Precision Predictions for Top-quark Width

王烨凡 (山东大学)

arxiv: 2212.06341

in cooperation with 陈龙斌, 李海涛, 王健

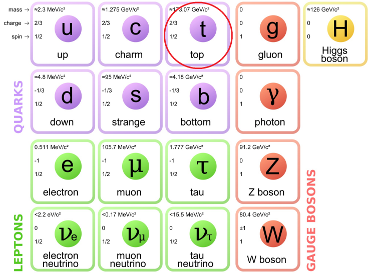
第一届量子场论数学结构讲习班

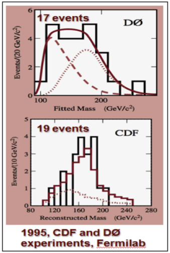
2023-05-17

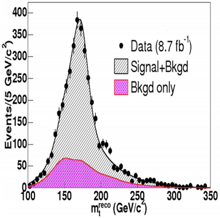
Outline

- Background of Top-quark
- High-order Corrections of Top-quark Width
- Summary and Outlook

Background of Top-quark







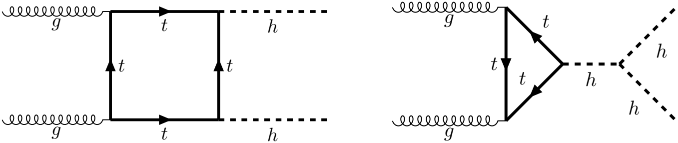
[Denisov, Vellidis 2015]

Due to the large mass of top-quark, until 1995 it was discovered by CDF and D0 cooperation at Tevatron.

Background of Top-quark

Top-quark is the heaviest elementary particle in the Standard Model.

Top-quark provides the strongest coupling to the SM Higgs boson and opens doors to new physics.

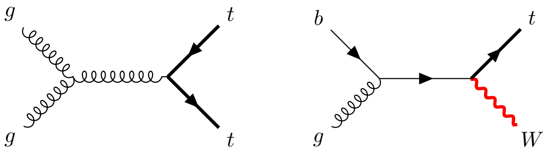


gluon fusion

Background of Top-quark

At hardron colliders, the dominant contribution to the top quark production is top-pair production.

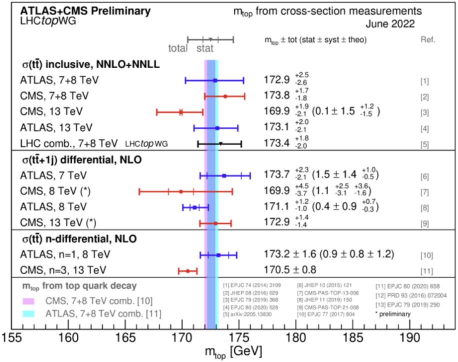
The next largest contribution is single-top production, which was observed in 2009 at the Tevatron [CDF 2009, D0 2009].



Background of Top-quark

Top-quark mass is the one of the fundamental parameters in Standard Model.

Summary of the top-mass analyses at the LHC.

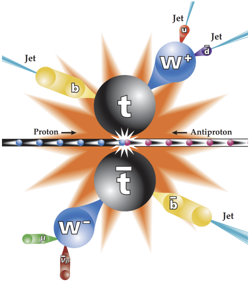


Background of Top-quark

Top decay width Γ_t is one of fundamental properties of top-quark.

Due to its large mass, Γ_t is expected to be very large.

The measurement of Γ_t could hint at new-physics.



[Denisov, Vellidis 2015]

Motivation

The top-quark decays almost exclusively to Wb. $\Gamma_t = \Gamma_t(t \to Wb)$.

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$

[PDG 2022]

At LHC, the direct measurement is model independent but less precise, $\Gamma_t = 1.9 \pm 0.5$ GeV by ATLAS [ATLAS 2019].

The indirect measurement is model dependent but more precise, $\Gamma_t = 1.36 \pm 0.02\ (\text{stat.})^{+0.14}_{-0.11}\ (\text{syst.})$ GeV by CMS [CMS, 2014], which is the most precise measurement for Γ_t by now.

In the future e^+e^- collider, Γ_t can be measured with an uncertainty of 30 MeV [Martinez, Miquel 2019].

