Loop Operators in three-dimensional $\mathcal{N}=2$ fishnet theories

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Wilson loops

- In non-abelian gauge theory, the definition of usual Wilson loops is

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- WLs were also used in three-dimensional gravity/higher spin theories, p-adic string theory. [talks by Ling-Yan Hung, Yi-Hao Ying]

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- It play an important roles since the vev of the circular BPS Wilson loops can be computed exactly using localization. [Pestun, 07]
- It can also been studied using integrability, especially for the cusped Wilson loops. [Drukker, 12][Correa, Maldacena, Sever, 12]

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- In ABJM theory, $1/6$-BPS WLs was constructed based on previous constructions by Gaiotto and Yin (07) in 3d $\mathcal{N} = 2, 3$ Chern-Simons-matter theories. [Drukker, Plefka, Young][Chen, JW][Rey, Suyama, Yamaguchi] 08

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- In the gravity side, half-BPS membrane/F-string was found which should be dual to WLs.
- One year later, such half-BPS WLs was constructed. It is fermionic! [Drukker, Trancanelli]

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- These WLs can be understood as marginal deformation of half-BPS WLs. This is consistent of classification of superconformal lines as defects. [Agmon, Wang, 20]

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- One should choose a regularization scheme consistent with localization.
- For BPS WLs, this should be at framing -1. [Kapustin, Willett, Yaakov, 09]

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- Integrability of open chain from (cusped) WLs in ABJM theory is **another** interesting unsolved problem.
- We want to study these two problems in a simpler setup.

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- The worldsheet theory of dual string is 2d integrable field theory in the free limit.
- Integrability permits us to compute certain quantities non-perturbatively even in the non-BPS sector.
- Example $<\mathcal{D}_1\mathcal{D}_2 tr(...)>$ [Jiang, Komatsu, Vescovi, 19]*2[Yang, Jiang,Komatsu, JW, to appear]

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- Orbifolding: [Wang, Y.-S. Wu, 03][Beisert, Roiban, 0510][Bai, Chen, Ding, Li, JW, 16]

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- One can also adding only N_f D7's with $N_f \ll N_c$. The gauge theory is $\mathcal{N} = 4$ SYM coupled to N_f $\mathcal{N} = 2$ chiral multiplets in the fundamental and anti-fundamental representations. Now there are degrees of freedom at the boundaries, similar more or less to what happens in Kondo effects. [Erler and Mann, 05]

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- Flavored ABJM theory: [Bai, Chen, He, JW Yang, Zhu, 17]

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- β- and γ- deformations, defined by certain star products are among them. [Beisert, Roiban, 04][He, JW, 13][Chen, Liu, JW, 16]
- By taking special double scaling limit, the gauge fields are decoupled and one lead to integrable theories with only scalars (and fermions).[Gurdogan, Kazakov, 15][Caetano, Gurdogan, Kazakov, 16]

Fishnet diagrams

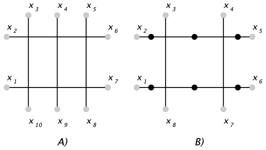

Figure: Examples of fishnet diagrams. Figure from [Bork etal., 20].

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- The matter fields are four scalars ϕ_I and four fermions ψ^I in the bi-fundamental representation $(N_1, \bar{N}_2)$ of $U(N_1) \times U(N_2)$.

