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Gravitational Wave

- Gravity is weak between common objects
- Need to look in the universe for strong gravity
- Need to rotate celestial very fast, better close to speed of light, to generate strong gravitational wave
- Gravitational wave has been detected when two blackholes merge
- LIGO has detected many cases
    - Frequency up to more than 100Hz

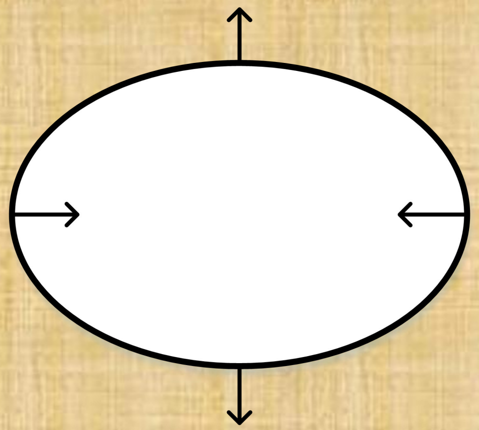
Gravitons

- Always assumed by physics community to exist
- E = hf
- Need high frequency to observe quantum properties
  - At least visible light frequency
  - Better f > 10^14 Hz
- Impossible for celestial objs.
- Need 10^19 Gev for particles
  - LHC 10^4 GeV

Gravitational Wave and Gravitons

- Plane wave

$$\boldsymbol{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}}$$
$$h_{\mu\nu} = e_{\mu\nu}e^{ikx} + e^{*}{}_{\mu\nu}e^{-ikx}$$



- Polarization $k_{\mu} = (\omega, 0, 0, \omega)$

$$e_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & e_{11} & e_{12} & 0 \\ 0 & e_{12} & -e_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Classical gravitational field is measured on $e_{\mu\nu}$
- $e_{\mu\nu}$ much smaller when frequency goes higher
- Single graviton energy higher

- Stress Energy Tensor

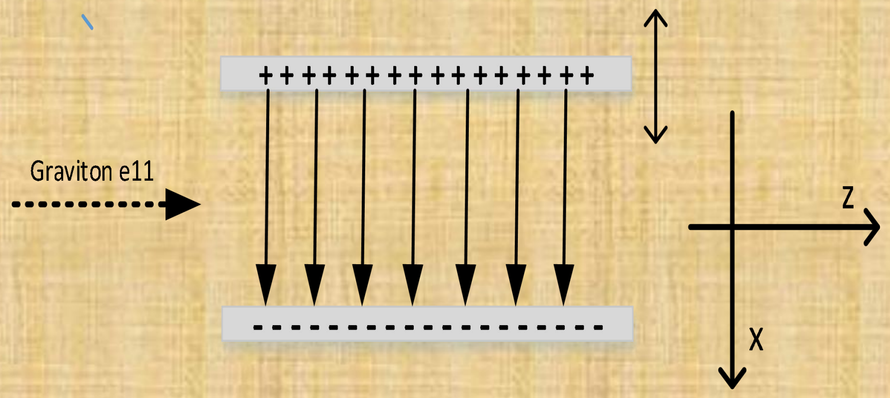
$$\langle t_{\mu\nu} \rangle = \frac{k_{\mu}k_{\nu}}{8\pi G}(|e_{11}|^2 + |e_{12}|^2)$$

From Gravitons to Photons

- EM Action with gravity

$$L=\int d^4x\sqrt{g}\left[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}+g^{\mu\nu}j_\mu(x)A_\nu\right]$$

$$D_\mu A_\nu-D_\nu A_\mu=\partial_\mu A_\nu-\partial_\nu A_\mu=F_{\mu\nu}$$



$$L(h,A)=\int d^4x\left(\frac{1}{8}h_\rho{}^\rho\bar{F}_{\mu\nu}F^{\mu\nu}-\frac{1}{2}\bar{F}_{\mu\rho}F_\nu{}^\rho h^{\mu\nu}-\frac{1}{2}h_\nu{}^\nu j_\mu A^\mu+j_\mu A_\nu h^{\mu\nu}\right)$$

