

Domain wall for RG flows

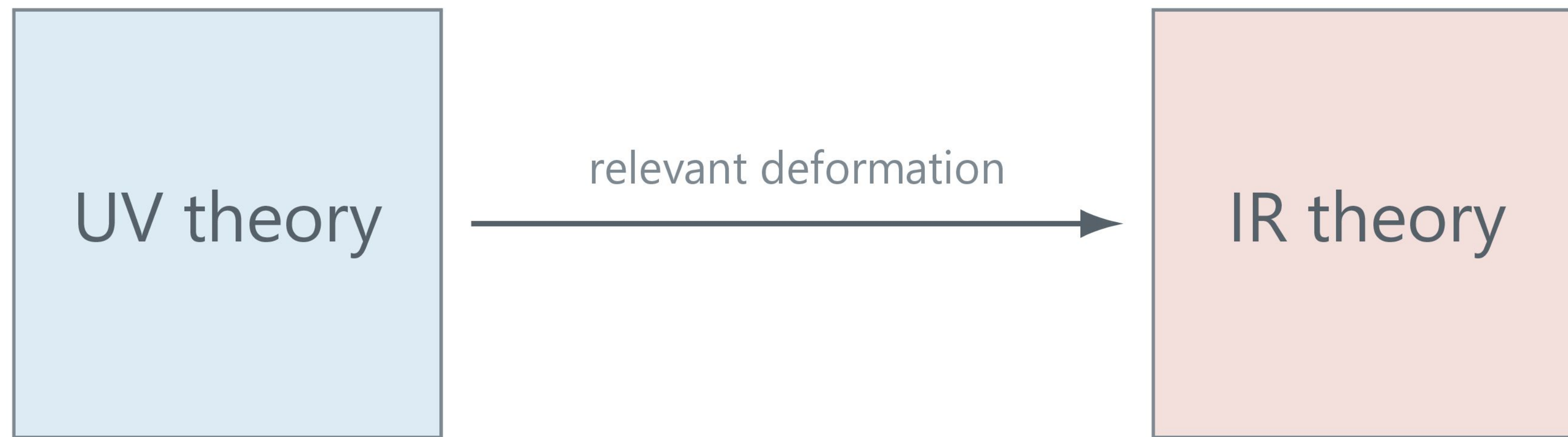
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The talk is based a recent work [arXiv: 2605.24978](#), with [Jin-Rui Zhang](#), [Jing-Hao Jin](#), and [Ting-Kai Chen](#), and a upcoming work with [Antonio Antunes](#) and [Junchen Rong](#)

I. Motivation:

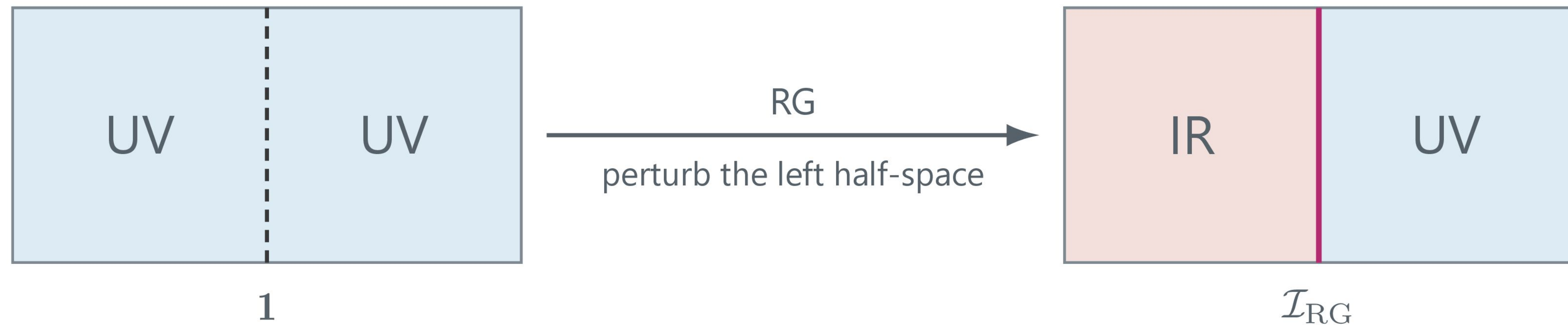
A quantum field theory looks different at different energy scales. The evolution of its effective description with scale is a renormalization-group flow.



$$S_{UV} \longrightarrow S_{UV} + \lambda \int_{\mathbb{R}^D} d^D x \, \Phi(x) \, , \quad \Delta_{\Phi} < D$$

At a nontrivial fixed point, the theory is scale invariant and, under the standard assumptions of relativistic QFT, is conformal.

An ordinary RG flow separates theories in scale; an RG wall separates them in space:



$$S_{\text{step}} = S_{UV} + \lambda \int_{\mathbb{R}^D} d^D x \, \theta(-x_{\perp}) \Phi(x)$$

The finite-scale step defect is not conformal. The RG wall is its infrared fixed-point limit, if that limit exists. The flow also includes wall-localized couplings:

$$S_{\text{eff}} = S_{\text{step}} + \sum_a \mu^a \int_{x_{\perp}=0} d^{D-1} y \, \hat{\mathcal{O}}_a(y) \quad \Longrightarrow \quad \beta_{\text{bulk}}(\lambda) = \beta_{\text{wall}}(\mu^a) = 0$$

A half-space deformation converts an RG trajectory into a codimension-one fixed point object.

Why package an RG flow into a wall?

A. A flow-dependent UV-IR dictionary

$$M_{ji}^{\text{RG}} = \langle \mathcal{O}_j^{\text{IR}} | \mathcal{I}_{\text{RG}} | \mathcal{O}_i^{\text{UV}} \rangle$$

$$\mathcal{O}_{\text{IR}} = \left(\mathcal{O}_{\text{UV}} \right)^{\mathcal{I}_{\text{RG}}}$$

$$\Rightarrow \mathcal{O}_j^{\text{IR}} = \sum_i M_{ji}^{\text{RG}} \mathcal{O}_i^{\text{UV}} + \text{descendants} + \text{decoupled UV operators}$$

The cross-interface data determine:

- UV-IR operator pairings, UV data lost or reorganized in the IR;
- selection rules that survive the flow, sectors allowed to mix.