The action of ABJM theory

$$
\begin{aligned}
\mathcal{L}_{ABJM} &= \mathcal{L}_{CS} + \mathcal{L}_k + \mathcal{L}_p + \mathcal{L}_Y, \\
\mathcal{L}_{CS} &= \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}\Big(A_\mu\partial_\nu A_\rho + \frac{2i}{3}A_\mu A_\nu A_\rho - B_\mu\partial_\nu B_\rho - \frac{2i}{3}B_\mu B_\nu B_\rho\Big), \\
\mathcal{L}_k &= \mathrm{Tr}(-D_\mu\bar{\phi}^I D^\mu\phi_I + i\bar{\psi}_I\gamma^\mu D_\mu\psi^I), \\
\mathcal{L}_p &= \frac{4\pi^2}{3k^2}\mathrm{Tr}(\phi_I\bar{\phi}^I\phi_J\bar{\phi}^J\phi_K\bar{\phi}^K + \phi_I\bar{\phi}^J\phi_J\bar{\phi}^K\phi_K\bar{\phi}^I + 4\phi_I\bar{\phi}^J\phi_K\bar{\phi}^I\phi_J\bar{\phi}^K \\
&-6\phi_I\bar{\phi}^J\phi_J\bar{\phi}^I\phi_K\bar{\phi}^K), \\
\mathcal{L}_Y &= \frac{2\pi\mathrm{i}}{k}\mathrm{Tr}(\phi_I\bar{\phi}^I\psi^J\bar{\psi}_J - 2\phi_I\bar{\phi}^J\psi^I\bar{\psi}_J - \bar{\phi}^I\phi_I\bar{\psi}_J\psi^J + 2\bar{\phi}^I\phi_J\bar{\psi}_I\psi^J \\
&\quad +\epsilon^{IJKL}\phi_I\bar{\psi}_J\phi_K\bar{\psi}_L - \epsilon_{IJKL}\bar{\phi}^I\psi^J\bar{\phi}^K\psi^L).
\end{aligned} \tag{2}
$$

Here A_μ, B_μ are the gauge fields corresponding to the first and the second $U(N)$, respectively. (Here $N_1 = N_2 \equiv N$)

γ-deformation

- To perform γ-deformation, we replace all product AB in the Lagrangian by the following star product

$$A \star B = e^{\frac{\mathrm{i}}{2}\mathbf{q}_A \wedge \mathbf{q}_B} AB. \tag{3}$$

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- Here the antisymmetric product of the two charge vectors $\mathbf{q}_A$ and $\mathbf{q}_B$ is given by

$$\mathbf{q}_A \wedge \mathbf{q}_B = \mathbf{q}_A^T \mathbf{C} \mathbf{q}_B, \quad \mathbf{C} = \begin{pmatrix} 0 & -\gamma_3 & \gamma_2 \\ \gamma_3 & 0 & -\gamma_1 \\ -\gamma_2 & \gamma_1 & 0 \end{pmatrix}. \qquad (4)$$

Three global $U(1)$'s

The $U(1)$ charges of the fields are given by the table below:

f	ϕ_1	ϕ_2	ϕ_3	ϕ_4	$\bar{\psi}_1$	$\bar{\psi}_2$	$\bar{\psi}_3$	$\bar{\psi}_4$
q_1	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$
q_2	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
q_3	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$

(5)

where $\pm \equiv \pm\frac{1}{2}$ (Note that the gauge fields A_μ and B_μ are neutral under these three $U(1)$'s.).

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- This the $3d$ $\mathcal{N} = 2$ fishnet theory we focus on in this talk.

Lagrangian of 3d fishnet theory

- The Lagrangian reads

$$\mathcal{L} = \mathcal{L}_k + \mathcal{L}_Y + \mathcal{L}_{scalar}. \tag{7}$$

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- The scalar potential term is

$$\begin{aligned} \mathcal{L}_{scalar} &= 16\pi^2 \frac{\xi}{N^2} \mathrm{Tr}(\bar{\phi}^1 \phi_3 \bar{\phi}^2 \phi_1 \bar{\phi}^3 \phi_2 + \bar{\phi}^1 \phi_3 \bar{\phi}^4 \phi_1 \bar{\phi}^3 \phi_4 \\ & \quad \bar{\phi}^1 \phi_2 \bar{\phi}^4 \phi_1 \bar{\phi}^2 \phi_4 + \bar{\phi}^2 \phi_4 \bar{\phi}^3 \phi_2 \bar{\phi}^4 \phi_3). \end{aligned} \tag{9}$$

