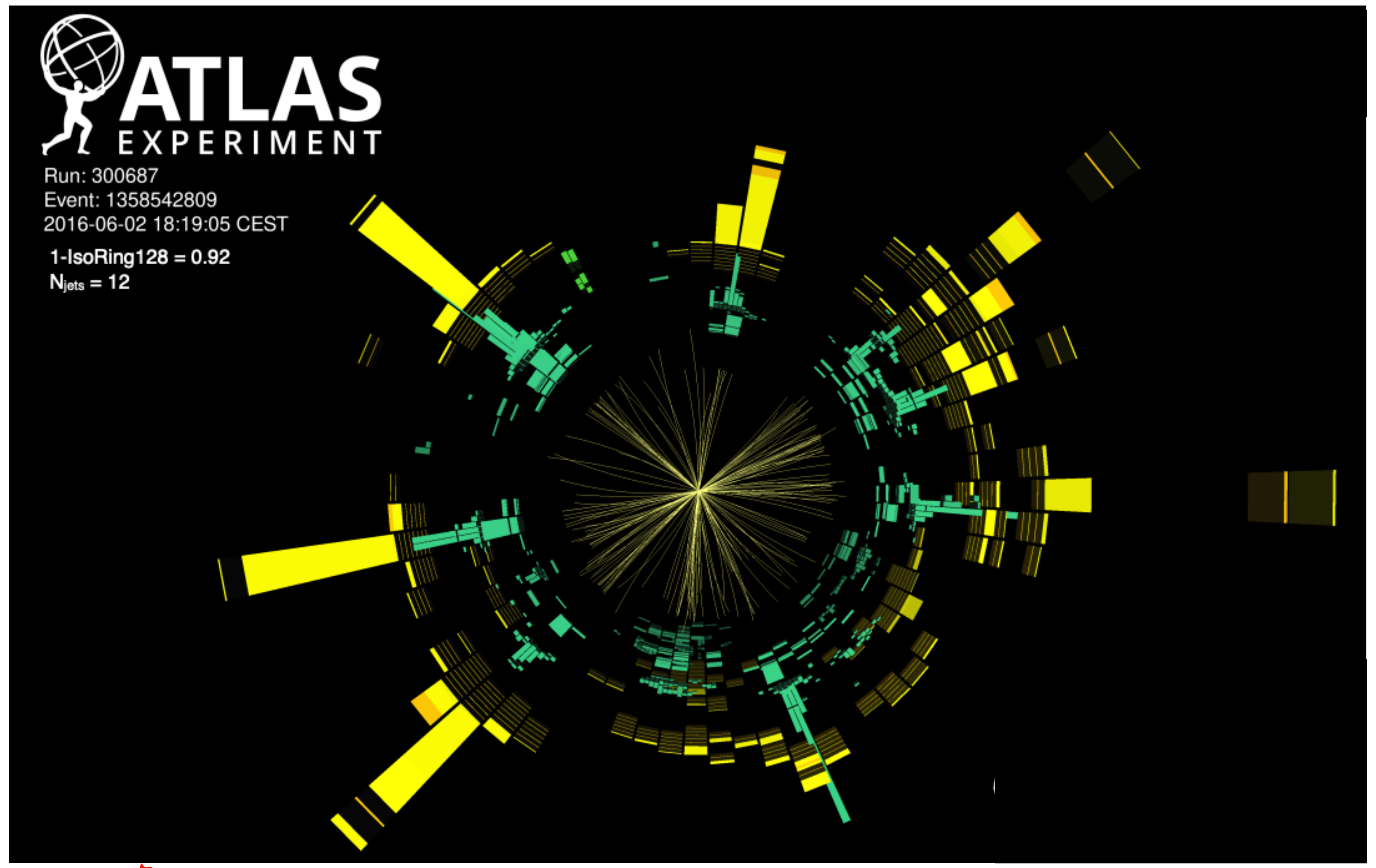
Conformal Collider Meet the LHC

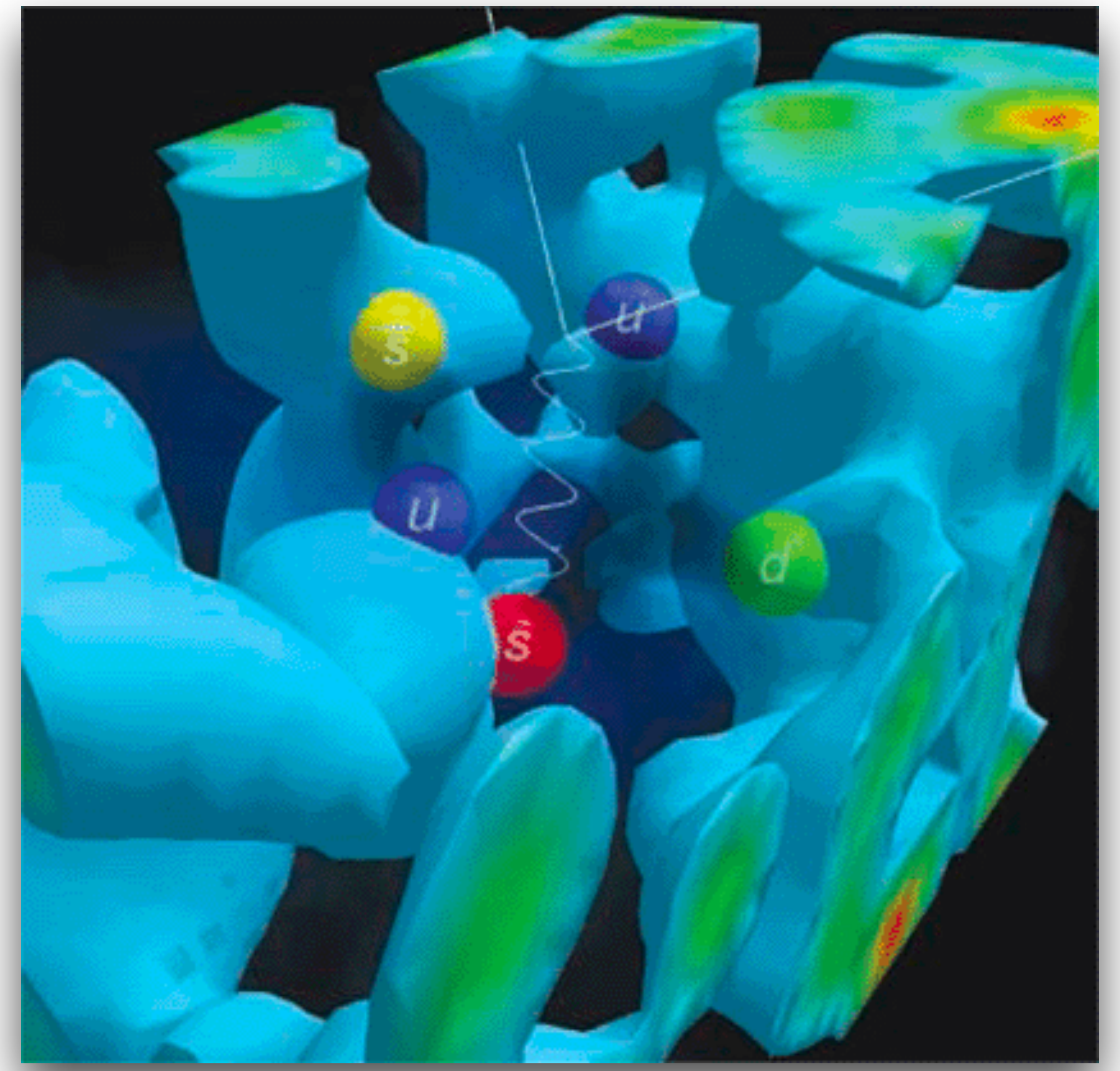
朱华星 (Hua Xing Zhu)
浙江大学

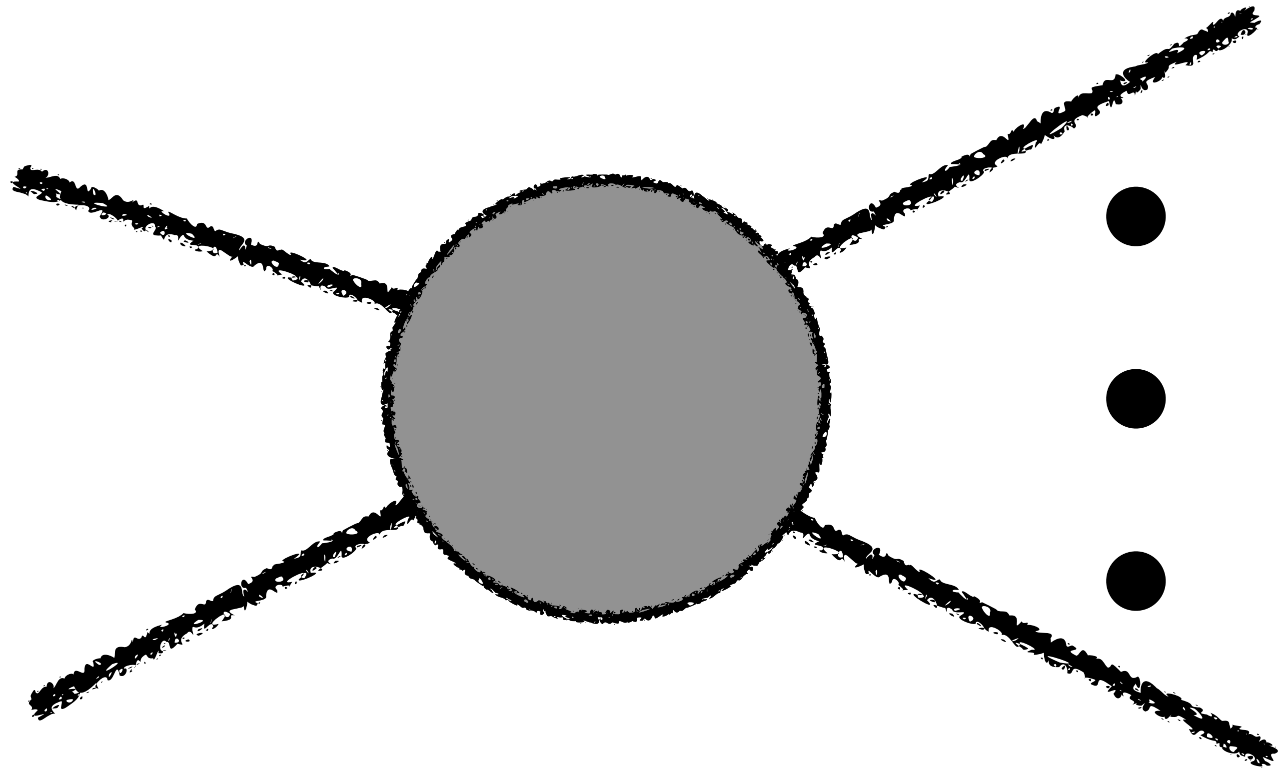
第三届全国场论与弦论学术研讨会
北京 2022年8月25日


0m
1m
2m
3m
4m
5m
6m
7m
Key:
Muon
Electron
Charged Hadron (e.g. Pion)
Neutral Hadron (e.g. Neutron)
Photon
4T
2T
Silicon Tracker
Electromagnetic Calorimeter
Hadron Calorimeter
Superconducting Solenoid
Iron return yoke interspersed with Muon chambers
Transverse slice through CMS


f(z)


ATLAS
EXPERIMENT
Run: 300687
Event: 1358542809
2016-06-02 18:19:05 CEST
1-IsoRing128 = 0.92
N_{jets} = 12

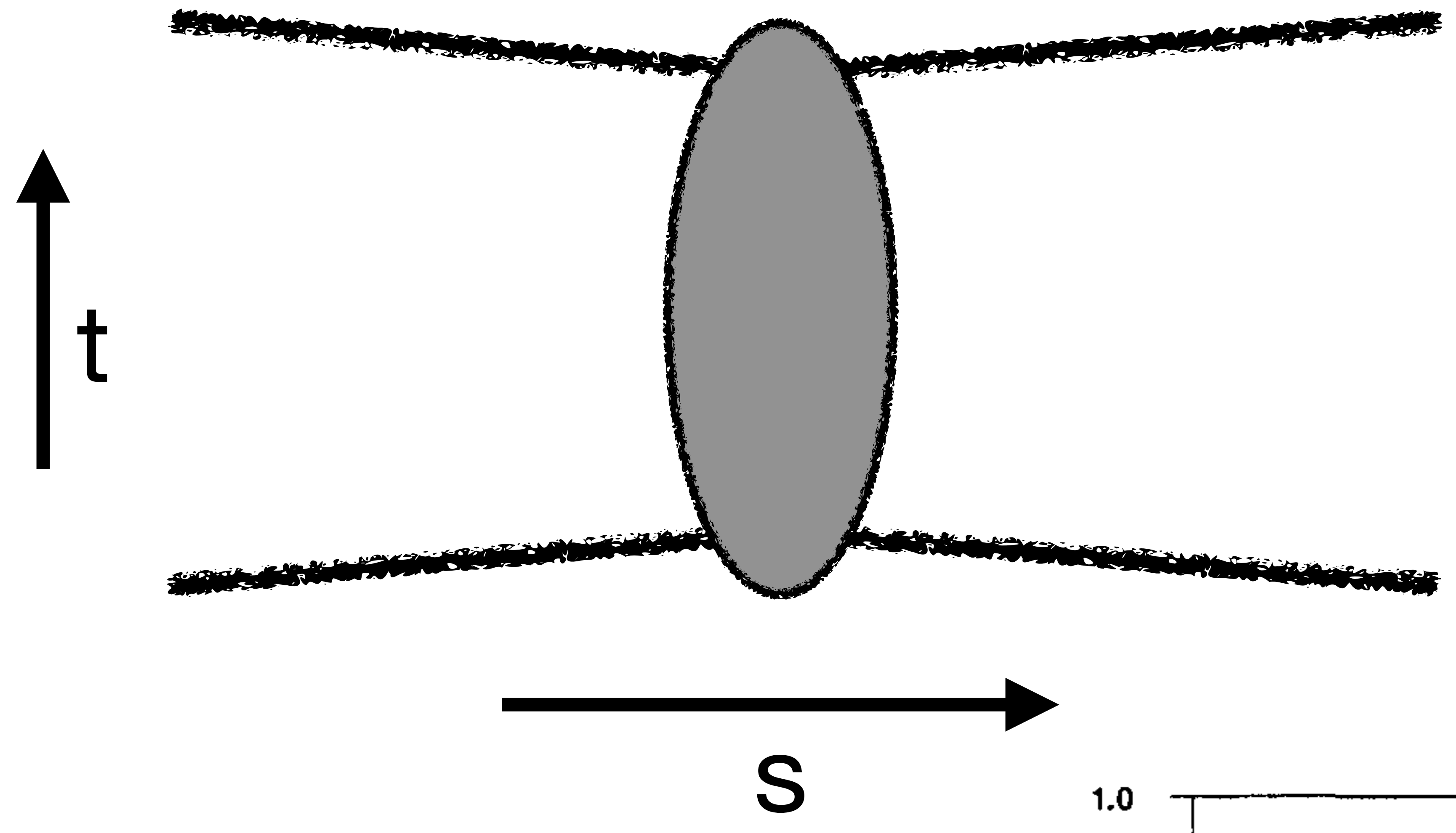




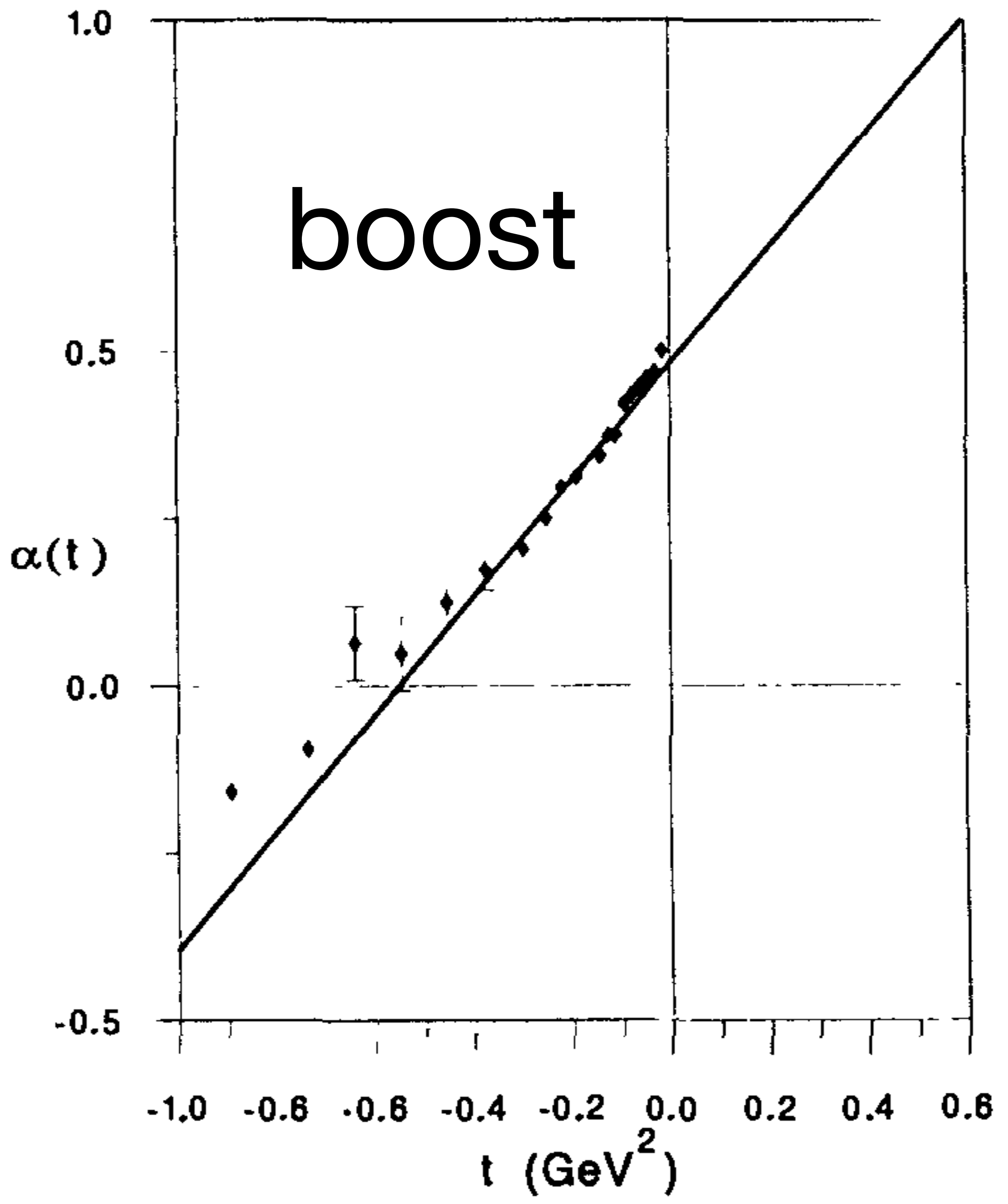
Outline

- String theory and high energy scattering
- Field theory description of multi-jet production
- Scaling phenomena in jet substructure
- Celestial block expansion and transverse spin phenomena
- Analyticity in transverse spin

High energy scattering in QCD



Unified description of t/s channel?



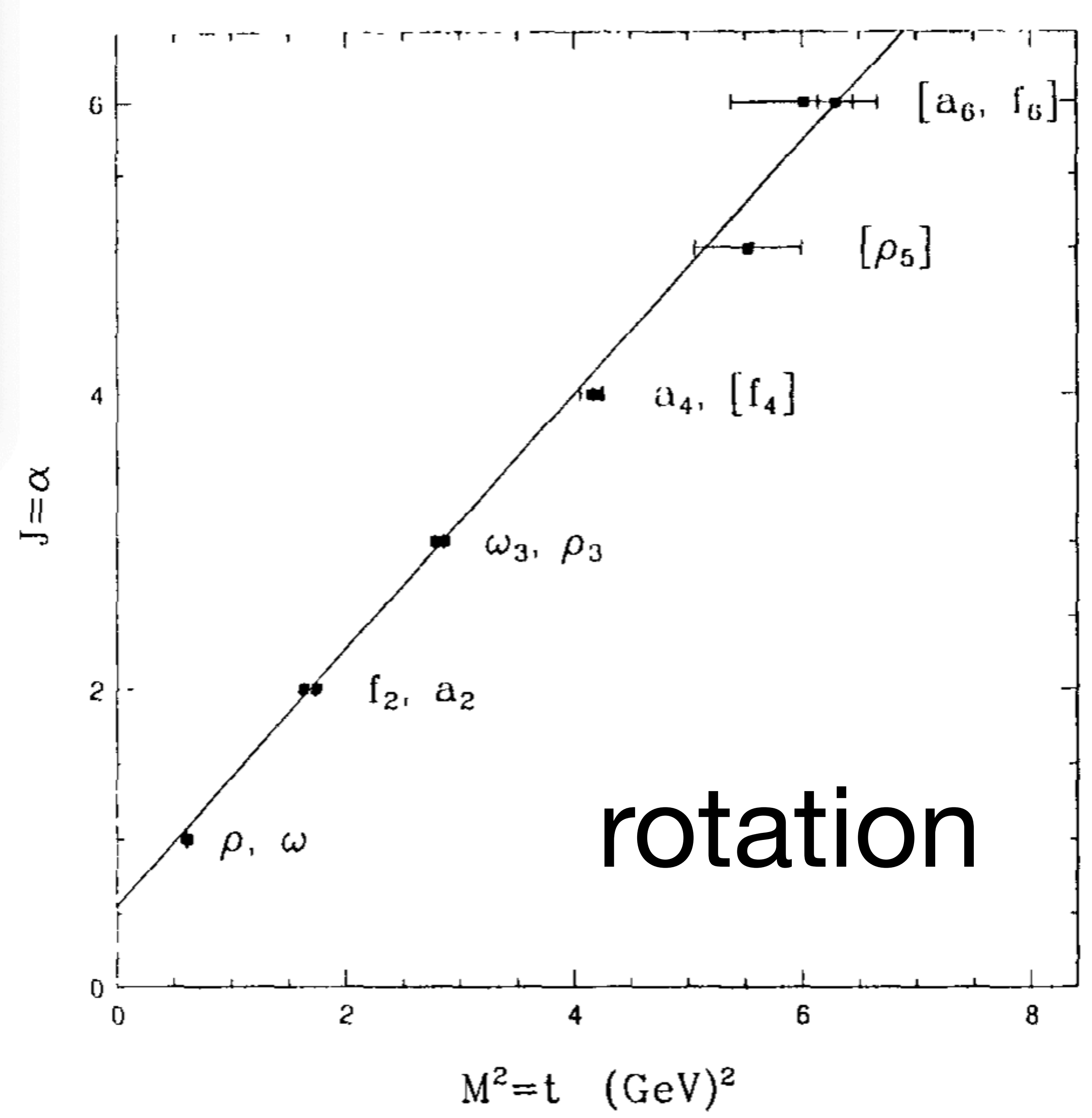
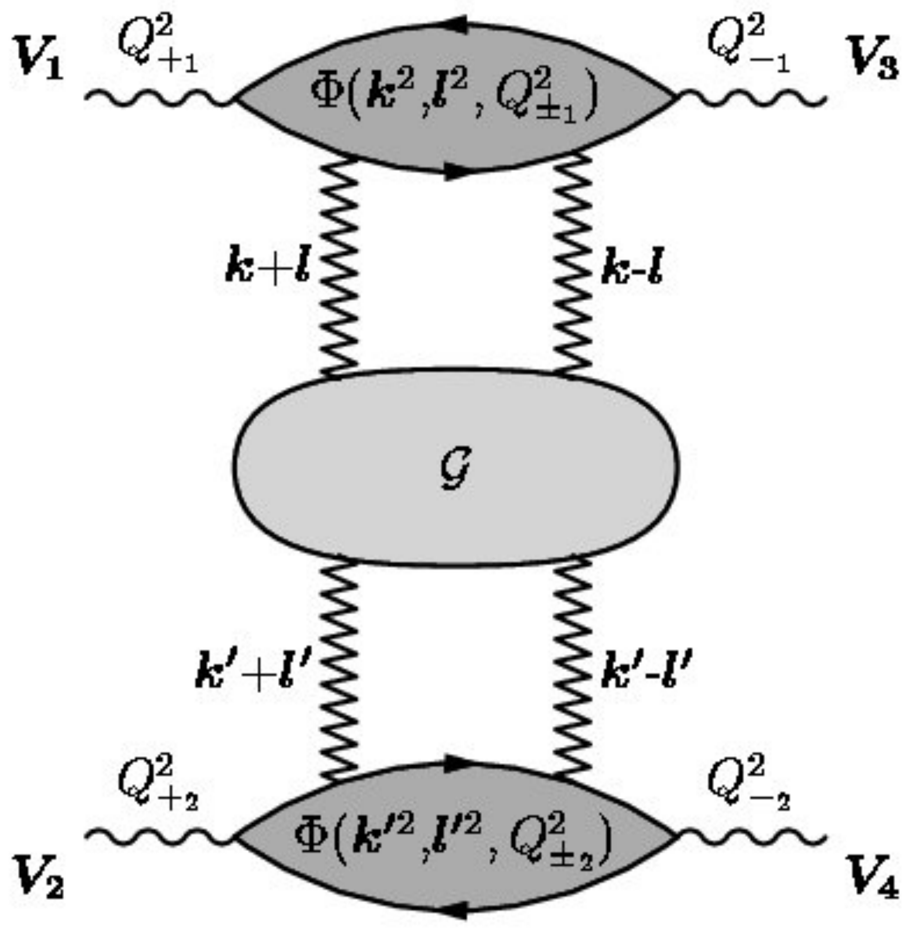


Fig. 1.6. The Chew–Frautschi plot.