Beautiful wall example in $4 - \epsilon$ free/Wilson-Fisher $O(N)$ reproduces leading UV-IR mixing; defect crossing provides a non-perturbative handle for perturbative CFTs (**1502.07217**).

The wall remembers more than the endpoints: it probes the UV-IR operator map.

B. Transport symmetries, defects and boundary conditions

A preserved topological defect can be deformed through the wall into its IR image:

$$D_{\rho(a)}^{\text{IR}} \circ \mathcal{J}_{\text{RG}} = \mathcal{J}_{\text{RG}} \circ D_a^{\text{UV}}$$

The wall acts on boundary conditions: fusing it onto a UV boundary produces its renormalized IR image:

$$B_{\text{IR}} = \text{Reg} \left(\mathcal{J}_{\text{RG}} \star B_{\text{UV}} \right)$$

where $\star$ denotes fusion, and “Reg” indicates short-distance counterterms needed if necessary.

The wall tracks:

- ordinary and generalized symmetries, and possible anomaly constraints on their transport;
- boundaries and other extended operators crossing or ending on the wall.

The wall transports the extended data of the theory, not only local operators.

C. The wall is a new physical system

At its fixed point, the interface is a defect CFT with data beyond the two endpoints of RG flow.

Wall spectra and bulk-to-wall OPE: The two bulk sides couple through,

$$\mathcal{O}_i^{(s)}(y, x_\perp) = \sum_{\alpha} \lambda_{i\alpha}^{(s)} |x_\perp|^{\hat{\Delta}_\alpha - \Delta_i^{(s)}} \hat{\mathcal{O}}_\alpha(y) + \cdots, \quad s \in \{\text{UV}, \text{IR}\}$$

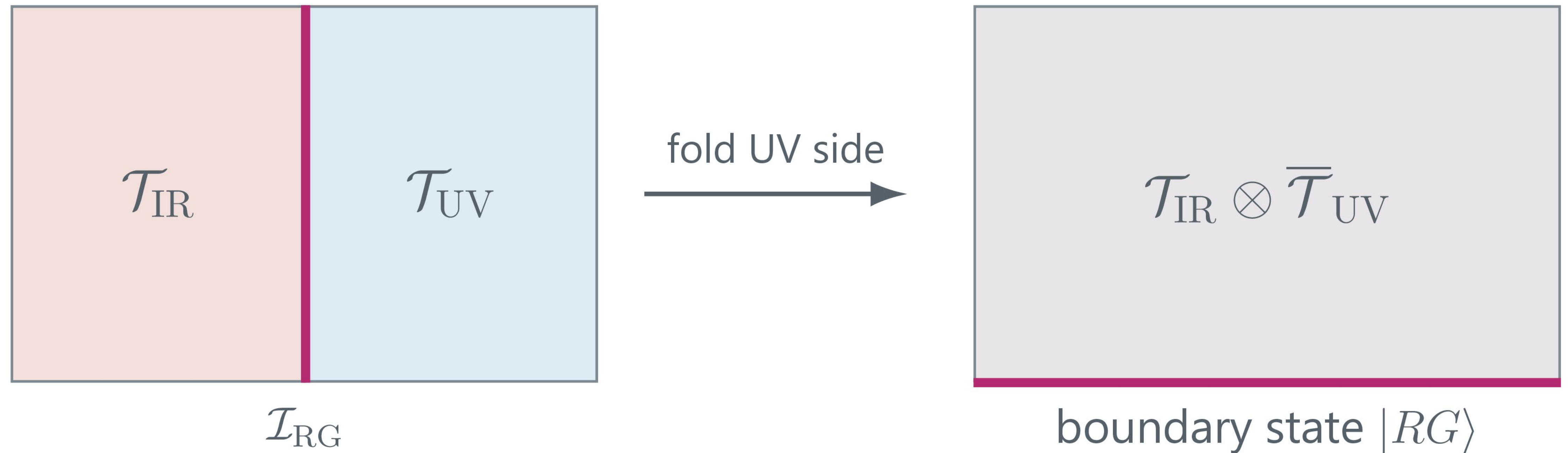
Displacement operator: $D(y)$ probes normal deformations of the wall; C_D , its 2-pt correlator coefficient, measures the coupling between the bulk and wall.

$$\partial_\mu T^{\mu\perp}(x) = \delta(x_\perp) D(y), \quad \Delta_D = D. \quad \langle D(y) D(0) \rangle = \frac{C_D}{|y|^{2D}}$$

Sphere free energy and defect Weyl anomaly: Map the planar wall to the equator of S^D . The universal logarithmic terms encode interface anomalies (in even dim.)

$$(\mathbb{S}^D, \mathcal{F}_{\text{RG}} \text{ on } \mathbb{S}_{\text{equator}}^{D-1}) \implies Z_{\mathbb{S}^D}(\mathcal{F}_{\text{RG}}) \propto a_{\text{bulk}} \log \frac{R}{\epsilon} + \mathcal{F}_{\text{interface}} \cdots$$

My talk focuses on RG walls in 2D RCFTs. Using folding trick, one turns the interface into a boundary conditions of the product theory, encoded in the closed channel by $|RG\rangle$:



Even for 2D RCFTs, constructing the wall non-perturbatively remains difficult. A generic RG boundary need preserve only the total Virasoro algebra, not the full product chiral algebra, and is therefore generally non-rational. The usual Cardy construction does not classify such boundaries. No general classification is known.

