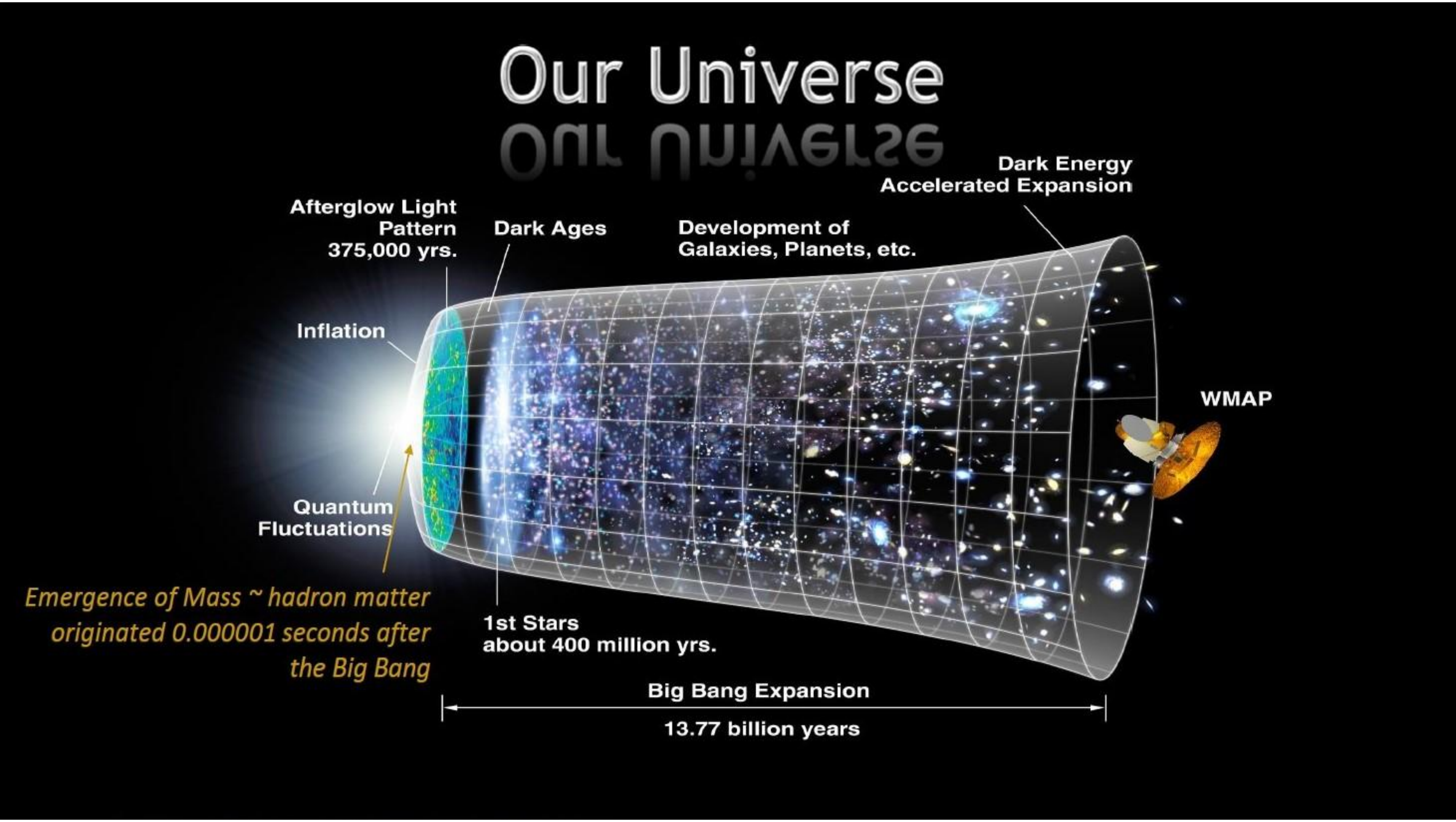
Continuum Schwinger Function Methods for Hadron Physics

Chen Chen

Interdisciplinary Center for Theoretical Study
&
Peng Huanwu Center for Fundamental Theory

March 16th, 2022

Background



Our Universe
Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
Emergence of Mass ~ hadron matter
originated 0.000001 seconds after
the Big Bang
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years

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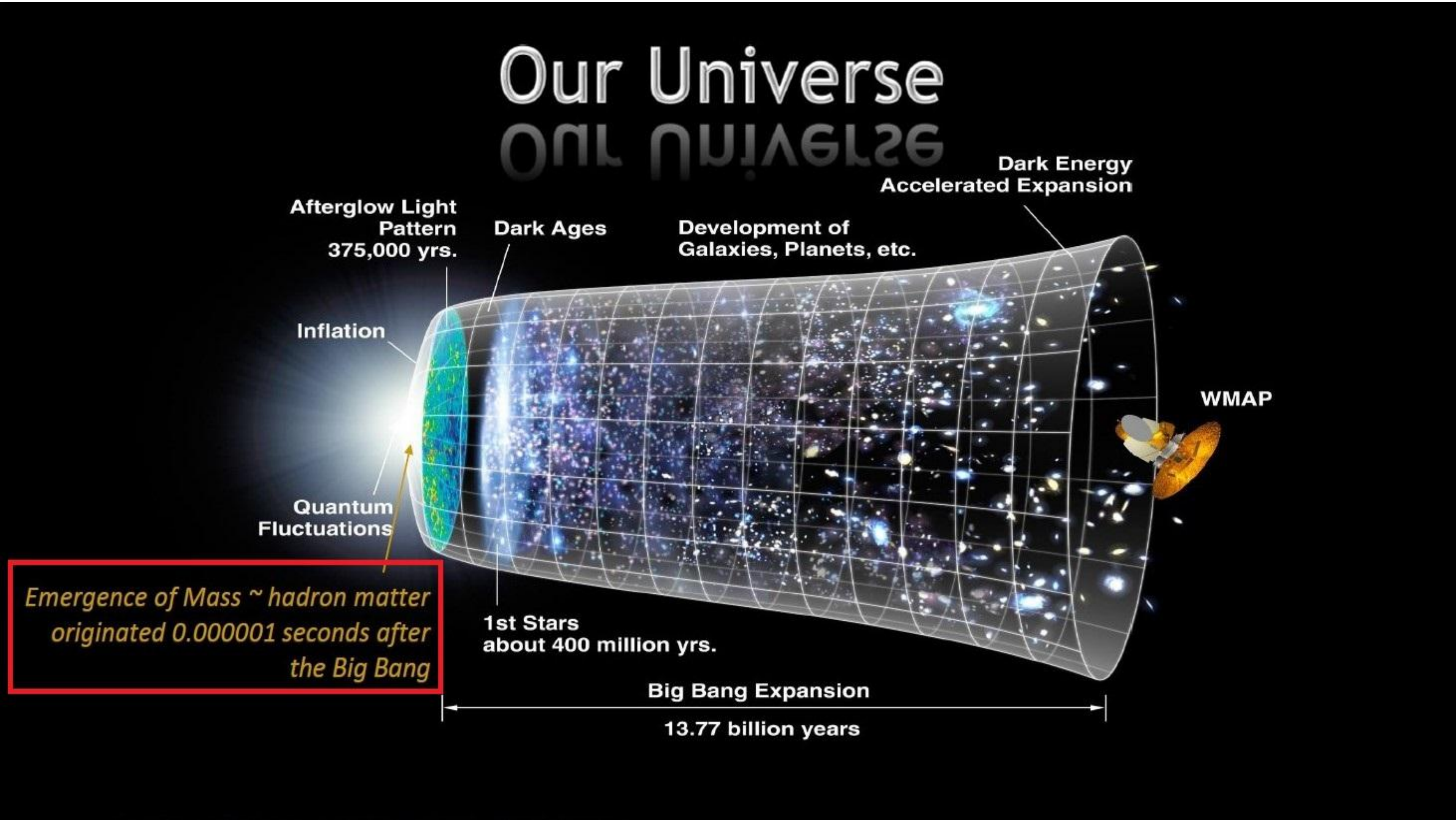

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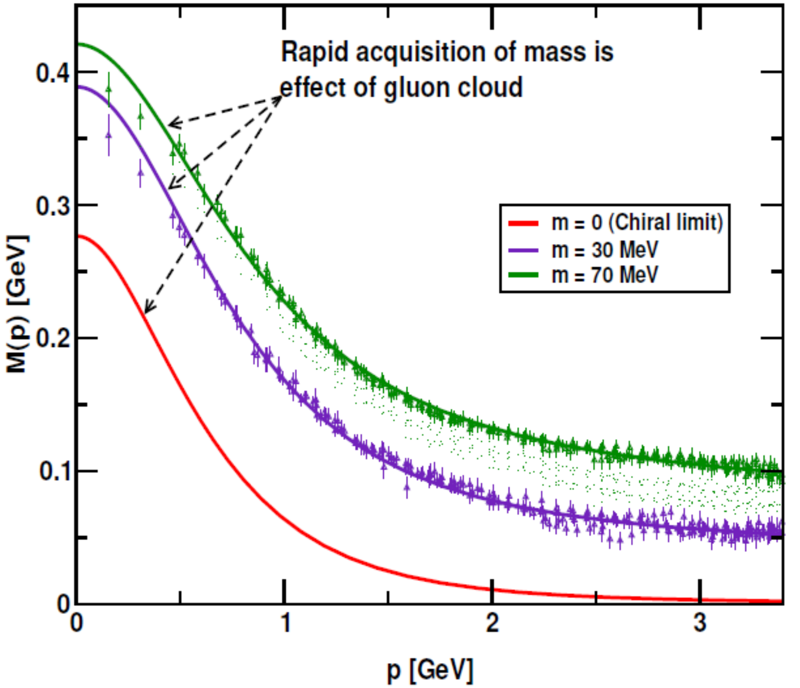
Non-Perturbative QCD:

- **Hadrons, as bound states, are dominated by non-perturbative QCD dynamics – *Two emergent phenomena***
  - **Confinement**: Colored particles have never been seen isolated
    - ✓ Explain how quarks and gluons bind together
  - **Dynamical Chiral Symmetry Breaking (DCSB)**: Hadrons do not follow the chiral symmetry pattern
    - ✓ Explain the most important mass generating mechanism for visible matter in the Universe
  - Neither of these phenomena is apparent in **QCD** 's Lagrangian, HOWEVER, They play a dominant role in determining the characteristics of real-world **QCD**!

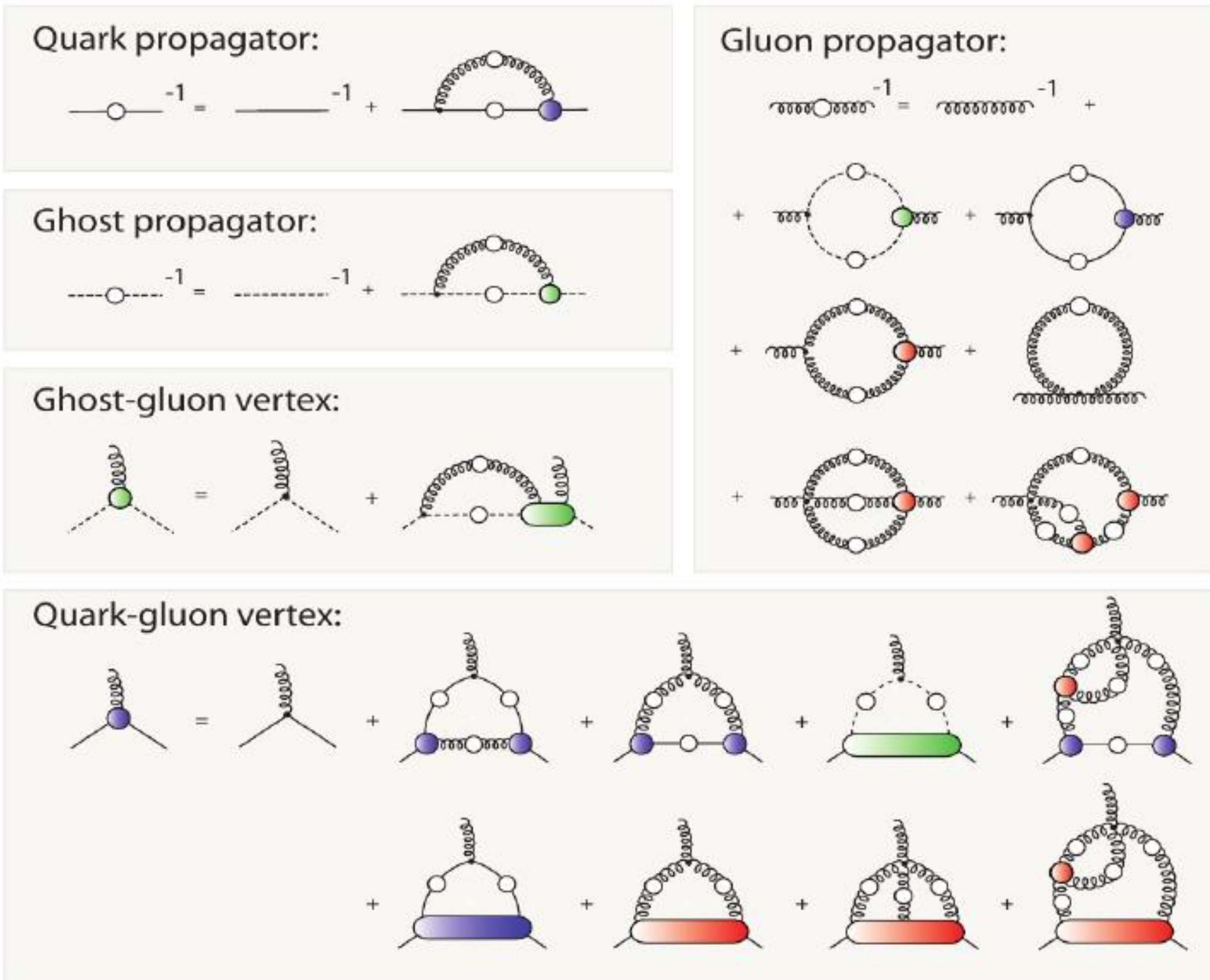
Non-Perturbative QCD:

- **From a quantum field theoretical point of view**, these emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD 's Schwinger functions (propagators and vertices). The Schwinger functions are solutions of the quantum equations of motion (**Dyson-Schwinger equations**).
- **Dressed-quark propagator:**



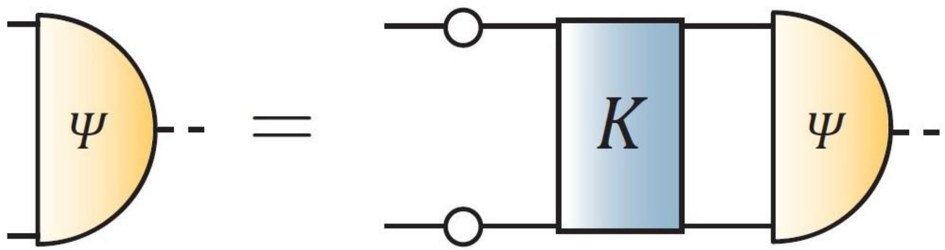
  - Mass generated from the interaction of quarks with the gluon.
  - Light quarks acquire a **HUGE** constituent mass.
  - Responsible of the 98% of the mass of the proton and the large splitting between parity partners.

Dyson-Schwinger equations (DSEs)

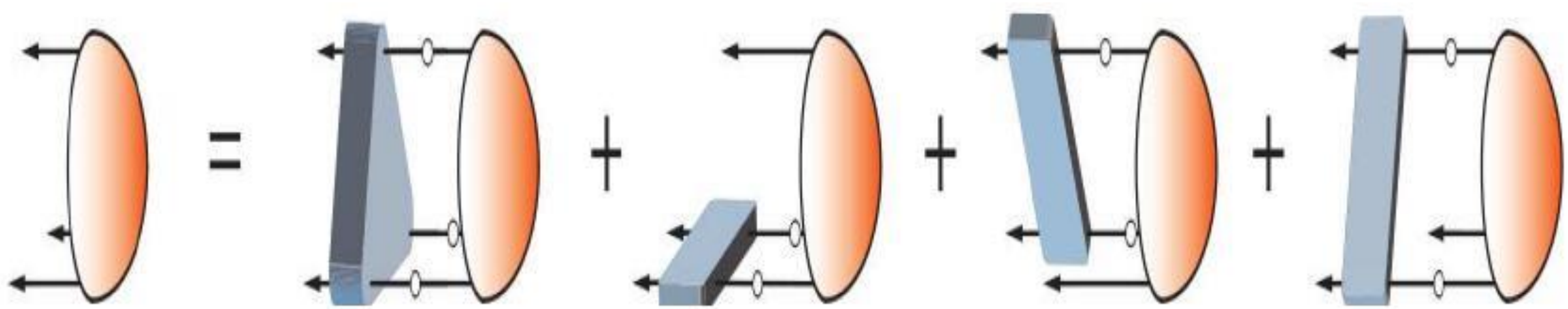


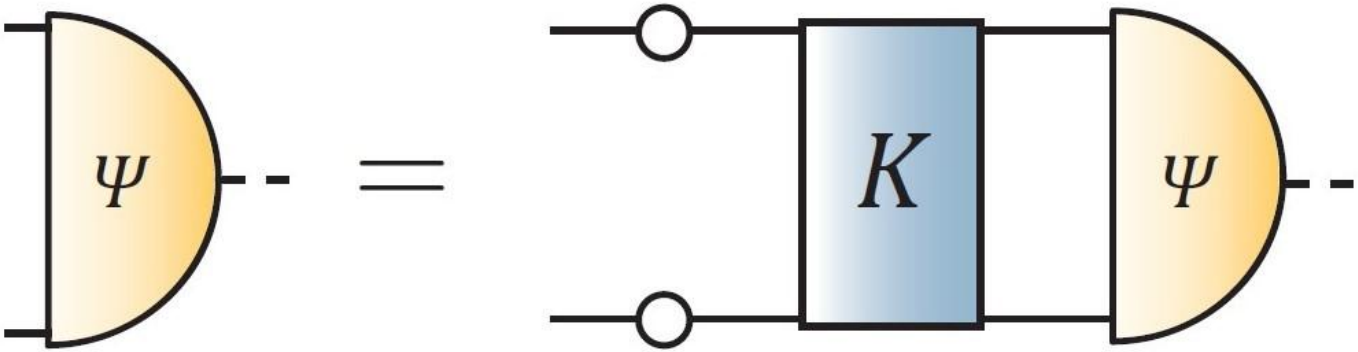
Hadrons: Bound-states in QFT

- **Mesons: a 2-body bound state problem in QFT**
  - ***Bethe-Salpeter Equation***
  - ***K*** **- fully amputated, two-particle irreducible, quark-antiquark scattering kernel**



- **Baryons: a 3-body bound state problem in QFT**
- ***Faddeev equation*: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.**





1. Mesons

Valence-quark distribution functions in the kaon and pion

***Phys. Rev. D* 93 (2016) 7, 074021 (91 citations)**

PHYSICAL REVIEW D **93,** 074021 (2016)

Valence-quark distribution functions in the kaon and pion

Chen Chen,[1,2,*] Lei Chang,[3,†] Craig D. Roberts,[4,‡] Shaolong Wan,[2,§] and Hong-Shi Zong[5,¶]

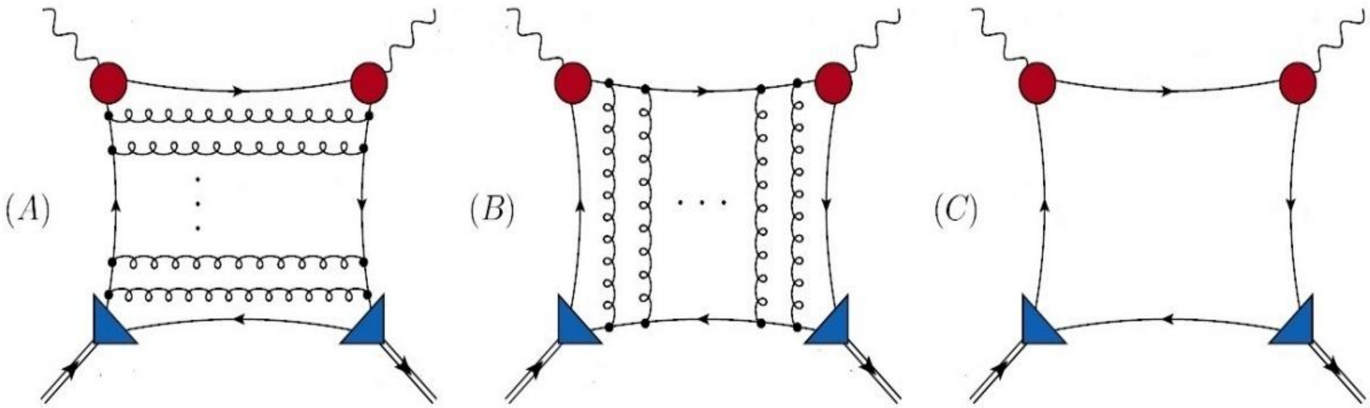
[1]*Hefei National Laboratory for Physical Sciences at the Microscale, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China*
[2]*Institute for Theoretical Physics and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China*
[3]*School of Physics, Nankai University, Tianjin 300071, China*
[4]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[5]*Department of Physics, Nanjing University, Nanjing, Jiangsu 210093, China*

