

INTEGRABILITY-CHAOS TRANSITION OF THE ADS_3 TENSIONLESS STRINGS

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Based on: 2605.06465, with Matthias Gaberdiel and Beat Nairz

Background

Integrability, Chaos, Holography

- Holographic principle conjectures, loosely speaking,

semi-classical gravity $\Leftrightarrow$ large- N QFT

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 - Semi-classical gravity : BH is inevitable in AdS, which is **chaotic**

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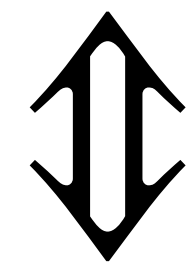
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 - large- N field theories : most known examples are **integrable** in the large- N limit
 - Semi-classical gravity : BH is inevitable in AdS, which is **chaotic**
- But **Integrability $\Rightarrow$ Chaos !?**
- More conclusive microscopical understanding in **proved/well understood examples** of holography

An example of explicitly derived holography

Tensionless string on $\text{AdS}_3 \times S^3 \times T^4$



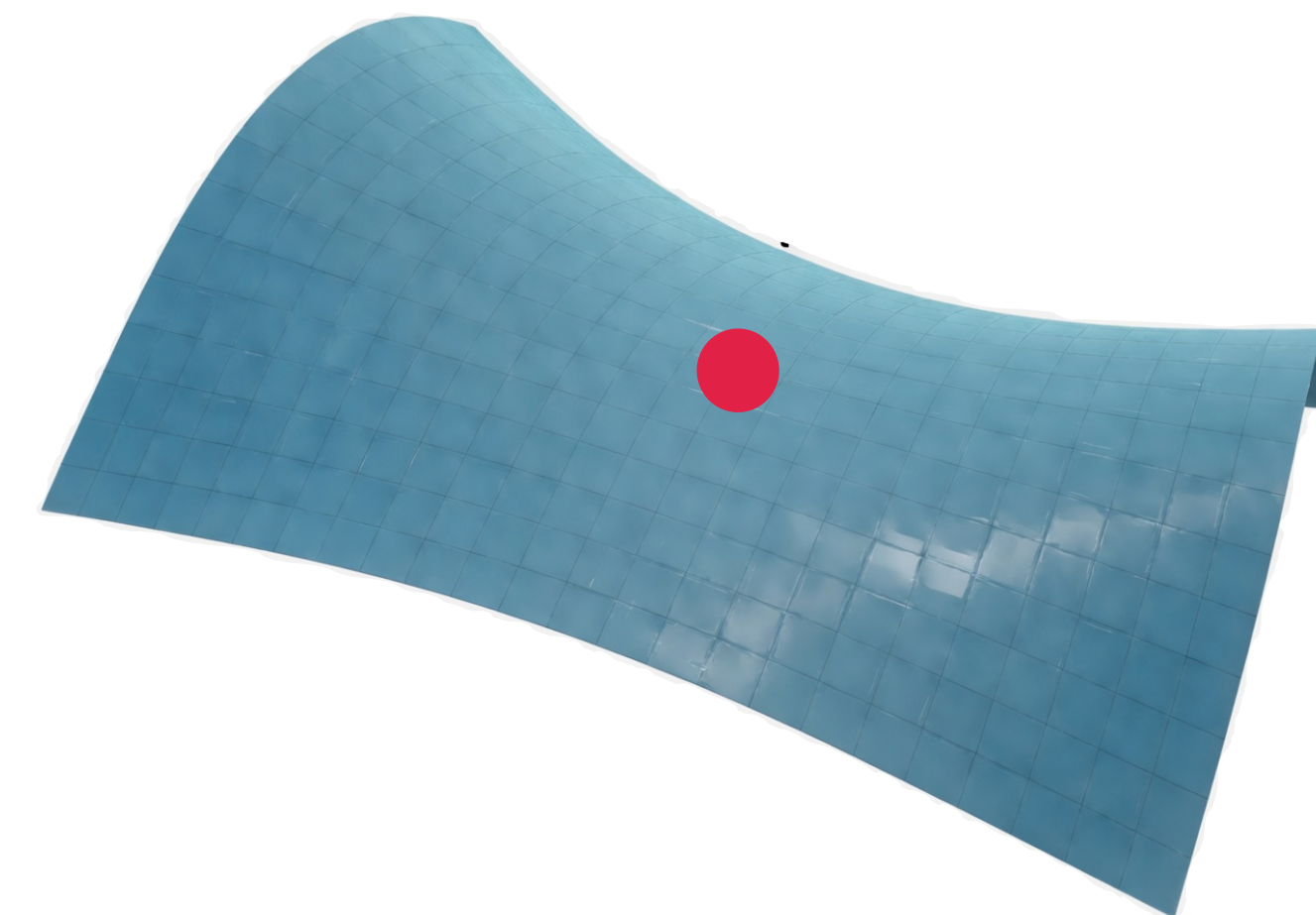
Deformed symmetric orbifold of T^4 CFT

Eberhardt, Gaberdiel, Gopakumar, 2018, 2019

$$\text{AdS}_3 \times S^3 \times T^4$$

- $\text{AdS}_3 \times S^3 \times T^4$ is the near horizon limit of the D1-D5 system
- Strings on $\text{AdS}_3 \times S^3 \times T^4$ with pure NS-NS fluxes can be described by WZW model. In the presence of R-R fluxes, hybrid formalism can be used
- The CFT dual to strings on this background is on the moduli space of the 2D symmetric orbifold CFT : $\text{Sym}^N(\mathbb{T}^4) = (\mathbb{T}^4)^N / S_N$
- A special “tensionless” point on the moduli space

$$k = 1 \quad \Leftrightarrow \quad \text{Sym}^N(\mathbb{T}^4)$$



The Symmetric Orbifold $\text{Sym}^N(\mathbb{T}^4)$

- $\mathbb{T}^4$: $\mathbb{X} = \{\alpha^i, \bar{\alpha}^i, \psi^\pm, \bar{\psi}^\pm, \tilde{\alpha}^i, \tilde{\bar{\alpha}}^i, \tilde{\psi}^\pm, \tilde{\bar{\psi}}^\pm\}, \quad i = 1,2$
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 - ◆ Untwisted sector: trivial monodromy on $\mathbb{C}P^1$
 - ◆ Twisted sector: non-trivial monodromy (elements of S_N) on $\mathbb{C}P^1$
Labelled by conjugacy classes of S_N : $\sigma_w = (1 \cdots w)$

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- Due to nontrivial monodromies, fields are fractionally moded, e.g. $\frac{n}{w}$, in twisted sectors
- Correlators in $\text{Sym}^N(\mathbb{T}^4)$ can be computed by lifting to covering surfaces that trivialize the monodromy

The Symmetric Orbifold $\text{Sym}^N(\mathbb{T}^4)$

- **Ground states** in the w -cycle twisted sector: σ_w

- **1/2-BPS** states, $h = j = \frac{w-1}{2} = \tilde{h} = \tilde{j}$

$$|\text{BPS}_-\rangle_w = \left(\bar{\psi}_{-\frac{w-2}{2w}}^+ \psi_{-\frac{w-2}{2w}}^+ \cdots \bar{\psi}_{-\frac{1}{2w}}^+ \psi_{-\frac{1}{2w}}^+ \right) \left(\tilde{\bar{\psi}}_{-\frac{w-2}{2w}}^+ \tilde{\psi}_{-\frac{w-2}{2w}}^+ \cdots \tilde{\bar{\psi}}_{-\frac{1}{2w}}^+ \tilde{\psi}_{-\frac{1}{2w}}^+ \right) \sigma_w, \quad \text{for odd } w$$

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as well as there $\bar{\psi}_{-1/2}^+, \psi_{-1/2}^+, \tilde{\bar{\psi}}_{-1/2}^+, \tilde{\psi}_{-1/2}^+$ descendants