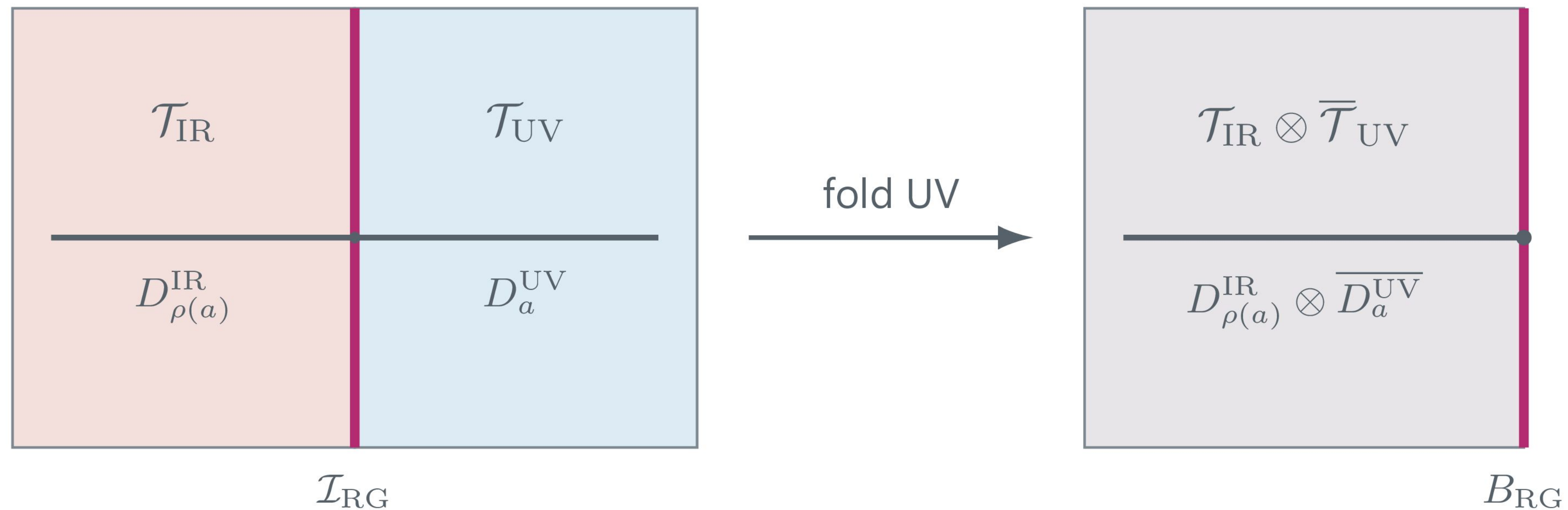
The talk will be given as follows:

- Motivation to study RG domain walls; 
- Bottom-up approach: generalized symmetries, phantom currents;
- Top-down approach: generalized orbifolding, automorphism, and brane transports;
- Outlook

II. Bottom-up Approach:

A RG wall is intrinsically non-perturbative, and hard to determine. However, if D_a is topological defect in $\mathcal{T}_{\text{UV}}$ and completely transparent to the perturbation, it will remain topological in $\mathcal{T}_{\text{IR}}$,

$$D_{\rho(a)}^{\text{IR}} \circ \mathcal{J}_{\text{RG}} = \mathcal{J}_{\text{RG}} \circ D_a^{\text{UV}}$$



$D_{\rho(a)}^{\text{IR}} \otimes \overline{D_a^{\text{UV}}}$ can end topologically on B_{RG}

Preserved defects constrain the allowed sectors and junctions. They are closed under the fusion. Together with their topological junctions, they define a preserved fusion category $\mathcal{C}_{\text{prev}}$.

$$D_a \cdot D_b = \bigoplus_c N_{ab}^c D_c, \quad \text{for } D_a \in \mathcal{C}_{\text{prev}}.$$

Here N_{ab}^c are fusion multiplicities. The fusion and crossing data are discrete RG invariants: The IR images of the preserved lines must reproduce their generalized 't Hooft anomaly.

$$s_\psi \equiv h_\psi - \bar{h}_\psi \pmod{\mathbb{Z}}, \quad \psi \in \mathcal{H}_a$$

The allowed values of s_ψ in the D_a -twisted Hilbert space $\mathcal{H}_a$, provide an independent spin selection rule. A mismatch of fusion, anomaly or defect-spin data rules out a candidate IR endpoint before the RG boundary states is known.

These are necessary matching conditions, but do not themselves construct the RG wall or prove that the flow exists. (**1201.0767, 1802.04445, 2407.21353, 2601.03879, 2603.09591**)

In the folded picture, Gaiotto noticed that the transmitted product line $D_{\rho(a)}^{\text{IR}} \otimes D_a^{\text{UV}}$ can end topologically on B_{RG} . So, shrinking the line to boundary will turn a chiral defect endpoint field into an operator on the RG wall,



In the above picture, the tether is deformed topologically. Taking the regular boundary OPE of the chiral field ψ_a will produce $\hat{\mathcal{O}}_{\psi_a} \in \mathcal{H}_{B_{\text{RG}}, B_{\text{RG}}}$, an open-channel wall operator.

The notation of a *phantom current* is a holomorphic integer-spin field in the folded theory, inherited from a defect endpoint but absent as an ordinary current of either UV/IR CFTs.
(1201.0767, 2505.09668, 2511.00356, 2605.24978)

Spin-1 Phantom current:

A spin-1 phantom current is a non-local $(1,0)$ - chiral current absent in either UV/IR theory satisfying

$$j_A(z) = \psi_{h_L,A}(z) \bar{\psi}_{h_R,A}(z) \in \mathcal{H}_{D_{A,L} \otimes \overline{D_{A,R}}}^{(1,0)}, \quad \text{with} \quad h_L + h_R = 1$$

The current $j_A(z)$ has an interesting feature that its tilt operator is **exactly marginal** on the wall

$$\bar{\partial} j_A(z) = \delta(x_\perp) t_A, \quad t_A = j_A|_B, \quad \hat{\Delta}_{t_A} = 1$$

