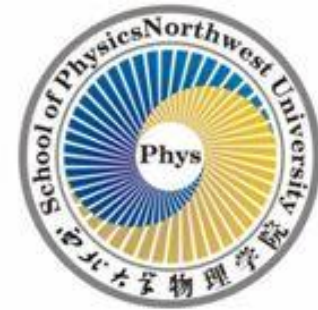




第七届全国“场论与弦论”学术研讨会



孤子中的几何相位和虚拟磁单极

报告人：赵立臣

2026年08月02-07日

合肥

报告结构

一、可积系统与孤子物理简介

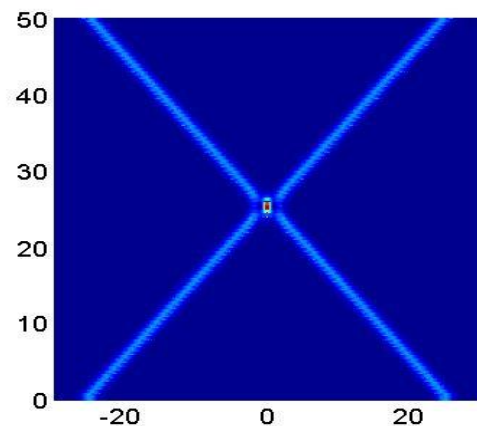
二、孤子中的几何相位

三、孤子中的虚拟磁单极

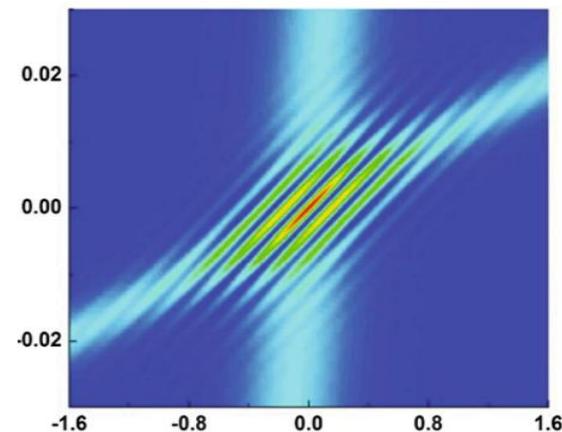
四、总结与讨论

一、可积系统与孤子物理简介

孤子的基本特征

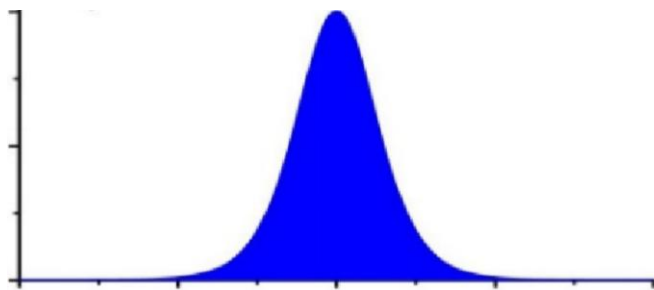


粒子性

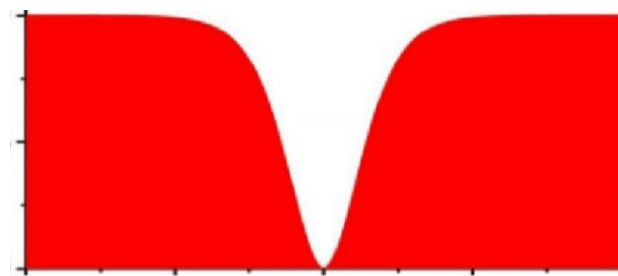


波动性

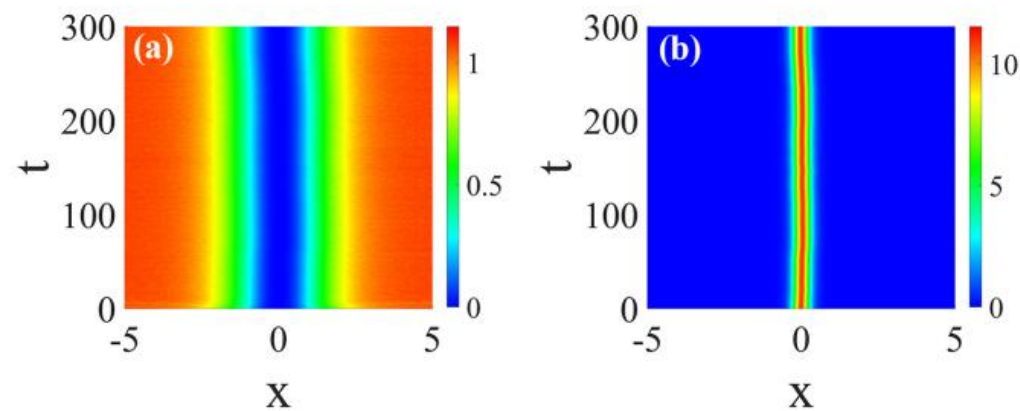
基本孤子类型



亮孤子



暗孤子



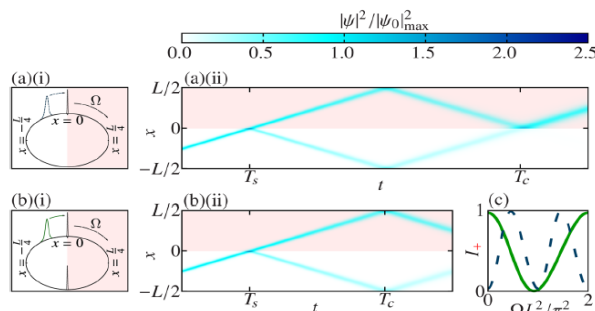
暗-亮孤子

一、可积系统与孤子物理简介

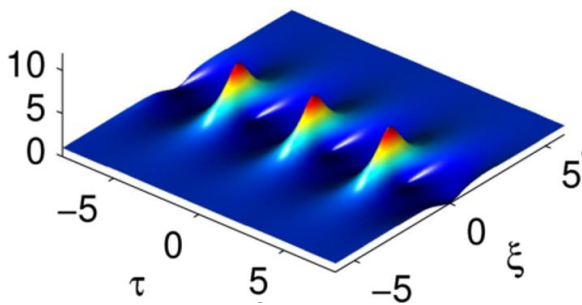
光学、玻色-爱因斯坦凝聚、等离子体等非线性系统具有丰富多样的局域波激发动力学，**可积系统理论**则为**解析探究**相关非线性物理及其应用提供了有力工具。



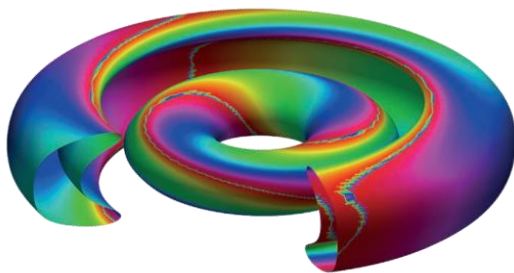
海洋中怪波现象



孤子干涉仪 *PRL* (2015)



呼吸子 *Nature Photon.* (2014)



拓扑孤子 *Nature* (2017)

玻色凝聚体中孤子的实验观测

暗孤子:

Burger S, et al, Phys. Rev. Lett. 83, 5198 (1999)
Denschlag J, et al, Science 287, 97 (2000)

亮孤子:

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暗-亮孤子:

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C. Hamner, et al., Phys. Rev. Lett. 106, 065302 (2011)
P. G. Kevrekidis et al., Rev. Phys. 1, 140 (2016)
A. Farolfi, et al, Phys. Rev. Lett. 125, 030401 (2020)

暗-亮-亮孤子:

T.M. Bersano, et al., Phys. Rev. Lett. 120, 063202 (2018)
X. Chai, et al, Phys. Rev. Lett. 125, 030402 (2020)

