

Exact and Finite Curved QFT from CFT

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Based on:

- ① *Entanglement entropy of conformal field theory in all dimensions*, Xin Jiang(蒋昕), **HY**, arXiv:2506.02786, JHEP.
- ② *Exact Bulk-Boundary Pairs in AdS/CFT*, Xin Jiang(蒋昕), Peng Wang(王鹏), **HY**, arXiv:2605.15776.
- ③ *Exact Holographic Kinematics in AdS/CFT*, **HY**, arXiv:2605.21252.
- ④ *Exact and Finite AdS QFT from CFT*, **HY**, has disturbingly remained under arXiv moderation for more than one and a half months.
- ⑤ *Exact and Finite de Sitter QFT from CFT*, **HY**, arXiv:2606.24382

1 Motivation and background

2 The Scale-Space of Open-Solid-Torus: AdS

3 The Ball Moduli Space: dS

- Euclidean Parent: Single-Time dS and Wavefunction Representation
- Lorentzian Parent: Double-Time dS and Parent-Time Unitarity
- Two Representations and dS Puzzles

4 Multiplicity of CMS Presentations, Dualities and Physical Time Polarizations

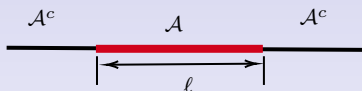
5 Summary and discussion

I have five long-standing confusions, some of them stem from possible misinterpretations of the canonical pictures.

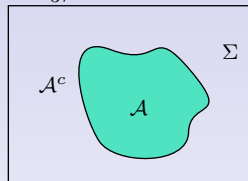
My puzzle one: CFT with a cutoff?

For adjacent entangling regions (the exclusive elementary EE in literature)

CFT₂, $t = 0$ time slice



CFT₃, $t = 0$ time slice



the entanglement entropies (EE) $S_{\text{adj}}(A : A^c)$ must be divergent due to the short-range correlations in contiguous fields ($\Delta x \Delta p \sim 1$). Say, for the left panel CFT₂,

$$S_{\text{adj}}(A : A^c) = \frac{c}{3} \log \frac{\ell}{\epsilon},$$

where ℓ is the length of A . ϵ is the UV cutoff **put by hand from dimension analysis**.

Puzzle one: Why a UV cutoff in a UV-complete theory?

My puzzle two: The standard holographic dictionary

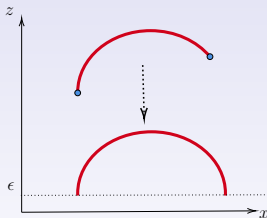
AdS/CFT says:

the bulk theory (quantum gravity) = the boundary theory (CFT)

In practice, however, entries of AdS/CFT are organized asymptotically. Boundary data are related to bulk fields through asymptotic **divergent** limits, such as:

$$\langle O(x)O(y) \rangle = \exp \left[-\Delta \left(L(x, y) + \log \epsilon^2 \right) \right], \quad S_{\text{adj}}(A^c : A) \simeq \frac{\text{Area}}{4G^{D+1}},$$

where the geodesic $L(x, y)$ and RT surface are **boundary-anchored** and divergent.



Semi-classical limits such as large- N , strong-coupling or heavy-operator approximation are imposed during this process.

Puzzle two: Why boundary-anchored?

My puzzle three: Kinematic vs Dynamics?

Some familiar pairs:

	CFT_D	AdS_{D+1}
RT formula:	$S_{\text{adj}}(A^c : A)$	codim-two minimal surfaces
correlators:	$\langle \mathcal{O}(x) \mathcal{O}(y) \rangle$	geodesics
Wilson:	Wilson loop	dim-two world sheet
determined by:	$SO(D, 2)$	$SO(D, 2)$

all have kinematic (symmetry) parts to match.

Logically, if the kinematic sector matches in the dynamical limits, since they are independent of dynamics, their matching should be independent of the dynamical limits.

