The Three-Family N=1 Supersymmetric Pati-Salam Models from Type IIA String Theory on $\mathrm{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifold with Intersecting D6-Branes

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Outline

Outline

Supersymmetry and the SSMs with R-Parity

- A natural solution to the gauge hierarchy problem in the SM.
- Gauge coupling unification can be achieved.
- The Lightest Supersymmetric Particle (LSP) such as the LSP neutralino etc can be a dark matter candidate.
- The electroweak gauge symmetry can be broken radiatively due to the large top quark Yukawa coupling.
- . . . .

The Grand Unified Theories: $SU(5)$ and $SO(10)$

- Gauge interaction unification.
- Fermion unification.

  In $SO(10)$ model, one family of the SM fermion forms a spinor **16** representation.
- Yukawa coupling unification.

  $b-\tau$ and $b-\tau-t$ Yukawa coupling unifications in $SU(5)$ and $SO(10)$ models, respectively.
- Charge quantization.
- Weak mixing angle at weak scale M_Z.
- Neutrino masses and mixings by seesaw mechanism.
- Prediction: dimension-six and dimension-five proton decays via heavy gauge boson and colored Higgsino exchanges, respectively.

Problems

- Gauge symmetry breaking.
- Doublet-triplet splitting problem.
- Dimension-five Proton decay problem.
- Fermion mass problem.

  The wrong prediction on the fermion mass ratios: $m_e/m_\mu = m_d/m_s$.
- Landau pole problem above the GUT scale.

String Models

- Calabi-Yau compactification of heterotic $E_8 \times E_8$ string theory
- Orbifold compactification of heterotic $E_8 \times E_8$ string theory
- D-brane models on Type II orientifolds
- Free fermionic string model builing
- $\mathcal{F}$-Theory Model Building

Supersymmetry is a bridge between the promising low energy phenomenology and high-energy fundamental physics.

Grand Particle Physics Paradigm

String Theory $\rightarrow$ String Models $\rightarrow$ SUSY GUTs $\rightarrow$ SSMs $\rightarrow$ SM

Type II String Theories with D-Branes

- D-branes located at orbifold singularities where the chiral fermions appear on the worldvolume of D-branes [1].
- Intersecting D-branes on Type II orientifolds where the open string spectrum contains chiral fermions localized at the D-brane intersections [2].

[1] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y. S. Stanev; M. Berkooz and R.G. Leigh

[2] M. Berkooz, M. R. Douglas and R. G. Leigh.

Intersecting D-brane models

- Type IIA theory on $\mathrm{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold.
- The possible realistic models are Pati-Salam models.
- A realistic intersecting D-brane model.

Outline

The $\mathrm{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifold

We consider T^6 to be a six-torus factorized as $T^6 = T^2 \times T^2 \times T^2$ whose complex coordinates are z_i, $i = 1,\ 2,\ 3$ for each of the 2-torus, respectively. The θ and ω generators for the orbifold group $Z_2 \times Z_2$, which are associated with their twist vectors $(1/2, -1/2, 0)$ and $(0, 1/2, -1/2)$ respectively, act on the complex coordinates of T^6 as

$$\theta : \quad (z_1, z_2, z_3) \to (-z_1, -z_2, z_3)\ ,$$
$$\omega : \quad (z_1, z_2, z_3) \to (z_1, -z_2, -z_3)\ .$$

The $\mathrm{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifold

- We mod out this theory by orientifold action ΩR.
- Ω is world-sheet parity

$$\sigma \longrightarrow \pi - \sigma \ ,$$

- R acts as

$$R : (z_1, z_2, z_3) \to (\overline{z}_1, \overline{z}_2, \overline{z}_3) \ .$$

The $\mathrm{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifold

- There are four kinds of orientifold 6-planes (O6-planes) for the actions of ΩR, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$, respectively.
- To cancel the RR charges of O6-planes, we introduce stacks of N_a D6-branes, which wrap on the factorized three-cycles.
- There are two kinds of complex structures consistent with orientifold projection for a torus – rectangular and tilted [3].

[3] R. Blumenhagen, B. Körs and D. Lüst, JHEP **0102** (2001) 030.

The Rectangular Orientifold

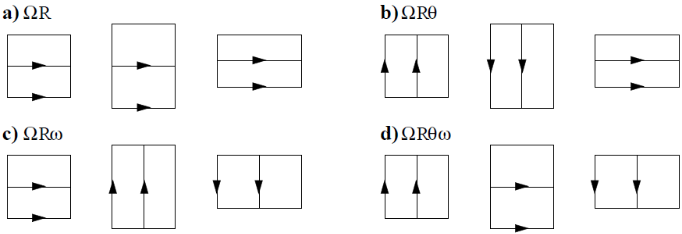

Figure: The rectangular orientifold.

The Tilted Orientifold

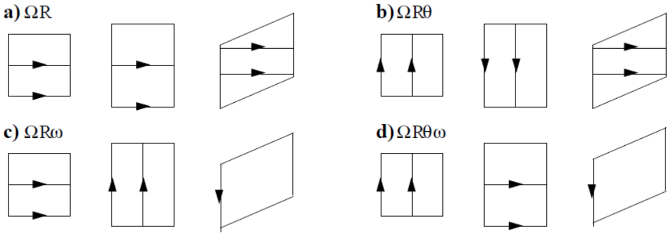

Figure: The tilted orientifold.

Homology Classes

If we denote the homology classes of the three cycles wrapped by the D6-brane stacks as $n_a^i[a_i] + m_a^i[b_i]$ and $n_a^i[a_i'] + m_a^i[b_i]$ with $[a_i'] = [a_i] + \frac{1}{2}[b_i]$ for the rectangular and tilted tori respectively, we can label a generic one cycle by (n_a^i, l_a^i) in either case, where in terms of the wrapping numbers $l_a^i \equiv m_a^i$ for a rectangular two-torus and $l_a^i \equiv 2\tilde{m}_a^i = 2m_a^i + n_a^i$ for a tilted two-torus. Note that for a tilted two-torus, $l_a^i - n_a^i$ must be even. For a stack of N_a D6-branes along the cycle (n_a^i, l_a^i), we also need to include their ΩR images $N_{a'}$ with wrapping numbers $(n_a^i, -l_a^i)$. For D6-branes on top of O6-planes, we count the D6-branes and their images independently.

Homology Three-Cycles

- The homology three-cycles for stack a of N_a D6-branes and its orientifold image a' are

$$[\Pi_a] = \prod_{i=1}^{3} \left(n_a^i [a_i] + 2^{-\beta_i} l_a^i [b_i] \right), \quad [\Pi_{a'}] = \prod_{i=1}^{3} \left(n_a^i [a_i] - 2^{-\beta_i} l_a^i [b_i] \right) .$$

Here, $\beta_i = 0$ if the i-th two-torus is rectangular and $\beta_i = 1$ if it is tilted.

