Reflected entropy for an evaporating black hole

Yang Zhou

Based on arXiv:2006.10846 (JHEP) with Tianyi Li and Jinwei Chu

第一届全国场论与弦论学术研讨会

Introduction

- Quantum gravity is the key to understand the origin of our universe

- A simpler object involving quantum gravity is black hole. They have a temperature that leads to Hawking radiation.

- Black holes also have entropy, given by the Area of the horizons.

- The question is whether black holes behave like ordinary quantum systems. People believe they do (string theory, AdS/CFT) but do not know how.

- Importantly, there is a paradox if they do: consider a black hole formed by a pure state, after evaporation it becomes a thermal state (according to Hawking)-> information is lost

- You may argue that strange things can happen at the end of the evaporation. But the paradox already shows up near the middle age of BH.

- To understand this, we first introduce 2 different notions of entropy: fine-grained entropy and coarse-grained entropy.

Fine-grained < coarse-grained [Review: arXiv:2006.06872]

- 1st : Fine-grained entropy is simply the von Neumann entropy. It is Shannon's entropy with distribution replaced by density matrix. It is invariant under unitary time evolution.

- 2nd: Coarse-grained entropy is defined as follows. We only measure simple observables A_i. And consider all possible density matrices which give the same result as our system.

$$Tr[\tilde{\rho} A_i] = Tr[\rho A_i]$$

We then choose the maximal von Neumann entropy over all possible density matrices $S(\tilde{\rho})$. It increases under unitary time evolution. -> entropy in thermodynamics.

Information paradox

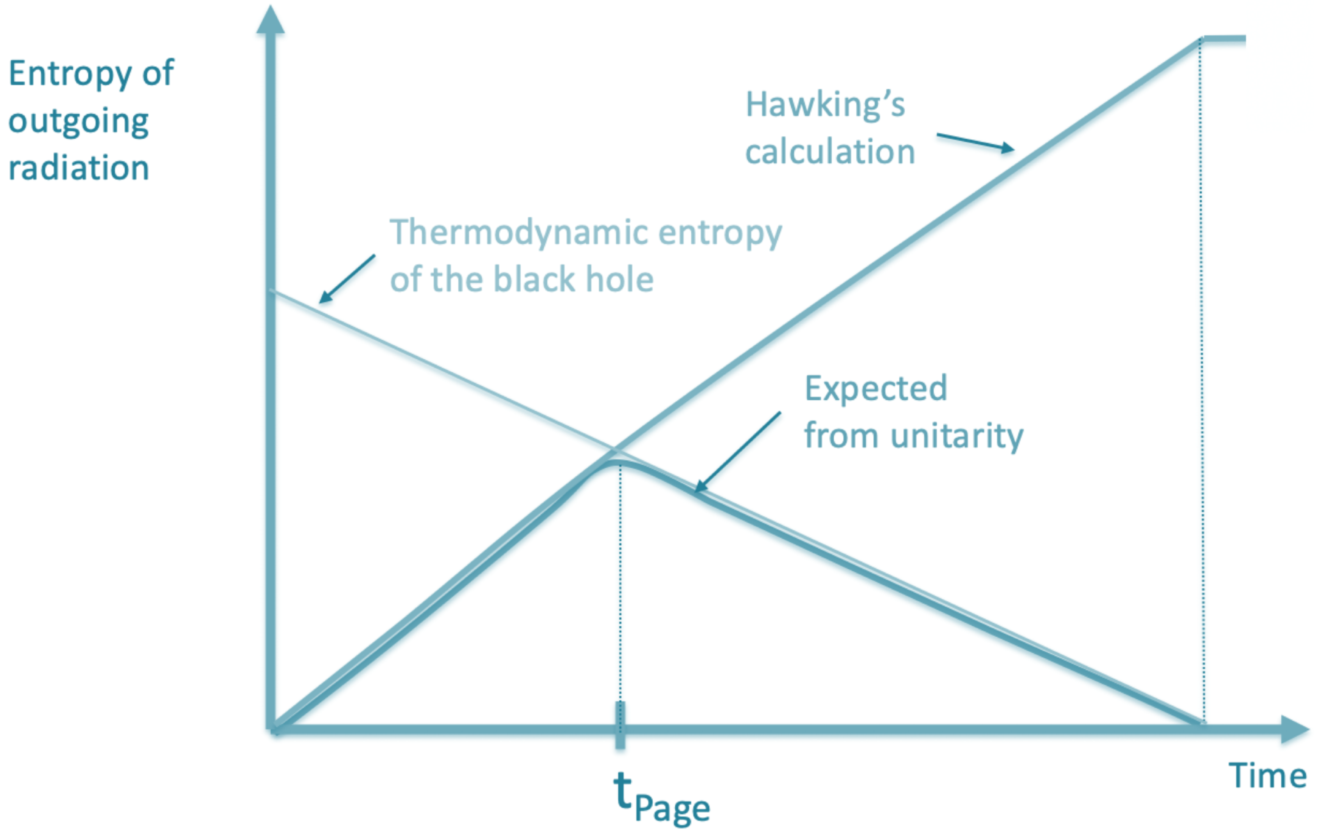
- Bekenstein-Hawking entropy is coarse-grained entropy.

- The thermal aspect of Hawking radiation comes from separating entangled outgoing Hawking quanta and interior Hawking quanta.

- As the entropy of radiation gets bigger and bigger, we run into trouble because, the entangled partners in black hole should have the same entropy, which exceeds the horizon entropy.

- In fact, the constantly increasing result was made by Hawking. Page suggested that the outgoing radiation entropy should follow Page curve

Page curve



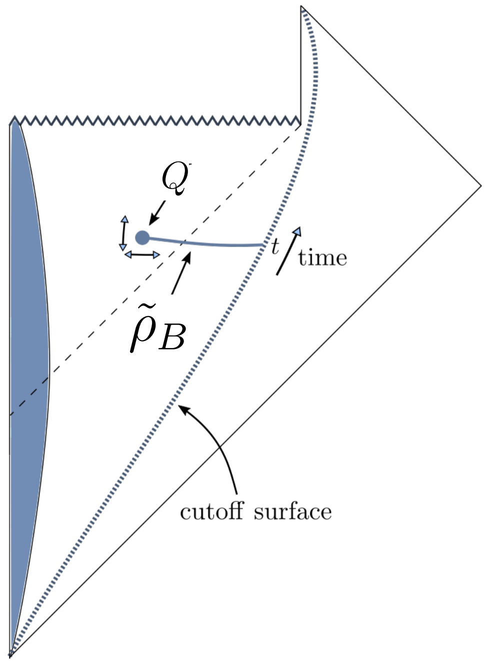
[Fig from arXiv:2006.06872]

How to reproduce Page curve?

