Fermionic BPS Wilson Loops in Four-Dimensional Superconformal Field Theories

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- These line operators can carry charges of 1-form global symmetries *[Gaoitto, Kapustin, Seiberg, Willett, 2014], [cf. Yi-Nan's talk]* .

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- On the field theory side, it was conjectured that the computations reduces to the ones in a Gaussian matrix model *[Erickson et. al.00]*. And the strong coupling results match with the ones from the string theory side *[Drukker et. al. 99][Berenstein et. al. 98]*.

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- This conjecture was later proved using supersymmetric localization. *[Pestun, 07]*

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- Later this puzzle was solved by including the coupling to fermions to make up half-BPS WLs. *[Drukker, Trancanelli, 09]*

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- They are dual to F-strings with complicated mixed boundary conditions. *[Correa, Giraldo-Rivera, Silva, 19]*

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- It is also supported by the fact that there are massless fermions on the worldsheet of F-string dual to half-BPS Wilson loops. *[Kim, Kim, Lee, 12][Aguilera-Damia et. al. 18][Correa, Giraldo-Rivera, Silva, 19]*

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- We also constructed fermionic half-BPS WLs in general quiver $\mathcal{N}=2$ super-Chern-Simons theories. *[Ouyang, **JW**, Zhang, 15][Mauri, Ouyang, Penati, **JW**, Zhang, 18]*

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- Yes, we can!
- We need to introduce extra dimensionful parameter in the construction. So even for case of straight line, scale invariance is lost. But this is fine.
- We constructed BPS WLs for lines and circular loops in $\mathcal{N} = 2$ quiver theories and $\mathcal{N} = 4$ SYM.

General discussions on WLs

- For closed contour C, the Wilson loop

$$W = \mathrm{Tr}_{\mathrm{R}} \mathcal{P} \exp \left(i \oint_C A_\mu(x(\tau)) \dot{x}^\mu d\tau \right) , \tag{1}$$

is gauge invariant.

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- For open contour C with both ends at infinity the Wilson loop

$$W = \mathrm{Tr}_{\mathrm{R}} \mathcal{P} \exp\left(i \int_C A_\mu(x(\tau)) \dot{x}^\mu d\tau \right) , \tag{2}$$

  is invariant under gauge transformation when the gauge transformation parameter vanishes at infinity.

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- So for straight WL, we mainly focus on the (super-)connection.

General discussions on BPS WLs

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- Such loop operators were named Maldacena-Wilson loops.

General discussions on BPS WLs

- For quiver gauge theory with gauge group $G_1 \times G_2$. Let us define the superconnection

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- And then L can be decomposed as

$$L = \begin{pmatrix} B_1 & F_1 \\ F_2 & B_2 \end{pmatrix}. \tag{6}$$

General discussions on WLs

- Assume under a supercharge Q_s (with Grassmann odd factor discarded), $Q_s L = \partial_\tau G_s - i[A_\mu \dot{x}^\mu + \tilde{B}(x), G_s] + i\{F(x), G_s\}$, and G_s is block anti-diagonal. Then a BPS Wilson loop preserving the supercharge $\mathcal{Q}_s$ can be defined as

$$W_{\rm fer} = {\rm sTr}\mathcal{P}\exp\left(i\oint L d\tau\right), \quad (7)$$

when G_s is periodic, or

$$W_{\rm fer} = {\rm Tr}\mathcal{P}\exp\left(i\oint L d\tau\right), \quad (8)$$

when G_s is anti-periodic. *[K. Lee, S. Lee, 10]*

$\mathcal{N} = 2$ superconformal $SU(N) \times SU(N)$ quiver theory

- Let us consider the $\mathcal{N} = 2$ superconformal $SU(N) \times SU(N)$ quiver theory which is a marginal deformation of the $\mathbb{Z}_2$ orbifold of $\mathcal{N} = 4$ SYM.