So one can define an exactly marginal deformation on the wall,

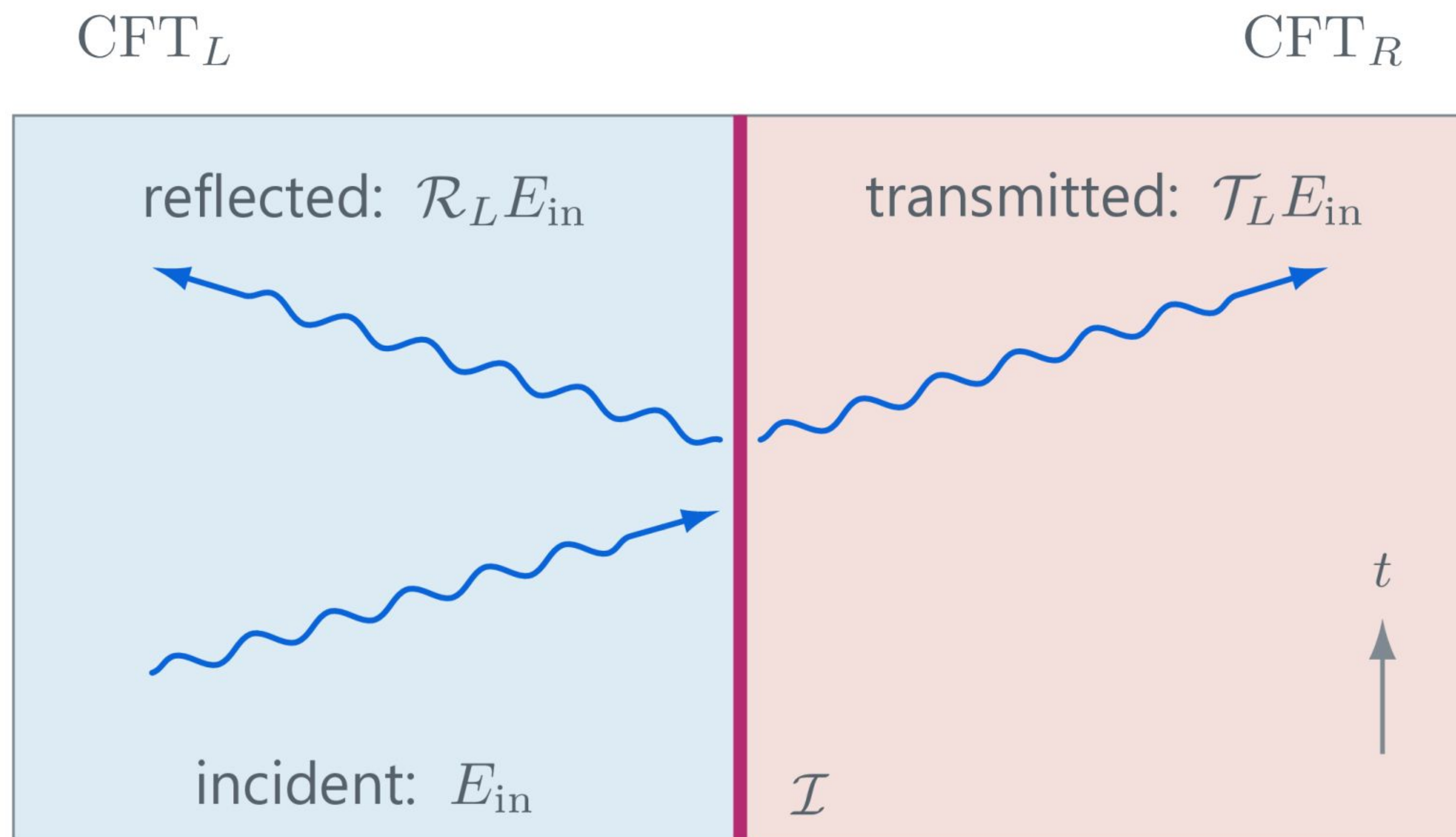
$$U_\theta = \exp i\theta \int_{x_\perp=0} dy t_A(y) \quad \Rightarrow \quad |B(0)\rangle \quad \Big| \quad \xrightarrow[U_\theta |B(\theta)\rangle = U_\theta |B(0)\rangle]{U_\theta} \quad \Big| \quad |B(\theta)\rangle$$

A spin-1 phantom current generates an exactly marginal defect conformal manifold.

Spin-2 Phantom current:

A spin-2 phantom current, on the other hand, will help determine the energy transmission on the interface.

In 2D, energy conservation leaves no third spatial channel along the wall. A conformal interface splits an incoming null-energy pulse into reflected and transmitted flux, completely determined by the central charges c_L , c_R and c_{LR} :



$$\langle T_L(z)T_R(w) \rangle_{\mathcal{F}} = \frac{c_{LR}}{2(z-w)^4}$$

$$\mathcal{T}_L = \frac{c_{LR}}{c_L}, \quad \mathcal{T}_R = \frac{c_{LR}}{c_R}; \quad \mathcal{T}_s + \mathcal{R}_s = 1.$$

$$\mathcal{T}_{\text{av}} = \frac{c_L \mathcal{T}_L + c_R \mathcal{T}_R}{c_L + c_R} = \frac{2c_{LR}}{c_L + c_R}$$

$$\mathcal{T}_{\text{av}} + \mathcal{R}_{\text{av}} = 1$$

hep-th/0611296, 1904.10974

When there is a spin-2 phantom current X , it will mix two stress-tensor channels on the RG interface,

$$X = \phi_L \bar{\phi}_R, \quad \bar{X} = \bar{\phi}_L \phi_R, \quad h_L + h_R = 2$$

$$T_L = \alpha \bar{T}_L + \beta T_R + \gamma \bar{X}, \quad \bar{T}_R = \alpha' \bar{T}_L + \beta' T_R + \gamma' \bar{X}$$

These $\alpha, \beta, \dots$ coefficients can be exactly determined by the following bootstrap constraints:

- **Energy-flux conservation (Diagonal Vir.):** the interface neither absorbs more emits energy.

$$T_L + \bar{T}_R = \bar{T}_L + T_R$$

- **Canonical normalization:**

$$\|T_s\|^2 = \frac{c_s}{2}, \quad \text{for } s = L, R$$

- **Cluster decomposition (2511.00356):** A stronger cluster decomposition principle is imposed that the two outgoing stress tensors in $\mathcal{T}_{L/R}$ don't talk to each other:

$$T_L(z) \cdot \bar{T}_R(w) \sim 0$$

Constraint equations can be read off by the poles of the above OPEs. The relative central charge c_{LR} can be thus determined purely by the local OPE data:

$$c_{LR} = \beta = \frac{c_L c_R (h_R - h_L)}{c_R h_R - c_L h_L}, \quad \text{when } \gamma \neq 0 \quad \text{and} \quad c_R h_R \neq c_L h_L$$

Even without knowing the details of the RG boundary state, a hidden spin-2 phantom current channel turns unknown wall dynamics into a finite-dimensional OPE problem on determining the transmission rate of the null-energy flux.