- **1/4-BPS** states : focus on magnon excitations of the 1/2-BPS states: $A_{\frac{p_1}{w}} \dots A_{\frac{p_k}{w}} |\text{BPS}\rangle_w$

◆ Before deformation: annihilated by $\tilde{Q}_1 = \tilde{G}_{1/2}^-$, $\tilde{S}_1 = \tilde{G}_{-1/2}^+$, $\tilde{Q}_2 = \tilde{G}_{1/2}'^-$, $\tilde{S}_2 = \tilde{G}_{-1/2}'^+$

where $G^+ = \bar{\alpha}^2 \psi^+ + \alpha^2 \bar{\psi}^+$, $G'^+ = -\bar{\alpha}^1 \psi^+ - \alpha^1 \bar{\psi}^+$, $G^- = \bar{\alpha}^1 \psi^- + \alpha^1 \bar{\psi}^-$, $G'^- = \bar{\alpha}^2 \psi^- + \alpha^2 \bar{\psi}^-$

◆ After deformation: non-vanishing anomalous dimension $\langle \phi' | \tilde{\gamma} | \phi \rangle_\Phi$, where $\tilde{\gamma} = \{\tilde{S}_2, \tilde{Q}_2\} = \tilde{L}_0 - \tilde{K}_0^3$

Marginal deformation of $\text{Sym}^N(\mathbb{T}^4)$

- At the tensionless/orbifold point, an infinitely large stringy higher-spin symmetry
- To compare with general SUGRA on $\text{AdS}_3 \times S^3$, deform away the tensionless point

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- Exactly marginal deformation:

$$\delta S = g \int d^2y \Phi(y, \bar{y}) , \quad \Phi(x, \bar{x}) = V \left((G_{-1/2}^- \tilde{G}_{-1/2}'^- - G_{-1/2}'^- \tilde{G}_{-1/2}^-) |2_-\rangle, x, \bar{x} \right)$$

where $|2_-\rangle$ is the BPS state with $h = j = \frac{1}{2} = \tilde{h} = \tilde{j}$ in the $w = 2$ twisted sector

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- The correlators are deformed

$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle_\Phi = \langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle_0 + g \int d^2y \langle \mathcal{O}_1 \dots \mathcal{O}_n \Phi(y, \bar{y}) \rangle + \frac{g^2}{2} \int d^2y_1 d^2y_2 \langle \mathcal{O}_1 \dots \mathcal{O}_n \Phi(y_1, \bar{y}_1) \Phi(y_2, \bar{y}_2) \rangle + \dots$$

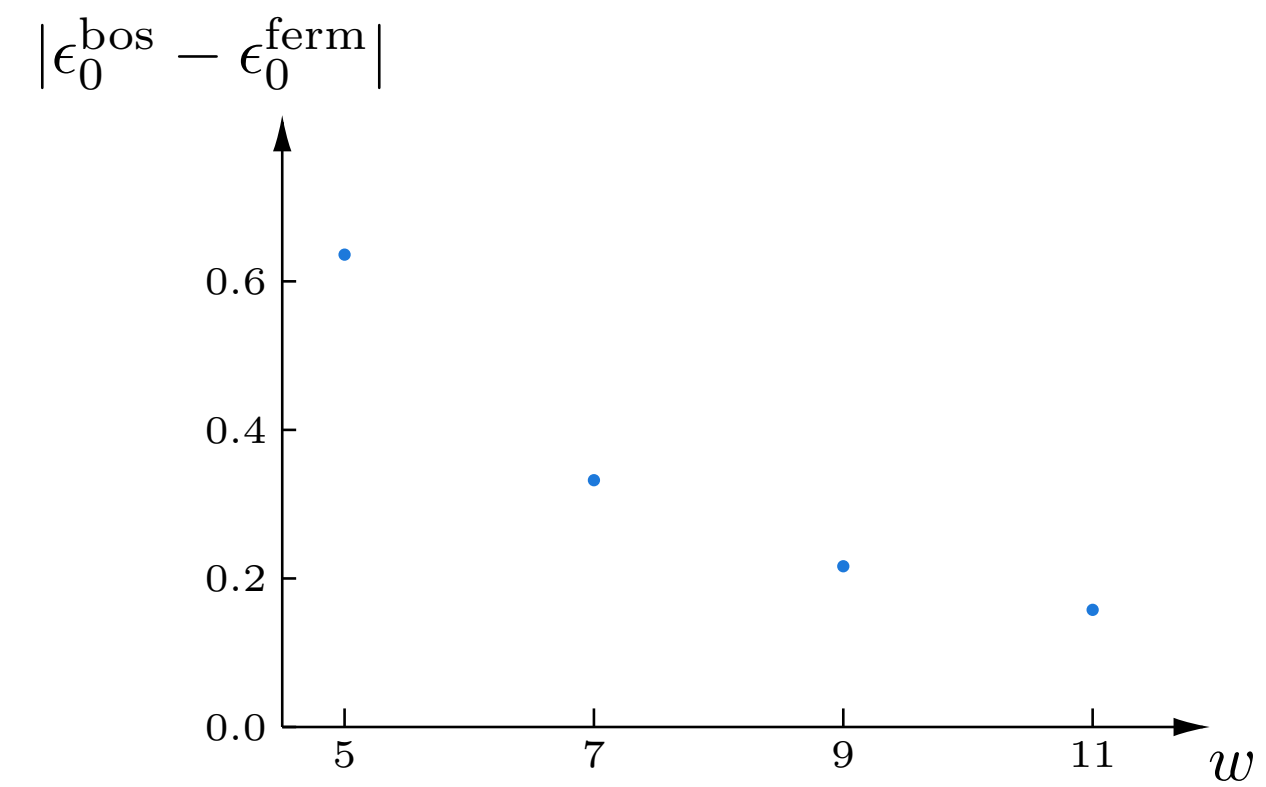
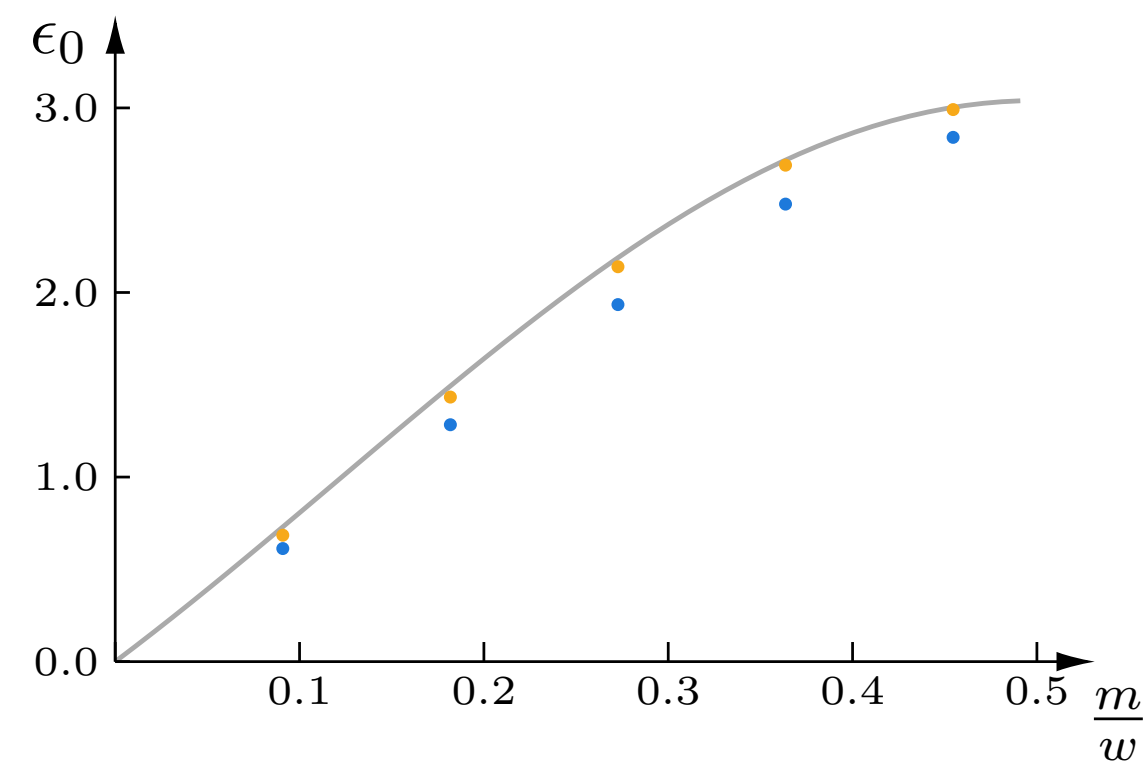
- The higher-spin symmetry is broken by this deformation
- Many states acquire anomalous dimensions