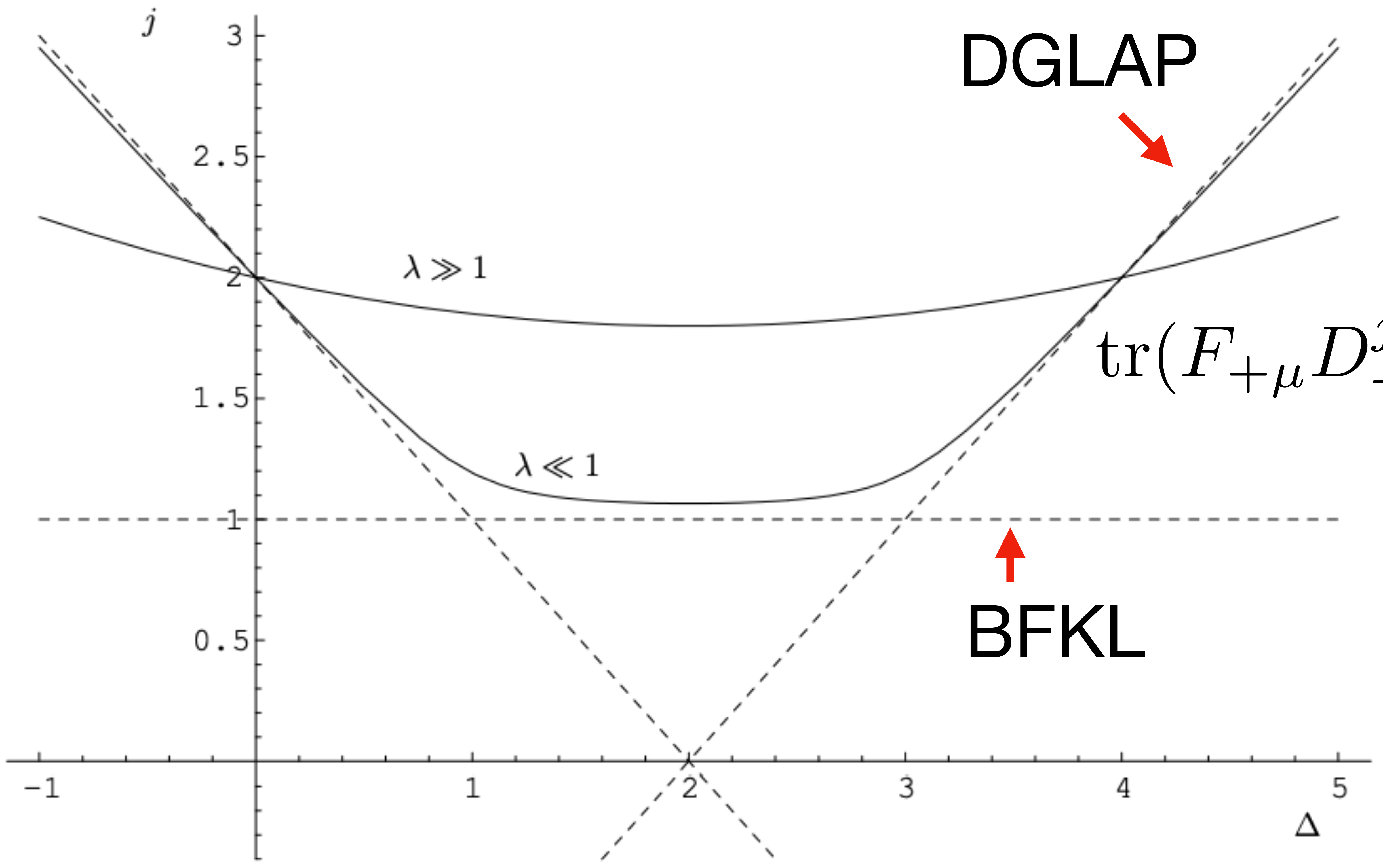
Unified description of hard and soft pompon?



BFKL pomeron

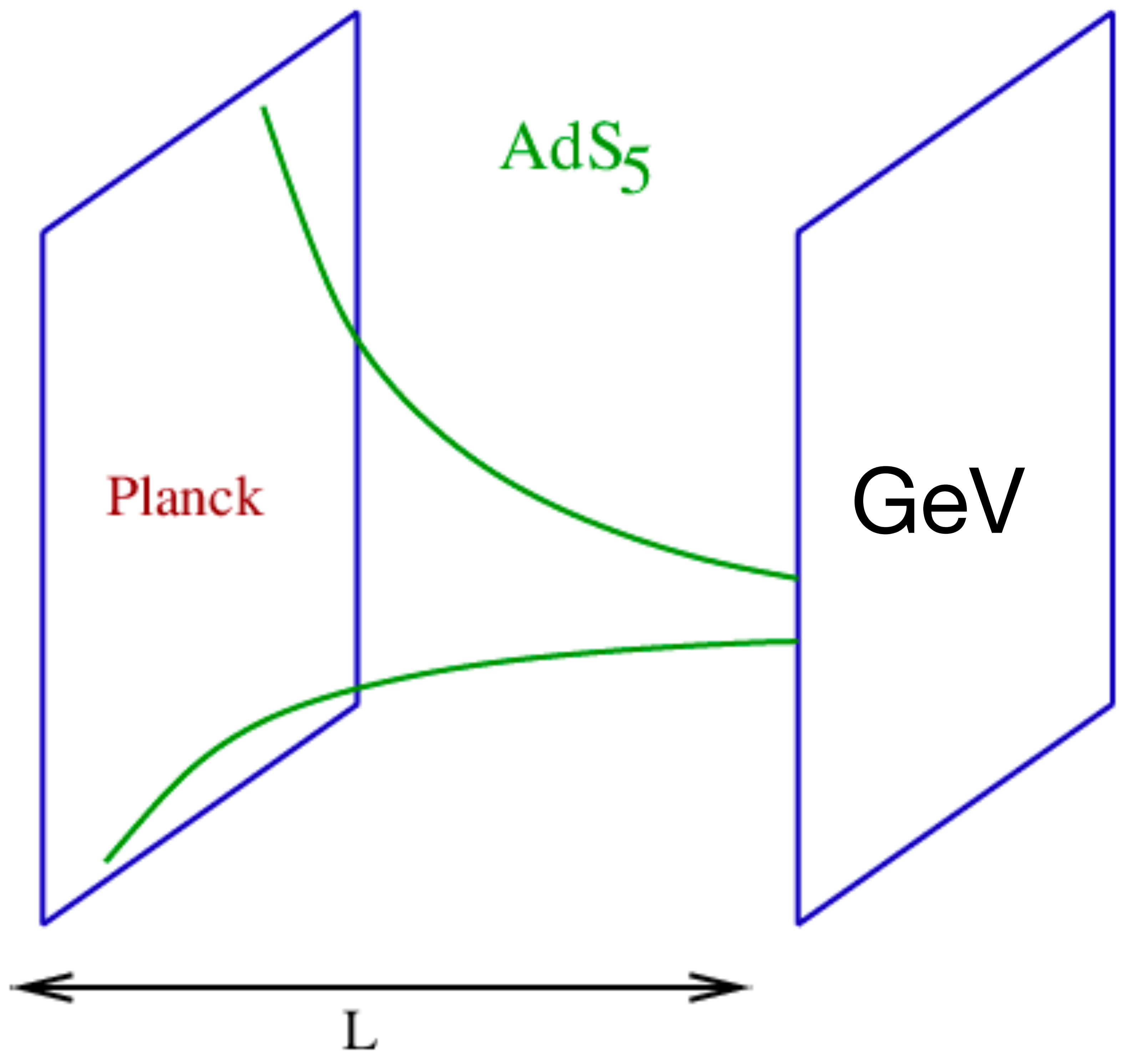
Stringy model for high energy scattering

The Pomeron and gauge/string duality
Brower, Polchinski, Strassler, Tan, 2006



$$\mathrm{tr}(F_{+\mu} D_+^{j-2} F_+^{\mu})$$

Kravchuk, Simmons-Duffin, 2018

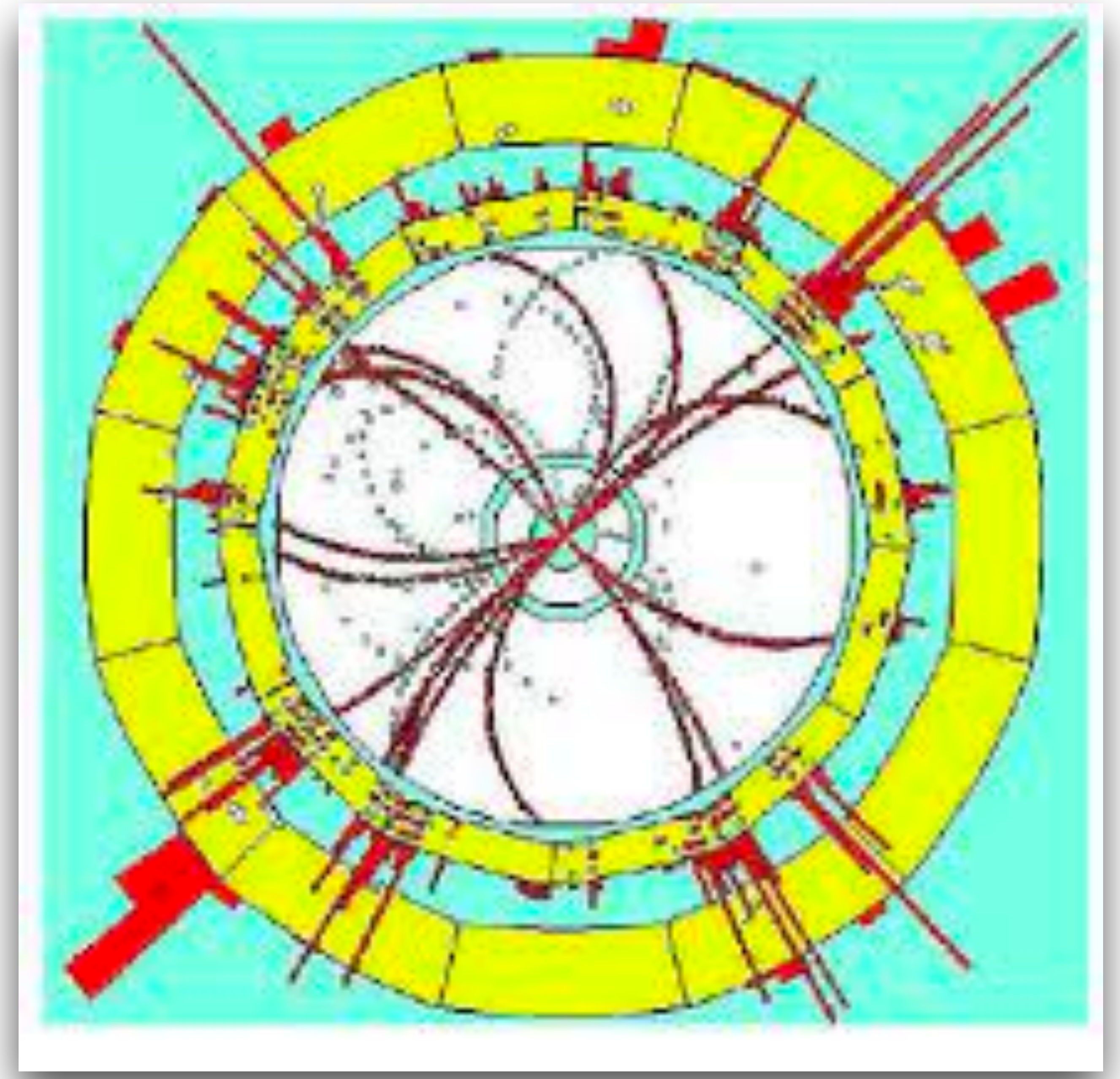


Analyticity in spin

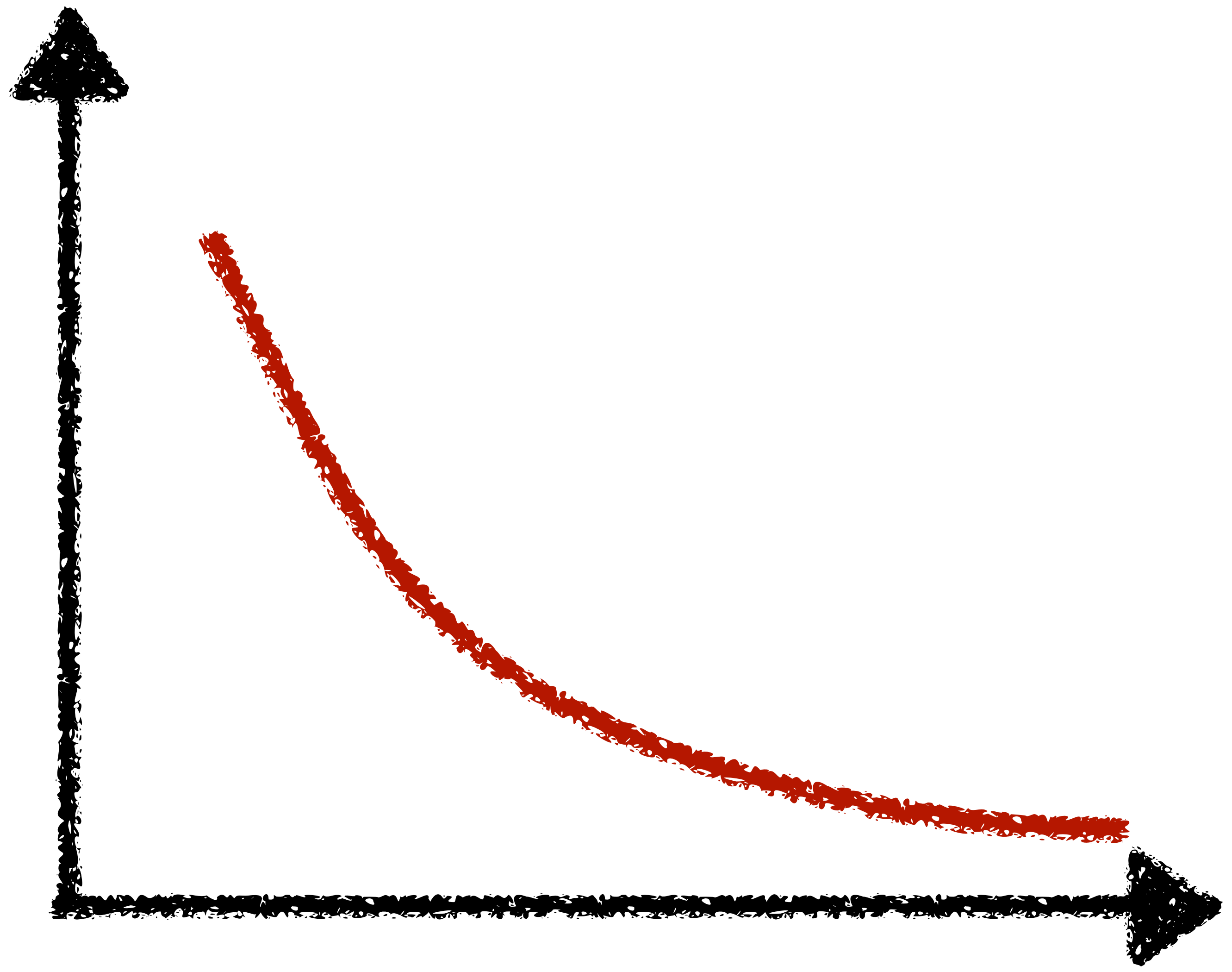
Jaroszewicz, 1982
Lipatov, 1997
Kotikov, Lipatov, 2002, 2004

Multi-particle/jet production in e+e-

- How do we theoretically describe the production of multi particles/jets in high energy collision?
- What D.O.F. to keep, and what to be integrated out?



Projection to low dimensional space that one can calculate, at least perturbatively



Two approaches to describe final state in e+e-

Event shape/jet cross section

$$f(O) = \sum_n \int dP.S.^{(n)} |\mathcal{M}_{2\to n}|^2$$
$$\times \delta(O - \hat{O}(k_1, \cdots, k_n))$$

O invariant under soft/collinear radiation

Infrared & collinear safety

Sterman, Weinberg, 1975

Mellin transformation

M-point energy correlators

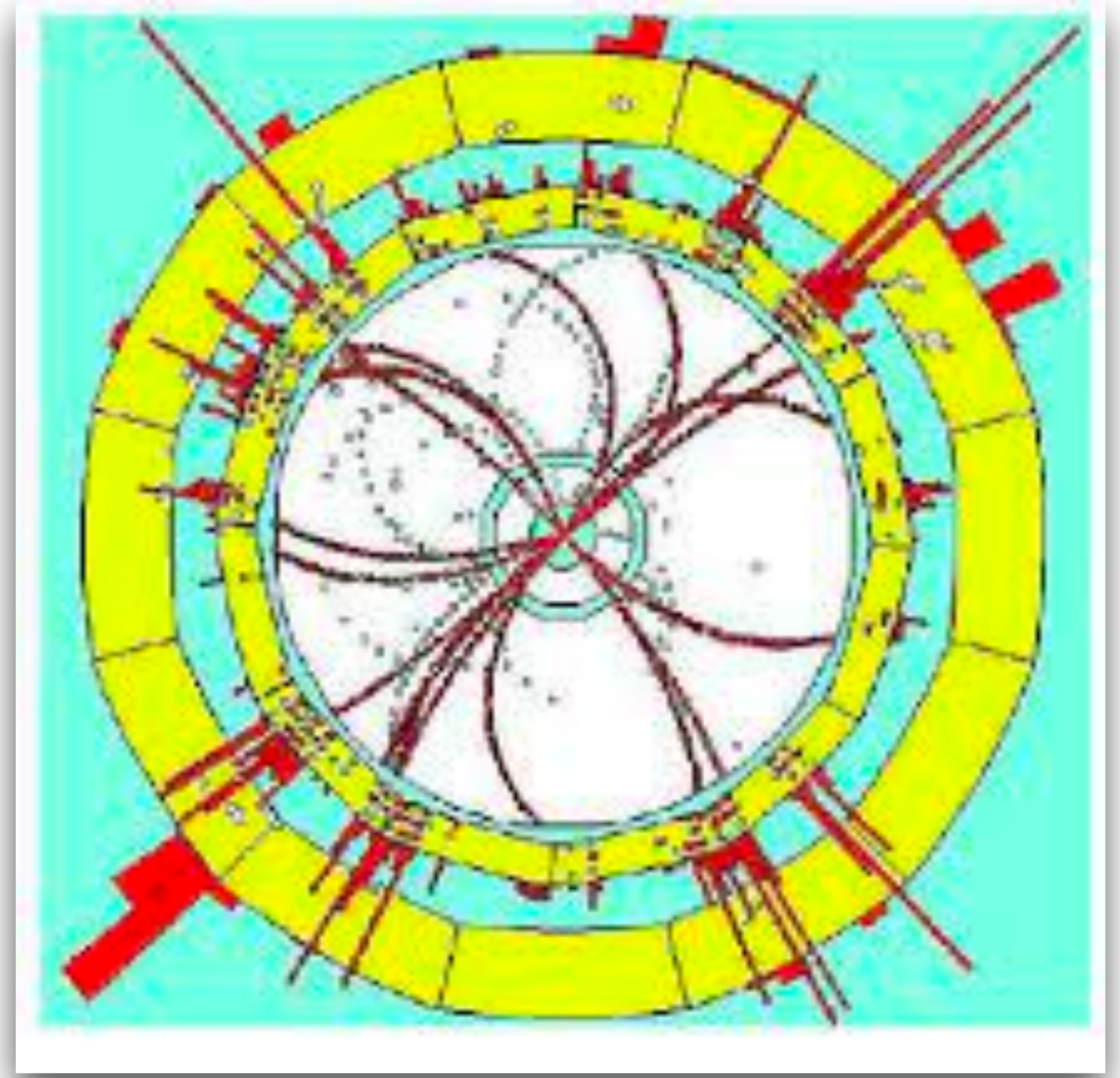
$$\Sigma(\theta_{12}, \theta_{13}, \cdots, \theta_{m-1,m}) = \sum_n \int dP.S.^{(n)} |\mathcal{M}_{2\to n}|^2$$

$$E_1 \cdots E_m \times \delta(\theta_{12} - \hat{\theta}(k_1, k_2)) \cdots \delta(\theta_{m-1,m} - \hat{\theta}(k_{m-1}, k_m))$$