Lagrangian of 3d fishnet theory

- The Yukawa-like terms involving the interactions among the scalars and the fermions are

$$\begin{aligned}\mathcal{L}_Y &= -2\pi \mathrm{i}\frac{\sqrt{\xi}}{N}\mathrm{Tr}(-2\bar{\phi}^1\phi_3\bar{\psi}_1\psi^3 - 2\bar{\phi}^2\phi_4\bar{\psi}_2\psi^4 \\ &\quad -2\bar{\phi}^3\phi_2\bar{\psi}_3\psi^2 - 2\bar{\phi}^4\phi_1\bar{\psi}_4\psi^1 + 2\phi_1\bar{\phi}^3\psi^1\bar{\psi}_3 + 2\phi_2\bar{\phi}^4\psi^2\bar{\psi}_4 \\ &\quad +2\phi_3\bar{\phi}^2\psi^3\bar{\psi}_2 + 2\phi_4\bar{\phi}^1\psi^4\bar{\psi}_1 + 2\bar{\phi}^1\psi^4\bar{\phi}^2\psi^3 - 2\bar{\phi}^3\psi^1\bar{\phi}^4\psi^2 \\ &\quad -2\phi_1\bar{\psi}_4\phi_2\bar{\psi}_3 + 2\phi_3\bar{\psi}_1\phi_4\bar{\psi}_2). \end{aligned} \tag{10}$$

Loop operators

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- Here we only give the constructions of BPS circular loop operators in Euclidean spacetime in $\mathcal{N}=2$ convention,

$$W_{cir.} = \mathrm{Tr}(\mathcal{P}\exp(-\mathrm{i}\oint d\tau L_{cir.}(\tau))), \tag{11}$$

$$\begin{aligned} L_{cir.} &= B + F, \quad B = -\mathrm{i}(\bar{M}_Z N_{\bar{Z}} + N_{\bar{Z}}\bar{M}_Z), \quad F = \bar{M}_\zeta - N_{\bar{\zeta}}, \\ [\bar{M}_Z]_{(ab)} &= \bar{m}_i^{ab} Z^i_{(ab)}, \quad [N_{\bar{Z}}]_{(ab)} = n^i_{ab}\bar{Z}_i^{(ab)}, \\ [\bar{M}_\zeta]_{(ab)} &= \bar{m}_i^{ab}\zeta^i_{(ab)+}, \quad [N_{\bar{\zeta}}]_{(ab)} = n^i_{ab}\bar{\zeta}^{(ab)}_{i-}. \end{aligned} \tag{12}$$

Note that $\zeta^i_{(ab)+} = \mathrm{i}u_+\zeta^i_{(ab)}$, $\bar{\zeta}^{(ab)}_{i-} = \mathrm{i}\bar{\zeta}^{(ab)}_i u_-$.

- The $u_\pm$ in the previous page are

$$u_{+\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-\frac{i\tau}{2}} \\ e^{\frac{i\tau}{2}} \end{pmatrix}, \quad u_{-\alpha} = \frac{i}{\sqrt{2}} \begin{pmatrix} -e^{-\frac{i\tau}{2}} \\ e^{\frac{i\tau}{2}} \end{pmatrix}, \tag{13}$$

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- We have two class of BPS loop operators with

$$\begin{aligned} \text{Class I:} &\qquad \bar{m}_i^{21}=n_{12}^{i}=0, \\ \text{Class II:} &\qquad \bar{m}_i^{12}=n_{21}^{i}=0, \end{aligned} \tag{15}$$

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- We can show that classically

$$W_{cir.} - (N_1 + N_2) \qquad (16)$$

is Q-exact, where the supercharge Q can be used to perform supersymmetric localization and N_1, N_2 are the ranks of the gauge groups. If this relation is preserved at quantum level, we will have

$$< W_{cir.} >= N_1 + N_2, \qquad (17)$$

One-loop diagram

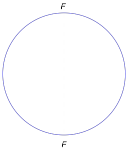

Figure: One-loop Feynman diagram.