QES formula for BH [Penington, Almheiri-Engelhardt-Marolf-Maxfield]

- The fine-grained entropy of black hole surround by quantum fields is given in terms of semiclassical entropy by

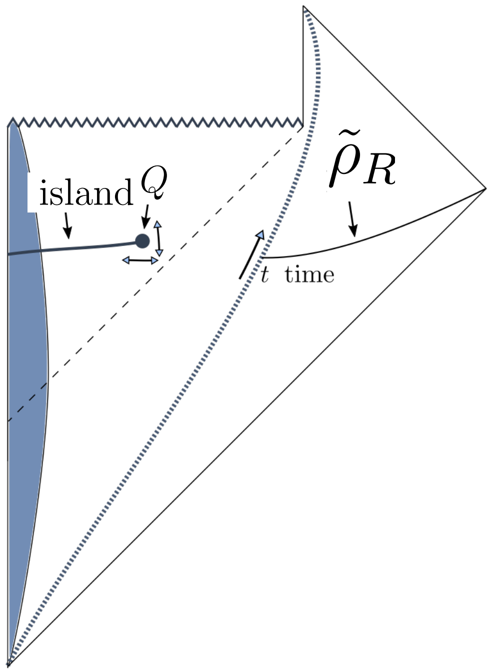
$$S_B = \text{ext}_Q \left\{ \frac{\text{Area}(Q)}{4G_N} + S(\tilde{\rho}_B) \right\} ,$$



Island formula for radiation [Almheiri-Mahajan-Maldacena-Zhao]

- Similarly, the fine-grained entropy of radiation is given in terms of semiclassical entropy by

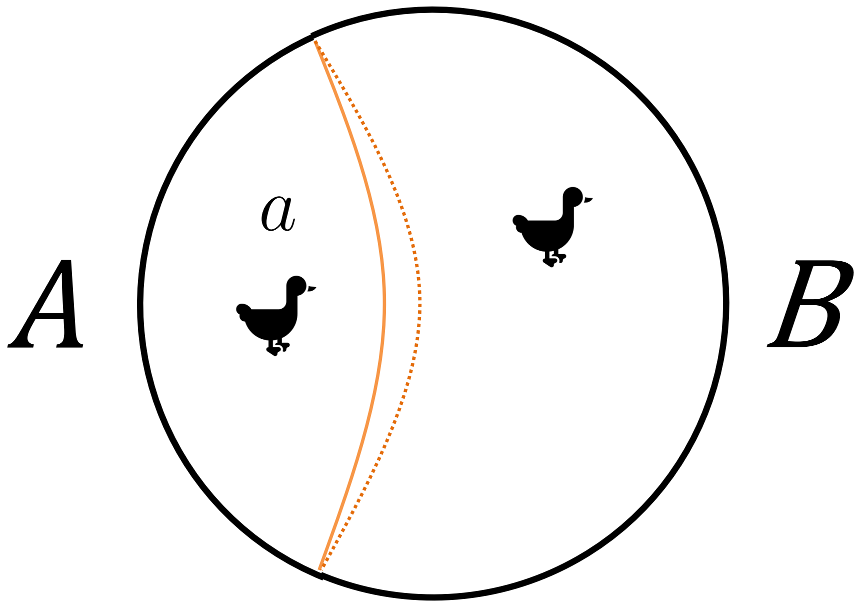
$$S(\rho_R) = \text{ext}_I \left\{ \frac{\text{Area}(\partial I = Q)}{4G_N} + S(\tilde{\rho}_{R \cup I}) \right\}$$



Quantum extremal surface [Engelhardt-Wall,RT,HRT]

- QES origins from holographic entanglement entropy in AdS/CFT with bulk matter

$$S(A) = \text{ext}_Q \left\{ \frac{\text{Area}(Q)}{4G_N} + S^{\text{bulk}}(a) \right\}$$



Motivations

- So far we only consider BH + radiation is pure, but what if BH + radiation is a mixed state?
- Are there other quantities which can have island formula?
- Can we read more information about the island?
- Can we compute the correlation between Hawking radiation A and B?

Von Neumann entropy
vs
Reflected entropy

(See also arXiv:2006.10754 by V.Chandrasekaran,M.Miyaji,P.Rath)

Outline

- Reflected entropy and the holographic dual
- Quantum extremal cross section
- Gravitational reflected entropy
- Eternal black hole + CFT model

Canonical purification [Dutta-Faulkner]

- Consider a mixed state on a bipartite Hilbert space

$$\rho_{AB}$$

- Flipping Bras to Kets for the basis

$$|i\rangle\langle j| \Rightarrow |i\rangle \otimes |j\rangle$$

- A canonical purification

$$|\sqrt{\rho_{AB}}\rangle \in (\mathcal{H}_A \otimes \mathcal{H}_A^\star) \otimes (\mathcal{H}_B \otimes \mathcal{H}_B^\star)$$

$$\mathrm{Tr}_{\mathcal{H}_A^\star \otimes \mathcal{H}_B^\star} |\sqrt{\rho_{AB}}\rangle\langle\sqrt{\rho_{AB}}| = \rho_{AB}$$

$$\rho_{AB} = \frac{1}{2}(|\uparrow\uparrow\rangle\langle\uparrow\uparrow|_{AB} + |\downarrow\downarrow\rangle\langle\downarrow\downarrow|_{AB})$$

$$\Downarrow$$

$$|\sqrt{\rho_{AB}}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\uparrow\rangle_{AA'BB'} + |\downarrow\downarrow\downarrow\downarrow\rangle_{AA'BB'})$$

- Reflected entropy

$$S_R(A:B) \equiv S(AA^\star)_{\sqrt{\rho_{AB}}}$$

Reflected entropy

- Properties

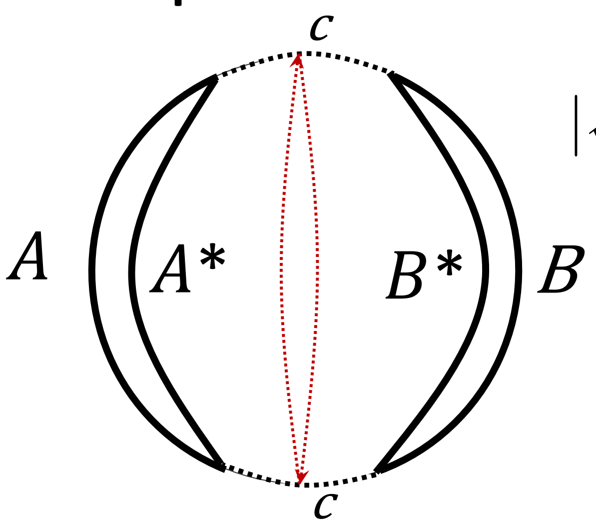
$$\text{pure state}: \ S_R(A:B) = 2S(A)\ ,$$