Gaberdiela, Peng, Zadeh, 2015

Planar Integrability

Gaberdiel, Gopakumar, Nairz, 2024

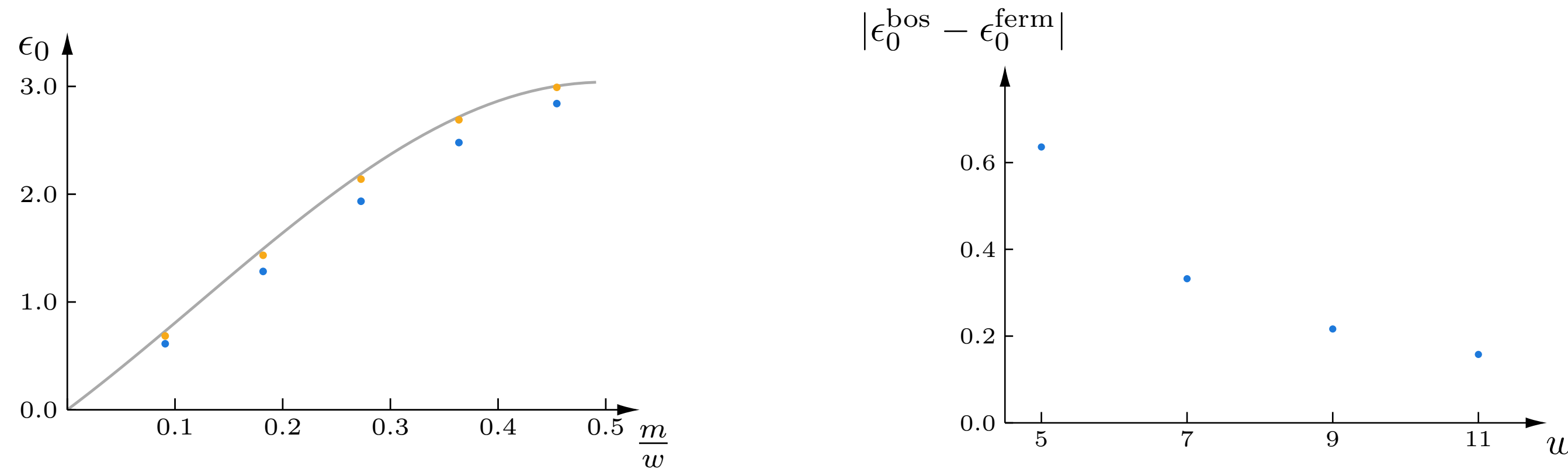
- Unexpected degeneracies among magnons, e.g. $\psi_{\frac{1}{2}-\frac{m}{w}}^- \psi_{-\frac{1}{2}+\frac{m}{w}}^- |w\rangle$, $\alpha_{-\frac{m}{w}}^1 \alpha_{-1+\frac{m}{w}}^1 |w\rangle$, $m = 1, \dots, w-1$



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- At large N and large w , the dispersion relation of the 1/4-BPS states

$$\gamma(p) = \sqrt{(1-p)^2 + 4g^2 \sin^2 \pi p}, \quad p = n/w$$

Berenstein, Maldacena, Nastase, 2002

Hoare, Stepanchuk, Tseytlin, 2013

- This resembles the dispersion relation of the BMN limit once we identify

$$p(n) = \frac{1}{\sqrt{1 + 4\pi^2 g^2}} \frac{n}{\alpha' p_+ \mu}, \quad 2\pi g = g_s q_R$$

- This is a clear indication that the theory is integrable in this limit

Our work

Motivation

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Non-Planar Corrections

Gaberdiel, Nairz, CP 2026

- By **chaotic** in the bulk, dynamics on **black hole** backgrounds are usually assumed
- By **integrable** on the boundary, strict $N \rightarrow \infty$ limits are usually assumed

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- By **integrable** on the boundary, strict $N \rightarrow \infty$ limits are usually assumed
- However, nontrivial dynamics in the bulk, e.g. formation of blackholes and scattering on that background, requires **non-vanishing gravitational/string couplings**, which corresponds to **deviating from the strict large N limit**

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- However, nontrivial dynamics in the bulk, e.g. formation of blackholes and scattering on that background, requires **non-vanishing gravitational/string couplings**, which corresponds to **deviating from the strict large N limit**
- Therefore, it is natural to try resolving the puzzle by considering $1/N$ corrections to the CFT, which describes effects from dynamical bulk gravity
- We expect an **integrable-chaos** transition as we turn on the $1/N$ corrections

Computation

Computation Scheme

Gaberdiel, Nairz, CP 2026

- Using $\tilde{\gamma} = \{\tilde{S}_2, \tilde{Q}_2\} = \tilde{L}_0 - \tilde{K}_0^3$, the deformed anomalous dimension matrix reads

$$\langle \phi' | \tilde{\gamma} | \phi \rangle_{\Phi} = \sum_{\chi} \langle \phi' | \tilde{S}_2 \Phi | \chi \rangle \langle \chi | \Phi \tilde{Q}_2 | \phi \rangle + \langle \phi' | \tilde{Q}_2 \Phi | \chi \rangle \langle \chi | \Phi \tilde{S}_2 | \phi \rangle$$

- The leading non-planar contribution comes from the subset of internal states χ that is from the fusion channel

$$(1 \ k + 1) \cdot (1 \cdots w) = (1 \cdots k)(k + 1 \cdots w), \quad k = 2, \dots, w - 2$$

