Codimension two Holography

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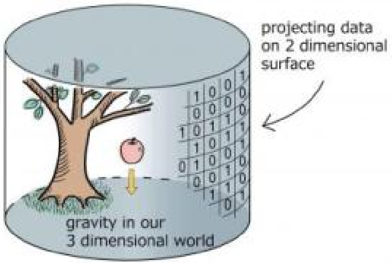
Outline

1. Background
   - Review of Holography
   - Codimension two Holography
2. Main Results
   - Exact Construction of Wedge Holography
   - Aspects of Wedge Holography
   - More general solution
   - Generalization to dS/CFT and flat holography
3. Summary and Open questions

Outline

Holography

- Holographic principle: 't Hooft, Susskind
  Duality between higher dimensional gravity and lower dimensional QFT
- AdS/CFT: Maldacena



- Generalizations
  dS/CFT, Kerr/CFT, flat holography, brane world holography, AdS/BCFT, doubly holographic model

Outline

Geometry Setup of Wedge Holography

The $d+1$ dimensional wedge N is bounded by two d dimensional branes Q_1 and Q_2 so that $\partial N = Q_1 \cup Q_2$. CFT lives on the corner of the wedge $\Sigma = \partial Q_1 = \partial Q_2$.

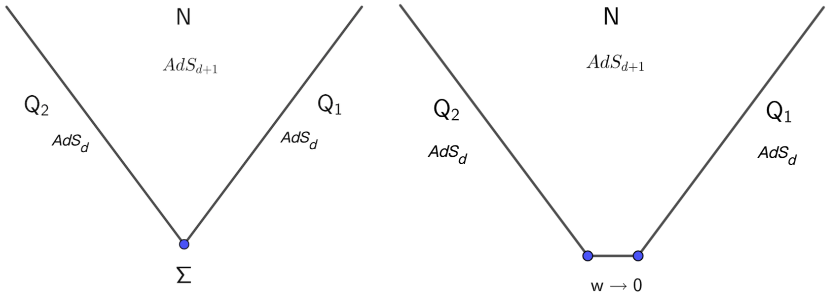


Figure: (left) Geometry of wedge holography; (right) Wedge holography from AdS/BCFT

Proposal of Wedge Holography

Akal, Kusuki,Takayanagi and Wei, [arXiv:2007.06800]

$$\begin{aligned}\text{Classical gravity on wedge } W_{d+1} &\simeq \text{Quantum gravity on two } AdS_d \text{ Q}\\ &\simeq \text{CFT}_{d-1} \text{ on } \Sigma\end{aligned}$$

- Gravitational action

$$I = \frac{1}{16\pi G_N}\int_N \sqrt{g}(R-2\Lambda) + \frac{1}{8\pi G_N}\int_{Q_1\cup Q_2}\sqrt{h}(K-T), \tag{1}$$

where T is the tension of branes.

- Neumann BC on Q:

$$K^i{}_j - (K-T)h^i{}_j = 0, \tag{2}$$

where K_{ij} are the extrinsic curvatures.

Comments on Wedge Holography

$$
\begin{aligned}
\text{Classical gravity on wedge } W_{d+1} &\simeq \text{Quantum gravity on two } AdS_d \text{ Q} \\
&\simeq \text{CFT}_{d-1} \text{ on } \Sigma
\end{aligned}
$$

- The first equivalence is due to the brane world holography and the second equivalence originates from AdS/CFT.
- Similar to so-called double holography developed for the resolution of information paradox.
- Can be regarded as a limit of AdS/BCFT with vanishing strip width.
- Support from Weyl anomaly, entanglement entropy, ...

$$
\mathcal{A} = \frac{1}{16\pi G_N} \int_{\Sigma} dx^2 \sqrt{\sigma} \Big(\sinh(\rho) R_{\Sigma} \Big) \tag{3}
$$

- Focus on (locally) AdS

Outline

Exact Construction of Solutions

A novel map from the solution to vacuum Einstein equations in AdS_d/CFT_{d-1} to the solution in wedge holography $AdSW_{d+1}/CFT_{d-1}$.

- Novel solution

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dx^2 + \cosh^2(x) h_{ij}(y) dy^i dy^j \tag{4}$$

- **Theorem I** : (4) is a solution to Einstein equation in $d+1$ dimensions

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \frac{d(d-1)}{2} g_{\mu\nu} \tag{5}$$

provided that h_{ij} obey Einstein equation in d dimensions

$$R_{h\ ij} - \frac{R_h}{2} h_{ij} = \frac{(d-1)(d-2)}{2} h_{ij}. \tag{6}$$

- **Theorem II**: (4) obey NBC (2) on the branes located at $x = \pm\rho$.