$$\text{factorized state}: \ S_R(A:B) = 0\ ,$$

$$\text{bounded from below}: \ S_R(A:B) \geq I(A:B)\ ,$$

$$\text{bounded from above}: \ S_R(A:B) \leq 2\text{min}\{S(A), S(B)\}$$

- Graph description



$$\psi_{ABc} \in \mathcal{H}_{ABc}$$

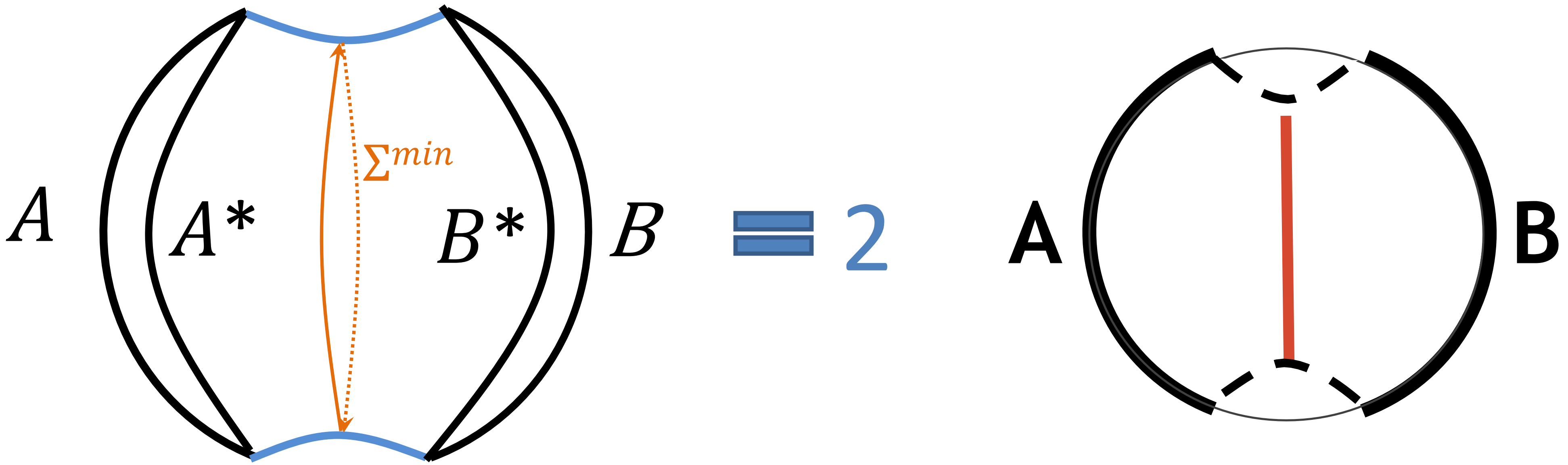
$$|\sqrt{\rho_{AB}}\rangle = |\sqrt{\text{Tr}_c|\psi\rangle\langle\psi|}\rangle \in (\mathcal{H}_A \otimes \mathcal{H}_{A^*}) \otimes (\mathcal{H}_B \otimes \mathcal{H}_{B^*})$$

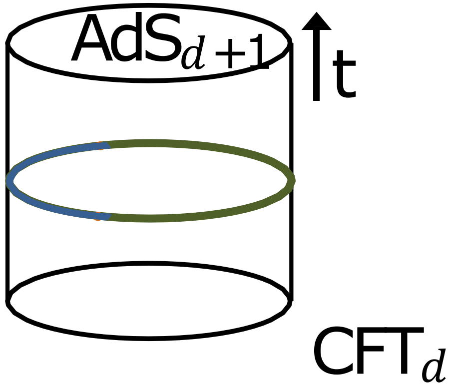
$$S_R(A:B) = S(AA^*:BB^*)_{\sqrt{\rho_{AB}}}$$

$$= \text{Entanglement Entropy of Red Curve}$$

Holographic reflected entropy

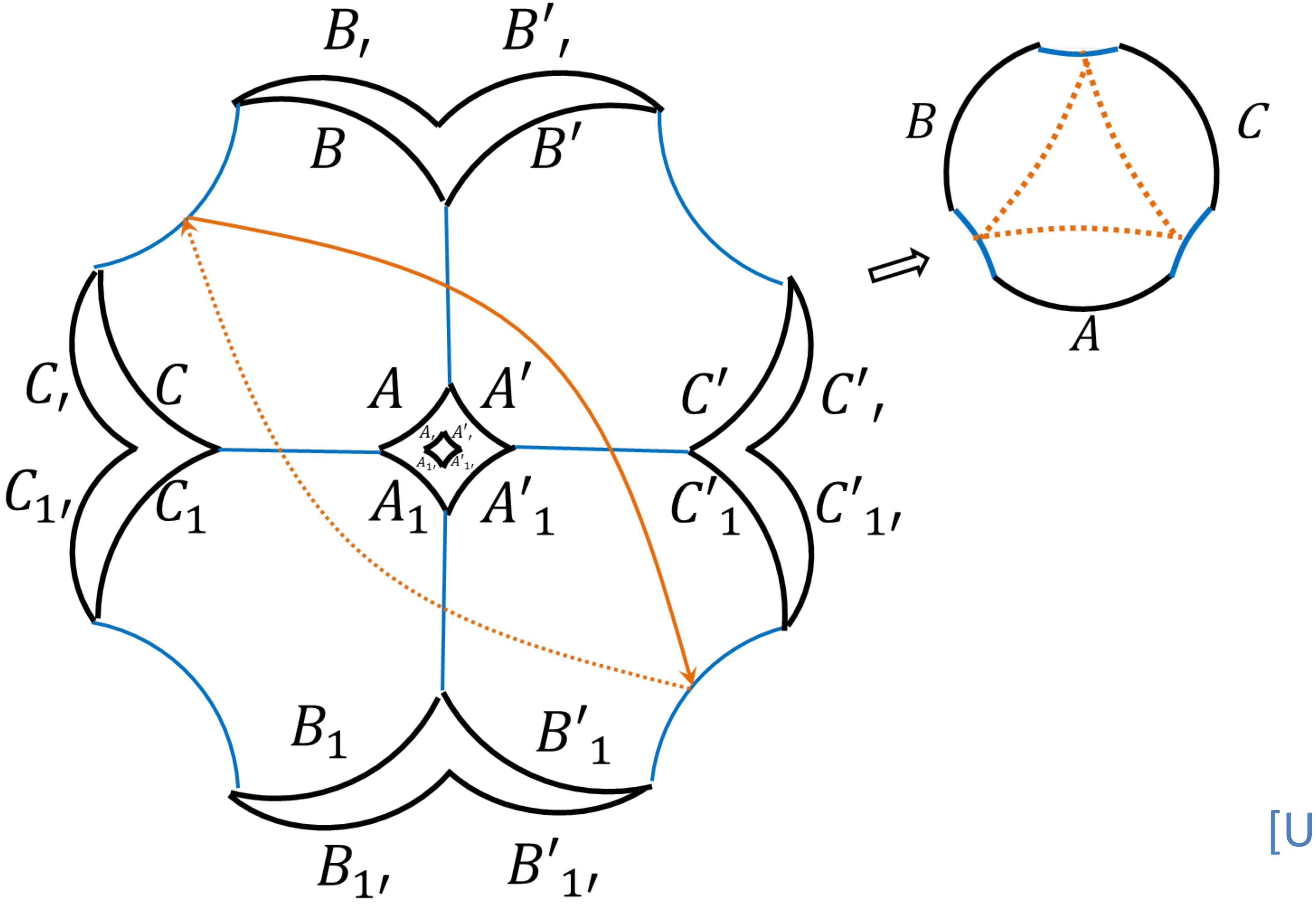
[Dutta-Faulkner]