due to the N -dependence in the twist operator correlators

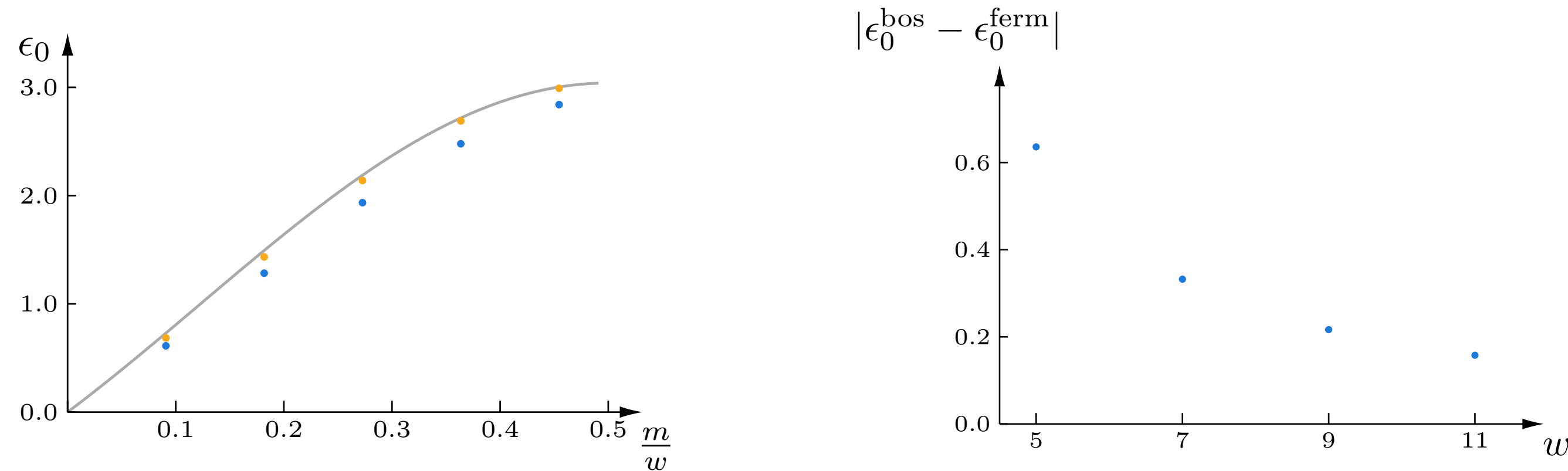
$$\langle \sigma_{w \pm 1} \sigma_2 \sigma_w \rangle \sim N^{-1/2} \quad \langle \sigma_{(w-k,k)} \sigma_2 \sigma_w \rangle \sim N^{-1}$$

The diagram illustrates the decomposition of a correlator on a genus-2 surface (a torus with two punctures) into a sum over internal states. On the left, a blue integral $\int d^2x$ is shown over a surface with punctures $\hat{\phi}$ and $\hat{\phi}'$. The surface is shaded gray and has a central white hole. On the right, the same correlator is expressed as a sum $\sum_{\chi}$ over internal states χ . The surface is split into two regions by a vertical dashed line. The left region is labeled $\tilde{S}_2 \hat{\Phi}$ and the right region is labeled $\tilde{Q}_2 \hat{\Phi}$. The internal states are labeled $\hat{\chi}_k$ and $\hat{\chi}_{w-k}$ in red.