(Received 3 February 2016; published 18 April 2016)

Valence-quark distribution functions in the kaon and pion

Theoretical Framework

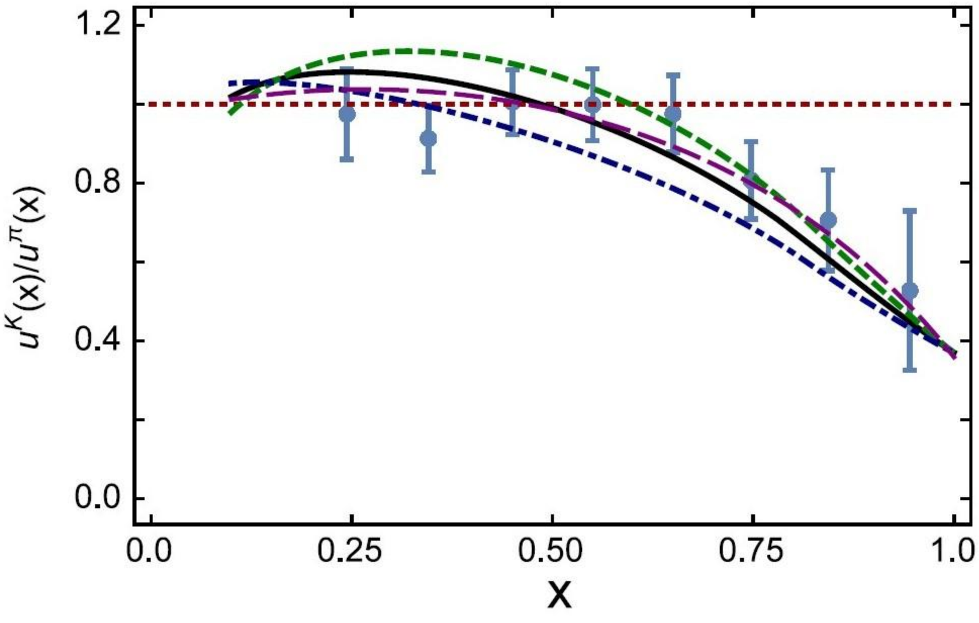
- The hadronic tensor relevant to inclusive deep inelastic lepton-meson scattering may be expressed in terms of the meson's quark distribution functions.
- Light pseudoscalar mesons are of great interest in hadron and nuclear physics, in large part because they are the *Nambu-Goldstone* modes which arise as a consequence of dynamical chiral symmetry breaking (DCSB).
- **Question: How to compute the mesons' parton distribution functions (PDFs) correctly?**
- In the framework of DSEs, the textbook *handbag* contribution to virtual Compton scattering:
  - The axial-vector Ward-Takahashi identites are lost and one violates momentum conservation within the bound-state.
  - The *minimal complete set* of the virtual-photon–meson forward Compton scattering amplitude:



Valence-quark distribution functions in the kaon and pion

Results

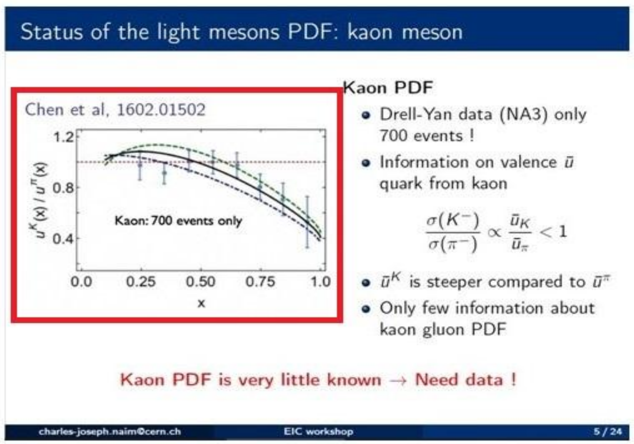
- By accounting for the gluon contribution, we presented the first symmetry-preserving predictions for the pion and kaon valence-quark distribution functions.
- We find that whereas roughly two thirds of the pion's light-front momentum is carried by valence dressed quarks at a characteristic hadronic scale; this fraction rises to 95% in the kaon; evolving distributions with these features to a scale typical of available Drell-Yan data produces a kaon-to-pion *ratio* of *u*-quark distributions that is in agreement with the single existing data set, and predicts a *u*-quark distribution within the pion that agrees with a modern of π*N* Drell-Yan data.

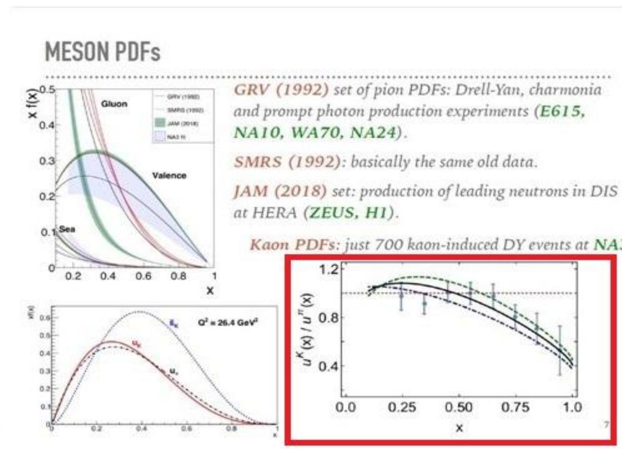


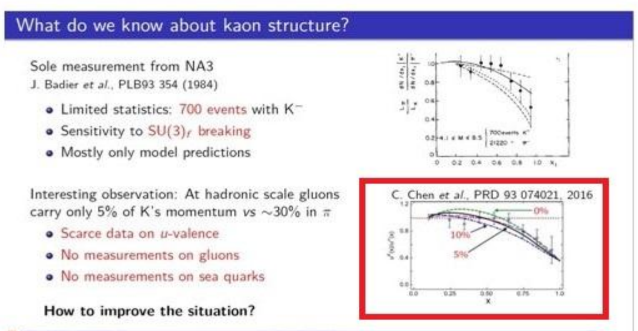
Valence-quark distribution functions in the kaon and pion

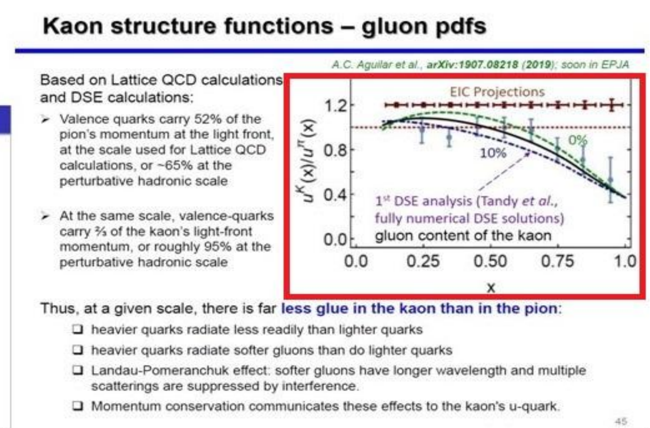
Impact

- The theorists and experimentalists showed great interest in our results.
- Especially, our computed *kaon-to-pion ratio* has been used in other researchers’ talks many times.









Valence-quark distribution functions in the kaon and pion

Impact

- This work was used to win approval from CERN management for the COMPASS++/AMBER Phase-1 project and provides crucial theory background for the development of Phase-2 plans.

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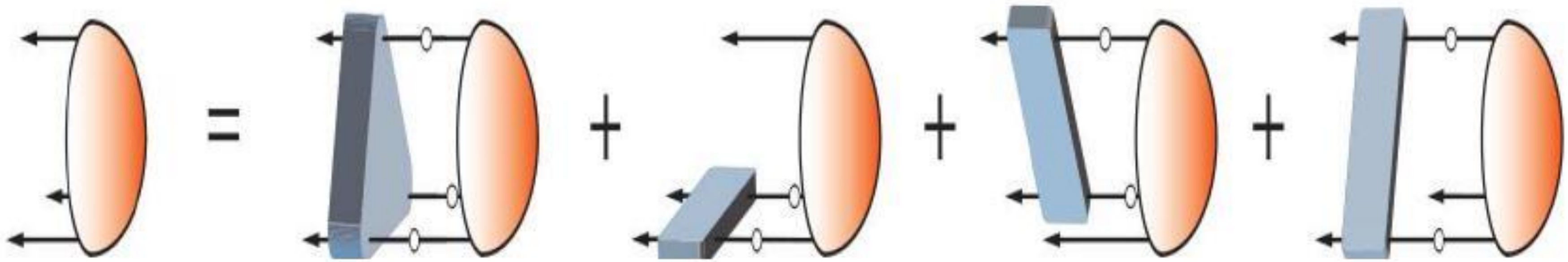
Letter of intent:

"… Using the Dyson-Schwinger-Equation (DSE) approach, the authors of Ref. [55] find that at the hadronic scale the gluons contribute to only 5% of the total momentum in the kaon, as compared to about one third in the pion. A stringent check of this prediction requires the measurement of the presently unknown gluon distribution in the kaon … "

Proposal for Phase-1:

"… A prediction for the ratio $u^K(x)/u^\pi(x)$ is available [74]: agreement with data [51] indicates that the gluon content of the kaon at the hadronic scale is just 5±5%, whereas that for the pion is more than 30% at this scale. … It is ***hence*** of utmost interest to upgrade the existing CERN M2 beam line by an RF-separation stage in a later phase of the Compass ++/Amber project. Such a unique high-energy high-purity kaon beam would for the first time allow a detailed measurement of the kaon's parton structure, which would pave the way to address the fundamental physics questions sketched above … "



2. Baryons

Diquarks

- **Mesons: quark-antiquark correlations -- color-singlets**
- **Diquarks: quark-quark correlations within a color-singlet baryon.**

- **Diquark correlations:**
  - In our approach: non-pointlike color-antitriplet and fully interacting.
  - Diquark correlations are soft, they possess an electromagnetic size.
  - Owing to properties of charge-conjugation, a diquark with spin-parity J^P may be viewed as a partner to the analogous J^{-P} meson.

$$\Gamma_{q\bar{q}}(p;P) = - \int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{q\bar{q}}(q;P)S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

$$\Gamma_{qq}(p;P)C^\dagger = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{qq}(q;P)C^\dagger S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

Diquarks

- Quantum numbers:
  - (I=0, J^P=0^+): isoscalar-scalar diquark
  - (I=1, J^P=1^+): isovector-pseudovector diquark
  - (I=0, J^P=0^-): isoscalar-pesudoscalar diquark
  - (I=0, J^P=1^-): isoscalar-vector diquark
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  - Tensor diquarks ?

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- Three-body bound states

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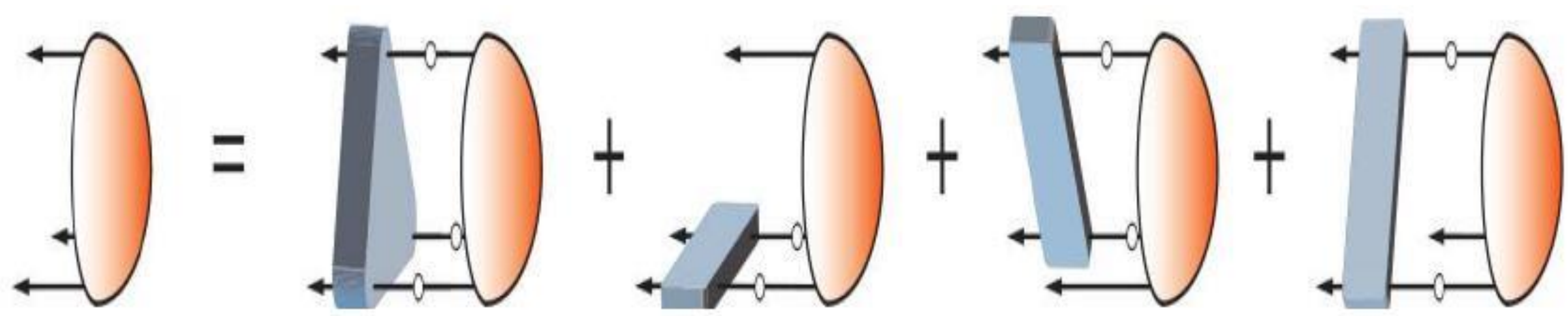

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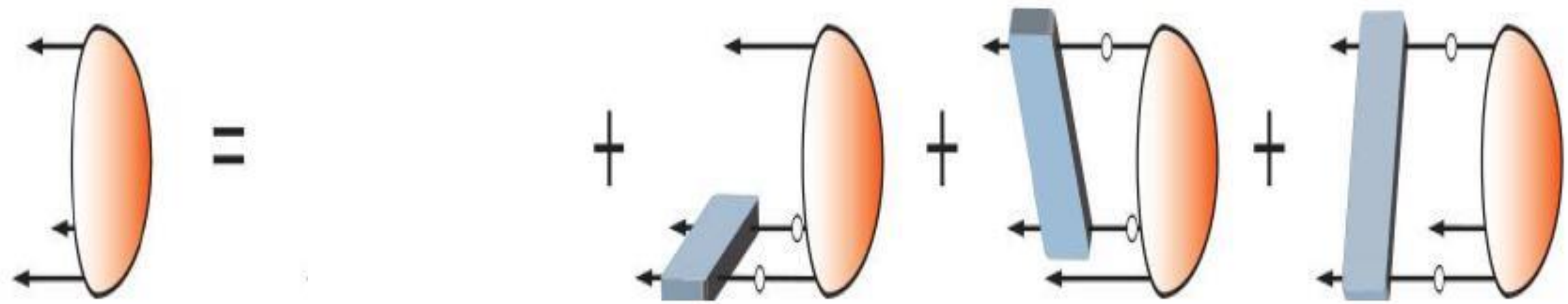

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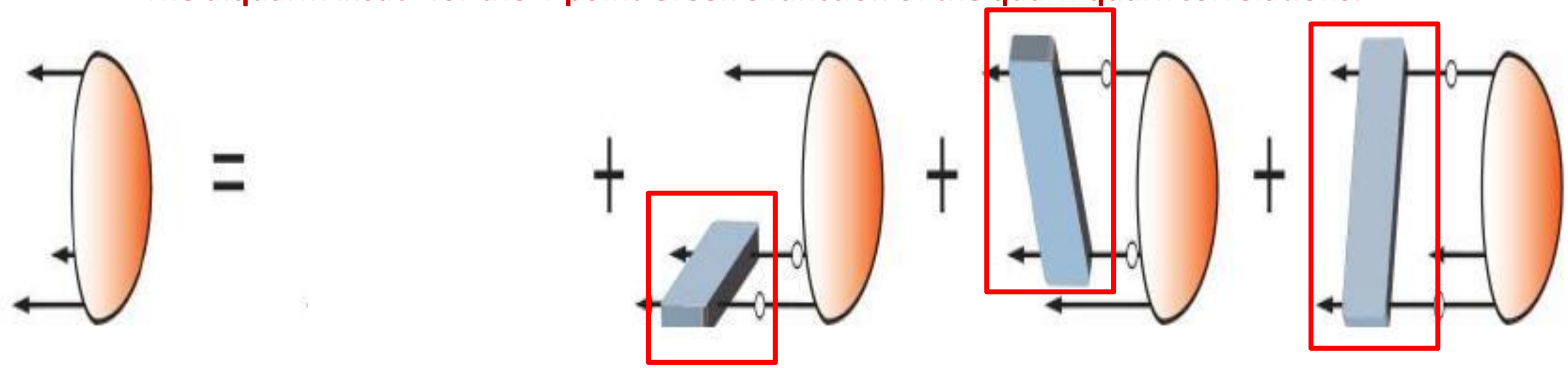
- Three-body bound states
  - **The diquark *Ansatz* for the 4-point Green's function of the quark-quark correlations:**



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  Quark-diquark two-body bound states

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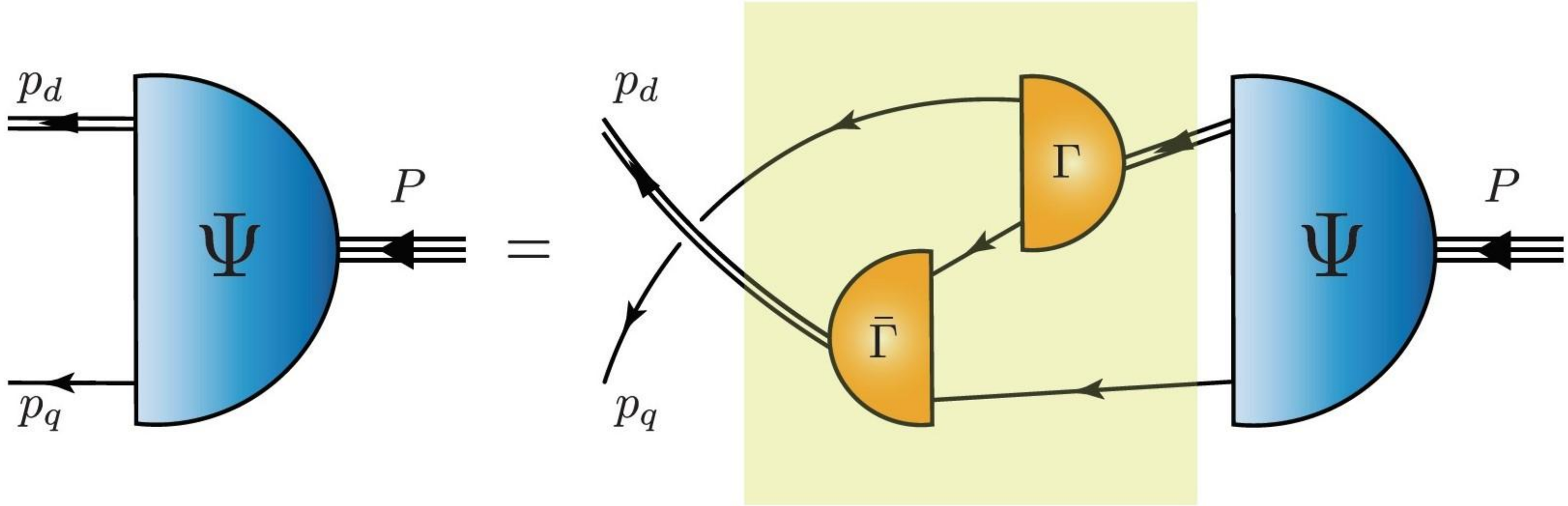
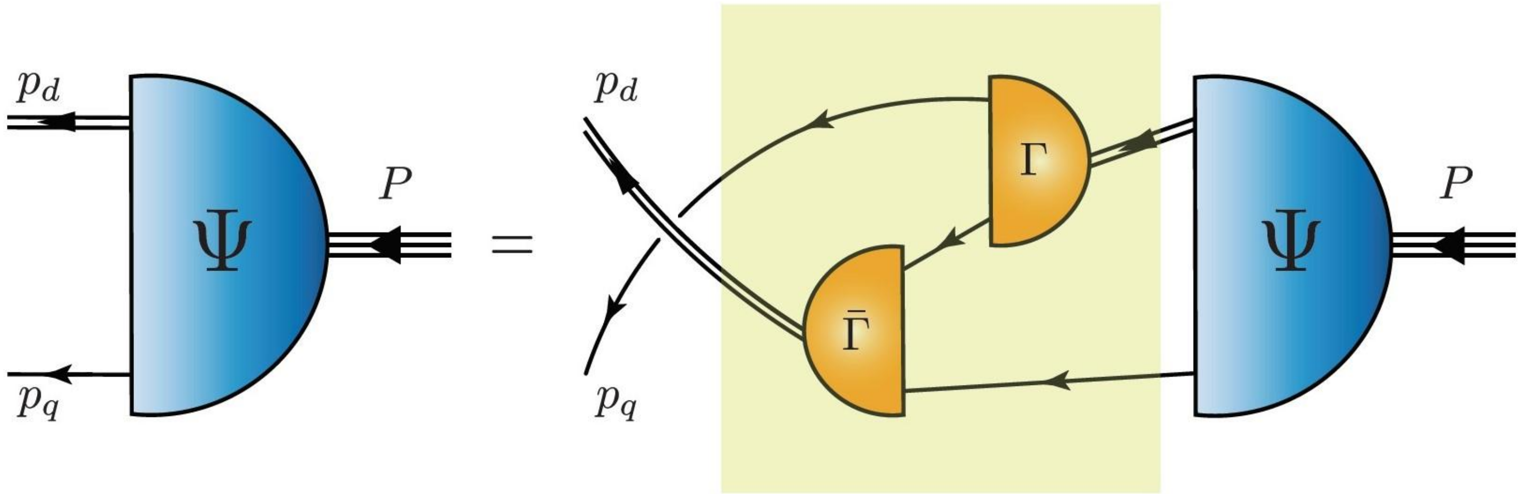