[Takayanagi-Umemoto]

Multipartite reflected entropy $\Delta_W(A:B:C)$



[Chu-Qi-YZ,2019]

[Umemoto-YZ,2018]

Replica trick

- Replica trick in canonical purifications

$$\psi_1 = |\sqrt{\mathrm{Tr}_c |\psi_{ABCabc}\rangle\langle\psi_{ABCabc}|}\,\rangle$$
$$\psi_2 = |\sqrt{\mathrm{Tr}_{bb'} |\psi_1\rangle\langle\psi_1|}\,\rangle$$
$$\psi_3 = |\sqrt{\mathrm{Tr}_{aa'a''a'''} |\psi_2\rangle\langle\psi_2|}\,\rangle$$

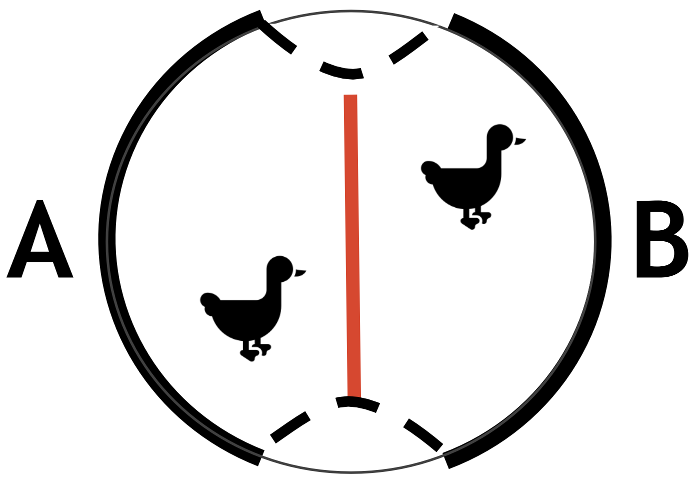
$\Rightarrow$

$$\psi_1^{(m)} = |(\mathrm{Tr}_c \rho_0)^{\frac{m}{2}}\rangle \ ,$$
$$\psi_2^{(m)} = |(\mathrm{Tr}_{bb'} \rho_1^{(m)})^{\frac{m}{2}}\rangle \ ,$$
$$\psi_3^{(m)} = |(\mathrm{Tr}_{aa'a''a'''} \rho_2^{(m)})^{\frac{m}{2}}\rangle$$

- Replica trick in Renyi index

$$\Delta_R(A:B:C) = \lim_{\substack{\boldsymbol{n}\to 1 \\ m \to 1}} S_{\boldsymbol{n}}, \qquad S_{\boldsymbol{n}} = \frac{1}{1-\boldsymbol{n}} \ln \frac{\mathrm{Tr}_R(\mathrm{Tr}_L \rho_3^{(m)})^{\boldsymbol{n}}}{(\mathrm{Tr}\rho_3^{(m)})^{\boldsymbol{n}}}$$

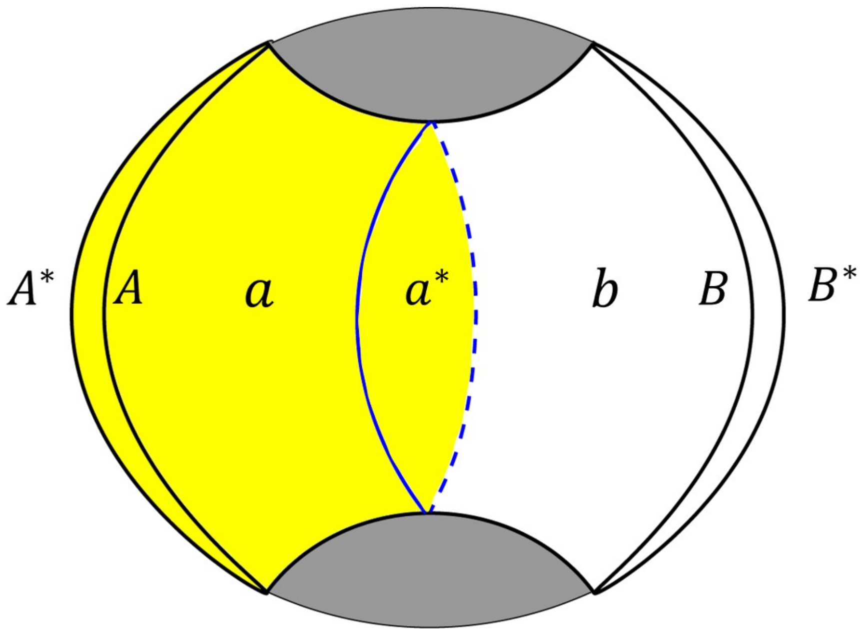
Quantum corrected reflected entropy



$$S_R(A:B) = \frac{2\langle \mathcal{A}[\partial a \cap \partial b]\rangle_{\tilde{\rho}_{ab}}}{4G_N} + S_R^{\text{bulk}}(a:b) + \mathcal{O}(G_N)$$

[Dutta-Faulkner]

FLM on double replicas



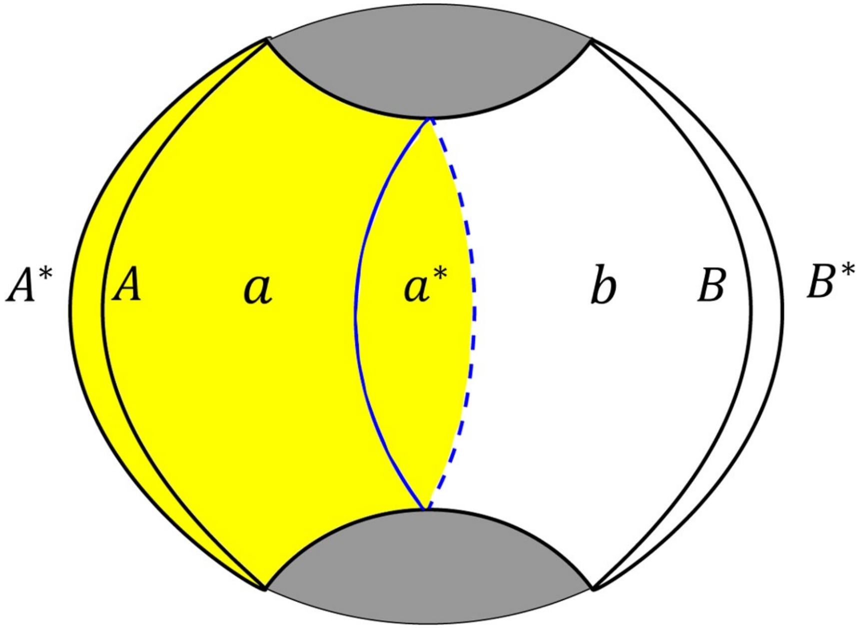
$$S(AA^*) = \frac{1}{4G_N}\langle \mathcal{A}[m(AA^*)]\rangle + S^{\text{bulk}}(aa^*) + \mathcal{O}(G_N)$$

$$\langle \mathcal{A}[m(AA^*)]\rangle = 2\langle \mathcal{A}[\partial a \cap \partial b]\rangle$$

$$S_R^{\text{bulk}}(a:b) = S^{\text{bulk}}(aa^*)$$

Quantum extremal cross section

[Li-Chu-YZ,2020]



$$S(AA^*) = \text{ext}_Q \left\{ \frac{\text{Area}(Q)}{4G_N} + S^{\text{bulk}}(aa^*) \right\}$$

