Parity violating gravity models

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Discrete symmetries: C (Charge conjugation), P (Parity), T (Time reversal)

Important role in discovering fundamental physical laws and in generating matter-antimatter asymmetry in our universe.

Parity violation in weak interaction, discovered in1956; CP violation discovered in 1964;……

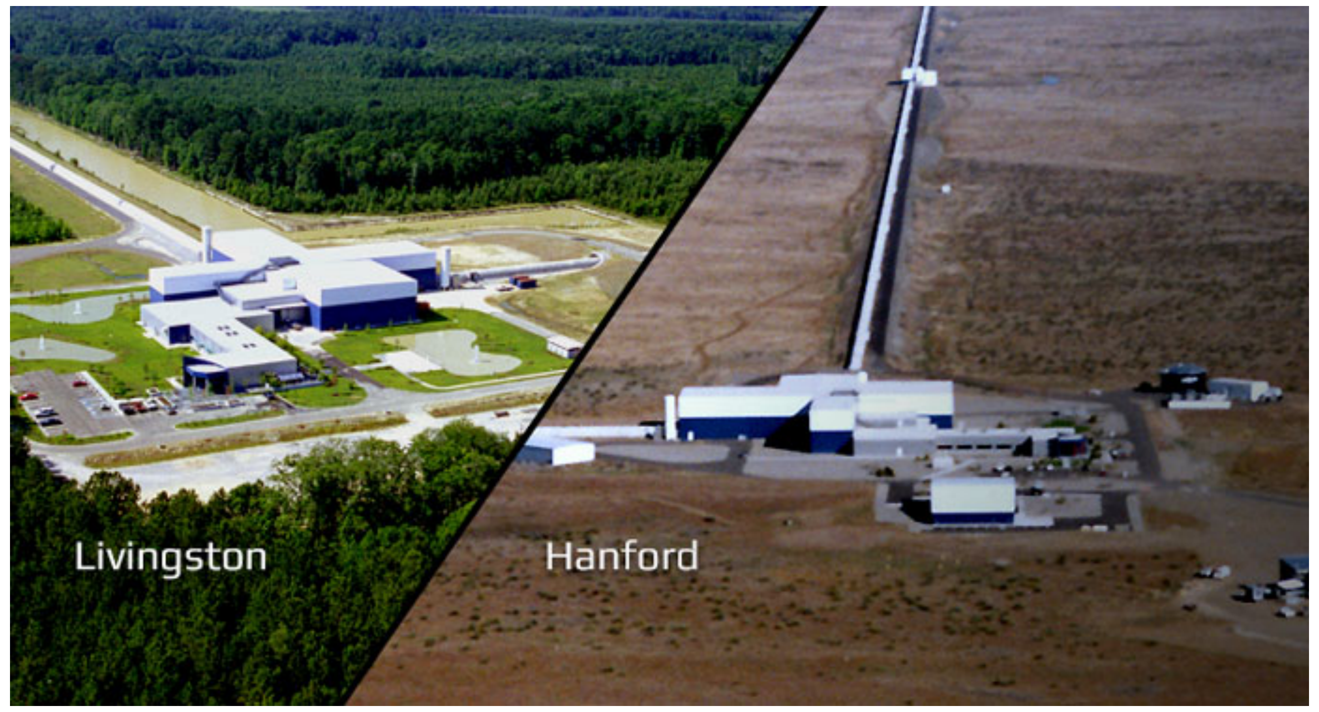
Some frequently discussed topics:

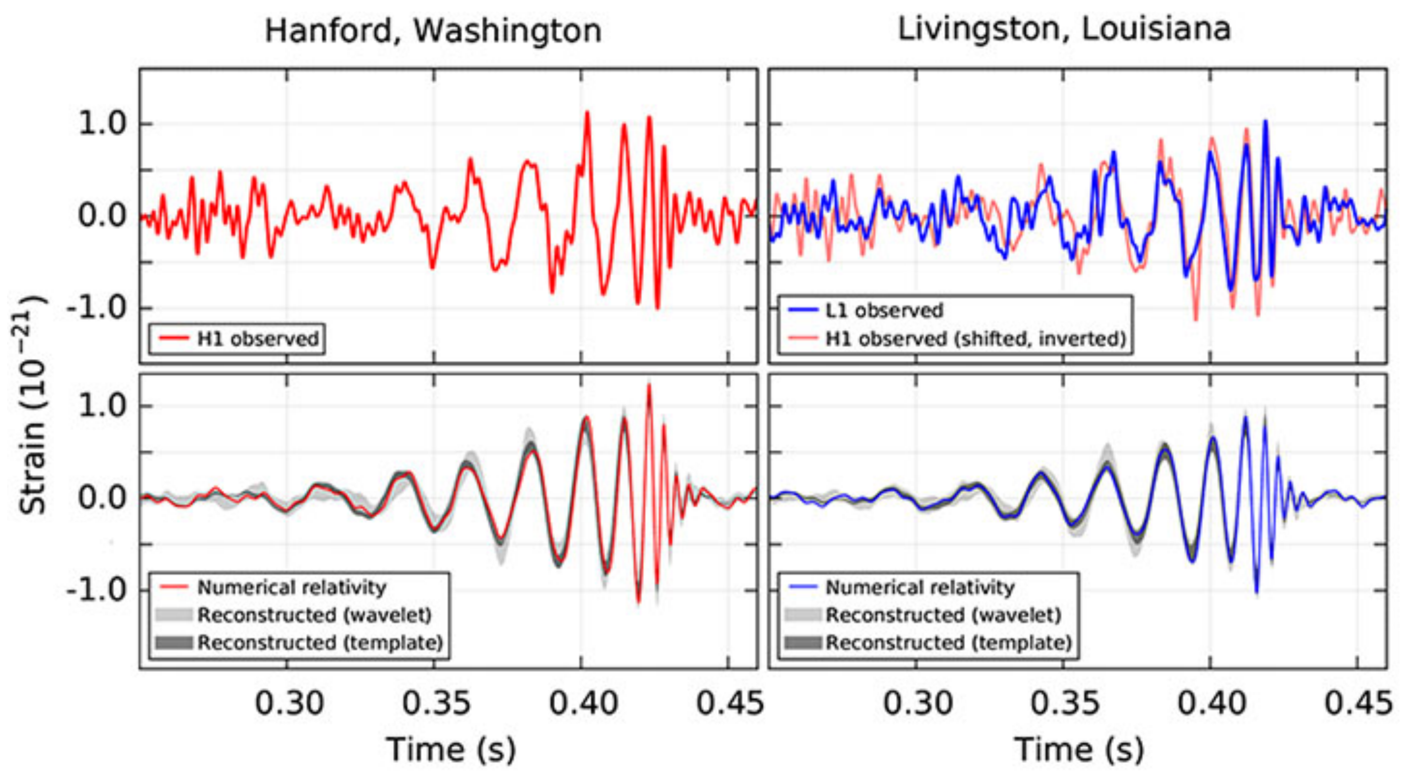
CP violation in strong interaction, θ term in QCD $\theta G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}$; P, CP & CPT violations in electrodynamics, Chern-Simons $\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}$; ……