Example: $\mathbb{Z}_N$ - parafermion flows to minimal $\mathcal{M}_{N+1,N+2}$

In **2605.24978**, we study the long RG flow from $\mathbb{Z}_N$ -parafermion $\mathcal{P}_N$, perturbed by the spin operator σ_P and $\sigma_P^\dagger$, to the minimal model $\mathcal{M}_{N+1}$, proposed by **Faddeev and Zamolodchikov**.

$$\mathcal{P}_N \xrightarrow[\sigma_P, \sigma_P^\dagger]{\text{RG flow}} \mathcal{M}_{N+1}, \quad \mathcal{C}_{\text{prev.}} = \mathfrak{so}(3)_N, \quad \text{for } N \geq 3$$

In the folded picture, the endpoint quasi-local fields on the TDLs of $\mathfrak{so}(3)_N$ contains non-local hidden phantom currents j_s with spin s ,

$$s = k^2, \quad 0 \leq k \leq \lfloor N/2 \rfloor \quad (s = 0, 1, 4, 9, \dots)$$

- **A spin-1 phantom current:** The lowest non-trivial fields, at $k = 1$, is a spin-1 current,

$$j_1 = \phi_{1,3}^{IR} \overline{\phi_{3,0}^{UV}}; \quad N = 3 : \quad j_1^{N=3} = \epsilon_{3/5}'^{\text{TIM}} \epsilon_{2/5}^{3\text{Potts}}$$

Indicating there would be a **defect conformal moduli** on the possible RG interface between $\mathbb{Z}_N$ -parafermion and minimal $\mathcal{M}_{N+1}$

- **A spin-2 phantom current:** Although there is no fundamental Virasoro spin-2 current, the $W^{(3)}$ -current in $\mathbb{Z}_N$ -parafermion secretly induces a **genuine spin-2 phantom current**,

$$X = \phi_{1,3}^{\text{IR}} \left(\overline{W_{-1}^{(3)} \phi_{3,0}^{UV}} \right)$$

So the space consisted of spin-2 (quasi-)primary operators is spanned by

$$\text{span} \left\{ T_{UV}, \bar{T}_{IR}, X, W^-(j_1) \right\} ,$$

where the field W^- is the diagonal Virasoro primary in the derivative family of j_1 . The proposed operator-mixing equations,

$$T_L = \alpha \bar{T}_L + \beta T_R + \gamma \bar{X} + \delta \bar{W}^-, \quad \bar{T}_R = \alpha' \bar{T}_L + \beta' T_R + \gamma' \bar{X} + \delta' \bar{W}^-$$

admitting precisely an **one-parameter (θ) family of solutions**, implying there must be defect conformal manifold. The transmission rate

$$\mathcal{T}_N(\theta) = \frac{8(N+1)(1+\cos\theta)}{3(N+2)^2} \implies \begin{cases} \theta = 0 : \text{transparent RG wall} \\ \theta = \pi : \text{factorized interface} \end{cases}$$

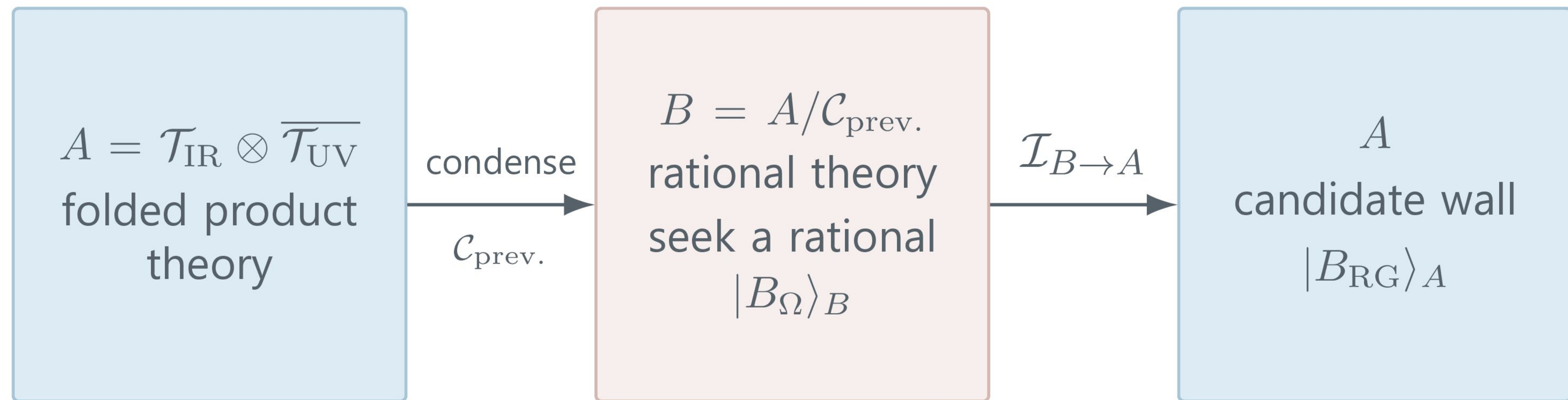
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III. Top-down Approach:

The bottom-up approach implies that T_{UV} and $\bar{T}_{IR}$ have to mix with those spin-two phantom currents.