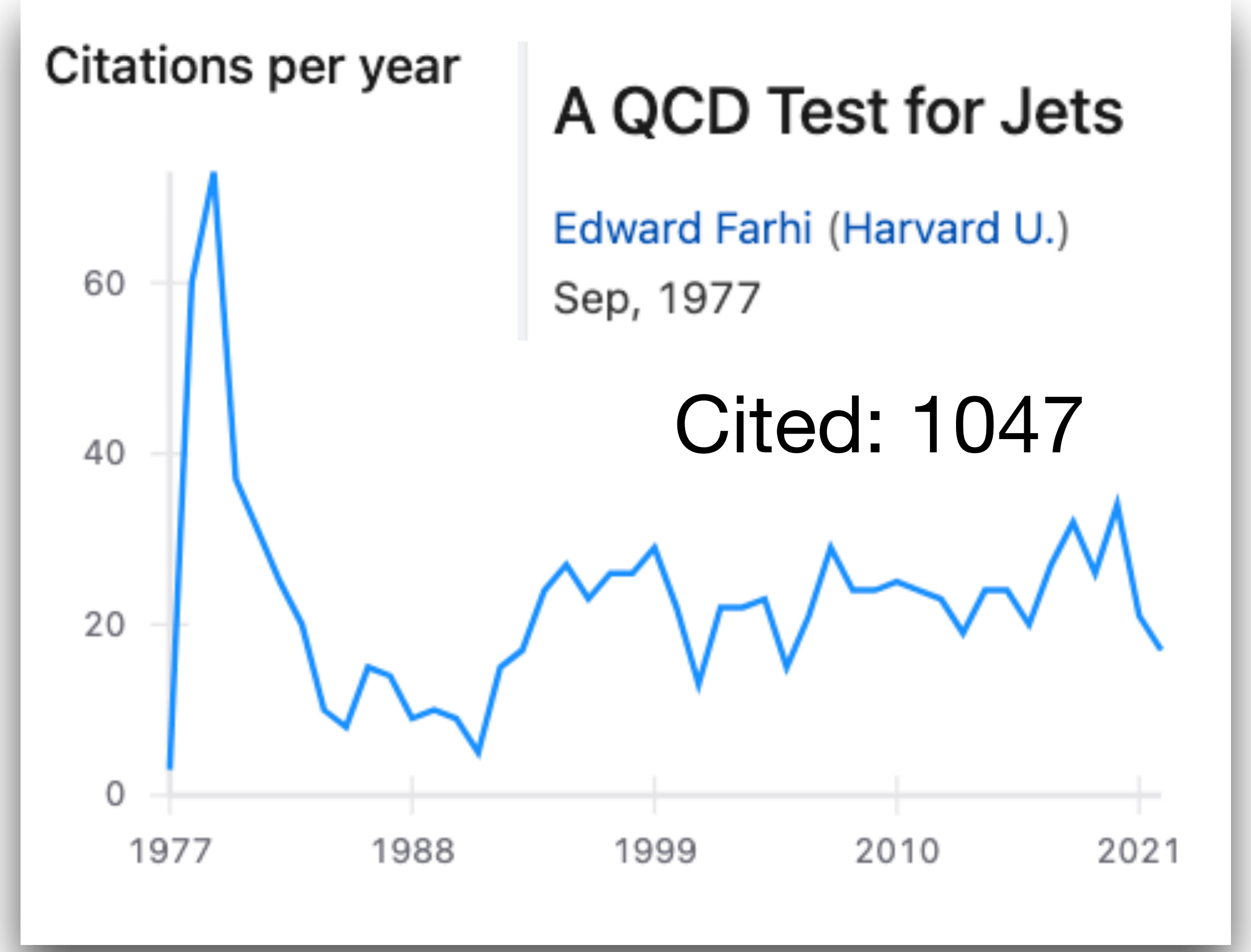
Linear energy weighting

Observable for testing QCD at earlier days

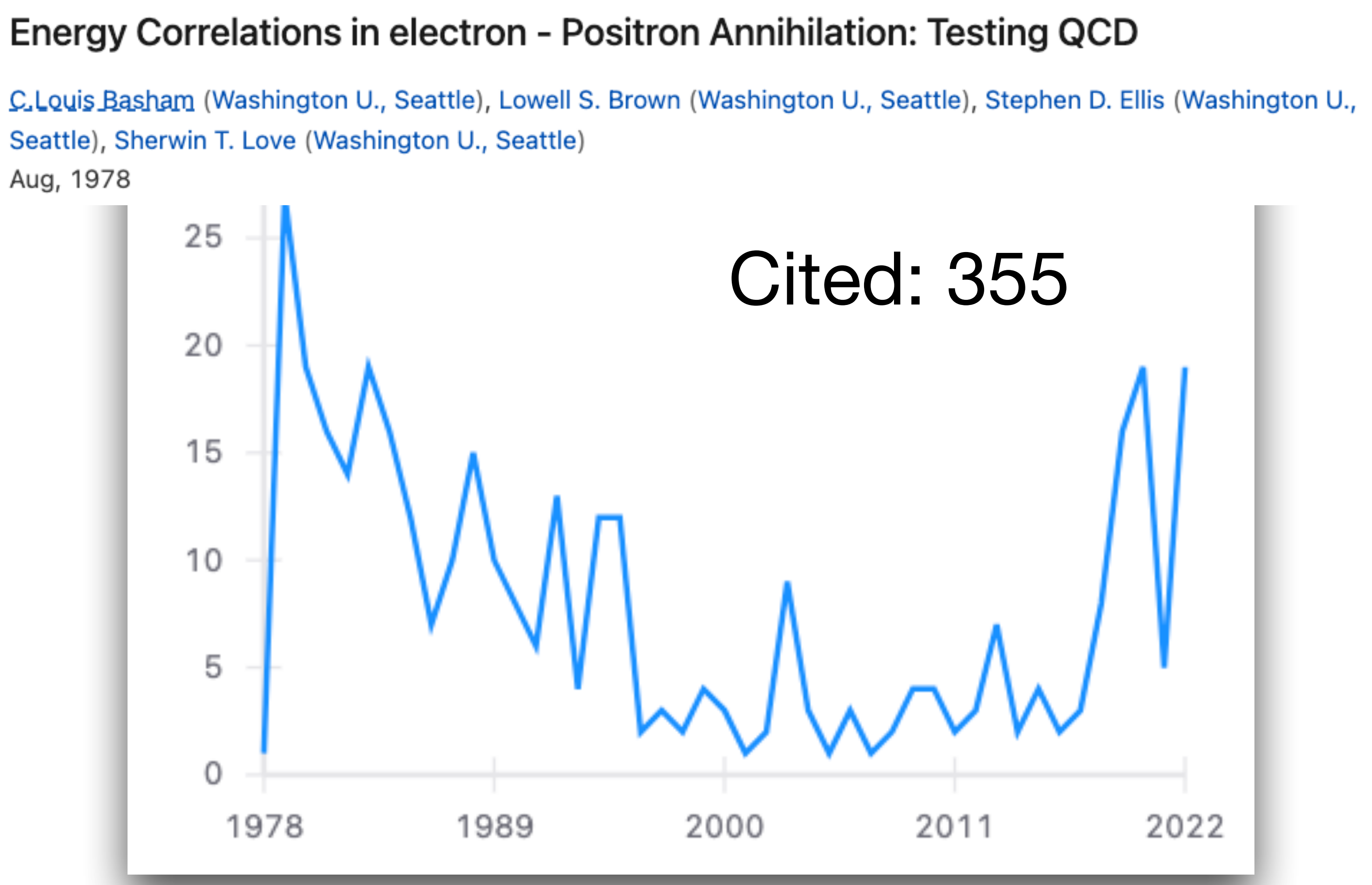
How do we define obsevable multi-particles production in e+e-?



Thrust



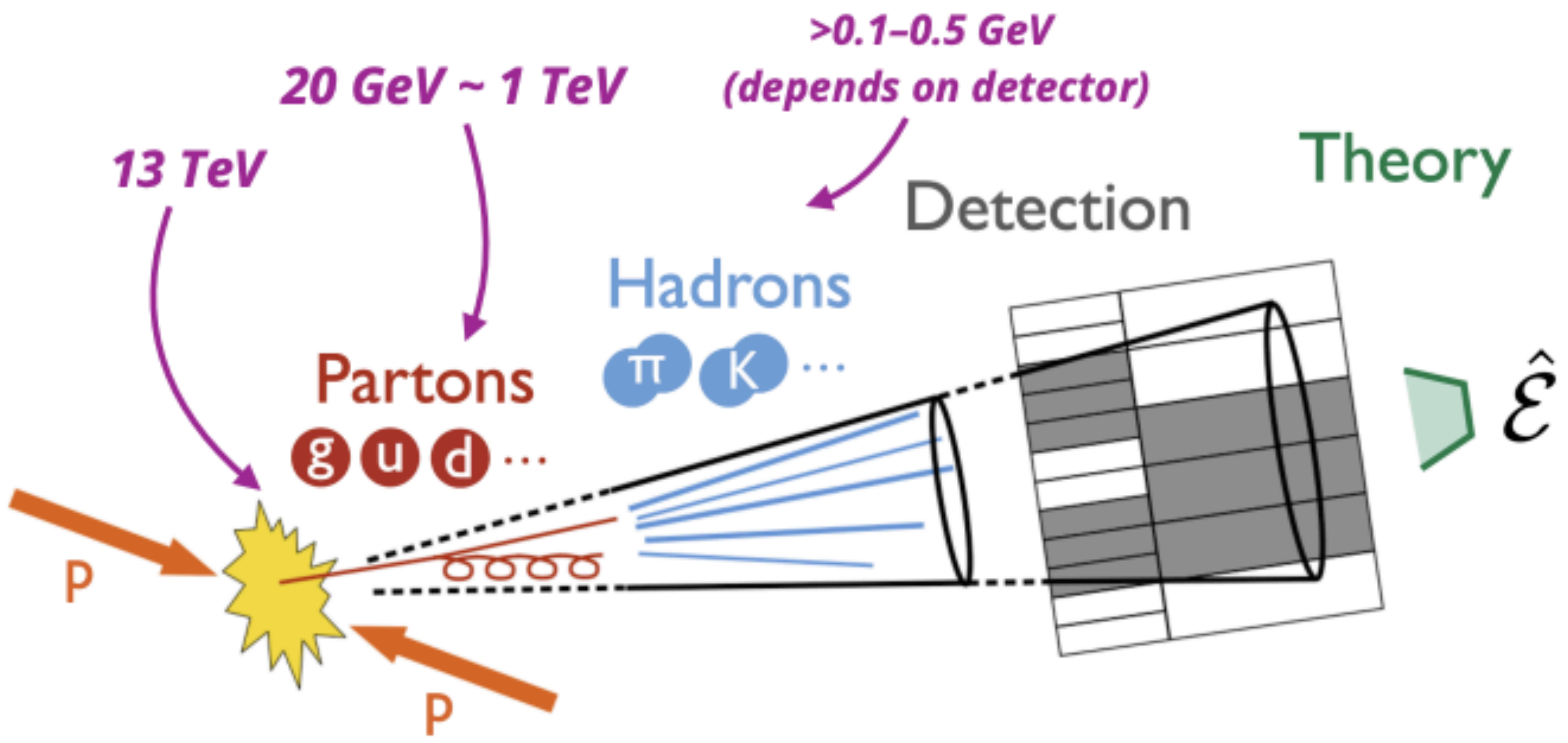
EEC



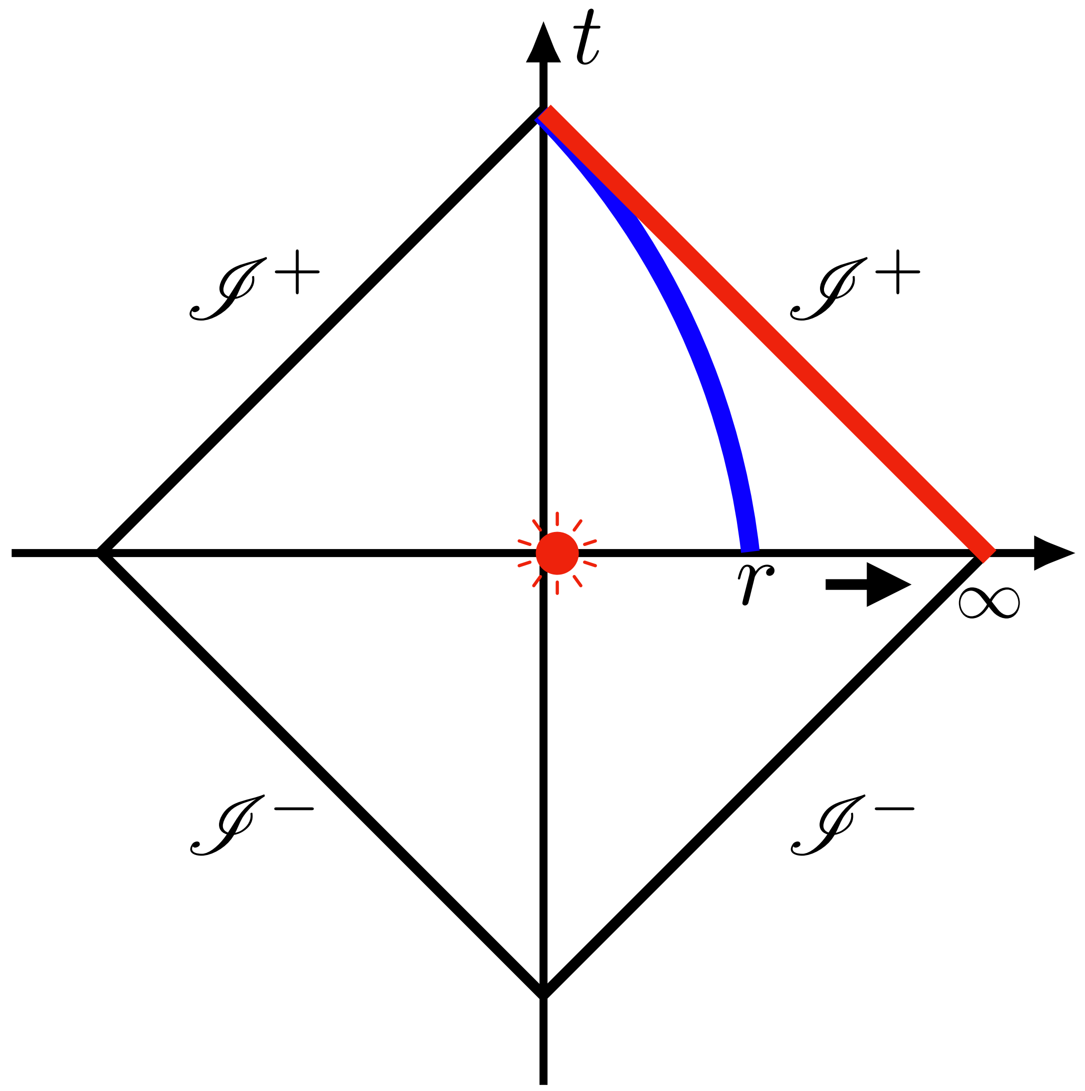
The revival of energy correlators, theoreticlly and experimentally

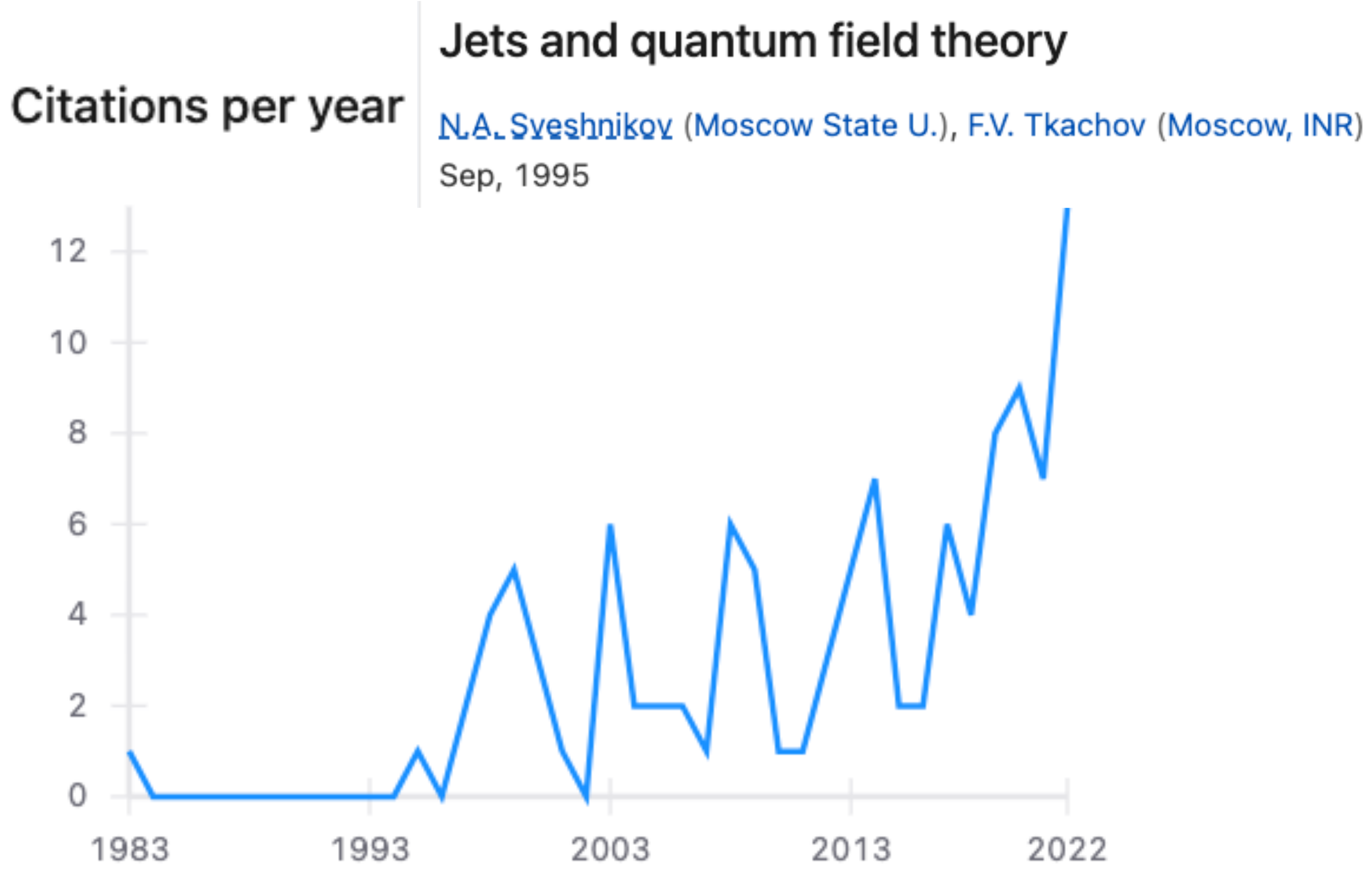
- Theoretically:
  - Operator definition for collider measurement: Sveshnikov, Tkachov, 1995; Hofman, Maldacena, 2008
  - Viability of analytic calculation: Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2013; Dixon, M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, 2018
  - OPE, factorization and resummation: Hofman, Maldacena 2008; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019; Korchemsky, 2019; Dixon, Moult, HXZ, 2019
  - Application in jet substructure: H. Chen, Moult, X.Y. Zhang, HXZ, 2020, H. Chen, Moult, HXZ, 2020
- Experimentally:
  - Superb energy reach and angular resolution at the LHC and open data program: Komiske, Moult, Thaler, HXZ, 2022

Quantum field definition for calorimetric detector



$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \vec{n}_i T^{0i}(t, r\vec{n})$$





cuss the experimental procedures employed e.g. in the recent discovery of the top quark [3], [4] without using the language of hadron jets. Yet apart from the early discussion of the issue of perturbative IR safety in connection with perturbative calculability [5], [6], remarkably little (if anything at all) has been done to integrate the jet paradigm into the framework of Quantum Field Theory. This is despite the fact that perturbative QFT is the only systematic calculational framework for obtaining theoretical predictions about jets. The conventional theory of jets was developed by trial and error within experimental and phenomenological communities and is based on the notion of jet definition algorithm which is foreign to QFT. On the other

Energy correlators as correlation function of ANEC operator

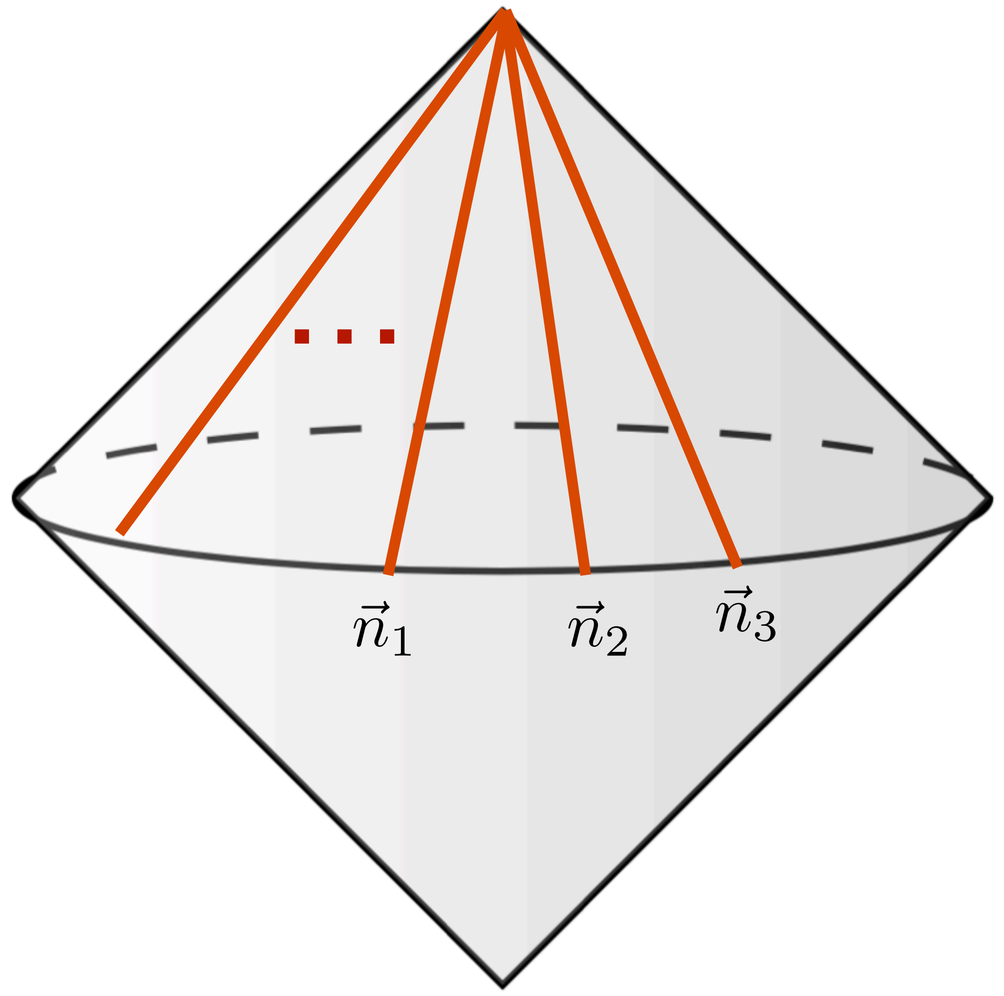
$$\Sigma(\theta_{12},\theta_{13},\cdots,\theta_{m-1,m}) = \sum_n \int dP.S.^{(n)} |\mathcal{M}_{2\to n}|^2$$

$$E_1 \cdots E_m \times \delta(\theta_{12} - \hat{\theta}(k_1,k_2)) \cdots \delta(\theta_{m-1,m} - \hat{\theta}(k_{m-1},k_m))$$

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\ \vec{n}_i T^{0i}(t, r\vec{n})$$

Sink   Source

$$\langle \Omega | O(x) \mathcal{E}(n_1) \mathcal{E}(n_2) \mathcal{E}(n_3) \cdots O^\dagger(0) | \Omega \rangle$$



a. **Manifest soft and collinear finite!**
b. **Analytic calculability** (allows to see analyticity in transverse spin)

Energy-Energy Correlator (EEC)

Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2013

$$\langle\Omega|O(x)T_{0\vec{n}_1}(y_1)T_{0\vec{n}_2}(y_2)O^\dagger(0)|\Omega\rangle_{\text{Euclidean}}$$

Analytic continuation

$$y_k^4 = -\epsilon_k + it_k$$

$$0 < \epsilon_0 < \cdots \epsilon_n$$

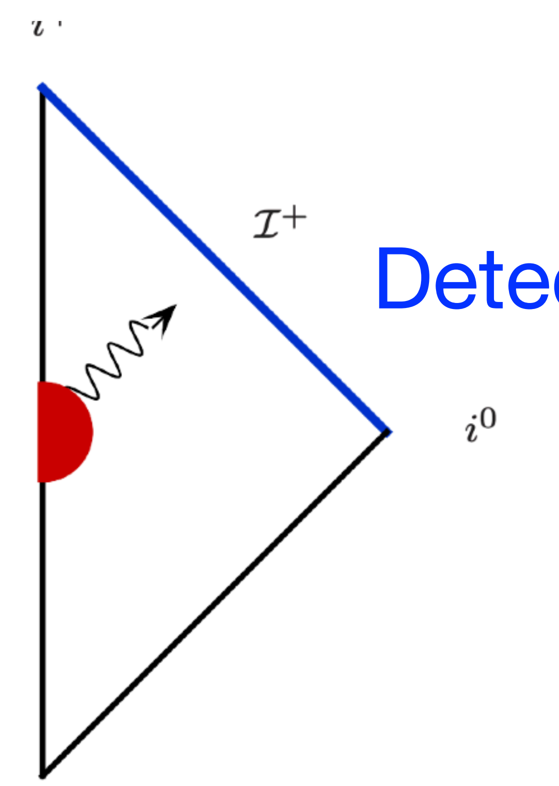
Mellin amplitude

$$\langle O^\dagger(x_1)\widetilde{O}(x_2)\widetilde{O}(x_3)O(x_4)\rangle_c = \int \prod_{1\le i<j\le 4} \frac{d\delta_{ij}}{2\pi i}(x_{ij}^2)^{\delta_{ij}} M(\delta_{ij})$$

Double discontinuity

$$\text{dDisc}_{w=w_0}\, g(w) = g(w) - \frac{1}{2}g(w^{\circlearrowleft}) - \frac{1}{2}g(w^{\circlearrowright})$$

$$\langle\Omega|O(x)\mathcal{E}(n_1)\mathcal{E}(n_2)O^\dagger|\Omega\rangle$$



Detector time integral

Analytic calculability

C parameter

$$\frac{1}{\sigma_0}\frac{d\sigma^{(3)}}{dC} = \frac{\alpha_s}{2\pi} C_F \int_{x_2^-(C)}^{x_2^+(C)} dx$$

Elliptic integral at tree-level!

$$\times \frac{6x\left[C\left(x^3+(x-2)^2\right)-6(1-x)(1+x^2)\right]}{C(C+6)^2(x-6/(C+6))\sqrt{(6/(C+6)-x)(x_2^+-x)(x-x_2^-)x}}$$

EEC

1. N=4 SYM

   One-loop, Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2013
   Two-loop, Henn, Sokatchev, K. Yan, 2019