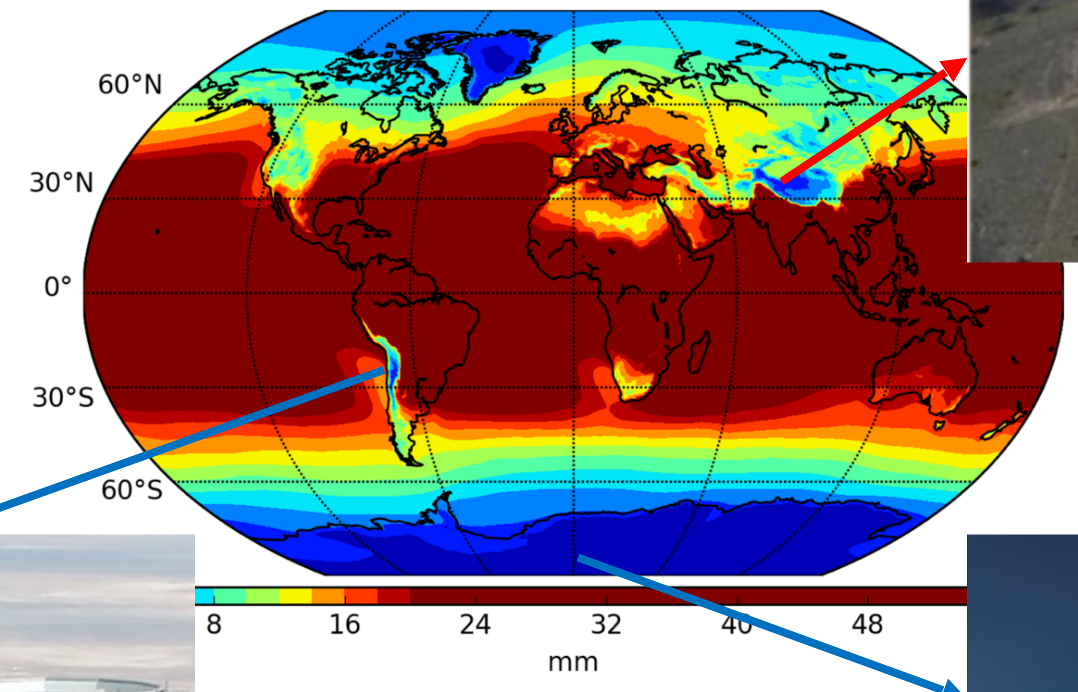
How about gravity?

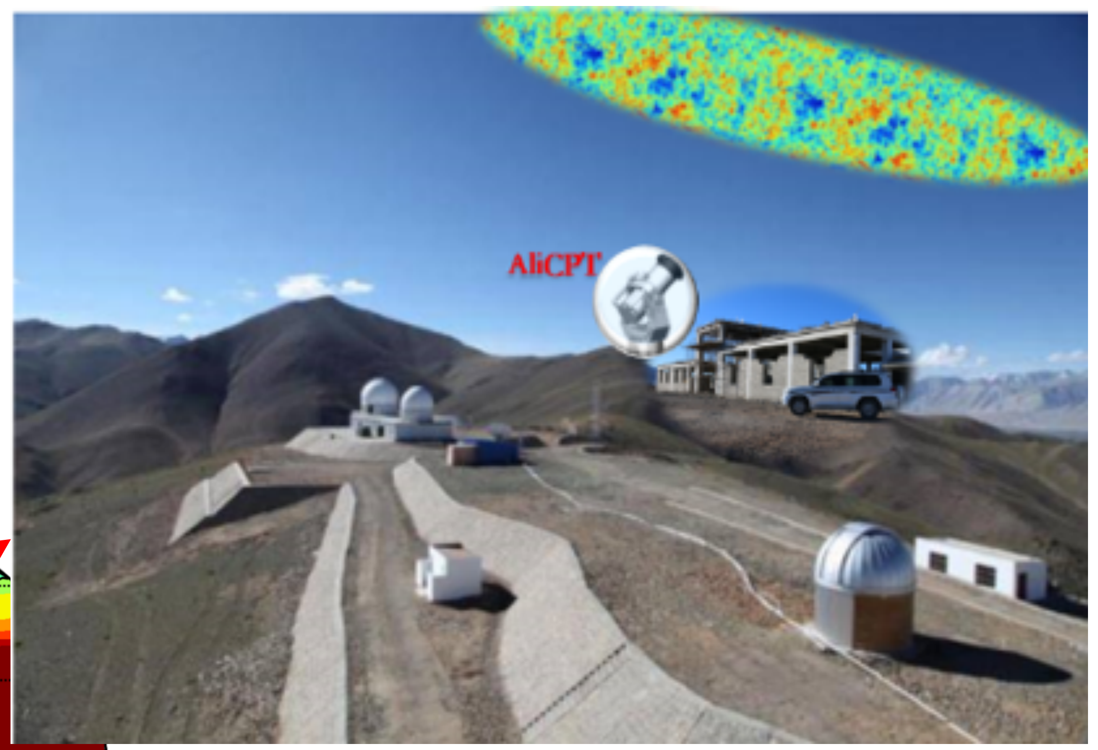
Parity-violating (PV) gravity as extension to general relativity (GR)

Subject of intense investigations in recent years, motivated by gravitational waves (GWs) detections, undergoing and planned CMB polarization experiments.









Outline

- Chern-Simons Modified General Relativity and Its Difficulties
- Teleparalel and Symmetric Teleparallel Gravities
- Nieh-Yan Modified Teleparallel Gravity
- Parity Violating Model in Symmetric Teleparallel Gravity
- Summary and Outlook

Chern-Simons Modified General Relativity & Its Difficulties

$$S = \int d^4x\sqrt{-g}[-\frac{1}{2}R + \frac{\theta}{4}R\widetilde{R}] + S_m$$

Lue, Wang, Kamionkowski, astro-ph/9812088;
Jackiw & Pi, gr-qc/0308071;
Alexander & Yunes, arXiv:0907.2562, Phys. Rept. (2009)

gravitational Chern-Simons (CS) term

Pontryagin density: $R\widetilde{R} \equiv \varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu}{}^{\alpha\beta}R_{\rho\sigma\alpha\beta} = \nabla_\mu K^\mu$, appears in gravitational anomalies, parity-odd

EOM $\quad G^{\mu\nu} + C^{\mu\nu} = T^{\mu\nu}$

Four dimensional Cotton tensor

$$C^{\mu\nu} = \frac{1}{2}[\nabla_\sigma\theta(\varepsilon^{\sigma\mu\alpha\beta}\nabla_\alpha R^\nu{}_\beta + \varepsilon^{\sigma\nu\alpha\beta}\nabla_\alpha R^\mu{}_\beta) + \frac{1}{2}\nabla_\sigma\nabla_\tau\theta(\varepsilon^{\sigma\mu\alpha\beta}R^{\tau\nu}{}_{\alpha\beta} + \varepsilon^{\sigma\nu\alpha\beta}R^{\tau\mu}{}_{\alpha\beta})]$$

Effects on GWs: left and right polarized GWs have the same velocity,
but different amplitudes (amplitude birefringence)

$$h^R = \frac{1}{\sqrt{2}}(h^+ - ih^\times)$$

$$h^L = \frac{1}{\sqrt{2}}(h^+ + ih^\times)$$

$$h^{A''} + 2\mathcal{H}\frac{1-\lambda_A\theta' k/(2a)}{1-\lambda_A\theta' k/a}h^{A'} + k^2 h^A = \frac{2a^2\sigma^A}{1-\lambda_A\theta' k/a}$$

$A = L, R$, with $\lambda^R = 1, \lambda^L = -1$

After inflation, left and right primordial GWs have power spectra with different amplitudes

$$P_h^R = \frac{1}{2}P_h(1-\epsilon)\ ,\ P_h^L = \frac{1}{2}P_h(1+\epsilon)\ ,\ P_h^R + P_h^L = P_h\ ,\ P_h^R - P_h^L = -\epsilon P_h$$