一、可积系统与孤子物理简介

当系统完全-可积时

达布变换

1886年, Darboux, 提出一维定态薛定谔方程的初等达布变换

1979年, Matveev, 给出KP和户田晶格方程的达布变换

1982年, Sall, 给出了Sin-Gordon 方程的达布变换

Lax 可积

反散射方法

1967年, Gardner, 利用线性薛定谔方程的本征问题及反问题构造了KdV方程的N-孤子解

1968年, P.D. Lax, Lax 可积性

1972年, Zakharov, Shabat, solve NLSE

1972年, Wadati, solve mKdV

1973年, Ablowitz, Kaup, Newell, Segur

AKNS系统

可求解不同类型的完全-可积非线性数理方程

一、可积系统与非线性局域波简介

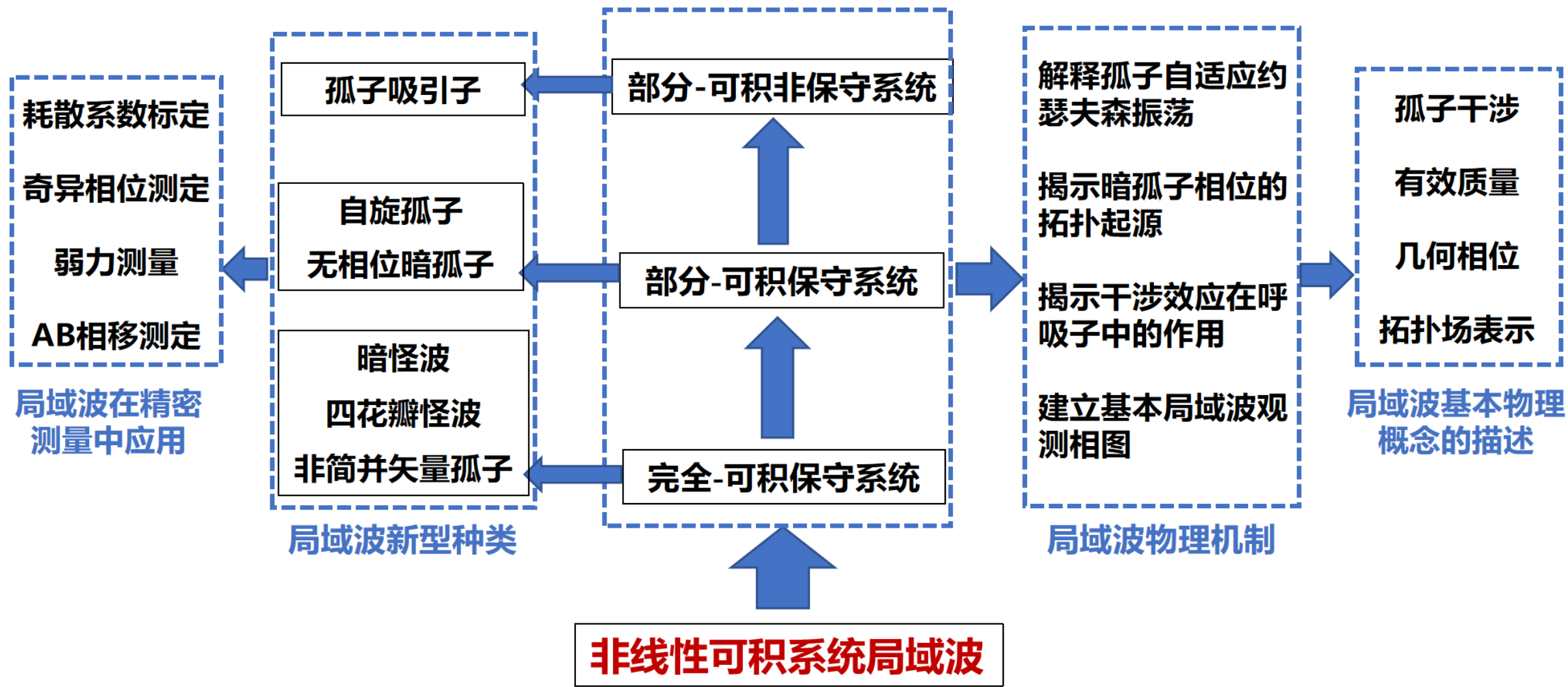
可积系统与非线性局域波研究还面临一些关键问题：

非线性系统可积性的探究仍缺乏成熟的一般性理论，亟待发展更具一般性可积系统分析方法

已发展相对成熟的可积系统理论大都需要特定且严格的边界与守恒量约束条件，远不能满足真实物理体系研究的需求

没能充分发挥可积系统在探究诸多基本物理问题的优势，亟需拓展它们的应用范围

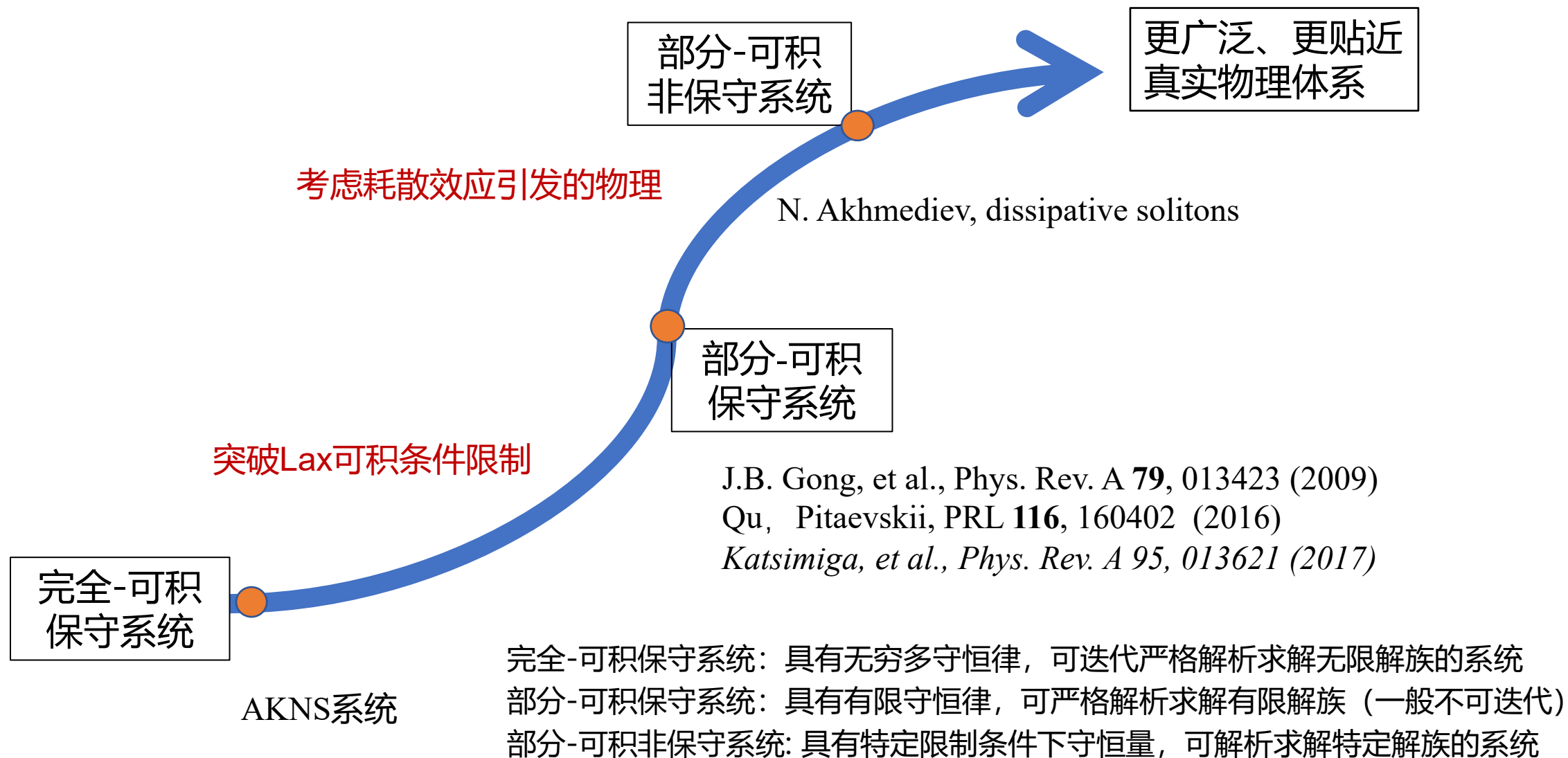
一、可积系统与孤子物理简介



研究工作主要内容和系统性示意图

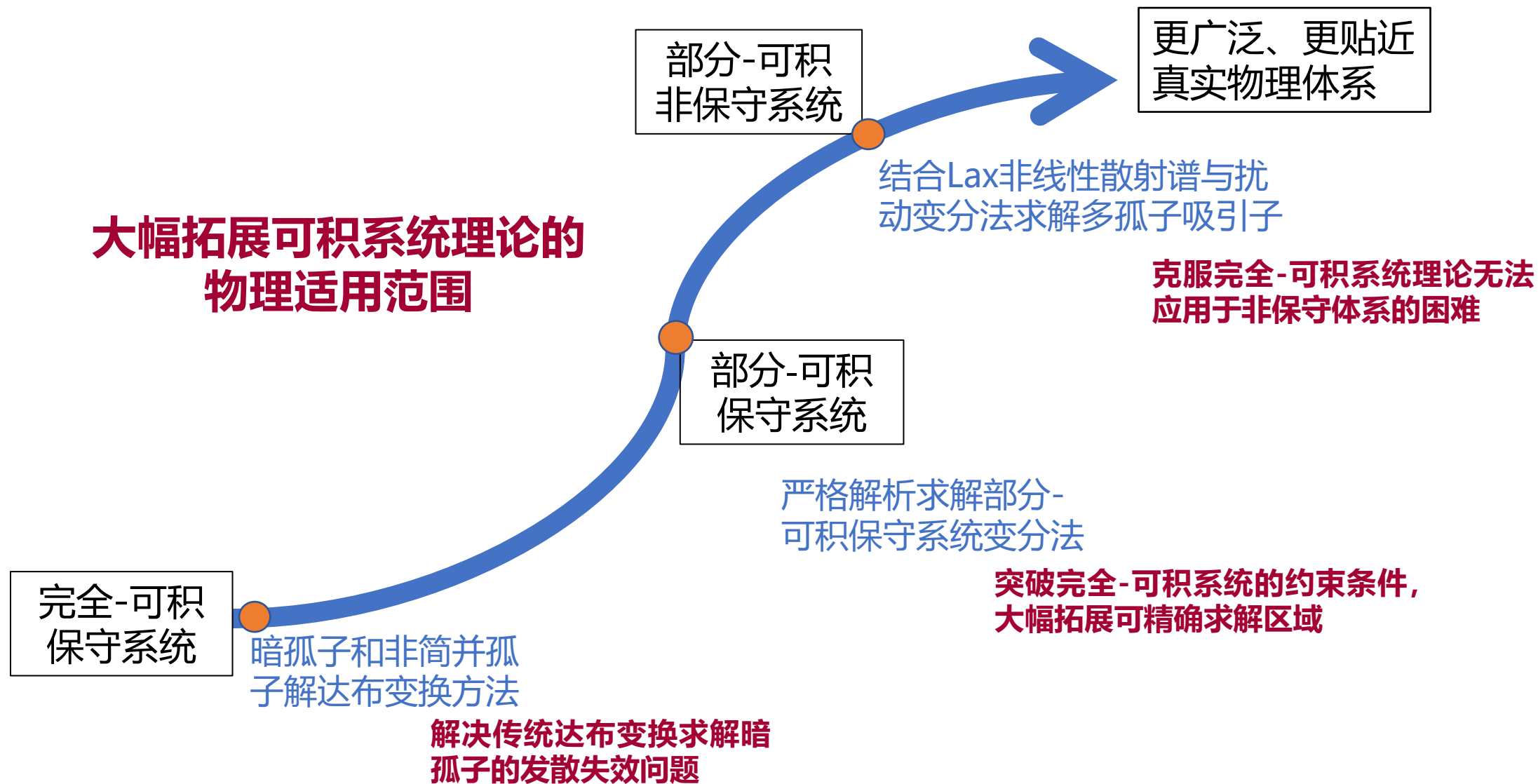
一、可积系统与孤子物理简介

1. 发展非线性系统解析求解方法



一、可积系统与孤子物理简介

1. 发展非线性系统解析求解方法



一、可积系统与孤子物理简介

可积系统理论方法方面

完全-可积系统：具有无穷多守恒律，可迭代严格解析求解无限解族的系统

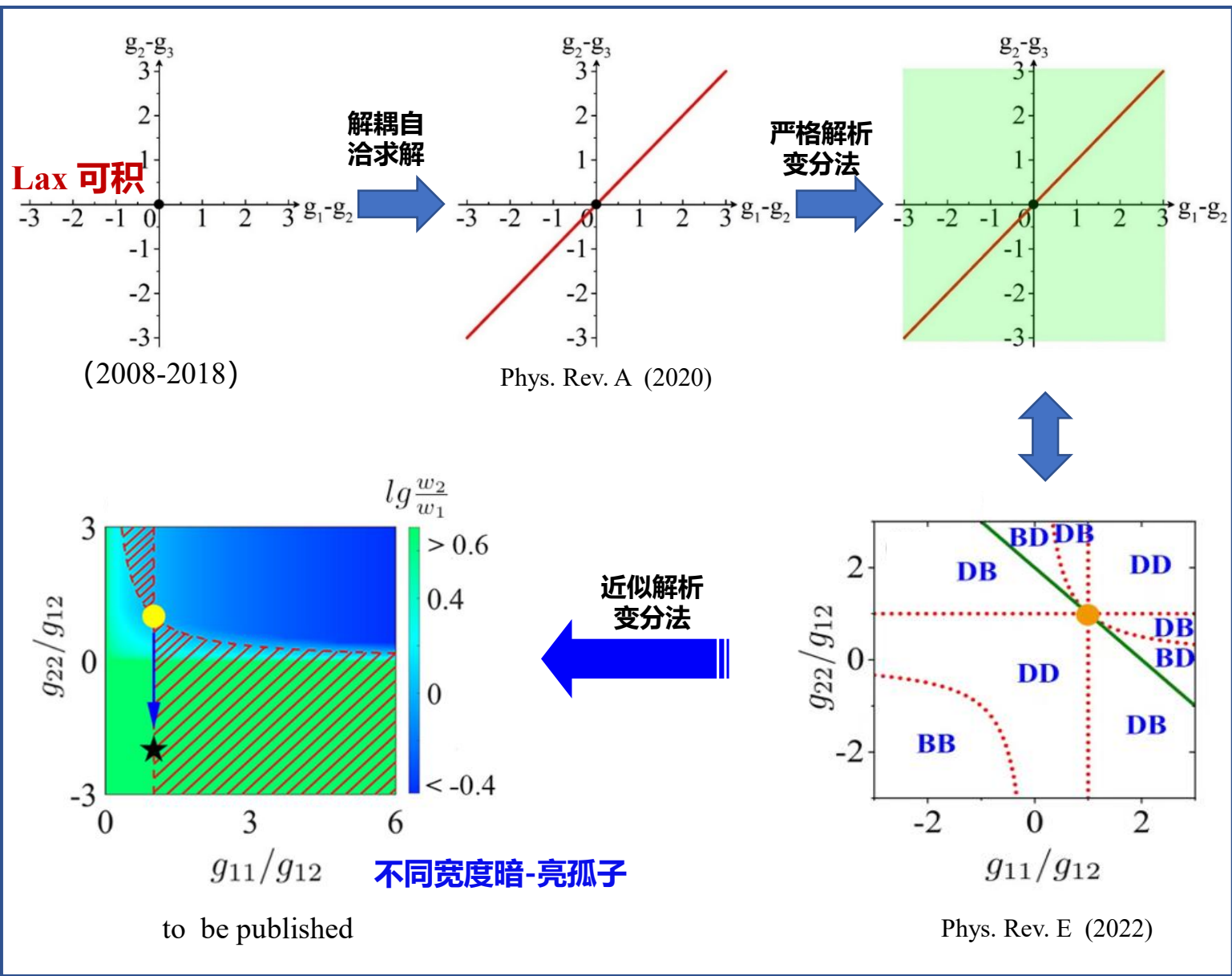
$$i\frac{\partial \psi}{\partial t} + \frac{1}{2}\frac{\partial^2 \psi}{\partial x^2} + \sigma|\psi|^2\psi = 0.$$