Puzzle three: For the Kinematic sector, why impose (dynamical) semi-classical limits such as large N , strong coupling or heavy operators?

My puzzle four and five: One to many? RG as extra dimension?

Puzzle four, from symmetry perspective, how to match:

A single AdS $\longleftrightarrow$ infinitely many CFTs

Puzzle five:

CFT (at least for vacuum) has no scale. Why claim non-existing RG scale as the bulk dimension?

Why then don't claim two, three, four... arbitrarily spurious directions as more "bulk" coordinates?

Holography contains an **exact kinematic sector** distinct from holographic dynamics.

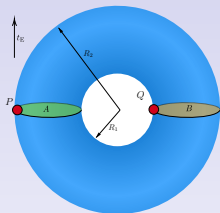


Figure: CFT_D

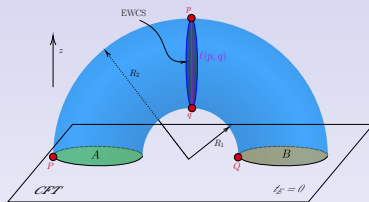


Figure: AdS_{D+1}

Exact pairs:

$$S_{\text{disj}}(A : B) \equiv E_W(A : B),$$

$$G_W(P, Q) \equiv C_\Delta \left[2 \cosh \frac{\ell(p, q)}{2} \right]^{-2\Delta}$$

and 3-point (triangle), 4-point (tetrahedron) ... correlators.

Q: What is the role of the Open-Solid-Torus (OST)? Why it works?

One of the most important and tempting claims of AdS/CFT is that it provides a nonperturbative definition of quantum gravity. This claim logically

Indicates:

quantum gravity must be some form of CFT data reorganization

since we do not know how to consistently quantize gravity.

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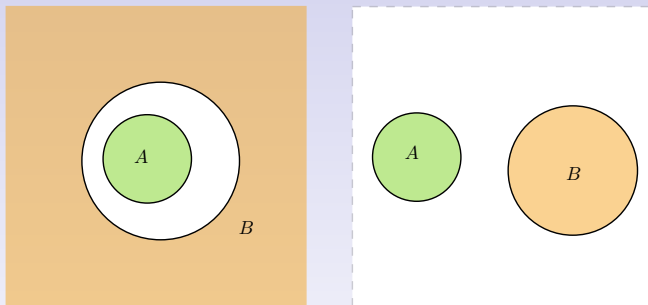
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Open-Solid-Torus (OST)

Generally, there exist two different configurations of OST, namely, the cavity configuration and the juxtaposed configuration, at $t = 0$ slice, looking like

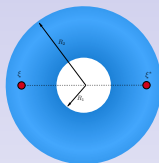


Define the inversive product ϱ of two spheres $\Sigma(\vec{x}, r)$ and $\Sigma(\vec{x}', r')$

$$\varrho = \left| \frac{r^2 + r'^2 - |\vec{x} - \vec{x}'|^2}{2rr'} \right|.$$

Any configuration (composed of two spheres) sharing the same ϱ must be conformally equivalent. So we only need consider the symmetric representative.

The Scale-Space of Open-Solid-Torus



The key is that points inside an OST are automatically paired. This is most transparent in the above symmetric OST, in which each point ξ^μ is paired with a symmetric partner $\xi^{*\mu} = -\xi^\mu$. Then we can make a linear parametrization,

$$X^\mu = \frac{\xi^\mu + \xi^{*\mu}}{2}, \quad Z = \frac{|\xi^\mu - \xi^{*\mu}|}{2}. \quad (1)$$

The pairing and parametrization in a generic asymmetric OST (last slide) are obtained by the associated conformal transformation, which are complicated nonlinear expressions. Therefore, the OST leads to a $(d+1)$ -dimensional Scale-Space

$$\boxed{Y^a(\xi) \equiv (X^\mu(\xi), Z(\xi)), \quad \mu = 0, \dots, d-1,} \quad (2)$$

entirely defined by the d -dimensional CFT.