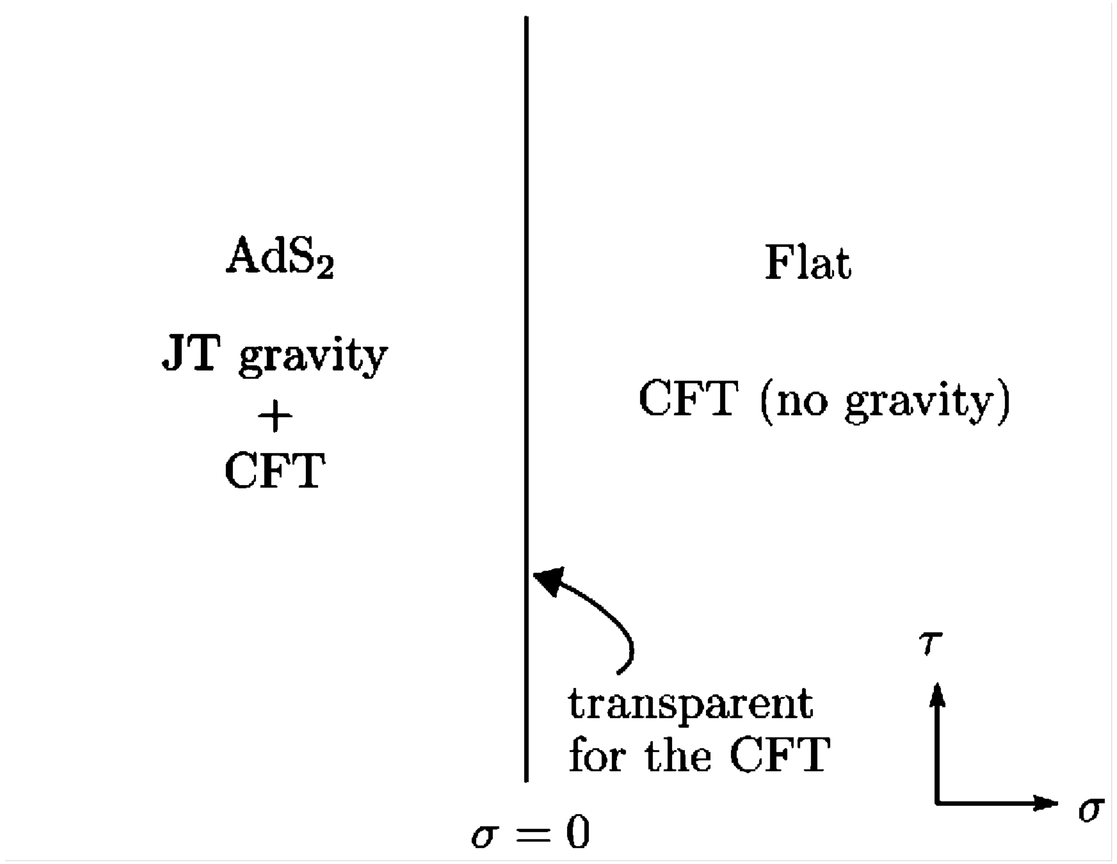
$$S_R(A:B) = \text{ext}_{Q'} \left\{ \frac{2\text{Area}(Q' = \partial a \cap \partial b)}{4G_N} + S_R^{\text{bulk}}(a:b) \right\}$$

Eternal black hole + CFT

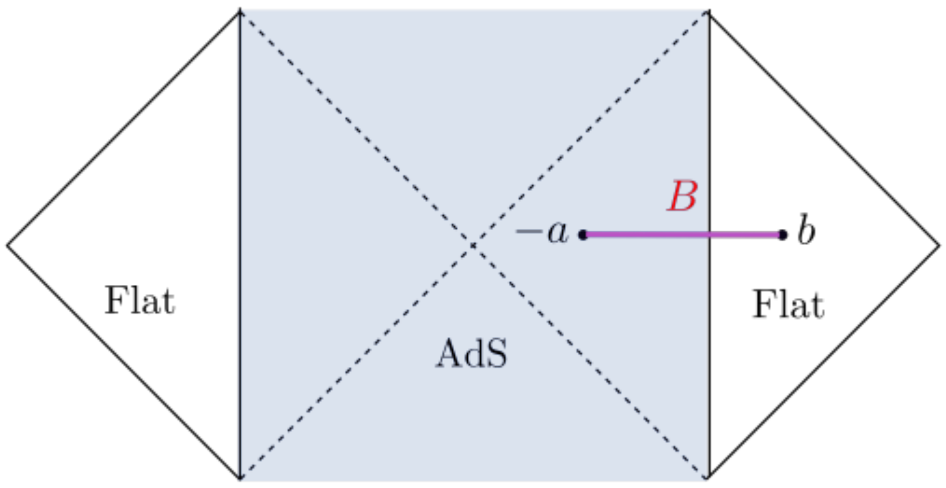
[Almheiri-Mahajan-Maldacena]

- AdS black holes do not evaporate.
- Information paradox can be realized in AdS spacetime joined to a Minkowski region, such that black hole can radiates into the attached Minkowski region
- Consider 2d Jackiw-Teiteboim gravity in AdS plus a CFT_2 (also in Minkowski), with a transparent boundary condition
- Explicit computations can be done in this model

$$I_{\mathrm{total}} = -\frac{S_0}{4\pi}\left[\int_\Sigma R + \int_{\partial\Sigma} 2K\right] - \int_\Sigma (R+2)\frac{\phi}{4\pi} - \frac{\phi_b}{4\pi}\int_{\partial\Sigma} 2K + S_{\mathrm{CFT}}$$



QES for a single interval



$$ds^2_{\text{in}} = \frac{4\pi^2}{\beta^2}\frac{dyd\bar{y}}{\sinh^2 \frac{\pi}{\beta}(y+\bar{y})}, \qquad ds^2_{\text{out}} = \frac{1}{\epsilon^2}dyd\bar{y}$$