- The homology three-cycles wrapped by the four O6-planes are

$$\Omega R : [\Pi_{\Omega R}] = 2^3 [a_1] \times [a_2] \times [a_3] ,$$

$$\Omega R\omega : [\Pi_{\Omega R\omega}] = -2^{3-\beta_2-\beta_3} [a_1] \times [b_2] \times [b_3] ,$$

$$\Omega R\theta\omega : [\Pi_{\Omega R\theta\omega}] = -2^{3-\beta_1-\beta_3} [b_1] \times [a_2] \times [b_3] ,$$

$$\Omega R\theta : [\Pi_{\Omega R}] = -2^{3-\beta_1-\beta_2} [b_1] \times [b_2] \times [a_3] .$$

The Intersecting Numbers

$$I_{ab} = [\Pi_a][\Pi_b] = 2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i - n_b^i l_a^i) ~,$$

$$I_{ab'} = [\Pi_a]\,[\Pi_{b'}] = -2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i + n_b^i l_a^i) ~,$$

The Intersecting Numbers

$$I_{aa'} = [\Pi_a]\,[\Pi_{a'}] = -2^{3-k}\prod_{i=1}^{3}(n_a^i l_a^i)\ ,$$

$$I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k}(-l_a^1 l_a^2 l_a^3 + l_a^1 n_a^2 n_a^3 + n_a^1 l_a^2 n_a^3 + n_a^1 n_a^2 l_a^3)\ .$$

Here, $[\Pi_{O6}] = [\Pi_{\Omega R}] + [\Pi_{\Omega R\omega}] + [\Pi_{\Omega R\theta\omega}] + [\Pi_{\Omega R\theta}]$ is the sum of O6-plane homology three-cycles wrapped by the four O6-planes, and $k = \beta_1 + \beta_2 + \beta_3$ is the total number of tilted two-tori.

The Intersecting D6-brane models

- The gauge fields are on the worldvolume of the D6-branes.
- The chiral fermions are at the intersections of D6-branes.

The General spectrum

Table: General spectrum on intersecting D6-branes at generic angles which is valid for both rectangular and tilted tori.

Sector	Representation
aa	$U(N_a/2)$ vector multiplet 3 adjoint chiral multiplets
$ab+ba$	I_{ab} $(\square_a, \overline{\square}_b)$ fermions
$ab'+b'a$	$I_{ab'}$ $(\square_a, \square_b)$ fermions
$aa'+a'a$	$\frac{1}{2}(I_{aa'}+\frac{1}{2}I_{a,O6})$ $\square\square$ fermions $\frac{1}{2}(I_{aa'}-\frac{1}{2}I_{a,O6})$ $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ fermions

Constaints on Model Building

- RR tadpole cancellation conditions The total RR charges of D6-branes and O6-planes must vanish since the RR field flux lines are conserved.

$$\sum_a N_a[\Pi_a] + \sum_a N_a\,[\Pi_{a'}] - 4[\Pi_{O6}] = 0\ .$$

- Four-dimensional $N = 1$ supersymmetric D6-brane configuration.

$$\theta_1 + \theta_2 + \theta_3 = 0 \mod 2\pi\ .$$

Here, θ_i is the angle between the $D6$-brane and the orientifold-plane in the i-th two-torus.

- K-theory anomaly free conditions: Witten anomaly cancellations.

No K-theory anomaly for the Pati-Salam model building since its gauge symmetry is $SU(4)_C \times SU(2)_L \times SU(2)_R$.

RR tadpole cancellation conditions

- We define the products of wrapping numbers

$$
\begin{array}{llll}
A_a \equiv -n_a^1 n_a^2 n_a^3, & B_a \equiv n_a^1 l_a^2 l_a^3, & C_a \equiv l_a^1 n_a^2 l_a^3, & D_a \equiv l_a^1 l_a^2 n_a^3, \\
\tilde{A}_a \equiv -l_a^1 l_a^2 l_a^3, & \tilde{B}_a \equiv l_a^1 n_a^2 n_a^3, & \tilde{C}_a \equiv n_a^1 l_a^2 n_a^3, & \tilde{D}_a \equiv n_a^1 n_a^2 l_a^3 .
\end{array}
$$

- To cancel the RR tadpoles, we can also introduce an arbitrary number of D6-branes wrapping cycles along the orientifold planes, the so called "filler branes". Thus, the tadpole conditions are

$$
-2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a =
$$

$$
-2^k N^{(3)} + \sum_a N_a C_a = -2^k N^{(4)} + \sum_a N_a D_a = -16.
$$

Here, $2N^{(i)}$ are the number of filler branes wrapping along the i-th O6-plane.

RR tadpole cancellation conditions

The tadpole cancellation conditions directly lead to the $SU(N_a)^3$ cubic non-Abelian anomaly cancellation. And the cancellation of U(1) mixed gauge and gravitational anomaly or $[SU(N_a)]^2 U(1)$ gauge anomaly can be achieved by Green-Schwarz mechanism mediated by untwisted RR fields.

Four-dimensional $N = 1$ supersymmetric D6-brane configuration

The SUSY conditions can also be written as

$$x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0 \ ,$$

$$A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0 \ .$$

Here, $x_A = \lambda, \ x_B = \lambda 2^{\beta_2+\beta_3}/\chi_2\chi_3, \ x_C = \lambda 2^{\beta_1+\beta_3}/\chi_1\chi_3, \ x_D = \lambda 2^{\beta_1+\beta_2}/\chi_1\chi_2$, and $\chi_i = R_i^2/R_i^1$ are the complex structure moduli. The positive parameter λ has been introduced to put all the variables A, B, C, D on an equal footing.

Four-dimensional $N = 1$ supersymmetric D6-brane configuration

- Filler brane with the same wrapping numbers as one of the O6-planes: among coefficients A, B, C and D, one and only one of them is non-zero and negative.
- Z-type D6-brane which contains one zero wrapping number: among A, B, C and D, two are negative and two are zero.
- NZ-type D6-brane which contains no zero wrapping number: among A, B, C and D, three are negative and the other one is positive.

Equivalent Classes in D-Brane Model Building

- The D6-brane Sign Equivalent Principle (DSEP).
- T-duality.

The D6-brane Sign Equivalent Principle (DSEP)

- Two models are equivalent if their three two-tori and the corresponding wrapping numbers for all the D6-branes are related by an element of permutation group S_3 which acts on three two-tori.
- Two D6-brane configurations are equivalent if their wrapping numbers on two arbitrary two-tori have the same magnitude but opposite sign, and their wrapping numbers on the third two-torus are the same.

T-duality

- Type I T-duality: T-duality transformation happens on two two-tori simultaneously, for example, the j-th and k-th two-tori

$$(n_x^j, l_x^j) \longrightarrow (-l_x^j, n_x^j), \quad (n_x^k, l_x^k) \longrightarrow (l_x^k, -n_x^k) \ , \quad .$$

Here, x runs over all stacks of D6-branes in the model.

- Type II T-duality: Under it, the transformations of the wrapping numbers for any stacks of D6-branes in the model are

$$n_x^i \rightarrow -n_x^i, \ \ l_x^i \rightarrow l_x^i, \quad n_x^j \leftrightarrow l_x^j, \quad n_x^k \leftrightarrow l_x^k, \quad .$$

Here, $i \neq j \neq k$, and x runs over all D6-branes in the model.

T-duality

- By combining with type I T-duality and DSEP, we obtain an variation of type II T-duality. Under it, the transformations of the wrapping numbers for any stacks of D6-branes in the model are

$$l_x^1 \to -l_x^1, \quad l_x^2 \to -l_x^2, \quad l_x^3 \to -l_x^3,$$

Here, x runs over all D6-branes in the model.

General Model Building

- No E_6 and E_8.
- No $SO(10)$.
- Georgi-Glashow $SU(5)$ [4]: no up-type quark Yukawa couplings.
- Flipped $SU(5) \times U(1)'$ [5]: no down-type quark Yukawa couplings.
- Trinification Model [6]: no lepton-type Yukawa couplings, and strong constraints from K-theory anomaly cancellation conditions.
- Standard-like model [7].
- Pati-Salam Model [8]: the best models.