部分-可积系统：具有有限守恒律，可严格解析求解有限解族（一般不可迭代）

$$\begin{aligned} i\frac{\partial \psi_+}{\partial t} &= -\frac{1}{2}\frac{\partial^2 \psi_+}{\partial x^2} + (g_1|\psi_+|^2 + g_2|\psi_-|^2)\psi_+, \\ i\frac{\partial \psi_-}{\partial t} &= -\frac{1}{2}\frac{\partial^2 \psi_-}{\partial x^2} + (g_2|\psi_+|^2 + g_3|\psi_-|^2)\psi_-. \end{aligned}$$

任意N组分任意非线性的一般性推广

$$i\partial_t \Psi = \left(-\frac{1}{2}\partial_{xx} + \sum_{j=1}^N g_{ij}|\psi_j|^2\right)\Psi,$$
$$\Psi = (\psi_1, \dots, \psi_i, \dots, \psi_N)^T$$



一、可积系统与孤子物理简介

N组分玻色凝聚中的孤子解

非线性系数都相等时Lax可积

$$i\partial_t \Psi = \left(-\frac{1}{2}\partial_{xx} + \sum_{j=1}^N g_{ij}|\psi_j|^2 \right) \Psi,$$

$$\Psi = (\psi_1, \dots, \psi_i, \dots, \psi_N)^T$$

$$\psi_{iD} = \left(i\sqrt{a_i^2 - f_i^2(t)} + f_i(t) \tanh\{w_i(t)[x - b(t)]\} \right) e^{i\theta_i(t)},$$

$$\psi_{iB} = f_i(t) \operatorname{sech}\{w_i(t)[x - b(t)]\} e^{i\{\xi_i(t) + [x - b(t)]\phi_i(t)\}},$$

$$\mathcal{L} = \sum_{i=1}^N \left[\frac{i}{2} (\tilde{\psi}_i^* \partial_t \tilde{\psi}_i - \tilde{\psi}_i \partial_t \tilde{\psi}_i^*) \left(1 - \frac{a_i^2}{|\tilde{\psi}_i|^2} \delta_{D,i} \right) - \frac{1}{2} |\partial_x \tilde{\psi}_i|^2 - \sum_{j=1}^N \frac{g_{ij}}{2} (|\tilde{\psi}_j|^2 - a_j^2 \delta_{D,j}) (|\tilde{\psi}_i|^2 - a_i^2 \delta_{D,i}) \right]$$

$$\delta_{D,i} = \begin{cases} 0, & \tilde{\psi}_i \text{ denotes bright soliton,} \\ 1, & \tilde{\psi}_i \text{ denotes dark soliton,} \end{cases}$$

一、可积系统与孤子物理简介

$$G_{ij} = \int_{-\infty}^{+\infty} \left(\pm \operatorname{sech}^2 \{w_i(t)[x - b(t)]\} \right) \left(\pm \operatorname{sech}^2 \{w_j(t)[x - b(t)]\} \right) dx,$$

$$\begin{pmatrix} g_{11} & \cdots & g_{1j} & \cdots & g_{1N} & 1 \\ \vdots & & \vdots & & \vdots & \vdots \\ g_{i1} & \cdots & g_{ij} & \cdots & g_{iN} & 1 \\ \vdots & & \vdots & & \vdots & \vdots \\ g_{N1} & \cdots & g_{Nj} & \cdots & g_{NN} & 1 \end{pmatrix} \begin{pmatrix} \pm f_1^2 \\ \vdots \\ \pm f_i^2 \\ \vdots \\ \pm f_N^2 \\ w^2 \end{pmatrix} = 0,$$

$$b'(t) = v, \quad b(t) = vt, \quad \phi_i = v, \quad \xi_i(t) = \frac{1}{2}(w^2 + v^2)t + \theta_i(t).$$

$$\psi_{iD} = \{i\sqrt{a_i^2 - f_i^2} + f_i \tanh[w(x - vt)]\} e^{i\theta_i(t)},$$

$$\psi_{iB} = f_i \operatorname{sech}[w(x - vt)] e^{i[\frac{1}{2}(w^2 + v^2)t + (x - vt)v + \theta_i(t)]}.$$

可用于求解任意非线性N-组分耦合薛定谔方程的孤子类型解析解

一、可积系统与孤子物理简介

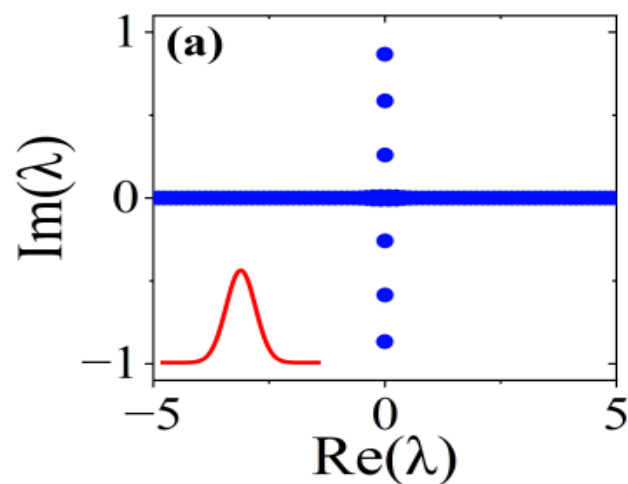
部分-可积**非保守**系统

拉格朗日微扰理论



非线性散射谱

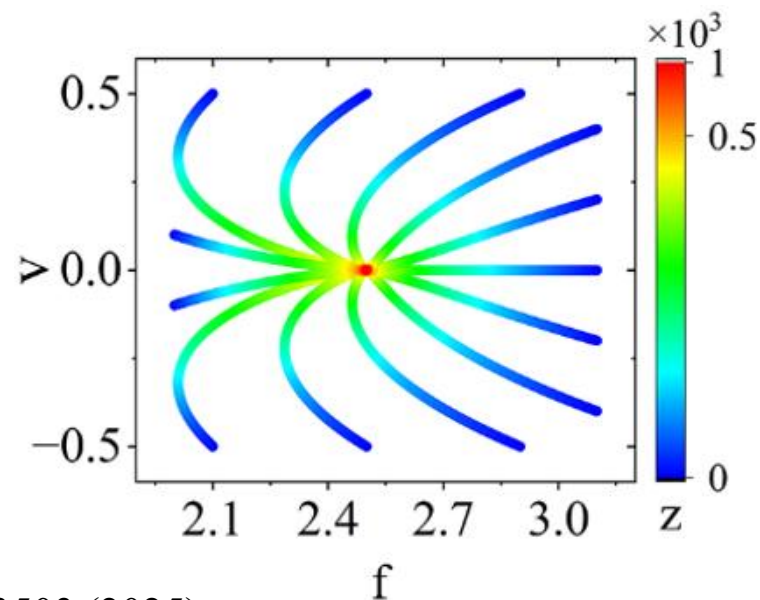
$$\begin{pmatrix} -\partial_\tau & \psi \\ \psi^* & \partial_\tau \end{pmatrix} \Phi = i\lambda \Phi$$



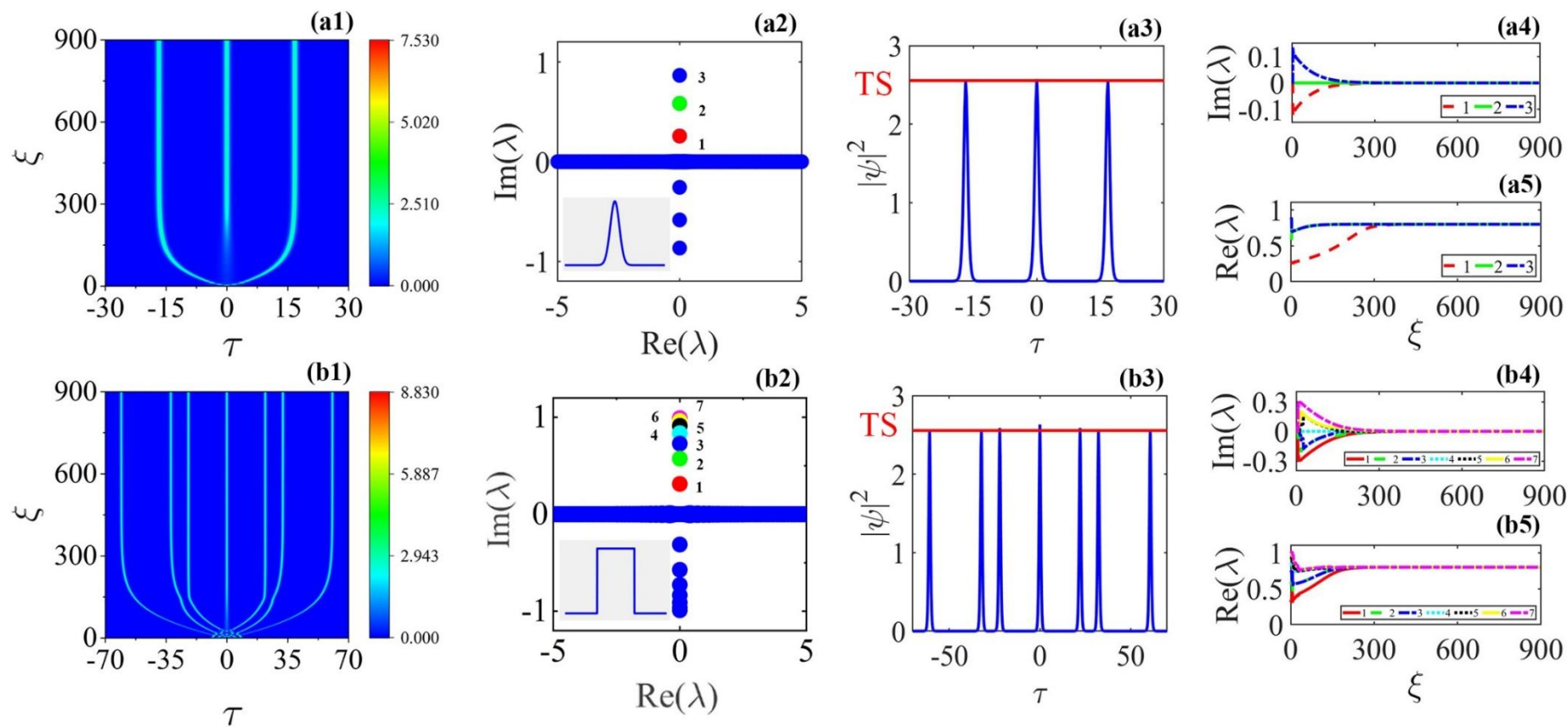
多孤子吸引子的解析解

任意初态

每一个离散谱点
拉格朗日微扰理论研究



一、可积系统与孤子物理简介



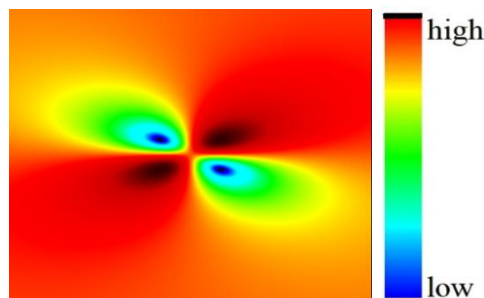
不同初态在耗散系统中最终形成具有相同非线性散射谱的多孤子吸引子；且离散谱数据与变分结果一致。