Motivation

On the theoretical side,

NLO QCD corrections [Jezabek, Kuhn 1989, Czarnecki 1990, Li, Oakes, Yuan 1991]

NLO EW corrections [Denner, Sack 1991, Eilam, Mendel, Migneron, Soni 1991]

Asymptotic analytical results of NNLO QCD corrections using $m_W \to 0$ and $m_W \to m_t$ [Czarnecki, Melnikov 1999, Chetyrkin, Harlander, Seidensticker, Steinhauser 1999, Blokland, Czarnecki, Slusarczyk, Tkachov 2004 2005]

Numerical result of full NNLO QCD corrections [Gao, Li, Zhu 2013, Brucherseifer, Caola, Melnikov 2013]

The full analytical results of NNLO QCD corrections are unknown.

Optical Theorem

Unitarity implies the S-matrix

$$S^\dagger S = 1, \quad S = \mathbb{1} + iT \tag{1}$$

where T is transfer matrix

$$\langle f|T|i\rangle = (2\pi)^4\delta^4(p_i - p_f)\mathcal{M}(i \to f). \tag{2}$$

The generalized optical theorem is

$$\mathcal{M}(i \to f) - \mathcal{M}^\star(i \to f) = i\Sigma_X \int d\prod_X (2\pi)^4\delta^4(p_i - p_f)\mathcal{M}(i \to X)\mathcal{M}^\star(f \to X) \tag{3}$$

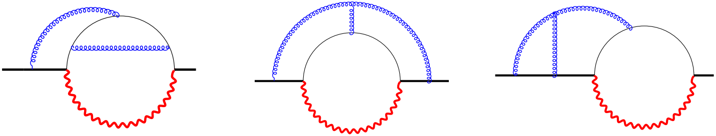
If $|i\rangle = |f\rangle = |A\rangle$ and $|A\rangle$ is one-particle state,

$$\mathrm{Im}\mathcal{M}\left(A \to A\right) = m_A\Sigma_X\Gamma\left(A \to X\right) = m_A\Gamma_{\text{tot}}. \tag{4}$$

Consider the three-loop self-energy diagrams Σ for $t \to Wb \to t$

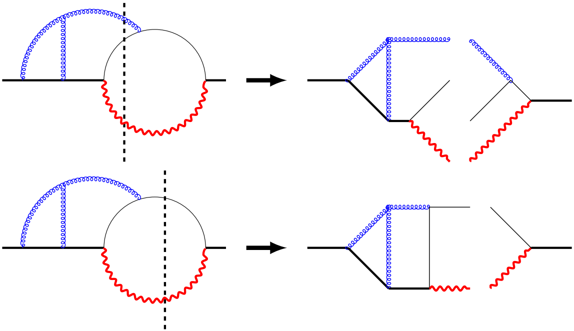
$$\Gamma_t = \frac{\mathrm{Im}(\Sigma)}{m_t} \tag{5}$$

Some typical three-loop diagrams in Σ



Optical Theorem

The imaginary part comes from cut diagrams. For example,



The separated virtual and real corrections are combined.

The complicated phase space integration can be avoided.

Scalar Integrals

For $t \to Wb \to t$, b quark is assumed massless. Kinematic variable is $w = m_W^2/m_t^2$

After spin summation

$$u(k, m_t)\bar{u}(k, m_t) = \not{k} + m_t \tag{6}$$

the amplitudes can be written as the linear combination of scalar integrals.

It means the numerator of integrals are scalar products, such as

$$\int \mathcal{D}^D q_1 \ \mathcal{D}^D q_2 \ \mathcal{D}^D q_3 \frac{(k \cdot q_1)(q_1 \cdot q_2) q_3^2}{D_1 D_2 D_3 D_4 D_5 D_6 D_7 D_8 D_9}, \tag{7}$$

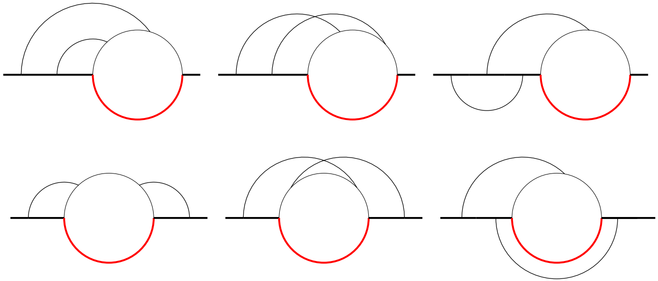
where q_1, q_2, q_3 are loop momenta, k is external momentum. The Lorentz tensor are vanished.