- Background field help to generate EM radiation, charge doesn’t

$$L(h,A)=-2Re\int d^4x\,[\bar{E}E_1e_{11}+\bar{E}E_2e_{12}]e^{i\omega(z-t)}$$

- Background electric or magnetic field perpendicular to gravitational wave propagation generate EM radiation, e_{11} and e_{12} generate different polarization

From Photon to Graviton

- **The stress-energy tensor of electro-magnetic field:**

$$T_{EM\mu\nu} = F_{\mu\lambda}F^{\lambda}{}_{\nu} - \frac{1}{4}\eta_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$$

- **Background field breaks angular momentum conservation**

- **Plane wave energy tensor**

$$T_{EM\mu\nu} = \begin{pmatrix} -\omega^2A^2 & 0 & 0 & \omega^2A^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \omega^2A^2 & 0 & 0 & \omega^2A^2 \end{pmatrix}$$

- **With a back ground electric field in x direction, and EM wave polarized in x direction**

$$\begin{pmatrix} -\omega\bar{E}A\cos(\omega z-\omega t) & 0 & 0 & \omega\bar{E}A\cos(\omega z-\omega t) \\ 0 & -\omega\bar{E}A\cos(\omega z-\omega t) & 0 & 0 \\ 0 & 0 & \omega\bar{E}A\cos(\omega z-\omega t) & 0 \\ \omega\bar{E}A\cos(\omega z-\omega t) & 0 & 0 & \omega\bar{E}A\cos(\omega z-\omega t) \end{pmatrix}$$

- With a back ground electric field in x direction, and EM wave polarized in y direction, the linear term of stress energy tensor is:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega\bar{E}A\cos(\omega z-\omega t) & 0 \\ 0 & -\omega\bar{E}A\cos(\omega z-\omega t) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Computation

- **The effective current:**

$$\boldsymbol{j_x = 2\omega\bar{E}|e_{11}|cos(\omega z - \omega t)}$$
$$\boldsymbol{j_y = 2\omega\bar{E}|e_{12}|cos(\omega z - \omega t)}$$

- **Use Green's Function**

$$A_x\left(r, \vec{\hat{k}}, t\right) \approx \int_{inside} d^3x' \frac{\omega\bar{E}|e_{11}|}{2\pi r} \cos\left[\omega\left(z' - t + r - \vec{\hat{k}} \cdot \vec{x}'\right)\right]$$

- **Probability for a graviton turning into photon in background electric field**

$$\epsilon_{G-EM} = P_{EM}/P_G = 4\pi G\epsilon_0\bar{E}^2L^2/c^4$$

- **In background magnetic field**

$$\epsilon_{G-EM} = P_{EM}/P_G = 4\pi GL^2\bar{B}^2/\mu_0c^4$$

Analysis

- As a graviton travels in background EM field, it slowly turns into a photon, the probability amplitude is proportional to distance, the probability is proportional to distance square.
- In the lab and in the universe, it is easier to have a stronger magnetic field than electric field.

$$\epsilon_{G-EM} = 8.2 \times 10^{-38} (\frac{\mathrm{BL}}{\mathrm{T \cdot m}})^2$$

- A photon travels in the background EM field will have the same probability to turn into graviton. Given a long enough distance, there will be oscillation
- The key is that both photon and graviton travels in the speed of light