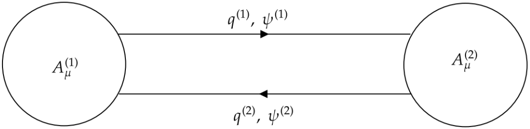

Figure: Quiver diagram.

Fields in the vector multiplets

- The fields in the two $\mathcal{N} = 2$ vector multiplets corresponding to two gauge group factors can be arranged into 2×2 block matrices:

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\begin{aligned}
A_\mu &= \begin{pmatrix} A_\mu^{(1)} & 0 \\ 0 & A_\mu^{(2)} \end{pmatrix}, \quad \mu = 0, ..., 5 \\
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- Here A_m with $m = 0, ..., 3$ is the gauge field and $A_{4,5}$ are two real scalars.
- We use $6d$ spinorial notations for the spinors. The $SO(1,5)$ Weyl spinors λ_1 and λ_2 have chirality -1 for Γ^{012345} and satisfy the reality condition $\bar{\lambda}^\alpha = -\epsilon^{\alpha\beta}\lambda^c_\beta$.

Fields in the hyper multiplets

- The matter content consists of two bifundamental hypermultiplets with component fields:

$$q^{\alpha} = \begin{pmatrix} 0 & q^{(1)\alpha} \\ q^{(2)\alpha} & 0 \end{pmatrix}, \quad \psi = \begin{pmatrix} 0 & \psi^{(1)} \\ \psi^{(2)} & 0 \end{pmatrix}. \tag{10}$$

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- We denote by q_α the complex conjugate of q^α.

Action

- The action of the $\mathcal{N}=2$ gauge theory is

$$
\begin{aligned}
S_{\mathcal{N}=2} \quad = \quad & \int d^4x \left(-\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{i}{2}\mathrm{Tr}(\bar{\lambda}^{\alpha}\Gamma^{\mu}D_{\mu}\lambda_{\alpha}) - D_{\mu}q_{\alpha}D^{\mu}q^{\alpha} \right. \\
& - i\bar{\psi}\Gamma^{\mu}D_{\mu}\psi + \sqrt{2}g\bar{\lambda}^{\alpha A}q_{\alpha}T_A\psi - \sqrt{2}g\bar{\psi}T_A q^{\alpha}\lambda^{A}_{\alpha} \\
& \left. - g^2(q_{\alpha}T^{A}q^{\beta})(q_{\beta}T_A q^{\alpha}) + \frac{1}{2}g^2(q_{\alpha}T_A q^{\alpha})(q_{\beta}T^{A}q^{\beta}) \right) , \quad (11)
\end{aligned}
$$

where T^A are the generators of the gauge group.

Coupling constants

The coupling constants for the two gauge group factors can be independently varied while preserving $\mathcal{N} = 2$ superconformal symmetry. We assemble them into a matrix:

$$g = \begin{pmatrix} g^{(1)} I_N & 0 \\ 0 & g^{(2)} I_N \end{pmatrix}, \tag{12}$$

where we denote by I_N the $N \times N$ identity matrix.

Superconformal transformation

- The above action is invariant under the following superconformal transformation,

$$\begin{aligned}
\delta A_\mu &= -i\bar{\xi}^\alpha\Gamma_\mu\lambda_\alpha = i\bar{\lambda}^\alpha\Gamma_\mu\xi_\alpha, \\
\delta q^\alpha &= -i\sqrt{2}\bar{\xi}^\alpha\psi, \\
\delta q_\alpha &= -i\sqrt{2}\bar{\psi}\xi_\alpha, \\
\delta\lambda^A_\alpha &= \frac{1}{2}F^A_{\mu\nu}\Gamma^{\mu\nu}\xi_\alpha+2igq_\alpha T^A q^\beta\xi_\beta - igq_\beta T^A q^\beta\xi_\alpha-2A^A_a\Gamma^a\vartheta_\alpha, \\
\delta\bar{\lambda}^{\alpha A} &= -\frac{1}{2}\bar{\xi}^\alpha F^A_{\mu\nu}\Gamma^{\mu\nu}-2igq_\beta T^A q^\alpha\bar{\xi}^\beta + igq_\beta T^A q^\beta\bar{\xi}^\alpha+2\bar{\vartheta}^\alpha A^A_a\Gamma^a, \\
\delta\psi &= -\sqrt{2}D_\mu q^\alpha\Gamma^\mu\xi_\alpha-2\sqrt{2}q^\alpha\vartheta_\alpha, \\
\delta\bar{\psi} &= \sqrt{2}\bar{\xi}^\alpha\Gamma^\mu D_\mu q_\alpha-2\sqrt{2}\bar{\vartheta}^\alpha q_\alpha.
\end{aligned} \tag{13}$$