2.1 Spectrum



- *Phys. Rev. D* 100 (2019) 3, 034008, 37 citations
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- *Phys. Rev. C* 96 (2017) 1, 015208, 36 citations
- *Phys. Rev. D* 92 (2015) 11, 114034, 26 citations
- *Few Body Syst.* 53 (2012) 293-326, 83 citations

2.1 Spectrum

PHYSICAL REVIEW D **97**, 034016 (2018)

Structure of the nucleon's low-lying excitations

Chen Chen,[1,*] Bruno El-Bennich,[2,†] Craig D. Roberts,[3,‡] Sebastian M. Schmidt,[4,§]
Jorge Segovia,[5,∥] and Shaolong Wan[6,¶]

[1]*Instituto de Física Teórica, Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271, 01140-070 São Paulo, São Paulo, Brazil*
[2]*Universidade Cruzeiro do Sul, Rua Galvão Bueno, 868, 01506-000 São Paulo, São Paulo, Brazil*
[3]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[4]*Institute for Advanced Simulation, Forschungszentrum Jülich and JARA, D-52425 Jülich, Germany*
[5]*Institut de Física d'Altes Energies (IFAE) and Barcelona Institute of Science and Technology (BIST), Universitat Autònoma de Barcelona, E-08193 Bellaterra (Barcelona), Spain*
[6]*Institute for Theoretical Physics and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China*

(Received 9 November 2017; published 15 February 2018)

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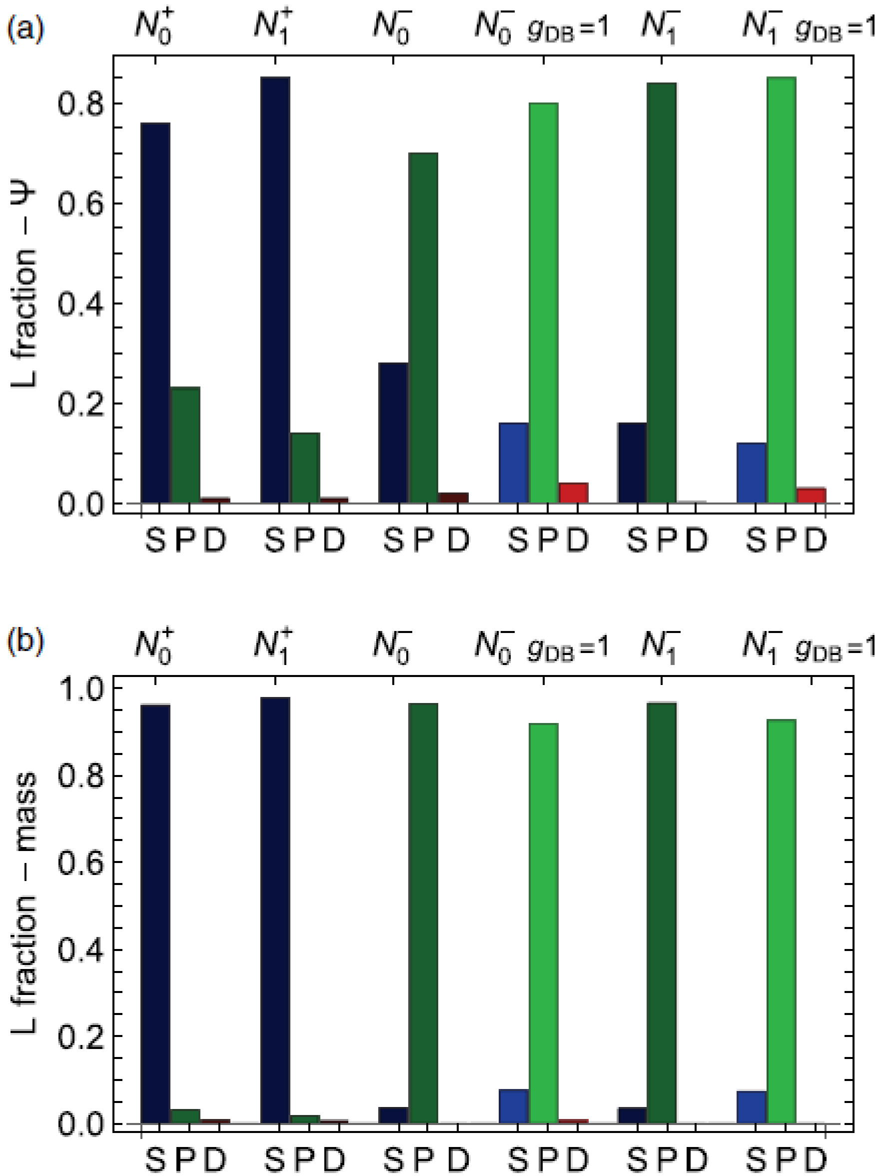
Phys. Rev. D **97** (2018) 3, 034016

$m_N \quad m^{1/2^+}_{N(1440)} \quad m^{1/2^-}_{N(1535)} \quad m^{1/2^-}_{N(1650)}$

- By including all kinds of diquarks, we performed a comparative study of the four lightest baryon (I=1/2, J^P=1/2^{+-}) isospin doublets in order to both elucidate their structural similarities and differences.
- The first **ON-SHELL** DSE treatment of these systems.
- The two lightest (I=1/2, J^P=1/2^+) doublets are dominated by scalar and pseudovector diquarks; the associated rest-frame Faddeev wave functions are primarily S-wave in nature; and the first excited state in this 1/2^+ channel has very much the appearance of a radial excitation of the ground state.
- In the two lightest (I=1/2, J^P=1/2^-) systems, **TOO**, scalar and pseudovector diquarks play a material role. In their rest frames, the Faddeev amplitudes describing the dressed-quark cores of these negative-parity states contain roughly equal fractions of even and odd parity diquarks; the associated wave functions of these negative-parity systems are predominantly P-wave in nature, but possess measurable S-wave components; and, the first excited state in this negative parity channel has little of the appearance of a radial excitation.

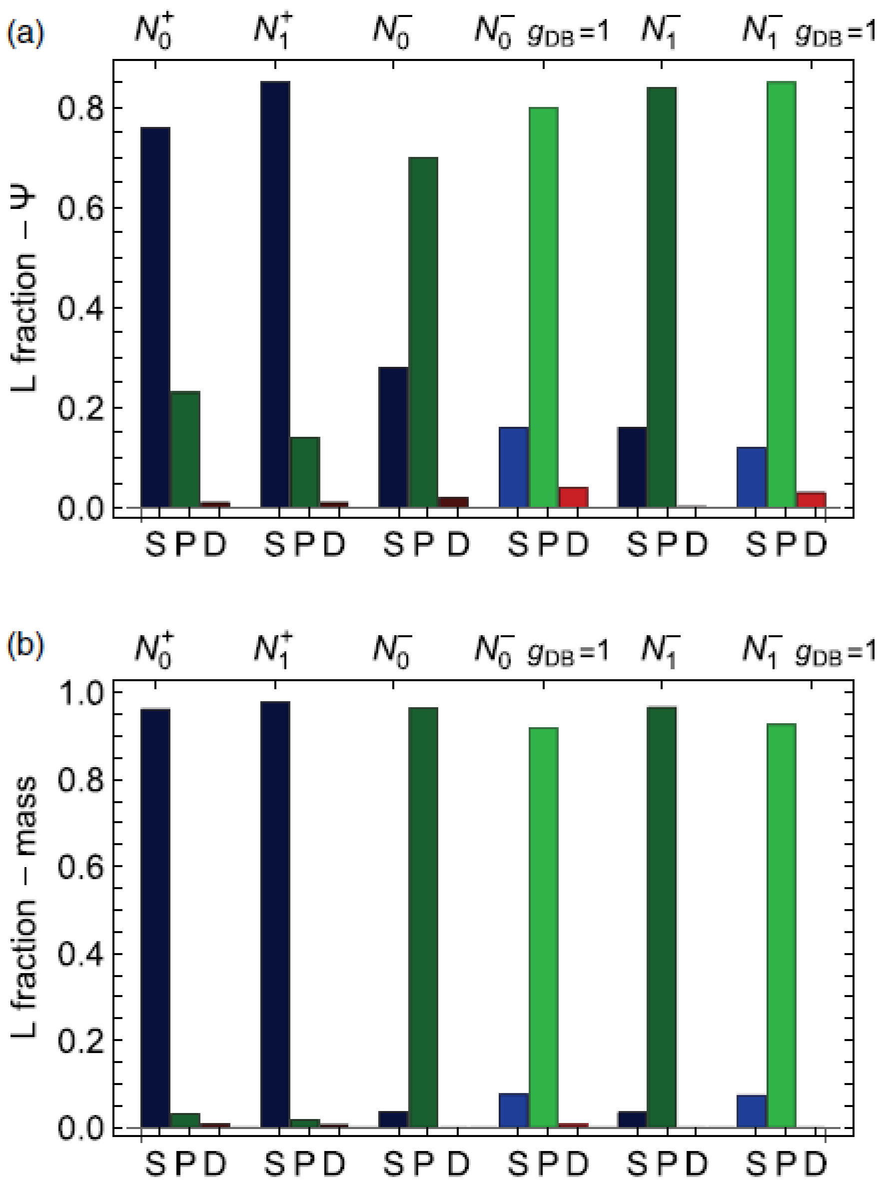
SOLUTIONS & THEIR PROPERTIES:
Rest-frame orbital angular momentum



- (a) Computed from the wave functions directly.

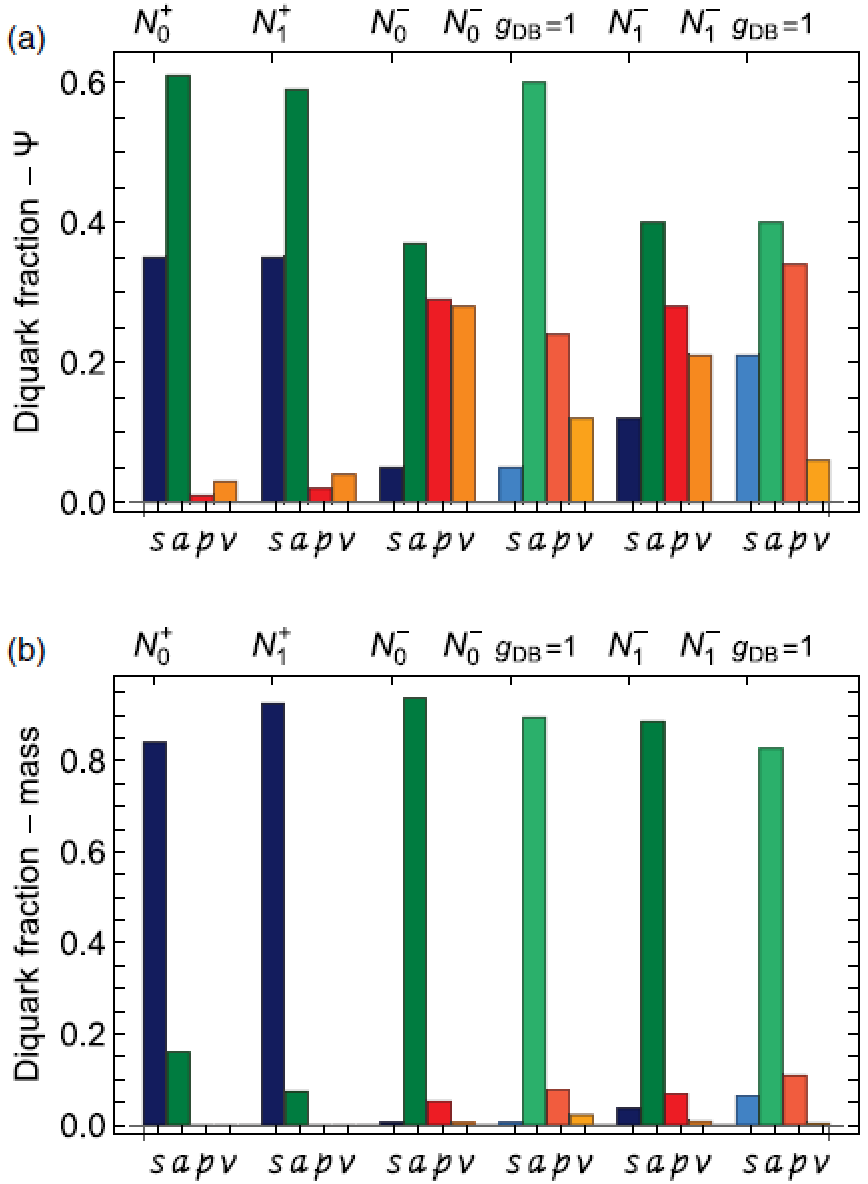
  (b) Computed from the relative contributions to the masses.
- (b) delivers the same qualitative picture of each baryon's internal structure as that presented in (a). Therefore, there is **little** mixing between partial waves in the computation of a baryon's mass.

SOLUTIONS & THEIR PROPERTIES:
Rest-frame orbital angular momentum



- The nucleon and Roper are primarily **S-wave** in nature, since they are not supported by the Faddeev equation unless **S-wave** components are contained in the wave function. On the other hand, the N(1535)1/2^- ,N(1650)1/2^- are essentially **P-wave** in character.
- These observations provide support in quantum field theory for the constituent-quark model classifications of these systems.

SOLUTIONS & THEIR PROPERTIES:
Diquark content

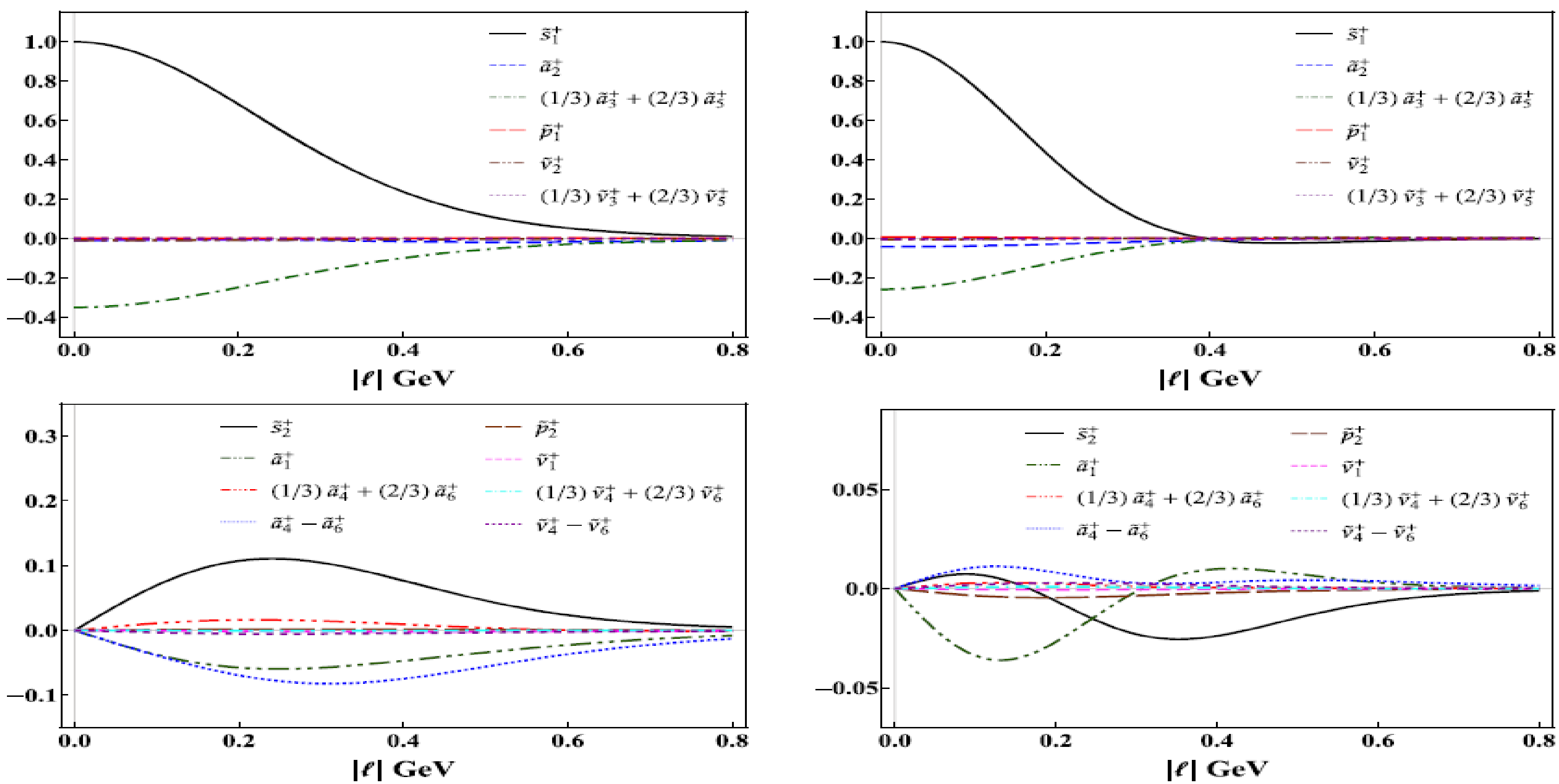


- (a) Computed from the amplitudes directly.

  (b) Computed from the relative contributions to the masses.
- From (a): although gDB < 1 has little impact on the nucleon and Roper, it has a significant effect on the structure of the negative parity baryons, serving to enhance the net negative-parity diquark content. The amplitudes associated with these negative-parity states contain roughly ***equal fractions*** of even and odd parity diquarks.
- From (b): In each case depicted in the lower panel, there is a single dominant diquark component. There are **significant interferences** between different diquarks.