- If we make a graviton detector with 30T magnetic field and 10km length

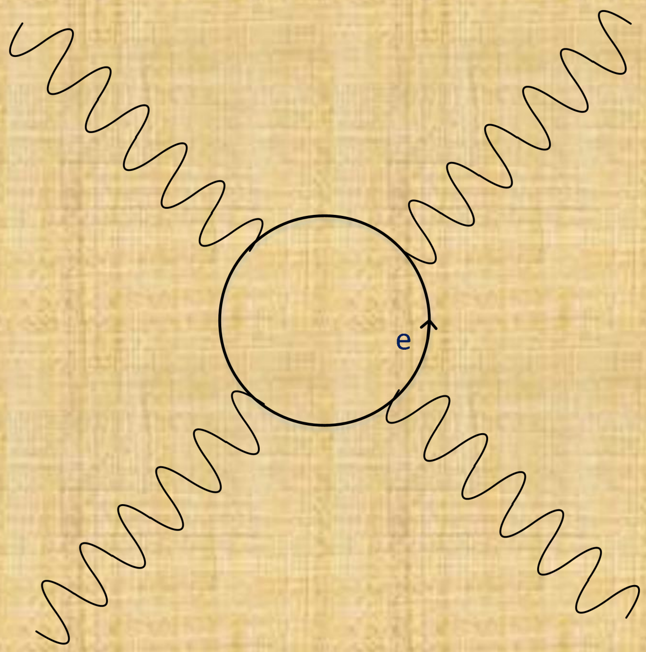
$$\epsilon_{G-EM} = 7.2 \times 10^{-27}$$

- for 1W wave of $\omega = 10^{14} Hz$, we will get about 2 events a month.

- For non-constant magnetic field, when wave length small compares to the distance scale of variation

$$\epsilon_{G-EM} = 4\pi G/\mu_0 c^4 \left[(\int B_x dl)^2 + (\int B_y dl)^2 \right]$$

QED 1-loop Correction



- Quantum field theory effects makes EM non-linear
- Euler-Heisenberg Lagrangian:

$$L_{1QED} = -\frac{2\alpha^2}{45{m_e}^4}\int d^4x\left[\left(\vec{E}^2-\vec{B}^2\right)^2+7\left(\vec{E}\cdot\vec{B}\right)^2\right]$$

- Consider background field, $A_\nu \to \overline{A_\nu} + A_\nu$, with a background magnetic field

$$F_{23} = F^{23} = -F_{32} = -F^{32} = \overline{B}$$

$$L_{QED}(\overline{B}) = -\int d^4x\left[\left(\frac{1}{4}-\frac{2\alpha^2\overline{B}^2}{45{m_e}^4}\right)F_{\mu\nu}F^{\mu\nu}+\frac{\alpha^2\overline{B}^2}{45{m_e}^4}\left(8{B_1}^2+14{E_1}^2\right)\right]$$

- For non-constant magnetic field, when wave length small compares to the distance scale of variation

$$\epsilon_{G-EM} = 4\pi G/\mu_0 c^4\left[\left(\int B_x dl\right)^2+\left(\int B_y dl\right)^2\right]$$

QED 1-loop Correction

- Speed of light is changed under background EM field.

$$v_{x-photon} = 1 - \frac{14\alpha^2\overline{B}^2}{45{m_e}^4}$$

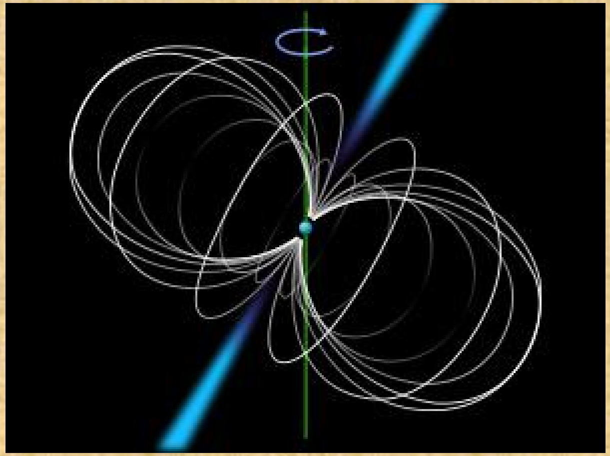
$$v_{y-photon} = 1 - \frac{8\alpha^2\overline{B}^2}{45{m_e}^4}$$