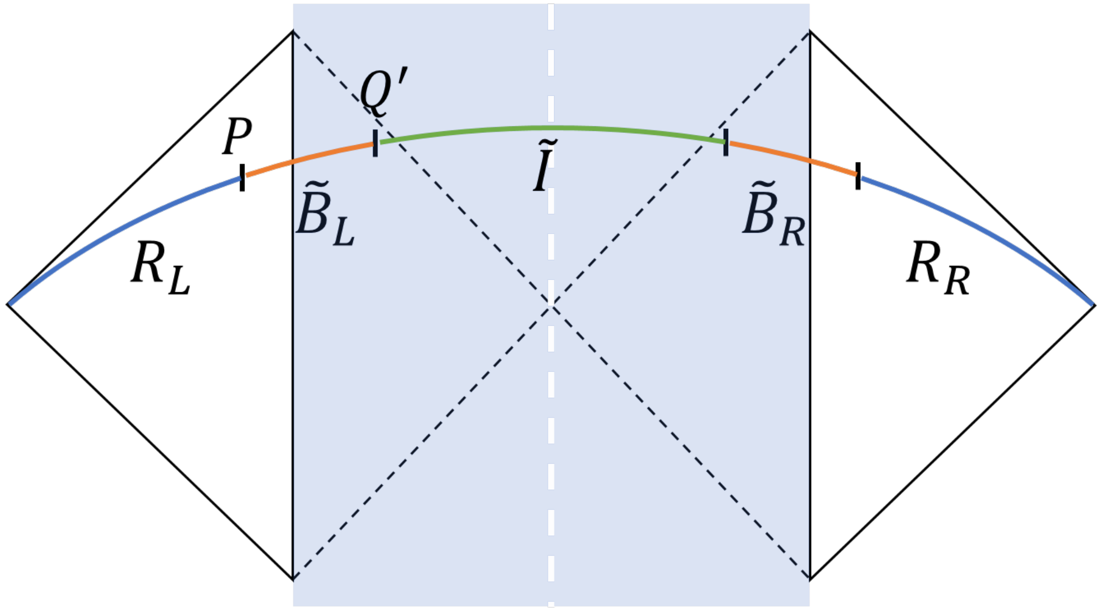
$$y = \sigma + i\tau, \qquad \bar{y} = \sigma - i\tau\ , \qquad \tau = \tau + \beta\ .$$

$$S_{\text{gen}} = S_0 + \phi(-a) + S_{\text{CFT}}([-a,b])$$

- –a is determined following QES condition

$$\partial_a S_{\text{gen}} = 0 \quad \rightarrow \qquad \sinh\left(\frac{2\pi a}{\beta}\right) = \frac{12\pi\phi_r}{\beta c}\frac{\sinh\left(\frac{\pi}{\beta}(b+a)\right)}{\sinh\left(\frac{\pi}{\beta}(a-b)\right)}$$

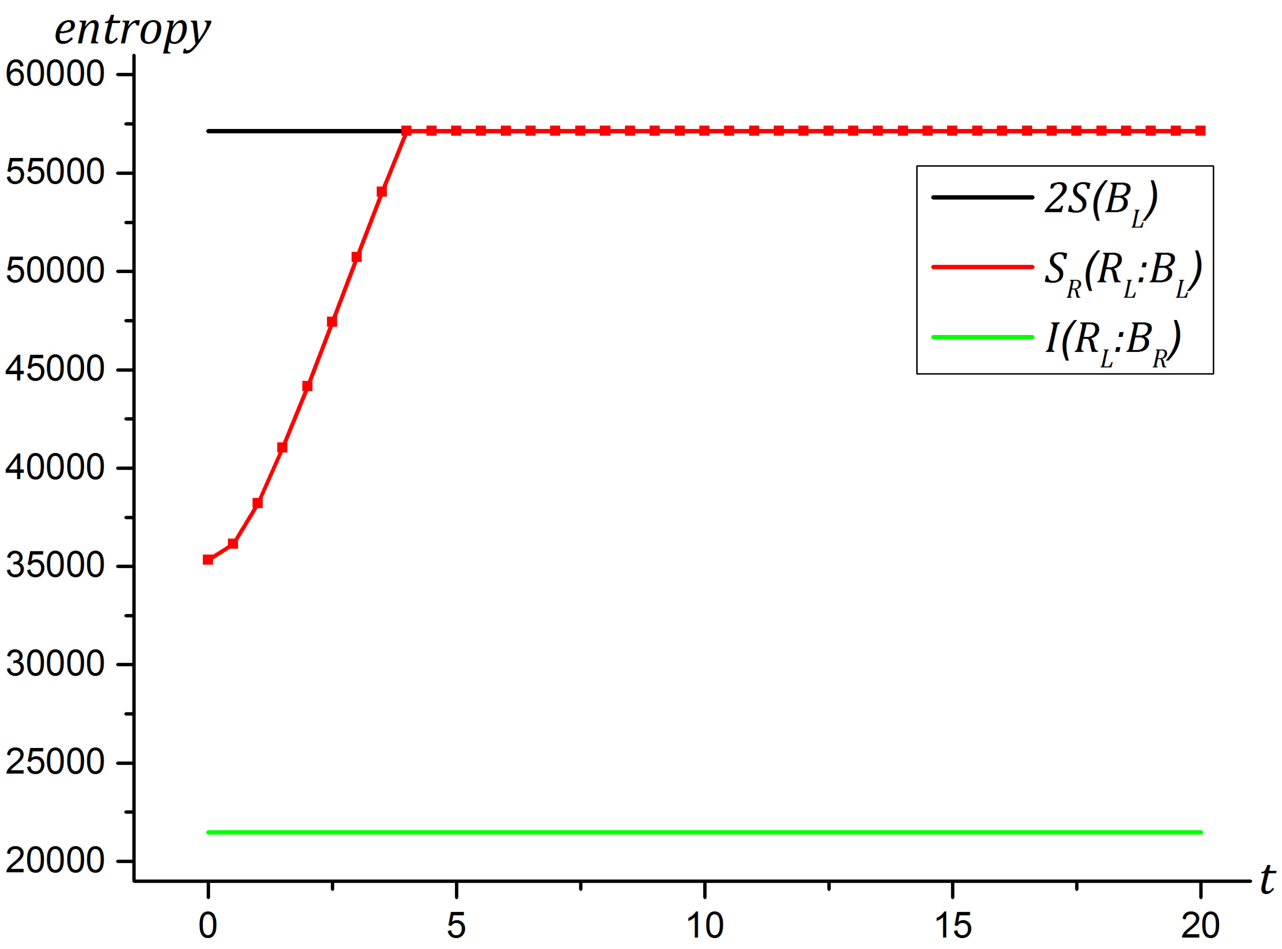
Gravitational reflected entropy (B-R)