Master Integrals

After integral reduction, the scalar integrals can expressed by minimal set of integrals called master integrals.

In this step we used integration-by-parts (IBP) identities and package `FIRE` [Smirnov, Chuharev 2019].

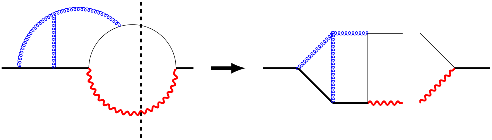
The typologies of master integrals



Master Integrals

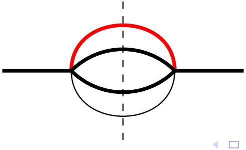
Imaginary part only from cuts of W boson and b quark.

Cannot get imaginary part from cuts crossing internal top quark.



Each master integral requires one W propagator.

The requirement leads to simplification in the calculation. For example,



Canonical Differential Equations

The key is to analytically calculate the master integrals.

Canonical differential equation [Henn 2013] is a powerful tool in analytical calculations.

The differential equations of canonical basis $\mathbf{F}$ can be written as

$$\frac{\partial \mathbf{F}(w,\epsilon)}{\partial w} = \epsilon \left[\sum_{i=1}^{4} \mathbf{R}_i \mathrm{d}\log(l_i)\right] \mathbf{F}(w,\epsilon), \quad w = \frac{m_W^2}{m_t^2}, \quad D = 4-2\epsilon \tag{8}$$

$l_i \in \{w-2, w-1, w, w+1\}$ and $\mathbf{R}_i$ being rational matrices. For example,

$$\frac{\partial F_4(w,\epsilon)}{\partial w} = \frac{\epsilon\,(F_5 - 2F_4)}{w-1} - \frac{\epsilon\,(F_4+F_5)}{w} \tag{9}$$

Canonical basis construction at three-loop is nontrivial.

By this canonical form, the differential equations can be solved recursively.

Boundary Conditions

Most of the basis integrals are regular at $w = 0$. For example,

$$\frac{\partial F_4(w,\epsilon)}{\partial w} = \frac{\epsilon\,(F_5 - 2F_4)}{w-1} - \frac{\epsilon\,(F_4 + F_5)}{w} \tag{10}$$

$$\Longrightarrow \quad F_4|_{w=0} + F_5|_{w=0} = 0 \tag{11}$$

The analytical results of some master integrals in $w = 0$ can be found in [Blokland, Czarnecki, Slusarczyk, Tkachov 2005, Ritbergen, Stuart 2000].

Boundary expressions can be reconstructed by numerical results with package `AMFlow` [Liu, Ma 2022].

HPLs

The analytical results of master integrals can be written as multiple polylogarithms (GPLs)

$$G_{a_1,a_2,\ldots,a_n}(x) \equiv \int_0^x \frac{\mathrm{d}t}{t-a_1} G_{a_2,\ldots,a_n}(t)\,, \tag{12}$$

$$G_{\vec{0}_n}(x) \equiv \frac{1}{n!} \ln^n x\,. \tag{13}$$

In our problem, we only need harmonic polylogarithms (HPLs).

$$H_{a_1,a_2,\ldots,a_n}(x) = G_{a_1,a_2,\ldots,a_n}(x)|_{a_i\in\{-1,0,1\}}. \tag{14}$$

For example,

$$H_0(x) = \ln x, \quad H_{1,0}(x) = \int_0^x \frac{\mathrm{d}t}{t-1} \ln t, \quad H_{-1,1,0}(x) = \int_0^x \frac{\mathrm{d}t}{t+1} H_{1,0}(t). \tag{15}$$

HPLs have good mathematical properties.

Analytical Results

Combing analytical results of master integrals and IBP relations, the bare amplitudes are obtained.