Planar Integrability I: Degeneracy

Gaberdiel, Gopakumar, Nairz, 2024

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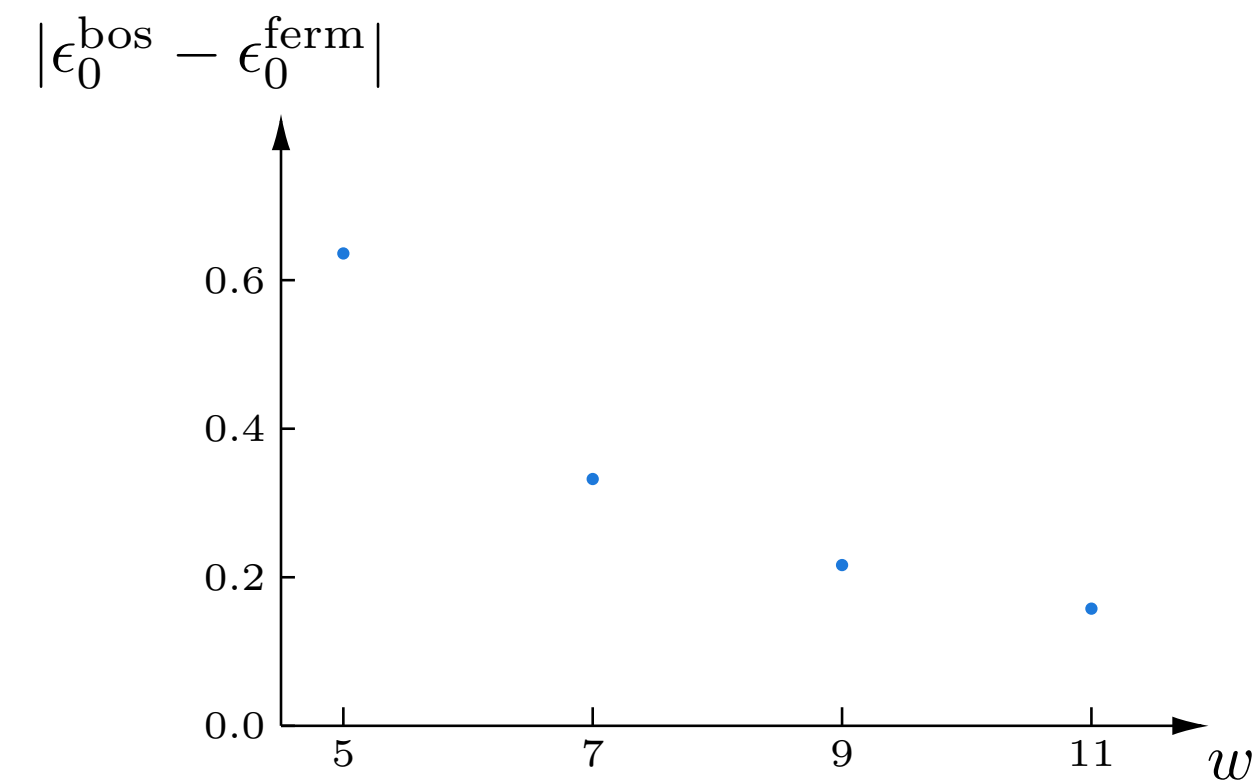
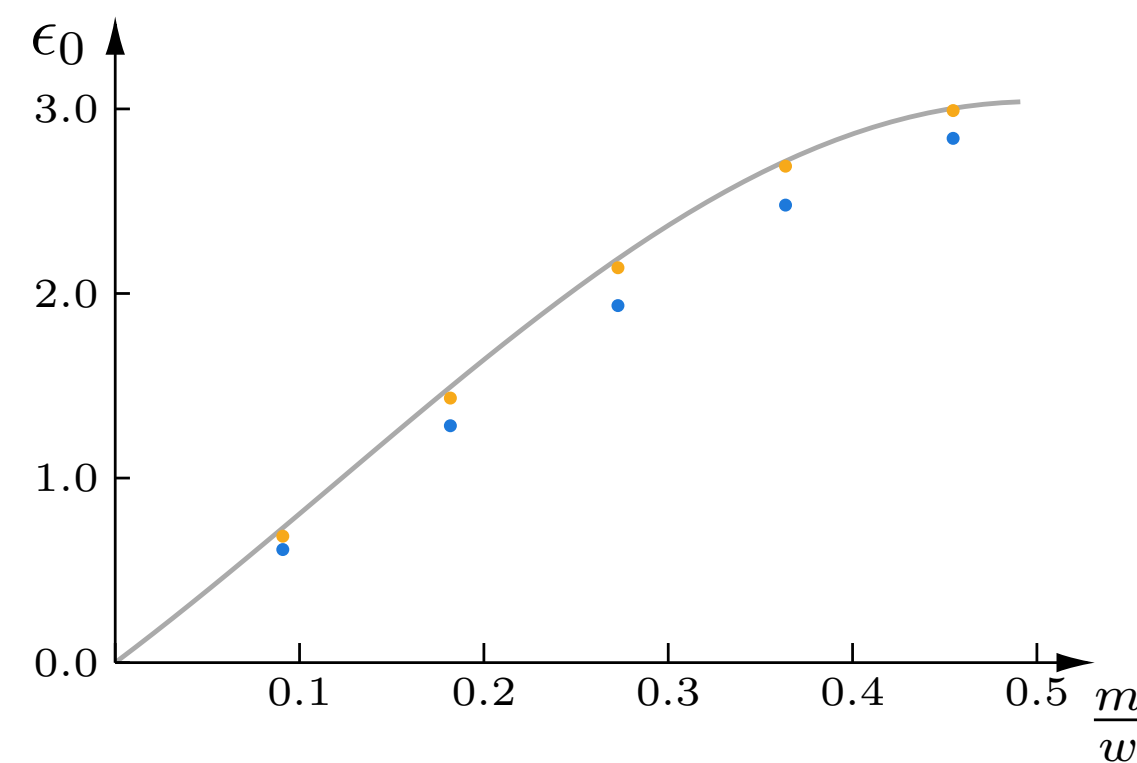
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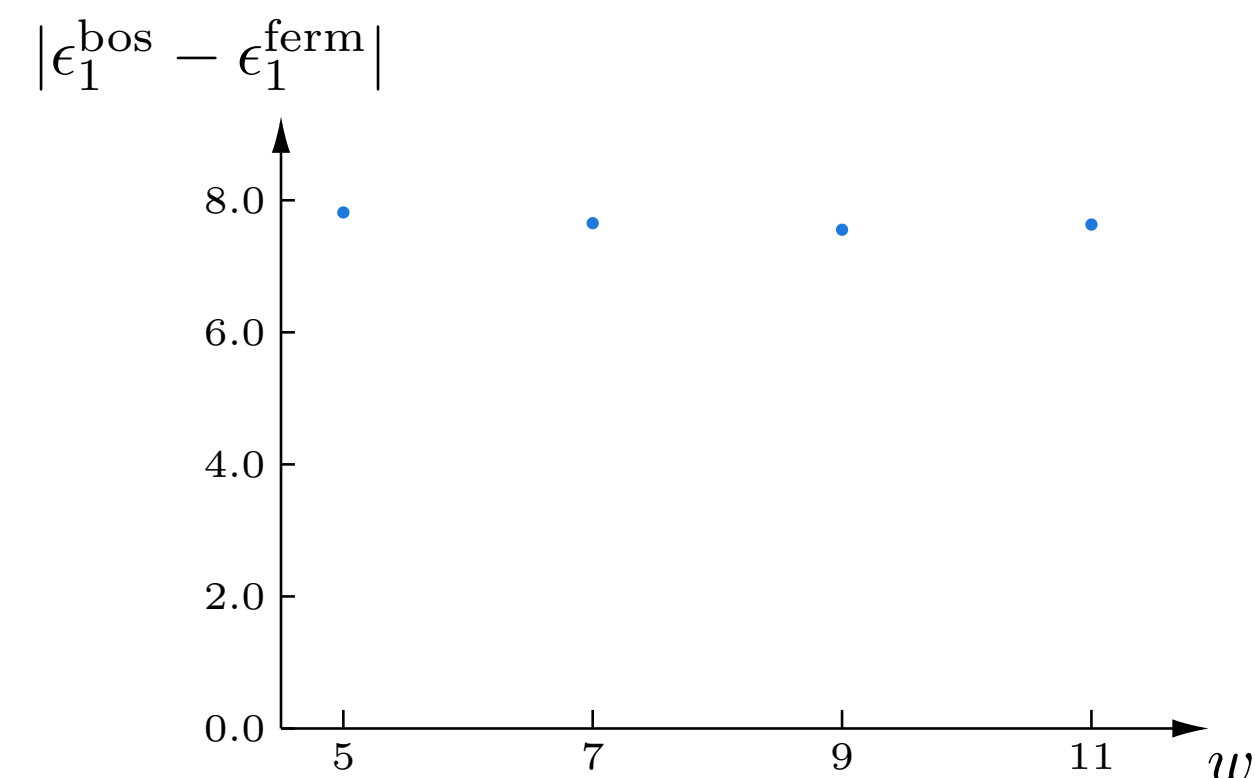
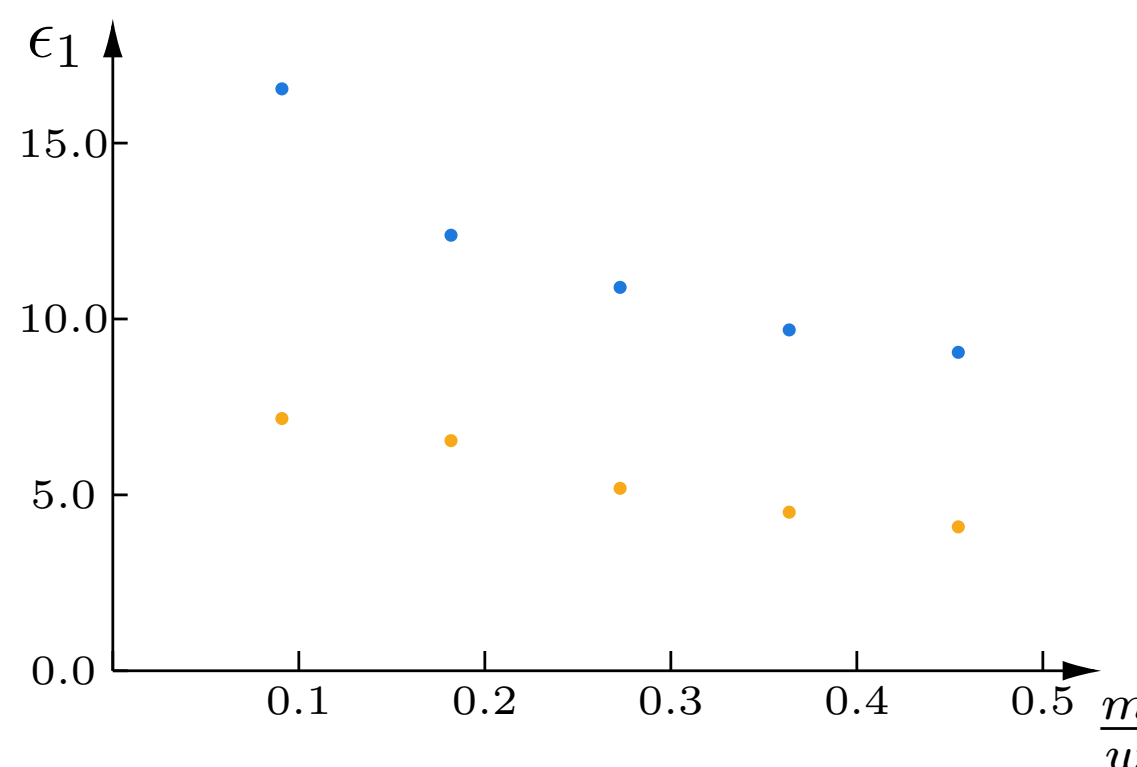
Breaking of the Planar Integrability I: Degeneracy