CMB TT, EE, BB, and TE spectra only depend on the sum $P_h^L + P_h^R$, but TB and EB depend on the difference

$$C_l^{XX(T)} = (4\pi)^2 \int k^2 dk P_h(k) \left[\Delta_{Xl}^{(T)}(k)\right]^2$$
$$C_l^{TE(T)} = (4\pi)^2 \int k^2 dk P_h(k) \Delta_{Tl}^{(T)}(k) \Delta_{El}^{(T)}(k)$$

$$C_l^{TB(T)} = -(4\pi)^2 \epsilon \int k^2 dk P_h(k) \Delta_{Tl}^{(T)}(k) \Delta_{Bl}^{(T)}(k)$$
$$C_l^{EB(T)} = -(4\pi)^2 \epsilon \int k^2 dk P_h(k) \Delta_{El}^{(T)}(k) \Delta_{Bl}^{(T)}(k)$$

$X = T, E, B$

CMB B mode experiment can be used to test parity symmetry of gravity! astro-ph/9812088

Instability in CS modified gravity

$$S_h = \frac{1}{4}\int d\eta d^3k \sum_{A=L,R} a^2 \underline{(1+\lambda_A \theta' \frac{k}{a})} [|h^{A'}|^2 - k^2 |h^A|^2]$$

$$1 + \lambda_L \theta' \frac{k}{a} < 0, \text{ for } \frac{k}{a} > 1/\theta' \text{ assuming } \theta' > 0$$

Ghost, wrong sign for the kinetic term, vacuum instability

CS contains two Riemann tensors, is a higher derivative term $R \sim \partial^2 g,\ R\widetilde{R} \sim (\partial^2 g)(\partial^2 g)$

Higher time derivative terms introduce extra propagating modes, some of them are ghosts (Ostrogradsky ghost modes)

The ghost mode presented here is not the Ostrogradsky type

Extensions of CS modified gravity

More examples can be found in: Zhao et al., arXiv:1909.10887

Crisostomi et al, arXiv:1710.04531

$$\mathcal{L}_{\rm PV} = \sum_{\rm A=1}^{4} a_{\rm A}(\phi, \phi^\mu \phi_\mu) L_{\rm A}$$

$$L_1 = \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\nu}{}^{\rho}{}_{\lambda} \phi^\sigma \phi^\lambda, \; L_2 = \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\lambda}{}^{\rho\sigma} \phi_\nu \phi^\lambda, \; L_3 = \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R^{\sigma}{}_{\nu} \phi^\rho \phi_\mu, \; L_4 = \varepsilon^{\mu\nu\rho\sigma} R_{\rho\sigma\alpha\beta} R^{\alpha\beta}{}_{\mu\nu} \phi^\lambda \phi_\lambda$$

No extra propagating modes from higher derivatives if $4a_1 + 2a_2 + a_3 + 8a_4 = 0$

But the problem of ghost instability still exists, Bartolo et al., arXiv:2008.01715

Question:

Can we have a healthy parity violating (PV) gravity ?

Can we make PV extension to GR from the viewpoint of non-metric theory?

Example of GR equivalent non-metric theory:
Teleparallel & Symmetric Teleparallel Gravities

Teleparallel & Symmetric Teleparallel Gravities

General metric-affine theory, non-Riemannian geometry, $\hat{\Gamma}^{\rho}{}_{\mu\nu},\ g_{\mu\nu}$

Torsion $\mathcal{T}^{\rho}_{\mu\nu} = \hat{\Gamma}^{\rho}_{\mu\nu} - \hat{\Gamma}^{\rho}_{\nu\mu}$

Non-metricity $Q_{\mu}{}^{\rho\sigma} = \hat{\nabla}_{\mu} g^{\rho\sigma}$ $\qquad \hat{\nabla} = \partial + \hat{\Gamma}$

Curvature $\hat{R}^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\hat{\Gamma}^{\rho}_{\nu\sigma} + \hat{\Gamma}^{\rho}_{\mu\alpha}\hat{\Gamma}^{\alpha}_{\nu\sigma} - \{\mu \leftrightarrow \nu\}$

Christoffel symbol $\Gamma^{\rho}{}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}(\partial_{\mu} g_{\sigma\nu} + \partial_{\nu} g_{\sigma\mu} - \partial_{\sigma} g_{\mu\nu})$ $\qquad \nabla = \partial + \Gamma$ $\qquad \nabla_{\mu} g^{\rho\sigma} = 0$

Distorsion tensor $C^{\rho}{}_{\mu\nu} = \hat{\Gamma}^{\rho}{}_{\mu\nu} - \Gamma^{\rho}{}_{\mu\nu} = \frac{1}{2}(\mathcal{T}^{\rho}_{\mu\nu} + \mathcal{T}_{\nu}{}^{\rho}{}_{\mu} - \mathcal{T}_{\mu\nu}{}^{\rho} + Q_{\nu}{}^{\rho}{}_{\mu} + Q_{\mu\nu}{}^{\rho} - Q^{\rho}{}_{\mu\nu})$

$$R^{\rho}{}_{\sigma\mu\nu} = \hat{R}^{\rho}{}_{\sigma\mu\nu} + \nabla_{\nu} C^{\rho}{}_{\mu\sigma} - \nabla_{\mu} C^{\rho}{}_{\nu\sigma} + C^{\rho}{}_{\nu\lambda} C^{\lambda}{}_{\mu\sigma} - C^{\rho}{}_{\mu\lambda} C^{\lambda}{}_{\nu\sigma}$$

General Relativity: Riemannian geometry, torsionless, metric compatible

$$\mathcal{T}^{\rho}_{\mu\nu} = 0,\ Q_{\mu}^{\ \rho\sigma} = 0 \Rightarrow \hat{\Gamma}^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\mu\nu},\ R^{\rho}{}_{\sigma\mu\nu} = \hat{R}^{\rho}{}_{\sigma\mu\nu}$$

Teleparallel Gravity: no curvature, metric compatible

$$\hat{R}^{\rho}{}_{\sigma\mu\nu} = 0,\ Q_{\mu}^{\ \rho\sigma} = 0 \Rightarrow \hat{\Gamma}^{\rho}{}_{\mu\nu} = (\Omega^{-1})^{\rho}{}_{\sigma}\partial_{\mu}\Omega^{\sigma}{}_{\nu}\ ,\ g_{\mu\nu}(\Omega^{-1})^{\mu}{}_{\rho}(\Omega^{-1})^{\nu}{}_{\sigma} = \mathcal{G}_{\rho\sigma} = \text{const.}$$

$\Omega^{\sigma}{}_{\nu}$:elements of GL(4) with $\det|\Omega^{\sigma}{}_{\nu}| \neq 0$

Gravity is identified with torsion $\mathcal{T}^{\rho}_{\mu\nu} = \hat{\Gamma}^{\rho}_{\mu\nu} - \hat{\Gamma}^{\rho}_{\nu\mu}$

$R^{\rho}{}_{\sigma\mu\nu} = \nabla_{\nu}C^{\rho}{}_{\mu\sigma} - \nabla_{\mu}C^{\rho}{}_{\nu\sigma} + C^{\rho}{}_{\nu\lambda}C^{\lambda}{}_{\mu\sigma} - C^{\rho}{}_{\mu\lambda}C^{\lambda}{}_{\nu\sigma}$ with $C^{\rho}{}_{\mu\nu} = \frac{1}{2}(\mathcal{T}^{\rho}{}_{\mu\nu} + \mathcal{T}_{\nu}{}^{\rho}{}_{\mu} - \mathcal{T}_{\mu\nu}{}^{\rho})$