Comments on Novel Solutions

Novel solution: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dx^2 + \cosh^2(x)h_{ij}(y)dy^i dy^j$ (4)

- When h_{ij} is a AdS_d metric, $g_{\mu\nu}$ become a AdS_{d+1} metric.
- In fact h_{ij} can be relaxed to be any metric obeying Einstein equations.
- (4) can be used to construct black hole solutions in wedge holography and AdS/BCFT.
- (4) is not the most general solutions to vacuum Einstein equations in $d+1$ dimensions.
- (4) relate wedge holography to AdS/CFT clearly.

Equivalence to AdS/CFT

The wedge holography $AdSW_{d+1}/CFT_{d-1}$ with novel solution (4) is equivalent to AdS_d/CFT_{d-1} with vacuum Einstein gravity.

- Equivalence between classical gravitational actions

$$\begin{aligned} I_{AdSW_{d+1}} &= \frac{1}{16\pi G_N}\int_N \sqrt{g}(R-2\Lambda) + \frac{1}{8\pi G_N}\int_{Q_1\cup Q_2}\sqrt{h}(K-T) \\ &= \frac{1}{16\pi G_N^{(d)}}\int_{Q_1}\sqrt{h}\Big(R_h + (d-1)(d-2)\Big) = I_{AdS_d}\ , \end{aligned} \tag{7}$$

- Newton's constant

$$\frac{1}{G_N^{(d)}} = \frac{1}{G_N}\int_0^{\rho}\cosh^{d-2}(x)dx. \tag{8}$$

- Imposing only NBC (2) instead of EOM (off-shell).

A “proof” of Wedge Holography

The equivalence to AdS/CFT can be regarded as a “proof ” of wedge holography in a certain sense.

- Assuming AdS/CFT

$$\text{classical gravity in } \mathrm{AdS}_d \simeq \mathrm{CFT}_{d-1} \tag{9}$$

- Equivalence between action

$$\text{classical gravity on wedge } W_{d+1} \simeq \text{classical gravity in } \mathrm{AdS}_d \tag{10}$$

- A “proof” of wedge holography

$$\text{classical gravity on wedge } W_{d+1} \simeq \mathrm{CFT}_{d-1} \tag{11}$$

Comments on Equivalence

Wedge holography with novel solution (4) is equivalent to AdS/CFT with vacuum Einstein gravity.

- Equivalence still hold after holographic renormalization

$$I_C = \frac{1}{16\pi G_N^{(d)}} \int_\Sigma \sqrt{\sigma} \left(2K_\Sigma + 2(1-d) + \frac{1}{d-2} R_\Sigma + ... \right) \tag{12}$$

- Hayward term can be absorbed into the above counterterms

$$I_H = \frac{1}{8\pi G_N} \int_\Sigma \sqrt{\sigma} (\Theta - \pi) \tag{13}$$

- Most results of AdS/CFT apply directly to wedge holography.
- In general, wedge holography is expected to equivalent to AdS/CFT with matter fields (Kaluza-Klein modes).

Outline

Weyl anomaly

Weyl anomaly measures the breaking of scaling symmetry of conformal field theory (CFT) due to quantum effects.

- Weyl anomaly in even dimensions

$$\mathcal{A} = \int_\Sigma dx^{2p}\sqrt{\sigma}[\sum_n B_n I_n - 2(-1)^p A\ E_{2p}] \tag{14}$$

- 2d Weyl anomaly

$$\mathcal{A}_{2d} = \int_\Sigma dx^2\sqrt{\sigma}\frac{c_{2d}}{24\pi}R_\Sigma, \tag{15}$$

- c-theorem

$$A_{UV} \geq A_{IR}. \tag{16}$$

- Universal term of entanglement entropy for sphere

$$S_{EE}|_{\ln\frac{1}{\epsilon}} = 4(-1)^{(d+1)/2}A \tag{17}$$

Holographic Weyl anomaly

Holographic Weyl anomaly can be obtained from the UV logarithmic divergent term of the gravitational action.