Gaberdiel, Nairz, CP 2026

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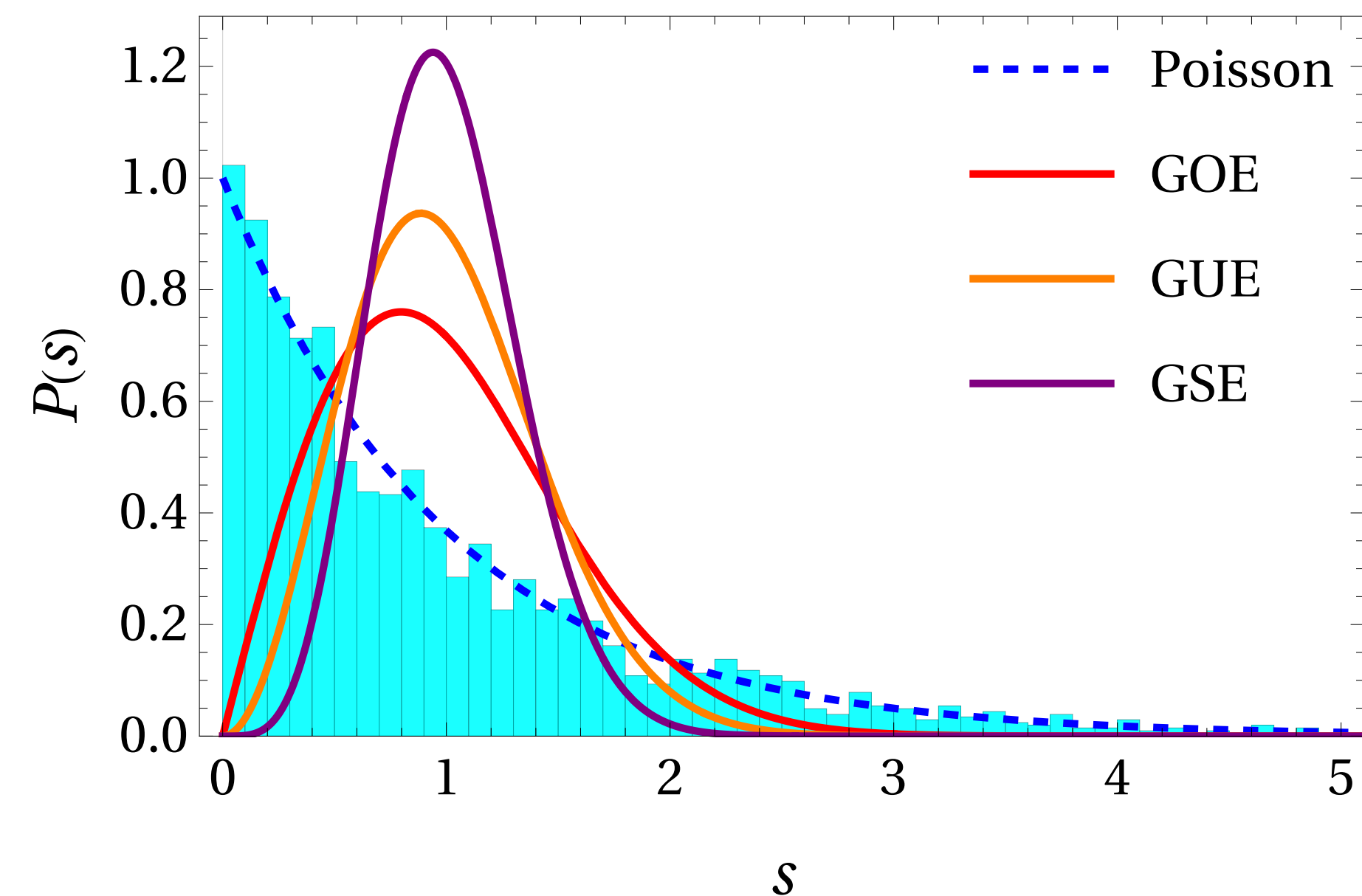
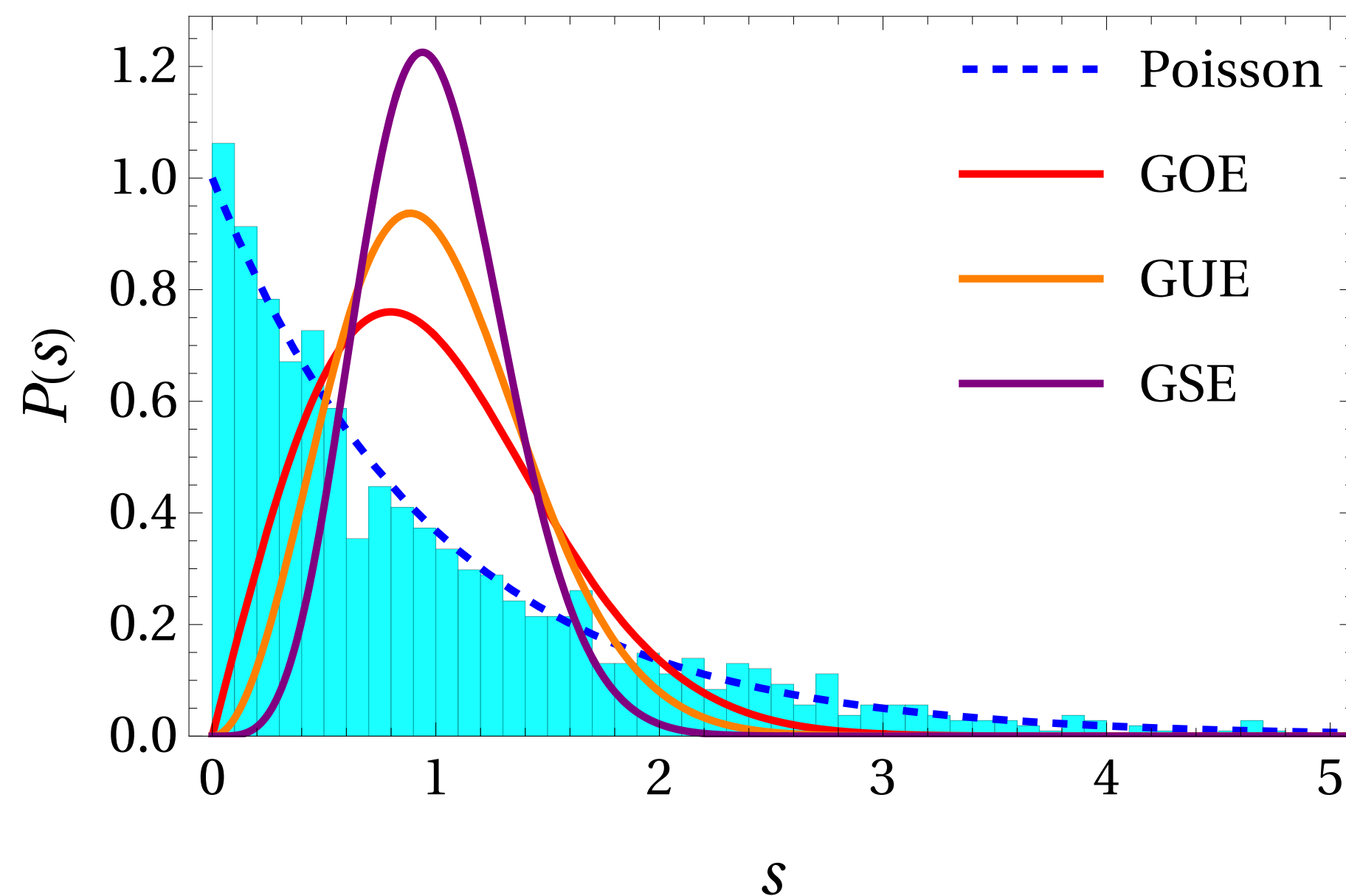
- If such degeneracies persist at the non-planar order as well ? **No.**