It is important that only in the symmetric OST, the scale interpretation of the extra coordinate Z is explicit as shown in eq. (1), whereas in asymmetric configurations, the relation between the scale and extra dimension is obscured.

Metric of the Scale-Space

The geometry of the Scale-Space can be computed by using entanglement entropy, correlators or stabilizer (little group). Alternatively, we use a direct ambient embedding approach as follows:

In the standard conformal embedding of a CFT into the projective null cone of $\mathbb{R}^{d+1,1}$, a CFT point $\xi \in \mathbb{R}^d$ is represented by

$$P^A(\xi) = \left(\frac{1 + \xi^2}{2}, \xi^\mu, \frac{1 - \xi^2}{2} \right), \quad P(\xi)^2 = 0,$$

with Poincaré gauge $P^{-1} + P^{d+1} = 1$. It is sufficient to consider the symmetric OST.

Mapping the pairing rule $X^\mu = \frac{\xi^\mu + \xi^{*\mu}}{2}$ onto the embedding space, with the unity normalization, we have

$$V^A(X, Z) = \frac{(P^A(\xi) + P^A(\xi^*)) / 2}{|P^A(\xi) + P^A(\xi^*)| / 2} = \left(\frac{1 + X^2 + Z^2}{2Z}, \frac{X^\mu}{Z}, \frac{1 - (X^2 + Z^2)}{2Z} \right), \quad V^2 = -1.$$

The metric of the Scale-Space is now induced from the ambient metric:

$$ds_{\text{SS}}^2 = dV^A dV_A = \frac{dX^\mu dX_\mu + dZ^2}{Z^2},$$

which is precisely AdS_{d+1} .

Now that we have an induced $(d + 1)$ -dimensional Scale-Space entirely defined by CFT_d , all physical objects, such as operators and correlators, can be lifted to this Scale-Space.

Let $O_\Delta(X)$ be a scalar primary operator of scaling dimension Δ . The quadratic conformal Casimir has eigenvalue

$$C_2(\Delta) = \Delta(\Delta - d).$$

Since the Scale-Space has AdS_{d+1} metric, the corresponding scalar Casimir is

$$\nabla_Y^2 = Z^2(\partial_Z^2 + \square_X) - (d - 1)Z\partial_Z.$$

The lifted **scalar** must belong to the conformal family of O_Δ , and should obey the Scale-Space Casimir equation,

$$[\nabla_Y^2 - \Delta(\Delta - d)] O_\Delta^{\text{SS}}(X, Z) = 0.$$

This yields

$$O_\Delta^{\text{SS}}(X, Z) = \mathcal{R}_\Delta(X, Z) O_\Delta(X) \equiv Z^\Delta {}_0F_1 \left(\Delta - \frac{d}{2} + 1; -\frac{Z^2}{4} \square_X \right) O_\Delta(X).$$

Let

$$G_{a_1 \dots a_n, c}^{\text{CFT}}(X_1, \dots, X_n) = \langle O_{a_1}(X_1) \cdots O_{a_n}(X_n) \rangle_c$$

be the CFT connected n -point correlator of scalar primary operators. The completed connected correlators in the Scale-Space are

$$G_{a_1 \dots a_n, c}^{\text{SS}}(Y_1, \dots, Y_n) = \prod_{i=1}^n \mathcal{R}_{\Delta_{a_i}}(X_i, Z_i) G_{a_1 \dots a_n, c}^{\text{CFT}}(X_1, \dots, X_n),$$

where $Y_i = (X_i, Z_i)$. This is the CFT-first construction. The dynamics is not introduced through a bulk bare action. It is inherited from the exact CFT connected correlators.