- Here $\xi_\alpha = \theta_\alpha + x^m\Gamma_m\vartheta_\alpha$ and the index $a = 4, 5$.

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- We fixed a typo in *[Rey, Suyama, 2010]*. This point is very crucial for us.

Bosonic BPS connection

- In Minkowski spacetime, one can define a 1/2 BPS Wilson line along the timelike infinite straight line straight line $x^m = \delta_0^m \tau$ as

$$W_{\rm bos} = \mathcal{P} e^{i \int {\rm d}\tau L_{1/2}(\tau)}, \quad L_{1/2} = gA_0 - gA_5. \tag{14}$$

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- This leads to

$$\Gamma_5 \Gamma_0 \theta_\alpha = \theta_\alpha, \, \Gamma_5 \Gamma_0 \vartheta_\alpha = -\vartheta_\alpha. \tag{16}$$

Supercharges

- We decompose ξ_α as $\xi_\alpha = \theta s_\alpha$ where θ is a real Grassmann variable and s_α are bosonic spinors. We focus on the Poincaré supercharges for superconnection along a line.
- We define Q_s using $\delta_\xi = \sqrt{2}\theta Q_s$.

Fermionic superconnections

- The BPS superconnection L (along the above line) is a supermatrix, analogous to the ones constructed in [Drukker, Trancanelli, 2009]:

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- The matrices B and F are defined as

$$B = \begin{pmatrix} B^{(1)} & 0 \\ 0 & B^{(2)} \end{pmatrix}, \tag{18}$$

$$F = \zeta^c \psi + \bar{\psi}\eta, \tag{19}$$

$$\zeta = \begin{pmatrix} \zeta^{(1)} I_N & 0 \\ 0 & \zeta^{(2)} I_N \end{pmatrix}, \tag{20}$$

$$\eta = \begin{pmatrix} \eta^{(2)} I_N & 0 \\ 0 & \eta^{(1)} I_N \end{pmatrix}, \tag{21}$$

Superconnection

- We fix a spinor s_α satisfying $\Gamma_5\Gamma_0 s_\alpha = s_\alpha$ and demand L to transform as

$$Q_s L = \mathcal{D}_0 G_s \equiv \partial_0 G_s - i[L_{1/2} + B, G_s] + i\{F, G_s\}, \qquad (22)$$

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  for some bosonic matrix G_s.
- Splitting this constraint into a fermonic and bosonic part, we find

$$Q_s B = i\{F, G_s\}, \qquad (23)$$

$$Q_s F = \partial_0 G_s - i[L_{1/2} + B, G_s]. \qquad (24)$$

Superconnection

- The solution is

$$L = L_{1/2} + \frac{2i}{(\bar{s}^\alpha \Gamma_0 s_\alpha)} Q_s G_s - \frac{2}{(\bar{s}^\alpha \Gamma_0 s_\alpha)} G_s^2, \tag{25}$$

where

$$G_s = \zeta^c \Gamma_0 s_\alpha q^\alpha - q_\alpha \bar{s}^\alpha \Gamma_0 \eta, \tag{26}$$

with η and ζ^c satisfying

$$\Gamma_5 \Gamma_0 \eta = \eta, \quad \zeta^c \Gamma_5 \Gamma_0 = -\zeta^c, \tag{27}$$

Relaxing the real condition in Euclidean signature

- In the Euclidean signature, the bars over the spinors now do not stand for Dirac conjugation. ψ and $\bar{\psi}$ are independent spinors.