一、可积系统与孤子物理简介

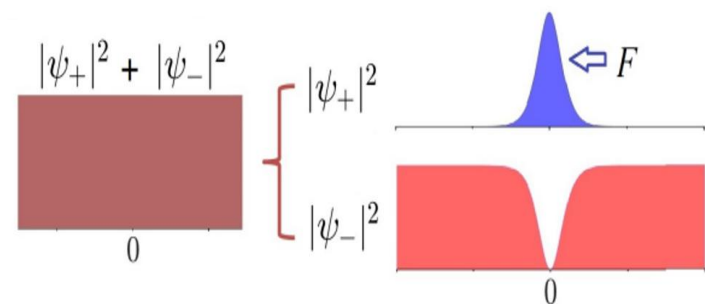
2. 预言新型局域波动力学

新型局域波类型

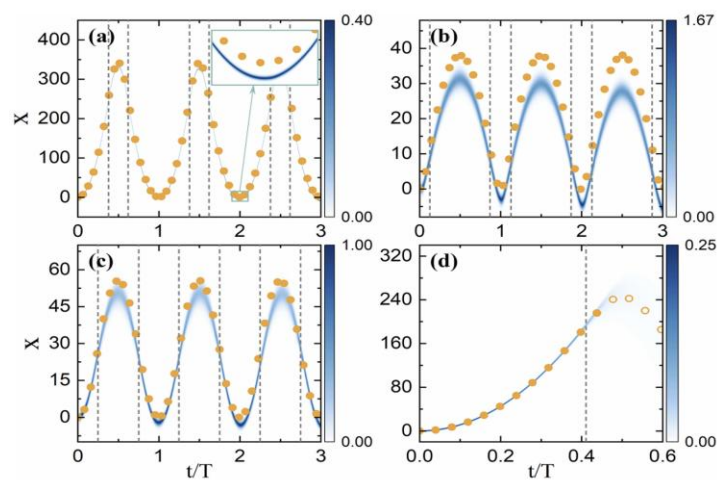
“四花瓣”怪波



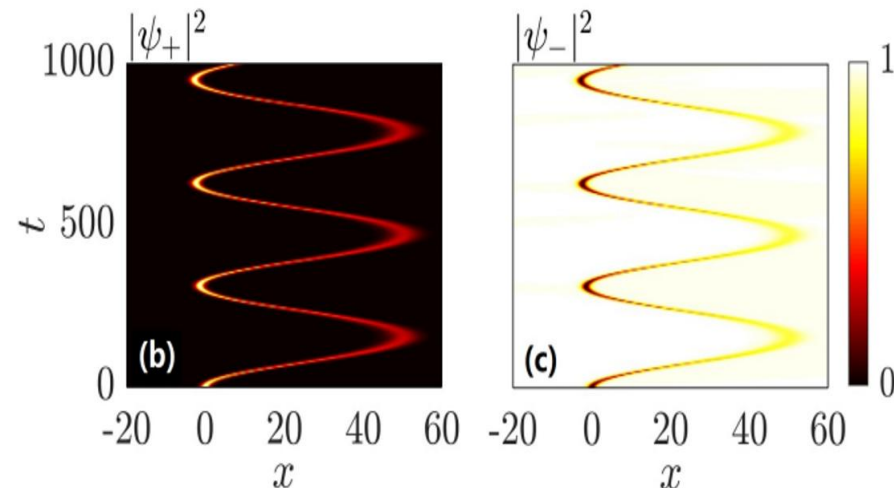
自旋孤子



新奇动力学现象



非线性自适应约瑟夫森振荡



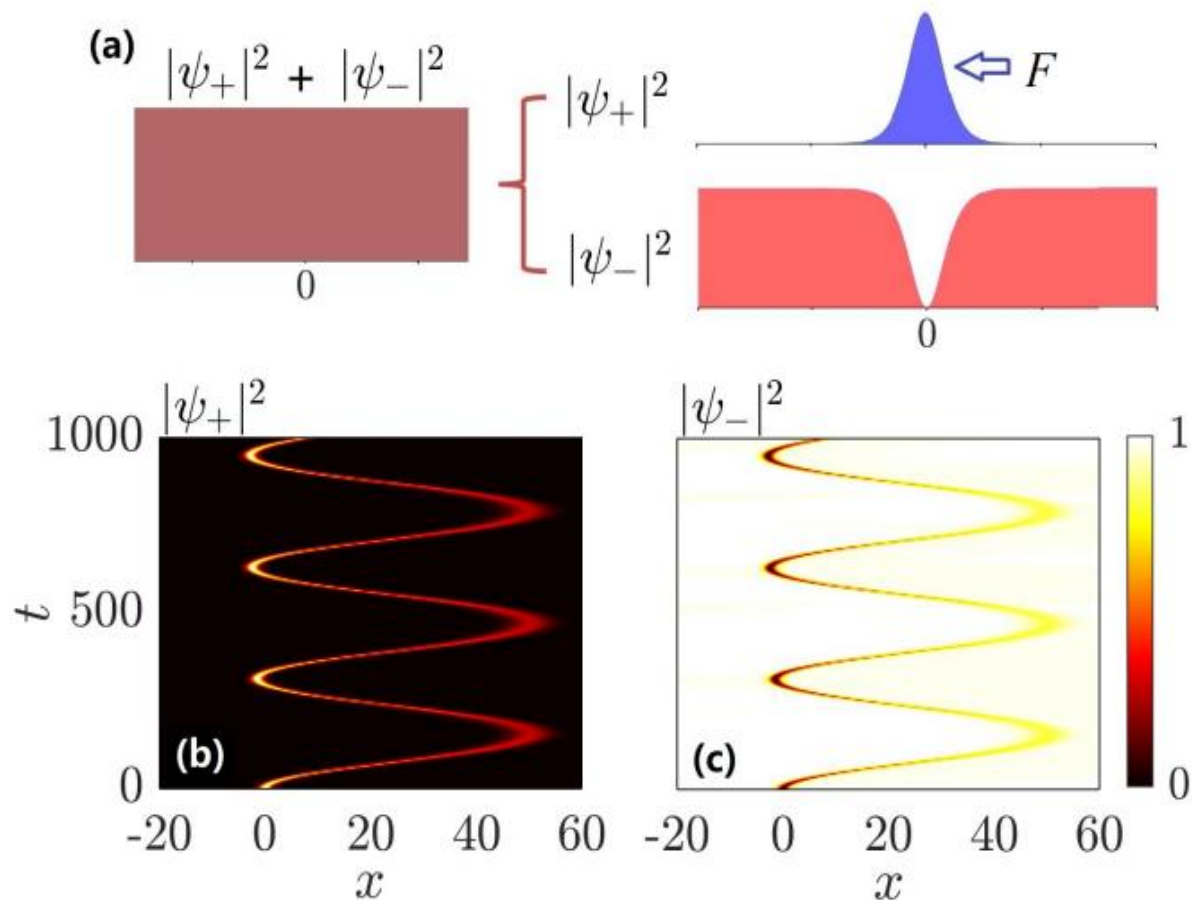
常力驱动自旋孤子的周期振荡

一、可积系统与孤子物理简介

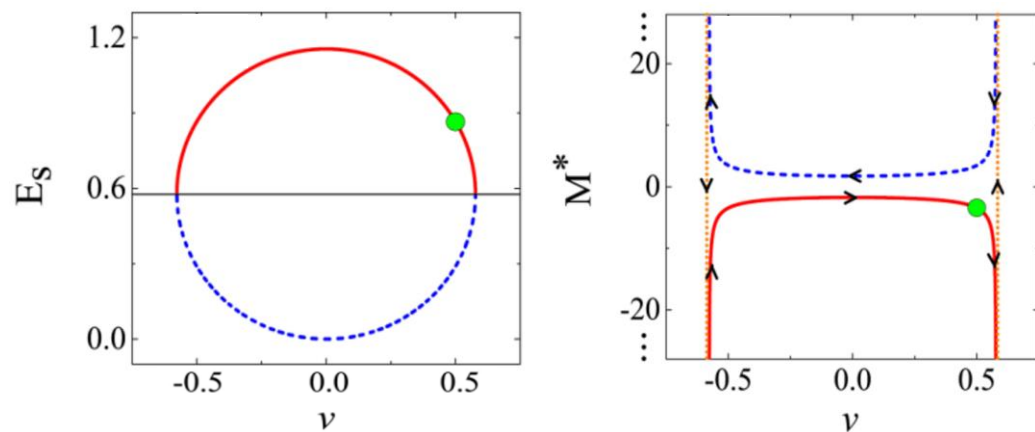
预言新型局域波动力学方面

自旋孤子振荡现象

无周期外势

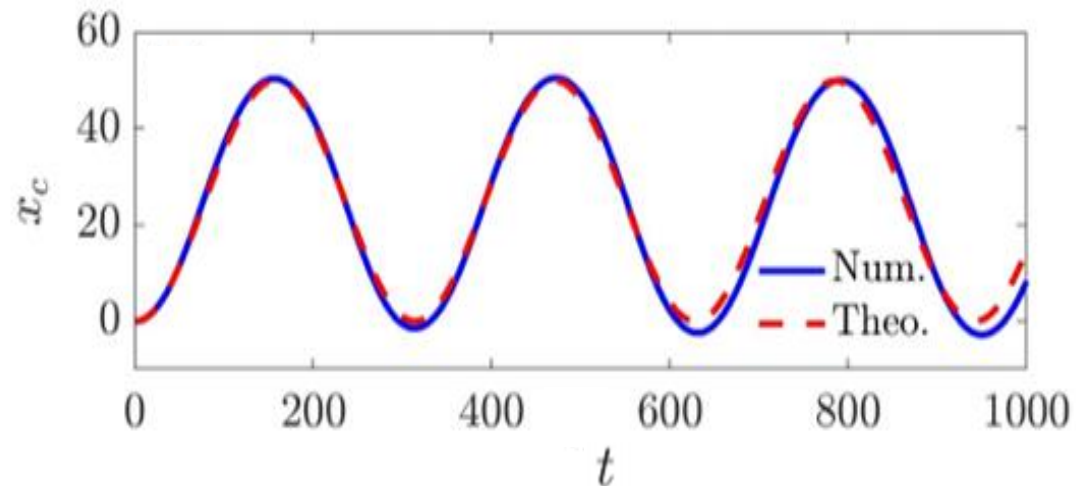


理论描述与解释

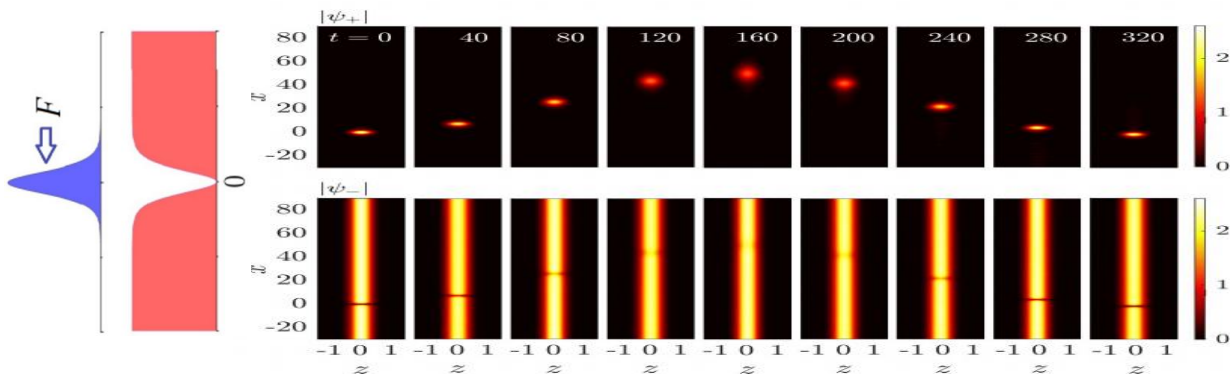


有效质量 $M^* = \pm \frac{M_0^*}{\sqrt{1 - \frac{v^2}{c_s^2}}}$

类似于狭义相对论中
质量-速度关系

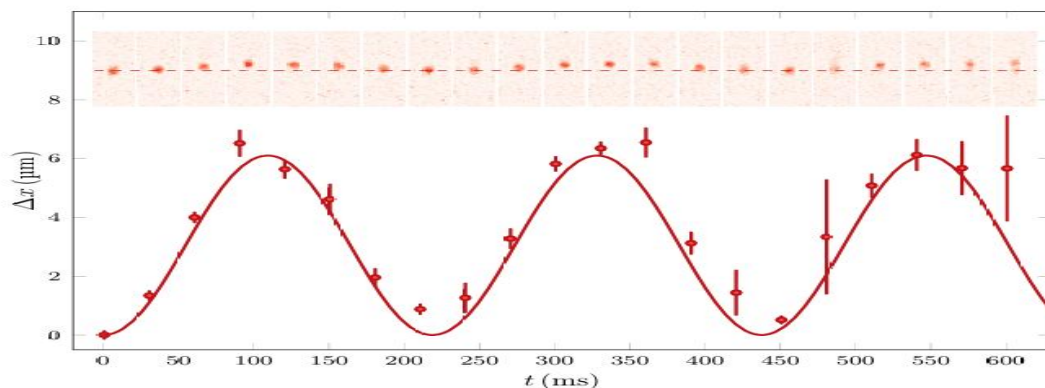


一、可积系统与孤子物理简介



我们的理论预言

Phys. Rev. A 101, 043621 (2020)



J. Dalibard小组的实验观测

nature physics

Article

<https://doi.org/10.1038/s41567-025-02970-1>

Bloch oscillations of a soliton in a one-dimensional quantum fluid

Received: 12 August 2024

F. Rabec^{1,2}, G. Chauveau^{1,2}, G. Brochier¹, S. Nascimbene¹, J. Dalibard¹ & J. Beugnon¹✉

Accepted: 9 June 2025

In this work, we study BOs of a large number of interacting particles without any lattice potential in a two-component 1D system. The possibility of observing BOs of solitons in multicomponent Bose gases has been recently discussed in refs. 34–37. We use here a weakly interacting quasi-1D Bose gas with two components, called |1⟩ and |2⟩. The mesoscopic system studied is a localized wave packet in component |2⟩ immersed in an extended quantum bath of atoms in component |1⟩, realizing a so-called magnetic soliton^{38–43}. The total ground-state energy of this system is a periodic function of its momentum. The application of a weak, constant and uniform force to the wave packet therefore results in a linear increase in momentum with time and in an

34. Zhao, L.-C. et al. Spin soliton with a negative-positive mass transition. *Phys. Rev. A* **101**, 043621 (2020).
35. Yu, X. & Blakie, P. Propagating ferrodark solitons in a superfluid: exact solutions and anomalous dynamics. *Phys. Rev. Lett.* **128**, 125301 (2022).
36. Bresolin, S., Roy, A., Ferrari, G., Recati, A. & Pavloff, N. Oscillating solitons and ac Josephson effect in ferromagnetic Bose-Bose mixtures. *Phys. Rev. Lett.* **130**, 220403 (2023).
37. Meng, L.-Z., Luo, X.-W. & Zhao, L.-C. Self-adapted Josephson oscillation of dark-bright solitons under constant forces. Preprint at <https://arxiv.org/abs/2501.15841> (2025).

一、可积系统与孤子物理简介

如何利用孤子动力学深入探究相关基础物理问题？

一、可积系统与孤子物理简介