After renomarization, QCD corrections of Γ_t up to NNLO.

$$\Gamma(t \to Wb) = \Gamma_0 \left[X_0 + \frac{\alpha_s}{\pi} X_1 + \left(\frac{\alpha_s}{\pi}\right)^2 X_2 \right], \tag{16}$$

$$\Gamma_0 = \frac{G_F m_t^3 |V_{tb}|^2}{8\sqrt{2}\pi}. \tag{17}$$

The LO and NLO corrections are

$$\begin{aligned} X_0 &= (2w+1)(w-1)^2, \\ X_1 &= C_F\Big(X_0 \Big(-2H_{0,1}(w) + H_0(w)H_1(w) - \frac{\pi^2}{3}\Big) + \frac{1}{2}(4w+5)(w-1)^2 H_1(w) \\ &\quad + w(2w^2+w-1)H_0(w) + \frac{1}{4}(6w^3 - 15w^2 + 4w + 5)\Big) \end{aligned} \tag{18}$$

Analytical Results

According to color structure,

$$\Gamma(t \to Wb) = \Gamma_0 \left[X_0 + \frac{\alpha_s}{\pi} X_1 + \left(\frac{\alpha_s}{\pi}\right)^2 X_2 \right], \tag{19}$$

$$X_2 = C_F (T_R n_l X_l + T_R n_h X_h + C_F X_F + C_A X_A) \tag{20}$$

$$X_l = -\frac{X_0}{3} \left[H_{0,1,0}(w) - H_{0,0,1}(w) - 2H_{0,1,1}(w) + 2H_{1,1,0}(w) - \pi^2 H_1(w) - 3\zeta(3) \right] + g_l(w),$$

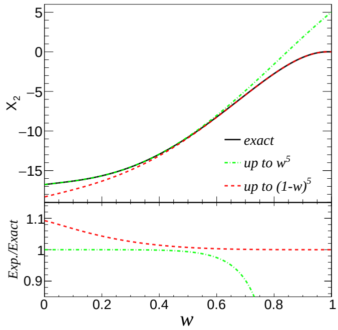
$$\begin{aligned} X_F = &\frac{1}{12} X_0 [-6 \left(2H_{0,1,0,1}(w) + 6H_{1,0,0,1}(w) - 3H_{1,0,1,0}(w) - 12\zeta(3) H_1(w)\right) - \pi^2 H_{1,0}(w)] \\ &+ (X_0 + 4w) \left(-\frac{1}{6}\pi^2 H_{0,-1}(w) - 2H_{0,-1,0,1}(w) \right) \\ &+ \frac{1}{12} \left(18w^3 - 3w^2 + 76w + 15\right) \pi^2 H_{0,1}(w) - \frac{1}{2} \left(4w^3 - 2w^2 + 4w + 3\right) H_{0,0,0,1}(w) \\ &+ \frac{1}{2} \left(4w^3 - 2w^2 + 16w + 3\right) H_{0,0,1,0}(w) + w \left(2w^2 - 7w - 16\right) H_{0,0,1,1}(w) \\ &- \frac{1}{2} \left(2w^3 - 11w^2 - 28w - 1\right) H_{0,1,1,0}(w) + \frac{1}{720}\pi^4 \left(42w^3 - 191w^2 - 328w - 11\right) + g_F(w). \end{aligned}$$

Cross Check

Master integrals are confirmed by numerical check with `AMFlow`.

Two different gauges have been used to cross check.

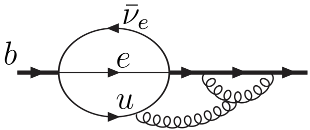
The result expanded in $w = 0$ and $w = 1$ $(w = m_W^2/m_t^2)$ coincides with [Blokland, Czarnecki, Slusarczyk, Tkachov 2004 2005].



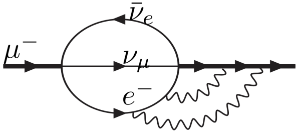
Relations With Other Process

Our results are proportional to the invariant mass spectrum in semileptonic $b \to uW^*$

Integrating over w $(w = m_W^2/m_t^2)$ from 0 to 1, reproduce NNLO QCD corrections in semileptonic decay $\Gamma(b \to X_u e\bar{\nu}_e)$ [Ritbergen 1999].