AdS QFT from CFT: 1PI

Introduce sources $J^a(Y)$ on the Scale-Space. The finite connected generating functional is defined by

$$W_{\text{SS}}[J] = \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{i=1}^n d\mu(Y_i) J^{a_i}(Y_i) G_{a_1 \dots a_n, c}^{\text{SS}}(Y_1, \dots, Y_n),$$

where the Scale-Space measure $d\mu(Y) = \frac{d^d X dZ}{Z^{d+1}}$. The response field is

$$\Phi_a(Y) = \frac{\delta W_{\text{SS}}[J]}{\delta J^a(Y)} = \langle O_a^{\text{SS}}(Y) \rangle_J.$$

The finite 1PI effective action is the Legendre transform

$$\Gamma_{\text{SS}}[\Phi] = \int d\mu(Y) J^a(Y) \Phi_a(Y) - W_{\text{SS}}[J].$$

This is a $(d+1)$ -dimensional finite Scale-Space (AdS_{d+1}) QFT, obtained by a data reorganization of a CFT_d .

The usual bulk-first logic begins with a bare bulk action and computes boundary correlators. Here the direction is reversed:

$$G_{n,c}^{\text{CFT}} \longrightarrow G_{n,c}^{\text{SS}} \longrightarrow W_{\text{SS}}[J] \longrightarrow \Gamma_{\text{SS}}[\Phi].$$

A local or Wilsonian bulk action, if it exists, is an additional representation of Γ_{SS} , not an input.

AdS QFT from CFT: GKPW as a Singular Limit

The lifted operator has the small-scale behavior

$$O_{\Delta}^{\text{SS}}(X, Z) \sim Z^{\Delta} O_{\Delta}(X), \quad Z \rightarrow 0.$$

The limit $Z \rightarrow 0$ is not obtained from generic interior OST points. This is most transparent in the cavity configuration,

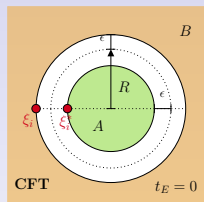
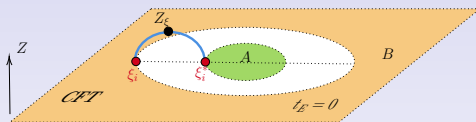


Figure: The cavity configuration. ξ_i and its partner ξ_i^* are on the boundary. The planar theory is a singular limit $\epsilon \rightarrow 0$ of the OST. This is the canonical pattern by which finite parent objects reduce to the usual singular quantities.

To recover the ordinary boundary theory, the relevant insertions, or equivalently the support of the source, must be restricted to a boundary-anchored cutoff surface of the OST, and the cutoff is then sent to zero.

In our current picture, the OST is not the boundary of the Scale-Space. Only in the singular limit, does the CFT appear as a boundary theory of the Scale-Space. This is the general pattern by which finite parent objects reduce to the usual singular quantities.

AdS QFT from CFT: Ordinary GKPW as a Singular Limit

To recover the ordinary boundary source deformation, a source localized at $Z = \epsilon$ should be chosen as

$$J_\epsilon(X, Z) = Z^{d+1-\Delta} j(X) \delta(Z - \epsilon).$$

Then

$$\begin{aligned} \int d\mu(Y) J_\epsilon(Y) O_\Delta^{\text{SS}}(Y) &= \int \frac{d^d X dZ}{Z^{d+1}} Z^{d+1-\Delta} j(X) \delta(Z - \epsilon) [Z^\Delta O_\Delta(X) + O(Z^{\Delta+2})] \\ &= \int d^d X j(X) O_\Delta(X) + O(\epsilon^2). \end{aligned}$$

Therefore

$$j \longmapsto J_\epsilon[j], \quad \lim_{\epsilon \rightarrow 0} W_{\text{SS}}[J_\epsilon[j]] = W_{\text{CFT}}[j]. \quad (3)$$

In the traditional GKPW, one starts from a bulk action for a field ϕ , and the CFT source $j(x)$ appears as the boundary value of ϕ :

$$Z_{\text{CFT}}[j] = Z_{\text{bulk}}[\phi \rightarrow j], \quad \phi(X, Z) \sim Z^{d-\Delta} j(X) + \dots, \quad Z \rightarrow 0. \quad (4)$$

This form arises because the two sides of the usual GKPW relation are represented **asymmetrically**: the CFT is described by a source functional $W_{\text{CFT}}[j]$, whereas the bulk side is described by a bare or classical action $Z_{\text{bulk}}[\phi]$ evaluated on the field ϕ .