Planar Integrability II: Spectral Statistics

Gaberdiel, Nairz, CP 2026

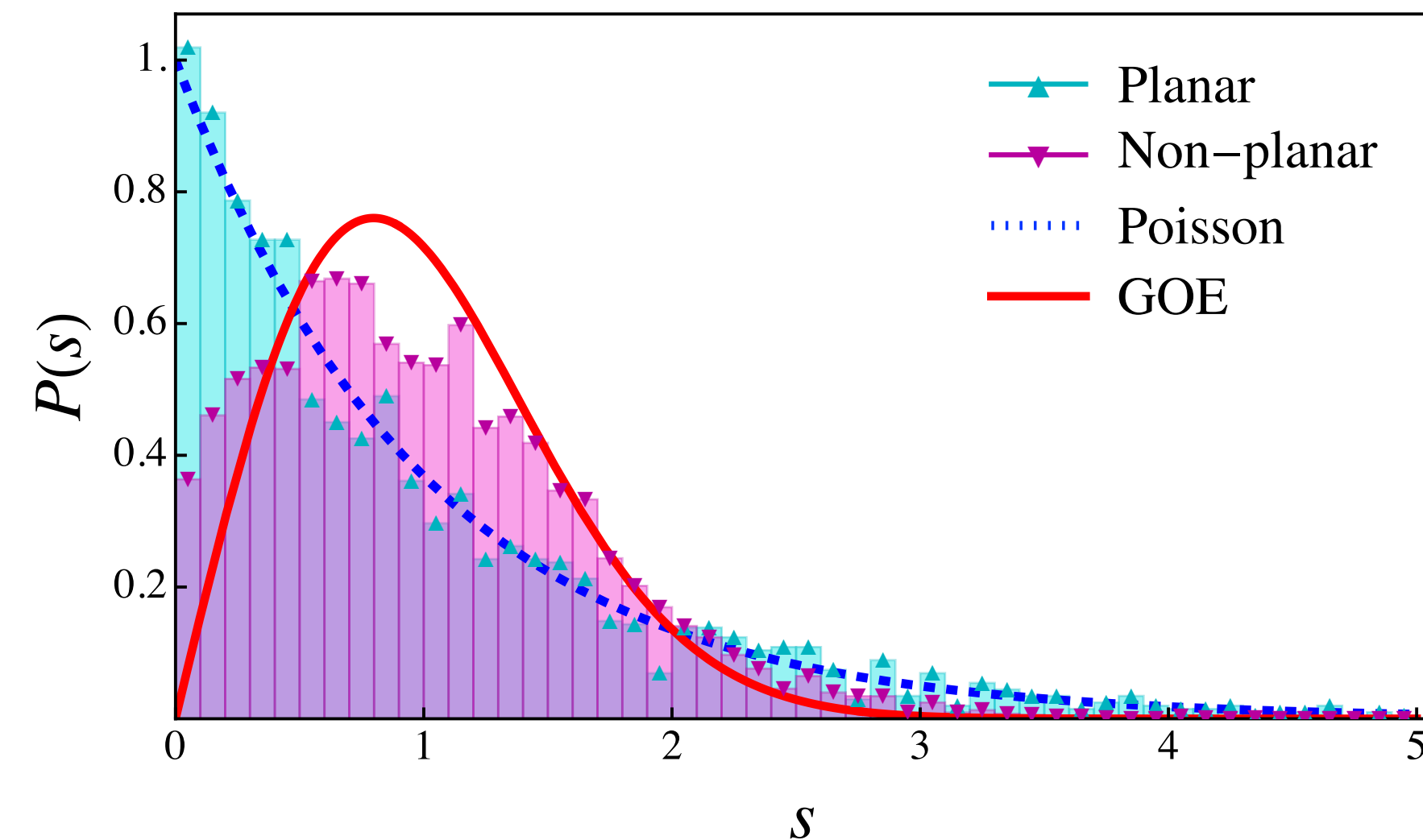
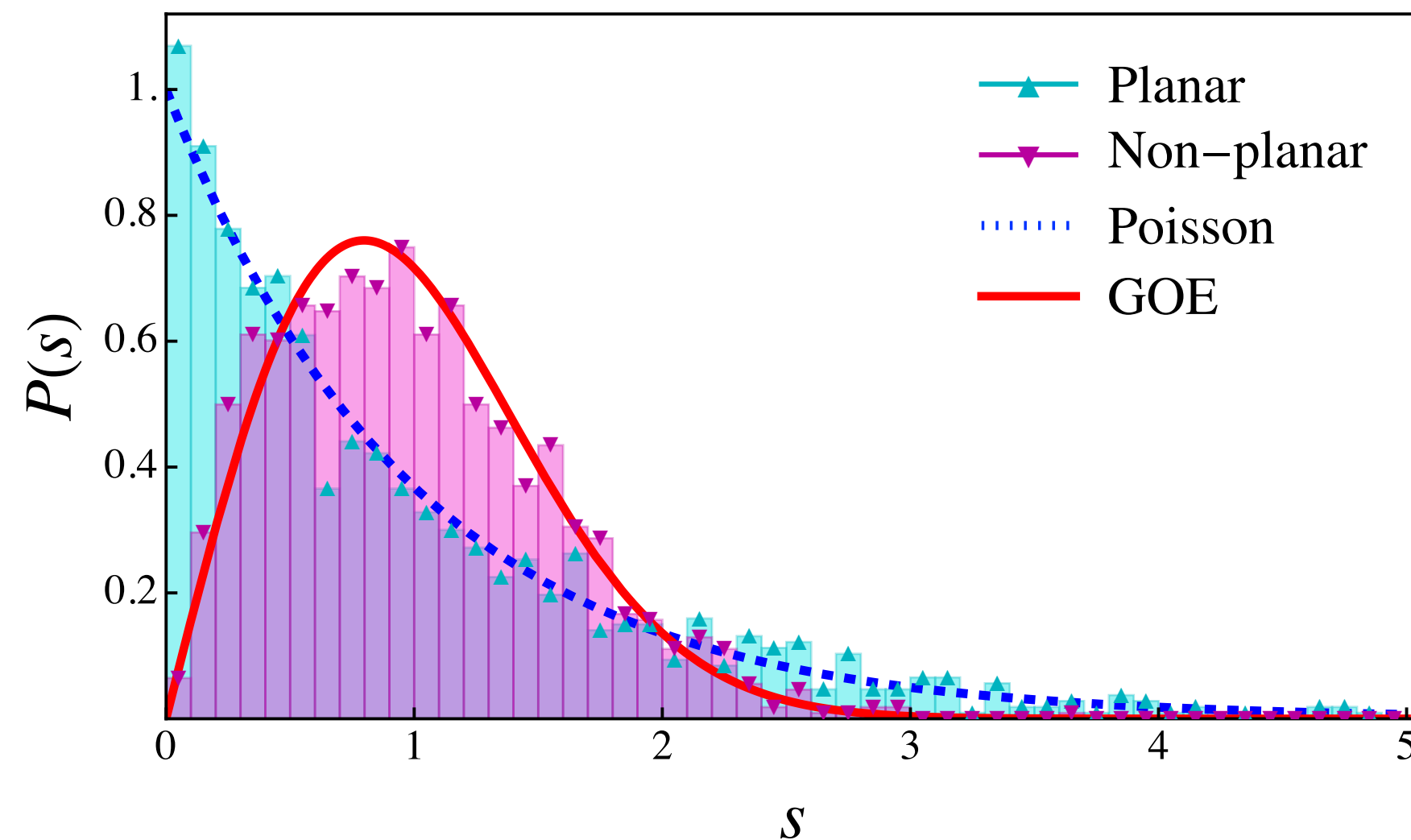
- Another diagnose of the integrability in the planar limit is the spectral statistics
 1. At the planar limit, **large degeneracies** of states: unrelated (multi-magnon) states share the same anomalous dimensions (in each charge sector)
 2. Apart from the degeneracies, the level spacing of the spectrum is **Poisson distributed**



Breaking of the Integrability II: Emergence of Chaos

Gaberdiel, Nairz, CP 2026

- However, the statistics of the non-planar corrections is dramatically different
 1. There large degeneracies of states in the planar limit are **lifted**: the anomalous dimensions are all distinct
 2. The level spacing of the non-planar anomalous dimensions exhibit clear level repulsion and is close to the **Wigner's surmise** observed in **RMT**



Comments

Comments on the Results I

Gaberdiel, Nairz, CP 2026

- Our computation is entirely in the **quantum** territory

$k = 1 \quad \Rightarrow \quad R_{\text{AdS}} \sim \ell_{\text{string}} \quad :$ spacetime is highly curved, “geometry” is no longer a good notion

$N < \infty \quad \Rightarrow \quad g_s^2 > 0 \quad :$ string loop corrections are considered