2. QCD

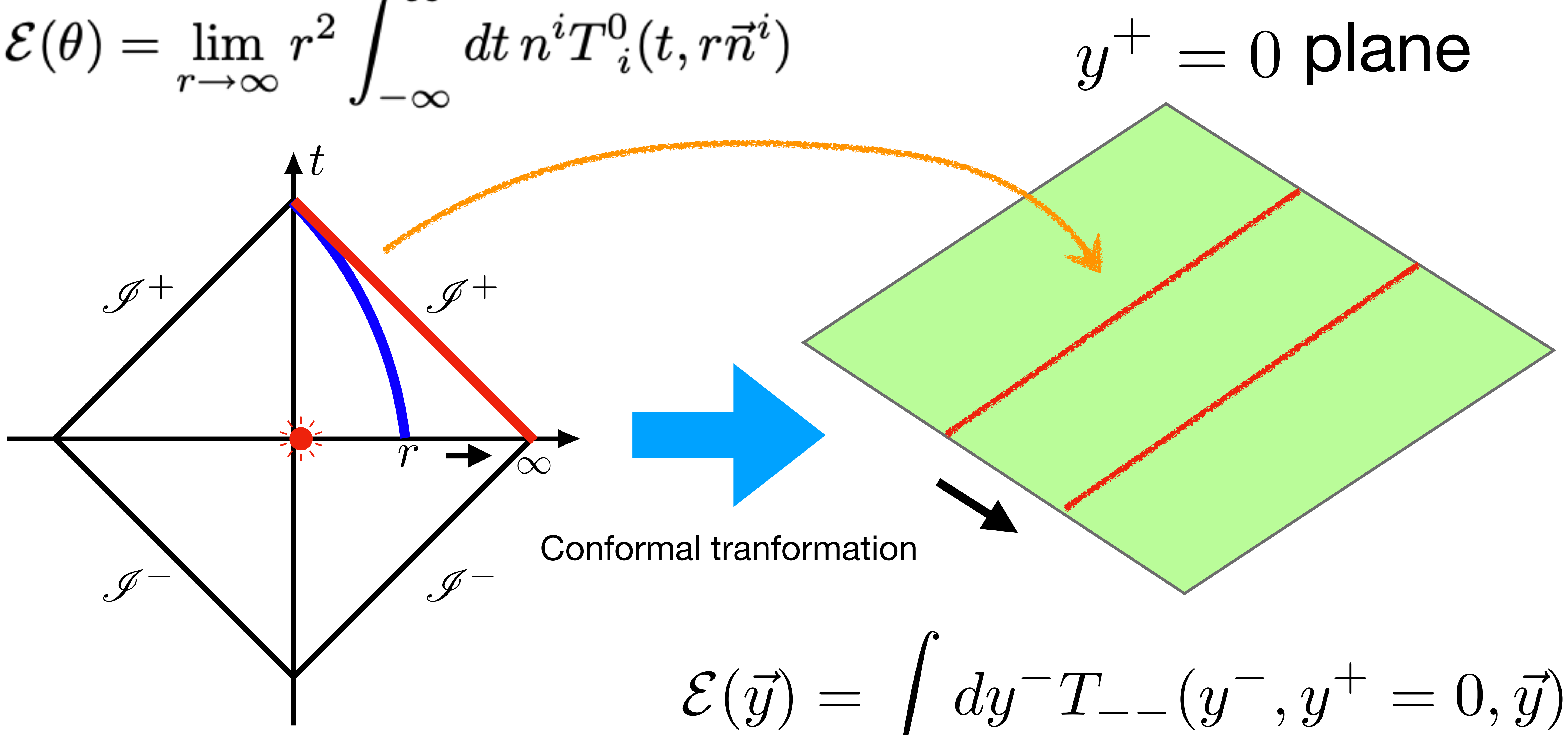
   One-loop, Dixon, M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, 2018

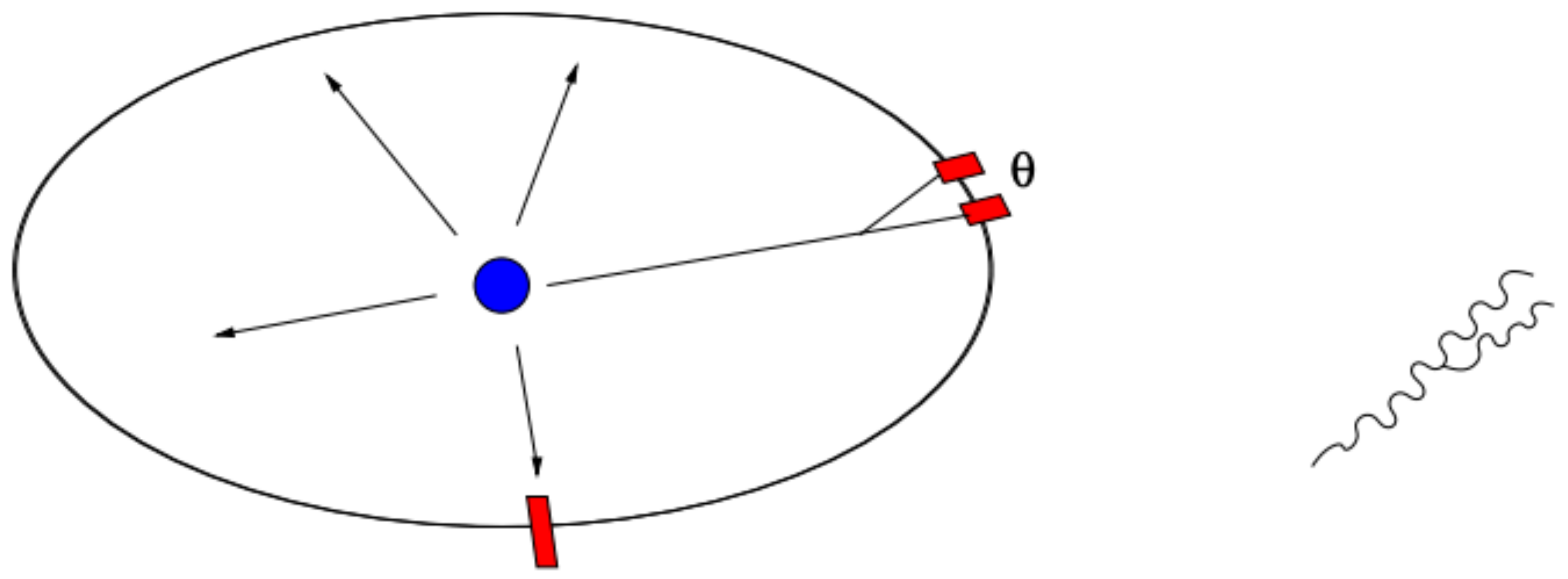
alphabet: $\left\{\zeta, 1-\zeta, \frac{1-\sqrt{\zeta}}{1+\sqrt{\zeta}}\right\}$ $\qquad \zeta = \frac{1}{2}(1-\cos\theta)$

Lightray OPE

Hofman, Maldacena, 2008

$$\mathcal{E}(\theta) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} dt\, n^i T^0_{\ i}(t, r\vec{n}^i)$$





$$\mathcal{E}(\vec{y}) = \int dy^- T_{--}(y^-, y^+ = 0, \vec{y})$$

ANEC operator

Collinear limit of EEC

Lightray OPE

$$\lim_{\vec{n}_2\to\vec{n}_1} \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) = \sum_i \theta_{12}^{\gamma_i} \mathbb{O}_i(\vec{n}_1)$$

Scaling phenomena 🤔

Null integrated J=3 operator 🤔

Scaling exponent from twist operator 🤔

Lightray OPE

Hofman, Maldacena 08; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 19

ensure finite, non-vanishing light transform

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} r^{\Delta-J} \int_0^\infty dt\, O^{\mu_1\ldots\mu_J}(t, r\vec{n})\bar{n}_{\mu_1}\ldots\bar{n}_{\mu_J}$$

Light-transform of $O_{(\Delta,J)}$

dimension $\widetilde{\Delta}$	$J-\Delta-1$	$+$	Δ	$= J-1$
collinear spin $\widetilde{J}$	$-\Delta+J+1$	$+$	$-J$	$= 1-\Delta$

for energy flow operator

$\Delta=4\,, J=2$

$$\lim_{\vec{n}_2\to\vec{n}_1} \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) = \sum_i \theta_{12}^{\gamma_i}\mathbb{O}_i(\vec{n}_1)$$

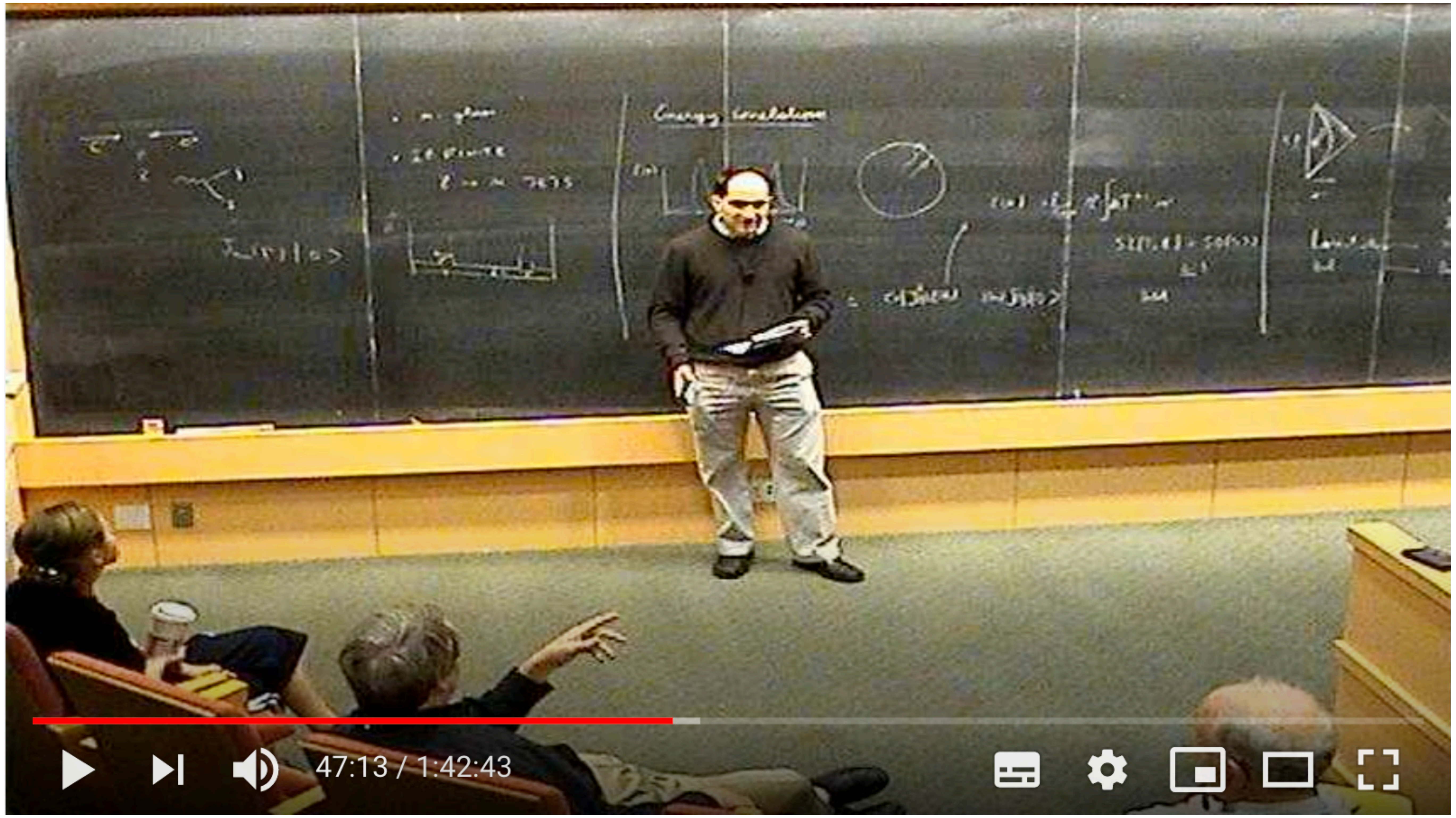
$\mathbb{O} = \mathbf{L}[O]$ $\quad \tau = \Delta - J$

celestial dimension $\widetilde{\Delta}$	$(2-1)+(2-1)$	$=$	$0 + (3-1)$	
collinear spin $\widetilde{J}$	$(1-4)+(1-4)$	$=$	$\gamma_i + 1-\Delta_i$	$\Rightarrow \gamma_i = \tau_i - 4$

Only J=3 local operator appear in the OPE

$$\mathcal{E}(\vec{n}_1)\,\mathcal{E}(\vec{n}_2) \sim \sum_i c_i\,\theta^{\tau_i-4}\mathbb{O}_i(\vec{n}_2)$$

Small angle expansion reduce to twist expansion of local operator

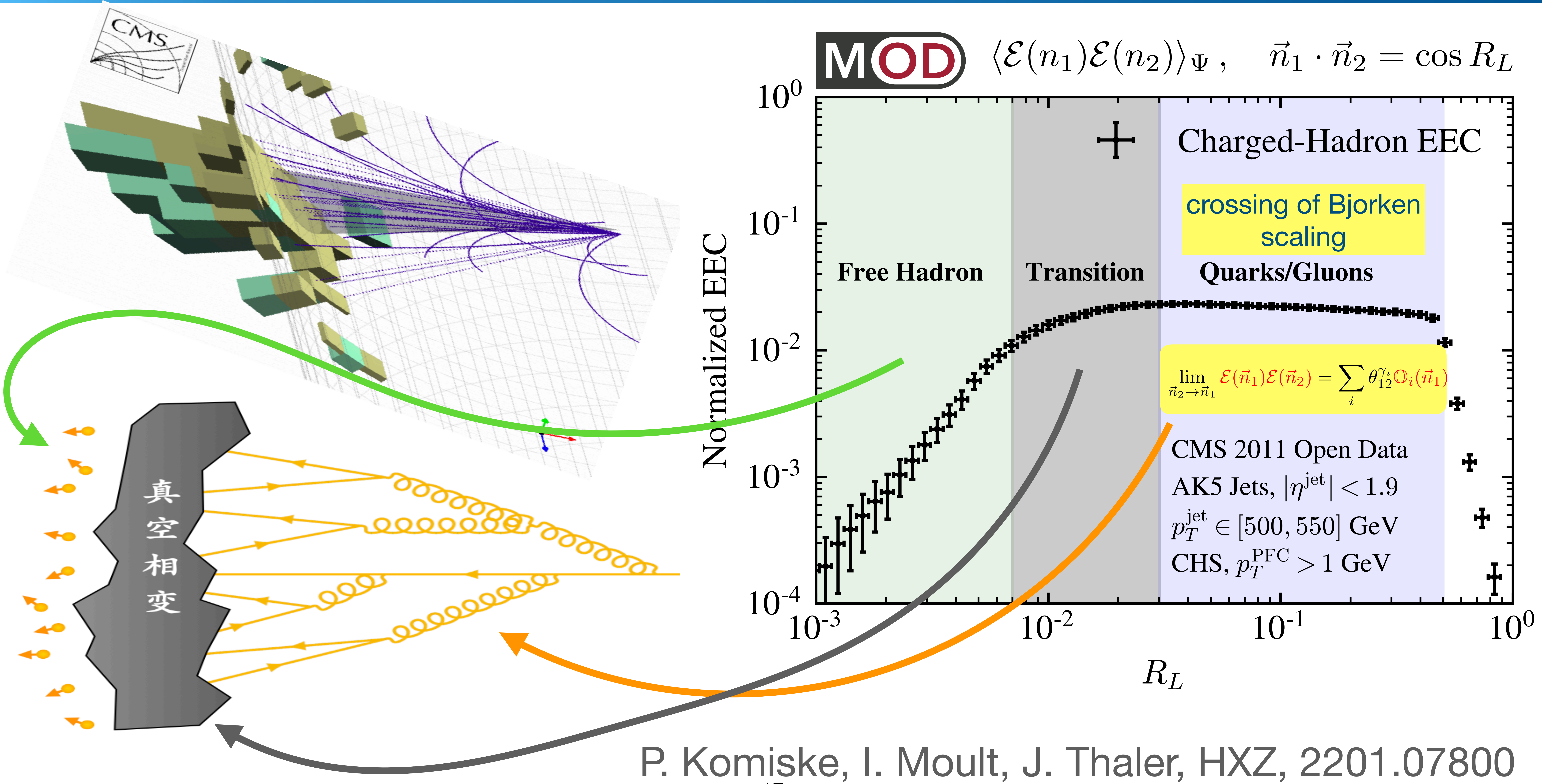


Polchinski: There is a lot of QCD data, can you see this (scaling behavior) there?

Maldacena: People do not do this. I haven't figured out why they don't. I think they just haven't thought about this. I was talking to people who did this calculation of two-point function at LEP, computing alpha_s and so on, and they focused mostly on the large angles. But they didn't study the small angles. And I asked him whether they had a good reason for not studying the small angles and they said well we didn't know the resummation formula, didn't study it.

KITP workshop, 2009

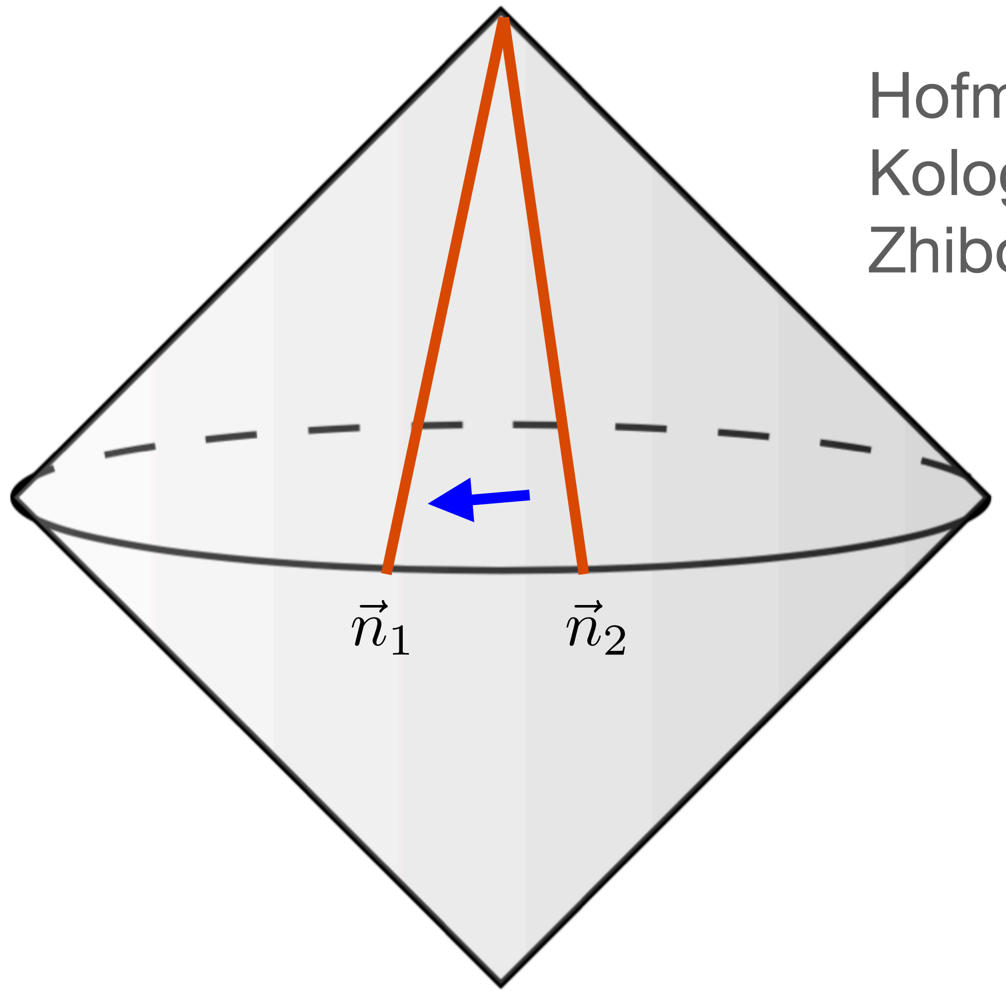
Seeing hadronization phase transition from EEC



P. Komiske, I. Moult, J. Thaler, HXZ, 2201.07800

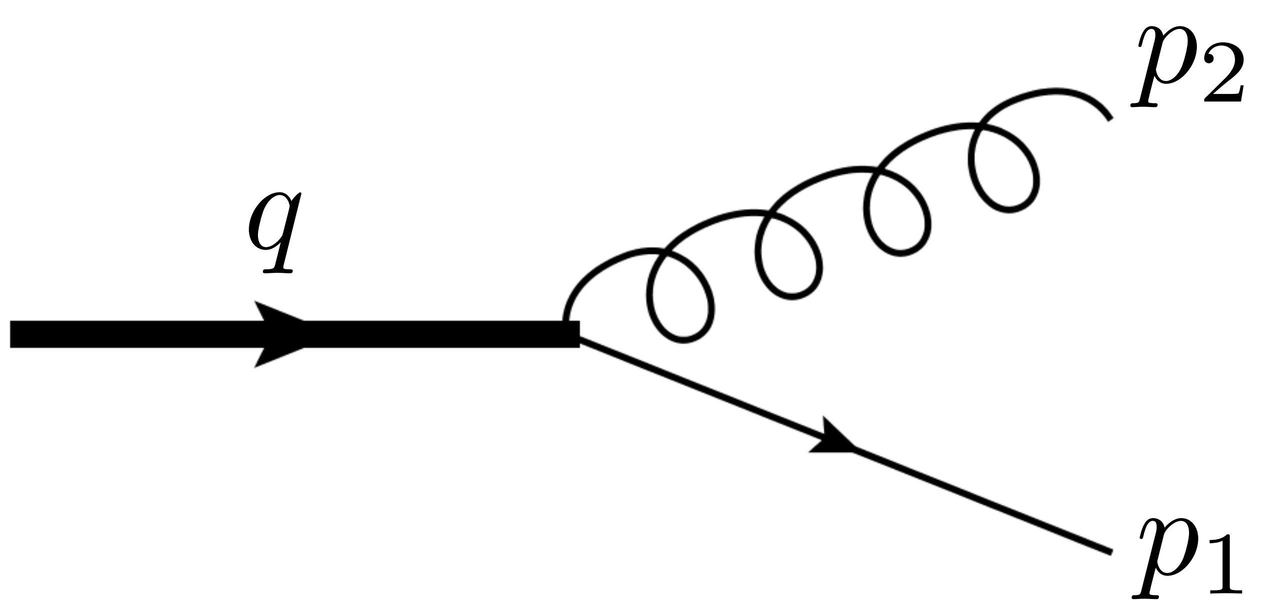
A puzzle from different viewpoints

Coordinate space OPE



Hofman, Maldacena, 2008
Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019

Momentum space splitting



$q^2 > 0$

Konishi, Ukawa, Veneziano, 1979
Korchemsky, 2019
Dixon, Moult, HXZ, 2019

DGLAP spacelike kernel $P_S(z)$

$$\gamma_S(J) = \int_0^1 dz\, z^{J-1} P_S(z)$$

$$\theta^{\gamma_S(J)}$$

DGLAP timelike kernel $P_T(z)$

$$\gamma_T(J) = \int_0^1 dz\, z^{J-1} P_T(z)$$

$$\theta^{\gamma_T(J)}$$

How to reconcile?