Relaxing the real condition in Euclidean signature

- In the Euclidean signature, the bars over the spinors now do not stand for Dirac conjugation. ψ and $\bar{\psi}$ are independent spinors.
- It is convenient to define $\bar{s}^{\alpha} = -\epsilon^{\alpha\beta} s^{c}_{\beta}$ for any spinors with an α index.

Circular Wilson loop

- Consider the circle $(x^0, x^1, x^2, x^3) = r(\cos\tau, \sin\tau, 0, 0)$ in the $x^0 - x^1$ plane.

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$$L_{1/2} = g\dot{x}^m A_m + igrA_5, \tag{28}$$

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where dot denotes derivation with respect to τ.

- The supersymmetries preserved by the bosonic Wilson loop $W_{\rm bos} = \mathcal{P}\exp(i\int_0^{2\pi} d\tau L_{1/2}(\tau))$ satisfy

$$r^{-1}\dot{x}^m \Gamma_m \Gamma_5 \xi_\alpha = i\xi_\alpha, \quad \Rightarrow \quad \vartheta_\alpha = -ir^{-1}\Gamma_{015}\theta_\alpha. \tag{29}$$

Fermionic BPS circular WL

- We would like to construct a Wilson loop on the same contour which is invariant under a supercharge $\mathcal{Q}_s$ parameterized by

$$\theta_\alpha = \frac{1}{2\sqrt{2}}\theta s_\alpha, \quad \vartheta_\alpha = -\frac{i}{2\sqrt{2}r}\Gamma_{015}\theta s_\alpha \tag{30}$$

where θ is a complex Grassman variable and s^α is a fixed bosonic spinor.

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  where θ is a complex Grassman variable and s^α is a fixed bosonic spinor.
- As the previous case, we want to find G_s and L such that $\mathcal{Q}_s L = \mathcal{D}_\tau G_s$.

Fermionic BPS circular WL

- Assuming that s_1 and s_2 are linearly independent, we find the solutions are

$$L = L_{1/2} + \frac{2r}{\bar{s}^{\alpha}\Pi_{-}\Gamma_5 s_{\alpha}} \mathcal{Q}_s G_s + i \frac{2r}{\bar{s}^{\alpha}\Pi_{-}\Gamma_5 s_{\alpha}} G_s^2, \tag{31}$$

$$G_s = i\zeta^c \Pi_{-}\Gamma_5 s_{\alpha} q^{\alpha} - i q_{\alpha} \bar{s}^{\alpha} \Gamma_5 \Pi_{+} \eta, \tag{32}$$

with $\Pi_{\pm} = \frac{1}{2} \pm \frac{i}{2r}\Gamma_5 \dot{x}^m \Gamma_m$, and η, ζ^c are τ-independent and satisfying

$$\zeta^c \Gamma_{015} s_{\alpha} = \bar{s}^{\alpha} \Gamma_{015} \eta = 0. \tag{33}$$

Fermionic BPS circular WL

- Because G_s is periodic on the contour, the trace of the holonomy of L does not preserve the supercharge $\mathcal{Q}_s$, which is different from their three-dimensional counterparts *[Drukker, Trancanelli, 2009]*.

Fermionic BPS circular WL

- Because G_s is periodic on the contour, the trace of the holonomy of L does not preserve the supercharge $\mathcal{Q}_s$, which is different from their three-dimensional counterparts *[Drukker, Trancanelli, 2009]*.
- Since L has a natural supermatrix structure, we can define the Wilson loop by using the supertrace:

$$W_{\text{fer}} = \text{sTr}\mathcal{P}\exp\left(i\oint Ld\tau\right), \tag{34}$$

which preserves the supercharge $\mathcal{Q}_s$.