Two-loop diagrams

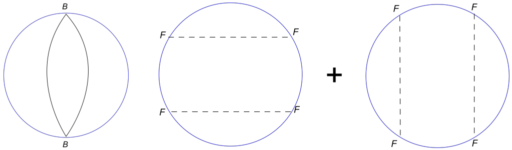

Figure: Two-loop Feynman diagrams.

Framing

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- The link number of Wilson loop contour and the auxiliary contour used for point-splitting is called framing.
- To compare with the results from supersymmetric localization, the perturbative calculations should be done at framing -1.
- But this has not yet taken into account the bosonic spinor used in the definition of fermionic WLs!

Hopf fiberation

- We parametrize the round $S^3 = \{X^i \in \mathbb{R}^4 | X^i X_i = 1\}$ as

$$X^i = (\cos\eta\cos(\tau-\phi), \cos\eta\sin(\tau-\phi), \sin\eta\sin(\tau+\phi), \sin\eta\cos(\tau+\phi))\,. \tag{18}$$

Here (ϕ, η) parametrize a S^2 and for each fixed pair (ϕ, η), the τ–circle is the Hopf fiber.

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- We use the following stereographic projection

$$x^\mu(X^i) = \left(\frac{X^1}{1-X^4}, \frac{X^2}{1-X^4}, \frac{X^3}{1-X^4}\right), \tag{19}$$

  to map $S^3 \backslash \{(0,0,0,1)\}$ to $\mathbb{R}^3$.

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- This gives the following parametrization for $\mathbb{R}^3$,

$$x^\mu = \left(\frac{\cos\eta\cos(\tau-\phi)}{1-\sin\eta\cos(\tau+\phi)}, \frac{\cos\eta\sin(\tau-\phi)}{1-\sin\eta\cos(\tau+\phi)}, \frac{\sin\eta\sin(\tau+\phi)}{1-\sin\eta\cos(\tau+\phi)}\right)$$

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- For the auxiliary contour, we can choose $\phi = 0, \eta \to 0$ and keep the terms up to the linear order of η. The result is [The same contour in Bianchi etal, 2016]

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- The BPS conditions for the spinors $u_{\pm}$ along the auxiliary contour are

$$\gamma_{\mu}\dot{x}^{\mu}_{\eta}u_{\eta\pm} = \pm|\dot{x}_{\eta}|u_{\eta\pm}, \quad u_{\eta+}u_{\eta-} = -\mathrm{i}, \quad u_{\eta\pm}\partial_{\tau}u_{\eta\mp} = 0. \tag{22}$$

The first equation is a Killing spinor equation along the auxiliary contour.

- Demanding that when $\eta \to 0$ these spinors go back to the spinors in eq. (13), we get that

$$u_\eta(\tau)_{+\alpha} = \frac{1}{\sqrt{2}} \left(\begin{array}{c} e^{-\frac{i\tau}{2}}(1 - \frac{i}{2}\eta\sin\tau) + \frac{\eta}{2}e^{\frac{i\tau}{2}} \\ e^{\frac{i\tau}{2}}(1 + \frac{i}{2}\eta\sin\tau) - \frac{\eta}{2}e^{-\frac{i\tau}{2}} \end{array} \right) + \mathcal{O}(\eta^2),$$

$$u_\eta(\tau)_{-\alpha} = \frac{i}{\sqrt{2}} \left(\begin{array}{c} -e^{-\frac{i\tau}{2}}(1 - \frac{i}{2}\eta\sin\tau) + \frac{\eta}{2}e^{\frac{i\tau}{2}} \\ e^{\frac{i\tau}{2}}(1 + \frac{i}{2}\eta\sin\tau) + \frac{\eta}{2}e^{-\frac{i\tau}{2}} \end{array} \right) + \mathcal{O}(\eta^2).$$