Resolution: generalized Gribov-Lipatov Reciprocity

Time-like factorization formula   Dixon, Moult, HXZ, 2019

$$\mu^2 \frac{d\Sigma(\zeta,\mu)}{d\mu^2} = \int_0^1 dy\, y^2 \Sigma(\zeta y^2, \mu) P_T(y,\mu) \quad \Rightarrow \quad \theta^{\gamma_T(J+\gamma_S(J))} = \theta^{\gamma_S(J)}$$

$$\gamma_S(J) = \gamma_T(J + \gamma_S(J))$$

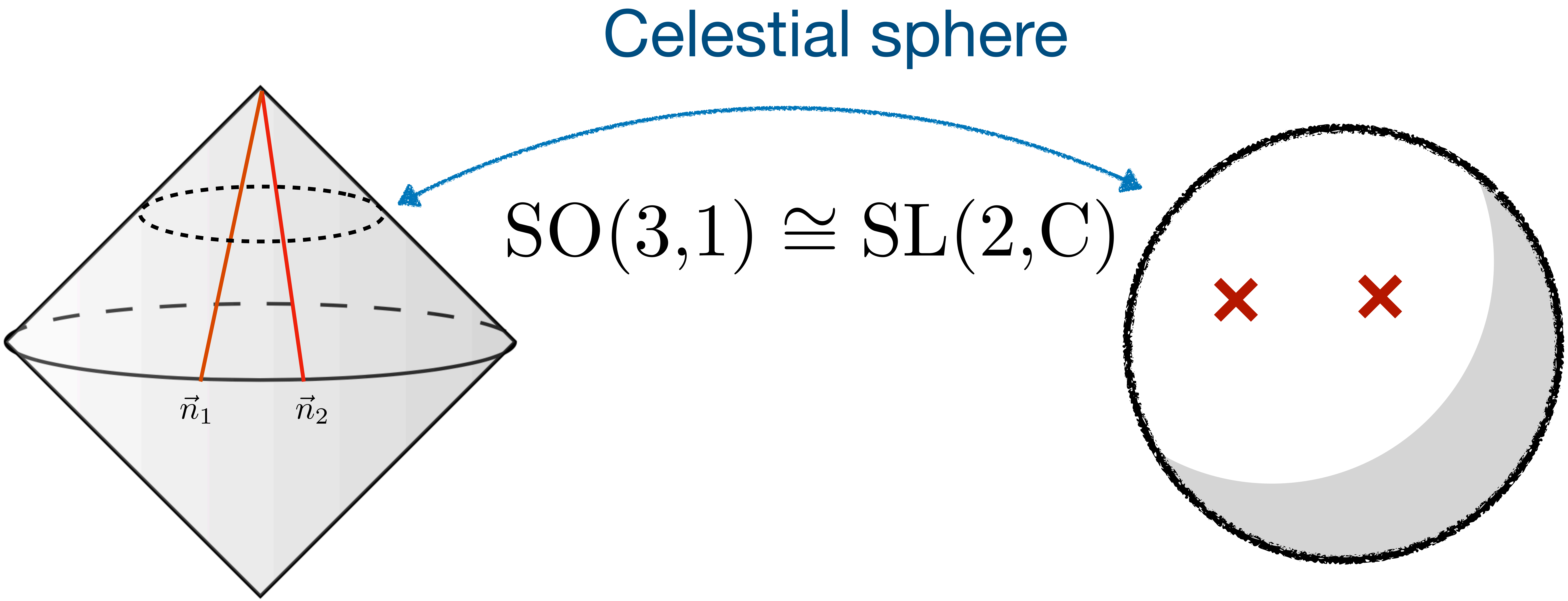
Basso, Korchemsky, 2005

Analyticity in spin at work! $\gamma_S(J - \gamma_T(J)) = \gamma_T(J)$

Note: This relation also holds in QCD, at least to three loops

H. Chen, T.Z. Yang, HXZ, Y.J. Zhu, 2020

EEC as a defect conformal field theory

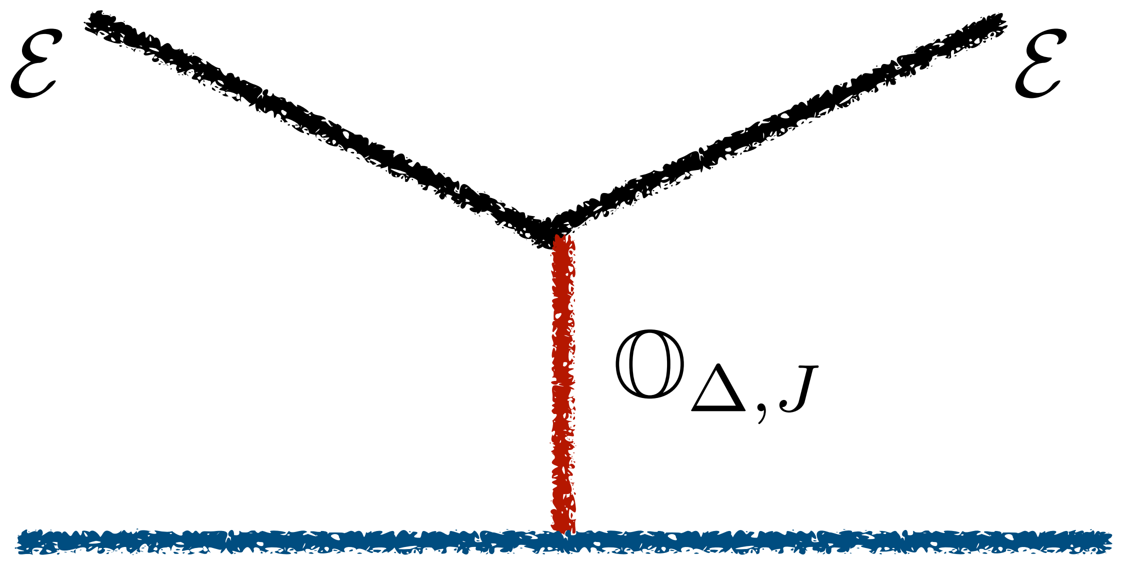


Lightray operator as a local operator living on a fictitious
2D Eucildean defect CFT

Application of symmetry: conformal block expansion

Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019

Lightray OPE



$$\langle\Omega|O(x)\mathcal{E}(n_1)\mathcal{E}(n_2)O^\dagger|\Omega\rangle = \sum_\Delta \mathcal{C}_{\delta,0}(z_1,z_2,\partial_{z_2})\langle\Omega|O(x)\mathbb{O}_{\Delta,J}(z_2)O^\dagger|\Omega\rangle$$

Eigenvector of Casimir operator

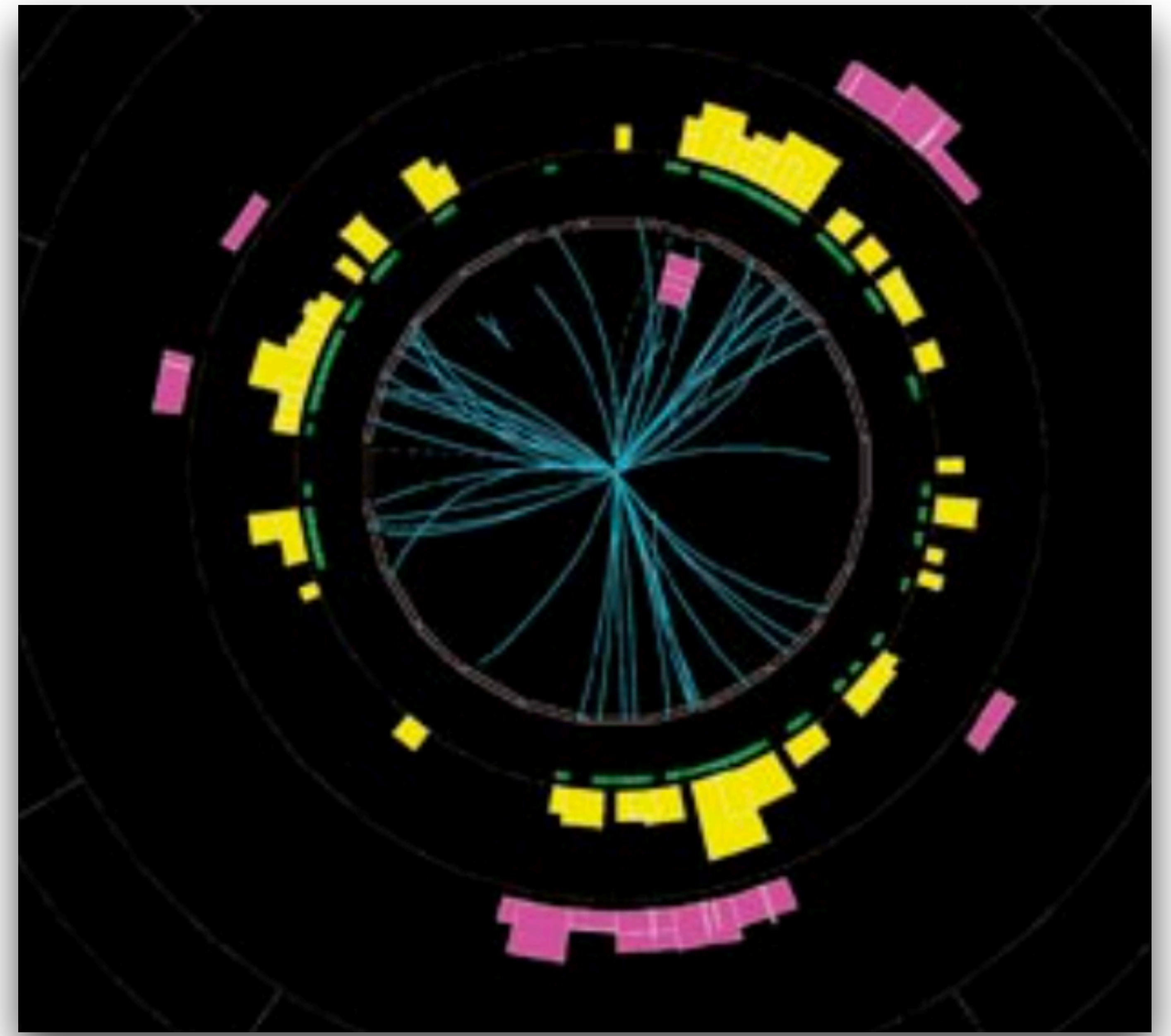
$$C_{\delta,0}(z_1,z_2,\partial_{z_2}) = -\frac{1}{2}\left(p_\mu\frac{\partial}{\partial p^\nu} - p_\nu\frac{\partial}{\partial p^\mu}\right)\left(p^\mu\frac{\partial}{\partial p_\nu} - p^\nu\frac{\partial}{\partial p_\mu}\right)$$

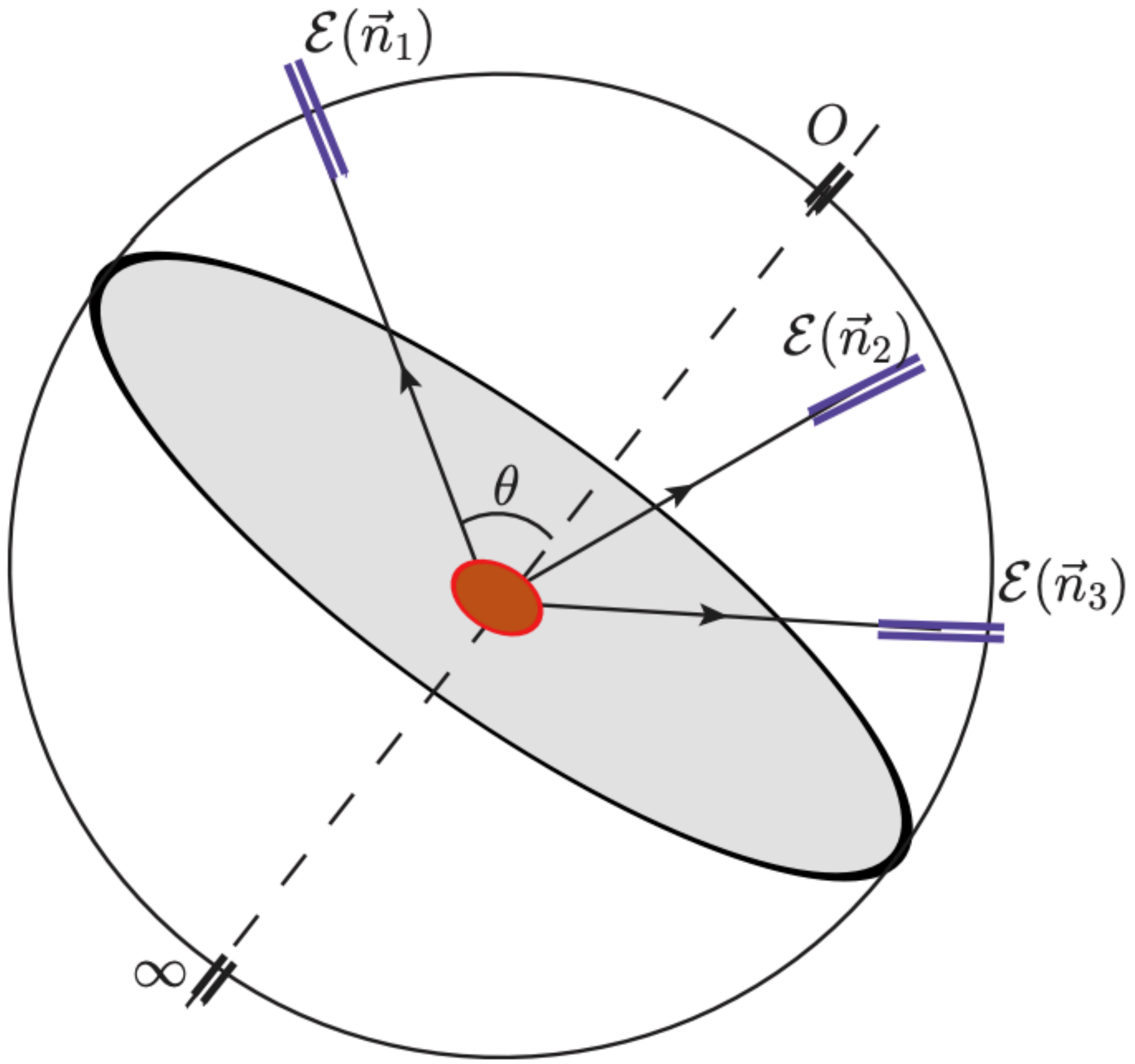
Celestial Block

$$f_\Delta^{\Delta_1,\Delta_2}(\zeta) = \zeta^{\frac{\Delta-\Delta_1-\Delta_2+1}{2}}\,{}_2F_1\left(\frac{\Delta-1+\Delta_1-\Delta_2}{2},\frac{\Delta-1-\Delta_1+\Delta_2}{2},\Delta+1-\frac{d}{2},\zeta\right)$$

Identical to conformal block for 2D CFT with co-dimension 1 boundary

EEEC: Three-point energy correlator





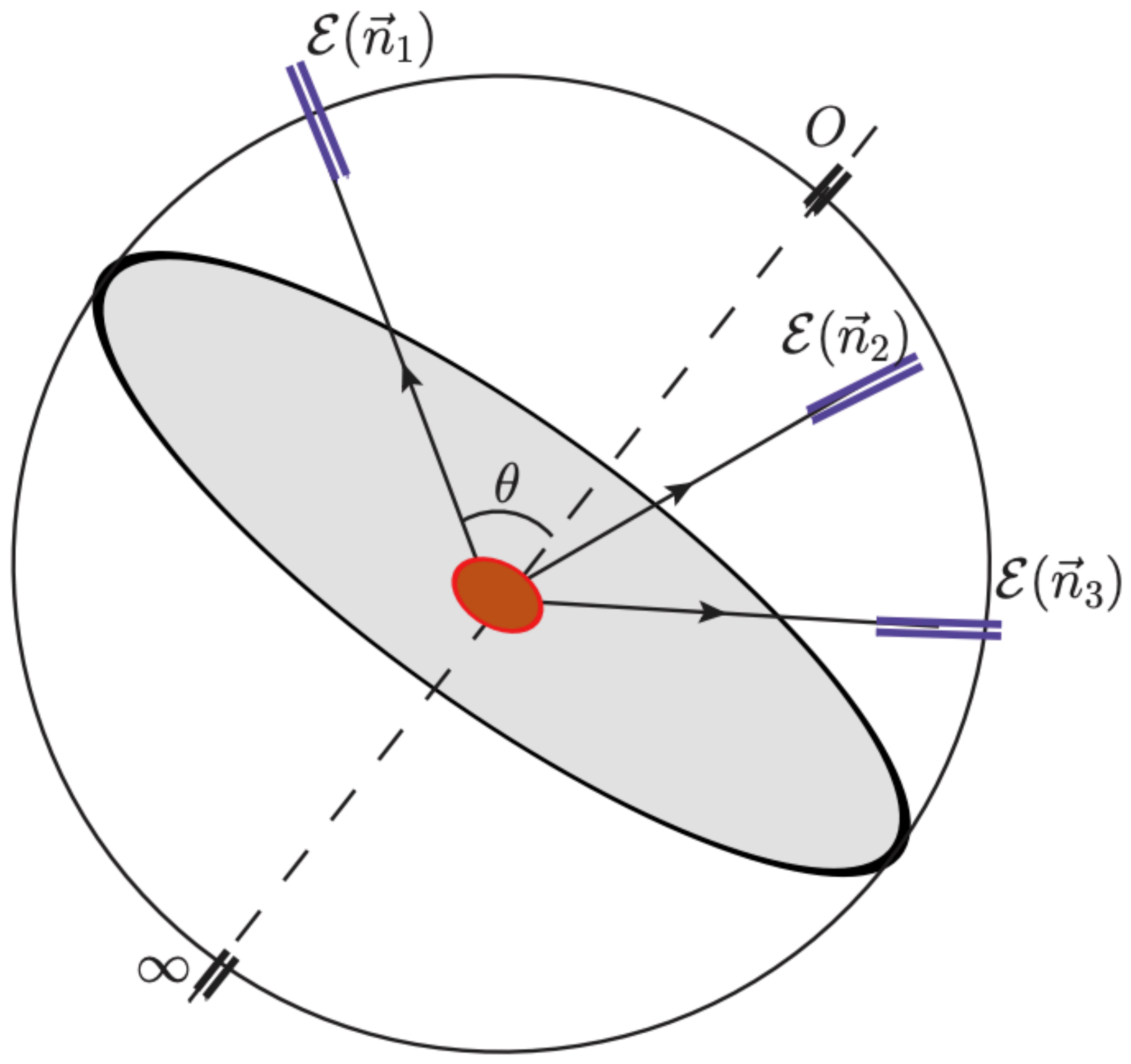
$$
\begin{aligned}
g_1 &= \mathrm{Li}_2(-v_2) \\
g_2 &= \mathrm{Li}_2(1+w_3) + \mathrm{Li}_2(1+\bar{w}_3) + 2\,\mathrm{Li}_2(-v_3) \\
&\quad - \mathrm{Li}_2(1+w_1) - \mathrm{Li}_2(1+\bar{w}_1) - 2\,\mathrm{Li}_2(-v_1) \\
g_3 &= \mathrm{Li}_2(-z_2) - \mathrm{Li}_2(-\bar{z}_2) + \frac{1}{2}\ln|z_2|^2 \ln\frac{1+z_2}{1+\bar{z}_2} \\
g_4 &= \mathrm{Li}_2(1+w_1) - \mathrm{Li}_2(1+\bar{w}_1) + \mathrm{Li}_2(1+w_2) \\
&\quad - \mathrm{Li}_2(1+\bar{w}_2) + \mathrm{Li}_2(1+w_3) - \mathrm{Li}_2(1+\bar{w}_3) \\
g_5 &= \pi^2 \\
g_6 &= \ln^2\frac{\bar{w}_1}{w_1} \\
g_7 &= \ln\frac{\bar{w}_1}{w_1}\ln|z_2|^2 \\
g_8 &= \ln(1+v_3)\ln|z_1|^2 - \ln(1+v_1)\ln|z_3|^2
\end{aligned}
$$

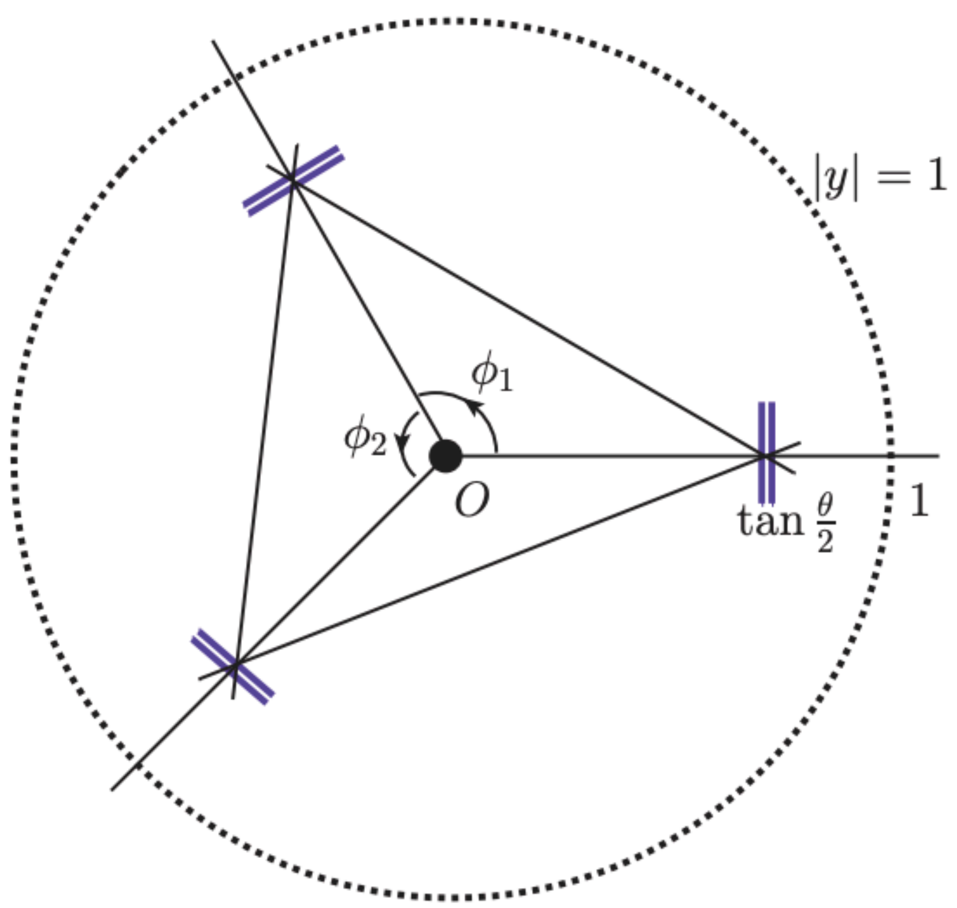
K. Yan, X.Y. Zhang, 2203.04349;
T.Z. Yang, X.Y. Zhang, 2208.01051

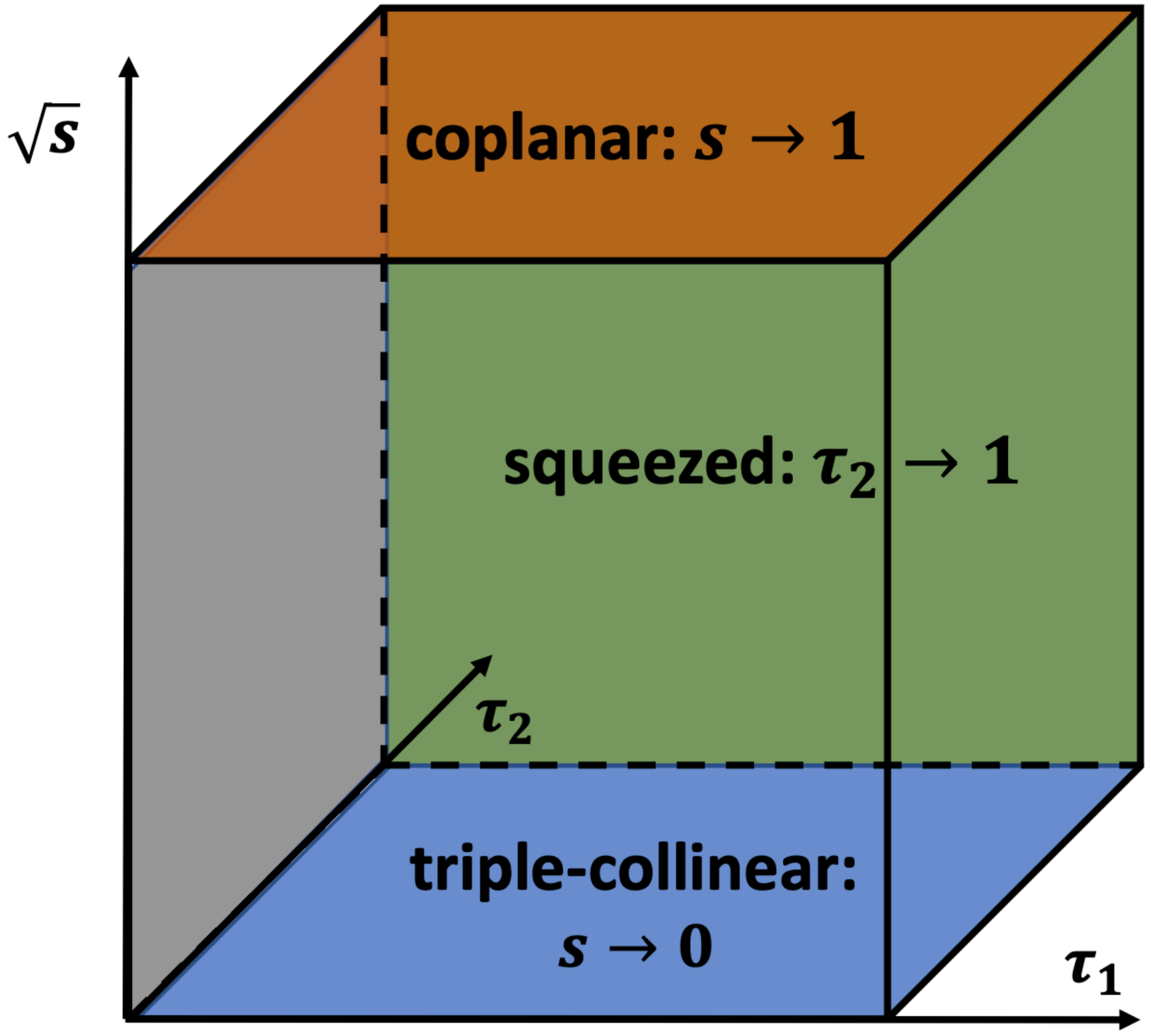
Appearance of a A_3 cluster algebra

$$
x_1,\, x_2,\, x_3 = \frac{1+x_2}{x_1},\; x_4 = \frac{1+x_3}{x_2} = \frac{1+x_1+x_2}{x_1 x_2},
$$
$$
x_5 = \frac{1+x_4}{x_3} = \frac{1+x_1}{x_2},\; x_6 = \frac{1+x_5}{x_4} = x_1,\; x_7 = \frac{1+x_6}{x_5} = x_2,\; \ldots
$$