[4] M. Cvetič, I. Papadimitriou and G. Shiu

[5] J. R. Ellis, P. Kanti and D. V. Nanopoulos; C. Kokorelis; C. M. Chen, G. V. Kraniotis, V. E. Mayes, D. V. Nanopoulos and J. W. Walker

[6] Ching-Ming Chen, Tianjun Li and Dimitri V. Nanopoulos

[7] Ching-Ming Chen, Tianjun Li and Dimitri V. Nanopoulos

[8] M. Cvetič, Li, Liu

Outline

The Pati-Salam models

- The gauge symmetry is $SU(4)_C \times SU(2)_L \times SU(2)_R$.
- All the SM fermions and Higgs fields form bifundamental representations

$$F_L : (\mathbf{4}, \overline{\mathbf{2}}, 1) ~;~~~ F_R : (\overline{\mathbf{4}}, 1, \mathbf{2}) ~;~~~ \Phi : (1, \mathbf{2}, \overline{\mathbf{2}}) ~.$$

- All the SM fermion Yukawa couplings can be allowed by the global $U(1)$ at the stringy tree level.

Classifications of Pati-Salam model building [9]

- Type I Pati-Salam Models: $U(4)_C \times U(2)_L \times U(2)_R$.
- Type II Pati-Salam Models: $U(4)_C \times USp(2n)_L \times USp(2m)_R$.
- Type III Pati-Salam Models: Mixed Type I and II PS, e.g, $U(4)_C \times U(2)_L \times USp(2m)_R$ or $U(4)_C \times USp(2n)_L \times U(2)_R$.
- Type II and Type III Pati-Salam Models have rank one problem for Yukawa couplings.

We consider the Type I Pati-Salam model building.

[9] Mirjam Cvetič, Tianjun Li and Tao Liu, Nucl. Phys. B **698**, 163 (2004); Mirjam Cvetič, Paul Langacker, Tianjun Li and Tao Liu Nucl. Phys. B **709**, 241 (2005).

The Pati-Salam model Model Building

- We introduce three stacks of D6-branes, a, b, and c with numbers of D6-branes 8, 4, and 4. So, a, b, and c stacks give us the gauge symmetries $U(4)_C$, $U(2)_L$ and $U(2)_R$, respectively.
- RR tadpole cancellation conditions are

$$\sum_{i=a,b,c} N_i[\Pi_i] + \sum_{j=a,b,c} N_j \left[\Pi_{j'}\right] - 4[\Pi_{O6}] \leq 0 \ .$$

- For the homology class of the i-th orientifold 6-plane, if the above sum is negative, we introduce the additional filler D6-branes wrapping along the i-th O6-plane to cancel the RR tadpole. And thus we shall have $USp(2n)$ gauge symmetry for the filler branes, which can be considered as hidden sector.

The Pati-Salam model Model Building

- In principle, the hidden gauge symmetry can be $U(n)$ gauge symmetry.
- Three observable gauge symmetries $U(4)_C$, $U(2)_L$ and $U(2)_R$ determines three complex structure moduli from four-dimensional $N = 1$ supersymmetry conditions, and then it is very difficult to have additional $U(n)$ hidden gauge symmetry.
- Therefore, in our model building, we assume that the hidden gauge symmtries are $USp(2n)$, and prove that there are only 33 independent models.
- After we complete the searches, we shall discuss the hidden $U(n)$ gauge symmetry to complete the model building, and find three new models.

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The Pati-Salam model Model Building

- we require that the intersection numbers satisfy

$$I_{ab} + I_{ab'} \ = \ 3 \ ,$$

$$I_{ac} \ = \ -3 \ , \ I_{ac'} \ = \ 0 \ .$$

- The intersection numbers $I_{ac} = 0$ and $I_{ac'} = -3$ are equivalent to $I_{ac} = -3$ and $I_{ac'} = 0$ due to the symmetry transformation $c \leftrightarrow c'$.
- The conditions $I_{ab} + I_{ab'} = 3$ and $I_{ac} = -3$ give us three families of the SM fermions with quantum numbers $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ under $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry.

The Pati-Salam model Model Building

- The condition $I_{ac'} = 0$ gives us the vector-pairs of the chiral multiplets with quantum numbers $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ from $N = 2$ subsector. These particles are the Higgs fields which are needed to break the Pati-Salam gauge symmetry down to the SM gauge symmetry.

The Pati-Salam model Model Building

- $I_{ac'} = 0$ implies that *a* stack of D6-branes is parallel to the orientifold (ΩR) image c' of the c stack of D6-branes along at least one tow-torus, for example, the third two-torus. Then, there are open strings which stretch between the a and c' stacks of D6-branes. If the minimal distance squared $Z^2_{(ac')}$ (in $1/M_s$ units) between these two stacks of D6-branes on the third two-torus is small, *i.e.*, the minimal length squared of the stretched string is small, we have the light scalars with squared-masses $Z^2_{(ab')}/(4\pi^2\alpha')$ from the NS sector, and the light fermions with the same masses from the R sector. These scalars and fermions form the 4-dimensional $N = 2$ hypermultiplets, so, we obtain the $I^{(2)}_{ac'}$ (the intersection numbers for a and c' stacks on the first two two-tori) vector-pairs of the chiral multiplets with quantum numbers $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$. These particles are the Higgs fields needed to break the Pati-Salam gauge symmetry down to the SM gauge symmetry.

The Gauge Symmetry Breaking

- The a, b, and c stacks give us the gauge symmetry $U(4)_C$, $U(2)_L$ and $U(2)_R$, respectively. The anomalies from three $U(1)$s are cancelled by the generalized Green-Schwarz mechanism, and the gauge fields of these $U(1)$s obtain masses via the linear $B \wedge F$ couplings. So, the effective gauge symmetry is $SU(4)_C \times SU(2)_L \times SU(2)_R$.
- To break the gauge symmetry, we split the a stack of D6-branes into a_1 and a_2 stacks with 6 and 2 D6-branes, respectively. And then the $SU(4)_C$ gauge symmetry is broken down to the $SU(3)_C \times U(1)_{B-L}$ gauge symmetry.

The Gauge Symmetry Breaking

- We split the c stack of D6-branes into c_1 and c_2 stacks with 2 D6-branes for each one. And then the $SU(2)_R$ gauge symmetry is broken down to the $U(1)_{I_{3R}}$ gauge symmetry.
- After D6-brane splittings, we obtain that the gauge symmetry is $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$.
- We obtain the vector-pairs of the chiral multiplets with quantum numbers $(\mathbf{1}, \mathbf{1}, -\mathbf{1}, \mathbf{1/2})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{1}, -\mathbf{1/2})$ under $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ from the a_2 and c'_1 intersections or $N = 2$ subsector. These particles can break the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ down to the SM gauge symmetry and keep the D- and F-flatness.