3. 发展非线性局域波物理理论

通过定义孤子的有效波矢和本征能量等物理量，基于严格解析解定量刻画了孤子的干涉等波动性质

孤子干涉

利用变分法解决部分-可积系统中孤子色散关系分析困难（暗孤子背景引起的能量发散、粒子亏缺数不一定守恒）

有效质量

将狄拉克磁单极理论推广至复空间，揭示了局域波相位背后普遍存在的狄拉克磁单极势。

几何/拓扑性质

可测量重力加速度、原子间散射长度等物理量的物质波孤子干涉仪

常力驱动孤子周期振荡中周期和外力的定量关系可以实现弱力测量

精密测量

孤子中的拓扑或几何性质？

二、孤子中的几何相位

周期外场驱动孤子中的拓扑效应

Thouless pump of solitons

线性能带拓扑适用

M. Jürgensen, S. Mukherjee, and M. C. Rechtsman, Quantized nonlinear Thouless pumping, *Nature* **596**, 63 (2021).

M. Jürgensen, S. Mukherjee, C. Jörg and M. C. Rechtsman, Quantized fractional Thouless pumping of solitons, *Nat. Phys.* **19**, 420 (2023).

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Q. Fu, P. Wang, Y. V. Kartashov, V. V. Konotop, and F. Ye, Nonlinear Thouless Pumping: Solitons and Transport Breakdown, *Phys. Rev. Lett.* **128**, 154101 (2022).

Q. Fu, P. Wang, Y. V. Kartashov, V. V. Konotop, and F. Ye, Two-Dimensional Nonlinear Thouless Pumping of Matter Waves, *Phys. Rev. Lett.* **129**, 183901 (2022).

T. Tuloup, R. W. Bomantara, and J. Gong, Breakdown of quantization in nonlinear Thouless pumping, *New J. Phys.* **25**, 083048 (2023).

N. Mostaan, F. Grusdt and N. Goldman, Quantized topological pumping of solitons in nonlinear photonics and ultracold atomic mixtures, *Nat. Commun.* **13**, 5997 (2022).

X. Cao, C. Jia, Y. Hu and Z. Liang, Nonlinear Thouless pumping of solitons across an impurity, *Phys. Rev. A* **110**, 013305 (2024).

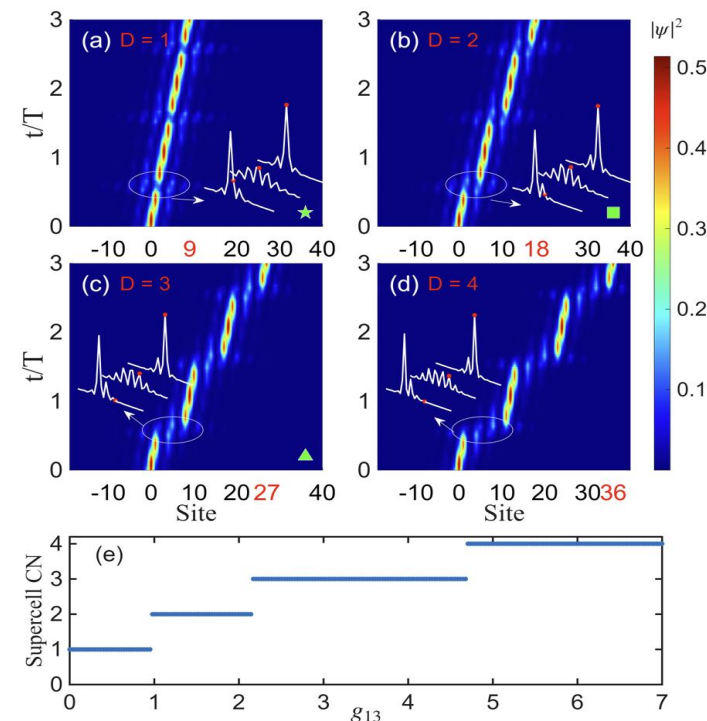
超越线性能带拓扑

Chen number of the three linear bands are +1, -2, +1

K. Sone, M. Ezawa, Y. Ashida, N. Yoshioka and T. Sagawa, Nonlinearity-induced topological phase transition characterized by the nonlinear Chern number, *Nat. Phys.* **20**, 1164 (2024).

Y.-L. Tao, Y.-P Zhang, and Y. Xu, Nonlinearity-Induced Fractional Thouless Pumping of Solitons, *Phys. Rev. Lett.* **135**, 097202 (2025).

Y.-L. Tao, J.-H Wang, and Y. Xu, Nonlinearity-Induced Thouless Pumping of Solitons, [arXiv:2409.19515](https://arxiv.org/abs/2409.19515).



T. Jiang, LC Zhao, to be published.

拓扑背后一般跟虚拟磁场有关，虚拟磁场还可以产生几何相位等可观测效应

二、孤子中的几何相位

SOC

参数空间R的拓扑

$$\gamma_m(C) = \iint_S \mathbf{B}_m(\mathbf{R}) \cdot d\mathbf{S}.$$

晶格系统**电子波包**

$$\hbar \dot{\mathbf{r}}_c = \nabla_{\mathbf{k}} \epsilon_n(\mathbf{k}_c) - \hbar \dot{\mathbf{k}}_c \times \mathbf{B}_n(\mathbf{k}_c)$$

$$\hbar \dot{\mathbf{k}}_c = -e\mathbf{E} - e\dot{\mathbf{r}}_c \times \mathbf{B}(\mathbf{r}_c)$$