Boundary of EEEC



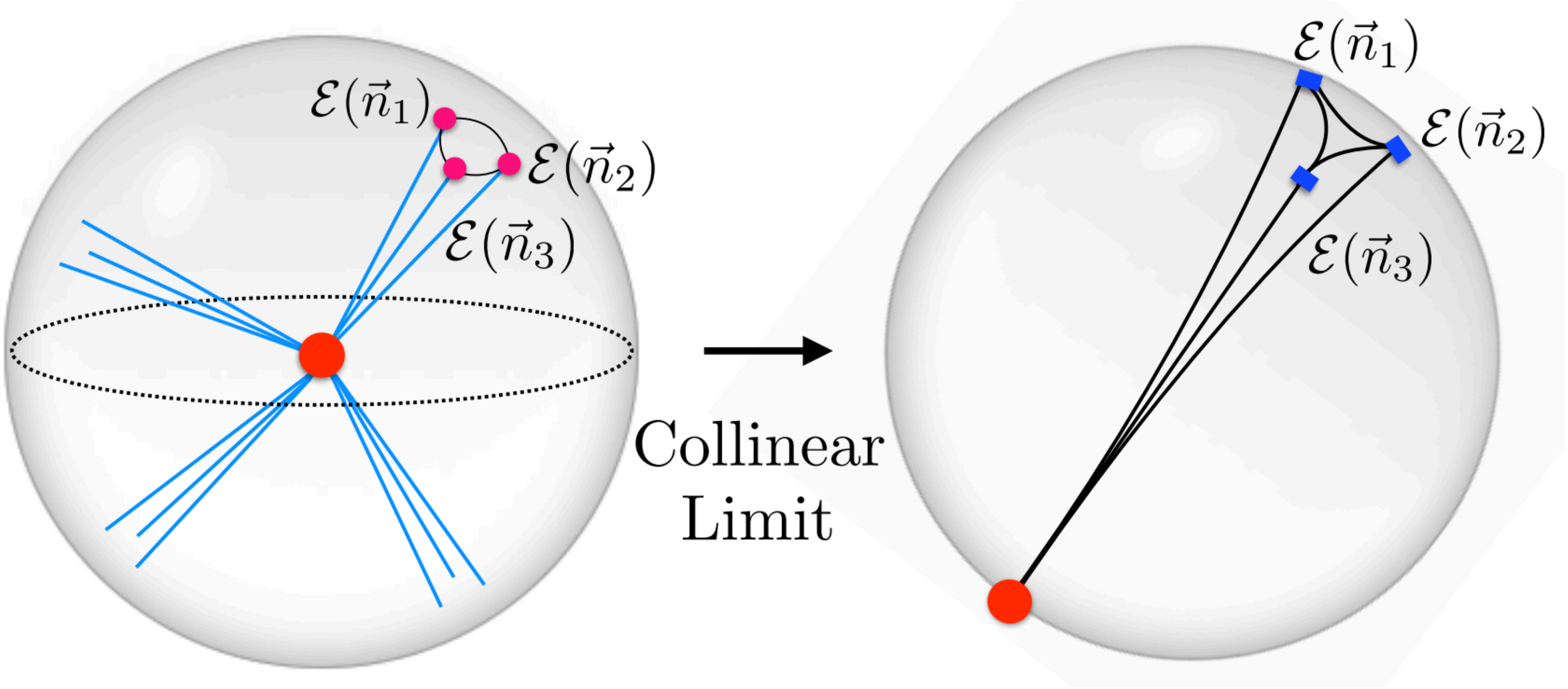




$$s = \tan^2\frac{\theta}{2}, \quad \tau_1 = e^{i\phi_1}, \quad \tau_2 = e^{i\phi_2}$$

EEEC cubic

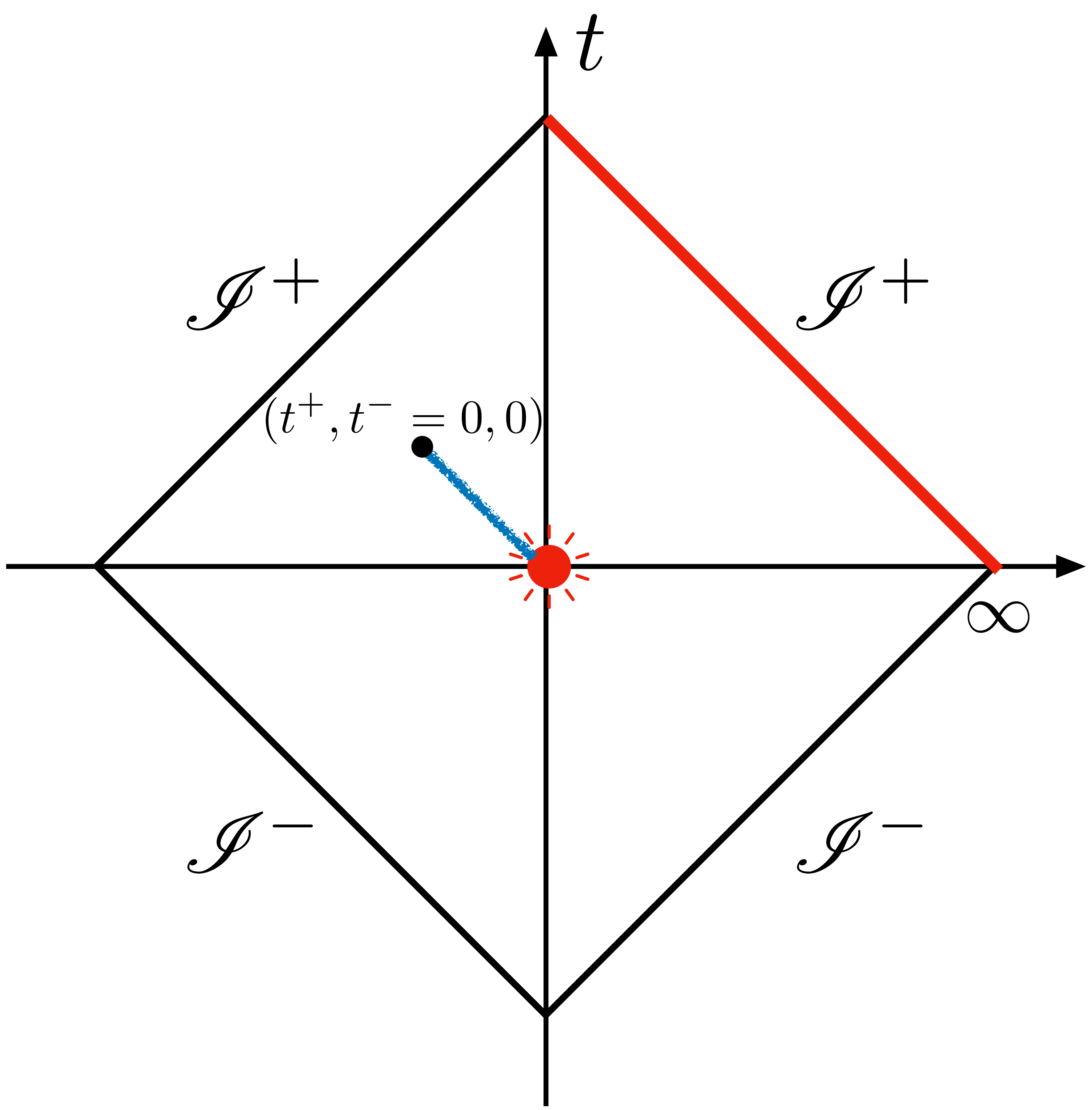
Triple collinear limit



The defect is boosted to a point at infinity on the celestial sphere

Boosted EEEC

$$\langle \mathcal{E}_1 \mathcal{E}_2 \mathcal{E}_3 \rangle_{\text{boost}} = \int dt\, e^{itP^+} \langle \Omega | \bar{\psi}((t^+, t^- = 0, 0)) \gamma^+ \mathcal{E}_1 \mathcal{E}_2 \mathcal{E}_3 U[(t^+, 0, 0), 0] \psi(0) | \Omega \rangle$$

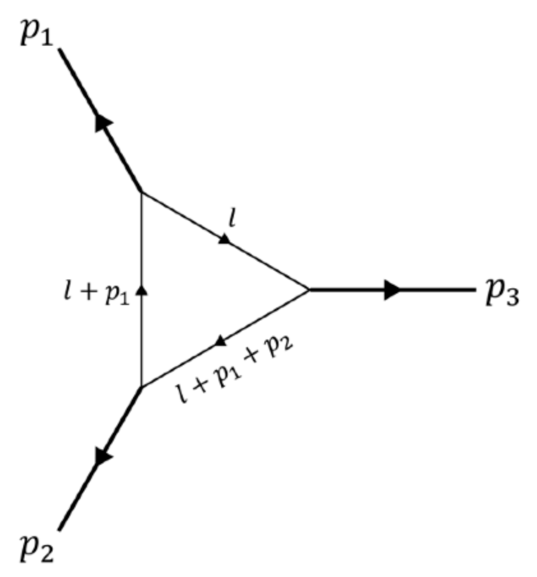


H. Chen, Moult, Sandor, HXZ, 2022

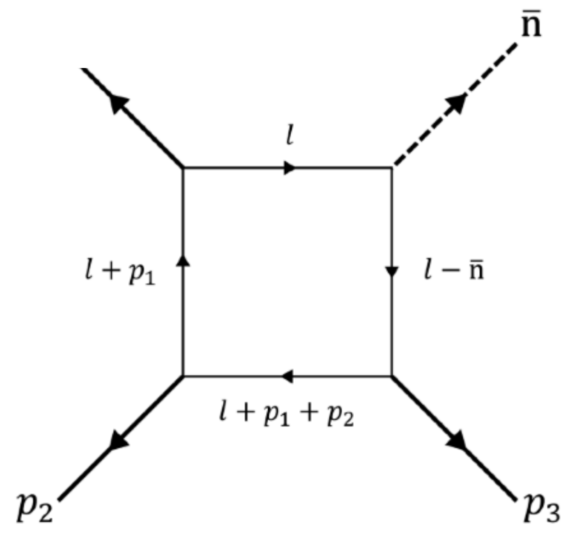
Explicit result in the triple collinear limit

H. Chen, M.X. Luo, Moult, T.Z. Yang, X.Y. Zhang, HXZ, 2019

$$
\begin{aligned}
G_{\mathcal{N}=4}(z) =& \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v) \\
&- (1+u+v)(\partial_u+\partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z) \\
&+ \frac{(u-1)(u+1)}{2uv^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right), \quad (2.12)
\end{aligned}
$$

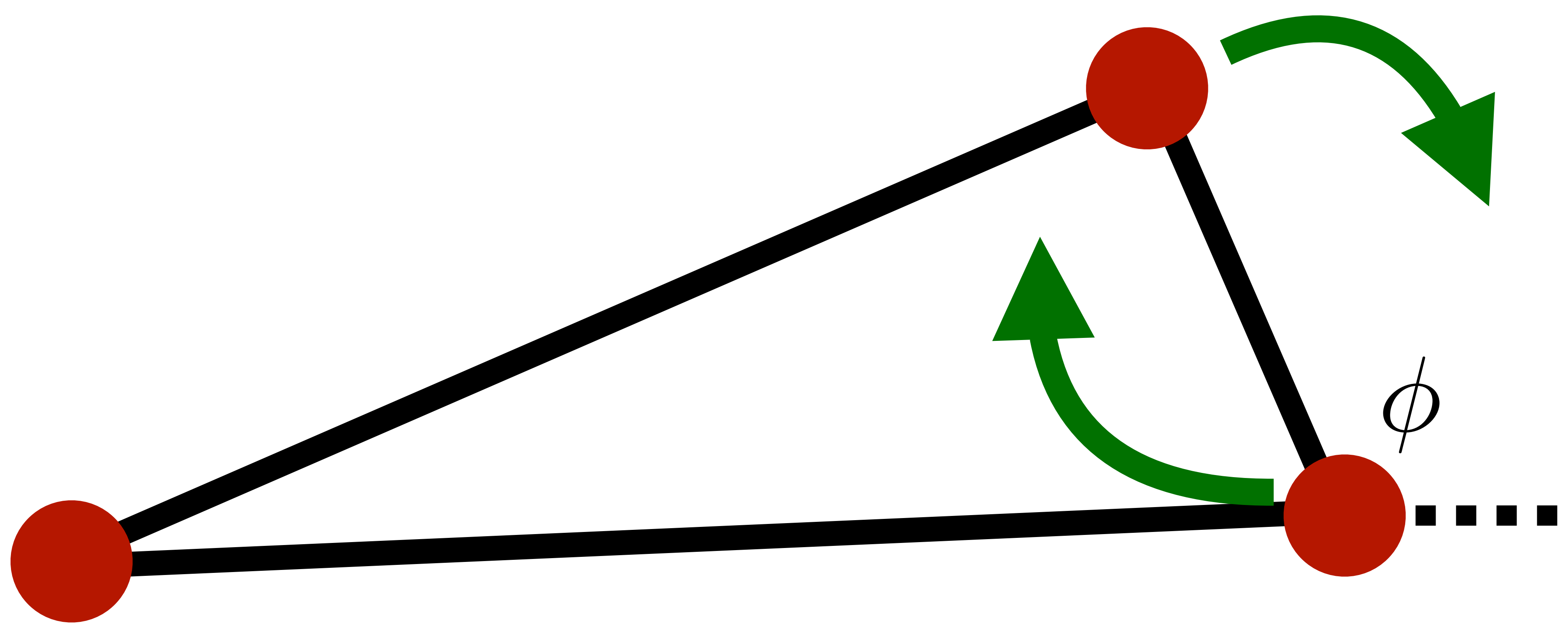


$$
\Phi(z) = \frac{2}{z-\bar{z}}\left(\mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z}) + \frac{1}{2}\left(\log(1-z) - \log(1-\bar{z})\right)\log(z\bar{z})\right)
$$



$$
D_2^+(z) = \mathrm{Li}_2(1-|z|^2) + \frac{1}{2}\log(|1-z|^2)\log(|z|^2)
$$

Where is the cos comes from?



Squeeze limit in QCD

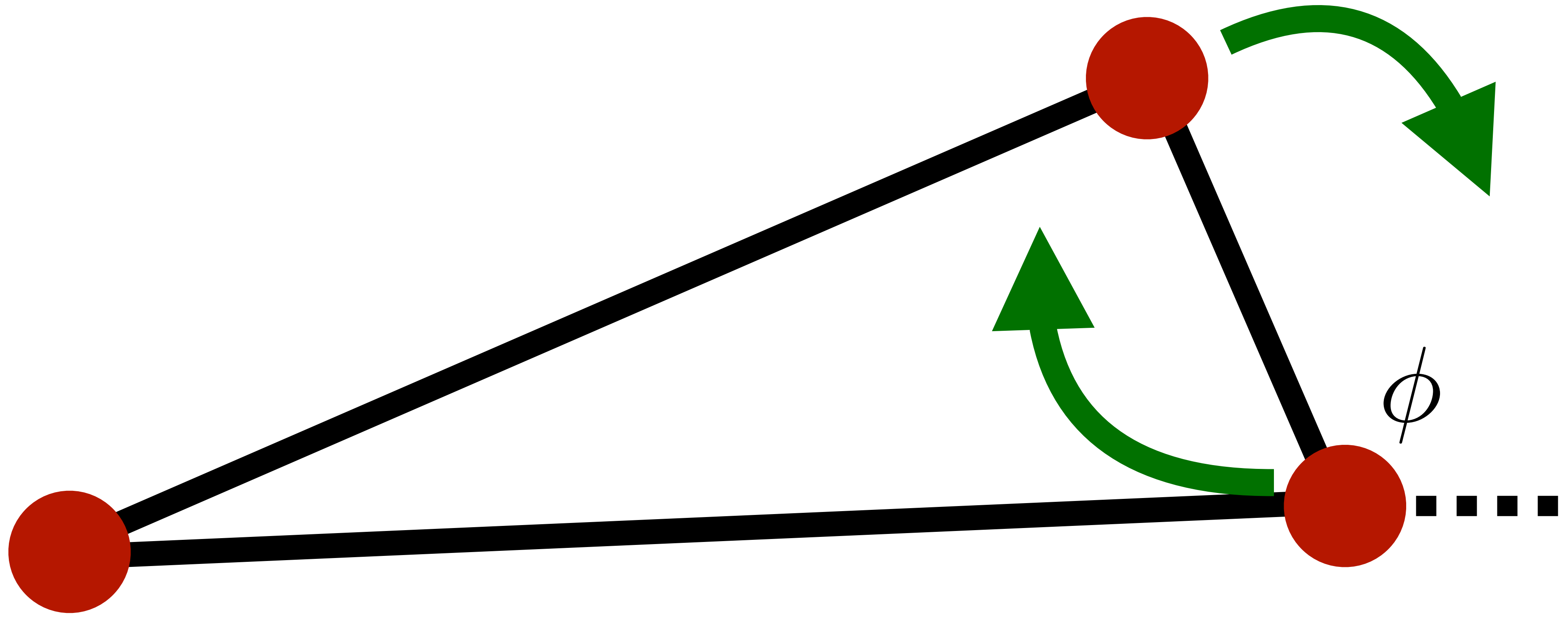
$$\frac{d^3\Sigma_i}{d\theta_L^2 d\theta_S^2 d\phi} \simeq \frac{1}{\pi}\left(\frac{\alpha_s}{4\pi}\right)^2 \frac{\mathrm{Sq}_i^{(0)}(\phi)}{\theta_L^2\theta_S^2} + \cdots$$

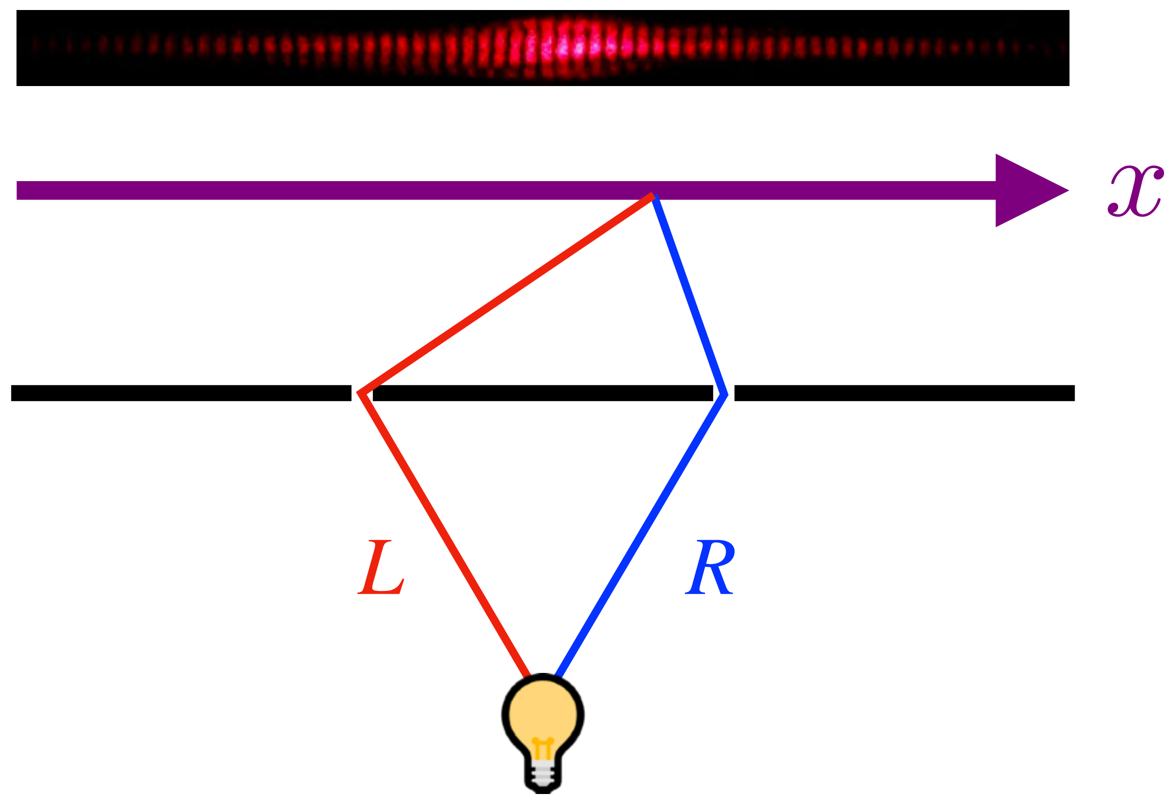
$$\mathrm{Sq}_q^{(0)}(\phi) = C_F n_f T_F \left(\frac{39 - 20\cos(2\phi)}{225}\right) + C_F C_A \left(\frac{273 + 10\cos(2\phi)}{225}\right) + C_F^2 \frac{16}{5}$$

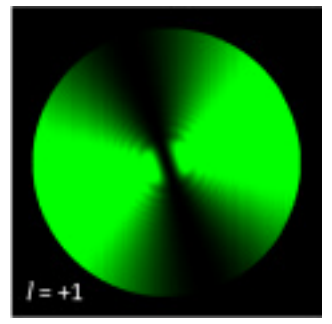
$$\mathrm{Sq}_g^{(0)}(\phi) = C_A n_f T_F \left(\frac{126 - 20\cos(2\phi)}{225}\right) + C_A^2 \left(\frac{882 + 10\cos(2\phi)}{225}\right) + C_F n_f T_F \frac{3}{5}$$