The Gauge Symmetry Breaking

The complete symmetry breaking chains are

$$
\begin{aligned}
& SU(4) \times SU(2)_L \times SU(2)_R \\
& \xrightarrow{a \to a_1 + a_2} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
& \xrightarrow{c \to c_1 + c_2} SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L} \\
& \xrightarrow{\text{Higgs Mechanism}} SU(3)_C \times SU(2)_L \times U(1)_Y \ .
\end{aligned}
$$

The Gauge Kinetic Functions

- The complex structure moduli U^i in the string theory basis are

$$U^i = \frac{iR_2^i}{R_1^i + \frac{\beta_i}{2} iR_2^i} = \frac{i\chi^i}{1 + \frac{\beta_i}{2} i\chi^i} , \qquad \text{where } \chi^i \equiv \frac{R_2^i}{R_1^i} .$$

- The dilaton s and complex structure moduli u^i in the supergravity theory basis are

$$\mathrm{Re}\,(s) = \frac{e^{-\phi_4}}{2\pi} \left(\frac{\sqrt{\mathrm{Im}\, U^1\, \mathrm{Im}\, U^2\, \mathrm{Im}\, U^3}}{|U^1 U^2 U^3|} \right) ,$$

$$\mathrm{Re}\,(u^j) = \frac{e^{-\phi_4}}{2\pi} \left(\sqrt{\frac{\mathrm{Im}\, U^j}{\mathrm{Im}\, U^k\, \mathrm{Im}\, U^l}} \right) \left| \frac{U^k\, U^l}{U^j} \right| , \quad (j,k,l) = (\overline{1,2,3}) .$$

where j denotes the j^{th} two-torus.

The Gauge Kinetic Functions

- ϕ_4 is the four dimensional dilaton which is related to the supergravity moduli as

$$2\pi e^{\phi_4} = \Big(\mathrm{Re}(s)\,\mathrm{Re}(u_1)\,\mathrm{Re}(u_2)\,\mathrm{Re}(u_3)\Big)^{-1/4} \ .$$

- Inverting the above formulas we can solve for U moduli in string theory basis in terms of s and u as

$$\frac{|U^j|^2}{\mathrm{Im}\,(U^j)} = \sqrt{\frac{\mathrm{Re}\,(u^k)\,\mathrm{Re}\,(u^l)}{\mathrm{Re}\,(u^j)\mathrm{Re}\,(s)}}, \quad (j,k,l) = (\overline{1,2,3}) \ .$$

The Gauge Kinetic Functions

- The holomorphic gauge kinetic function for any D6-brane stack x wrapping a calibrated 3-cycle is

$$f_x = \frac{1}{2\pi\ell_s^3}\left[e^{-\phi}\int_{\Pi_x}\mathrm{Re}(e^{-i\theta_x}\Omega_3) - i\int_{\Pi_x} C_3\right] ,$$

where the integral involving 3-form Ω_3 is

$$\int_{\Pi_x}\Omega_3 = \frac{1}{4}\prod_{i=1}^{3}(n_x^i R_1^i + 2^{-\beta_i} i l_x^i R_2^i) .$$

- The gauge kinetic function is

$$f_x = \frac{1}{4\kappa_x}(n_x^1\, n_x^2\, n_x^3\, s - \frac{n_x^1\, l_x^2\, l_x^3\, u^1}{2^{(\beta_2+\beta_3)}} - \frac{l_x^1\, n_x^2\, l_x^3\, u^2}{2^{(\beta_1+\beta_3)}} - \frac{l_x^1\, l_x^2\, n_x^3\, u^3}{2^{(\beta_1+\beta_2)}}) .$$

Here, $\kappa_x = 1$ for $\mathrm{U}(N_x)$ and $\kappa_x = 2$ for $USp(2N_x)$ or $SO(2N_x)$.

The Gauge Kinetic Functions

- The holomorphic gauge kinetic function for the SM hypercharge $\mathrm{U}(1)_Y$

$$f_1 = \frac{5}{3} f_Y = \frac{2}{3} f_a + f_c \ .$$

$$f_1 = \frac{5}{3} f_Y = \frac{1}{6} f_{a_1} + \frac{1}{2} f_{a_2} + \frac{1}{2} f_{c_1} + \frac{1}{2} f_{c_2} \ .$$

The Conditions for the Pati-Salam Model Building

- RR tadpole cancellation conditions.
- Four-dimensional $N = 1$ supersymmetric D6-brane configurations.
- Intersecting number conditions, or three generation conditions and Pati-Salam gauge symmetry breaking conditions.

The Pati-Salam model building is a pure mathematical problem.

Search Method for Supersymmetric Pati-Salam Models

- From a mathematical point of view, the search for wrapping numbers satisfying all the conditions is equivalent to solving a Diophantine equation.
- In general, there does not exist any algorithm to solve such equation, as implied by the negative solution of Hilbert's tenth problem.
- We devise an algorithm specially adapted to the form of our equations, and find all the common solutions for the RR tadpole cancellation conditions, four-dimensional $\mathcal{N} = 1$ supersymmetry conditions, and intersecting number conditions with this deterministic algorithm [10].

[10] W. He, T. Li and R. Sun, JHEP **08**, 044 (2022) [arXiv:2112.09632 [hep-th]].

Hilbert's tenth problem

- Given a Diophantine equation with any number of unknown quantities and with rational integral numerical coefficients: To devise a process according to which it can be determined in a finite number of operations whether the equation is solvable in rational integers.
- Provide a general algorithm which, for any given Diophantine equation (a polynomial equation with integer coefficients and a finite number of unknowns), can decide whether the equation has a solution with all unknowns taking integer values.

Hilbert's tenth problem has been solved, and it has a negative answer: such a general algorithm cannot exist.

Deterministic Algorithm

First step: we enumerate all the possible combinations of the signs of the twelve wrapping number products $A_a, B_a, \ldots, C_c, D_c$. Because of the SUSY condition, (A_a, B_a, C_a, D_a) contains three negative numbers and one positive number, or two negative numbers and two zeroes, or with one negative and three zeros. The same applies to the stack b and the stack c.

Deterministic Algorithm

Second step: for each possibility listed in the first step, we append the twelve corresponding inequalities to the our system and try to solve the new system. Inside the system, we look for the following kinds of equations or subsystems

1. Equation with only one non constant monome.
2. A system of linear equations of full rank.
3. A system of linear inequalities which has finitely many integer solutions.

Deterministic Algorithm

When we find inside our system equations or subsystems of these kinds, we can solve them with respect to the variables appearing in them. After the resolution, the total number of unknown variables decreases. This might lead to branching into subcases because the solution might not be unique. But there are only **finitely** many subcases because we specifically looked for subsystems admitting finite integer solution sets. Then, we repeat the procedure for each subcase.

Deterministic Algorithm

Third step: we repeat the procedure described in the second step until there are no subsystem of the three kinds. At this stage, each subcase either gives a solution or still has some unsolved varialbles (wrapping numbers). Now we make use of inequalities of degree larger than 1, such as those given by four-dimensional $N = 1$ supersymmetric D6-brane configurations. By enumerating all the possible signs of the remaining variables, we can determine whether four-dimensional $N = 1$ supersymmetric D6-brane configurations can be achieved.

The Final Pati-Salam Model Building

- Implementing this algorithm with the help of a computer program, we find that in total there are $202,752$ models.
- Because each equivalent class has 6,144 models, we obtain that there are only 33 independent models with different gauge coupling relations at string scale after moduloing the equivalent relations. There is one and only one model with string-scale gauge coupling unification.
- With hidden sector $U(n)$ gauge symmetry, we find three more models. Thus, the total number of the independent model is 36.
- For the string landscape study, it is very important to consider the equivalent relations.