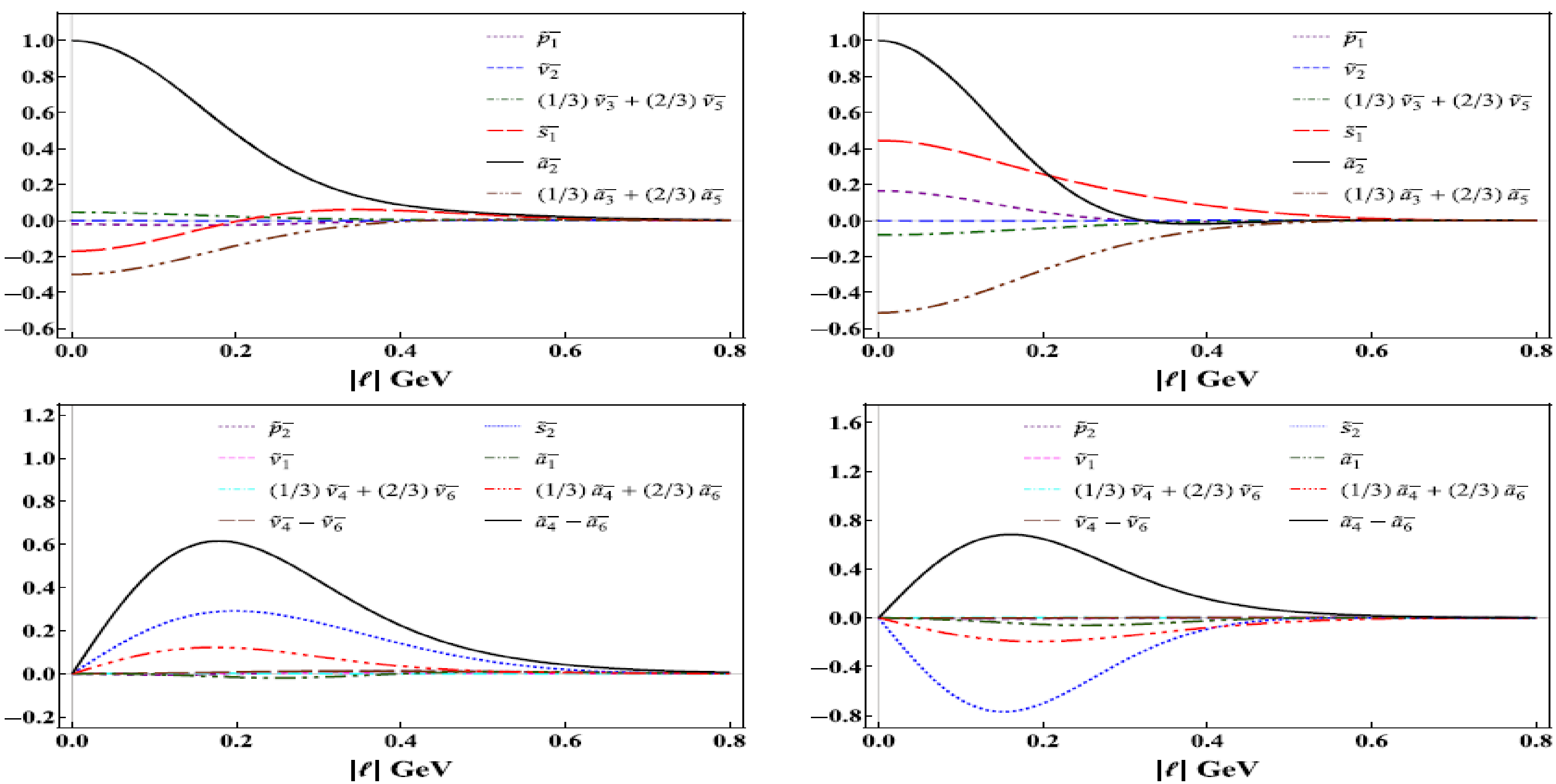
SOLUTIONS & THEIR PROPERTIES:
Pointwise structure

- We consider the zeroth Chebyshev moment of all S- and P-wave components in a given baryon's Faddeev wave function.
- Nucleon's first positive-parity excitation: all S-wave components exhibit a single zero; and four of the P-wave projections also possess a zero. This pattern of behavior for the first excited state indicates that it may be interpreted as a radial excitation.



SOLUTIONS & THEIR PROPERTIES:
Pointwise structure

- For N(1535)1/2^-,N(1650)1/2^- : the contrast with the positive-parity states is stark. In particular, there is no simple pattern of zeros, with all panels containing at least one function that possesses a zero.
- In their rest frames, these systems are predominantly P-wave in nature, but possess material S-wave components; and the first excited state in this negative parity channel—N(1650)1/2^−—has little of the appearance of a radial excitation, since most of the functions depicted in the right panels of the figure do not possess a zero.



Delta-baryons: coming soon…

Preprint nos. NJU-INP 057/22, USTC-ICTS/PCFT-22-11

Composition of low-lying $\mathrm{J} = \frac{3}{2}^{\pm}$ Δ-baryons

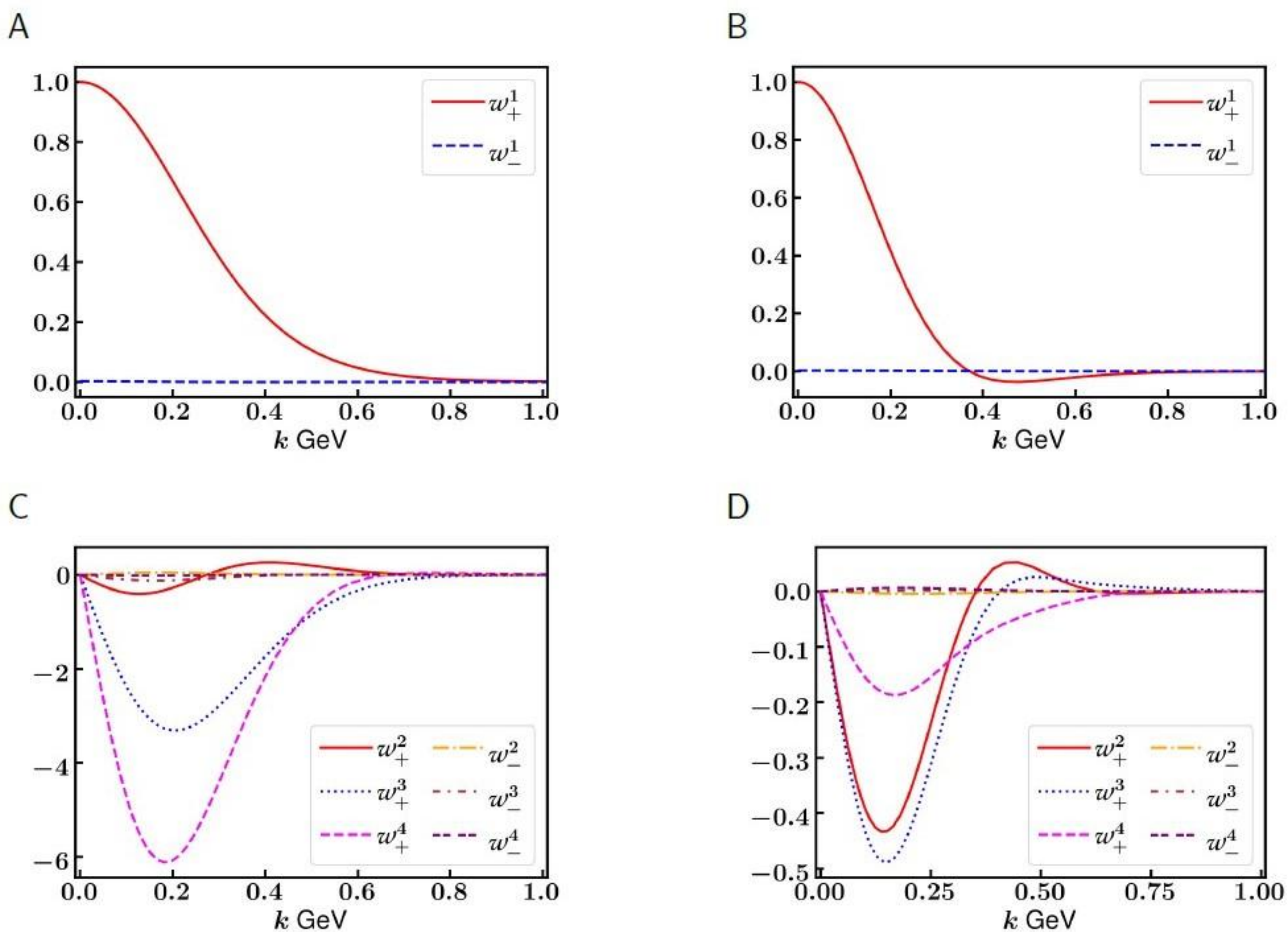
Langtian Liu (刘浪天),[1,2] Chen Chen (陈晨),[3,4,*] Ya Lu (陆亚),[1,2,5] Craig D. Roberts,[1,2,†] and Jorge Segovia[6,2]

[1] *School of Physics, Nanjing University, Nanjing, Jiangsu 210093, China*
[2] *Institute for Nonperturbative Physics, Nanjing University, Nanjing, Jiangsu 210093, China*
[3] *Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, China*
[4] *Peng Huanwu Center for Fundamental Theory, Hefei, Anhui 230026, China*
[5] *Department of Physics, Nanjing Tech University, Nanjing 211816, China*
[6] *Dpto. Sistemas Físicos, Químicos y Naturales, Univ. Pablo de Olavide, E-41013 Sevilla, Spain*

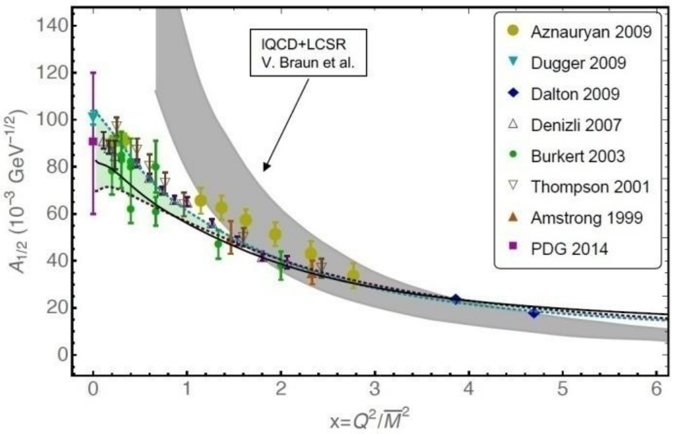
(Dated: 2022 March 15)

A Poincaré-covariant quark+diquark Faddeev equation is used to develop insights into the structure of the four lightest $(I, J = \frac{3}{2}, \frac{3}{2}^{\pm})$ baryon multiplets. Whilst these systems can contain isovector-pseudovector and isovector-vector diquark correlations, one may neglect the latter and still arrive at a reliable description. The $(\frac{3}{2}, \frac{3}{2}^{+})$ states are the simpler systems, with structural features that bear some resemblance to quark model pictures, *e.g.*, their most prominent rest-frame angular momentum component is S-wave and the $\Delta(1600)\frac{3}{2}^{+}$ may reasonably be viewed as a radial excitation of the $\Delta(1232)\frac{3}{2}^{+}$. On the other hand, the $(\frac{3}{2}, \frac{3}{2}^{-})$ states are somewhat more complex. The $\Delta(1940)\frac{3}{2}^{-}$ expresses little of the character of a radial excitation of the $\Delta(1700)\frac{3}{2}^{-}$; and whilst the rest-frame wave function of the latter is predominantly P-wave, the leading piece in the $\Delta(1940)\frac{3}{2}^{-}$ wave function is S-wave, presenting a conflict with quark model expectations. Experiments that can test these predictions, such as resonance electroproduction at large momentum transfers, may shed light on the character of emergent hadron mass.

Delta-baryons: coming soon…



The $\gamma^{(*)}p \to N(1535)\frac{1}{2}^{-}$ Transition



In process, will be submitted to *Physical Review Letters*

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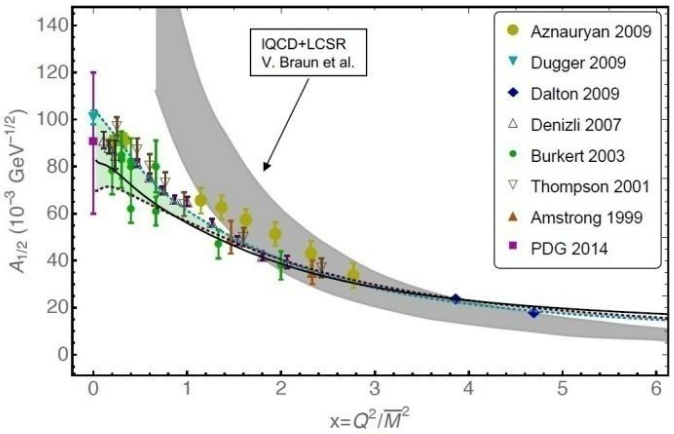

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PRL **115,** 171801 (2015) PHYSICAL REVIEW LETTERS week ending 23 OCTOBER 2015

Completing the Picture of the Roper Resonance

Jorge Segovia,[1] Bruno El-Bennich,[2,3] Eduardo Rojas,[2,4] Ian C. Cloët,[5] Craig D. Roberts,[5]
Shu-Sheng Xu,[6] and Hong-Shi Zong[6]

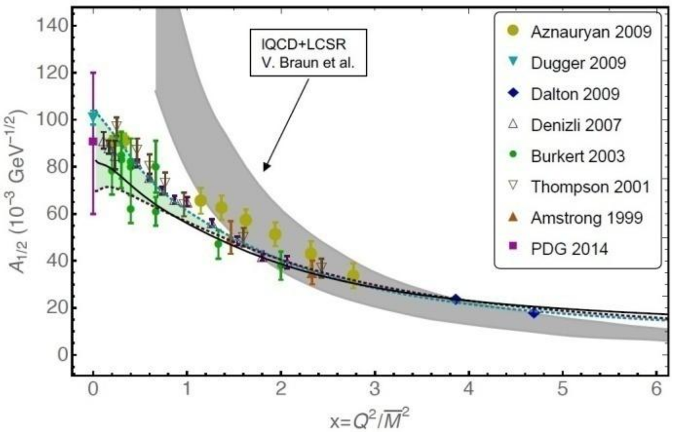
[1]*Grupo de Física Nuclear and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM), Universidad de Salamanca, E-37008 Salamanca, Spain*
[2]*Laboratório de Física Teórica e Computacional, Universidade Cruzeiro do Sul, 01506-000 São Paulo, SP, Brazil*
[3]*Instituto de Física Teórica, Universidade Estadual Paulista, 01140-070 São Paulo, SP, Brazil*
[4]*Instituto de Física, Universidad de Antioquia, Calle 70 No. 52-21, Medellín, Colombia*
[5]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[6]*Department of Physics, Nanjing University, Nanjing 210093, China*

(Received 16 April 2015; revised manuscript received 29 July 2015; published 21 October 2015)

We employ a continuum approach to the three valence-quark bound-state problem in relativistic quantum field theory to predict a range of properties of the proton's radial excitation and thereby unify them with those of numerous other hadrons. Our analysis indicates that the nucleon's first radial excitation is the Roper resonance. It consists of a core of three dressed quarks, which expresses its valence-quark content and whose charge radius is 80% larger than the proton analogue. That core is complemented by a meson cloud, which reduces the observed Roper mass by roughly 20%. The meson cloud materially affects long-wavelength characteristics of the Roper electroproduction amplitudes but the quark core is revealed to probes with $Q^2 \gtrsim 3m_N^2$.

DOI: 10.1103/PhysRevLett.115.171801 PACS numbers: 13.40.Gp, 14.20.Dh, 14.20.Gk, 11.15.Tk

The $\gamma^{(*)}p \to N(1535)\frac{1}{2}^{-}$ Transition



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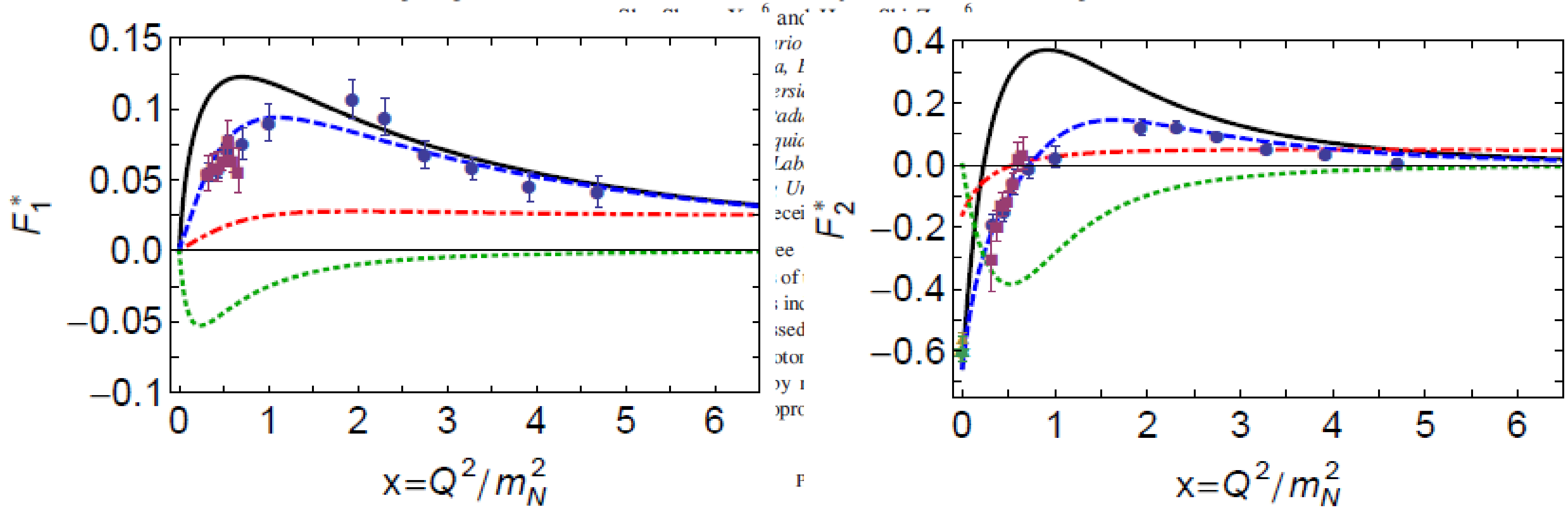

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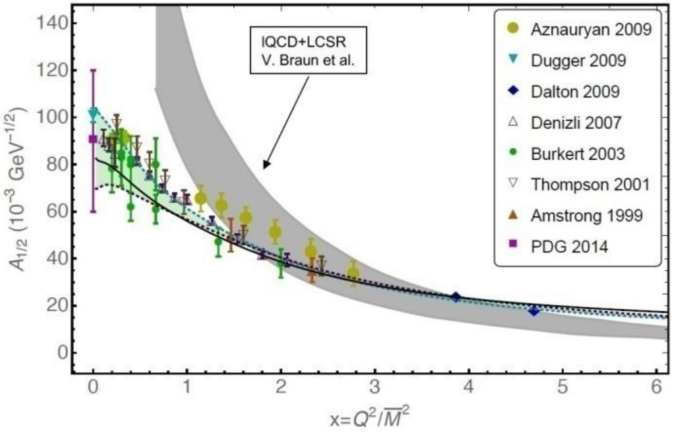

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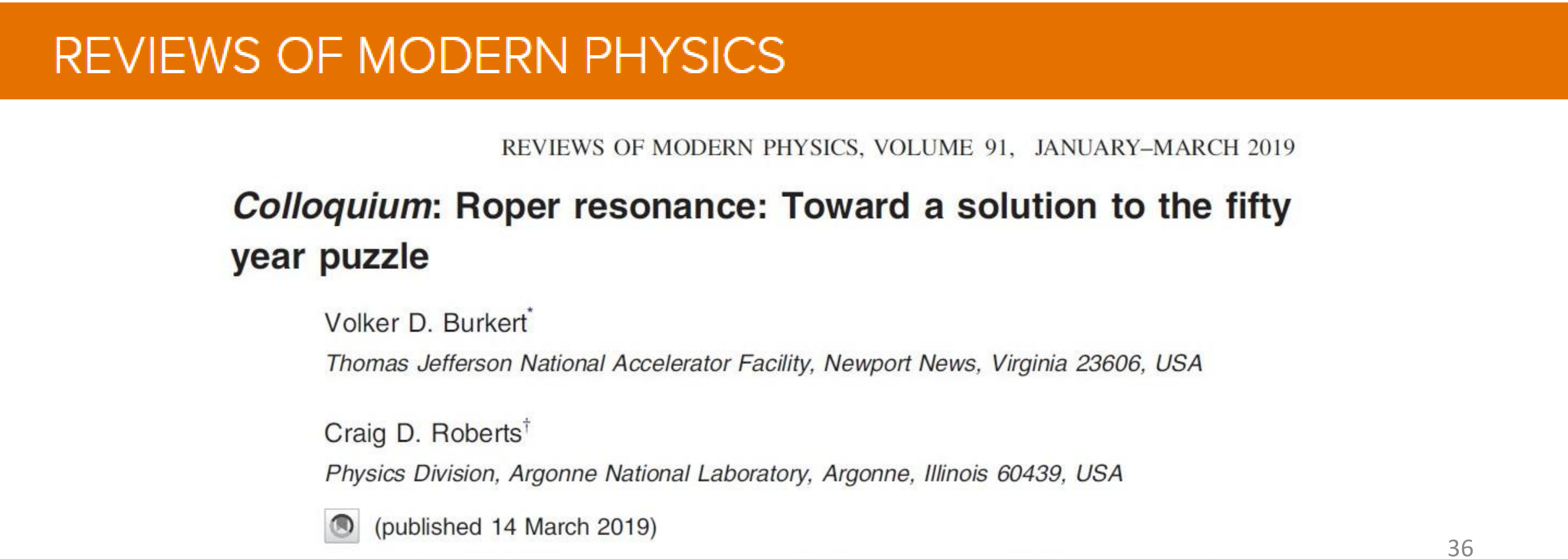