- Our computation cleanly **isolates the leading non-planar $1/N$ correction** from the planar integrable results, which is purely dual to **bulk gravitational corrections** without mixing with other effects.
- This supports the duality: turn on $1/N$ correction gives the expected chaotic behavior from the bulk perspective, namely turn on gravity coupling that leads to chaotic effects

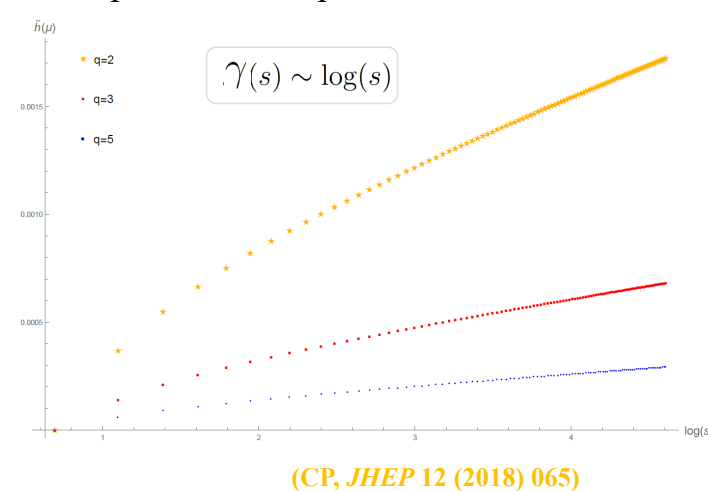
Comments on the Results II

Gaberdiel, Nairz, CP 2026

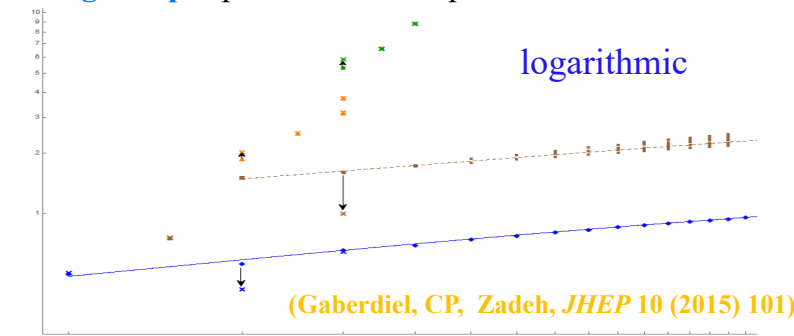
1. Previous studies about **integrable-chaos transition** that are not (trivially) due to competition between a chaotic subsector and an integrable subsector
2. We proposed that such transition should be related with the **deforming away from the tensionless/higher-spin limit of string theory**, and provided concrete examples
3. The computation in this work is a manifestation of such connections in an explicit stringy holographic example

CONNECTIONS WITH CONVENTIONAL THEORIES

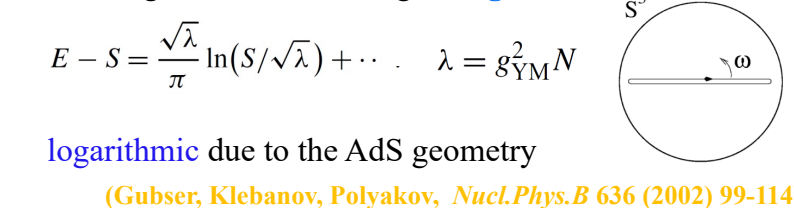
- Dispersion relation of this SYK model: the anomalous dimension γ logarithmically depends on the spin s



- Higher-spin perturbation computation



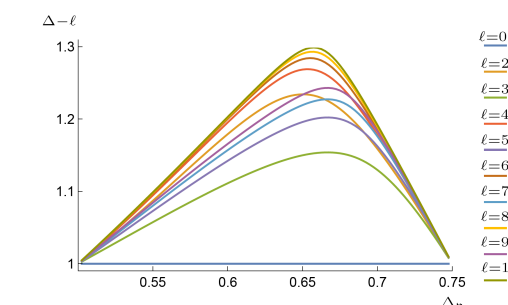
- Rotating folded closed long string in AdS S^3



3D DISORDERED MODELS: HIGHER-SPIN LIMITS

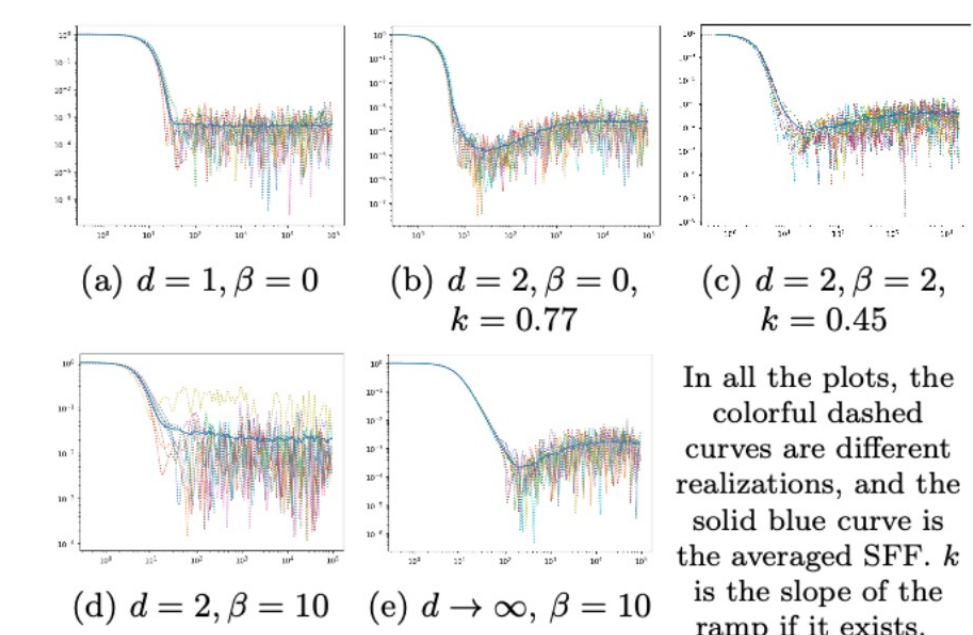
Chang, Colin-Ellerin, CP, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- Most of the details of the model are quite different from the 0+1d and 1+1d models
- Nevertheless there is again a **clear connection to higher-spin theories**
- There are special limits
 - ❖ Free $\not\equiv$ $\lambda \rightarrow 0$, $\Delta_s = \frac{1}{2}$, $\Delta_s = 1$ higher-spin
 - ❖ Free $\not\equiv$ $\lambda \rightarrow \infty$, $\Delta_s = \frac{3}{4}$, $\Delta_s = \frac{1}{2}$ higher-spin
 - ❖ Diagonal $\lambda \rightarrow \frac{1}{2}$, $\Delta_s = \frac{2}{3}$, $\Delta_s = \frac{2}{3}$ the 2+1D SYK model
- This indicates the connection to higher-spin theories is probably **universal**



MORE IS DIFFERENT !

- Spectral form factor



Comments on the Results III

Gaberdiel, Nairz, CP 2026

1. A bigger picture
2. Wormholes are important
3. They are dual to ensemble average of theories on the boundary
4. A general question is where does the ensemble come from, as conventional field theories in holography has a fixed lagrangian?
Schlenker, Witten 2022; Kudler-Flam, Witten 2026
5. Witten and collaborators argued that as there is no tunable parameter in well-known CFT in holography, the only thing that can be averaged is the number N
6. A general lesson, not necessarily related to the ensemble average, from their argument as well as many other evidence is that as N changes towards infinity, there could be dramatic non-trivial dynamics
7. The integrability-chaos transition we observe is a consequence of **changing N** , which is an explicit example of the dramatical difference of the Hilbert space as **N changes in the large- N process**
8. Hopefully more to come

Thank you!