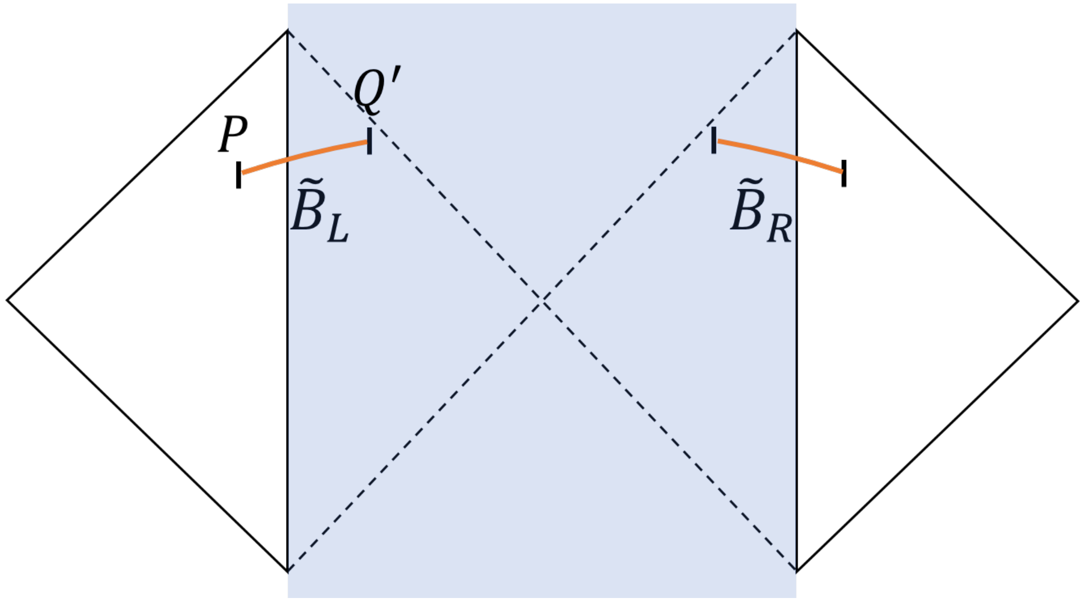
$$S_R(R_L : B_L) = S(R) = \min \text{ext}_I \left\{ \frac{A(\partial I)}{4G_N} + S(\tilde{\rho}_{R\cup I}) \right\}$$

$$S_R(R_L : B_L) = \min \text{ext}_{Q'} \left\{ \frac{2A(Q' = \partial \tilde{I}_L \cap \partial \tilde{B}_L)}{4G_N} + S_R(\tilde{\rho}_{R_L \cup \tilde{I}_L} : \tilde{\rho}_{\tilde{B}_L}) \right\}$$

$$b = 0.01, \phi_r = 100, S_0 = c = 20000 \text{ and } \epsilon_{UV} = 0.01$$



B-B reflected entropy

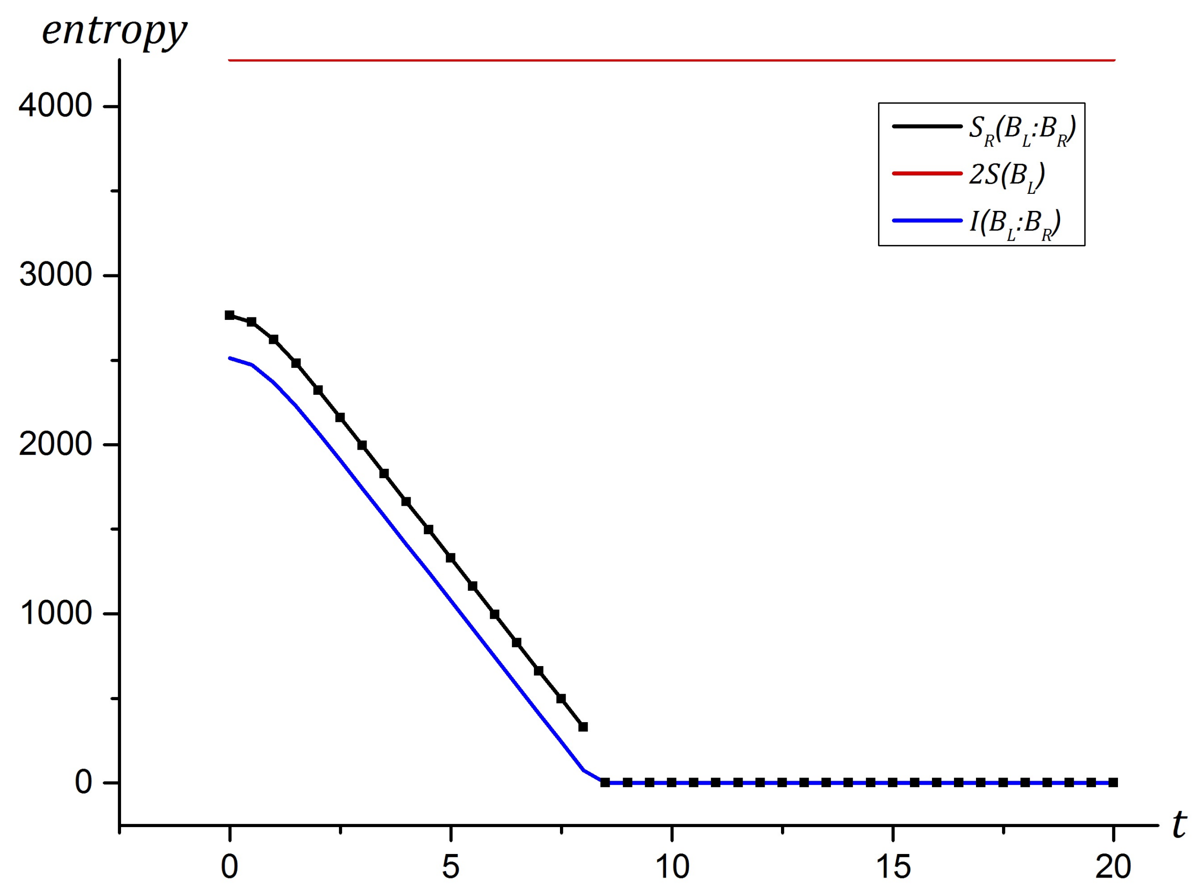


$$S_R(B_L : B_R) = \min \text{ext}_{Q'} \left\{ \frac{2A(Q' = \partial \tilde{B}_L \cap \partial \tilde{B}_R)}{4G_N} + S_R(\tilde{\rho}_{\tilde{B}_L} : \tilde{\rho}_{\tilde{B}_R}) \right\}$$