In the present construction, both sides are written in the correlator-functional language. The finite theory is described by $W_{\text{SS}}[J]$, and the singular CFT is described by $W_{\text{CFT}}[j]$. Therefore, the recovery of GKPW is naturally expressed as Eq. (3), rather than (4).

The Landscape of Scale-Space QFT

The construction applies to any CFT for which the connected correlator hierarchy is defined. In general, the AdS QFT obtained from an arbitrary CFT need not be local. However, since the starting CFTs are well-defined, we should treat all the induced QFTs, whether local or nonlocal, weak or strong, on an equal footing. This opens up a vast new arena for QFT. Ambiguities of curved QFT are shifted to choices of CFT data, state, and finite boundary geometry.

The traditional holographic CFTs should be viewed as a special class, whose Scale-Space lifted QFTs are local, weakly coupled, and gravitational. More generally, one may classify traditional holographic CFTs by the phase of their Scale-Space partners as summarized in Table 1:

CFT property	Scale-Space QFT property
arbitrary CFT	exact finite renormalized Scale-Space effective action
local CFT	local or quasi-local 1PI kernels
weak CFT	controlled interaction hierarchy
gravitational CFT	quantum gravity interpretation
traditional holographic CFT	local + weak + gravitational

Table: Classification of CFT by the properties of their Scale-Space QFT.

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Now that the moduli space of OST provides AdS geometry,

are there other Conformal-Moduli-Spaces (CMS)?

How about the moduli space of a ball?

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Geometry of the Ball-Moduli Space

Again, in $\mathbb{R}^{d+1,1}$, a Euclidean CFT_d point $x \in \mathbb{R}^d$ is represented by

$$P^A(x) = \left(\frac{1+x^2}{2}, x^\mu, \frac{1-x^2}{2} \right), \quad P(x)^2 = 0,$$

Let $V^A = (V^{-1}, V^\mu, V^{d+1})$ be a non-null ambient vector. Since

$$V \cdot P(x) = -\frac{V^{-1} + V^{d+1}}{2}x^2 + V_\mu x^\mu + \frac{-V^{-1} + V^{d+1}}{2} \equiv ax^2 + b_\mu x^\mu + c,$$

the incidence relation $V \cdot P(x) = 0$ defines a sphere or a plane. To get a ball $B(X, T)$ with center X and radius T , we choose the normalization and orientation by imposing

$$V(X, T) \cdot P(x) = \frac{T^2 - |x - X|^2}{2T}.$$

Then $V \cdot P(x) = 0$ fixes

$$V = \left(\frac{1 + X^2 - T^2}{2T}, \frac{X^\mu}{T}, \frac{1 - X^2 + T^2}{2T} \right), \quad V^2 = 1.$$

The metric on the ball-moduli space $\mathcal{M}$ is induced from the ambient metric:

$$ds_{\mathcal{M}}^2 = dV \cdot dV = \frac{-dT^2 + dX^\mu dX_\mu}{T^2},$$

precisely dS_{d+1} .

Following the similar steps in the OST AdS, we lift the primaries of CFT to the Ball-Moduli dS,

$$O_{\Delta}^{\mathcal{M}}(X, T) = T^{\Delta} {}_0F_1 \left(\Delta - \frac{d}{2} + 1; \frac{T^2}{4} \square_X \right) O_{\Delta}(X).$$

Comparing with the OST AdS lift

$$O_{\Delta}^{\text{SS}}(X, Z) = Z^{\Delta} {}_0F_1 \left(\Delta - \frac{d}{2} + 1; -\frac{Z^2}{4} \square_X \right) O_{\Delta}(X),$$

the sign difference is fixed by the signature of the scale direction. In OST Scale-Space the scale coordinate is [spacelike](#), while in the ball-moduli space the radius coordinate is [timelike](#).