In stead of $\Omega^{\rho}{}_{\mu}$, using the language of tetrad $e^{A}{}_{\mu}$

$$g_{\mu\nu} = \eta_{AB} e^{A}{}_{\mu} e^{B}{}_{\nu} \qquad \hat{\Gamma}^{\rho}{}_{\mu\nu} = e_{A}{}^{\rho}(\partial_{\mu} e^{A}{}_{\nu} + e^{B}{}_{\nu}\omega^{A}{}_{B\mu}) \qquad \omega^{A}{}_{B\nu} = (\Lambda^{-1})^{A}_{C}\partial_{\nu}\Lambda^{C}_{B}$$

Λ^{A}_{B} : Lorentz matrix element

Weitzenböck connection (a gauge choice)

$\omega^{A}{}_{B\nu} = 0$ breaks local Lorentz symmetry, but diffeomorphism invariance is preserved

Torsion tensor or torsion two form have simple forms

$$\mathcal{T}^{\rho}_{\mu\nu} = e_{A}^{\rho}(\partial_{\mu} e^{A}{}_{\nu} - \partial_{\nu} e^{A}{}_{\mu}) \ , \ \mathcal{T}^{A}_{\mu\nu} = e^{A}{}_{\rho}\mathcal{T}^{\rho}_{\mu\nu} = \partial_{\mu} e^{A}{}_{\nu} - \partial_{\nu} e^{A}{}_{\mu}$$

GR Equivalent Teleparallel Gravity (TGR)

$$S_g = \frac{1}{2}\int d^4x \, \mathrm{e}\mathbb{T} = \int d^4x \, \mathrm{e}\left(-\frac{1}{2}\mathcal{T}_{\mu}\mathcal{T}^{\mu} + \frac{1}{8}\mathcal{T}_{\alpha\beta\mu}\mathcal{T}^{\alpha\beta\mu} + \frac{1}{4}\mathcal{T}_{\alpha\beta\mu}\mathcal{T}^{\beta\alpha\mu}\right) \qquad \mathcal{T}_{\mu} = \mathcal{T}^{\alpha}_{\mu\alpha}$$

Equivalent to GR $\quad S_g = \int d^4x \sqrt{-g}\left[-\frac{1}{2}R(e) - \nabla_{\mu}\mathcal{T}^{\mu}\right]$

→ from metric or from tetrad

Symmetric Teleparallel Gravity: no curvature, torsionless

$$\hat{R}^{\rho}{}_{\sigma\mu\nu} = 0,\ \mathcal{T}^{\rho}_{\mu\nu} = 0 \Rightarrow \hat{\Gamma}^{\rho}{}_{\mu\nu} = (\Omega^{-1})^{\rho}{}_{\sigma}\partial_{\mu}\Omega^{\sigma}{}_{\nu} \text{ with } \Omega^{\sigma}{}_{\nu} = \frac{\partial y^{\sigma}}{\partial x^{\nu}}$$

Gravity is identified with non-metricity, $Q_{\mu}{}^{\rho\sigma} = \hat{\nabla}_{\mu} g^{\rho\sigma}$

$$R^{\rho}{}_{\sigma\mu\nu} = \nabla_{\nu} C^{\rho}{}_{\mu\sigma} - \nabla_{\mu} C^{\rho}{}_{\nu\sigma} + C^{\rho}{}_{\nu\lambda} C^{\lambda}{}_{\mu\sigma} - C^{\rho}{}_{\mu\lambda} C^{\lambda}{}_{\nu\sigma} \text{ with } C^{\rho}{}_{\mu\nu} = \frac{1}{2}(Q_{\nu}{}^{\rho}{}_{\mu} + Q_{\mu\nu}{}^{\rho} - Q^{\rho}{}_{\mu\nu})$$

Building blocks: $g_{\mu\nu}$ y^{ρ}

Freedoms by y^{ρ} can be gauged away

Transform to the y-coordinate (coincident gauge)

$\hat{\Gamma}^{\rho}{}_{\mu\nu} = 0,\ \hat{\nabla} = \partial$ breaks diffeomorphism invariance

GR Equivalent Symmetric Teleparallel Gravity (TGR)

$$S_g = \frac{1}{2}\int d^4x\sqrt{-g}\mathbb{Q} \equiv \frac{1}{2}\int d^4x\,\sqrt{-g}\left[\frac{1}{4}Q_{\alpha\mu\nu}Q^{\alpha\mu\nu} - \frac{1}{2}Q_{\alpha\mu\nu}Q^{\mu\nu\alpha} - \frac{1}{4}Q^{\alpha}Q_{\alpha} + \frac{1}{2}\bar{Q}^{\alpha}Q_{\alpha}\right]$$

$$Q_\alpha = g^{\sigma\lambda}Q_{\alpha\sigma\lambda},\ \bar{Q}_\alpha = g^{\sigma\lambda}Q_{\sigma\alpha\lambda}$$

Equivalent to GR

$$S_g = \frac{1}{2}\int d^4x\sqrt{-g}\left[-R(g) + \nabla_\mu(Q^\mu - \bar{Q}^\mu)\right]$$

Nieh-Yan Modified Teleparallel Gravity (NYTG)

In Teleparallel Gravity, parity violation can be realized by the Nieh-Yan term [*H.T. Nieh & M.L. Yan, J. Math. Phys. 23, 373 (1982)*] coupled with an axion-like scalar field

$$S = \frac{1}{2}\int d^4x \mathrm{e}\mathbb{T} + S_{NY} + ...$$

$$S_{NY} = \int d^4x \; \mathrm{e} \; \frac{c\theta}{4}(\mathcal{T}_{A\mu\nu}\widetilde{\mathcal{T}}^{A\mu\nu} - \varepsilon^{\mu\nu\rho\sigma}\hat{R}_{\mu\nu\rho\sigma}) \xrightarrow{\text{Teleparallelism}} \int d^4x\sqrt{-g}\frac{c\theta}{4}\mathcal{T}_{A\mu\nu}\widetilde{\mathcal{T}}^{A\mu\nu}$$

$$\widetilde{\mathcal{T}}^{A\mu\nu} = (1/2)\varepsilon^{\mu\nu\rho\sigma}\mathcal{T}^{A}{}_{\rho\sigma}$$

With Weitzenböck condition

$$S_{NY} = \int d^4x\sqrt{-g}\;\frac{c\theta}{4}\mathcal{T}_{A\mu\nu}\widetilde{\mathcal{T}}^{A\mu\nu} = \int d^4x\sqrt{-g}\;\frac{c\theta}{8}\eta_{AB}\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu e^A_{\ \nu} - \partial_\nu e^A_{\ \mu})(\partial_\rho e^B_{\ \sigma} - \partial_\sigma e^B_{\ \rho})$$

Similar to electromagnetic Chern-Simons term

Local Lorentz invariance is broken, diffeomorphism is preserved

ML, Haomin Rao & Dehao Zhao, arXiv:2007.08038

In all, NYTG model is prosed as a minor extension to GR

$$S = S_g + S_{NY} + S_\theta + S_m = \int d^4x\sqrt{-g}[-\frac{R}{2} + \frac{c\theta}{4}\mathcal{T}_{A\mu\nu}\widetilde{\mathcal{T}}^{A\mu\nu} + \frac{1}{2}\nabla_\mu\theta\nabla^\mu\theta - V(\theta)] + S_m$$

with $\mathcal{T}^A_{\mu\nu} = \partial_\mu e^A_{\ \nu} - \partial_\nu e^A_{\ \mu}$ Weitzenböck connection is taken here

EOMs

10 field equations $G^{\mu\nu} + N^{\mu\nu} = T^{\mu\nu} + T^{\mu\nu}_\theta$ $N^{\mu\nu} = ce_A^{\ \nu}\partial_\rho\theta\widetilde{\mathcal{T}}^{A\mu\rho} = c\partial_\rho\theta\widetilde{\mathcal{T}}^{\nu\mu\rho}$