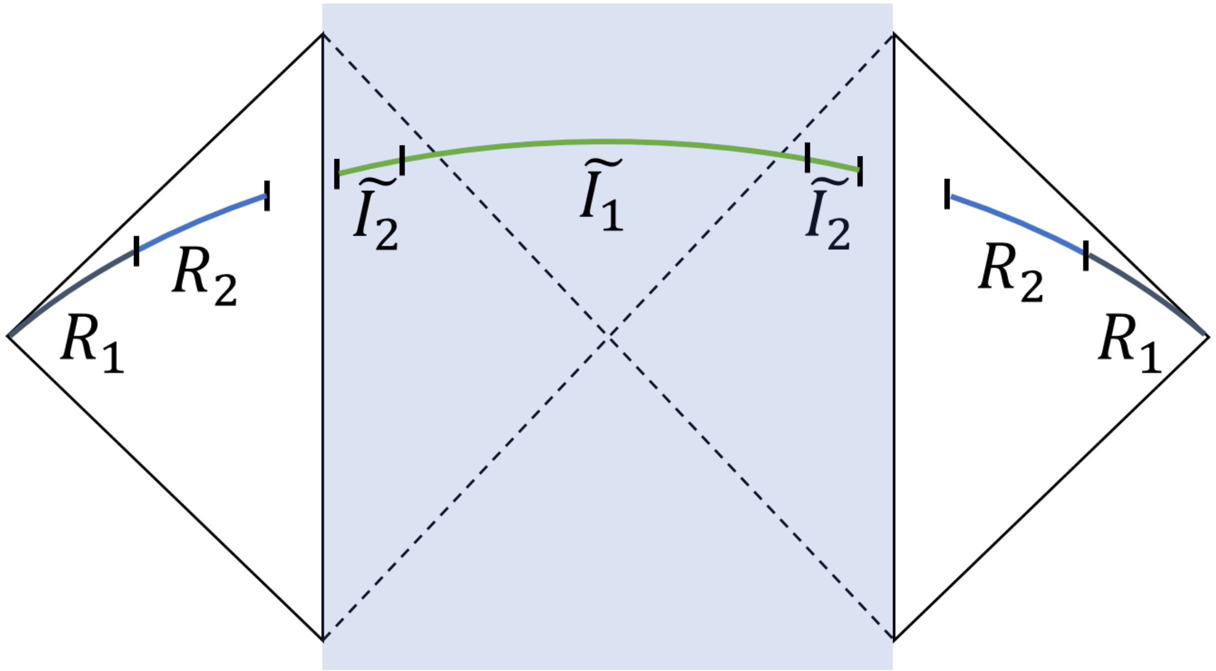
$$S_R(B_L : B_R) = 2S_0 + 2\phi_r + \frac{c}{3}(b - \ln\cosh t + \ln 2)$$

$$S_R(\tilde{\rho}_{\tilde{B}_L} : \tilde{\rho}_{\tilde{B}_R}) \sim c(-0.15\eta \ln \eta + 0.67\eta)$$

$$b = 1, \phi_r = 100, S_0 = c = 1000$$

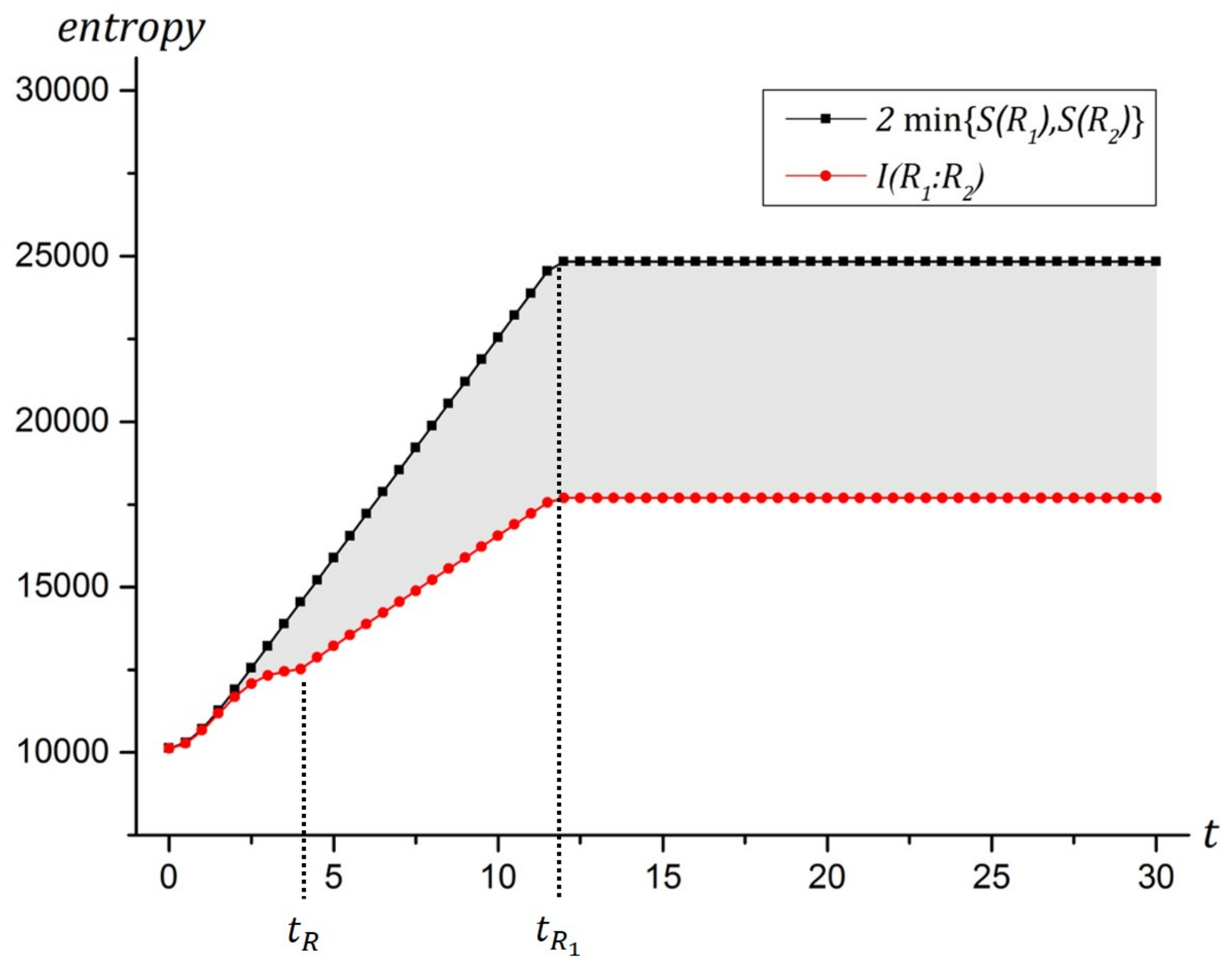


R-R reflected entropy



$$S_R(R_1 : R_2) = \min \text{ext}_{Q'} \left\{ \frac{2A(Q' = \partial \tilde{I}_1 \cap \partial \tilde{I}_2)}{4G_N} + S_R(\tilde{\rho}_{R_1 \cup \tilde{I}_1} : \tilde{\rho}_{R_2 \cup \tilde{I}_2}) \right\}$$

$$b_1 = 0.01, b_2 = 5, \phi_r = 10, S_0 = c = 2000 \text{ and } \epsilon_{UV} = 0.001$$



Paradox for initial mixed state

- Von Neumann entropy contains redundancy
- Reflected entropy is good measure of information transfer
- The paradox $S_R > 2\ S_{BH}$, Page time is defined at $S_R = 2\ S_{BH}$
- The island formula of reflected entropy resolved the paradox

Summary

- Reflected entropy curve is the analogy of Page curve for a globally mixed state
- Reflected entropy can be computed for R-R, B-B and B-R
- Reflected entropy has island cross-section as its area term
- Future direction: multipartite generalization (in progress)

Thank You!