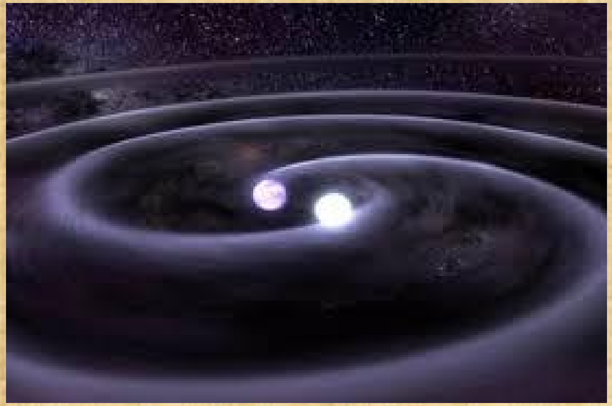
$$\frac{\boldsymbol{\alpha^2\overline{B}^2}}{\boldsymbol{{m_e}^4}} \to \frac{\boldsymbol{\alpha^2\hbar^3\overline{B}^2}}{\boldsymbol{\mu_0 c^5 {m_e}^4}} = \mathbf{3 \times 10^{-23}}\left(\frac{\overline{\boldsymbol{B}}}{\boldsymbol{T}}\right)^2$$

- QED effect has no impact on ground experiment
- QED significantly impact Magnetars and Neutron stars

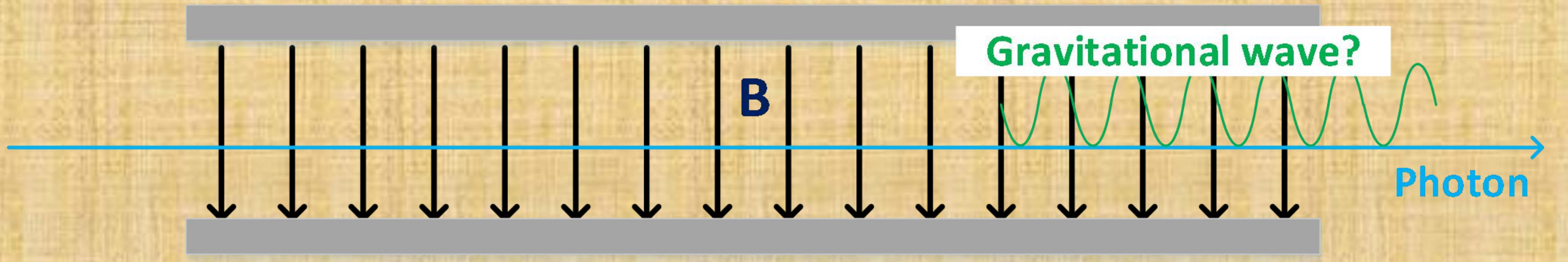
Implications

- There are more gravitons in the universe than we believed. a typical galaxy has a size of 100000 light years ($10^{21} m$) and an average magnetic field of $10^{-9} T$, and most of the field are not turbulent, the switching probability is on the order of 10^{-14} (versus $\sim 10^{-40}$ thermal radiation).
- Neutron stars and Magnetars
- Creating/detecting gravitons in the lab
  - 60's Hertz Experiment considered
  - Quantum optics from 80-90's
  - Receive gravitons from the universe
    - Fixed direction, background problems
  - Detecting missing photons
    - Relative low cost





Is Gravitational Field a Quantum Field?