6 constraint equations $N^{\mu\nu} = N^{\nu\mu}$

Using Bianchi identity $\nabla_\mu N^{\mu\nu} = \nabla_\mu T^{\mu\nu}_\theta$

consistent with Klein-Gordon eq $\Box\theta + V_\theta - \frac{c}{4}\mathcal{T}_{A\mu\nu}\widetilde{\mathcal{T}}^{A\mu\nu} = 0$

because $\nabla_\mu N^{\mu\nu} = (c/4)\mathcal{T}_{A\rho\sigma}\widetilde{\mathcal{T}}^{A\rho\sigma}\nabla^\nu\theta$ $\nabla_\mu T^{\mu\nu}_\theta = (\Box\theta + V_\theta)\nabla^\nu\theta$

Applications in cosmology

Spatially flat FRW universe: background

$$ds^2 = a^2(d\eta^2 - \delta_{ij}dx^i dx^j)$$

Tetrads in the background

$$e^A_{\ \mu} = a(\eta)\delta^A_\mu$$

This is a particular choice

EOMs

$$3\mathcal{H}^2 = a^2(\rho_\theta + \rho)\ ,\ 2\mathcal{H}' + \mathcal{H}^2 = -a^2(p_\theta + p)\ ,\ \theta'' + 2\mathcal{H}\theta' + a^2 V_\theta = 0$$

Nieh-Yan term does not affect the background evolution!

Perturbations around the background — This is also a particular choice

$$e^0{}_0 = a(1 + A)\ ,\ e^0{}_i = a(\partial_i\beta + \beta_i^V)\ ,\ e^a{}_0 = a\delta_{ai}(\partial_i\gamma + \gamma_i^V)\ ,$$
$$e^a{}_i = a\delta_{aj}[(1-\psi)\delta_{ij} + \partial_j\partial_i\alpha + \partial_i\alpha_j^V + \epsilon_{ijk}(\partial_k\lambda + \lambda_k^V) + \frac{1}{2}h_{ij}^T]$$

Izumi & Ong, arXiv:1212.5774

Metric perturbations

$$g_{00} = a^2(1 + 2A)\ ,\ g_{0i} = -a^2(\partial_i(\gamma - \beta) + \gamma_i^V - \beta_i^V)\ ,$$
$$g_{ij} = -a^2[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j\alpha + \partial_i\alpha_j^V + \partial_j\alpha_i^V + h_{ij}^T]$$

In comparison with traditional perturbation theory based on metric:

Besides the scalar perturbations: $A, \gamma - \beta, \psi, \alpha$, vector perturbations: $\gamma_i^V - \beta_i^V, \alpha_i^V$, and tensor perturbation: h_{ij}^T in the metric, the parametrization of tetrad brings extra scalar perturbation λ and vector perturbation λ_i^V. All the

Matter perturbations

$$T^0_{\ 0} = \rho + \delta\rho\ ,\ T^0_{\ i} = (\rho + p)(\partial_i v + v^V_i)$$
$$T^i_{\ j} = -(p + \delta p)\delta^i_j + \Sigma^i_{\ j}\ .$$

$$\Sigma^i_{\ j} = \partial_i\partial_j\sigma - (1/3)\nabla^2\sigma\delta_{ij} + \partial_{(i}\sigma^V_{j)} + \sigma^T_{ij}$$

Perturbation equations (Scalar, Newtonian gauge $\gamma - \beta = 0,\ \alpha = 0$)

$$2k^2\psi + 6\mathcal{H}(\psi' + \mathcal{H}A) = -(\theta'\delta\theta' - \theta'^2 A + a^2 V_\theta\delta\theta) - a^2\delta\rho$$
$$2\psi' + 2\mathcal{H}A = \theta'\delta\theta + a^2(\rho + p)v$$
$$2\psi'' + 2\mathcal{H}(A' + 2\psi') + (2\mathcal{H}^2 + 4\mathcal{H}')A = (\theta'\delta\theta' - \theta'^2 A - a^2 V_\theta\delta\theta) + a^2(\delta p + \frac{2}{3}k^2\sigma)$$
$$\psi - A = c\theta'\lambda - a^2\sigma\ .$$
$$\delta\theta'' + 2\mathcal{H}\delta\theta' + k^2\delta\theta + a^2 V_{\theta\theta}\delta\theta - \theta'(A' + 3\psi') + 2a^2 V_\theta A = -2c\mathcal{H}k^2\lambda$$

Nieh-Yan contributes a viscosity

$\theta'\psi + \mathcal{H}\delta\theta = 0$ ⟶ A new constraint led by $N^{\mu\nu} = N^{\nu\mu}$

Curvature perturbation to hypersurfaces of homogeneous θ vanishes $\zeta_\theta = -\psi - \mathcal{H}\frac{\delta\theta}{\theta'} = 0$

The dynamical scalar field θ is not an independent dynamical DOF!

Perturbation equations (Vector)

$\beta_i^V = \lambda_i^V = 0$ ⟶ Constraint led by $N^{\mu\nu} = N^{\nu\mu}$

Other equations are the same with GR

Perturbation equations (Tensor)

$$h_{ij}^{T\,\prime\prime} + 2\mathcal{H} h_{ij}^{T\,\prime} + k^2 h_{ij}^T + c\theta'(ik_l)\epsilon_{lk(i}h_{j)k}^T = -2a^2\sigma_{ik}^T$$

$N^{\mu\nu} = N^{\nu\mu}$ puts no extra constraint on tensor perturbations

Using left and right circular polarization bases

$$h_{ij}^T = h^L \hat{e}_{ij}^L + h^R \hat{e}_{ij}^R$$
$$\sigma_{ij}^T = \sigma^L \hat{e}_{ij}^L + \sigma^R \hat{e}_{ij}^R$$

$$\boxed{h^{A\prime\prime} + 2\mathcal{H} h^{A\prime} + (k^2 + c\lambda_A \theta' k)h^A = -2a^2\sigma^A}$$

$n_l \epsilon_{lik} \hat{e}_{jk}^A = i\lambda_A \hat{e}_{ij}^A$, here $A = L, R$ and $\lambda_L = -1, \lambda_R = 1$, $\vec{n}$ is the unit vector of $\vec{k}$.

After renormalization $v^A = ah^A$

$$v^{A''} + (\omega_A^2 - a''/a)v^A = 0$$

modified dispersion relation

$$\omega_A^2 = k^2 + \lambda_A c\theta' k = k^2(1 + \lambda_A c\theta'/k)$$

Left and right polarization modes have different phase velocities, phenomenon called velocity birefringence, a property of **parity violation**

$$v_p^A = \omega_A/k \simeq 1 + \lambda_A c\theta'/(2k)$$

Same group velocity

$$v_g = d\omega_A/dk \simeq 1 + c^2\theta'^2/(8k^2)$$

Infrared effects, important when $k \to 0$

Quadratic actions for scalar and tensor perturbations

$$S = S_g + S_{NY} + S_\theta$$

Scalar perturbation

$$\zeta = -\psi - \mathcal{H}\delta\theta/\theta' = 0$$

$$S_\zeta^{(2)} = 0$$

The dynamical scalar field θ is not an independent dynamical DOF!