New regularization scheme

- Now, consider an unregularized integral from the computations of VEV of the loop operator,

$$\oint d\tau_{1>\cdots>n} I(x(\tau_m), u(\tau_m)), \tag{23}$$

where $x(\tau_m) = (\cos\tau_m, \sin\tau_m, 0)$ and $u(\tau_m)$ being given in eq. (13), and $\oint d\tau_{1>\cdots>n}$ means

$$\int_{2\pi>\tau_1>\cdots>\tau_n>0} \prod_{i=1}^{n} d\tau_i. \tag{24}$$

- To regularize it, we replace $x(\tau_m), u(\tau_m)$ with

$$x_m(\tau_m) = (\cos\tau_m + (m-1)\delta\cos^2\tau_m,$$
$$\sin\tau_m + (m-1)\delta\sin\tau_m\cos\tau_m, (m-1)\delta\sin\tau_m), \quad (25)$$

and

$$u_m(\tau_m)_{+\alpha} = \frac{1}{\sqrt{2}}\begin{pmatrix} u_{m+,1} \\ u_{m+,2} \end{pmatrix}, \quad (26)$$

$$u_{m+,1} = e^{-\frac{\mathrm{i}\tau_m}{2}}(1 - \frac{\mathrm{i}}{2}(m-1)\delta\sin\tau_m) + \frac{1}{2}(m-1)\delta e^{\frac{\mathrm{i}\tau_m}{2}}, \quad (27)$$

$$u_{m+,2} = e^{\frac{\mathrm{i}\tau_m}{2}}(1 + \frac{\mathrm{i}}{2}(m-1)\delta\sin\tau_m) - \frac{1}{2}(m-1)\delta e^{-\frac{\mathrm{i}\tau_m}{2}}, \quad (28)$$

with δ the regularization parameter (it is just the η in the previous discussions).

- The result for $u_m(\tau_m)_{-\alpha}$ can be found in our paper.

Results

We give numerical evidence that all the above one-loop and two-loop diagrams give vanishing contributions. This is consistent with the prediction from localization

$$< W_{cir.} >= N_1 + N_2, \tag{29}$$

and gives supports that these loops are BPS at quantum level.

Cusped Wilson Lines

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- In $\mathcal{N} = 4$ super Yang-Mills theory, the cusp anomlous dimension can be computed using integrability [Drukker, 12][Correa, Maldacena, Sever, 12] and some BPS case be computed using localization [Correa, Henn, Maldacena, Sever, 12].

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- Similar study in ABJM case was believed to give the dispersion relation of the magnons of the ABJM spin chain. A conjecture was given in [Gromov, Sizov, 14] . But even the integrability of the open spin chain from WL in ABJM theory is yet to be established.

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- Similar study in ABJM case was believed to give the dispersion relation of the magnons of the ABJM spin chain. A conjecture was given in [Gromov, Sizov, 14] . But even the integrability of the open spin chain from WL in ABJM theory is yet to be established.
- So we plan to study cusped Wilson lines in the simpler setup of 3d fishnet theories.

Cusped Line Operators in Fishnet Theory

- We also constructed cusped line operators in fishnet theory such that the left/right wing is half-BPS.
- But the cusped line operators are non-BPS as a whole.

One-loop Feynman diagrams for cusp

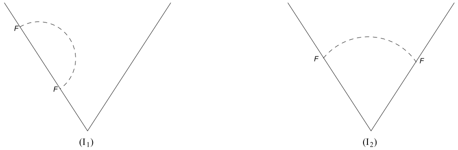

Figure: One-loop Feynman diagrams for cusp.

Two-loop Feynman diagrams for cusp

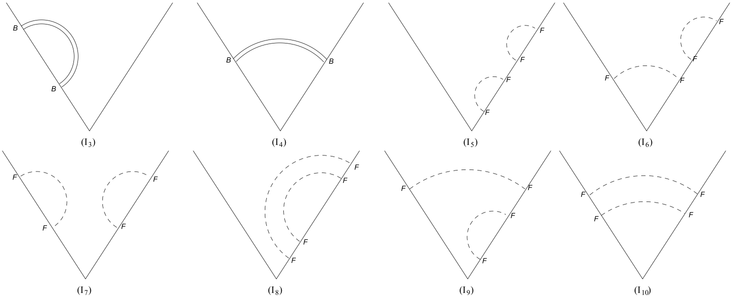

Figure: Two-loop Feynman diagrams for cusp.