2.2 Form Factors

- arXiv: 2103.02054 [hep-ph], submitted to Phys. Rev. D, 7 citations
- *Phys. Lett. B* 815 (2021) 136150, 9 citations
- *Phys. Rev. D* 102 (2020) 1, 014043, 19 citations
- *Phys. Rev. D* 100 (2019) 3, 034001, 19 citations
- *Phys. Rev. D* 99 (2019) 3, 034013, 35 citations
- *Few Body Syst.* 55 (2014) 1-33, 49 citations
- *Phys. Rev. C* 88 (2013) 3, 032201, 42 citations

- *Prog. Part. Nucl. Phys.* **116** (2021) 103835, 40 citations **(Review)**

Form Factors

- **Form factors: contain important information about the structure and the properties of hadrons.**
- **Different probes correspond to different form factors.**
- **The nucleon electromagnetic current:**

$$J_\mu^{\mathrm{EM}}(K,Q) = \bar{u}(P_f)\left[\gamma_\mu F_1(Q^2) + \frac{1}{2m_N}\sigma_{\mu\nu}Q_\nu F_2(Q^2)\right]u(P_i)$$

  - **A large number of experimental measurements, with high precision and up to large momentum transfer.**

- **The nucleon axial current:**

$$J_{5\mu}^j(K,Q) = \bar{u}(P_f)\frac{\tau^j}{2}\gamma_5\left[\gamma_\mu G_A(Q^2) + i\frac{Q_\mu}{2m_N}G_P(Q^2)\right]u(P_i)$$

  - **The relative measurements are much more difficult, since they are related to weak processes.**
  - ***G_A* – axial form factor: experimental data are rather sparse and with large uncertainties.**
  - ***G_P* – induced pseudoscalar form factor: ONLY 4 empirical results.**

- **The nucleon pseudoscalar current (pseudoscalar form factor):**

$$J_5^j(K,Q) = \bar{u}(P_f)\frac{\tau^j}{2}\gamma_5 G_5(Q^2)u(P_i)$$

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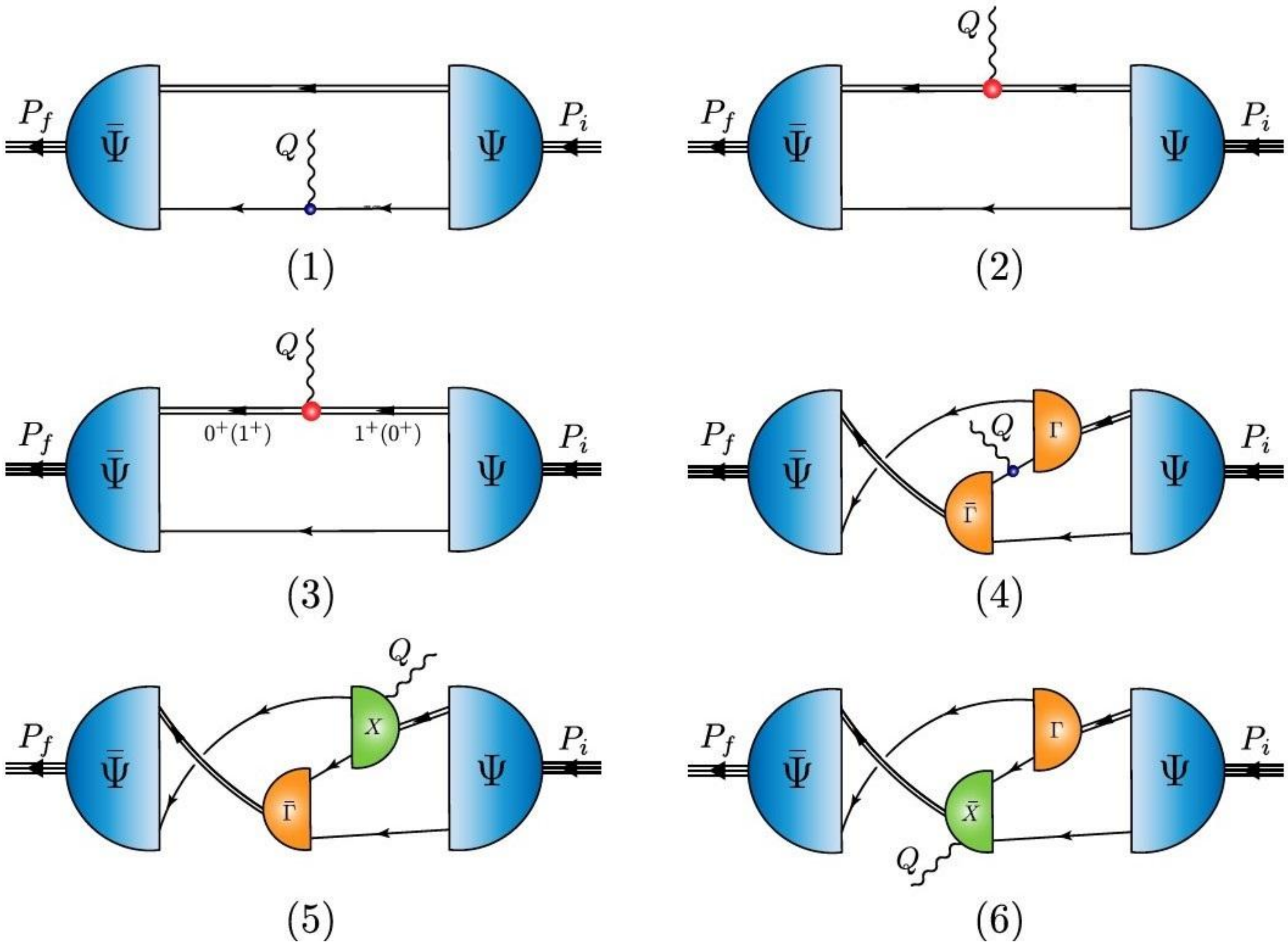
- **The Partially Conservation of the Axial Current (PCAC) relation:**

$$G_A(Q^2) - \frac{Q^2}{4m_N^2}G_P(Q^2) = \frac{m_q}{m_N}G_5(Q^2)$$

How to Compute Form Factors?

➢ **In the quark-diquark framework, the associated symmetry-preserving current:**



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Nucleon axial and pseudoscalar form factors from the covariant Faddeev equation

Gernot Eichmann and Christian S. Fischer
Institut für Theoretische Physik, Justus-Liebig-Universität Giessen, D-35392 Giessen, Germany
(Dated: November 2, 2018)

We compute the axial and pseudoscalar form factors of the nucleon in the Dyson-Schwinger approach. To this end, we solve a covariant three-body Faddeev equation for the nucleon wave function and determine the matrix elements of the axialvector and pseudoscalar isotriplet currents. Our only input is a well-established and phenomenologically successful ansatz for the nonperturbative quark-gluon interaction. As a consequence of the axial Ward-Takahashi identity that is respected at the quark level, the Goldberger-Treiman relation is reproduced for all current-quark masses. We discuss the timelike pole structure of the quark-antiquark vertices that enters the nucleon matrix elements and determines the momentum dependence of the form factors. Our result for the axial charge underestimates the experimental value by $20-25\%$ which might be a signal of missing pion-cloud contributions. The axial and pseudoscalar form factors agree with phenomenological and lattice data in the momentum range above $Q^2 \sim 1\dots 2$ GeV^2.

PACS numbers: 11.80.Jy 12.38.Lg, 11.40.Ha 14.20.Dh

I. INTRODUCTION

The nucleon's axial and pseudoscalar form factors are of fundamental significance for the properties of the nucleon that are probed in weak interaction processes. Their momentum dependence can be experimentally tested by (anti)neutino scattering off nucleons or nuclei, charged pion electroproduction and muon capture processes; see [1–3] for reviews. Both form factors are experimentally hard to extract and therefore considerably less well known than their electromagnetic counterparts. Precisely measured is only the low-momentum limit g_A of the axial form factor which is determined from neutron β-decay. Planned experiments at major facilities are expected to change this situation in the near future.

The theoretical calculation of the nucleon's axial and pseudoscalar form factors requires genuinely non-perturbative methods. Chiral perturbation theory has been successful in this respect [1, 4, 5] although it is generally limited to the region of low momentum transfer. Recent studies in lattice gauge theory are getting closer to the physical pion mass region [6–8] but finite-volume effects become increasingly important. Another non-perturbative approach is the one via functional meth-

The study of axial and pseudoscalar form factors in the functional approach has so far been limited to an approximation where the nucleon is treated as a bound object of a quark and a diquark that interact via quark exchange [12, 13]. The entire gluonic substructure appears here only implicitly within the dressing of quark and diquark propagators as well as diquark vertex functions. There are several conceptual issues that complicate the treatment of form factors in the quark-diquark model. First, the requirement of current conservation induces the appearance of intricate 'seagull' diagrams [14]. Such terms have been taken into account for electromagnetic form factors, but their implementation in the case of axial form factors has not yet been possible for technical reasons [13]. Second, to comply with chiral Ward identities, a current-conserving quark-diquark model requires vector diquarks in addition to the usual scalar and axialvector diquark degrees of freedom [15]. Such an elaborate treatment of the quark-diquark model has not yet been performed.

The situation is somewhat different when the nucleon is treated as a genuine three-body problem. The resulting Faddeev equation in rainbow-ladder truncation has been solved only recently for the nucleon and Δ

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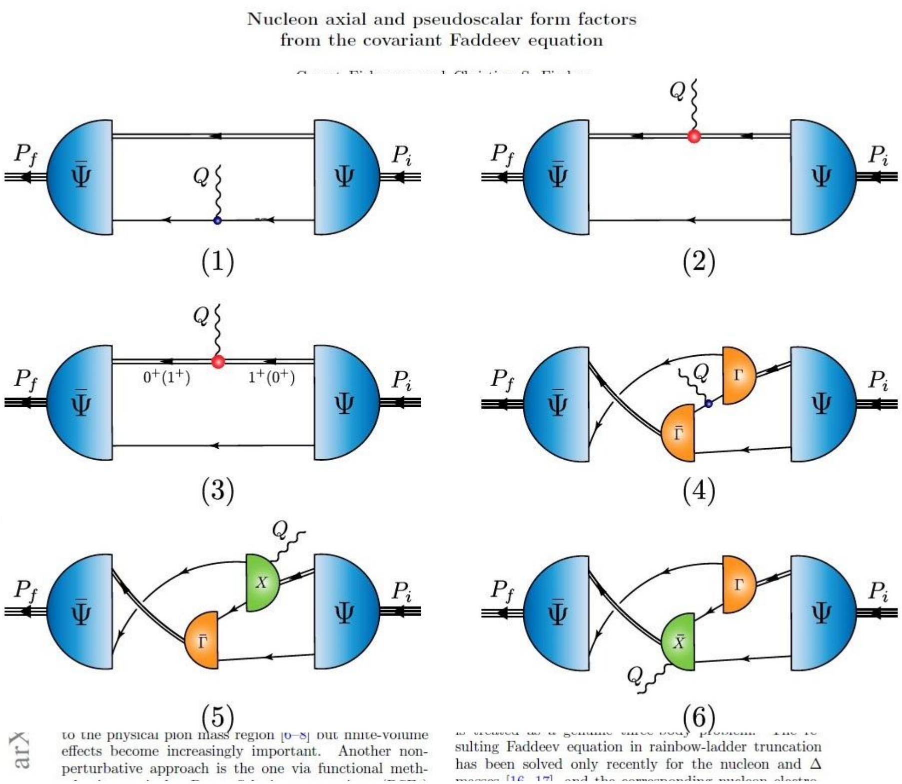

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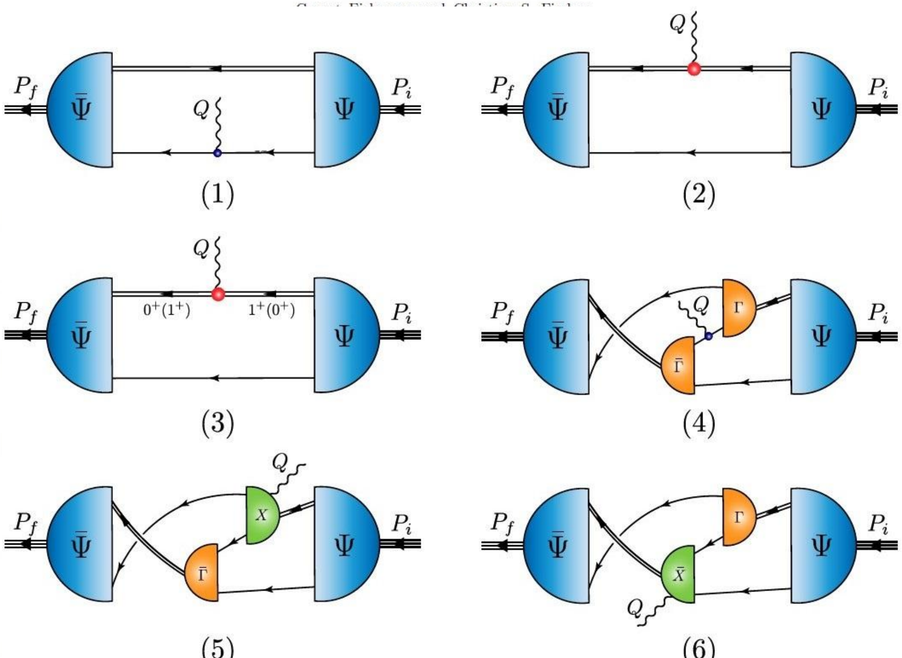

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Goldberger-Treiman relation and g pi N N from the three quark BS / Faddeev approach in the NJL model

Noriyoshi Ishii (Erlangen - Nuremberg U.) (Apr 28, 2000)

Published in: *Nucl.Phys.A* 689 (2001) 793-845 • e-Print: nucl-th/0004063 [nucl-th]

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- Chen Chen, C. S. Fischer, C. D. Roberts and J. Segovia (**2021**), Nucleon axial-vector and pseudoscalar form factors, and PCAC relations, arXiv: 2103.02054 [hep-ph], submitted to *Phys.Rev.D*.

Physics Letters B 815 (2021) 136150

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Form factors of the nucleon axial current

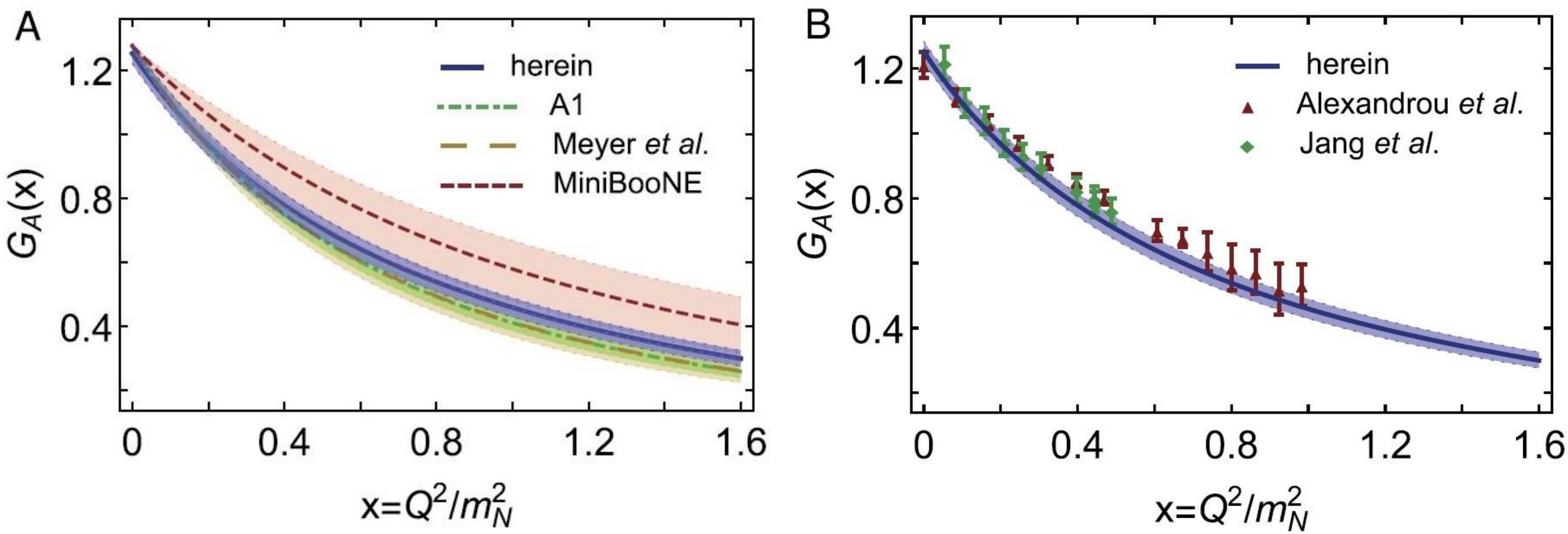
Chen Chen [a], Christian S. Fischer [a,b], Craig D. Roberts [c,d,*], Jorge Segovia [e,d]

[a] *Institut für Theoretische Physik, Justus-Liebig-Universität Gießen, D-35392 Gießen, Germany*
[b] *Helmholtz Forschungsakademie Hessen für FAIR (HFHF), GSI Helmholtzzentrum für Schwerionenforschung, Campus Gießen, 35392 Gießen, Germany*
[c] *School of Physics, Nanjing University, Nanjing, Jiangsu 210093, China*
[d] *Institute for Nonperturbative Physics, Nanjing University, Nanjing, Jiangsu 210093, China*
[e] *Dpto. Sistemas Físicos, Químicos y Naturales, Univ. Pablo de Olavide, E-41013 Sevilla, Spain*

The axial current – *GA* & *GP*

$$J^j_{5\mu}(K,Q) = \bar{u}(P_f)\frac{\tau^j}{2}\gamma_5\left[\gamma_\mu G_A(Q^2) + i\frac{Q_\mu}{2m_N}G_P(Q^2)\right]u(P_i)$$

- **Two form factors:**
  - ***GA* – *axial* form factor**

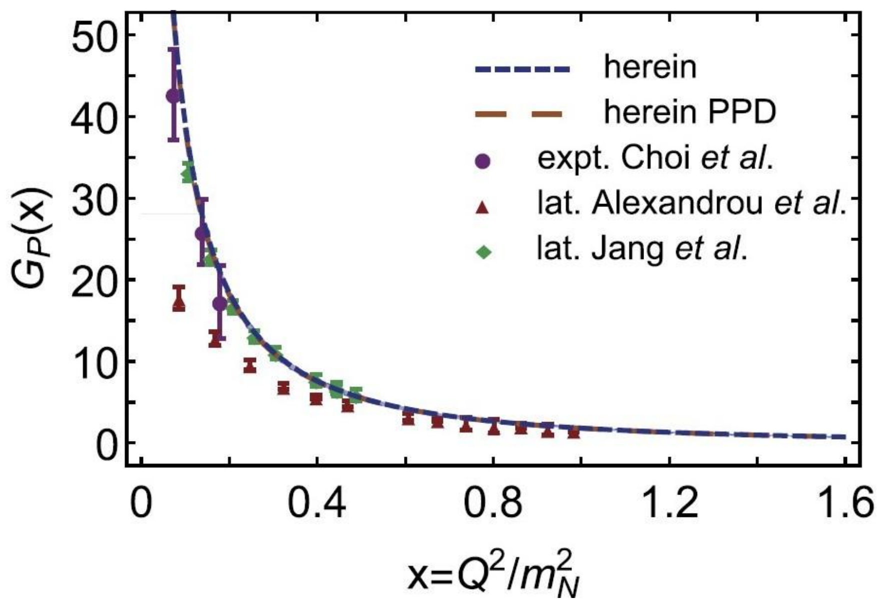


✓ Chen Chen, C. S. Fischer, C. D. Roberts and J. Segovia (2021), Phys.Lett.B 815 (2021) 136150