Correlator lift, generating functional, 1PI

Similarly, for the Euclidean CFT correlators

$$G_{a_1 \dots a_n, c}^E(X_1, \dots, X_n) = \langle O_{a_1}(X_1) \cdots O_{a_n}(X_n) \rangle_{E, c}, \quad (5)$$

the completed connected ball-moduli space lifts are

$$G_{a_1 \dots a_n, c}^{E-\mathcal{M}}(Y_1, \dots, Y_n) = \prod_{i=1}^n R_{\Delta_{a_i}}^E(X_i, T_i) G_{a_1 \dots a_n, c}^E(X_1, \dots, X_n). \quad (6)$$

The finite connected generating functional is

$$W_{E-\mathcal{M}}[J] = \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{i=1}^n d\mu(Y_i) J^{a_i}(Y_i) G_{a_1 \dots a_n, c}^{E-\mathcal{M}}(Y_1, \dots, Y_n). \quad (7)$$

The finite 1PI effective action is

$$\Gamma_{E-\mathcal{M}}[\Phi] = \int d\mu(Y) J^a(Y) \Phi_a(Y) - W_{E-\mathcal{M}}[J]. \quad (8)$$

The construction is therefore

$$G_{n, c}^E \longrightarrow G_{n, c}^{E-\mathcal{M}} \longrightarrow W_{E-\mathcal{M}}[J] \longrightarrow \Gamma_{E-\mathcal{M}}[\Phi]. \quad (9)$$

No bulk path integral or bare moduli space action is required to define these objects. A traditional local action on the ball-moduli space, if it exists, is an additional representation of the same completed kernel hierarchy.

Singular limit as the traditional results

Near $T = 0$, the completed operator behaves as

$$O_{\Delta}^{E-\mathcal{M}}(X, T) = T^{\Delta} O(X) + O(T^{\Delta+2}). \quad (10)$$

Choose a source localized at $T = \epsilon$,

$$J_{\epsilon}^a(X, T) = T^{d+1-\Delta_a} j^a(X) \delta(T - \epsilon). \quad (11)$$

Then

$$\int d\mu(Y) J_{\epsilon}^a(Y) O_a^{E-\mathcal{M}}(Y) = \int d^d X j^a(X) O_a(X) + O(\epsilon^2). \quad (12)$$

Therefore

$$\lim_{\epsilon \rightarrow 0} W_{E-\mathcal{M}}[J_{\epsilon}[j]] = W_E[j]. \quad (13)$$

This is the finite-parent origin of the usual dS/CFT boundary relation. In a semiclassical gravitational phase, one may write schematically

$$Z_E[j] \sim \Psi_{dS}[j]. \quad (14)$$

In the present formulation, this is not the definition of the theory. It is a singular boundary representation of the finite moduli-space functional.

- The Euclidean parent CFT naturally gives a wavefunction representation.
- This is compatible with the traditional dS/CFT picture, where a Euclidean CFT is associated with the spacelike boundary of de Sitter space.
- In this representation, the timelike direction of the dS moduli-space is generated by the radius.
- Since the parent theory is Euclidean, this representation does not by itself provide a Lorentzian Hilbert space or a preferred Hamiltonian time.
- The familiar features of dS/CFT, including state-selection ambiguity, contour dependence, and the wavefunction character of the dictionary, should therefore be understood as inheriting from the ill-defined parent (time absent) CFT.

Then how about starting from a well-defined (Lorentzian) parent CFT?