- We computed the vev of this cusped line operator and using the prescription

$$\langle W \rangle = N_1 \exp(V_{N_2}) + N_2 \exp(V_{N_1}), \tag{30}$$

in [Griguolo etal, 2012] to extract the generalized potential,

$$\begin{aligned} V_N \quad &= N \bar{m}_i^{12} n_{21}^i (\mu L)^{2\epsilon} \frac{\Gamma(\frac{1}{2}-\epsilon)}{4\pi^{3/2-\epsilon}} \left(\frac{1}{\epsilon} - \frac{\sec\phi}{2\epsilon} + \sec\phi \log(1+\sec\phi) \right) \\ &+ N^2 (\bar{m}_i^{12} n_{21}^i)^2 \frac{\Gamma(\frac{1}{2}-\epsilon)^2}{16\pi^{3-2\epsilon}} (\mu L)^{4\epsilon} \left(\frac{1}{2\epsilon^2} - \frac{\sec\phi}{4\epsilon^2} \right. \\ &\left. + \frac{1}{\epsilon} \sec\phi \log(1+\sec\phi) + \frac{1}{2\epsilon} \sec\phi \log\cos\phi \right). \end{aligned} \tag{31}$$

- Notice that there are $1/\epsilon^2$ terms in part the with $(\mu L)^{4\epsilon}$ factor even in the straight line limit. This is different from the ABJM case and is related to the fact that no diagrams with vertices appears at two loops in the fishnet theory. It indicates that there may be a better way to extract V_N from $\langle W \rangle$.

- It was found in [Bonini etal, 16] that the prescription in [Griguolo etal, 12] fails starting at three-loop order. An alternative prescription which works better at higher-loop order in the ladder limit was provided in [Bonini etal, 16]. This prescription is identical to the one in [Griguolo etal, 12] up to two-loop order.
- The suitable renormalization condition is that when $\phi = 0$ the renormalized V should vanish. From this condition, we get the renormalized generalized potential

$$
\begin{aligned}
V_N^{ren} &= V_N - V_N|_{\phi=0} \\
&= N\bar{m}_i^{12} n_{21}^i (\mu L)^{2\epsilon} \frac{\Gamma(\frac{1}{2}-\epsilon)}{4\pi^{3/2-\epsilon}} \Big(\frac{\sec\phi}{2\epsilon} - \frac{1}{2\epsilon} \\
&- \sec(\phi)\log(1+\sec(\phi)) + \log 2\Big) + N^2 (\bar{m}_i^{12} n_{21}^i)^2 \frac{\Gamma(\frac{1}{2}-\epsilon)^2}{16\pi^{3-2\epsilon}} \\
&\times (\mu L)^{4\epsilon} \Big(-\frac{\sec\phi}{4\epsilon^2} + \frac{1}{4\epsilon^2} + \frac{1}{\epsilon}\sec\phi\log(1+\sec\phi) - \frac{1}{\epsilon}\log 2 \\
&+ \frac{1}{2\epsilon}\sec\phi\log\cos\phi \Big) .
\end{aligned} \tag{32}
$$

The universal cusp anomalous dimension

The so-called universal cusp anomalous dimension, γ_{cusp}, is obtained from the large imaginary ϕ limit [Griguolo etal, 2012]:

$$\gamma_{cusp} = -\lim_{\phi\to\infty} \frac{2\epsilon(V_N^{ren}|_{\phi\to i\phi})}{\phi}. \tag{33}$$

If we still use prescription despite of the existence of $1/\epsilon^2$ terms, we get $\gamma_{cusp} = 0$ at two-loop order in our fishnet theory.

Conclusion

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- New regularization scheme for BPS fermionic loop operators were proposed. It also applies for BPS WLs in super-Chern-Simons theories.
- *This work shows that proper computations of Fermionic BPS WLs is complicated than people throught before.*
- We also studied cusped line operators and computed generalized cusp anomalous dimension.