Supersymmetry enhancement

- For general ζ and η, this WL is $1/16$-BPS.
- For special ζ and η, this WL is $1/8$- or $3/16$-BPS.
- It preserves quite fewer supersymmetries, comparing with bosonic circular WLs.

Relation with bosonic WLs

- Following similar steps as in the three-dimensional case *[Drukker, Trancanelli, 2009][Ouyang,* **JW***, Zhang, 2015]*, one can show that the condition $\mathcal{Q}_s L = \mathcal{D}_\tau G_s$ leads to a classical $\mathcal{Q}_s$-cohomological equivalence between the fermionic BPS Wilson loop and the bosonic one:

$$W_{\rm fer} - W_{\rm bos} = \mathcal{Q}_s V, \tag{35}$$

where

$$W_{\rm bos} = {\rm sTr}\mathcal{P}\exp\left(i\oint L_{1/2} d\tau\right), \tag{36}$$

and V is a complicated function of the gauge and matter fields.

$\mathcal{N} = 4$ super Yang-Mills

- The action of $\mathcal{N} = 4$ SYM is

$$S_{\mathcal{N}=4} = \int_{\mathbf{R}^4} d^4x \left(-\frac{1}{4}\text{Tr}(F_{MN}F^{MN}) - \frac{i}{2}\text{Tr}(\bar{\Psi}\Gamma^M D_M \Psi) \right). \quad (37)$$

Now Γ^M are 10d gamma matrices.

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  Now Γ^M are 10d gamma matrices.
- We use the index conventions $M, N = 0, ..., 9$ and $R, S = 5, ..., 9$. And A_R are six scalars in the adjoint representation of the gauge group.

$\mathcal{N} = 4$ superconformal symmetry

- The action is invariant under the superconformal transformations:

$$\begin{aligned} \delta A_M &= -i\xi^c \Gamma_M \Psi, \\ \delta \Psi &= \frac{1}{2} F_{MN} \Gamma^{MN} \xi - 2\Gamma^S A_S \vartheta. \end{aligned} \tag{38}$$

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- where $\xi = \theta + x^m \Gamma_m \vartheta$ with $m = 0, ..., 3$. The constant spinors θ and ϑ generate Poincaré supersymmetry transformations and special superconformal transformations respectively.

Half-BPS bosonic WLs

- In the Euclidean signature, the superconformal transformations are formally the same as (38), but there are no reality conditions for the spinors.

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$$L_{1/2} = g\dot{x}^\mu A_\mu + igrA_5 \tag{39}$$

on the circle contour $(x^0, x^1, x^2, x^3) = r(\cos\tau, \sin\tau, 0, 0)$ satisfy

$$\dot{x}^\mu \Gamma_\mu \Gamma_5 \xi = i\xi, \quad \Rightarrow \quad \vartheta = -ir^{-1}\Gamma_{015}\theta. \tag{40}$$

Selected supercharge

- We would like to construct a Wilson loop on the same contour which is invariant under a super-charge $\mathcal{Q}_s$ parameterized by

$$\theta = \frac{1}{2}\chi s, \quad \vartheta = -\frac{i}{2r}\Gamma_{015}\chi s, \tag{41}$$

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- where χ is a complex Grassmann variable and s is a fixed bosonic spinor.