The axial current – G_A & G_P

$$J^j_{5\mu}(K,Q) = \bar{u}(P_f)\frac{\tau^j}{2}\gamma_5\left[\gamma_\mu G_A(Q^2) + i\frac{Q_\mu}{2m_N}G_P(Q^2)\right]u(P_i)$$

- **Two form factors:**
  - ***G_A – axial* form factor**
  - ***G_P – induced pseudoscalar* form factor**

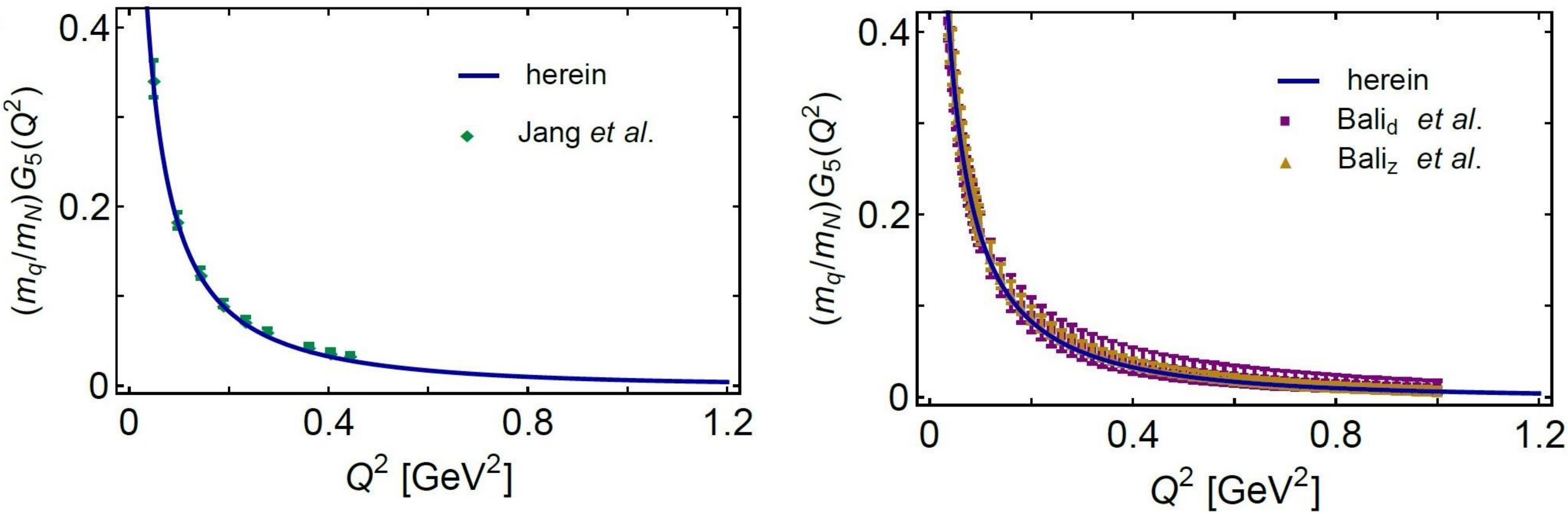


✓ Chen Chen, C. S. Fischer, C. D. Roberts and J. Segovia (2021), Phys.Lett.B 815 (2021) 136150

The pseudoscalar current – *G5* & *GπNN*

$$J_5^j(K,Q) = \bar{u}(P_f)\frac{\tau^j}{2}\gamma_5 G_5(Q^2)u(P_i)$$

- **One form factor:**
  - ***G5* – *pseudoscalar* form factor**



✓ Chen Chen, C. S. Fischer, C. D. Roberts and J. Segovia (2021), arXiv: 2103.02054 [hep-ph]

The pseudoscalar current – G_5 & $G_{\pi NN}$

$$J_5^j(K,Q) = \bar{u}(P_f)\frac{\tau^j}{2}\gamma_5 G_5(Q^2)u(P_i)$$

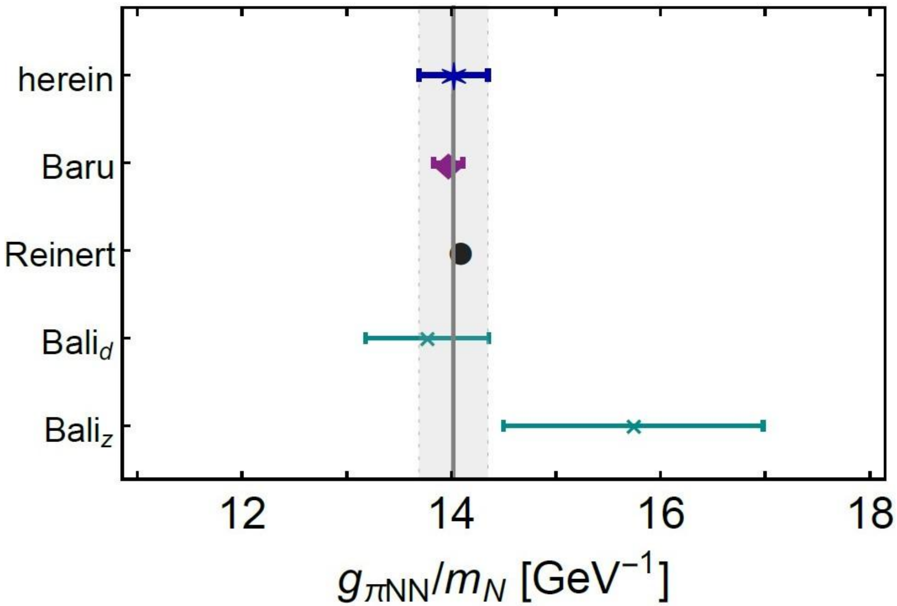
- **One form factor:**
  - ***G_5 – pseudoscalar* form factor**
- **At the pion mass pole, the residue of *G_5* is the *pion-nucleon coupling constant* $g_{\pi NN}$. Thus one can define the *pion-nucleon* form factor $G_{\pi NN}$:**

$$G_5(Q^2) =: \frac{m_\pi^2}{Q^2+m_\pi^2}\frac{f_\pi}{m_q}G_{\pi NN}(Q^2)$$

$$G_{\pi NN}(Q^2=-m_\pi^2) = g_{\pi NN}$$

- **The Goldberger-Treiman relation:**

$$G_A(0) = \frac{f_\pi}{m_N}G_{\pi NN}(0)$$



✓ Chen Chen, C. S. Fischer, C. D. Roberts and J. Segovia (2021), arXiv: 2103.02054 [hep-ph]

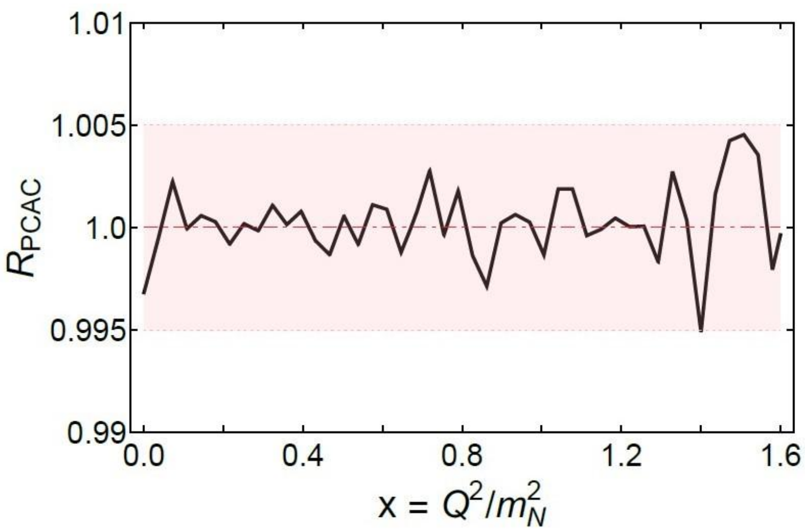
PCAC

- **The Partially Conservation of the Axial Current (PCAC) relation:**

$$G_A(Q^2) - \frac{Q^2}{4m_N^2} G_P(Q^2) = \frac{m_q}{m_N} G_5(Q^2)$$

- **Define: the PCAC ratio**

$$R_{\rm PCAC} := \frac{4m_N^2 G_A}{Q^2 G_P + 4m_q m_N G_5}$$



✓ Chen Chen, C. S. Fischer, C. D. Roberts and J. Segovia (2021), arXiv: 2103.02054 [hep-ph]

D. Proof of PCAC

We have specified all the necessary building blocks to construct the diagrams of $J^j_{5\mu}(K,Q)$ and $J^j_5(K,Q)$ depicted in Fig. 3, with the corresponding expressions given in Appendix B. Before we perform numerical computations, it is important to prove analytically the PCAC relation, Eq. (7), *i.e.* $J^j_{5\mu}(K,Q)$ and $J^j_5(K,Q)$ are both a sum of six terms (listed in Fig. 3):

$$\begin{aligned} J^j_{5(\mu)} =& J^{\rm q}_{5(\mu)} + J^{{\rm dq},aa}_{5(\mu)} + (J^{{\rm dq},sa}_{5(\mu)} + J^{{\rm dq},as}_{5(\mu)}) \\ &+ J^{\rm ex}_{5(\mu)} + J^{\rm sg}_{5(\mu)} + J^{\overline{\rm sg}}_{5(\mu)} \,. \end{aligned} \tag{68}$$

Note too that, in this proof, we shall consider either the neutral (τ^3) or the charged ($\tau^{1\pm i2}$) currents; in the isospin limit, their flavor coefficients are precisely the same.

Diagram 1: current coupling to quark line

For Diagram 1 in Fig. 3, contracting Eq. (B.2) with Q_μ and using Eq. (17), we obtain[3]

$$\begin{aligned} & Q_\mu J^{{\rm q},0^+}_{5\mu}(K,Q) + 2im_q J^{{\rm q},0^+}_5(K,Q) \\ =& \frac{1}{2}\int_p \bar\Psi^{0^+}(p_f';-P_f)\, S(p_{q+}) \left[Q_\mu \Gamma_{5\mu}(p_{q+},p_{q-}) + \right. \end{aligned}$$

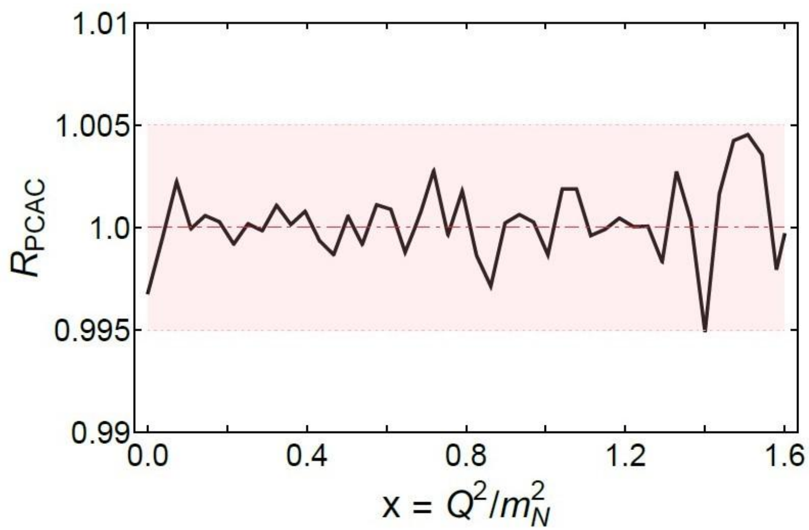
PCAC

- **The Partially Conservation of the Axial Current (PCAC) relation:**

$$G_A(Q^2) - \frac{Q^2}{4m_N^2}G_P(Q^2) = \frac{m_q}{m_N}G_5(Q^2)$$

- **Define: the PCAC ratio**

$$R_{\rm PCAC} := \frac{4m_N^2 G_A}{Q^2 G_P + 4m_q m_N G_5}$$



✓ Chen Chen, C. S. Fischer, C. D. Roberts and J. Segovia (2021), arXiv: 2103.02054 [hep-ph]

$$\begin{aligned} &Q_\mu J^{\overline{\rm sg},1^+1^+}_{5\mu} + 2im_q J^{\overline{\rm sg},1^+1^+}_5 \\ &= \int_p\int_k \bar\Phi^{1^+}_{\alpha,f}\Big[(\frac{1}{12})\Gamma^{1^+}_\beta(\tilde k_r)S^{\rm T}(\tilde q)\bar\Gamma^{1^+}_\alpha(\tilde p_r)i\gamma_5 + \\ &(-\frac{5}{12})\Gamma^{1^+}_\beta(\tilde k_r)S^{\rm T}(\tilde q)i\gamma_5^{\rm T}\bar\Gamma^{1^+}_\alpha(\tilde p'_r)\Big]\Phi^{1^+}_{\beta,i}\,; \qquad (95)\end{aligned}$$

The color/flavor coefficients in the first lines of Eqs. (92)-(95) are calculated via Eq. (C.10), *i.e.* the bystander legs of the seagulls' conjugations; and the coefficients in the second lines are calculated via Eq. (C.9), the exchange legs.

Sum of all contributions

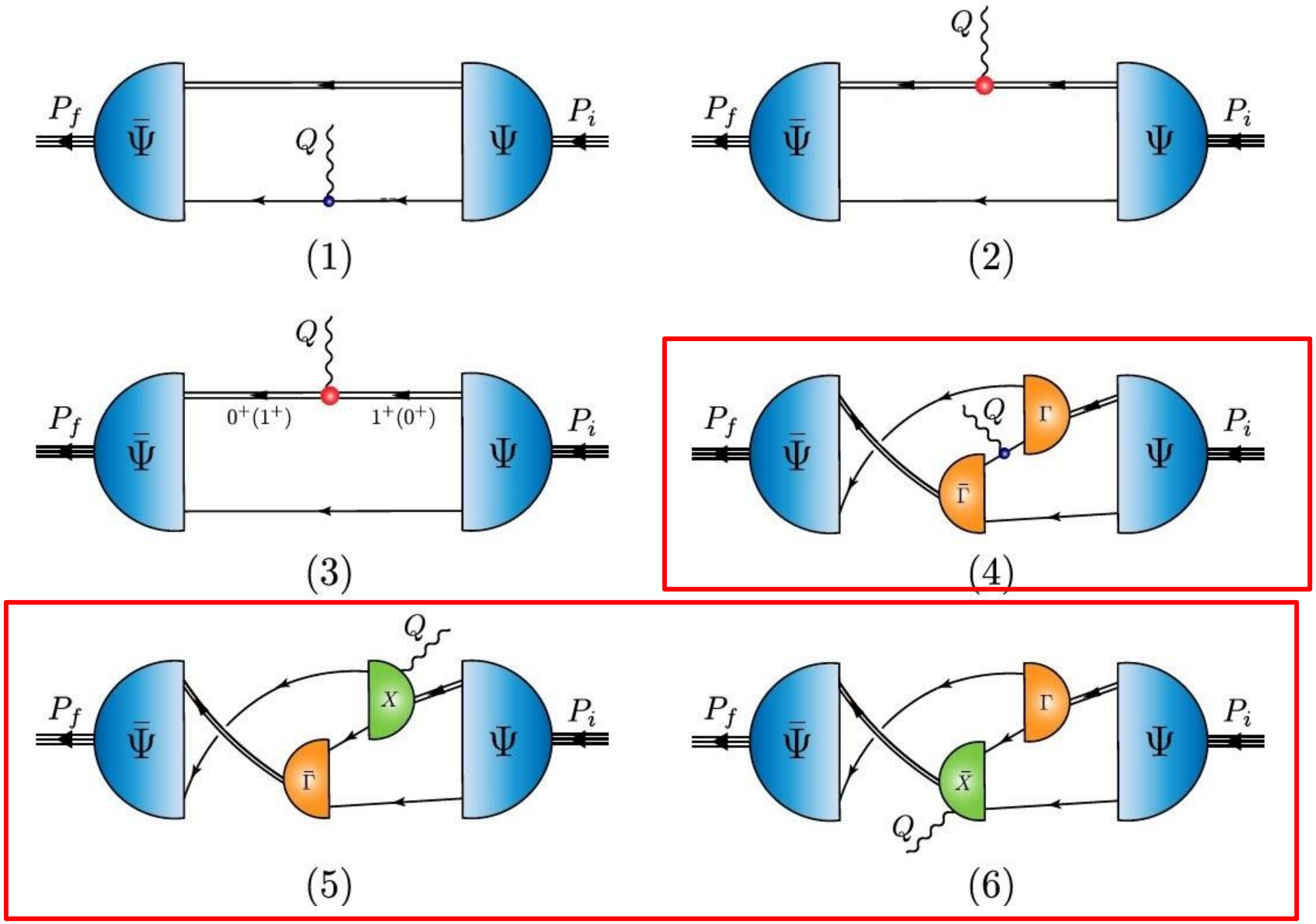
Using Eqs. (68), (73), (78), (79), (80), (81), (86) and (91), it is straightforward to obtain their sum:

$$\begin{aligned} &Q_\mu J^j_{5\mu}(K,Q) + 2im_q J^j_5(K,Q) = \sum_{J_1^{P_1},J_2^{P_2}=0+,1+} \\ &\Big[(Q_\mu J^{{\rm q},J_1^{P_1}J_2^{P_2}}_{5\mu}(K,Q) + 2im_q J^{{\rm q},J_1^{P_1}J_2^{P_2}}_5(K,Q)) \\ &+(Q_\mu J^{{\rm ex},J_1^{P_1}J_2^{P_2}}_{5\mu}(K,Q) + 2im_q J^{{\rm ex},J_1^{P_1}J_2^{P_2}}_5(K,Q)) \\ &+(Q_\mu J^{{\rm sg},J_1^{P_1}J_2^{P_2}}_{5\mu}(K,Q) + 2im_q J^{{\rm sg},J_1^{P_1}J_2^{P_2}}_5(K,Q)) \\ &+(Q_\mu J^{\overline{\rm sg},J_1^{P_1}J_2^{P_2}}_{5\mu}(K,Q) + 2im_q J^{\overline{\rm sg},J_1^{P_1}J_2^{P_2}}_5(K,Q))\Big] \\ &= 0\,, \qquad (96)\end{aligned}$$

where $j = 3$ for the neutral current, or $j = 1 \pm i2$ for the charged currents.

Form Factors at Large *Q^2*

- **In the quark-diquark framework, the associated symmetry-preserving current:**



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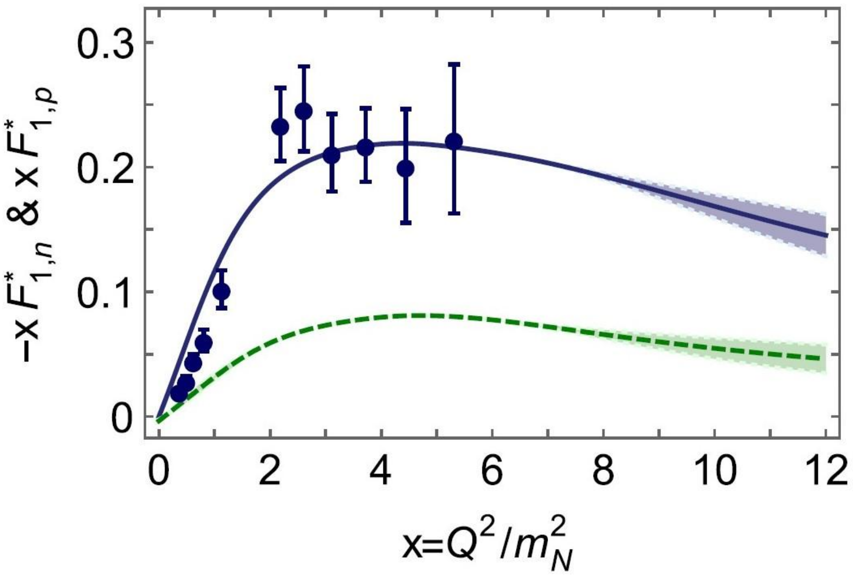
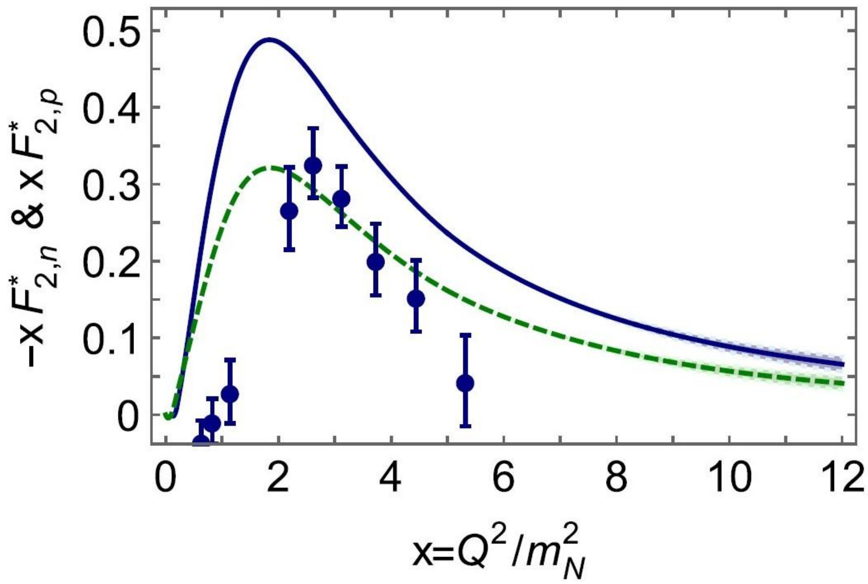
- The CLAS12 detector at JLab 12 will deliver data on the Roper-resonance electroproduction form factors out to *Q^2 ≈ 12 mN^2*.
- We use the *Schlessinger point method* (SPM) to interpolate the transition form factors, calculated on *Q^2* = [0, 6] *mN^2*, and then extrapolate the results on *Q^2* = [6, 12] *mN^2.*
- Our predictions will be tested in the foreseeable future.