Quadratic action for tensor perturbation

$$S = \int d^4x \, \frac{a^2}{8} \left(h_{ij}^{T\prime} h_{ij}^{T\prime} - \partial_k h_{ij}^T \partial_k h_{ij}^T - c\theta' \epsilon_{ijk} h_{il}^T \partial_j h_{kl}^T \right)$$

or in Fourier space

$$S = \sum_{A=L,R} \int d\eta d^3k \, \frac{a^2}{4} \left[h^{A*\prime} h^{A\prime} - (k^2 + c\theta' \lambda_A k) h^{A*} h^A \right]$$

$$h_{ij}^T(t, \vec{x}) = \sum_{A=L,R} \int \frac{d^3k}{(2\pi)^{3/2}} \, h^A(t, \vec{k}) \, \hat{e}_{ij}^A(\vec{k}) \, e^{ik_j x^j}$$

No ghost instability, the model is healthy.

Maybe there is gradient instability for extremely small k so that $1 + c\theta' \lambda_A / k < 0$

for small coupling and slow-rolling θ, this happens at scales well outside the horizon

Experimental constraints: $c\theta' < 6.5 \times 10^{-42}$ GeV, Wu, Zhu, Niu, Zhao & Wang, arXiv:2110.13872

Generalizations

1, Restore the local Lorentz symmetry,
i.e., consider the covariant version of the NYTG model;

2, Apply to the universes of all the three kinds of FRW backgrounds,
i.e., $K=0$, $K>0$, and $K<0$.

ML, Haomin Rao & Yeheng Tong, arXiv: 2104.05917

Covariantization

Restore the local Lorentz covariance by giving up the Weitzenböck condition

$$\mathcal{T}^A{}_{\mu\nu} = 2(\partial_{[\mu} e^A{}_{\nu]} + \omega^A{}_{B[\mu} e^B{}_{\nu]}) \qquad \omega^A{}_{B\nu} = (\Lambda^{-1})^A_C \partial_\nu \Lambda^C_B$$

Fundamental variables $e^A{}_\mu \quad \Lambda^A_B$

The action

$$S = \int d^4x \sqrt{-g}\left[-\frac{R(e)}{2} + \frac{c}{4}\theta\, \mathcal{T}_{A\mu\nu}\widetilde{\mathcal{T}}^{A\mu\nu} + \frac{1}{2}\nabla_\mu\theta\nabla^\mu\theta - V(\theta)\right] + S_m$$

invariant under the local Lorentz transformation $e^A{}_\mu \to (L^{-1})^A{}_B e^B{}_\mu\,,\ \Lambda^A{}_B \to \Lambda^A{}_C L^C{}_B$

Variation with the tetrad $G^{\mu\nu} + N^{\mu\nu} = T^{\mu\nu} + T^{\mu\nu}_\theta$

Variation with Λ^A_B : $N^{[\mu\nu]} = 0$

Advantage: we can always start with the tetrad with familiar and simple form

Applications to the cosmology

Background

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a(\eta)\left(d\eta^2 - \frac{dr^2}{1-Kr^2} - r^2 d\Omega^2\right)$$

or $$ds^2 = a^2(d\eta^2 - \gamma_{ij}dx^i dx^j)$$

We can always take $$e^0{}_0 = a\ ,\ e^0{}_i = 0\ ,\ e^a{}_0 = 0\ ,\ e^a{}_i = a\gamma^a{}_i$$

Impose symmetry requirement (this may be relaxed, we will come back to it in the future)

$$\mathcal{L}_{\hat{\xi}}\hat{\Gamma}^\rho{}_{\mu\nu} = 0$$ $\mathcal{L}_{\hat{\xi}}$ is the Lie derivative along the Killing vector field $\hat{\xi}$.

(1) $K = 0$

The solution is parametrized as

$$e^A{}_\mu = a(\eta) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin\varphi\cos\phi & r\cos\varphi\cos\phi & -r\sin\varphi\sin\phi \\ 0 & \sin\varphi\sin\phi & r\cos\varphi\sin\phi & r\sin\varphi\cos\phi \\ 0 & \cos\varphi & -r\sin\varphi & 0 \end{pmatrix} \,, \quad \Lambda = \mathring{\Lambda}$$

$$3\mathcal{H}^2 = a^2(\rho_\theta + \rho) \;,\; 2\mathcal{H}' + \mathcal{H}^2 = -a^2(p_\theta + p) \;,\; \theta'' + 2\mathcal{H}\theta' + a^2 V_\theta = 0$$

Nieh-Yan term has no effect on the background of the flat universe, confirmed our previous result

(2) $K > 0$

The solution is parametrized as

$$e^A{}_\mu = a(\eta)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin\varphi\cos\phi/\mathcal{R} & r\left(\mathcal{R}\cos\varphi\cos\phi + \mathcal{K}r\sin\phi\right) & r\sin\varphi\left(\mathcal{K}r\cos\varphi\cos\phi - \mathcal{R}\sin\phi\right) \\ 0 & \sin\varphi\sin\phi/\mathcal{R} & r\left(\mathcal{R}\cos\varphi\sin\phi - \mathcal{K}r\cos\phi\right) & r\sin\varphi\left(\mathcal{R}\cos\phi + \mathcal{K}r\cos\varphi\sin\phi\right) \\ 0 & \cos\varphi/\mathcal{R} & -r\mathcal{R}\sin\varphi & -\mathcal{K}r^2(\sin\varphi)^2 \end{pmatrix}, \quad \Lambda = \mathring{\Lambda}$$

$\mathcal{R} = \sqrt{1 - Kr^2}$ and the real parameter $\mathcal{K}$ satisfies $\mathcal{K}^2 = K$

$$3(\mathcal{H}^2 + \mathcal{K}^2) = a^2\left(\rho_\theta + \rho\right)\ ,\ 2\mathcal{H}' + \mathcal{H}^2 + \mathcal{K}^2 - 2c\mathcal{K}\theta' = -a^2\left(p_\theta + p\right)\ ,\ \theta'' + 2\mathcal{H}\theta' + a^2 V_\theta - 6c\mathcal{K}\mathcal{H} = 0$$

Nieh-Yan modification has effect on the background of closed universe

(3) $K < 0$

The solution is parametrized as

$$e^A{}_\mu = a(\eta)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\mathcal{R} & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r\sin\varphi \end{pmatrix} \; , \quad \Lambda = \mathring{\Lambda}\cdot\begin{pmatrix} \mathcal{R} & \mathcal{K}r & 0 & 0 \\ \mathcal{K}r\sin\varphi\cos\phi & \mathcal{R}\sin\varphi\cos\phi & \cos\varphi\cos\phi & -\sin\phi \\ \mathcal{K}r\sin\varphi\sin\phi & \mathcal{R}\sin\varphi\sin\phi & \cos\varphi\sin\phi & \cos\phi \\ \mathcal{K}r\cos\varphi & \mathcal{R}\cos\varphi & -\sin\varphi & 0 \end{pmatrix}$$

$\mathcal{R} = \sqrt{1 - Kr^2}$ and the real parameter $\mathcal{K}$ satisfies $\mathcal{K}^2 = -K$

$$3(\mathcal{H}^2 - \mathcal{K}^2) = a^2\,(\rho_\theta + \rho) \;\; , \;\; 2\mathcal{H}' + \mathcal{H}^2 - \mathcal{K}^2 = -a^2\,(p_\theta + p) \;\; , \;\; \theta'' + 2\mathcal{H}\theta' + a^2 V_\theta = 0$$

Nieh-Yan term has no effect on the background of the open universe

Perturbations

Tetrad perturbations

$$e^0{}_0 = a(1+A)\ ,\ e^0{}_i = a(D_i\beta + \beta^V{}_i)\ ,\ e^a{}_0 = a\gamma^{ai}(D_i\chi + \chi^v_i)\ ,$$
$$e^a{}_i = a\gamma^{aj}\left[(1-\psi)\gamma_{ij} + D_iD_j\alpha + D_i\alpha^V_j - \varepsilon_{ij}{}^k(D_k\lambda + \lambda^V_k) + \frac{1}{2}h^T_{ij}\right]$$