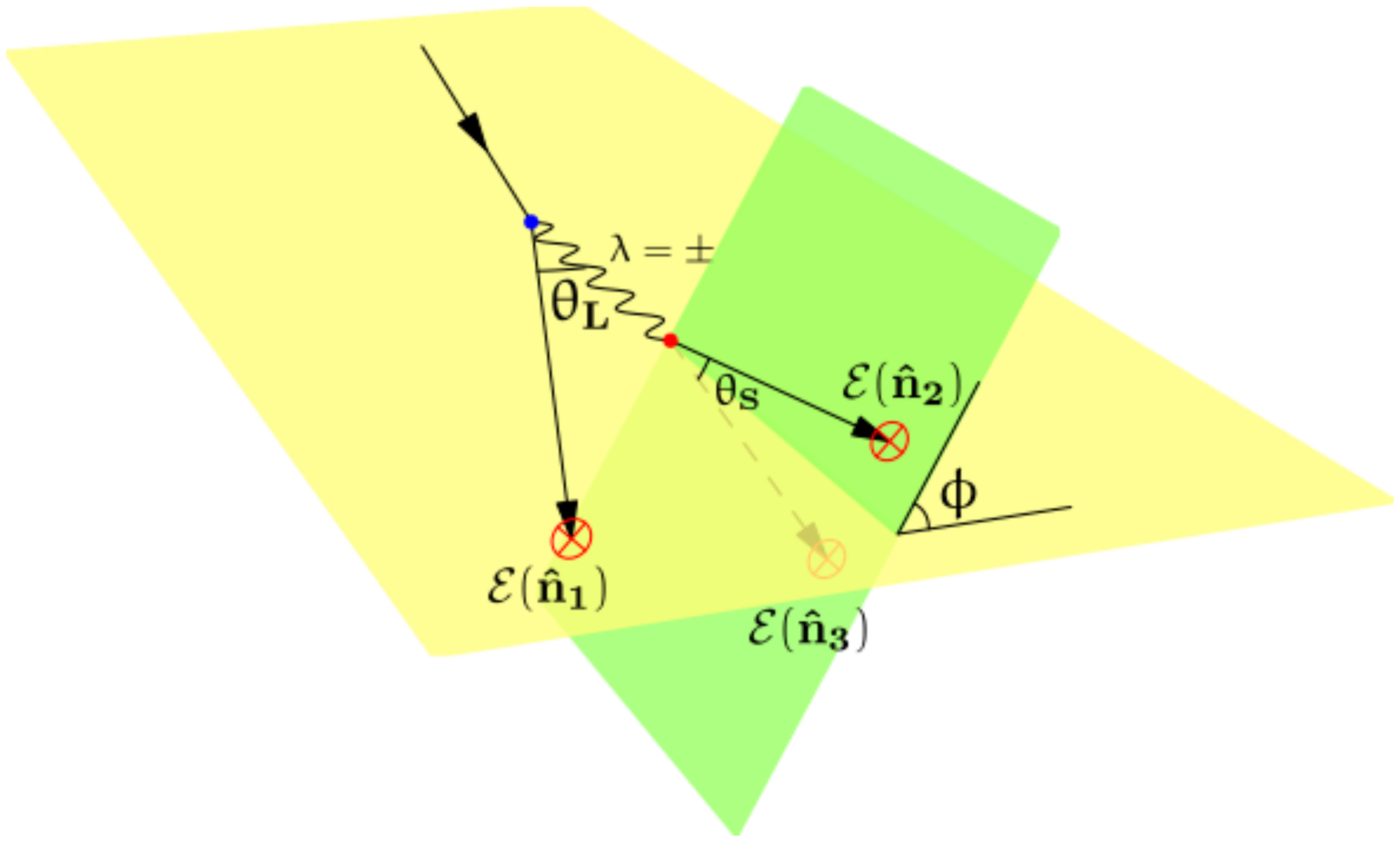
Intriguing cos(2ϕ) modulation

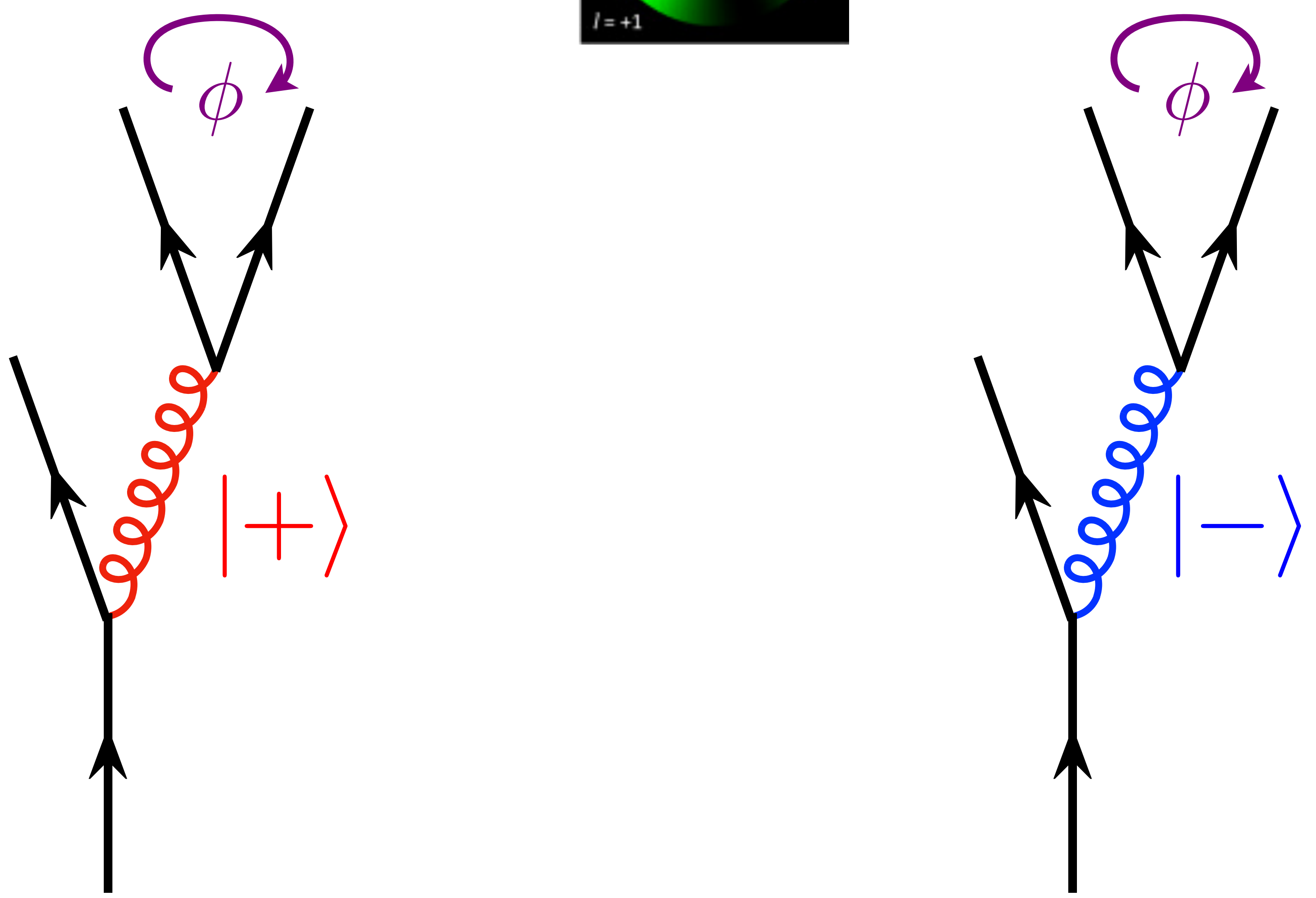
Particle interpretation: spin double slit experiment











Helicity flip interference

Interpretation from operator transverse spin

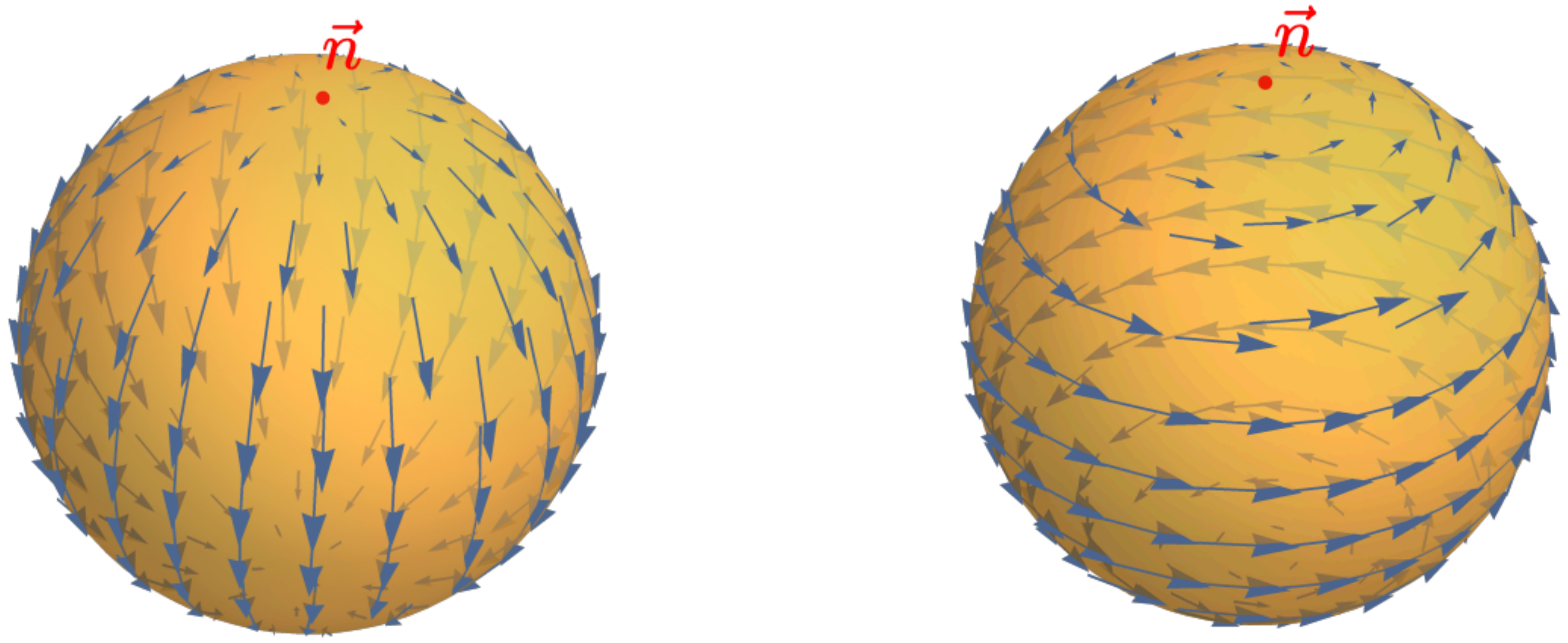
$$\mathcal{E}(\vec{n}_1)\,\mathcal{E}(\vec{n}_2) \sim \sum_i c_i\, \theta^{\tau_i - 4}\mathbb{O}_i(\vec{n}_2)$$

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$$

$$\mathcal{O}_{\tilde{g},\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

helicity $\pm$

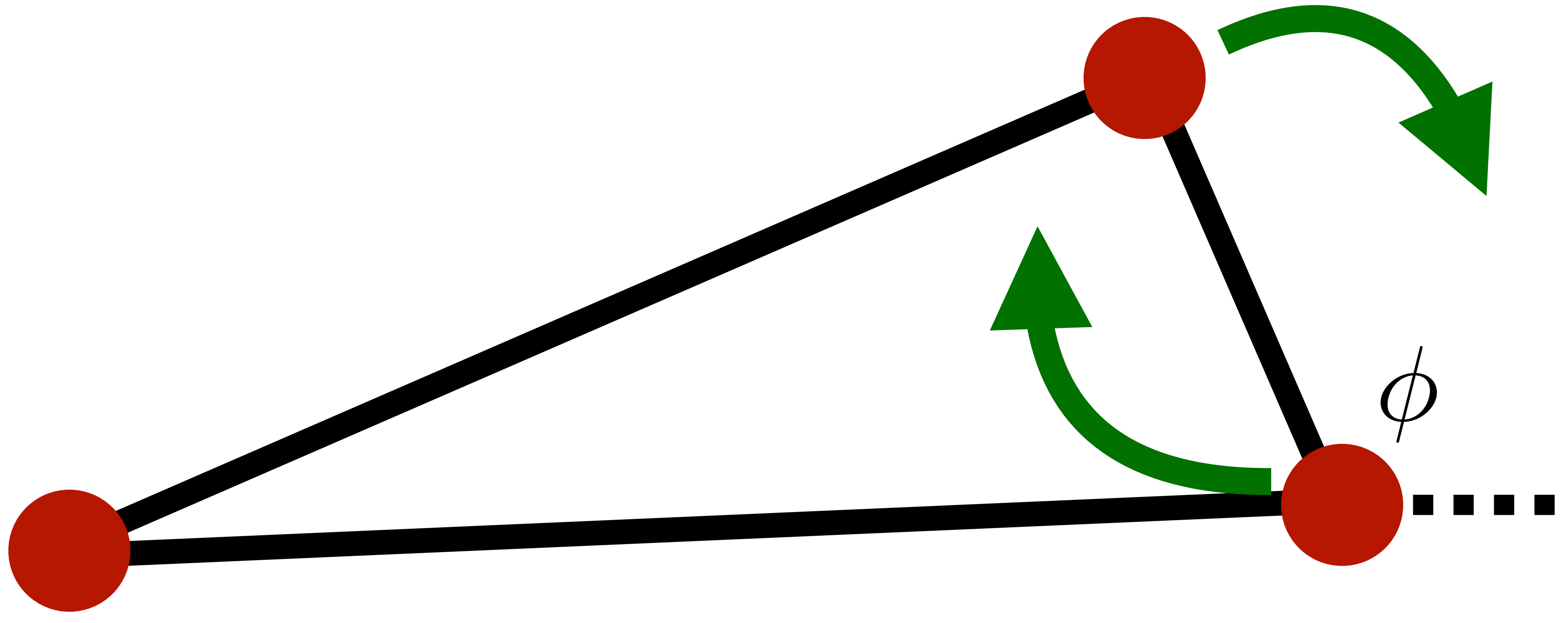


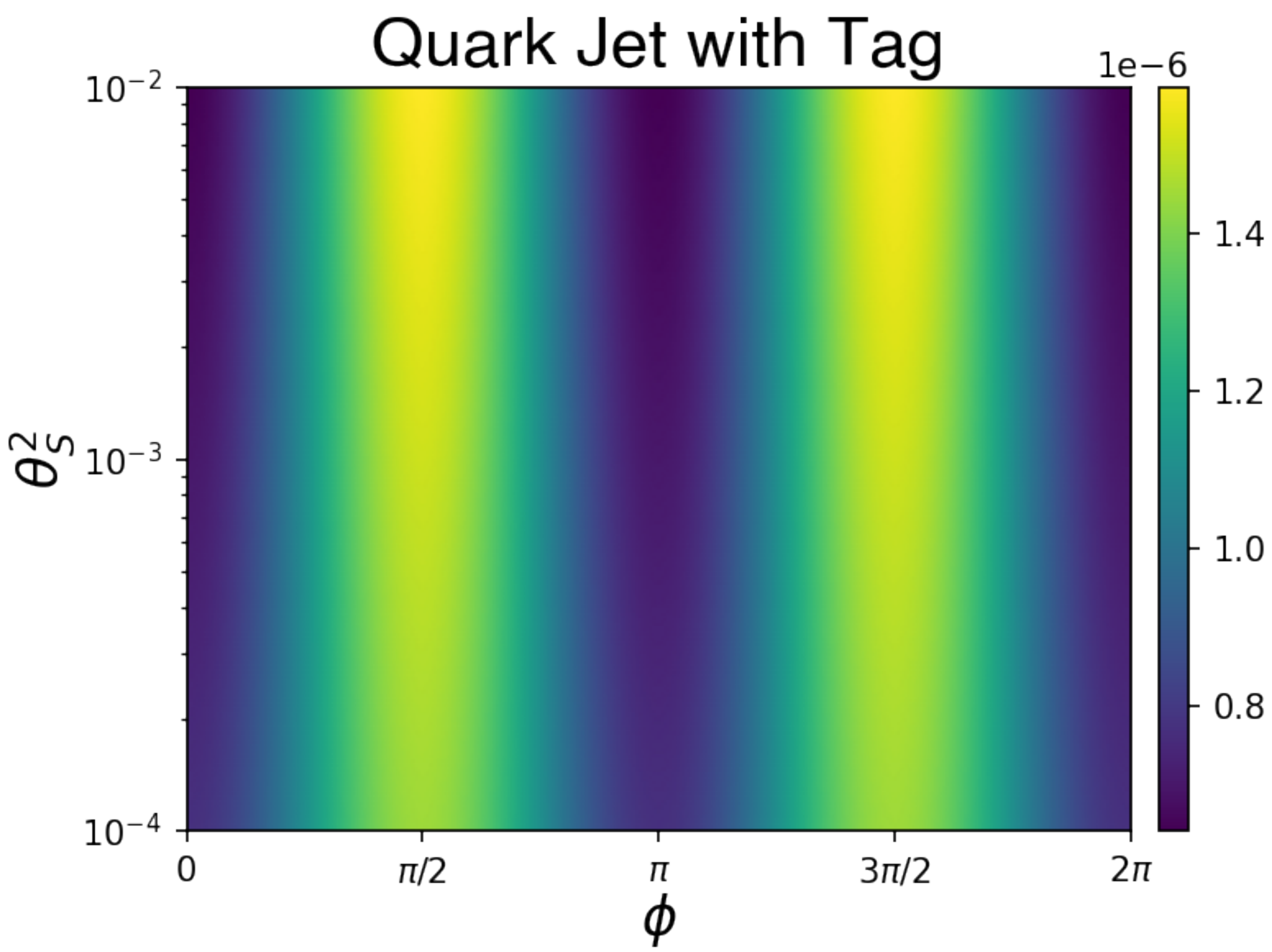
δ = Δ-1 j

$$\frac{d}{d\ln\mu^2}\vec{\mathcal{O}}^{[J]} = -\widehat{\gamma}(J)\cdot\vec{\mathcal{O}}^{[J]}$$

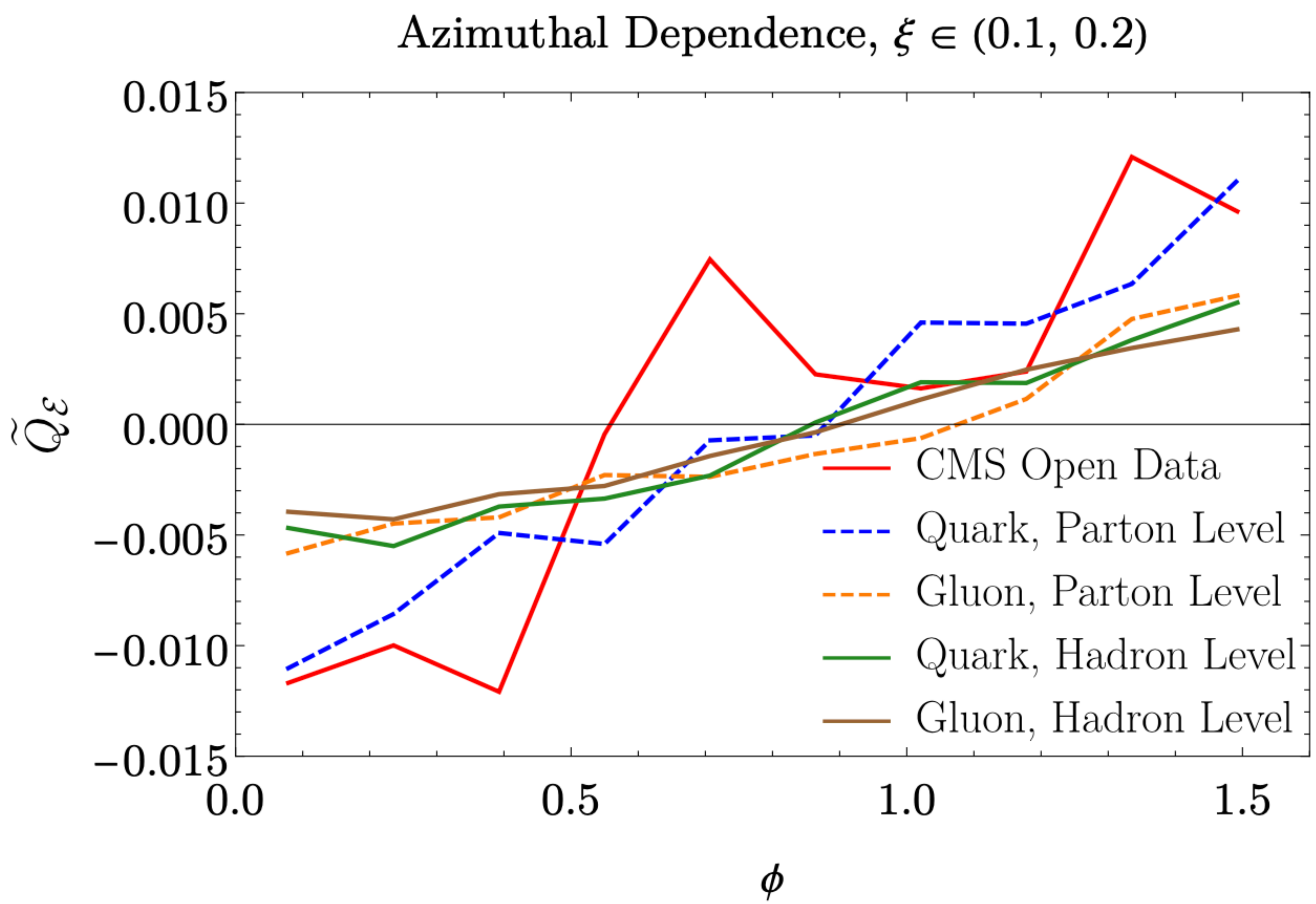
$$\widehat{\gamma}(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 0 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & 0 \\ 0 & 0 & \gamma_{\tilde{g}\tilde{g}}(J)\mathbf{1} \end{pmatrix}$$

In principle observable at the LHC!