The Pati-Salam Model: Model 1

Table: D6-brane configurations and intersection numbers of Model 1, and its gauge coupling relation is $g_a^2 = g_b^2 = g_c^2 = (\frac{5}{3}g_Y^2) = 4\sqrt{\frac{2}{3}}\pi e^{\phi^4}$.

Model 1	$U(4) \times U(2)_L \times U(2)_R \times USp(2)^4$											
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	n_S	n_A	b	b'	c	c'	1	2	3	4
a	8	$(1, 1) \times (1, 0) \times (1, -1)$	0	0	3	0	0	-3	0	1	0	-1
b	4	$(-1, 0) \times (-1, 3) \times (1, 1)$	2	-2	-	-	0	0	0	0	-3	1
c	4	$(0, 1) \times (-1, 3) \times (-1, 1)$	2	-2	-	-	-	-	-3	1	0	0
1	2	$(1, 0) \times (1, 0) \times (2, 0)$	$x_A = \frac{1}{3}x_B = x_C = \frac{1}{3}x_D$									
2	2	$(1, 0) \times (0, -1) \times (0, 2)$	$\beta_1^g = \beta_2^g = \beta_3^g = \beta_4^g = -3$									
3	2	$(0, -1) \times (1, 0) \times (0, 2)$	$\chi_1 = 1,\ \chi_2 = \frac{1}{3},\ \chi_3 = 2$									
4	2	$(0, -1) \times (0, 1) \times (2, 0)$										

The Pati-Salam Model: Model 2

Table: D6-brane configurations and intersection numbers of Model 2, and its gauge coupling relation is
$g_a^2 = \frac{7}{6}g_b^2 = \frac{5}{6}g_c^2 = \frac{25}{28}(\frac{5}{3}g_Y^2) = \frac{8}{27}5^{3/4}\sqrt{7}\pi e^{\phi^4}$.

Model 2	$U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$									
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	n_S	n_A	b	b'	c	c'	1	4
a	8	$(1, -1) \times (-1, 1) \times (1, -1)$	0	4	0	3	0	-3	-1	1
b	4	$(0, 1) \times (-2, 1) \times (-1, 1)$	-1	1	-	-	0	-1	-1	0
c	4	$(-1, 0) \times (5, 2) \times (-1, 1)$	3	-3	-	-	-	-	0	-5
1	2	$(1, 0) \times (1, 0) \times (2, 0)$	$x_A = 2x_B = \frac{14}{5}x_C = 7x_D$							
4	2	$(0, -1) \times (0, 1) \times (2, 0)$	$\beta_1^g = -3$, $\beta_4^g = 1$							
			$\chi_1 = \frac{7}{\sqrt{5}}$, $\chi_2 = \sqrt{5}$, $\chi_3 = \frac{4}{\sqrt{5}}$							

The Pati-Salam Model: Model 3

Table: D6-brane configurations and intersection numbers of Model 3, and its gauge coupling relation is
$g_a^2 = \frac{5}{6} g_b^2 = \frac{7}{6} g_c^2 = \frac{35}{32}\left(\frac{5}{3} g_Y^2\right) = \frac{8}{27} 5^{3/4} \sqrt{7} \pi e^{\phi^4}$.

Model 3	$U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$										
stack	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	n_S	n_A	b	b'	c	c'	1	4
a	8	$(-1, -1) \times (1, 1) \times (1, 1)$	0	-4	0	3	0	-3	1	-1
b	4	$(-5, 2) \times (-1, 0) \times (1, 1)$	-3	3	-	-	0	1	0	5
c	4	$(-2, -1) \times (0, 1) \times (1, 1)$	1	-1	-	-	-	-	1	0
1	2	$(1, 0) \times (1, 0) \times (2, 0)$	$x_A = \frac{14}{5} x_B = 2x_C = 7x_D$							
4	2	$(0, -1) \times (0, 1) \times (2, 0)$	$\beta_1^g = -3,\ \beta_4^g = 1$							
			$\chi_1 = \sqrt{5},\ \chi_2 = \frac{7}{\sqrt{5}},\ \chi_3 = \frac{4}{\sqrt{5}}$							

Decoupling of the Exotic Particles

- In general, we can decouple all the exotic particles via Higgs mechanism and instanton effects, etc, except the chiral multiplets under $SU(4)_C$ symmetric representation. The key point is the gauge anomaly cancellation.
- We can decouple the chiral multiplets under $SU(4)_C$ fundamental or anti-fundamental representations by Higgs mechanism.
- The chiral multiplets under $SU(4)_C$ anti-symmetric representation do not contribute to the gauge anomaly. Their mass terms are forbidden by the anomalous $U(1)$ gauge symmetries, and can be generated via the instanton effects. Thus, they can be decoupled.

Decoupling of the Exotic Particles

- For the models with the chiral multiplets under $SU(4)_C$ symmetric representation, it seems to us that we cannot decouple the exotic particles. Otherwise, we might break the $U(1)_{EM}$ gauge symmetry.
- Therefore, we cannot decouple the exotic particles only in Model 4, 23, and 32 in our 33 models.

String-scale Gauge Coupling Relation

- The generic gauge coupling relations at string scale for intersecting D6-brane models can be written as $g_a^2 = k_2\, g_b^2 = k_Y\, g_Y^2 = g_U^2$, where k_Y and k_2 are constants for each model.
- For simplicity, we can call these gauge coupling relations as gauge coupling unification $\alpha_1 = \alpha_2 = \alpha_3$ by redefining $\alpha_1 \equiv k_Y g_Y^2/4\pi$, $\alpha_2 \equiv k_2 g_b^2/4\pi$, and $\alpha_3 \equiv g_a^2/4\pi$.
- To define the unification scale, we choose the evolution under the conditions $\alpha_U^{-1} \equiv \alpha_1^{-1} = (\alpha_2^{-1} + \alpha_3^{-1})/2$ and $\Delta = |\alpha_1^{-1} - \alpha_2^{-1}|/\alpha_1^{-1}$. In our study, the difference between α_U^{-1} and α_2^{-1} or α_3^{-1} is limited to be less than 1.0%.
- The string-scale gauge coupling relations can be realized by introducing vector-like particles from $N = 2$ subsector.

String-scale Gauge Coupling Relation

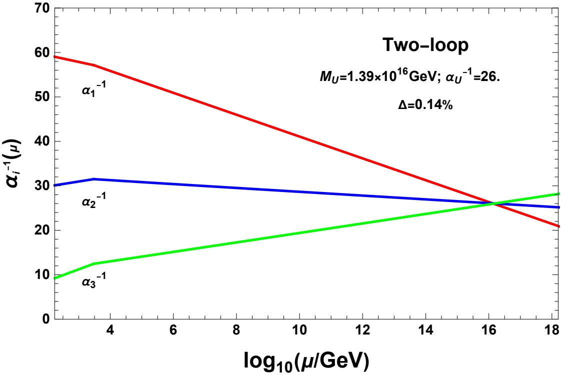


Figure: Gauge coupling unification at 1.39×10^{16} GeV in Model 1 by employing the current precision electroweak data and setting $M_{\rm SUSY} = 3$ TeV.