几何相位

反常速度漂移

单粒子情形的主要问题：波函数的**快速色散**

BEC

孤子

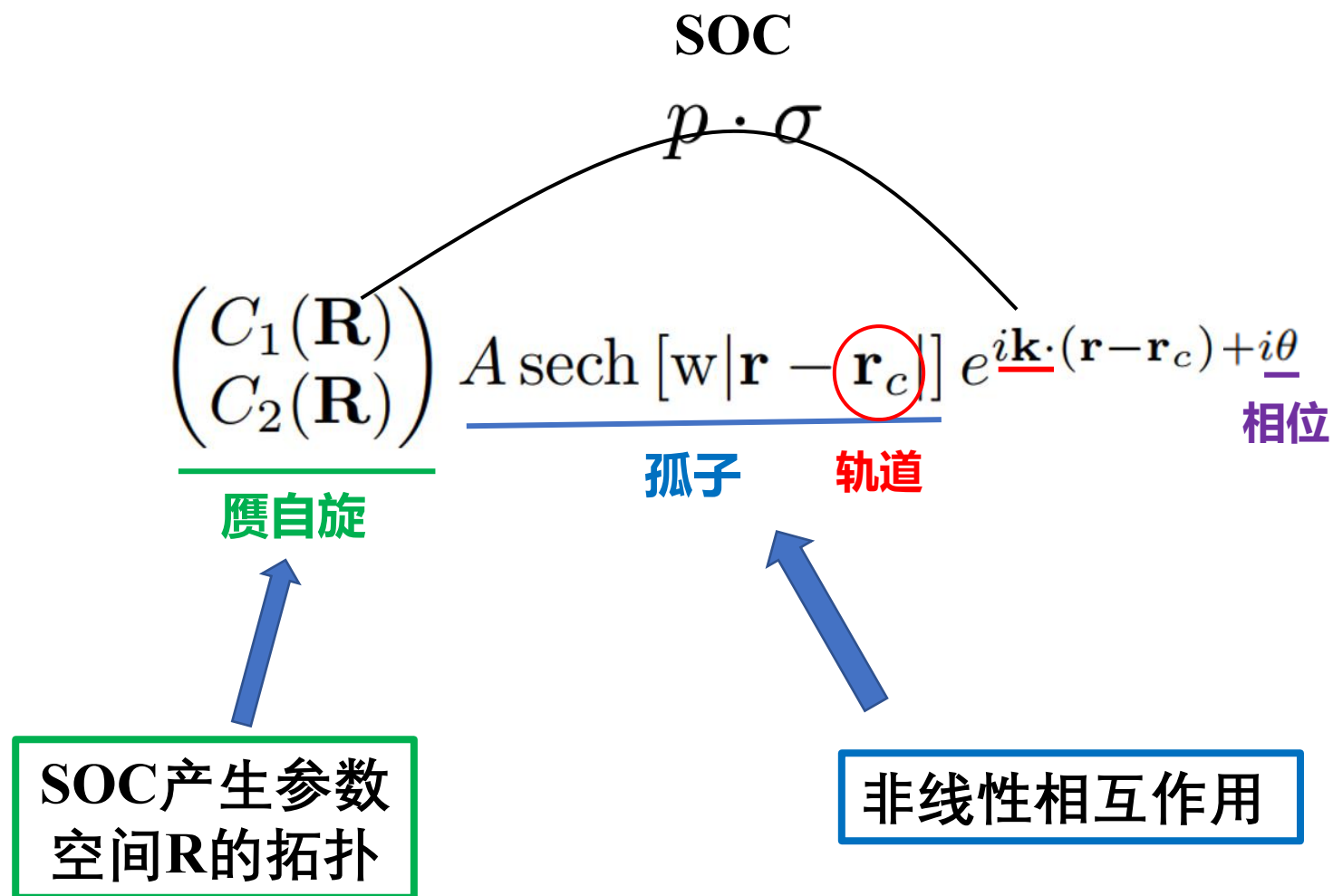
$$A \operatorname{sech} [w|\mathbf{r} - \mathbf{r}_c|] e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}_c)}$$

鲜明的粒子性和波动性
可方便观测轨道和相位

如何发挥**孤子的优势**探究几何相位和反常速度漂移效应？

二、孤子中的几何相位

SOC-BEC



二、孤子中的几何相位

$$\underbrace{\begin{pmatrix} C_1(\mathbf{R}) \\ C_2(\mathbf{R}) \end{pmatrix}}_{\text{赝自旋}} \underbrace{A \operatorname{sech}[w|\mathbf{r} - \mathbf{r}_c|]}_{\text{孤子}} \underbrace{e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}_c) + i\theta}}_{\text{相位}}$$

$p \cdot \sigma$

$\mathbf{r}_c$ 轨道

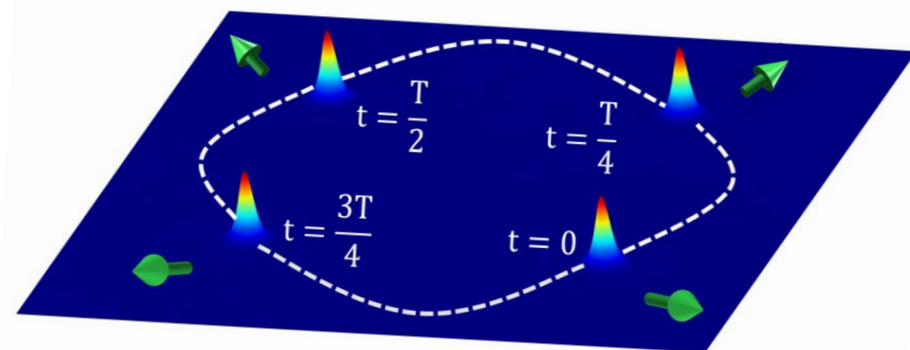
θ 相位

巡回演化

几何相位

$\mathbf{k}$ 和 $\mathbf{r}_c$ 的关联性

反常轨道漂移

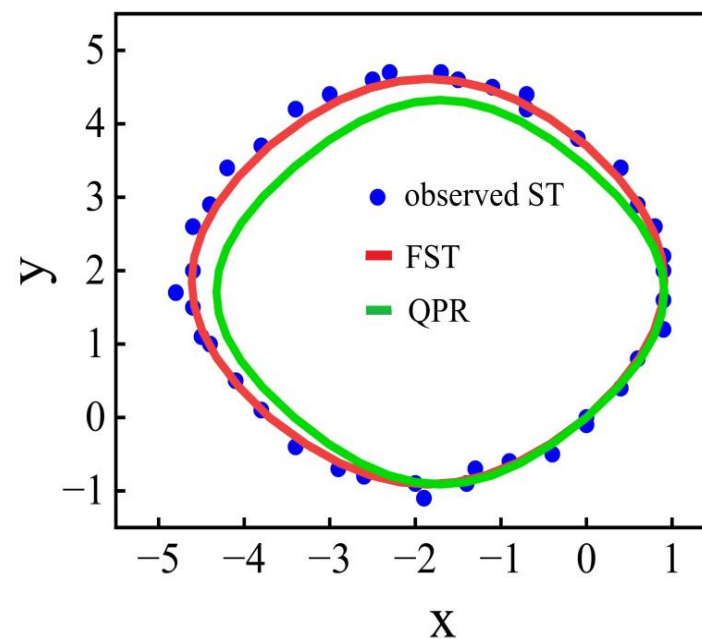
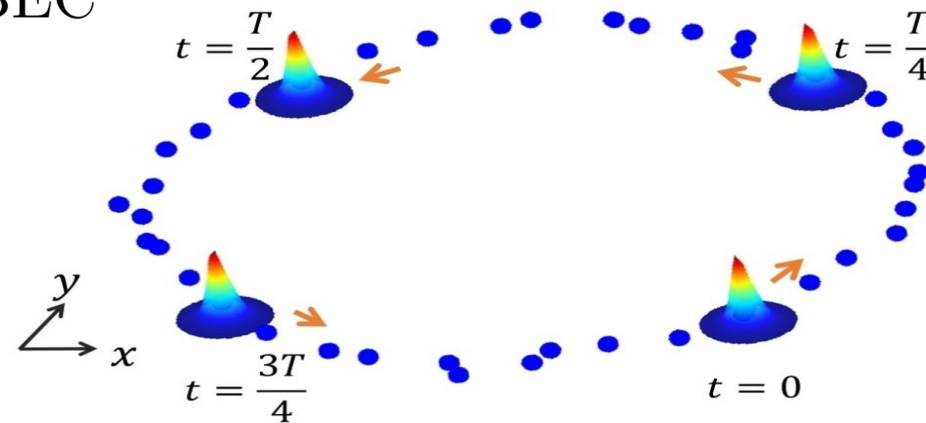


二、孤子中的几何相位

孤子运动中的反常速度漂移

$$\begin{aligned}\dot{\mathbf{k}} &= -\nabla V(\mathbf{r}_c), \\ \dot{\mathbf{r}}_c &= \nabla_{\mathbf{k}} E(\mathbf{k}) - \boxed{\dot{\mathbf{k}} \times \mathbf{B}},\end{aligned}$$

SOC-BEC



二、孤子中的几何相位

孤子运动中的反常速度漂移与Niu方程的区别及联系

对比的方面	Niu 方程	孤子方程
物理情景	周期势系统	无周期势
波包特征	扩散的波包	不扩散的孤子
方程形式	波包质心的动力学方程 $\hbar \dot{\mathbf{r}}_c = \nabla_{\mathbf{k}} \epsilon_n(\mathbf{k}_c) - \hbar \dot{\mathbf{k}}_c \times \mathbf{B}_n(\mathbf{k}_c)$$\hbar \dot{\mathbf{k}}_c = -e\mathbf{E} - e\dot{\mathbf{r}}_c \times \mathbf{B}(\mathbf{r}_c)$	孤子中心的动力学方程 $\dot{\mathbf{r}}_c = \nabla_{\mathbf{k}} E(\mathbf{k}) - \dot{\mathbf{k}} \times \mathbf{B},$$\dot{\mathbf{k}} = -\nabla V(\mathbf{r}_c)$

二、孤子中的几何相位

超越线性能带拓扑的理论刻画？

利用反常速度漂移重构参数空间的有效磁场

$$\dot{\mathbf{r}}_c = \nabla_{\mathbf{k}} E(\mathbf{k}) - \boxed{\dot{\mathbf{k}} \times \mathbf{B}},$$

$$\dot{\mathbf{k}} = -\nabla V(\mathbf{r}_c)$$

可验证理论推导

或可帮助反向构建理论

二、孤子中的几何相位

孤子巡回运动的几何相位

Berry phase

dynamical phase

$$\dot{\theta} = \boxed{\mathbf{A} \cdot \dot{\mathbf{R}}} + \boxed{\mathbf{k}(t) \cdot \dot{\mathbf{r}}_c(t)} \boxed{- \mu}$$

action induced phase

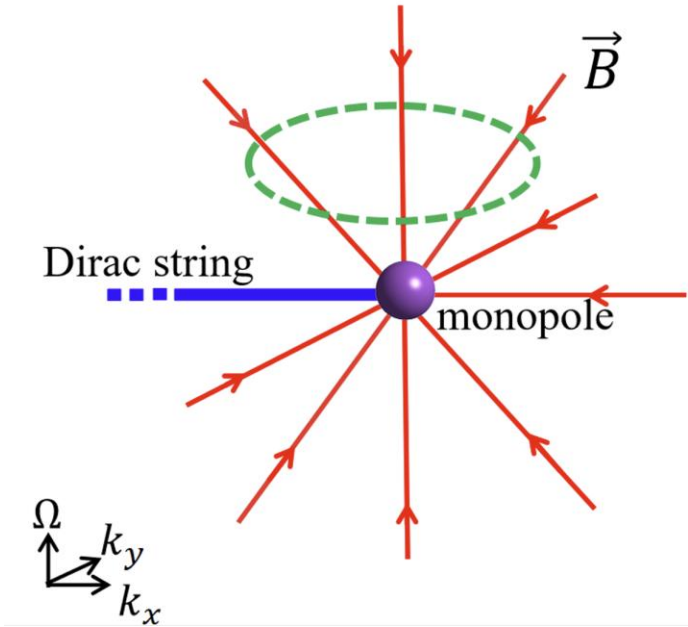
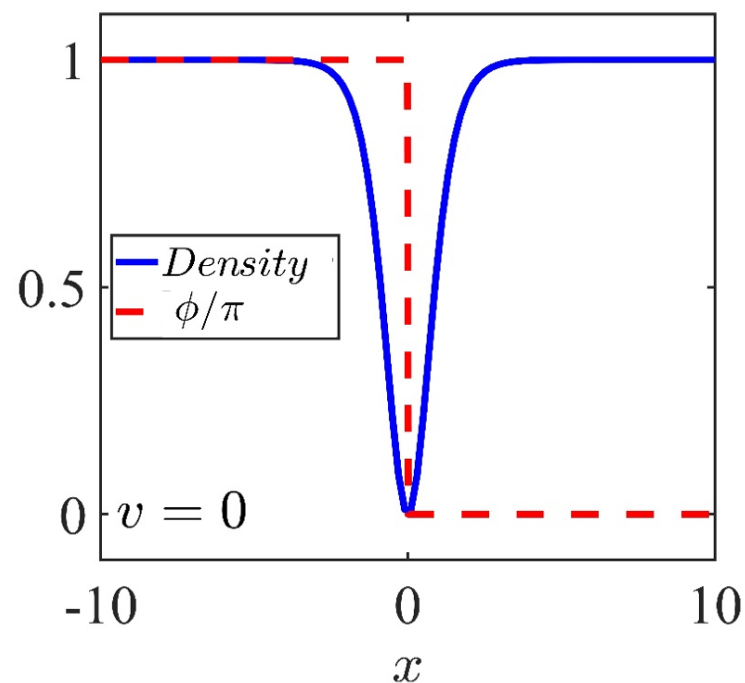


TABLE I: The observed total phases of soliton during one cyclic evolution and the corresponding theoretical values.