Integrating X_F over w, obtain the analytic two-loop QED correction to the muon lifetime $\Gamma(\mu^- \to \nu_\mu e^- \bar{\nu}_e)$ [Ritbergen, Stuart 1999].



Off-Shell W Boson

W boson has the width of $\Gamma_W = 2.085$ GeV, the Γ_t become [Jezabek, Kuhn 1989]

$$\tilde{\Gamma}_t \equiv \Gamma(t \to W^* b) = \frac{1}{\pi}\int_0^{m_t^2} dq^2 \frac{m_W \Gamma_W}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} \Gamma_t(q^2/m_t^2), \tag{21}$$

In the narrow width limit, $\Gamma_W \to 0$, $\tilde{\Gamma}_t \to \Gamma_t$.

$$\tilde{\Gamma}_t = \Gamma_0 \left[\tilde{X}_0 + \frac{\alpha_s}{\pi}\tilde{X}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \tilde{X}_2\right], \quad r = \frac{\Gamma_W}{m_W}, \quad w = \frac{m_W^2}{m_t^2}$$

$$\begin{aligned} \tilde{X}_0 = {} & \frac{1}{2\pi}\big(- (2(r-i)w - i((r-i)w+i)^2 G(w+irw,1)) \\ & - ((r+i)w-i)^2 2(r+i)w + iG(w-irw,1) - 4r(1-2w)w\big), \end{aligned} \tag{22}$$

$$\begin{aligned} \tilde{X}_1 = {} & \frac{1}{18\pi}\big(((r+i)w-i)(2(4\pi^2-9)(r+i)^2w^2 + (4\pi^2-27)(1-ir)w + 4\pi^2 - 15)G(w-iw,1) \\ & + (r-i)w-i)(2(4\pi^2-9)(r-i)^2w^2 + (4\pi^2-27)(1+ir)w + 4\pi^2 - 15)G(w+iw,1) \\ & + \cdots\big) \end{aligned} \tag{23}$$

Numerical Results

Input parameters from [P.D.G 2022]

$$
\begin{aligned}
m_t &= 172.69\ \text{GeV}, \quad m_b = 4.78\ \text{GeV}, \\
m_W &= 80.377\ \text{GeV}, \quad \Gamma_W = 2.085\ \text{GeV}, \\
m_Z &= 91.1876\ \text{GeV}, \quad G_F = 1.16638 \times 10^{-5}\ \text{GeV}^{-2}, \\
|V_{tb}| &= 1, \quad \alpha_s(m_Z) = 0.1179.
\end{aligned} \tag{24}
$$

$\Gamma_t^{(0)} = 1.486$ GeV with $m_b = 0$ and on-shell W.

$$
\begin{aligned}
\Gamma_t = \Gamma_t^{(0)} [& (1 + \delta_b^{(0)} + \delta_W^{(0)}) \\
& + (\delta_b^{(1)} + \delta_W^{(1)} + \delta_{\rm EW}^{(1)} + \delta_{\rm QCD}^{(1)}) \\
& + (\delta_b^{(2)} + \delta_W^{(2)} + \delta_{\rm EW}^{(2)} + \delta_{\rm QCD}^{(2)} + \delta_{\rm EW \times QCD}^{(2)})]
\end{aligned} \tag{25}
$$

Numerical Results

Corrections in percentage ($\%$) normalized by the LO width $\Gamma_t^{(0)} = 1.486$ GeV with $m_b = 0$ and on-shell W.

	$\delta_b^{(i)}$	$\delta_W^{(i)}$	$\delta_{\rm EW}^{(i)}$	$\delta_{\rm QCD}^{(i)}$	Γ_t [GeV]
LO	-0.273	-1.544	—	—	1.459
NLO	0.126	0.132	1.683	-8.575	1.361
NNLO	$*$	0.030	$*$	-2.070	1.331

QCD corrections are dominant.

NLO EW correction is 1.683%.

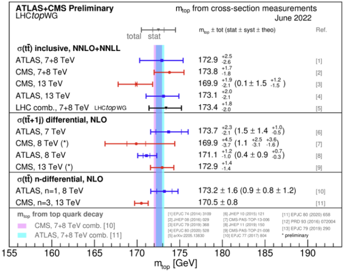
The off-shell W boson effect at NNLO is further suppressed.