Consistent with but more than the metric perturbations

$$g_{00} = a^2(1+2A)\ ,\ g_{0i} = -a^2(D_iB + B^V_i)\ ,$$
$$g_{ij} = -a^2\left[(1-2\psi)\gamma_{ij} + 2D_iD_j\alpha + D_i\alpha^V_j + D_j\alpha^V_i + h^T_{ij}\right]$$

Perturbations to the spin connection or Λ^A_B

$$\Lambda = \bar{\Lambda}\exp(\epsilon)$$

$$\epsilon^0{}_a = \gamma_a{}^i(D_i\kappa + \kappa^V_i)\ ,\ \epsilon^a{}_b = \gamma^{ai}\gamma_b{}^j\varepsilon_{ij}{}^k(D_k\tau + \tau^V_k)$$

Taking the gauge $\delta\theta = 0, \ \alpha = 0, \ \kappa = 0, \ \tau = 0.$

Gauge-invariant scalar perturbation $\zeta = -\psi - \mathcal{H}\delta\theta/\theta' = -\psi$

Quadratic actions for scalar perturbations

(1) $K = 0$

$$S^{(2)} = -\int d^4x \ a^2\gamma \left\{3\zeta'^2 - 6\mathcal{H}\zeta' A + (2A+\zeta)D^2\zeta + a^2 V A^2 + 2(\mathcal{H}A - \zeta')D^2 B + 2c\theta'\zeta D^2\lambda\right\}$$

Constraints

$$\zeta = 0 \ ,$$
$$\zeta' - \mathcal{H}A = 0 \ ,$$
$$-3\mathcal{H}\zeta' + D^2\zeta + a^2 V A + \mathcal{H}D^2 B = 0$$

No dynamical scalar perturbation in this case, confirmed our previous result.

(2) $K > 0$

$$S^{(2)} = -\int d^4x\, a^2\gamma \Big\{3\zeta'^2 - 6\mathcal{H}\zeta' A + (2A+\zeta)D^2\zeta + (a^2V - 3\mathcal{K}^2)A^2 + 3(\mathcal{K}^2 - 3c\mathcal{K}\theta')\zeta^2 + 6\mathcal{K}^2 A\zeta + 2(\mathcal{H}A - \zeta')D^2B - \mathcal{K}^2 BD^2B + c\theta'(2\zeta + \mathcal{K}\lambda)D^2\lambda - c\mathcal{K}\theta'\beta D^2\beta\Big\} .$$

constraints

$$\beta = 0\,,$$
$$\zeta + \mathcal{K}\lambda = 0\,,$$
$$\zeta' - \mathcal{H}A + \mathcal{K}^2 B = 0\,,$$
$$-3\mathcal{H}\zeta' + D^2\zeta + 3\mathcal{K}^2\zeta + (a^2V - 3\mathcal{K}^2)A + \mathcal{H}D^2B = 0$$

$$\zeta(\eta,\vec{x}) = \sum_{n\geq 2}^{\infty}\sum_{l=0}^{n-1}\sum_{m=-l}^{l}\zeta_{nlm}(\eta)\mathcal{S}_{nlm}(\vec{x})$$

$$\mathcal{S}_{nlm}(\vec{x}) = K^{\frac{3}{4}}\sin^l\chi\,\frac{d^{l+1}(\cos n\chi)}{d(\cos\chi)^{l+1}}\,Y_{lm}(\varphi,\phi)$$

After lifting the constraints

$$S^{(2)} = \sum_{n\geq 2}^{\infty}\sum_{l=0}^{n-1}\sum_{m=-l}^{l}\int d\eta\, z^2\left(\frac{1}{2}|\zeta'_{nlm}|^2 - \frac{1}{2}\omega^2|\zeta_{nlm}|^2\right)$$

where

$$z^2 = \frac{a^2\theta'^2}{\mathcal{H}^2}\left(1 - \frac{\theta'^2}{\mathcal{F}_n}\right)$$

$$\omega^2 = k^2 + \frac{4\mathcal{K}^2}{3\mathcal{F}_n}\left[a^2V_n - (n^2-1)\theta'^2\right] + \frac{c\mathcal{K}}{\theta'}\left[2(n^2-4)\mathcal{H}^2 - 12\mathcal{K}^2 + \theta'^2\left(1 + \frac{4\mathcal{K}^2}{\mathcal{F}_n}\right)\right]$$

$k^2 = (n^2-1)\mathcal{K}^2$, $V_n = (n^2-4)(V + 3\mathcal{H}V_\theta/\theta')$ and $\mathcal{F}_n = 2(n^2-4)\mathcal{H}^2 + \theta'^2$

(3) $K < 0$

$$S^{(2)} = -\int d^4x\, a^2\gamma \left\{3\zeta'^2 - 6\mathcal{H}\zeta' A + (2A+\zeta)D^2\zeta + (a^2V + 3\mathcal{K}^2)A^2 - 3\mathcal{K}^2\zeta^2 - 6\mathcal{K}^2 A\zeta + 2(\mathcal{H}A - \zeta')D^2B + \mathcal{K}^2 BD^2B + 2c\theta'(\zeta - \mathcal{K}\beta)D^2\lambda\right\} .$$

constraints

$$\lambda = 0 ,$$
$$\zeta - \mathcal{K}\beta = 0 ,$$
$$\zeta' - \mathcal{H}A - \mathcal{K}^2 B = 0 ,$$
$$-3\mathcal{H}\zeta' + D^2\zeta - 3\mathcal{K}^2\zeta + (a^2V + 3\mathcal{K}^2)A + \mathcal{H}D^2B = 0$$

After lifting the constraints

$$S^{(2)} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\int_0^{\infty} dn \int d\eta\, z^2 \left(\frac{1}{2}|\zeta'_{nlm}|^2 - \frac{1}{2}\omega^2|\zeta_{nlm}|^2\right)$$

$$\zeta(\eta,\vec{x}) = \sum_{l=0}^{\infty}\sum_{m=l}^{l}\int_0^{\infty} dn\; \zeta_{nlm}(\eta)\mathcal{Z}_{nlm}(\vec{x})$$

$$\mathcal{Z}_{nlm}(\vec{x}) = K^{\frac{3}{4}}\frac{\Gamma(in+l+1)}{\Gamma(in)}(\sinh\chi)^{-\frac{1}{2}}\, P^{-l-\frac{1}{2}}_{in-\frac{1}{2}}(\cosh\chi)\, Y_{lm}(\varphi,\phi)$$

$$z^2 = \frac{a^2\theta'^2}{\mathcal{H}^2}\left(1 + \frac{\theta'^2}{\mathcal{F}_n}\right) , \quad \omega^2 = k^2 + \frac{4\mathcal{K}^2}{3\mathcal{F}_n}\left[(n^2+1)\theta'^2 - a^2V_n\right]$$

$k^2 = (n^2+1)\mathcal{K}^2$, $V_n = (n^2+4)(V + 3\mathcal{H}V_\theta/\theta')$ and $\mathcal{F}_n = 2[3\mathcal{K}^2 + (n^2+1)\mathcal{H}^2 + a^2V]$.