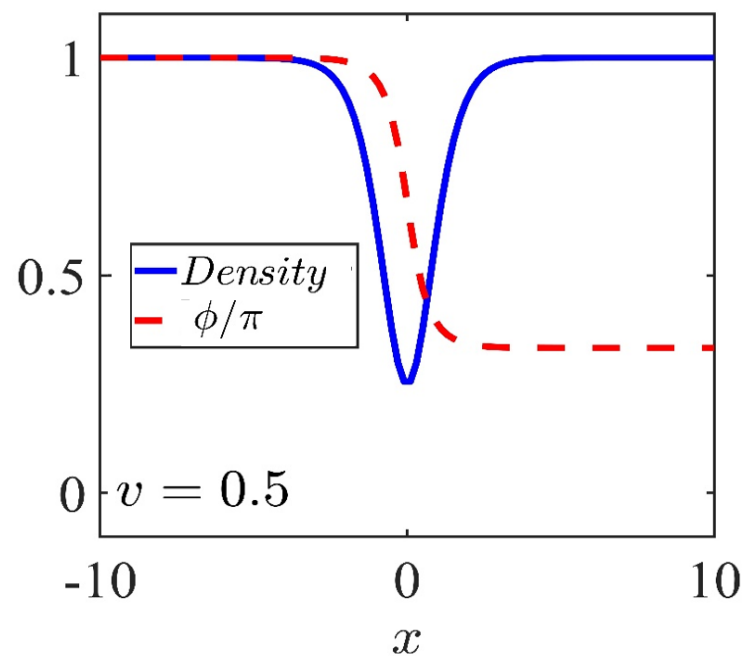
Ω	observed values	theoretical values			
	θ_{total}	θ_{total}	θ_{Berry}	$\theta_{\text{A-A}}$	$\theta_{\text{dyn.}}$
0.5	14.52π	14.41π	-0.59π	7.50π	7.50π
0	11.41π	11.32π	$-\pi$	4.72π	7.60π
-0.5	15.69π	15.59π	0.59π	6.82π	8.18π

三、孤子中的虚拟磁单极

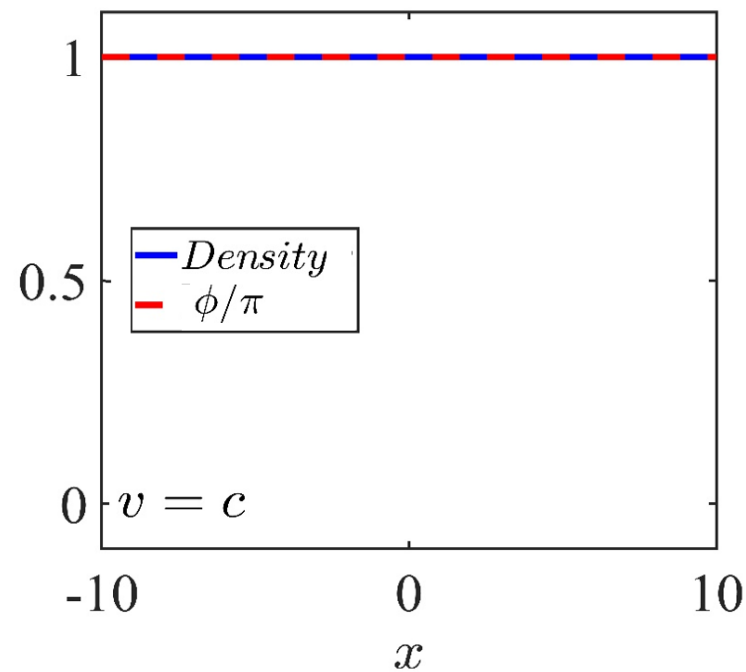
暗孤子：常见的一维拓扑孤子



暗孤子相位的拓扑起源？



它的拓扑场表示？



三、孤子中的虚拟磁单极

暗孤子：

$$\bar{\psi}(x, v) = \left[-iv + \sqrt{1 - v^2} \tanh(\sqrt{1 - v^2} x) \right] e^{ivx}$$

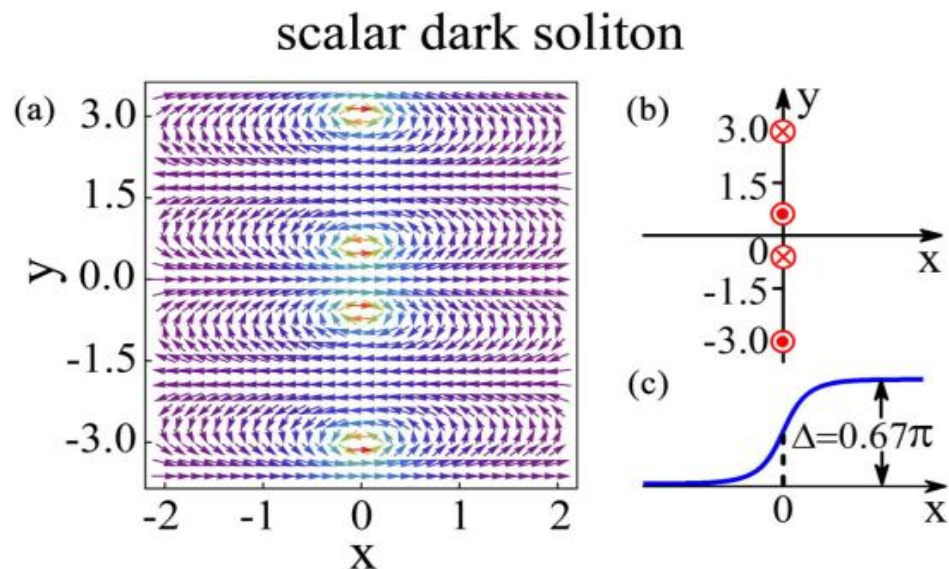


FIG. 1. (a) Topological vector potential $\mathbf{A}$ for a scalar dark soliton. (b) Corresponding magnetic field $\mathbf{B}$, and (c) the phase jump induced by the magnetic field. The soliton speed is set to $v = 0.50$. The pointlike magnetic fields are located at $(0, \pm 0.61)$ with a period of 1.15π along the imaginary axis.

暗孤子相位的拓扑起源

$$\mathbf{A} = \sum_N \frac{\pm \Omega [(x - x_N) \mathbf{e}_y - (y - y_N) \mathbf{e}_x]}{2\pi [(x - x_N)^2 + (y - y_N)^2]},$$

速度为零时相位跃变解释：

$$\begin{aligned} \lim_{v \rightarrow 0} \mathbf{A} &= \lim_{|y_0| \rightarrow 0} \left[\frac{\pi (x \mathbf{e}_y + |y_0| \mathbf{e}_x)}{2\pi (x^2 + |y_0|^2)} + \frac{-\pi (x \mathbf{e}_y - |y_0| \mathbf{e}_x)}{2\pi (x^2 + |y_0|^2)} \right] \\ &= \pi \delta[x] \mathbf{e}_x. \end{aligned}$$

三、孤子中的虚拟磁单极

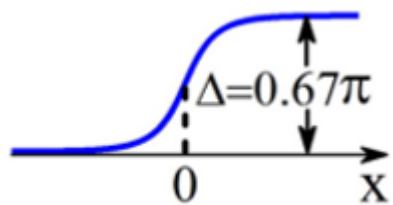
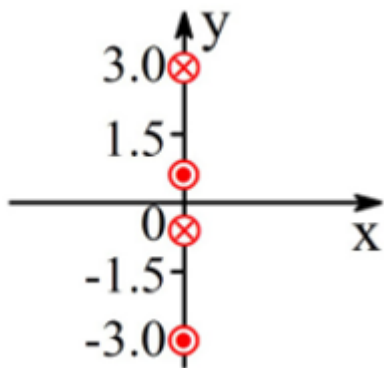
与Aharonov-Bohm效应的比较

$$\tilde{\psi} = \psi_0 \exp \left[-i \int \mathbf{A}(x, y=0) \cdot \mathbf{e}_x dx \right]$$

$$i\partial_t \tilde{\psi} = \left(\frac{(\hat{p}_x + \mathbf{A}(x, y=0) \cdot \mathbf{e}_x)^2}{2} + (g|\tilde{\psi}|^2 + \frac{\partial \int \mathbf{A}(x, y=0) \cdot \mathbf{e}_x dx}{\partial t}) \right) \tilde{\psi}$$

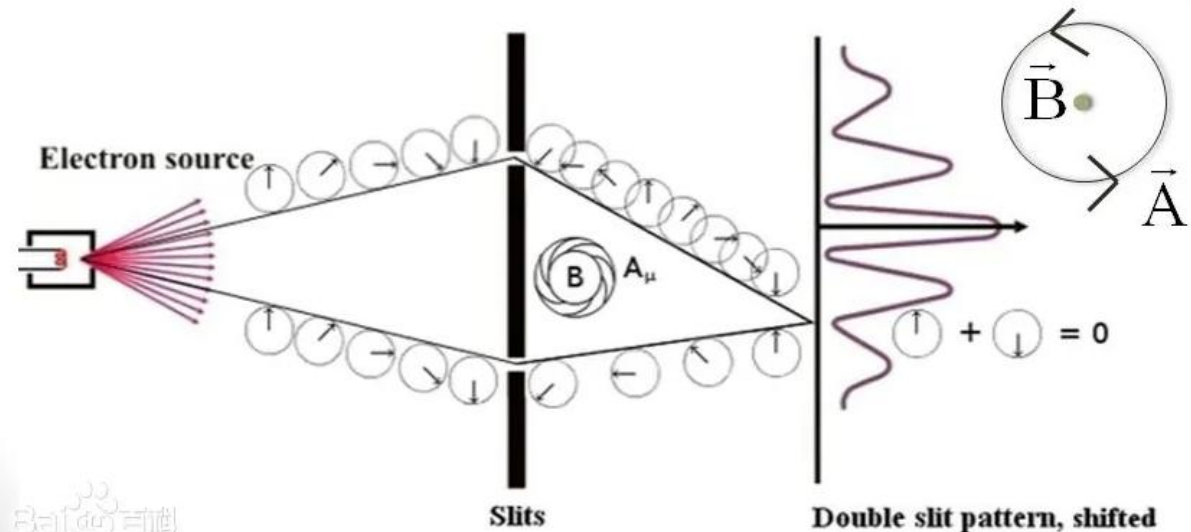
$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \frac{\partial \int \mathbf{A}(x, y=0) \cdot \mathbf{e}_x dx}{\partial t}$$



Aharonov-Bohm Effect

- A solenoid with the current is placed just behind the slits
- Uniform magnetic field is inside the solenoid
- Magnetic field outside is equal to zero
- But vector potential of the magnetic field is non-zero outside!