String-scale Gauge Coupling Relation

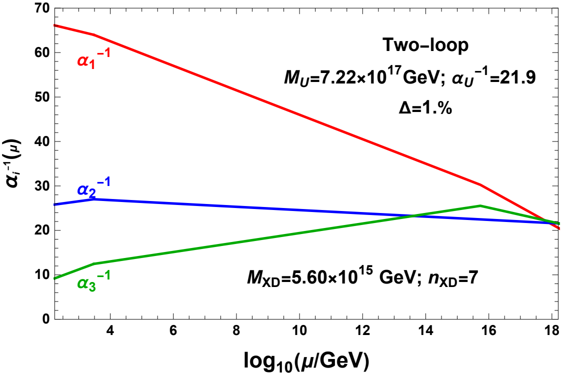


Figure: Gauge coupling unification at 7.22×10^{17} GeV in Model 2 by introducing seven pairs of $(XD, \overline{XD})$ from $N = 2$ subsector.

String-scale Gauge Coupling Relation

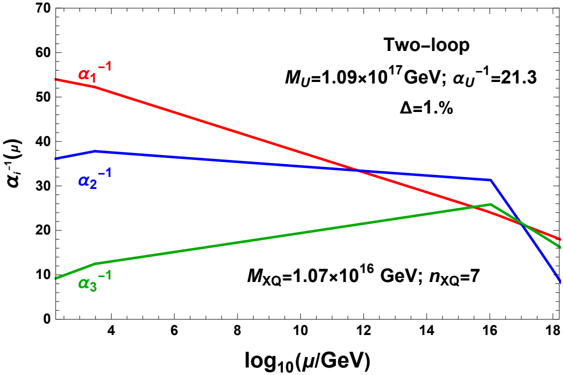


Figure: Gauge coupling unification at 1.09×10^{17} GeV in Model 3 by introducing seven pairs of $(XQ, \overline{XQ})$ from $N = 2$ subsector.

The Phenomenological Consequences of the Pati-Salam Models

- The SM fermion masses and mixings can be explained via three-point and four-point functions.
- We can calculate the gauge kinetic functions and Kähler potential, as well as supersymmetry breaking soft terms, and then study the low energy supersymmetry phenomenology.

Outline

A Realistic Pati-Salam Model

Table: D6-brane configurations and intersection numbers.

	$U(4)_C \times U(2)_L \times U(2)_R \times USp(2)^4$											
	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	n_S	n_A	b	b'	c	c'	1	2	3	4
a	8	$(0, -1) \times (1, 1) \times (1, 1)$	0	0	3	0	-3	0	1	-1	0	0
b	4	$(3, 1) \times (1, 0) \times (1, -1)$	2	-2	-	-	0	0	0	1	0	-3
c	4	$(3, -1) \times (0, 1) \times (1, -1)$	-2	2	-	-	-	-	-1	0	3	0
1	2	$(1, 0) \times (1, 0) \times (2, 0)$	$\chi_1 = 3, \ \chi_2 = 1, \ \chi_3 = 2$									
2	2	$(1, 0) \times (0, -1) \times (0, 2)$	$\beta_1^g = -3, \ \beta_2^g = -3$									
3	2	$(0, -1) \times (1, 0) \times (0, 2)$	$\beta_3^g = -3, \ \beta_4^g = -3$									
4	2	$(0, -1) \times (0, 1) \times (2, 0)$										

Table: The chiral and vector-like superfields in the observable sector, and their quantum numbers under the gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R \times USp(2)_1 \times USp(2)_2 \times USp(2)_3 \times USp(2)_4$.

	Quantum Number	Q_4	Q_{2L}	Q_{2R}	Field
ab	$3 \times (4, \overline{2}, 1, 1, 1, 1, 1)$	1	-1	0	$F_L(Q_L, L_L)$
ac	$3 \times (\overline{4}, 1, 2, 1, 1, 1, 1)$	-1	0	1	$F_R(Q_R, L_R)$
ac'	$3 \times (4, 1, 2, 1, 1, 1, 1)$	1	0	1	Φ_i
	$3 \times (\overline{4}, 1, \overline{2}, 1, 1, 1, 1)$	-1	0	-1	$\overline{\Phi}_i$
bc	$6 \times (1, 2, \overline{2}, 1, 1, 1, 1)$	0	1	-1	H_u^i, H_d^i
	$6 \times (1, \overline{2}, 2, 1, 1, 1, 1)$	0	-1	1	

Table: The additional exotic particles.

	Quantum Number	Q_4	Q_{2L}	Q_{2R}	Field
$a1$	$1 \times (4, 1, 1, 2, 1, 1, 1)$	1	0	0	X_{a1}
$a2$	$1 \times (\overline{4}, 1, 1, 1, 2, 1, 1)$	-1	0	0	X_{a2}
$b2$	$1 \times (1, 2, 1, 1, 2, 1, 1)$	0	1	0	X_{b2}
$b4$	$3 \times (1, \overline{2}, 1, 1, 1, 1, 2)$	0	-1	0	X_{b4}^i
$c1$	$1 \times (1, 1, \overline{2}, 2, 1, 1, 1)$	0	0	-1	X_{c1}
$c3$	$3 \times (1, 1, 2, 1, 1, 2, 1)$	0	0	1	X_{c3}^i
b_S	$2 \times (1, 3, 1, 1, 1, 1, 1)$	0	2	0	T_L^i
b_A	$2 \times (1, \overline{1}, 1, 1, 1, 1, 1)$	0	-2	0	S_L^i
c_S	$2 \times (1, 1, \overline{3}, 1, 1, 1, 1)$	0	0	-2	T_R^i
c_A	$2 \times (1, 1, 1, 1, 1, 1, 1)$	0	0	2	S_R^i

Gauge Symmetry Breaking

- The anomalies from three global $U(1)$s of $U(4)_C$, $U(2)_L$ and $U(2)_R$ are cancelled by the Green-Schwarz mechanism, and the gauge fields of these $U(1)$s obtain masses via the linear $B \wedge F$ couplings.
- On the first torus, we split the a stack of D6-branes into a_1 and a_2 stacks with 6 and 2 D6-branes, respectively, and split the c stack of D6-branes into c_1 and c_2 stacks with two D6-branes for each one, we break the gauge symmetry further down to $SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$.

Gauge Symmetry Breaking

- The $U(1)_{I_{3R}} \times U(1)_{B-L}$ gauge symmetry may be broken to $U(1)_Y$ by giving vacuum expectation values to the vector-like particles with the quantum numbers $(\mathbf{1}, \mathbf{1}, \mathbf{1/2}, \mathbf{-1})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{-1/2}, \mathbf{1})$ under the $SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ gauge symmetry from $a_2 c_1'$ intersections.

The Pati-Salam gauge symmetry can be broken down to the SM gauge symmetry close to the string scale.

The Gauge Symmetry Breaking

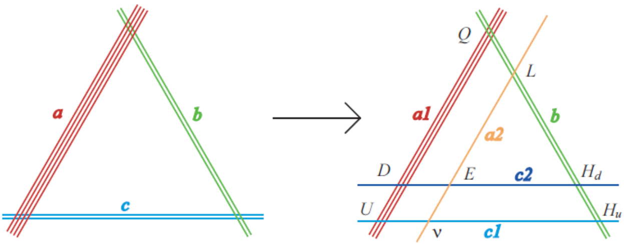


Figure: Breaking of the effective gauge symmetry $SU(4) \times SU(2)_L \times SU(2)_R$ down to $SU(3)_C \times U(2)_L \times U(1)_{I3R} \times U(1)_{B-L}$ via brane splitting.