- metric

$$ds^2 = dx^2 + \cosh^2(x)\frac{dz^2 + \sigma_{ij}dy^i dy^j}{z^2}, \tag{18}$$

where $\sigma_{ij} = \sigma_{ij}^{(0)} + z^2\sigma_{ij}^{(1)} + ... + z^{d-1}(\sigma_{ij}^{(d-1)} + \lambda_{ij}^{(d-1)} \ln z) + ...$

- Einstein equations yield

$$\sigma_{ij}^{(1)} = \frac{-1}{d-3}(R_{\Sigma\ ij} - \frac{R_\Sigma}{2(d-2)}\sigma_{ij}^{(0)}), \tag{19}$$

- Derive Weyl anomaly with central charge

$$A = \frac{\pi^{\frac{d-3}{2}}}{8\Gamma\left(\frac{d-1}{2}\right)}\frac{1}{G_N}\int_0^\rho \cosh^{d-2}(x)dx. \tag{20}$$

- We can also derive B-type Weyl anomaly.

Holographic c theorem

A-type central charges of Wedge holography obey c-theorem $A_{UV} \geq A_{IR}$.

$$A = \frac{\pi^{\frac{d-3}{2}}}{8\Gamma\left(\frac{d-1}{2}\right)} \frac{1}{G_N} \int_0^\rho \cosh^{d-2}(x) dx.$$

Null energy condition on branes yields $\rho_{UV} \geq \rho_{IR}$, which leads to $A_{UV} \geq A_{IR}$.

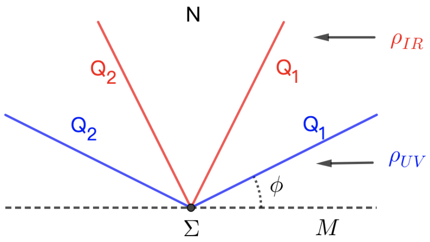


Figure: The smaller the tension $T = (d-1)\tanh\rho$ is, the bigger $\tan\phi = \text{csch}(\rho)$ is, the deeper the brane bends into the bulk (IR). Thus $\rho_{UV} \geq \rho_{IR}$.

Rényi entropy

As a generalization of von Neumann entropy, Rényi entropy is a complete measure of the quantum entanglement .

- Definition

$$S_n = \frac{1}{1-n} \ln \mathrm{tr}\rho_A^n, \tag{21}$$

where n is a positive number, $\rho_A = \mathrm{tr}_{\bar{A}}\, \rho$ is the induced density matrix of a subregion A. Here $\bar{A}$ denotes the complement of A and ρ is the density matrix of the whole system.

- Deduce to entanglement entropy in the limit $n \to 1$

$$S_{\mathrm{EE}} = -\mathrm{tr}\rho_A \ln \rho_A. \tag{22}$$

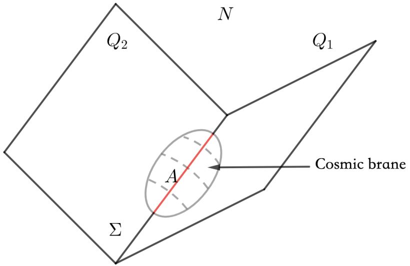
Holographic Rényi entropy

Holographic Rényi entropy can be calculated by the area of cosmic brane.

- Dong's proposal

$$n^2\partial_n\left(\frac{n-1}{n}S_n\right)=\frac{\text{Area(Cosmic Brane}_n)}{4G_N},\qquad (23)$$

- Backreact on geometry due to non-zero tension $T_n=\frac{n-1}{4nG_N}$
- Ending on the end-of-world branes Q



Holographic Rényi entropy

- Hyperbolic black hole on the branes

$$ds^2 = dx^2 + \cosh^2(x)\left(\frac{dr^2}{f(r)} + f(r)d\tau^2 + r^2 dH_{d-2}^2\right), \qquad (24)$$

where $f(r) = r^2 - 1 - \frac{(r_h^2-1)r_h^{d-3}}{r^{d-3}}$, dH_{d-2}^2 is the line element of $(d-2)$-dimensional hyperbolic space with unit curvature.