三、孤子中的虚拟磁单极

狄拉克磁单极理论

$$\psi(\mathbf{r}, t) = |\psi|e^{i\phi}$$

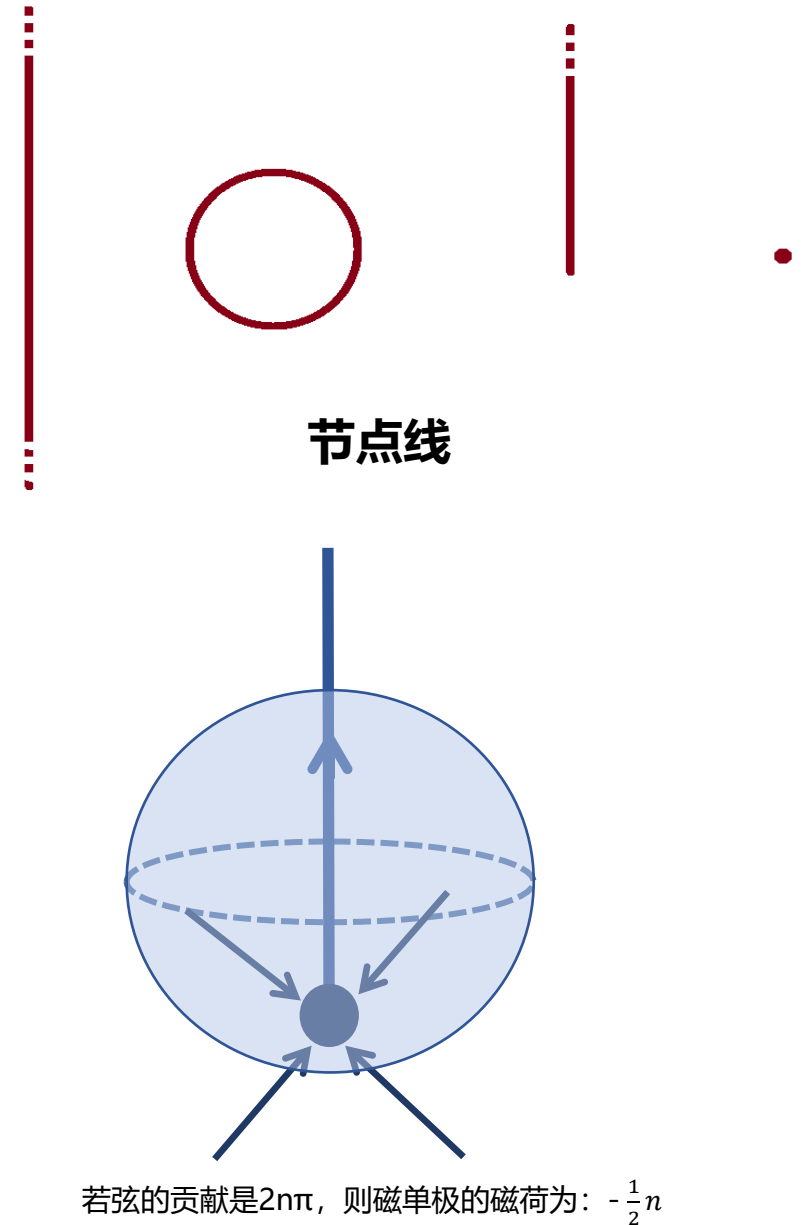
$$\mathbf{A} = (A_x, A_y, A_z) = \frac{\partial\phi}{\partial x}\mathbf{e}_x + \frac{\partial\phi}{\partial y}\mathbf{e}_y + \frac{\partial\phi}{\partial z}\mathbf{e}_z$$

$$\frac{\partial A_\mu}{\partial \nu} \neq \frac{\partial A_\nu}{\partial \mu}$$

$$\mathbf{A}_{3D} = \frac{y\mathbf{e}_x - x\mathbf{e}_y}{2r(r \pm z)}$$

狄拉克弦的端点对应磁单极

P. A. M. Dirac, Proc. R. Soc. Lond. A 133, 60-72 (1931)



三、孤子中的虚拟磁单极

$$\psi(x, t)$$

$$\begin{aligned}\mathcal{A}_c &= \frac{\partial\phi(Z)}{\partial x}\mathbf{e}_x + \frac{\partial\phi(Z)}{\partial y}\mathbf{e}_y \\ &= \frac{\partial\phi(Z)}{\partial Z}(\mathbf{e}_x + i\mathbf{e}_y).\end{aligned}$$

$$\mathbf{A} = \text{Re}[\mathcal{A}_c] = \sum_N \frac{\pm\mu[(x - x_N)\mathbf{e}_y - (y - y_N)\mathbf{e}_x]}{(x - x_N)^2 + (y - y_N)^2},$$

$$\mu = 1/2 \quad \frac{\partial A_x}{\partial y} \neq \frac{\partial A_y}{\partial x}$$

密度零点对应拓扑矢势的奇点(磁单极)

Lee–Yang 零点：固定温度，把磁场 / 逸度变复数得到的配分函数零点，刻画磁场驱动相变；

Fisher 零点：固定外场，把温度 (β) 变复数得到的配分函数零点，刻画温度驱动相变。

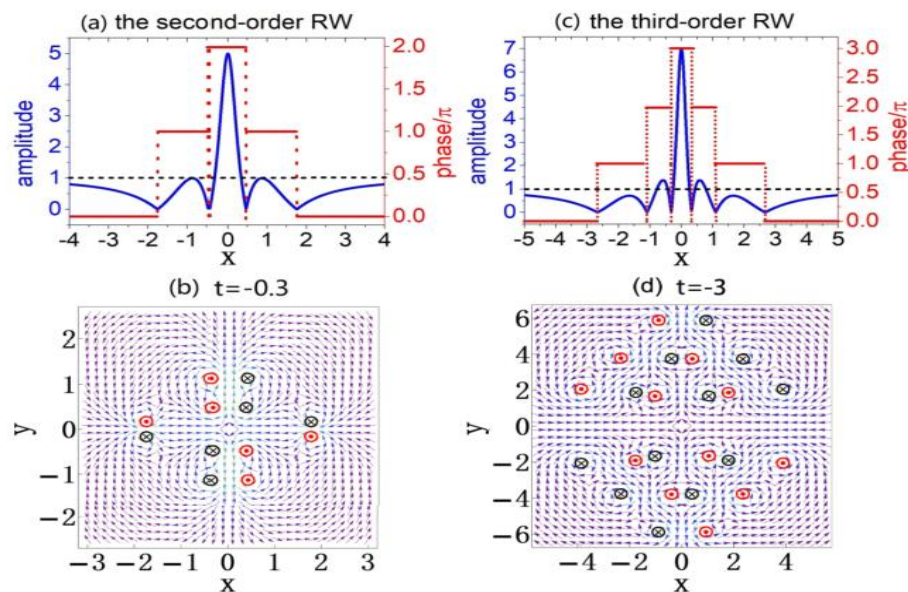
一维波函数**密度零点**一般地分布在**复平面**，我们推广狄拉克在实空间定义矢势的方式到复平面。结果表明在复空间广泛存在着**荷为1/2**的磁单极子势。

三、孤子中的虚拟磁单极

对比的方面	狄拉克磁单极理论	我们的工作
空间维度	三维	一维
不可积相因子	可定义相位空间梯度的 不可交换性	无法定义相位空间梯度的 不可交换性
零点分布特征	实空间节点线	复空间的离散零点
磁单极特征	可独立出现 $n/2$ 的荷	成对出现 $1/2$ 的荷

三、孤子中的虚拟磁单极

磁单极的碰撞可解释怪波动力学

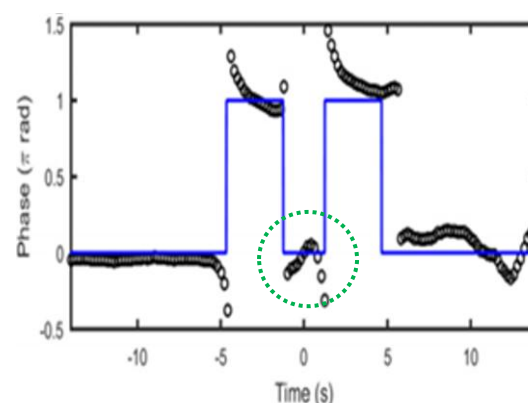


高阶怪波背后的磁单极势

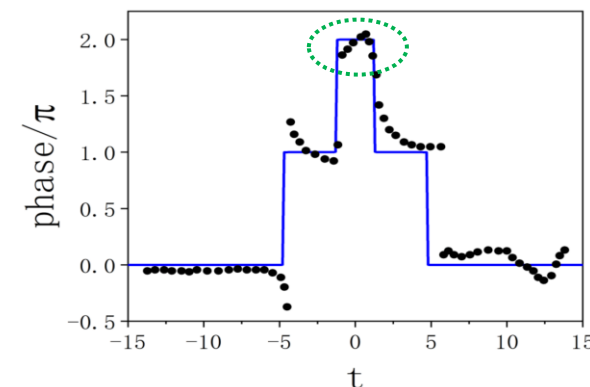
相反磁荷磁单极的碰撞产生 π 相位跃变
碰撞后交换磁荷使得相位跃变反转

Zhao, Meng, Qin, Yang, Liu, Chin. Phys. Lett. 42, 110002 (2025)

解决Dudley实验组怪波相位数据处理结果的不自洽问题



Dudley组实验和理论对比



我们修正后实验和理论对比
Phys. Rev. A (2023)

为实验测量涉及相位奇性问题提供理论参考

$$\begin{aligned} \lim_{t \rightarrow \pm 0} \mathbf{A} &= \lim_{b \rightarrow 0} \left\{ \frac{-\pi[(x \pm a_0)\mathbf{e}_y + b\mathbf{e}_x]}{2\pi[(x \pm a_0)^2 + b^2]} + \frac{\pi[(x \pm a_0)\mathbf{e}_y - b\mathbf{e}_x]}{2\pi[(x \pm a_0)^2 + b^2]} \right. \\ &\quad \left. + \frac{\pi[(x \mp a_0)\mathbf{e}_y + b\mathbf{e}_x]}{2\pi[(x \mp a_0)^2 + b^2]} + \frac{-\pi[(x \mp a_0)\mathbf{e}_y - b\mathbf{e}_x]}{2\pi[(x \mp a_0)^2 + b^2]} \right\} \\ &= (-\pi\delta[x \pm a_0] + \pi\delta[x \mp a_0]) \mathbf{e}_x. \end{aligned}$$

四、总结与讨论

- 1 孤子作为可获得的解析解研究各种相位问题颇具便捷性
- 2 拓展复平面中虚拟磁单极存在具有普遍性

谢谢大家！