Superconnection

- We found a connection L which satisfies $\mathcal{Q}_s L = \mathcal{D}_\tau G_s$ is

$$L = L_{1/2} + \frac{r}{s^c \Pi_- \Gamma^5 s} \mathcal{Q}_s G_s + \frac{ir}{s^c \Pi_- \Gamma^5 s} G_s^2, \quad (42)$$

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- with $\Pi_- = \frac{1}{2} - \frac{i}{2r} \Gamma_5 \dot{x}^m \Gamma_m$, $G_s = m^S A_S$,

Superconnection

- $$m^S(\tau) = c^R s^c \Pi_- \Gamma_5 s [\exp\left(-\frac{2iM_{015}}{\sqrt{v_0^2+v_1^2+v_5^2}}\right.$$
$$\left.\tanh^{-1}\left(\frac{v_0+(v_1+iv_5)\tan\left(\frac{\tau}{2}\right)}{\sqrt{v_0^2+v_1^2+v_5^2}}\right)\right)]_R^S,$$
  with $v_\mu = s^c\Gamma_\mu s$ and $(M_{015})_R^S = s^c\Gamma_{015}\Gamma_R\Gamma^S s$.
- For $m^S(\tau)$ to be periodic, we need to impose
$$\sqrt{-1-\frac{\mathrm{Tr}M_{015}^2}{2v^2}} \in \mathbb{Z}. \qquad (43)$$
- It is impossible to make $m^S(\tau)$ anti-periodic.

“Mutiple copy” or “replica trick”

- One can generalize m^S to an $r \times r$ matrix-valued vector M^S and the connection becomes

$$L = I_r \otimes L_{1/2} + \frac{r}{s^c \Pi_- \Gamma^5 s} M^S \otimes \mathcal{Q}_s A_S + \frac{ir}{s^c \Pi_- \Gamma^5 s} (M^S \otimes A_S)^2. \quad (44)$$

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- And then L can be decomposed as

$$L = \begin{pmatrix} B_1 & F_1 \\ F_2 & B_2 \end{pmatrix}. \quad (46)$$

"Multi-copy" or "replica trick"

- Now a BPS Wilson loop preserving the supercharge $\mathcal{Q}_s$ can be defined as

$$W_{\rm fer} = {\rm sTr}\mathcal{P}\exp\left(i\oint L d\tau\right). \tag{47}$$

Relation with bosonis BPS WLs

- One can prove that, at the classical level, $W_{\rm fer} - W_{\rm bos} = Q_s V$ where
$$W_{\rm bos} = {\rm sTr}\mathcal{P}\exp\left(i\oint(I_r\otimes L_{1/2})d\tau\right), \tag{48}$$
with ${\rm sTr}$ defined as the one in the previous slide, and V is a complicated function of gauge fields and matter fields.

Conclusion

- We constructed fermionic BPS Wilson loops in $\mathcal{N} = 2$ superconformal $SU(N) \times SU(N)$ quiver theory and $\mathcal{N} = 4$ super Yang-Mills theory.

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- These Wilson loops involve dimensionful parameters.
- For generic values of parameters, they preserve one real (complex) supercharge in Lorentzian (Euclidean) signature.
- Supersymmetry enhancement for Wilson loops happens when the parameters satisfy certain constraints.

Outlook

- Our Fermionic BPS circular WL is in the same $\mathcal{Q}_s$-cohomology of a corresponding bosonic BPS WL at the classical level. If this is still true at the quantum level, The fermonic loop will have the same vev as the corresponding bosonic one.

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- The vev of bosonic BPS circular WLs has been computed by localization.
- It is valuable to check these predictions by direct perturbative computaitons.

Outlook

- Further constructions starting with bosonic WLs with fewer supersymmetries: Zarembo loops *(2000)* and DGRT loops *[Drukker, Gimobi, Ricci, Trancanelli, 2007]*.

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- S-dual and holographic dual of our new fermionic BPS WLs?

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- When we consider the correlators of a half-BPS circular WL (in the fundamental or antisymmetric representations) and a non-BPS single trace operator in the 't Hooft limit, this WL will provide an integrable matrix product state *[Jiang, Komatsu, Vescovi, to appear]*.

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- It is appealing to explore whether the fermionic WLs constructed here also have such integrable structure.

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- Any hints about possible UV completion?

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- Should the BPS Wilson-'t Hooft loop operators be included in this set when we study supersymmetric gauge theories?

Thanks for Your Attention !