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Lorentzian Parent: Double-Time dS

We now consider a Lorentzian parent CFT, with coordinates and metric

$$x^\alpha = (t, \vec{X}), \quad dx^\alpha dx_\alpha = -dt^2 + d\vec{X}^2. \quad (15)$$

The corresponding d'Alembertian is

$$\square_M = -\partial_t^2 + \nabla_{\vec{X}}^2. \quad (16)$$

The key difference from the Euclidean parent is that the Minkowski CFT is unitary and already has a physical time t , a Hilbert space $\mathcal{H}_M$, and a Hamiltonian H_t ,

$$U(t) = e^{-iH_t t}, \quad H_t |0_M\rangle = 0. \quad (17)$$

Following the same steps, for a Lorentzian parent CFT, the Moduli-Space metric is

$$ds_{\text{dt}}^2 = \frac{-dT^2 + dx^\alpha dx_\alpha}{T^2} = \frac{-dT^2 - dt^2 + d\vec{X}^2}{T^2}. \quad (18)$$

Thus the induced geometry has two timelike directions. We can lift all the operators and correlators to this double-time dS. The generating functional and 1PI are defined accordingly.

The metric

$$ds_{\text{dt}}^2 = \frac{-dT^2 - dt^2 + d\vec{X}^2}{T^2} \quad (19)$$

has two negative directions. In ordinary double-time field theory, this would raise immediate questions about ghosts, causality, and unitarity.

However, the present construction has a different status. It does not begin with an autonomous local field theory on a double-time manifold. It begins with a unitary Minkowski CFT.

The double-time coordinate T is introduced through conformal family completion. The Hilbert space is not enlarged by a second canonical time evolution. Thus

$$\mathcal{H}_{\text{dt}} = \mathcal{H}_M, \quad H_{\text{phys}} = H_t. \quad (20)$$

The T -direction is timelike in the moduli-space metric, but it is not the Hamiltonian time used to define the Hilbert-space inner product. It is a **moduli-time** direction appearing in the Casimir equation.

In this precise sense, the double-time construction is parent-time unitary. Its unitarity is inherited from the Minkowski CFT, not established by independently quantizing a double-time local action.

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Two Representations and de Sitter Puzzles

The construction gives two related but distinct representations:

Euclidean CFT $\longrightarrow$ single-time dS moduli space $\longrightarrow$ wavefunction rep.,

and

Minkowski CFT $\longrightarrow$ double-time dS moduli space $\longrightarrow$ parent-time unitary rep..

The first representation is close to traditional dS/CFT. It is naturally Euclidean and wavefunction-like. The second representation keeps a Lorentzian parent Hilbert space and a physical Hamiltonian H_t . Therefore it gives a different interpretation of the same finite moduli-space idea.

This distinction suggests that some familiar dS difficulties may stem from the pathology of Euclidean CFT.

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Multiplicity of CMS Presentations

For a Lorentzian CFT_d ,

$$\boxed{\text{CFT}_d \xrightarrow{\text{OST}} \text{AdS}_{d+1}}$$

$$\boxed{\text{CFT}_d \xrightarrow{\text{Ball Moduli-Space}} \text{double-time dS}_{d+1} \longrightarrow \text{parent-time unitary rep.}}$$

So the CMS framework suggests an instructive separation between

(CFT data, its finite presentations, their possible spacetime interpretations)

Multiplicity of CMS Presentations, Dualities

- Let

$$\mathcal{D}_{\mathcal{C}} = \{\mathcal{A}_{\mathcal{C}}, \langle O_{a_1} \cdots O_{a_n} \rangle, C_{abc}, \Delta_a, \ell_a, \text{Ward identities}, \dots\}$$

denote the full data of the underlying CFT $\mathcal{C}$.

- Schematically, for a CMS $\mathcal{M}$, we lift

$$\mathcal{U}_{\mathcal{M}} : \mathcal{D}_{\mathcal{C}} \longrightarrow \mathcal{Q}[\mathcal{M}],$$

where $\mathcal{Q}[\mathcal{M}]$ is not an independent theory, but a finite presentation of the same CFT data.