Summary

- ✓ By accounting for the gluon contribution, I presented the first symmetry-preserving predictions for the pion and kaon valence-quark distribution functions. This work was used to win approval from CERN management for the COMPASS++/AMBER Phase-1 project and provides crucial theory background for the development of Phase-2 plans.

- ✓ By including all kinds of diquarks, I performed a comparative study of the four lightest baryon (I=1/2, J^P=1/2^{+-}) isospin doublets in order to both elucidate their structural similarities and differences. This is the first **ON-SHELL** DSE treatment of these systems.

- ✓ Using the quark+diquark Faddeev equation description of baryon structure, I supplied the first predictions for the complete array of nucleon axial and pseudoscalar form factors. In the process, I solved a problem that had escaped understanding for more than **20** years. This work opens the door to an entirely new array of hadron structure studies using continuum Schwinger function methods.

- ✓ I developed and refined a novel method for use in the interpolation of hadron form factors, calculated using continuum Schwinger function methods, and their subsequent reliable extrapolation to very large momentum transfers with quantifiable uncertainty estimates.

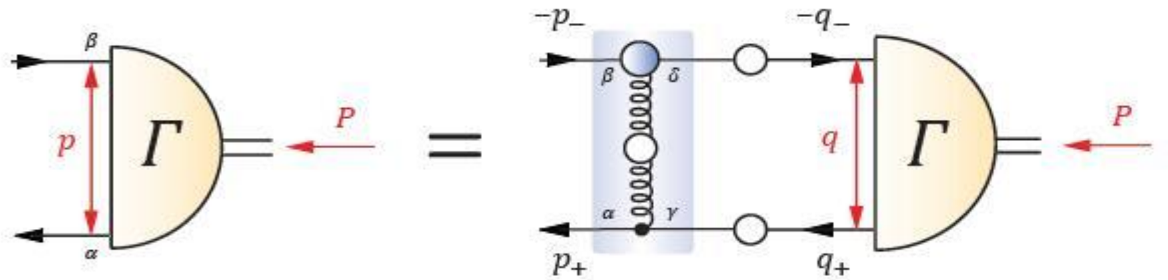
Thank you!

Dyson-Schwinger equations (DSEs)

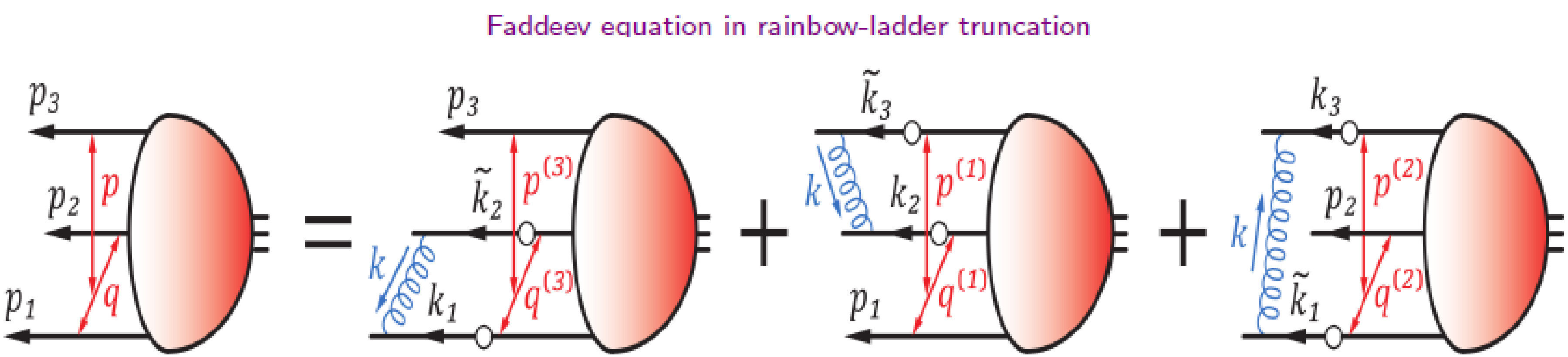
- **Dyson-Schwinger equations**
  - ✓ A Nonperturbative symmetry-preserving tool for the study of Continuum-QCD
  - ✓ Well suited to Relativistic Quantum Field Theory
  - ✓ A method connects observables with long-range behaviour of the running coupling
  - ✓ Experiment $\leftrightarrow$ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling

Hadrons: Bound-states in QFT

- **Mesons: a 2-body bound state problem in QFT**
  - ***Bethe-Salpeter Equation***
  - **K - fully amputated, two-particle irreducible, quark-antiquark scattering kernel**

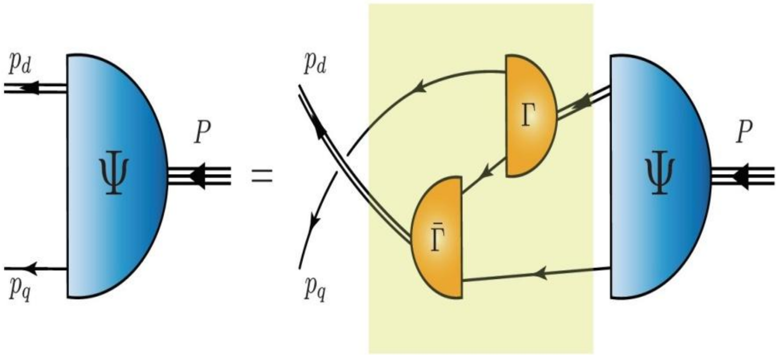


- **Baryons: a 3-body bound state problem in QFT.**
- ***Faddeev equation*: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.**



Quark-diquark picture

- A baryon can be viewed as a Borromean bound-state, the binding within which has two contributions:
  - ✓ Formation of tight diquark correlations.
  - ✓ Quark exchange depicted in the shaded area.



- The exchange ensures that diquark correlations within the baryon are fully dynamical: no quark holds a special place.
- The rearrangement of the quarks guarantees that the baryon's wave function complies with Pauli statistics.
- Modern diquarks are different from the old static, point-like diquarks which featured in early attempts to explain the so-called missing resonance problem.
- The number of states in the spectrum of baryons obtained is similar to that found in the three-constituent quark model, just as it is in today's LQCD calculations.

QCD-kindred model

- **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$$

- **algebraic form:**

$$\bar{\sigma}_S(x) = 2\bar{m}\mathcal{F}(2(x+\bar{m}^2)) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)[b_0 + b_2\mathcal{F}(\epsilon x)], \tag{A3a}$$

$$\bar{\sigma}_V(x) = \frac{1}{x+\bar{m}^2}[1 - \mathcal{F}(2(x+\bar{m}^2))], \tag{A3b}$$

with $x = p^2/\lambda^2$, $\bar{m} = m/\lambda$,

$$\mathcal{F}(x) = \frac{1-e^{-x}}{x}, \tag{A4}$$

$\bar{\sigma}_S(x) = \lambda\sigma_S(p^2)$ and $\bar{\sigma}_V(x) = \lambda^2\sigma_V(p^2)$. The mass scale, $\lambda = 0.566$ GeV, and parameter values,

$$\begin{array}{ccccc} \bar{m} & b_0 & b_1 & b_2 & b_3 \\ \hline 0.00897 & 0.131 & 2.90 & 0.603 & 0.185 \end{array}, \tag{A5}$$

associated with Eq. (A3) were fixed in a least-squares fit to light-meson observables [79,80]. [$\epsilon = 10^{-4}$ in Eq. (A3a) acts only to decouple the large- and intermediate-p^2 domains.]

QCD-kindred model

- **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$$

  - Based on solutions to the gap equation that were obtained with a dressed gluon-quark vertex.
  - Mass function has a real-world value at $p^2 = 0$, NOT the highly inflated value typical of ~~RL~~ truncation.
  - Propagators are entire functions, consistent with sufficient condition for confinement and completely unlike known results from ~~RL~~ truncation.
  - Parameters in quark propagators were fitted to a diverse array of meson observables. **ZERO** parameters changed in study of baryons.
  - Compare with that computed using the DCSB-improved gap equation kernel (DB). The parametrization is a sound representation numerical results, although simple and introdu long beforehand.

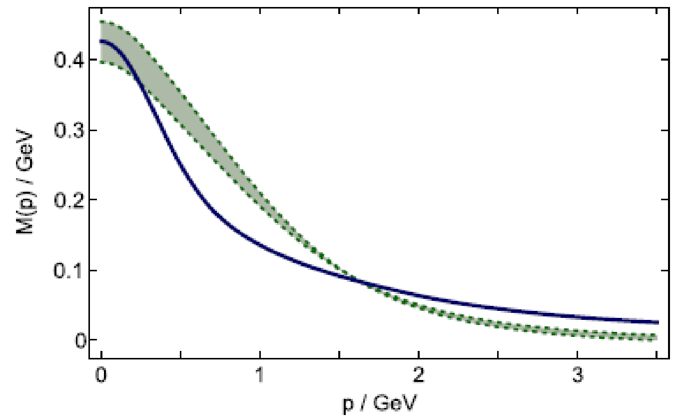


FIG. 6. Solid curve (blue)—quark mass function generated by the parametrization of the dressed-quark propagator specified by Eqs. (A3) and (A4) (A5); and band (green)—exemplary range of numerical results obtained by solving the gap equation with the modern DCSB-improved kernels described and used in Refs. [16,81–83].

QCD-kindred model

- **Diquark amplitudes: five types of correlation are possible in a *J=1/2* bound state: isoscalar scalar(I=0,J^P=0^+), isovector pseudovector, isoscalar pseudoscalar, isoscalar vector, and isovector vector.**
- **The LEADING structures in the correlation amplitudes for each case are, respectively (Dirac-flavor-color),**

$$\Gamma^{0^+}(k;K) = g_{0^+}\gamma_5 C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{0^+}^2),$$

$$\vec{\Gamma}_\mu^{1^+}(k;K) = i g_{1^+}\gamma_\mu C\vec{\tau}\vec{H}\,\mathcal{F}(k^2/\omega_{1^+}^2),$$

$$\Gamma^{0^-}(k;K) = i g_{0^-} C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{0^-}^2),$$

$$\Gamma_\mu^{1^-}(k;K) = g_{1^-}\gamma_\mu\gamma_5 C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{1^-}^2),$$

$$\vec{\Gamma}_\mu^{\bar{1}^-}(k;K) = i g_{\bar{1}^-}[\gamma_\mu, \gamma\cdot K]\gamma_5 C\vec{\tau}\vec{H}\,\mathcal{F}(k^2/\omega_{\bar{1}^-}^2),$$

- **Simple form. Just one parameter: diquark masses.**
- **Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.**

QCD-kindred model

- **Diquark masses (in GeV):**

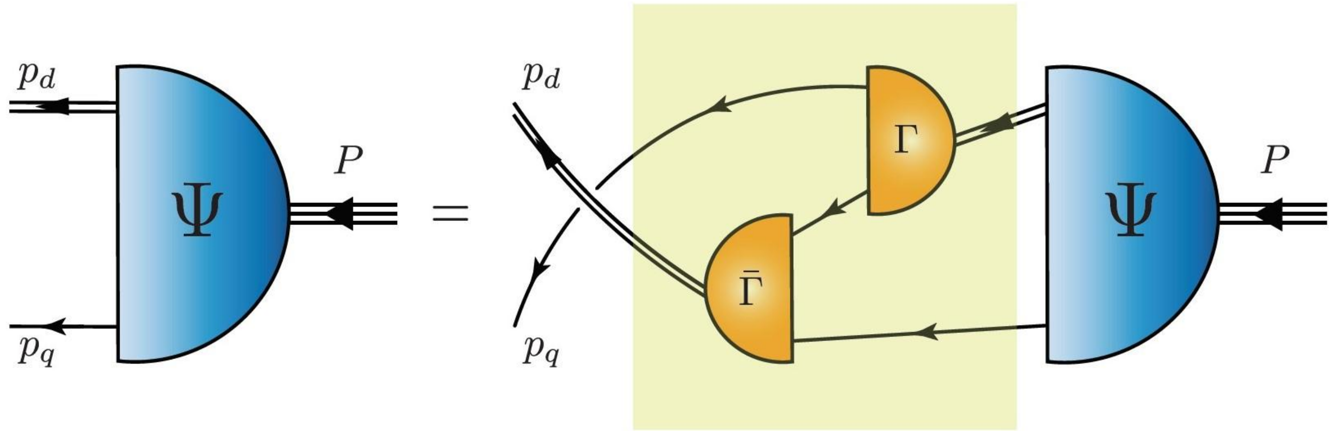
$$m_{0^+} = 0.8, \quad m_{1^+} = 0.9, \quad m_{0^-} = 1.2, \quad m_{1^-} = 1.3,$$

- **The first two values (positive-parity) provide for a good description of numerous dynamical properties of the nucleon, Δ-baryon and Roper resonance.**
- **Masses of the odd-parity correlations are based on those computed from a contact interaction.**
- **Such values are typical; and in truncations of the two-body scattering problem that are most widely used (RL), isoscalar- and isovector-vector correlations are degenerate.**
- **Normalization condition → couplings:**

$$g_{0^+} = 14.8, \qquad g_{1^+} = 12.7,$$
$$g_{0^-} = 12.8, \qquad g_{1^-} = 5.4, \qquad \cancel{g_{\bar{1}^-} = 2.5}.$$

- **Faddeev kernels: 22 × 22 matrices are reduced to 16 × 16 !**

A parameter: gDB



- **There is an absence of <u>spin-orbit repulsion</u> owing to an oversimplification of the gluon-quark vertex when formulating the RL bound-state equations. We therefore employ a simple artifice in order to implement the missing interactions.**

  - ✓ **We introduce a single parameter into the Faddeev equation for J^P=1/2^P baryons: gDB, a linear multiplicative factor attached to each opposite-parity (-P) diquark amplitude in the baryon's Faddeev equation kernel.**
  - ✓ **gDB is the single free parameter in our study.**

QCD-kindred model

- **The diquark propagators**

$$\Delta^{0^\pm}(K) = \frac{1}{m_{0^\pm}^2}\mathcal{F}(k^2/\omega_{0^\pm}^2),$$

$$\Delta_{\mu\nu}^{1^\pm}(K) = \left[\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^\pm}^2}\right]\frac{1}{m_{1^\pm}^2}\mathcal{F}(k^2/\omega_{1^\pm}^2).$$

- **The *$\mathcal{F}$-functions*: Simplest possible form that is consistent with infrared and ultraviolet constraints of confinement (IR) and 1/q^2 evolution (UV) of meson propagators.**

- **Diquarks are confined.**
  - **free-particle-like at spacelike momenta**
  - **pole-free on the timelike axis**
  - **This is NOT true of ~~RL~~ studies. It enables us to reach arbitrarily high values of momentum transfer.**

QCD-kindred model

- **The Faddeev amplitudes:**

$$\psi^{\pm}(p_i,\alpha_i,\sigma_i) = [\Gamma^{0^+}(k;K)]^{\alpha_1\alpha_2}_{\sigma_1\sigma_2}\Delta^{0^+}(K)[\boxed{\varphi^{\pm}_{0^+}(\ell;P)}u(P)]^{\alpha_3}_{\sigma_3}$$
$$+ [\Gamma^{1^+j}_{\mu}]\Delta^{1^+}_{\mu\nu}[\varphi^{j\pm}_{1^+\nu}(\ell;P)u(P)]$$
$$+ [\Gamma^{0^-}]\Delta^{0^-}[\varphi^{\pm}_{0^-}(\ell;P)u(P)]$$
$$+ [\Gamma^{1^-}_{\mu}]\Delta^{1^-}_{\mu\nu}[\varphi^{\pm}_{1^-\nu}(\ell;P)u(P)], \qquad (9)$$

- **Quark-diquark vertices:**

$$\varphi^{\pm}_{0^+}(\ell;P) = \sum_{i=1}^{2} \mathcal{s}^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{j\pm}_{1^+\nu}(\ell;P) = \sum_{i=1}^{6} \mathcal{a}^{j\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_\nu(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{\pm}_{0^-}(\ell;P) = \sum_{i=1}^{2} \mathcal{p}^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\mp},$$

$$\varphi^{\pm}_{1^-\nu}(\ell;P) = \sum_{i=1}^{6} \mathcal{v}^{\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_\nu(\ell;P)\mathcal{G}^{\mp},$$

where $\mathcal{G}^{+(-)} = \mathbf{I}_{\rm D}(\gamma_5)$ and

$$\mathcal{S}^1 = \mathbf{I}_{\rm D}, \qquad \mathcal{S}^2 = i\gamma\cdot\hat{\ell} - \hat{\ell}\cdot\hat{P}\mathbf{I}_{\rm D}$$
$$\mathcal{A}^1_\nu = \gamma\cdot\ell^{\perp}\hat{P}_\nu, \qquad \mathcal{A}^2_\nu = -i\hat{P}_\nu\mathbf{I}_{\rm D}, \qquad \mathcal{A}^3_\nu = \gamma\cdot\hat{\ell}^{\perp}\hat{\ell}^{\perp}_\nu$$
$$\mathcal{A}^4_\nu = i\hat{\ell}^{\perp}_\nu\mathbf{I}_{\rm D}, \quad \mathcal{A}^5_\nu = \gamma^{\perp}_\nu - \mathcal{A}^3_\nu, \quad \mathcal{A}^6_\nu = i\gamma^{\perp}_\nu\gamma\cdot\hat{\ell}^{\perp} - \mathcal{A}^4_\nu,$$

QCD-kindred model

- **Both the Faddeev amplitude and wave function are Poincare covariant, i.e. they are qualitatively identical in all reference frames.**
- **Each of the scalar functions that appears is frame independent, but the frame chosen determines just how the elements should be combined.**
- **In consequence, the manner by which the dressed quarks' spin, *S*, and orbital angular momentum, *L*, add to form the total momentum *J,* is frame dependent: *L*, *S* are not independently Poincare invariant.**
- **The set of baryon rest-frame quark-diquark angular momentum identifications:**

$$^2S:\ \mathcal{S}^1, \mathcal{A}_\nu^2, (\mathcal{A}_\nu^3 + \mathcal{A}_\nu^5),$$

$$^2P:\ \mathcal{S}^2, \mathcal{A}_\nu^1, (\mathcal{A}_\nu^4 + \mathcal{A}_\nu^6),$$

$$^4P:\ (2\mathcal{A}_\nu^4 - \mathcal{A}_\nu^6)/3,$$

$$^4D:\ (2\mathcal{A}_\nu^3 - \mathcal{A}_\nu^5)/3,$$

- **The scalar functions associated with these combinations of Dirac matrices in a Faddeev wave function possess the identified angular momentum correlation between the quark and diquark.**