Quadratic actions for tensor perturbations

$K = 0$

$$S^{(2)} = \sum_{A=L,R} \int d\eta d^3k \, \frac{a^2}{4} \left[h^{A*'} h^{A'} - \left(k^2 + c\theta' \lambda_A k\right) h^{A*} h^A \right]$$

The same with our previous result

$K \neq 0$

$$h^T_{ij}(\eta, \vec{x}) = \sum_{A=L,R} \sum_k h^A_{nlm}(\eta) (Q^A_{nlm})_{ij}(\vec{x})$$

$$S^{(2)} = \sum_{A=L,R} \sum_k \int d\eta \, \frac{a^2}{4} \left[(h^{A'}_{nlm})^2 - \omega_A^2 (h^A_{nlm})^2 \right] ,$$

$$\sum_k \equiv \begin{cases} \sum_{n=3}^{\infty} \sum_{l=2}^{n-1} \sum_{m=-l}^{l} & (K > 0) \\ \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \int_0^{\infty} dn & (K < 0) \end{cases}$$

$$\omega_A^2 = \begin{cases} k^2 + 2\mathcal{K}^2 + c\theta' \mathcal{K} + \lambda_A c\theta' k \sqrt{1 + \frac{3\mathcal{K}^2}{k^2}} & (K > 0) \\ k^2 - 2\mathcal{K}^2 + \lambda_A c\theta' k \sqrt{1 - \frac{3\mathcal{K}^2}{k^2}} & (K < 0) \end{cases}$$

Circular tensor base

$$D^2 (Q^A_{nlm})_{ij} = -k^2 (Q^A_{nlm})_{ij}$$

$$\varepsilon_{kl(i} D^k (Q^A_{nlm})^l{}_{j)} = -\lambda_A k \sqrt{1 + \frac{3K}{k^2}} \, (Q^A_{nlm})_{ij}$$

Parity Violating Model in Symmetric Teleparallel Gravity

$$S = \frac{1}{2}\int d^4x\sqrt{-g}(\mathbb{Q} - 2c\phi Q\tilde{Q}) + S_\phi + S_m \qquad Q\tilde{Q} \equiv \epsilon^{\mu\nu\rho\sigma}Q_{\mu\nu\alpha}Q_{\rho\sigma}{}^{\alpha}$$

Proposed first by Conroy & Koivisto, arXiv: 1908.04313

$$\hat{\Gamma}^{\rho}{}_{\mu\nu} = (\Omega^{-1})^{\rho}{}_{\sigma}\partial_\mu\Omega^{\sigma}{}_{\nu} \text{ with } \Omega^{\sigma}{}_{\nu} = \frac{\partial y^\sigma}{\partial x^\nu}$$

Fundamental variables $g_{\mu\nu},\ y^\rho$

Coincident gauge $\hat{\Gamma}^{\rho}{}_{\mu\nu} = 0$, breaks diffeomorphism invariance

Cosmological perturbations

$$ds^2 = a^2\left\{(1+2A)d\eta^2 + 2(\partial_i B + B_i)d\eta dx^i - [(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E + \partial_i E_j + \partial_j E_i + h_{ij}]dx^i dx^j\right\}$$

$$y^\mu = x^\mu + u^\mu \text{ with } u^\mu = (u^0, \partial_i u + u_i)$$

ML & Dehao Zhao, arXiv:2108.01337

Background and scalar perturbation equations are unchanged

Vector perturbations are promoted to be dynamical degrees of freedom

$$(f_A u^A)'' + (k^2 + 2\lambda_A k\mathcal{M} - \frac{f_A''}{f_A})(f_A u^A) = 0$$

$$f_A \equiv a\sqrt{|\mathcal{M}/(k^2 + 2\lambda_A k\mathcal{M})|}$$

$$\mathcal{M} \equiv 2c(2\mathcal{H}\phi + \phi') \qquad \lambda_A = +1(A = R), -1(A = L)$$

Amplitude and velocity birefringence phenomena, parity violation

Tensor perturbations

$$h^{A''} + 2\mathcal{H}h^{A'} + k^2 h^A - 4\lambda_A \mathcal{M} k h^A = -2a^2\sigma^A$$

$$\omega_A^2 = k^2 - 4\lambda_A \mathcal{M} k \qquad v_p^A = \frac{\omega_A}{k} \approx 1 - \frac{2\lambda_A \mathcal{M}}{k}$$

Velocity birefringence, parity violation

Quadratic action for scalar perturbation

$$S_S^{(2)} = \int d^4x \, z^2 \left(\zeta'^2 - \partial_i\zeta\partial_i\zeta\right) \qquad z^2 = a^2\phi'^2/(2\mathcal{H}^2).$$

Quadratic action for vector perturbations

Taking Newtonian gauge $E_i = 0$

$$S_V^{(2)} = \int d^4x \, a^2 \left\{\frac{1}{4}\partial_j B_i \partial_j B_i - \frac{\mathcal{M}}{2}\epsilon_{ijk}\left(B_i\partial_j B_k + 2B_i\partial_j u_k' + u_i'\partial_j u_k' - \partial_m u_i \partial_j \partial_m u_k\right)\right\}$$

$$-\Delta B_i + 2\mathcal{M}\epsilon_{ijk}\left(\partial_j B_k + \partial_j u_k'\right) = 0$$

$$B_i(\eta, \vec{x}) = \sum_{A=L,R} \int \frac{d^3k}{(2\pi)^{3/2}} B^A(\eta, \vec{k}) e_i^A(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$$

$$S_V^{(2)} = \frac{1}{2} \sum_{A=L,R} \int d\eta d^3k \, z^2 \left(u^{A\prime} u^{A*\prime} - \omega_A^2 u^A u^{A*} \right)$$

$\omega_A^2 = k^2 + 2\lambda_A \mathcal{M} k$, and $\quad z^2 = \dfrac{\lambda_A a^2 \mathcal{M}\, k^3}{k^2 + 2\lambda_A \mathcal{M} k}$

Amplitude and velocity birefringence

$k \ll |2\lambda_A \mathcal{M}|$, $z^2 \simeq a^2 k^2/2$

$k \gg |2\lambda_A \mathcal{M}|$, and $z^2 \simeq \lambda_A a^2 \mathcal{M} k$ Ghost mode for λ_L (λ_R) if $\mathcal{M} > 0$ (< 0)

Taking coincident gauge $\hat{\Gamma}=0, i.e., u_k=0$

$$S_V^{(2)}=\int d^4x\, a^2[\frac{1}{4}\left(\partial_j B_i\partial_j B_i+2\partial_j B_i\partial_j E_i'+\partial_j E_i'\partial_j E_i'\right)-\frac{1}{2}\mathcal{M}\epsilon_{ijk}\left(B_i\partial_j B_k-\partial_m E_i\partial_j\partial_m E_k\right)]$$

$$-\Delta B_i-\Delta E_i'+2\mathcal{M}\epsilon_{ijk}\partial_j B_k=0$$

$$S_V^{(2)} \quad = \frac{1}{2}\sum_{A=L,R}\int d\eta d^3k\, z^2\left(E^{A\prime}E^{A*\prime}-\omega_A^2 E^A E^{A*}\right)$$

Gauge-invariant variable $V_i=E_i-u_i$, $V_i=-u_i$ (Newtonian), $V_i=E_i$ (coincident)

Quadratic action for tensor perturbation

$$S_T^{(2)} = \sum_{A=L,R} \int d\eta d^3k \frac{a^2}{8} \left[h^{A\prime} h^{A*\prime} - \left(k^2 - 4\lambda_A \mathcal{M} k\right) h^A h^{A*} \right]$$

No dangerous mode, velocity birefringence

Summary

	$R+\theta R\tilde{R}$	$\mathbb{T}+\theta T\tilde{T}$	$\mathbb{Q}+\theta Q\tilde{Q}$
Scalar Perturbation	OK	OK	OK
Vector Perturbations	OK	OK	Amplitude and velocity birefringence, ghost mode, pathological
Tensor Perturbations	Amplitude birefringence, ghost mode, pathological	Velocity birefringence, healthy	Velocity birefringence, healthy

A naive conjecture: amplitude birefringence is accompanied by ghost!

Thanks!