Celestial non-gaussianities

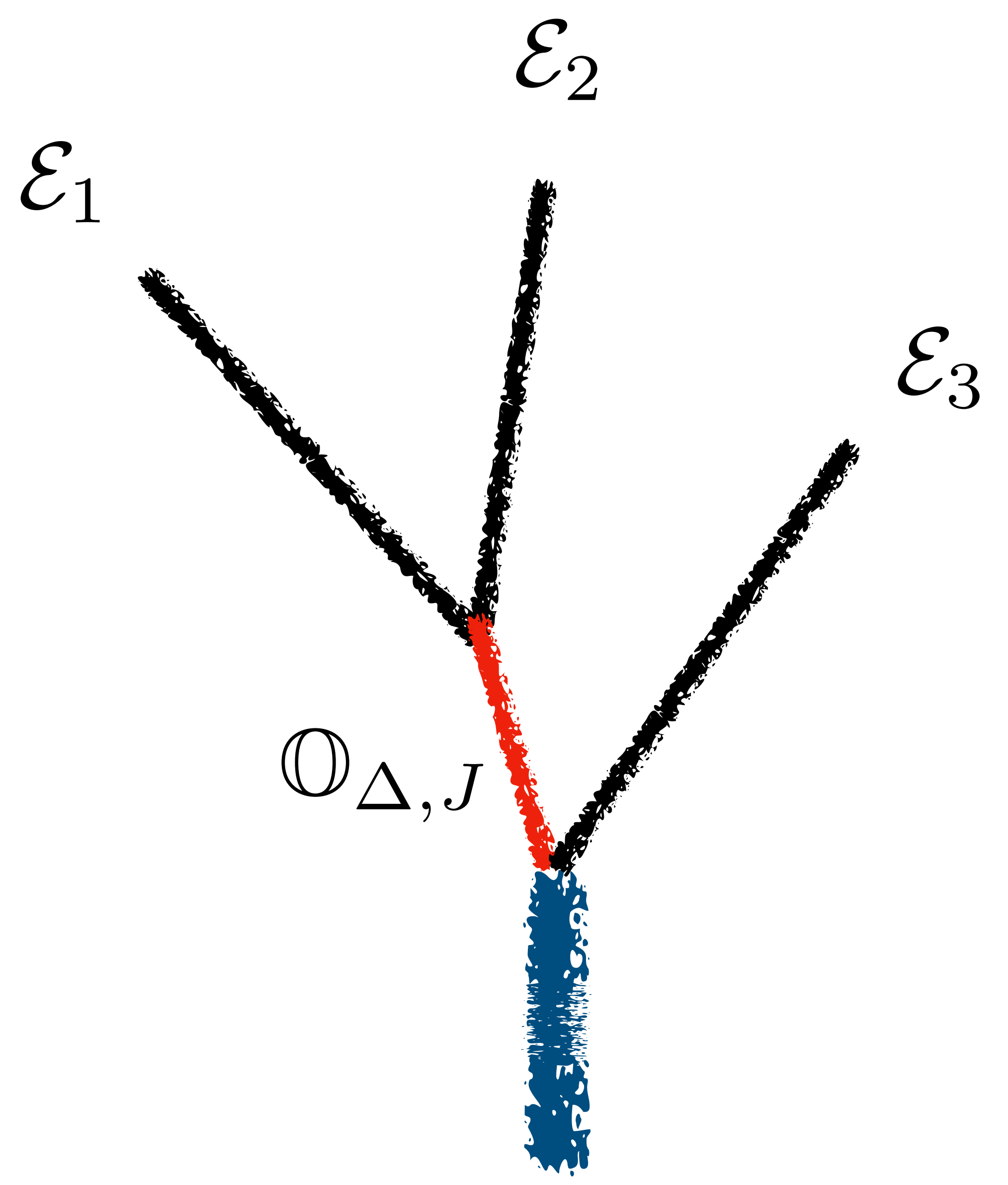


H. Chen, Moult, HXZ, 2020

H. Chen, Moult, Thaler, HXZ, 2022

Conformal block expansion for boosted EEEC

H. Chen, Moult, Sandor, HXZ, 2022
Chang, Simmons-Duffin, 2022



$\int dt\, e^{itP^+}\bar{\psi}((t^+, t^- = 0, 0))U[(t^+, 0, 0), 0]\psi(0)$

$$\langle\Omega|\bar{\psi}((t^+, t^- = 0, 0))\gamma^+\mathcal{E}_1\mathcal{E}_2\mathcal{E}_3 U[(t^+, 0, 0), 0]\psi(0)|\Omega\rangle$$
$$= \sum_{\Delta}\underbrace{\langle\Omega|\bar{\psi}\gamma^+\mathbb{O}_{\Delta,J}\mathcal{E}_3\psi(0)|\Omega\rangle}_{c_{\delta,j}G_{\delta,j}}$$

$$\mathcal{C}_2 = -2u^2(u - v - 1)\partial_u^2 - 4uv(u - v + 1)\partial_u\partial_v - 2v(u(1 + v) - (1 - v)^2)\partial_v^2$$

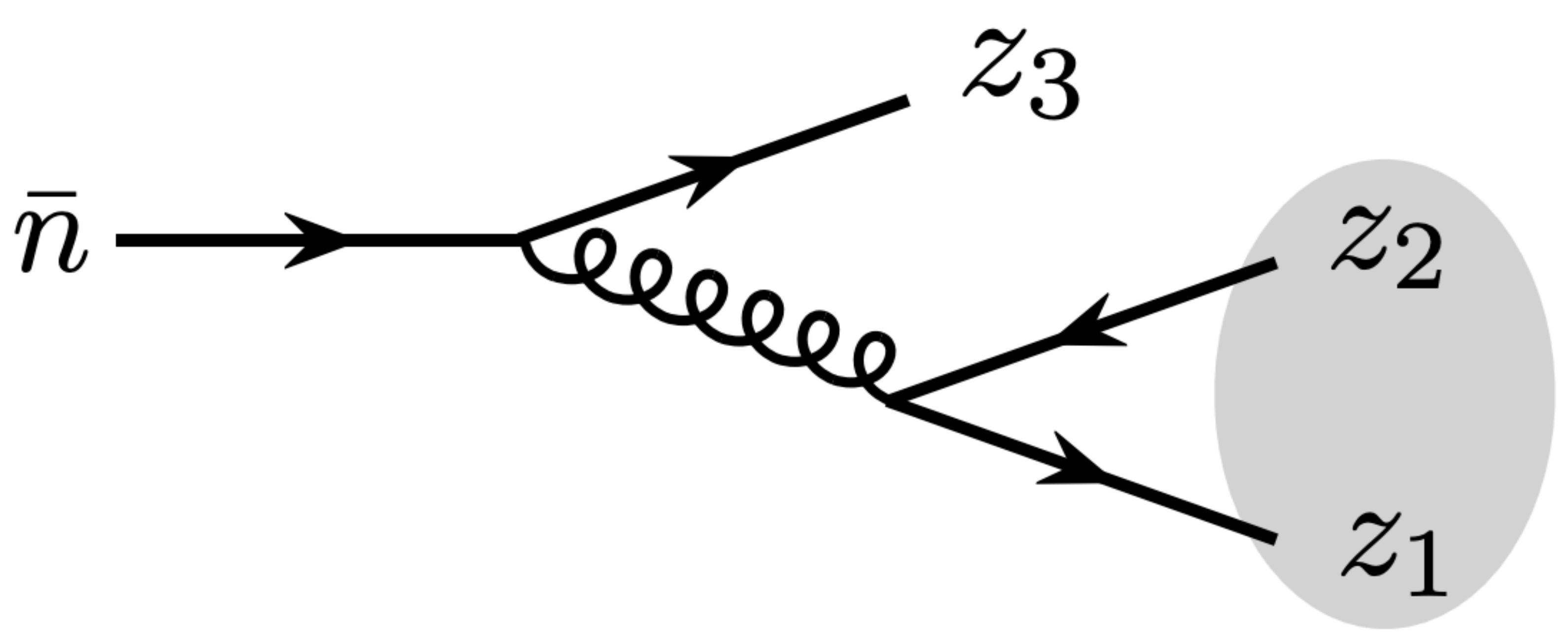
$$G_{\delta,j}(u,v) \equiv G_{\delta,j}(z,\bar{z}) = \frac{1}{1+\delta_{j,0}}\left(k_{\frac{\delta-j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z}) + k_{\frac{\delta+j}{2}}(z)k_{\frac{\delta-j}{2}}(\bar{z})\right)$$

Euclidean 2D conformal block

$$k_h(x) \equiv x^h\ {}_2F_1\ (h + a, h + b, 2h, x)$$

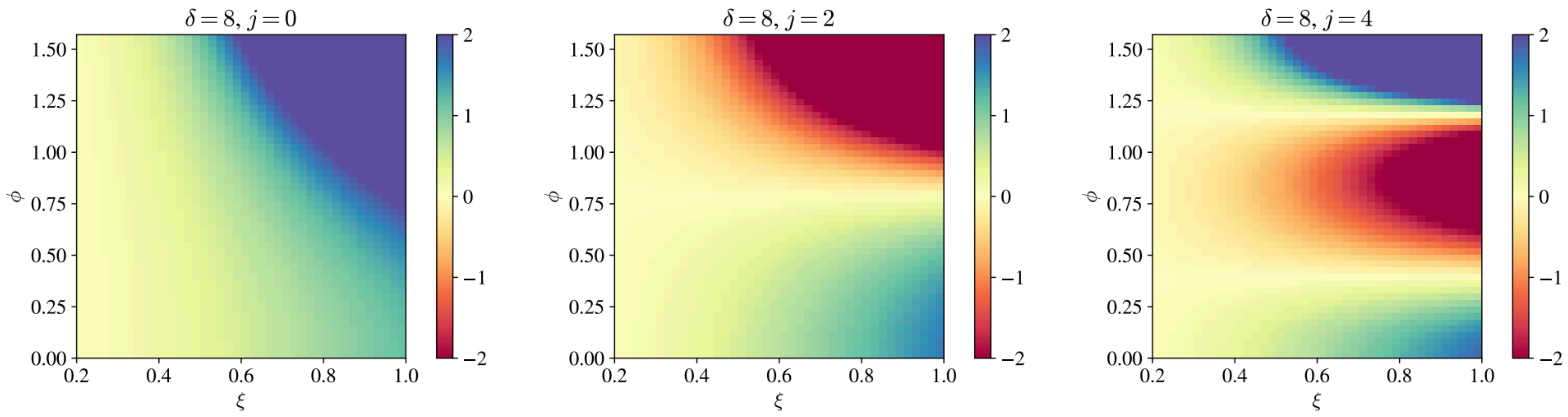
Explicit example of confomral block expansion

H. Chen, Moult, Sandor, HXZ, 2022

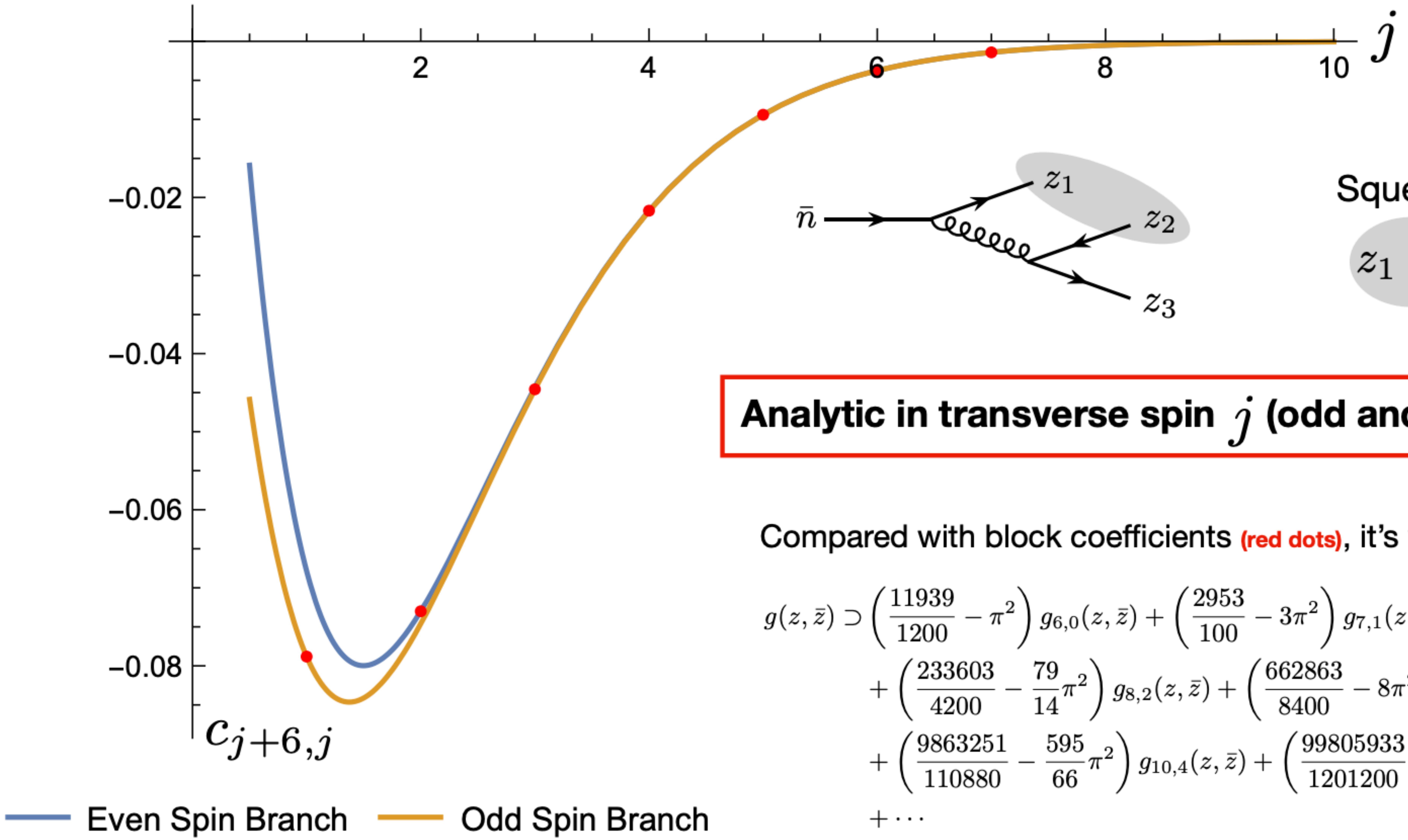


$G_{\delta,j}$

$$g_q(z) = C_F n_f T_F \Big[-\frac{1}{360} G_{4,2} + \frac{13}{1200} G_{4,0} + \frac{163}{126000} G_{6,2} + \left(\frac{111199}{33600} - \frac{\pi^2}{3}\right) G_{6,0}$$
$$- \frac{67}{420} \partial_\delta G_{8,0} + \left(\frac{39243247}{2116800} - \frac{79\pi^2}{42}\right) G_{8,2} + \left(\frac{201264317}{8820000} - \frac{7\pi^2}{3}\right) G_{8,0}$$
$$- \frac{751}{4620} \partial_\delta G_{10,2} - \frac{12317}{18480} \partial_\delta G_{10,0} + \left(\frac{9863251}{332640} - \frac{595\pi^2}{198}\right) G_{10,4}$$
$$+ \left(\frac{2801569019}{64033200} - \frac{40\pi^2}{9}\right) G_{10,2} + \left(\frac{168438023821}{3585859200} - \frac{937\pi^2}{196}\right) G_{10,0} + \cdots \Big]$$



Analyticity of transverse spin



H. Chen, Moult, Sandor, HXZ, 2022
Chang, Simmons-Duffin, 2022

Squeezed limit

$$z_1 \cdot z_2 \to 0$$

Analytic in transverse spin j (odd and even, resp.)

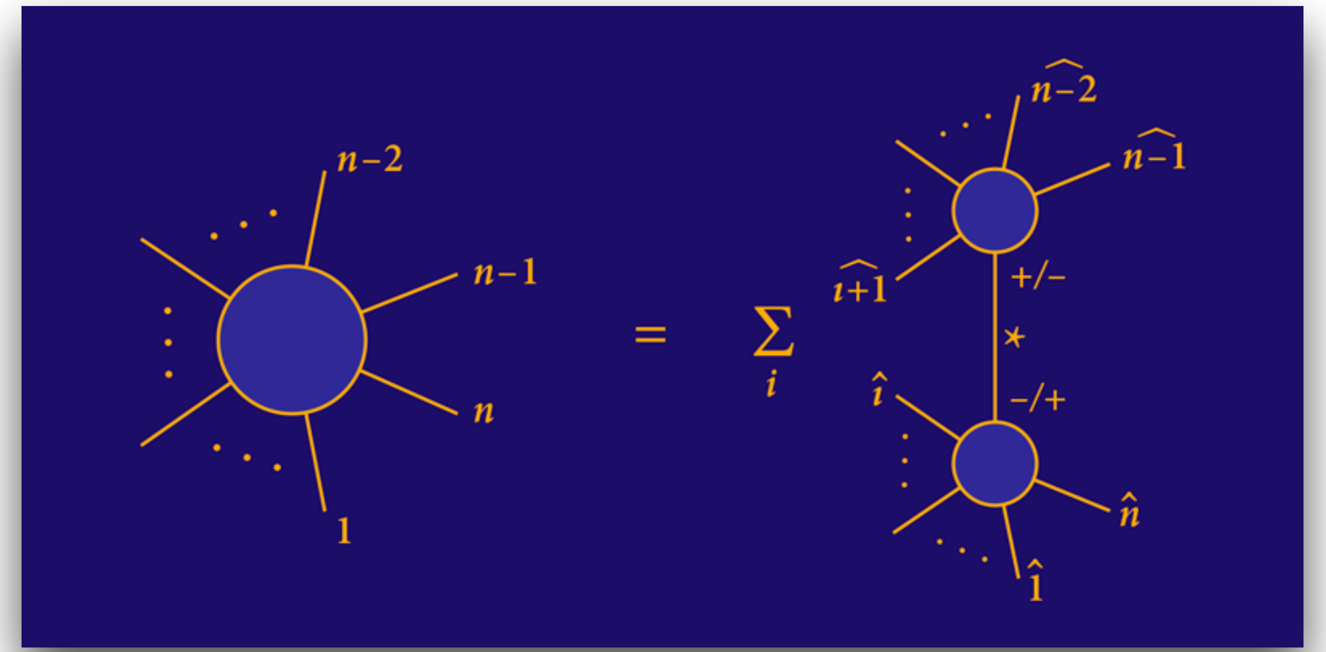
Compared with block coefficients (red dots), it's valid for $j \geq 1$

$$g(z,\bar{z}) \supset \left(\frac{11939}{1200} - \pi^2\right) g_{6,0}(z,\bar{z}) + \left(\frac{2953}{100} - 3\pi^2\right) g_{7,1}(z,\bar{z})$$
$$+ \left(\frac{233603}{4200} - \frac{79}{14}\pi^2\right) g_{8,2}(z,\bar{z}) + \left(\frac{662863}{8400} - 8\pi^2\right) g_{9,3}(z,\bar{z})$$
$$+ \left(\frac{9863251}{110880} - \frac{595}{66}\pi^2\right) g_{10,4}(z,\bar{z}) + \left(\frac{99805933}{1201200} - \frac{1204}{143}\pi^2\right) g_{11,5}(z,\bar{z})$$
$$+ \cdots$$

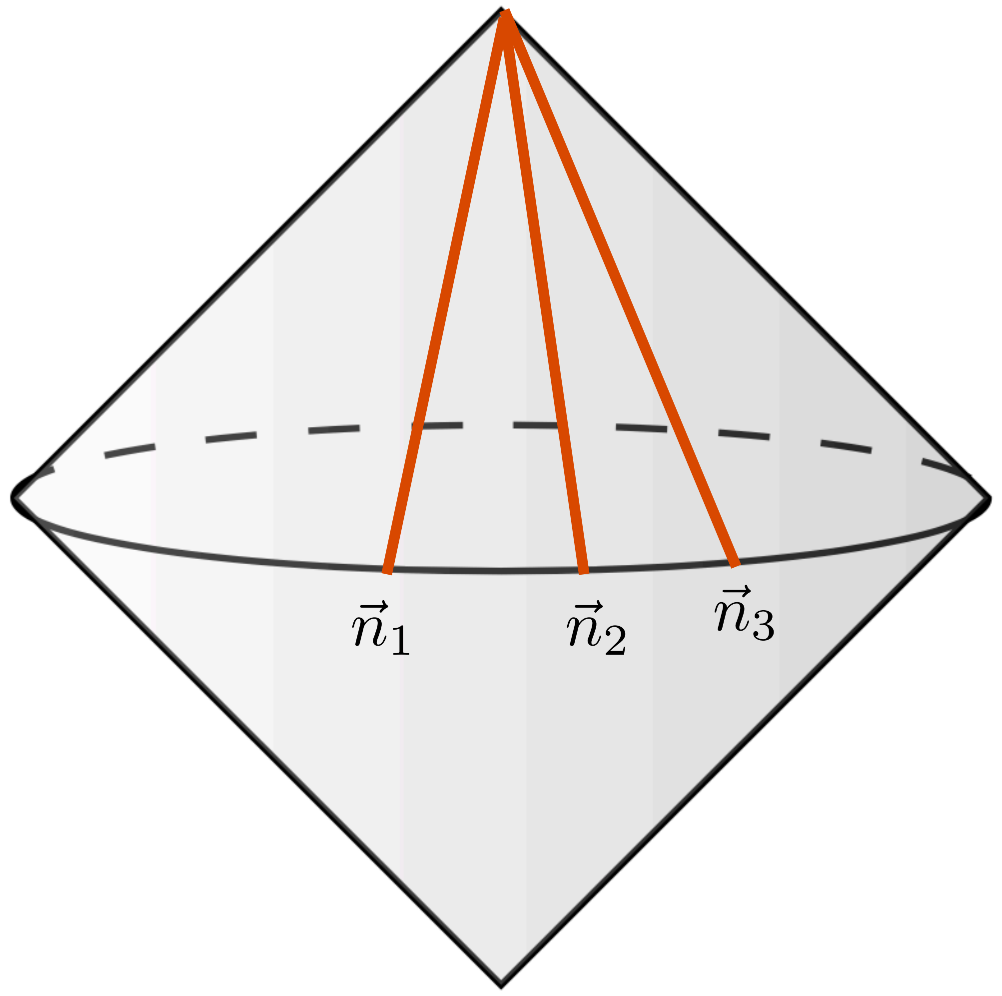
See also CFT with Wilson line defect
Barret, Liendo, Plefka, 2020

Summary

Scattering Amplitudes
Form factor

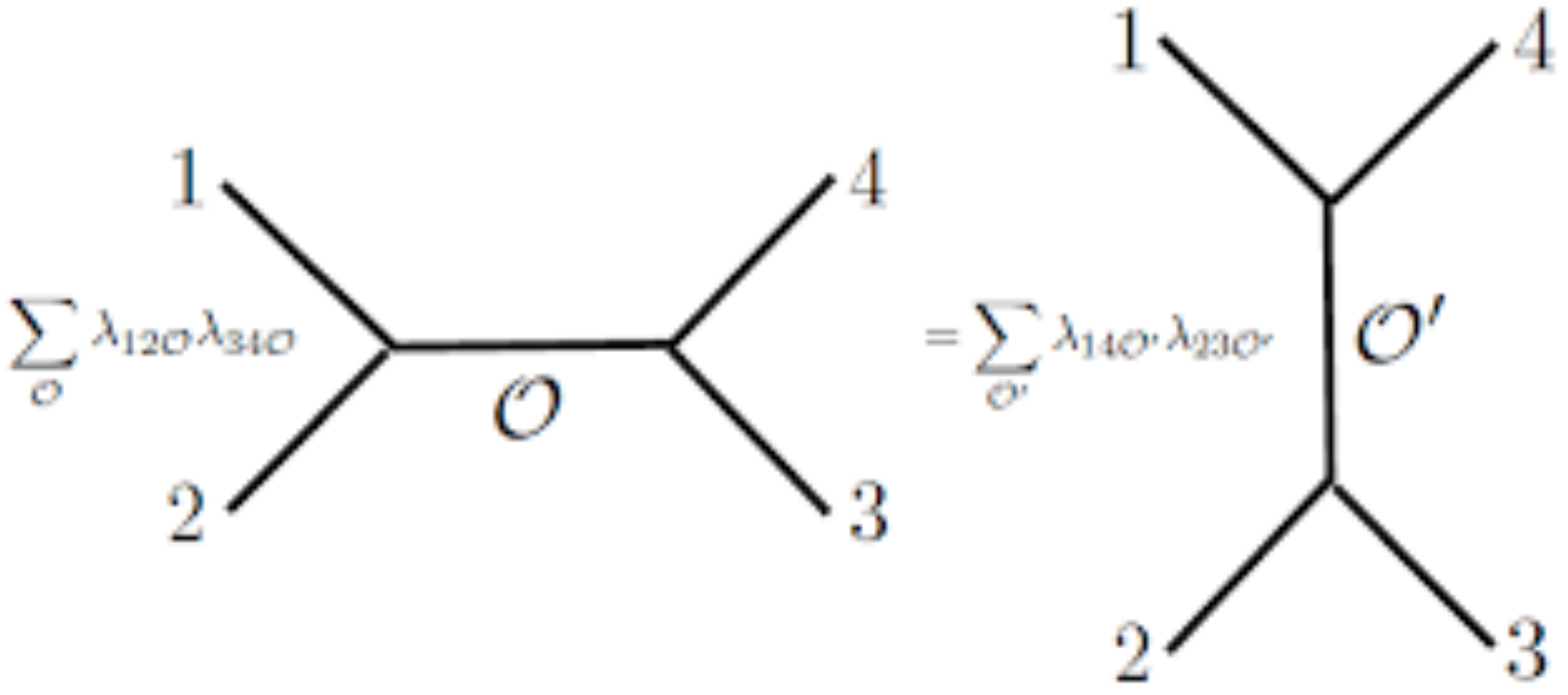


Correlation function of
lightray operator



Intrinsically IR finite
Admit OPE
Manifest analyticity in spin
Directly measurable!

Correlation Functions



~~ON~~ OFF - SHELL ~~BAD~~ GOOD

or Why String Field Theory is Needed to Determine the Effects of D-instantons

Backup slides

Lorentzian Inversion Formula [Caron-Huot, 2017]

Extracting block coefficients from **double discontinuity** of CFT 4-point correlator

Gribov-Froissart Formula

Partial wave expansion

$$A(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1)\, a_\ell(s)\, P_\ell(z), \quad z = \cos\theta$$

Gribov-Froissart formula

$$a_\ell(s) = \frac{1}{2\pi} \int_1^\infty dz\, Q_\ell(z) [\mathrm{Disc}_t A(s,z) + (-1)^\ell \mathrm{Disc}_u A(s,-z)]$$

partial wave ← **Discontinuity of amplitude**

Conformal block expansion $g(z,\bar{z}) = \sum_{\delta,j} c_{\delta,j}\, g_{\delta,j}(z,\bar{z})$

Lorentzian inversion $c(\delta,j) = c^t(\delta,j) + (-1)^j c^u(\delta,j)$

$$c^t(\delta,j) = \frac{\kappa_{\delta+j}}{4} \int_0^1 dz d\bar{z} \mu(z,\bar{z}) g_{j+d-1,\delta+1-d}(z,\bar{z}) \mathrm{dDisc}\,[g(z,\bar{z})]$$

$A(s,t)$	$g(z,\bar{z})$
$a_\ell(s)$	$c_{\delta,j}$
$\mathrm{Disc} A$	$\mathrm{dDisc}\, g$

$P_\ell(z)$	⟶	$Q_\ell(z)$
$g_{\delta,j}$	⟶	$g_{j+d-1,\delta+1-d}$

Double Discontinuity

$$\mathrm{dDisc}\, g(z,\bar{z}) = \cos(\pi(a+b)) g(z,\bar{z}) - \frac{1}{2} e^{i\pi(a+b)} g^{\circlearrowleft}(z,\bar{z}) - \frac{1}{2} e^{-i\pi(a+b)} g^{\circlearrowright}(z,\bar{z})$$