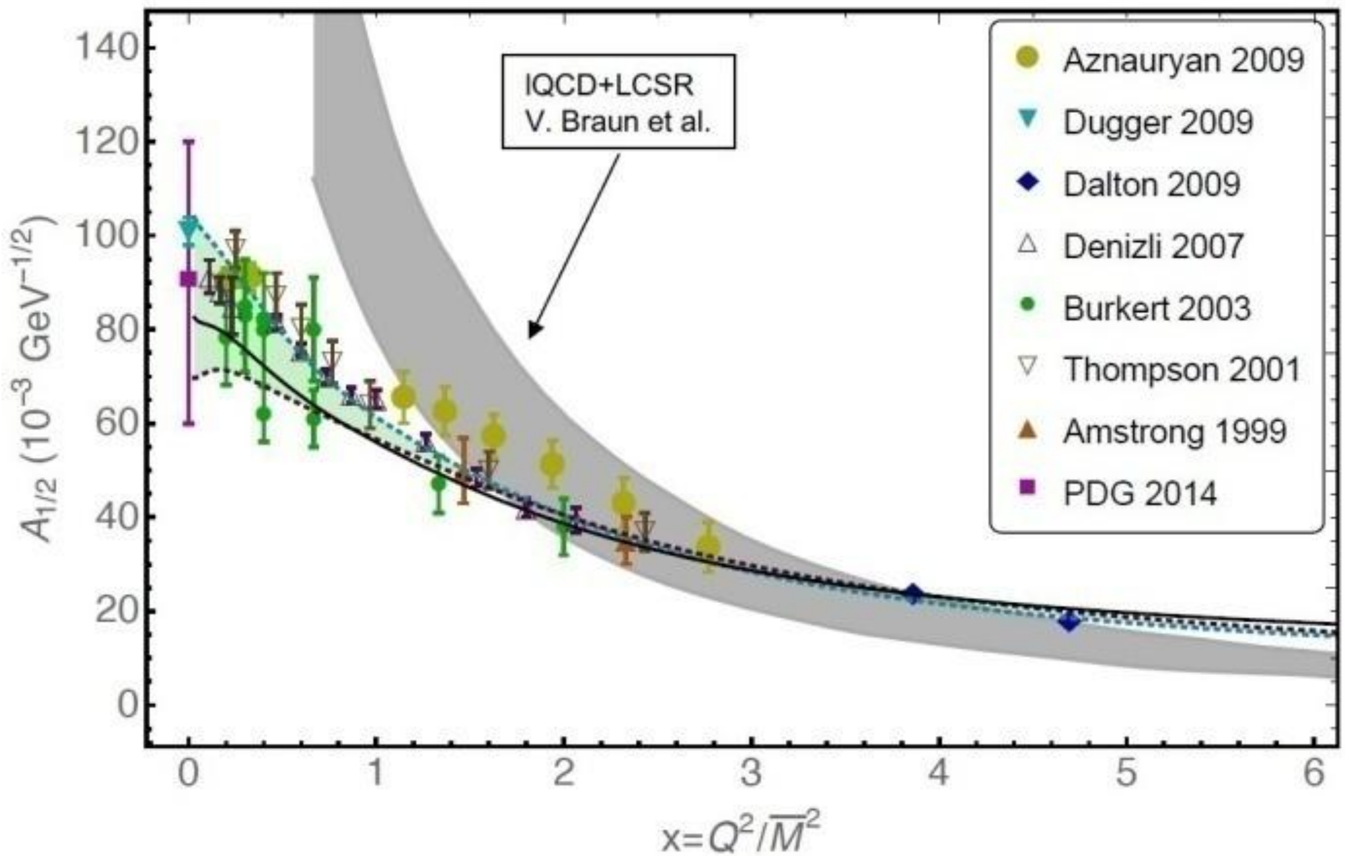
SOLUTIONS & THEIR PROPERTIES:
Masses

- We choose gDB=0.43 so as to produce a mass splitting of 0.1 GeV between the lowest-mass P=- state and the first excited P=+ state, viz. the empirical value.
- Our computed values for the masses of the four lightest 1/2^{+-} baryon doublets are listed here, in GeV:

$g_{\rm DB}$	m_N	$m_{N(1440)}^{1/2^+}$	$m_{N(1535)}^{1/2^-}$	$m_{N(1650)}^{1/2^-}$
0.43	1.19	1.73	1.83	1.91
1.0	1.19	1.73	1.43	1.61

- Pseudoscalar and vector diquarks have no impact on the mass of the two positive-parity baryons, whereas scalar and pseudovector diquarks are important to the negative parity systems.
- Although 1/2^- solutions exist even if one eliminates pseudoscalar and vector diquarks, 1/2^+ solutions do not exist in the absence of scalar and pseudovector diquarks.
- It indicates that, with our Faddeev kernel, the so-called P-wave (negative-parity) baryons can readily be built from positive-parity diquarks.

The $\gamma^{(*)}p \to N(1535)\frac{1}{2}^-$ Transition



In process, will be submitted to <u>*Physical Review Letters*</u>

Building blocks (I)

➢ **The current-quark vertices**

- **The axial-vector Ward-Takahashi identity:**

$$Q_\mu \Gamma^j_{5\mu}(k_+,k_-) + 2im_q\Gamma^j_5(k_+,k_-) = S^{-1}(k_+)i\gamma_5\frac{\tau^j}{2} + \frac{\tau^j}{2}i\gamma_5 S^{-1}(k_-)$$

- **The Bethe-Salpeter Amplitude of the pion:**

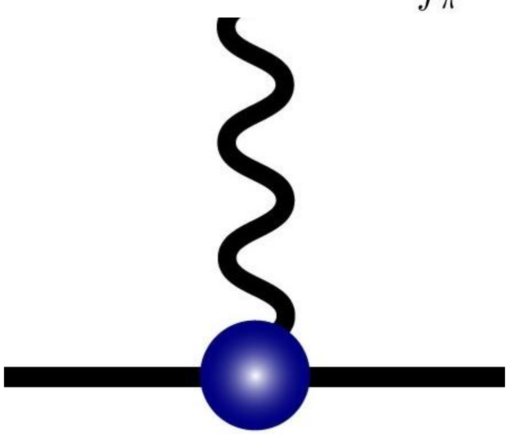
$$\Gamma^j_\pi(k,Q) = \tau^j\gamma_5\left[\boxed{iE_\pi(k,Q)}\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\right]$$

- **One *Ansatz*:** $E_\pi(k,Q) = \dfrac{1}{2f_\pi}\left(B(k_+^2) + B(k_-^2)\right)$

$$S^{-1}(k) = i\gamma\cdot kA(k^2) + B(k^2)$$

in the chiral limit:

$$E_\pi(k,0) = \frac{B(k^2)}{f_\pi}$$



Therefore, we finally arrive at

$$\Gamma^j_{5\mu}(k_+,k_-) = \frac{\tau^j}{2}\gamma_5\Big[\gamma_\mu\Sigma_A(k_+^2,k_-^2) + 2\gamma\cdot kk_\mu\Delta_A(k_+^2,k_-^2)$$

$$+ 2i\frac{Q_\mu}{Q^2+m_\pi^2}\Sigma_B(k_+^2,k_-^2)\Big], \qquad (28)$$

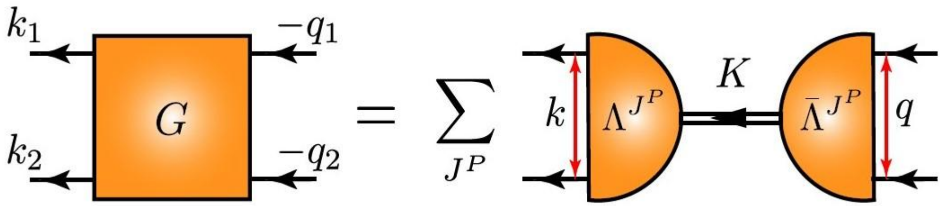
and

$$i\Gamma^j_5(k_+,k_-) = \frac{m_\pi^2}{Q^2+m_\pi^2}\frac{f_\pi}{2m_q}\Gamma^j_\pi(k,Q)$$

$$\equiv \frac{\tau^j}{2}\frac{m_\pi^2}{Q^2+m_\pi^2}\frac{1}{m_q}i\gamma_5\Sigma_B(k_+^2,k_-^2), \qquad (29)$$

Building blocks (II)

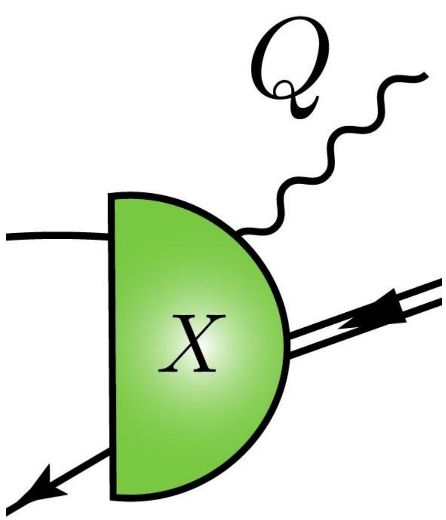
- **The seagull terms**
  - **The diquark *Ansatz* for the 4-point Green's function of the quark-quark correlations:**



  - **The equaltime commutators of the axial current operator:**

$$[\mathcal{A}^j_{5\mu=4}(x), \psi(y)]_{x_4=y_4} = \frac{\tau^j}{2}\gamma_5\psi(x)\delta^{(4)}(x-y)$$

$$[\mathcal{A}^j_{5\mu=4}(x), \bar{\psi}(y)]_{x_4=y_4} = \bar{\psi}(x)\gamma_5\frac{\tau^j}{2}\delta^{(4)}(x-y)$$



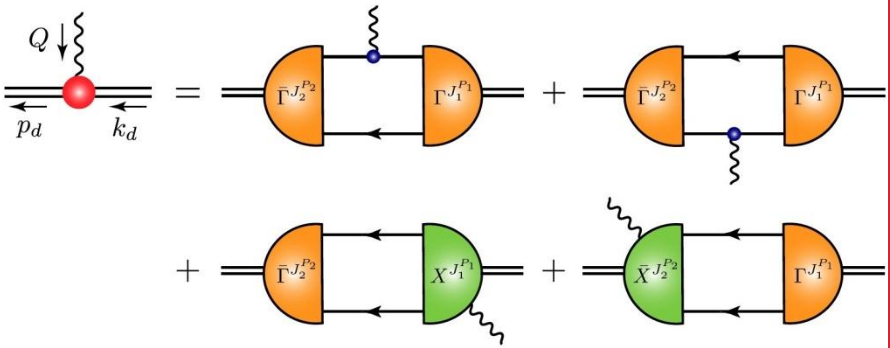
$$\chi^{j,J^P}_{5\mu,[\mathrm{sg}]}(k,Q) = -\frac{Q_\mu}{Q^2+m_\pi^2}\left[\frac{\tau^j}{2}i\gamma_5\Gamma^{J^P}(k-Q/2)+\right.$$
$$\left.\Gamma^{J^P}(k+Q/2)(i\gamma_5\frac{\tau^j}{2})^{\mathrm{T}}\right], \tag{57}$$

and

$$i\chi^{j,J^P}_{5,[\mathrm{sg}]}(k,Q) = -\frac{1}{2m_q}\frac{m_\pi^2}{Q^2+m_\pi^2}\left[\frac{\tau^j}{2}i\gamma_5\Gamma^{J^P}(k-Q/2)+\right.$$
$$\left.\Gamma^{J^P}(k+Q/2)(i\gamma_5\frac{\tau^j}{2})^{\mathrm{T}}\right]. \tag{58}$$

Building blocks (III)

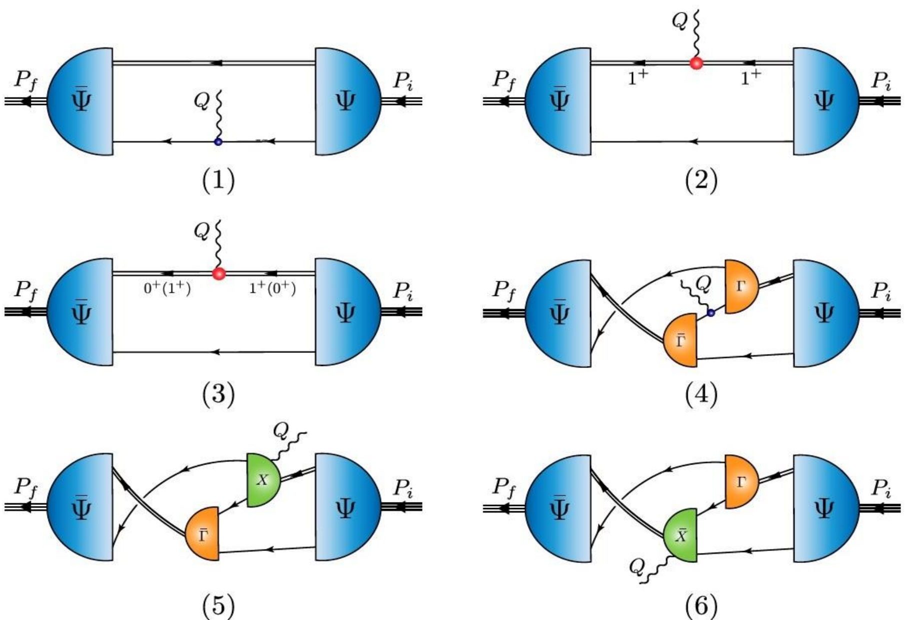
➢ The current-diquark vertices



➢ AXWTIs:

$$Q_\mu\,\Gamma^{aa}_{5\mu,\alpha\beta}(p_d,k_d)+2im_q\,\Gamma^{aa}_{5,\alpha\beta}(p_d,k_d)=0$$

$$Q_\mu\,\Gamma^{sa}_{5\mu,\beta}(p_d,k_d)+2im_q\,\Gamma^{sa}_{5,\beta}(p_d,k_d)=0$$



i) The $\{qq\}_{1^+}$–pseudoscalar-current vertex

$$\Gamma^{aa}_{5,\alpha\beta}(p_d,k_d)= \\ =\frac{1}{2m_q}\frac{m_\pi^2}{Q^2+m_\pi^2}\left(\kappa^{aa}_{\rm ps}\frac{M_q^E}{m_N}\epsilon_{\alpha\beta\gamma\delta}(p_d+k_d)_\gamma Q_\delta\right)d(\tau^{aa})\,, \tag{61}$$

ii) The $\{qq\}_{1^+}$–axial-current vertex

$$\Gamma^{aa}_{5\mu,\alpha\beta}(p_d,k_d)=\left(\frac{\kappa^{aa}_{\rm ax}}{2}\epsilon_{\mu\alpha\beta\nu}(p_d+k_d)_\nu+\right. \\ \left.+\frac{Q_\mu}{Q^2+m_\pi^2}\left(\kappa^{aa}_{\rm ps}\frac{M_q^E}{m_N}\epsilon_{\alpha\beta\gamma\delta}(p_d+k_d)_\gamma Q_\delta\right)\right)d(\tau^{aa})\,, \tag{62}$$

iii) The pseudoscalar-current induced $0^+\leftarrow 1^+$ transition vertex

$$\Gamma^{sa}_{5,\beta}(p_d,k_d)= \\ =\frac{1}{2m_q}\frac{m_\pi^2}{Q^2+m_\pi^2}\left(-2i\kappa^{sa}_{\rm ps}M_q^E Q_\beta\right)d(\tau^{sa})\,, \tag{63}$$

iv) The axial-current induced $0^+\leftarrow 1^+$ transition vertex

$$\Gamma^{sa}_{5\mu,\beta}(p_d,k_d)=\left(im_N\kappa^{sa}_{\rm ax}\delta_{\mu\beta}+\right. \\ \left.+\frac{Q_\mu}{Q^2+m_\pi^2}\left(-2i\kappa^{sa}_{\rm ps}M_q^E Q_\beta\right)\right)d(\tau^{sa})\,. \tag{64}$$

The Schlessinger point method (SPM)

- SPM: based on the Pade approximation.

- For a computed form factor *F(Q^2)*, one has a collection of *N* results, each associated with the form factor at a different value of *Q^2* = [0, *Q^2*_max].

- One randomly chooses first one point, then two, etc., until reaching that minimal number of points, *M<N*, for which the analytic approximation produced by the SPM approximation from any randomly chosen set of *M* points typically delivers a valid fit to the output.

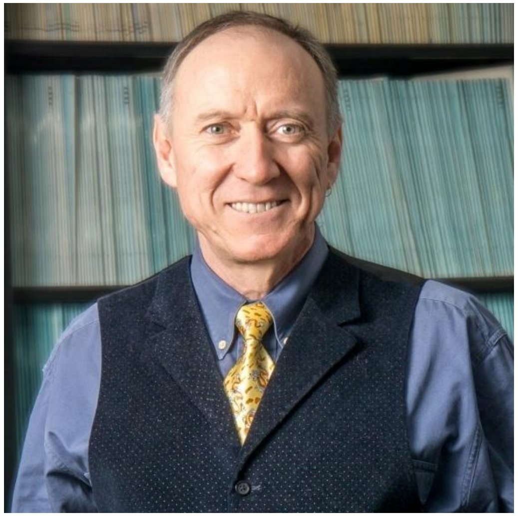
- One then defines the extrapolation by randomly choosing a large number of *M*-point samples; determining the SPM approximation from each collection; applying any known physical constraints (such as continuity, known scaling behaviour, etc.) to eliminate those functions which are unacceptable; and then drawing the associated extrapolation curve for each surviving approximation. This procedure generates a band of extrapolated curves whose collective reliability at any *Q^2>Q^2_max* is expressed by the width of the band at that point, which is itself determined by the precision of the original output on [0, *Q^2*_max].

- *Phys. Rev. D* **99** (2019) 3, 034013: ***M=12* & *1000 times***.

Education & Academic Employment

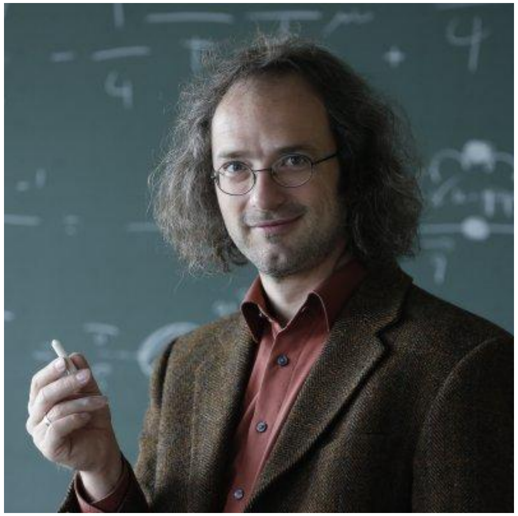
Dec 2021 – Present	**Visiting Scientist**, *Institute for Nonperturbative Physics, Nanjing University*, Nanjing, China.
May 2019 – Oct 2021	**Research Associate**, *Institute for Theoretical Physics, Justus Liebig University Giessen*, Giessen, Germany.
May 2016 – Mar 2019	**FAPESP Fellow**, *The Institute for Theoretical Physics, UNESP*, São Paulo, Brazil. Sponsored by FAPESP Foundation.
Nov 2013 – Apr 2016	**Postdoctoral Fellow**, *University of Science and Technology of China*, Hefei, China. As stipulated by the conditions attached to my Study Abroad program from the China Scholarship Council.
Jan 2011 – Jan 2013	**Visiting Scholar**, *Illinois Institute of Technology*, Chicago, US. Sponsored by Illinois Institute of Technology.
Jan 2011 – Jan 2013	**Visiting Graduate Student**, *Argonne National Laboratory*, Chicago, US. Sponsored by China Scholarship Council.
Sep 2007 – Aug 2013	**Ph.D., Theoretical Physics**, *University of Science and Technology of China*, Hefei, China.
Sep 2000 – Jun 2004	**B.Sc., Information Management and Information System**, *Nanjing University of Aeronautics and Astronautics*, Nanjing, China.

International Cooperation



➢ **Prof. Dr. *Craig D. Roberts***

- 2019-Present: International Distinguished Professor and Head: Institute for Nonperturbative Physics at Nanjing University
- 2001 ~ 2017: Group Leader, Theory, Argonne National Laboratory
- The Friedrich Wilhelm Bessel Research Prize
- The Helmholtz International Fellow Award
- The University of Chicago – Distinguished Performance Award
- Citations: 20,000+



➢ **Prof. Dr. *Christian S. Fischer***

- W3 professor, Justus Liebig University Giessen (JLU)
- Editor-in-Chief of ***Progress in Particle and Nuclear Physics***
- Helmholtz Young Investigator Award
- Citations: ~9,000

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