- Thus two admissible CMS presentations $\mathcal{M}$ and $\mathcal{N}$ are related by the map

$$\mathcal{T}_{\mathcal{M} \rightarrow \mathcal{N}} = \mathcal{U}_{\mathcal{N}} \mathcal{U}_{\mathcal{M}}^{-1}.$$

These maps obey the composition law

$$\mathcal{T}_{\mathcal{N} \rightarrow \mathcal{P}} \mathcal{T}_{\mathcal{M} \rightarrow \mathcal{N}} = \mathcal{T}_{\mathcal{M} \rightarrow \mathcal{P}}, \quad \mathcal{T}_{\mathcal{M} \rightarrow \mathcal{M}} = \text{Id}.$$

Therefore, admissible CMS presentations form a group. The group elements form a “**duality**” web. The “dualities” are not independent dynamical postulates; they are induced by changing the finite presentation of $\mathcal{D}_{\mathcal{C}}$.

One lesson from the double-time dS construction above is that a physical spacetime interpretation requires an additional polarization. In particular, the presence of a Lorentzian signature in a CMS metric does not automatically identify the corresponding CMS coordinate as physical time. A physical time interpretation requires data of the form

$$\left(\mathcal{H}, H_{\text{phys}}, \tau, \text{positivity, causality/locality conditions} \right), \quad (21)$$

where τ is the chosen physical time parameter and

$$\frac{d}{d\tau} O(\tau) = i [H_{\text{phys}}, O(\tau)] \quad (22)$$

defines the physical evolution. This time polarization is not determined by the CMS metric alone, but should inherit from the well-defined parent CFT.

The resulting hierarchy is therefore as follows:

- ① There is an invariant layer, namely the CFT data $\mathcal{D}_C$.
- ② There is a presentation layer, consisting of admissible CMS representations $\mathcal{Q}[\mathcal{M}]$, with their induced algebra, source functional, effective action, and kinematic geometries.
- ③ There is a physical-polarization layer, where one selects a Hilbert-space interpretation, a physical time, and possibly an effective spacetime geometry for a particular state or sector.

Thus the multiplicity of admissible CMS choices should not be read as multiple independent physical theories. Rather, it expresses the fact that the same CFT data can admit many finite geometric organizations. Physical spacetime, when present, is a further polarization or effective interpretation of such organized data.

1 Motivation and background

2 The Scale-Space of Open-Solid-Torus: AdS

3 The Ball Moduli Space: dS

- Euclidean Parent: Single-Time dS and Wavefunction Representation
- Lorentzian Parent: Double-Time dS and Parent-Time Unitarity
- Two Representations and dS Puzzles

4 Multiplicity of CMS Presentations, Dualities and Physical Time Polarizations

5 Summary and discussion

Summary:

- We introduced the CMS framework.
- We used OST to construct AdS QFT, used Ball-Moduli-Space to construct dS QFT.
- This provides the CFT methodology (correlators first) to define QFT, rather than the traditional bare action formalism.
- It provides an alternative way to consistently define curved QFT or double-time QFT.
- The problems in dS can be traced as the pathology of the underlying Euclidean CFT.

Suggestions:

- CFT-first rather than QFT-first.
- Do not regularize the singular object; find its finite parent.
- Finite geometry first, planar limit later.
- Abandon Bulk-Boundary picture. It arises only in the singular limit.
- Dualities might be interpreted as the CMS transformations.

Future directions:

- Lift various CFTs to CMS space.
- Construct traditional bare action QFT from the lifted QFT.
- Figure out what kinds of CFT lead to local, weak CMS QFT.
- Are there more nontrivial CMS? What are the corresponding geometries?
- How to get the $(d + 1)$ -dim flat QFT from CFT_d ?
- How to get **asymptotic** AdS/dS QFT from CFT excited states, say, thermal? — This is a crucial step to justify the CMS framework.
- Does the double-time have any inspiration for dS cosmology, dark matter and dark energy?
- QFT/CFT dimension reduction — to appear.

Thank you!

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