This suggests that, in an enlarged theory where these quasi-local operators are condensed in the vacuum, one may **rationalize the wall** by defining **automorphism** acting on the corresponding enlarged chiral algebra



Such a procedure was pioneered in Gaiotto's seminal work **1201.0767**.

- **Generalized orbifolding:** In modern language, those phantom currents are endpoint fields of TDLs in $\mathcal{C}_{\text{prev.}}$, condensing them into vacuum is equivalently to gauging the fusion category $\mathcal{C}_{\text{prev.}}$.

$$\hat{D}_a \equiv D_{\rho(a)}^{\text{IR}} \otimes \overline{D_a^{\text{UV}}}, \quad \mathfrak{U}_{\text{cond}} = \bigoplus_a n_0^a \hat{D}_a, \quad \implies \quad B = A/\mathcal{C}_{\text{prev.}}$$

Such a gauging implies a conformal embedding of the chiral algebra $\mathcal{V}_A$ in the product theory A to $\mathcal{V}_B$ in B . The total stress tensor is also unchanged:

$$c_A = c_B, \quad \mathcal{V}_A \subset \mathcal{V}_B, \quad \text{and} \quad T_A = T_B = T_{\text{tot}} = T_{UV} + \bar{T}_{IR}$$

In addition, the generalized orbifolding also assign a topological interface $\mathcal{J}_{B \rightarrow A}$ between theory A and B , satisfying

$$\mathcal{J}_{B \rightarrow A} \circ \mathcal{J}_{B \rightarrow A}^\dagger = \mathfrak{U}_{\text{cond}}$$

- **Finding an automorphism in $\mathcal{V}_B$** : This is the most crucial and delicate step. A possible RG interface/boundary must assign an automorphism $\Omega \in \text{Aut}(\mathcal{V}_B)$ on $\mathcal{V}_B$.

The Ω is necessarily keeping the total stress tensor T_{tot} invariant. But its action on $\mathcal{V}_A$ must be non-invariant, so that it can play the role of mixing the two stress tensor T_{UV} and $\bar{T}_{IR}$,

$$\Omega(T_{\text{tot}}) = T_{\text{tot}}, \quad \Omega(\mathcal{V}_A) \neq \mathcal{V}_A$$

In addition, the minimal requirement for Ω also needs

$$\mathcal{V}ir(T_{\text{tot}}) \subset \Omega(\mathcal{V}_A) \cap \mathcal{V}_A$$

- **Pullback to theory A** : Once Ω is defined, one can construct an Ω -twisted Cardy brane, and pull back to the product theory A by the topological interface $\mathcal{J}_{B \rightarrow A}$, one has a RG brane

$$|B_\Omega\rangle_B \in B, \quad \implies \quad |B_{\text{RG}}\rangle_A = \mathcal{J}_{B \rightarrow A} |B_\Omega\rangle_B$$

A review on Gaiotto's prime example: TIM flows to Ising

In the simplest RG flow from TIM to Ising triggered by the thermal operator $\phi_{1,3}^{\text{TIM}} = \epsilon'_{3/5} \bar{\epsilon}'_{3/5}$,
 The preserved fusion category is

$$\mathcal{C}_{\text{prev.}} \simeq \text{TY}(\mathbb{Z}_2) = \{1, g, \sigma\} : \quad g^2 = 1, \quad g\sigma = \sigma g = \sigma, \quad \sigma^2 = 1 + g$$

$$g = D_{\epsilon}^{\text{Ising}} \otimes \overline{D_{\epsilon''}^{\text{TIM}}}, \quad X = \epsilon_{1/2}^{\text{Ising}} \bar{\epsilon}_{3/2}''^{\text{TIM}}, \quad \text{satisfying } h_X = \frac{1}{2} + \frac{3}{2} = 2$$

Symmetry g is simply the $\text{diag}(\mathbb{Z}_2^{\text{Ising}} \times \mathbb{Z}_2^{\text{TIM}})$. Gauging it gives,

$$B = A / \langle g \rangle, \quad \mathcal{V}_A = \frac{\mathfrak{su}(2)_1^{(a)} \times \mathfrak{su}(2)_1^{(b)}}{\mathfrak{su}(2)_2} \otimes \frac{\mathfrak{su}(2)_2 \times \mathfrak{su}(2)_1^{(c)}}{\mathfrak{su}(2)_3}$$

$$c_A = c_B = c_{\text{Ising}} + c_{\text{TIM}} = \frac{6}{5}, \quad \subset \quad \mathcal{V}_B = \frac{\mathfrak{su}(2)_1^{(a)} \times \mathfrak{su}(2)_1^{(b)} \times \mathfrak{su}(2)_1^{(c)}}{\mathfrak{su}(2)_3}$$

- $\text{Out}(\mathcal{V}_B) = S^3$: The outer automorphism group of $\mathcal{V}_B$ is the permutation of the three $\mathfrak{su}(2)_1^{(a,b,c)}$. Especially, the emergent quantum $\mathbb{Z}_2$ symmetry is the transposition $\tau_q = (ab)$:

$$\tau_q \equiv (ab), \quad \implies \quad A = B / \langle \tau_q \rangle$$

- **RG automorphism Ω** : The interesting RG automorphism Ω is the transposition $\Omega \equiv (bc)$:

$$\Omega(T_{\text{Ising}}) = \frac{1}{8}T_{\text{Ising}} + \frac{5}{8}T_{\text{TIM}} + \frac{1}{48}X, \quad \Omega(T_{\text{TIM}}) = \frac{7}{8}T_{\text{Ising}} + \frac{3}{8}T_{\text{TIM}} - \frac{1}{48}X$$

Clearly, Ω keeps the total stress tensor T_{tot} , but provides non-trivial mixing between individual factor stress tensors through the hidden spin-two phantom current X .