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- RG flow/marginal deformation among different WLs.
- BPS fermionic WLs in higher dimensions (4d, 5d).

Thanks for Your Attention !

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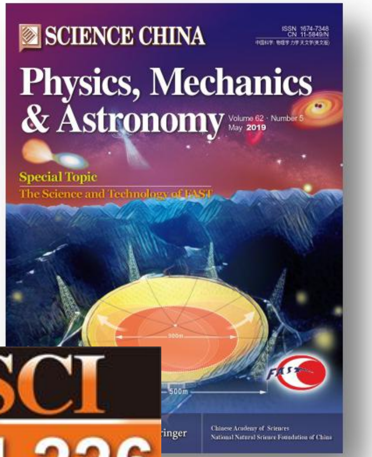

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Backup slides

Cusped Lines Operators in Fishnet Theory

We consider a cusp which is parametrized by

$$x^1 = \tau \cos\phi, \quad x^2 = |\tau| \sin\phi, \quad x^3 = 0, \quad -L \leq \tau \leq L. \tag{34}$$

$$W_{cusp} = \mathcal{P} \exp(-\mathrm{i} \int d\tau L_{line}(\tau)), \tag{35}$$

with

$$\begin{aligned} & L_{line} = B + F, \;\; B = \mathrm{i}(\bar{M}_Z N_{\bar{Z}} + N_{\bar{Z}} \bar{M}_Z), \;\; F = \bar{M}_\zeta + N_{\bar{\zeta}} \\ & [\bar{M}_Z]_{(ab)} = \bar{m}_i^{ab} Z^i_{(ab)}, \;\; [N_{\bar{Z}}]_{(ab)} = n^i_{ab} \bar{Z}_i^{(ab)} \\ & [\bar{M}_\zeta]_{(ab)} = \bar{m}_i^{ab} \zeta^i_{(ab)+}, \;\; [N_{\bar{\zeta}}]_{(ab)} = n^i_{ab} \bar{\zeta}_{i-}^{(ab)} \end{aligned} \tag{36}$$

Note that $\zeta^i_{(ab)+} = \mathrm{i}u_+\zeta^i_{(ab)}$, $\bar{\zeta}^{(ab)}_{i-} = \mathrm{i}\bar{\zeta}^{(ab)}_i u_-$ with $u_\pm$ being

$$\text{Right-half}: \; u_{+\alpha,R} = \frac{1}{\sqrt{2}}\begin{pmatrix} s_- \\ -\mathrm{i}s_+ \end{pmatrix}, \quad u_{-\alpha,R} = \frac{1}{\sqrt{2}}\begin{pmatrix} s_- \\ \mathrm{i}s_+ \end{pmatrix} \tag{37}$$

$$\text{Left-half}: \; u_{+\alpha,L} = \frac{1}{\sqrt{2}}\begin{pmatrix} s_+ \\ -\mathrm{i}s_- \end{pmatrix}, \quad u_{-\alpha,L} = \frac{1}{\sqrt{2}}\begin{pmatrix} s_+ \\ \mathrm{i}s_- \end{pmatrix}, \tag{38}$$

where we have defined $s_\pm = \exp(\pm\mathrm{i}\phi/2)$.

- It is easy to check that along both the left and the right half the following BPS conditions are satisfied

$$\gamma_\mu \dot{x}^\mu u_\pm = \pm|\dot{x}|u_\pm, \quad u_+u_- = -\mathrm{i}, \quad u_\pm \partial_\tau u_\mp = 0. \tag{39}$$

  Finally, the cusp operators are defined by taking the trace of eq. (35) with the superconnection in eq. (36), the contour in eq. (34) and the spinors in eq. (37). We also need to keep in mind solutions $\bar{m}_i^{21} = n_{12}^i = 0$ and $\bar{m}_i^{12} = n_{21}^i = 0$ of BPS equations lead to nontrivial cusp operators.

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- We will perform the computations in the framing 0 using the dimensional regularization with dimensional reduction and take the internal data $\bar{m}_i^{ab}, n^i_{ab}$ being the same along the cusp.