- The Rényi index $n = \frac{1}{2\pi T_{tem}} = \frac{2}{f'(r_h)}$
- Cosmic brane is the horizon of black hole
- Correct Rényi entropy

$$S_n = \frac{r_h^{d-2} + r_h^d - 2r_h}{(r_h-1)\left((d-1)r_h + d - 3\right)} \frac{V_{H_{d-2}}}{4G_N} \int_0^{\rho} \cosh^{d-2}(x)dx. \qquad (25)$$

Comments on Holographic Rényi entropy

- 2d CFT has the correct n dependence

$$S_n = \frac{n+1}{n}\frac{V_{H_1}}{8G_N}\int_0^\rho \cosh(x)dx, \tag{26}$$

- Entanglement entropy obey RT formula

$$S_{\text{EE}} = \frac{V_{H_{d-2}}}{4G_N}\int_0^\rho \cosh^{d-2}(x)dx. \tag{27}$$

- Correct universal term of entanglement entropy (20,17)

$$V_{H_{d-2}}|_{\ln\frac{1}{\epsilon}} = \frac{2\pi^{(d-3)/2}}{\Gamma(\frac{d-1}{2})}(-1)^{(d-3)/2} \tag{28}$$

Holographic Correlation Function

For simplicity, we focus on the two point functions of stress tensors.

- Consider metric fluctuations H_{ij} on the AdS brane

$$ds^2 = dx^2 + \cosh^2(x)\frac{dz^2 + \delta_{ab}dy^a dy^b + H_{ij}dy^i dy^j}{z^2} \tag{29}$$

- Choose the gauge

$$H_{zz}(z=0,\mathbf{y}) = H_{za}(z=0,\mathbf{y}) = 0 \tag{30}$$

- Following approach of AdS/CFT, we get

$$< T_{ab}(\mathbf{y})T_{cd}(\mathbf{y}') >= C_T \frac{\mathcal{I}_{ab,cd}}{|\mathbf{y}-\mathbf{y}'|^{2(d-1)}}, \tag{31}$$

with central charge

$$C_T = \frac{2\Gamma[d+1]}{\pi^{(d-1)/2}\Gamma[(d-1)/2](d-2)}\frac{1}{16\pi G_N}\int_0^{\rho}\cosh^{d-2}(x)dx. \tag{32}$$

Outline

General spacetime on brane

The spacetime on the brane is the one with constant Ricci scalar.

- Momentum constraint and Hamiltonian constraint

$$D_i(K^{ij} - Kh^{ij}) = 0, \tag{33}$$

$$R_h + K^{ij}K_{ij} - K^2 + d(d+1) = 0 \tag{34}$$

- Constraint of brane spacetime

$$R_h = \frac{d}{d-1}\left(T^2 - (d-1)^2\right). \tag{35}$$

- Three types of spacetime

$$\begin{cases} R_h < 0, & \text{if } \ |T| < (d-1), \\ R_h > 0, & \text{if } \ |T| > (d-1), \\ R_h = 0 & \text{if } \ |T| = (d-1). \end{cases} \tag{36}$$

Perturbation Solutions

- Solution near branes

$$ds^2 = dx^2 + \left((1 + 2x \tanh \rho) h_{ij} + \sum_{n=2}^{\infty} x^n h_{ij}^{(n)}(y) \right) dy^i dy^j, \qquad (37)$$

- Solving Einstein equations

$$h_{ij}^{(2)} = R_{h\ ij} + \left(d + (2-d) \tanh^2 \rho \right) h_{ij}, \qquad (38)$$

- Rewritten into more enlightening form

$$R_{h\ ij} - \frac{R_h}{2} h_{ij} - \frac{(d-1)(d-2)}{2\cosh^2 \rho} h_{ij} = 8\pi G_N^{(d)} T_{ij}, \qquad (39)$$

where $h_{ij}^{(2)} = (1 + \tanh^2(\rho)) h_{ij} + 8\pi G_N^{(d)} T_{ij}$.

- Effective matter fields on branes are CFTs $T^i{}_i = 0$

Comments on general solutions

We argue that wedge holography with general asymptotically AdS branes is equivalent to AdS/CFT with suitable matter fields such as KK modes.

- The brane metric need not to obey Einstein equations. The only constraint is that the Ricci scalar is a constant.
- On one hand, the novel solution is not the general solution of wedge holography.
- On the other hand, the novel solution and general solutions must correspond to the same CFTs with the same central charges.
- To resolve the "mismatch", we propose there are effective matter fields on the branes.
- In the dual viewpoint, the same CFTs up to some background fields.

Outline

General solutions

Remarkably, AdS/CFT, dS/CFT and flat holography can be unified in the framework of codimension two holography in asymptotically AdS.