Using Ω , one defines a Ω -twisted brane $|B_\Omega\rangle_B$, and pullback to theory A via the $\mathbb{Z}_2$ -interface

$$|B_{\text{RG}}\rangle_A = \mathcal{J}_{B \rightarrow A} |B_\Omega\rangle_B$$

Revisiting Ising-TIM RG brane:

An recent paper **(2605.23848)**, by **Antunes and Rong**, revisit the construction of the RG interface between Ising and TIM. They found that Gaiotto's RG brane $|B_{RG}\rangle_A$ is **not simple** from its open string partition function:

$$Z_{\text{RG}}^{\text{open}} = 3\chi_{1,1} + 43\chi_{\sigma,\sigma'} + \chi_{\epsilon,1} + \chi_{1,\epsilon''} + 3\chi_{\epsilon,\epsilon''}$$

where $\chi_a \equiv \chi_a^{\text{Ising}} \chi_b^{\text{TIM}}$ are the product characters.

The number of vacua,

$$n_{\text{vac}} = 3$$

is interesting, as it supports a 3 dimensional NIM-rep of $\mathcal{C}_{\text{prev.}} = \text{TY}(\mathbb{Z}_2)$. However it seems more likely that the $\mathcal{C}_{\text{prev.}}$ is **spontaneously broken**, and thus in a gapped phase.

More importantly, **AR** actually found a [new conformal interface](#) from a concrete lattice model,

$$H_L = -J^* \left(\sum_{i=1}^{N/2-1} Z_i Z_{i+1} + \sum_{i=1}^{N/2} X_i \right) + K^* \sum_{i=1}^{N/2-1} \mathcal{O}_i, \quad \mathcal{O}_i = Z_i Z_{i+1} X_{i+2} + X_i Z_{i+1} Z_{i+2};$$

$$H_R = - \left(\sum_{i=N/2+1}^{N-1} Z_i Z_{i+1} + \sum_{i=N/2+1}^N X_i \right), \quad H_b = h_b (Z_{N/2} Z_{N/2+1} + Z_N Z_1)$$

$$H = H_L + H_R + H_b$$

The Hamilton H preserves an diagonal $\mathbb{Z}_2$ -symmetry and the Kramer-Wannier duality, and thus the diagonal $\text{TY}(\mathbb{Z}_2)$ in $\text{Ising} \otimes \text{TIM}$.

$H_L \longrightarrow H_{\text{TIM}}$, when $K^*/J^* \simeq 0.428$;

$H_R \longrightarrow H_{\text{Ising}}$, by default;

A DMRG simulation on $N = 256$ sites find a critical point:

$H_b \longrightarrow \text{critical}$, when $h_b \simeq 1.379$

Defect spectra: $\Delta_+ \simeq 0.50$, $\Delta'_- \simeq 0.77$, $\Delta \simeq 1.50$ (nearly twofold)

There is no any primary in A or B matching the above spectra. Rather, **AR** proposed a new brane $|B_{RG}^{\text{new}}\rangle$ in the second $W^{(3)}$ -minimal model:

$$C \equiv W_3(5, 6) = \frac{\mathfrak{su}(3)_1 \times \mathfrak{su}(3)_2}{\mathfrak{su}(3)_3}, \quad c_C = \frac{6}{5} = c_A = c_B$$

The open string partition function of $|B_{RG}^{\text{new}}\rangle$, to match the DMRG spectra is given by

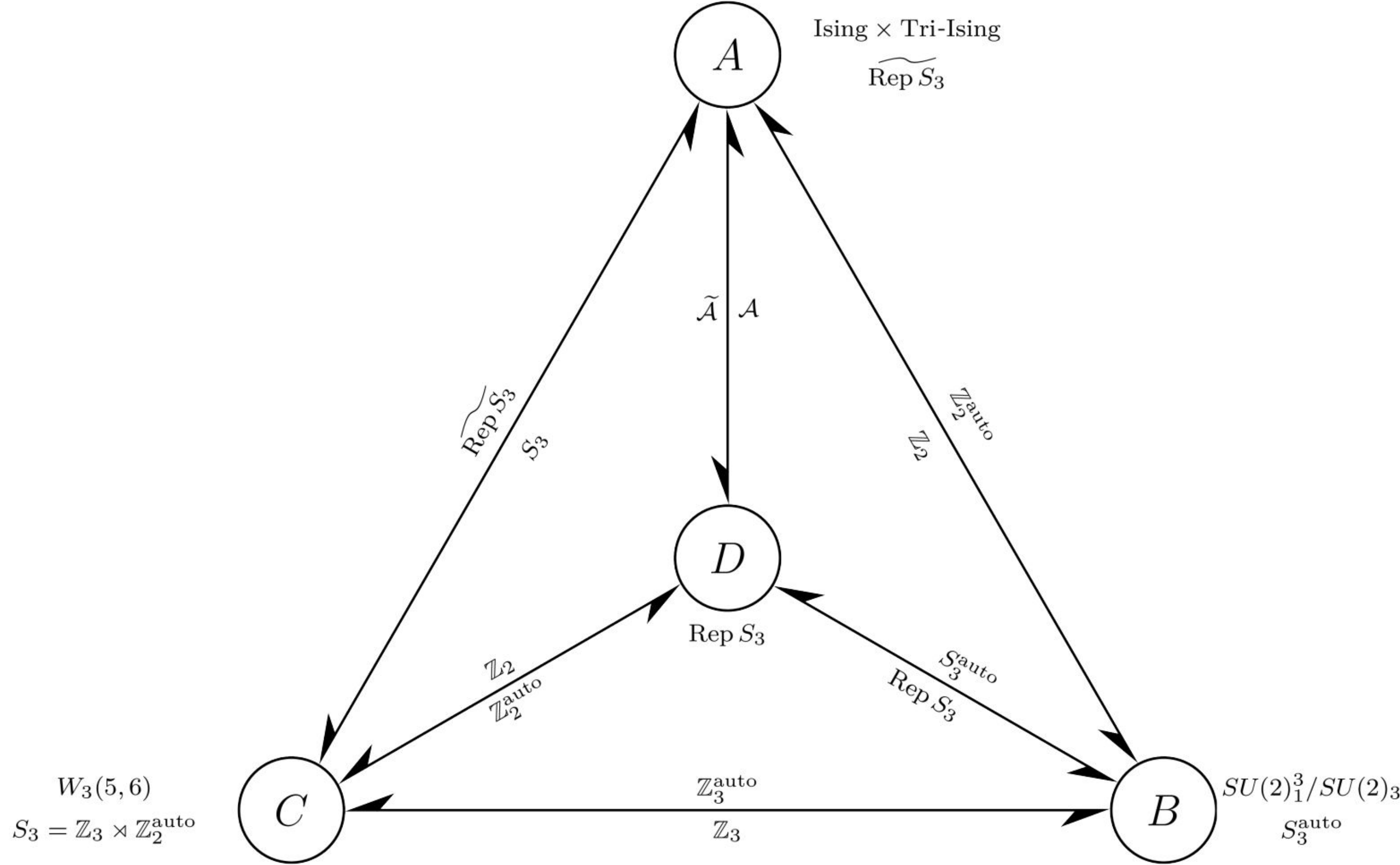
$$Z_{\text{new}}^{\text{open}} = \chi_0^{W_3} + \chi_{1/2}^{W_3} + 2\chi_2^{W_3} + \chi_{1/9}^{W_3} + \chi_{7/9}^{W_3} + \chi_{13/9}^{W_3}$$