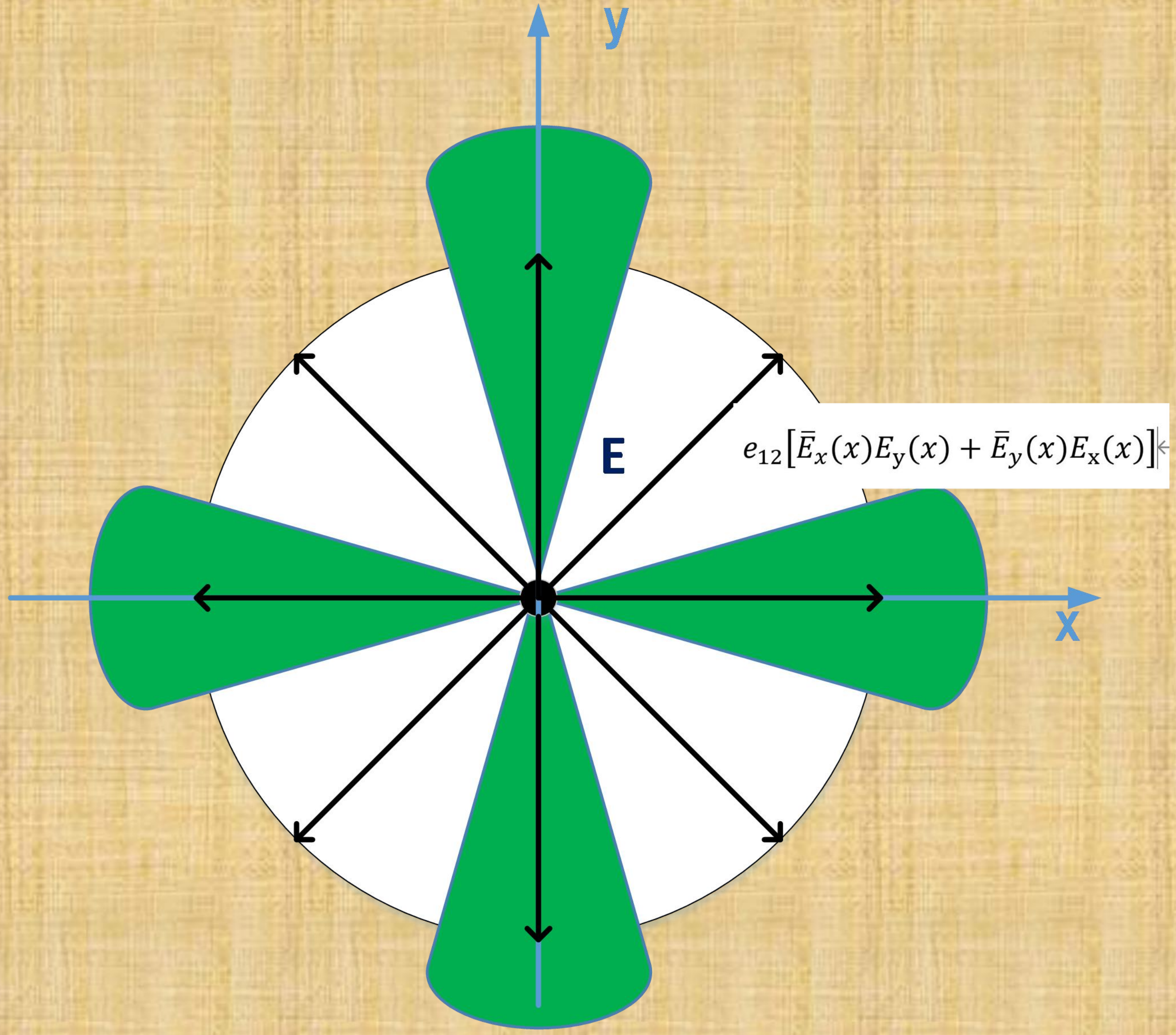
Can gravitational field remain classic while others are quantized?

Blackbody radiation?
Weak

- From this phenomenon, we know that classical general relativity + Quantum Electro-Magnetism is not consistent
- Quantum general relativity is not consistent so far
- This is THE experiment that can prove the existence of gravitons!
- More convincing than seeing photon from gravitons

On-going Research

- Graviton-Atom interfaction
- Crystal
- Ordinary matters
- Neutrino Experiment



Conclusion

- In gravity, experiments lags behind theory for ~100 years
  - Newton's gravitational law – Cavendish experiment
  - Einstein general relativity – discovery of gravitational wave
- Quantum gravity phenomenology and experiment can make important progress in this century
- No need to go to Planck Energy