- General ansatzs of metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dx^2 + f(x) h_{ij}(y) dy^i dy^j. \quad (40)$$

- Solving $R_{xx} = -d g_{xx}$ yield three types of solutions

$$f(x) = \begin{cases} \cosh^2(x), & |T| < (d-1), & \text{asymptotically AdS} \\ \sinh^2(x), & |T| > (d-1), & \text{asymptotically dS} \\ e^{\pm 2x}, & |T| = (d-1), & \text{asymptotically flat} \end{cases} \quad (41)$$

- Three kinds of solutions correspond to AdS/CFT, dS/CFT and flat holography, respectively.

Equivalence to dS/CFT

Codimension two holography with asymptotically dS branes is equivalent to dS/CFT.

- Ansatz of metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dx^2 + \sinh^2(x)h_{ij}(y)dy^i dy^j. \tag{42}$$

- The metric (42) is a solution to wedge holography, provided that h_{ij} obey Einstein equation with a positive cosmological constant

$$R_{h\ ij} - \frac{R_h}{2}h_{ij} = -\frac{(d-1)(d-2)}{2}h_{ij}. \tag{43}$$

- Equivalence to dS/CFT

$$\bar{I}_{\text{AdSW}_{d+1}} = \frac{1}{16\pi G_N^{(d)}}\int_{Q_1}\sqrt{h}\Big(R_h - (d-1)(d-2)\Big) = I_{\text{dS}_d}\ , \tag{44}$$

if Newton's constants are related by $\frac{1}{G_N^{(d)}} = \frac{1}{G_N}\int_0^\rho \sinh^{d-2}(x)dx$.

Equivalence to flat holography

Codimension two holography with asymptotically flat branes is equivalent to flat holography.

- Ansatz of metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dx^2 + \exp(\pm 2x)h_{ij}(y)dy^i dy^j. \quad (45)$$

- The metric (45) is a solution to wedge holography, provided that h_{ij} obey Einstein equation with zero cosmological constant

$$R_{h\ ij} - \frac{R_h}{2}h_{ij} = 0. \quad (46)$$

- Equivalence to flat holography

$$\bar{I}_{\text{AdSW}_{d+1}} = \frac{1}{16\pi G_N^{(d)}}\int_{Q_1}\sqrt{h}\Big(R_h\Big) = I_{\text{Min}_d}, \quad (47)$$

if Newton's constants are related by $\frac{1}{G_N^{(d)}} = \frac{1}{G_N}\int_{-\infty}^{\rho}\exp\left((d-2)x\right)dx$.

Summary

Summary:

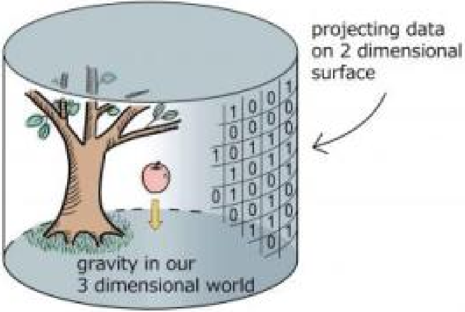
- We construct a class of exact gravitational solutions for wedge holography from the the ones in AdS/CFT.
- We prove that the wedge holography with this novel class of solutions is equivalent to AdS/CFT with vacuum Einstein gravity.
- By applying this powerful equivalence, we derive Weyl anomaly, Rényi entropy and correlation functions for wedge holography.
- We argue that wedge holography with general solutions correspond to AdS/CFT with suitable matter fields (Kaluza-Klein modes).
- AdS/CFT, dS/CFT and flat holography can be unified in the framework of codimension two holography in asymptotically AdS.

Outlook

Outlook:

- Find out more general solutions beyond the novel solutions.
- Generalize discussions to Einstein gravity coupled with matters.
- Discuss holographic complexity, condensed matter, chaos and so on in Wedge holography.
- Generalize wedge holography to higher derivative gravity.
- Study island and the Page curve of Hawking radiations.
- ...

Only one piece of the boundary can reconstruct all the apples in the bulk.



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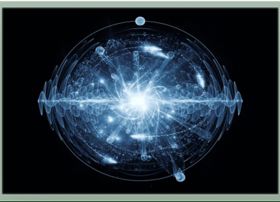

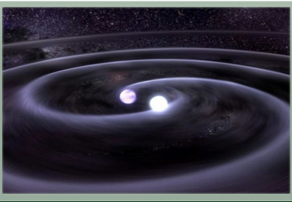

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