where $\chi_h^{W_3}$ are the characters of theory C . Apparently, $|B_{RG}^{\text{new}}\rangle$ is a simple brane, with

$$n_{\text{vac}} = 1$$

The missing towers complete a tetrahedral duality web:

The spectra tower discovered in **AR**: $h = 1/9, 7/9, 13/9$ occur in neither the theory A nor B primary spectrum, but rather in C and also branch into a **fourth theory D** :



$$\text{Spec}(B_{\text{RG}}) \subset \mathcal{V}_A \cap \Omega(\mathcal{V}_A)$$

$$= \mathcal{V}_B^{\langle \tau_q \rangle} \cap \mathcal{V}_B^{\langle \Omega \rangle}$$

$$= \mathcal{V}_B^{S_3} \equiv \mathcal{V}_D$$

$$\implies D = B/S_3$$

To appear, with Antunes and Rong

The new RG brane B_{RG}^{new} is the **gauge-invariant orbit** of τ_q :

Recall that the construction of the seed brane $|B_\Omega\rangle_B$ in B is from the transposition:

$$\Omega = (bc) \in \text{Out}(\mathcal{V}_B) , \quad \mathcal{V}_B = \frac{\mathfrak{su}(2)_1^{(a)} \times \mathfrak{su}(2)_1^{(b)} \times \mathfrak{su}(2)_1^{(c)}}{\mathfrak{su}(2)_3}$$

However gauging B back to A is rather using another transposition:

$$\tau_q = (ab) : \quad A = B / \langle \tau_q \rangle$$

The seed brane $|B_\Omega\rangle_B$ is **not gauge invariant** respect to τ_q

$$|B_{(bc)}\rangle_B \equiv |B_\Omega\rangle_B , \quad \tau_q \cdot |B_\Omega\rangle_B = |B_{(ac)}\rangle_B$$

$|B_{(ac)}\rangle_B$ is a (ac) -twisted brane. A gauge invariant way to transport $|B_\Omega\rangle_B$ back to A is to sum the $\mathbb{Z}_2^{\tau_q}$ -orbit

$$|B_{RG}^{\text{new}}\rangle = \frac{1}{\sqrt{2}} \left(|B_{(bc)}\rangle_B + |B_{(ac)}\rangle_B \right)$$

The open string partition function, proposed by **AR**,

$$Z_{\text{new}}^{\text{open}} = \chi_0^{W_3} + \chi_{1/2}^{W_3} + 2\chi_2^{W_3} + \chi_{1/9}^{W_3} + \chi_{7/9}^{W_3} + \chi_{13/9}^{W_3}$$

can be automatically obtained in this gauge invariant fashion, as

$$Z_{(bc),(ac)}^{\text{closed}} = Z_{r=(abc)}^{\text{twined}} \implies Z_{(bc),(ac)}^{\text{open}} = Z_{r=(abc)}^{\text{twist}} = \chi_{1/9}^D + \chi_{7/9}^D + \chi_{13/9}^D$$

$$Z_{(bc),(bc)}^{\text{open}} = Z_{(bc),(bc)}^{\text{open}} = \chi_0^D + \chi_{1/2}^D + \chi_2^D = \chi_0^C + \chi_{1/2}^C + 2\chi_2^C$$

The boundary entropy computation also suggests that $|B_{\text{RG}}^{\text{new}}\rangle$ is consistent with $\mathbb{Z}_2^{\tau_q}$ -interface:

$$\mathfrak{g}(B_{\Omega}) = \gamma, \quad \mathfrak{g}(B_{\text{RG}}^{\text{new}}) = \sqrt{2} \gamma, \quad \mathfrak{g}(B_{\text{RG}}^{\text{Gaiotto}}) = 2 \gamma, \quad \text{with} \quad \gamma = \left(\frac{3 + \sqrt{5}}{2} \right)^{-1/4}$$

III. Outlook:

- **Complete the TIM-3Potts RG interface:** In **ongoing work** with Antunes and Rong, we are matching a concrete lattice junction to the field-theoretical RG wall, including its interface spectrum and observables.
- **Entanglement through interacting RG interface:** Computing another observable, c_{eff} the logarithmic coefficient of EE across the interface, is challenging. Only free theory results are known. An interesting bound was proposed in **2404.01515**, $c_{LR} \leq c_{\text{eff}} \leq \min(c_L, c_R)$
- **Construct RG walls for long flows:** Extend the top-down construction to the family $\mathcal{P}_N \longrightarrow \mathcal{M}_{N+1}$. At large N , $c_{\text{UV}} \rightarrow 2$, $c_{\text{IR}} \rightarrow 1$ is a genuine long flow with genuinely non-perturbative RG wall **(in progress)**

Thank you for your attention!