The Additional Particles

- X_{a1}, X_{a2}, X_{c1}, X^i_{c3}, T^i_R, and S^i_R can have masses close to the string scale.
- X_{b2}, X^i_{b4}, T^i_L, and S^i_L can have masses about 5×10^{12} GeV.
- With some fine-tuning, we can have one pair of the light Higgs doublets.
- We can generate the correct $\mu H_d H_u$ term and the right-handed neutrino masses. And then the active neutrino masses and mixings can be explained via the seesaw mechanism.

Gauge Coupling Unification

$$\frac{1}{g_a^2} = \frac{1}{4}\left(n_a^1\, n_a^2\, n_a^3\, s - n_a^1\, l_a^2\, l_a^3\, u^1 - n_a^2\, l_a^1\, l_a^3\, u^2 - n_a^3\, l_a^1\, l_a^2\, u^3\right) .$$

$$\mathrm{Re}\,(s) = \frac{e^{-\phi_4}}{2\pi\sqrt{\chi_1\chi_2\chi_3}} \quad , \qquad \mathrm{Re}\,(u^i) = \frac{e^{-\phi_4}\sqrt{\chi_j\chi_k}}{2\pi\sqrt{\chi_j}} .$$

$$g_3^2 = g_{2L}^2 = \frac{5}{3}g_Y^2 = \left[\frac{e^{-\phi_4}}{2\pi}\frac{\sqrt{6}}{4}\right]^{-1} .$$

The gauge couplings are automatically unified at the string scale.

The Supersymmetry Breaking Soft Terms

- We assume that supersymmetry is broken by the F-terms of s and u_i fields: $\sum_{i=1}^{3} \Theta_i^2 + \Theta_s^2 = 1$.

$$F^{u^i,s} = \sqrt{3} m_{3/2} [(s + \bar{s}) \Theta_s e^{-i\gamma_s} + (u^i + \bar{u}^i) \Theta_i e^{-i\gamma_i}] \ .$$

- The gaugino masses are

$$M_a = \frac{-\sqrt{3} m_{3/2}}{\mathrm{Re} f_a} \left[\left(\sum_{j=1}^{3} \mathrm{Re}(u^j) \, \Theta_j \, e^{-i\gamma_j} \, n_a^j m_a^k m_a^l \right) + \Theta_s \mathrm{Re}(s) e^{-i\gamma_0} n_a^1 \, n_a^2 \, n_a^3 \right] \ .$$

The Supersymmetry Breaking Soft Terms

- The Higgs scalar mass-squared are

$$m_H^2 = m_{3/2}^2 \left[1 - \frac{3}{2} (|\Theta_3|^2 + |\Theta_s|^2) \right] ~.$$

- The supersymmetry breaking soft masses for squarks, sleptons, and sneutrinos, and the trilinear soft terms are very complicated.

The Supersymmetry Breaking Soft Terms

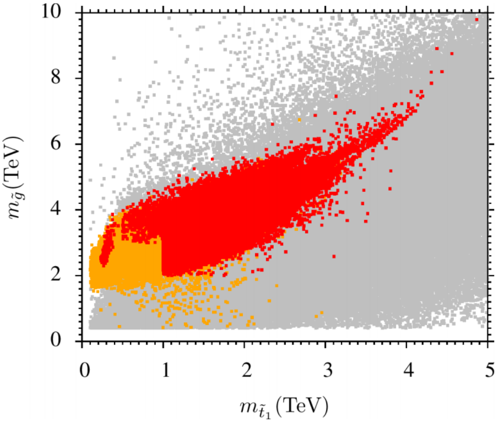
$$
\begin{aligned}
M_1 &= (0.519\Theta_1 + 0.346\Theta_2 + 0.866\Theta_3) \times m_{3/2} ~, \\
M_2 &= (0.866\Theta_2 - 0.866\Theta_4) \times m_{3/2} ~, \\
M_3 &= (0.866\Theta_2 + 0.866\Theta_3) \times m_{3/2} ~, \\
A_0 &= (-1.111\Theta_1 - 0.621\Theta_2 + 0.245\Theta_3 - 0.245\Theta_4) \times m_{3/2} ~, \\
\widetilde{m}_L &= \sqrt{1.0 + 0.899\Theta_1^2 - 0.518\Theta_2^2 - 0.849\Theta_3^2 - 1.418\Theta_4^2 - 0.557\Theta_1\Theta_2 - 0.557\Theta_3\Theta_4} \times m_{3/2} ~, \\
\widetilde{m}_R &= \sqrt{1.0 - 1.418\Theta_1^2 - 0.849\Theta_2^2 - 0.518\Theta_3^2 + 0.899\Theta_4^2 - 0.557\Theta_1\Theta_2 - 0.557\Theta_3\Theta_4} \times m_{3/2} ~, \\
\widetilde{m}_{H_u} &= \widetilde{m}_{H_d} = \sqrt{1.0 - 1.5\Theta_3^2 - 1.5\Theta_4^2} \times m_{3/2} ~, \qquad (5)
\end{aligned}
$$

Figure: The Supersymmetry Breaking Soft Terms with numerical parametrization.

The Properties

- No gaugino mass unification: the gluinos, or the winos, or the bino can be the lightest.
- All the left-handed squarks and sleptons/sneutrinos have the same masses.
- All the right-handed squarks and sleptons/sneutrinos have the same masses.

The Viable Parameter Space



The Viable Parameter Space

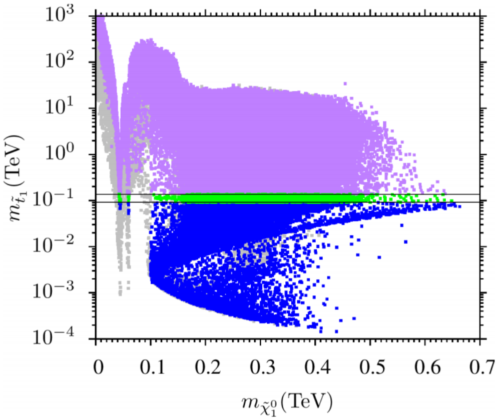


Figure:

The Viable Parameter Space

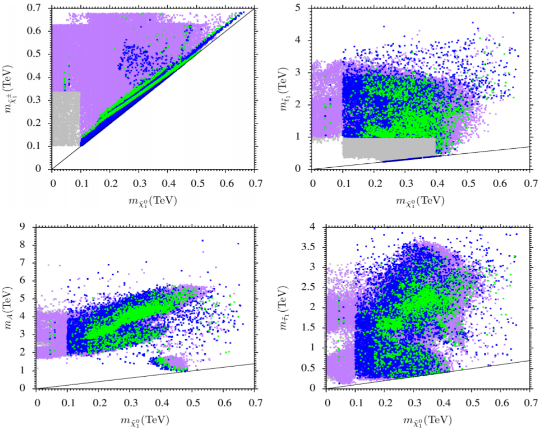


Figure:

The Viable Parameter Space

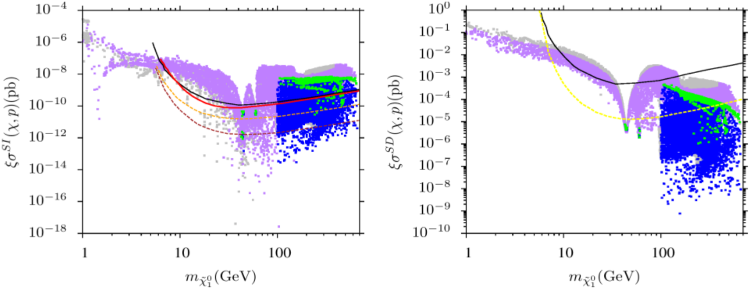


Figure:

The Benchmark Points

	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7
m_L	1805	2048.4	2071.4	2167.7	5517	1559.4	4141.3
m_R	1839.7	1793	3038.5	1974.9	3383.8	1463.4	2356.7
M_1	72.02	105.01	-1065.2	267.71	-1608.6	-635.52	-607.34
M_2	-3090.8	-3057	-4888.4	-3158.2	3038.5	-3237.7	3142.1
M_3	-1473.2	-1566.7	-1479.1	-1489.1	1887.7	-1185.6	2305.1
A_0	1045.3	959.36	3753.5	658.41	-931.66	2692.1	-744.84
$\tan\beta$	17.3	18.8	54.6	18.3	44.7	12.8	46.5
$m_{H_u} = m_{H_d}$	2397.4	2551.8	2886.1	2519.8	4682.9	2321.6	4137.2
μ	384	425	638	158	199	375	88
$\Delta_{\rm EW}$	**37**	**43**	**99**	**32**	**14**	**63**	88
m_h	122	122	125	122	122	126	123
m_H	2972	3048	1064	3053	3115	3042	2749
m_A	2953	3028	**1057**	3094	3196	3022	2730
$m_{H^\pm}$	2973	3049	1069	3054	3116	3043	2750
$m_{\tilde\chi^0_{1,2}}$	**45**, 396	**61**, 438	**472**, 653	**117**, 166	**205** ,206	**270**, 387	**294**, 620
$m_{\tilde\chi^0_{3,4}}$	400, 2546	441, 2524	656, 4042	186, 2606	753, 2564	390, 2688	624, 2638
$m_{\tilde\chi^\pm_{1,2}}$	375, 2515	414, 2491	621, 4022	**154**, 2572	**213**, 2531	372, 2684	637, 2603
$m_{\tilde g}$	3204	3390	3255	3248	4128	2627	4866
$m_{\tilde u_{L,R}}$	3743, 3254	3965, 3358	4509, 4052	3965, 3351	6411, 4765	3344 ,2639	6085, 4683
$m_{\tilde t_{1,2}}$	1953, 3260	1999, 3463	1970, 3183	1027, 2978	2275, 5308	**272**, 2774	2657,5115
$m_{\tilde d_{L,R}}$	3744, 3257	3966, 3361	4510, 4051	3966, 3354	64112, 4753	3345, 2639	6085, 4682
$m_{\tilde b_{1,2}}$	3156, 3270	3241, 3476	2695, 3193	3244, 3486	3788, 5354	2536, 2818	3785, 5145
$m_{\tilde\nu_{1,2}}$	2656	2810	3711	2944	5519	2576	4578
$m_{\tilde\nu_3}$	2625	2773	3213	2911	5170	2565	4269
$m_{\tilde e_{L,R}}$	2657, 1838	2811, 1790	3710, 3064	2944, 1975	5514, 3425	2573, 1479	4576,2356
$m_{\tilde\tau_{1,2}}$	1746, 2626	1673, 2774	1493, 3200	1870, 2910	2059, 5164	1386, 2555	**299**,4265
$\sigma_{\rm SI}$(pb)	4.44×10^{-11}	4.20×10^{-11}	1.48×10^{-10}	5.38×10^{-9}	$\mathbf{1.05\times10^{-11}}$	$\mathbf{7.42\times10^{-10}}$	$\mathbf{7.44\times10^{-11}}$
$\sigma_{\rm SD}$(pb)	5.53×10^{-6}	3.79×10^{-6}	3.45×10^{-6}	3.48×10^{-4}	3.49×10^{-7}	$\mathbf{2.17\times10^{-5}}$	$\mathbf{1.55\times10^{-6}}$
$\Omega_\chi h^2$	0.104	0.110	0.101	0.129	**0.007**	**0.002**	**0.128**

The Properties

- The Pati-Salam model from intersecting D-branes can not only satisfy all the current experimental constraints but also natural with $\Delta < 100$.
- The viable parameter space is tightly constrained by the requirements of naturalness and consistency with the observed dark matter relic density, so that it is fully testable at current and future dark matter searches, unless a non-thermal production mechanism of dark matter is at work.
- The Z-resonance, h-resonance, A-funnel, Higgsino LSP, and light stau/stop-neutralino coannihilation solutions are consistent with current LHC and dark matter constraints while the "well-tempered" neutralino scenario is ruled out.
- Prediction: only Bino, Higgsinos, right-handed staus, and stops can have mass below 1 TeV.

The SM Fermion Masses and Mixing:

- The Yukawa couplings at the string scale are

$$Y_{ijk} = h_{qu} \sum_{l \in Z} \exp(-\frac{A_{ijk}(l)}{2\pi\alpha'}) \ .$$

- The SM fermion Yukawa couplings and CKM mixing matrix at the GUT scale can be obtained via renormalization group equation running.
- The goal: try to obtain the SM fermion Yukawa couplings and CKM mixing matrix at the GUT scale.

The Brane Configuration

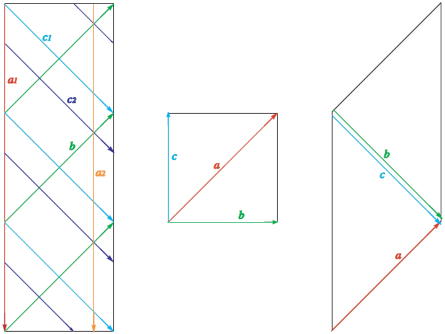


Figure: Brane configuration for the three two-tori. The SM fermion mass hierarchies primarily result from the intersections on the first torus.

Results

- The SM quark masses and the quark CKM mixing matrix can be obtained.
- The correct τ lepton mass can be obtained. However, the electron mass is about $5 \sim 6.5$ times larger than the expected value, while the muon mass is about $50 \sim 60\%$ too small. Similar to GUTs, we end up with roughly the wrong fermion mass relation $m_e/m_\mu \cong m_d/m_s$.
- The correct electron and muon masses can be generated via dimension-5 operators (four-point function in string theory) [11].

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[11] C. M. Chen, T. Li, V. E. Mayes and D. V. Nanopoulos, Phys. Rev. D **78**, 105015 (2008).

Outline

Summary

- We provide a systematic construction of three-family $N = 1$ supersymmetric Pati-Salam models from Type IIA string theory on $\mathrm{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds with intersecting D6-branes.
- All the gauge symmetries $SU(4)_C \times SU(2)_L \times SU(2)_R$ arise from the stacks of D6-branes with $U(n)$ gauge symmetries, while the hidden sector is specified by $USp(n)$ D6-branes.
- The Pati-Salam gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings, and further down to the SM via the D- and F-flatness preserving Higgs mechanism from massless open string states in a $N = 2$ subsector.

Summary

- We propose a deterministic algorithm to construct all the possible three-family N=1 supersymmetric Pati-Salam models by solving all the common solutions for the RR tadpole cancellation conditions, N=1 supersymmetry conditions, and three generation conditions with it.
- We show that there are only 33 independent models with different gauge coupling relations at string scale after moduloing out equivalent relations. In particular, there is one and only one independent model which has gauge coupling unification.
- We can decouple the exotic particles in these models, realize the gauge coupling relation at string scale, explain the SM fermion masses and mixings, calculate the supersymmetry breaking soft terms, and study the low energy supersymmetry

Thank You Very Much for Your Attention!