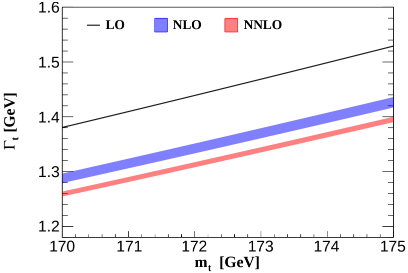
The b quark mass correction at NLO is not severely suppressed compared to the LO due to the large logarithms.

Top-quark Mass

Top-quark mass varies from 170 GeV to 175 GeV.

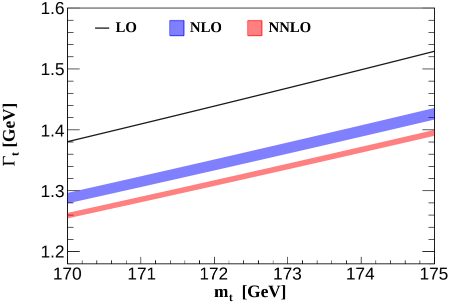
The width changes from 1.258 GeV to 1.394 GeV.





Theoretical Uncertainties

QCD renormalization scale $\mu \in [m_t/2, 2m_t]$, the variation is about $\pm 0.8\%$ and $\pm 0.4\%$ at NLO and NNLO.



$\overline{\mathrm{MS}}$ scheme differ from on-shell scheme -3.79% and 0.09% at NLO and NNLO.

Missing NNNLO QCD contribution would be of the order of 0.4%.

Theoretical Uncertainties

The uncertainties at NNLO from $\alpha_s(m_Z) = 0.1179 \pm 0.0009$ and $m_W = 80.377 \pm 0.012$ GeV [P.D.G 2022] are 0.1% and 0.01%.

The deviation between the α and G_F scheme in the EW correction is 0.1% at NLO.

The missing NNLO EW as well as the mixed $\mathrm{EW} \times \mathrm{QCD}$ corrections.

Considering all the possible uncertainties, the uncertainty at NNLO is less than 1%.

Mathematica program TopWidth

Mathematica program TopWidth can be downloaded from https://github.com/haitaoli1/TopWidth. The package HPL is required [Maitre 2006].

```
<< TopWidth`

        (****** TopWidth-1.0 ******)
    Authors: Long-Bin Chen, Hai Tao Li, Jian Wang, YeFan Wang
    TopWidth[QCDorder, mbCorr, WwidthCorr, EWcorr, mu] is provided for top width calculations
    Please cite the paper for reference: arXiv:2212.06341

*-*-*-*-*-* HPL 2.0 *-*-*-*-*-*

Author: Daniel Maitre, University of Zurich
Rules for minimal set loaded for weights: 2, 3, 4, 5, 6.
Rules for minimal set for + - weights loaded for weights: 2, 3, 4, 5, 6.
Table of MZVs loaded up to weight 6
Table of values at I loaded up to weight 6
$HPLFunctions gives a list of the functions of the package.
$HPLOptions gives a list of the options of the package.
More info in hep-ph/0507152, hep-ph/0703052 and at
 http://krone.physik.unizh.ch/~maitreda/HPL/

(* SetParameters[mt, mb, mw, Wwidth, mz, GF] *)
(* If the parameters are not set by the users the code will use the default ones *)
SetParameters[17269/100, 478/100, 80377/1000, 2085/1000, 911876/10000, 11663788×10^-12]

(* NNLO decay width *)
TopWidth[2, 1 (* with  mb effects *), 1 (* with  Γw effects*), 1 (* with NLO EW effects *), 17269/100]

1.33051
```

Summary and Outlook

We first provide full analytical result of top-quark width at NNLO in QCD.

It's the first NNLO analytic result for massive particle inclusive decay into massive particle.

The analytical result can be used to perform both fast and accurate evaluations.

The off-shell W boson contribution is calculated analytically up to NNLO in QCD.

The most precise top-quark width is predicted to be 1.331 GeV for mt = 172.69 GeV with the total theoretical uncertainty less than 1%.

The next target is NNNLO QCD corrections for top-quark width.