

On-shell Approaches for Gravitational Wave Physics

Waveform and beyond

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第七届“场论与弦论”学术研讨会（合肥）

Outline

Introduction: amplitudes meet GW

EFT matching

Observable based formalism (KMOC)

Outlook and conclusion

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Gravitational wave: new window to probe our Universe

GWTC-5: about 390 events

- ▶ Probe dynamics of black holes
- ▶ Test general relativity
- ▶ Probe new physics beyond GR
- ▶ Multimessenger astronomy/cosmology

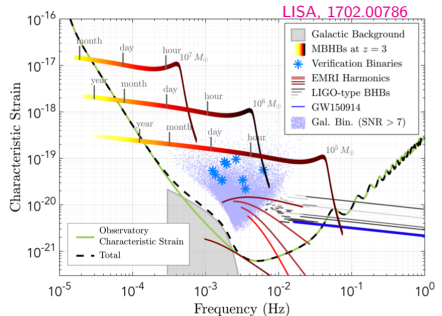
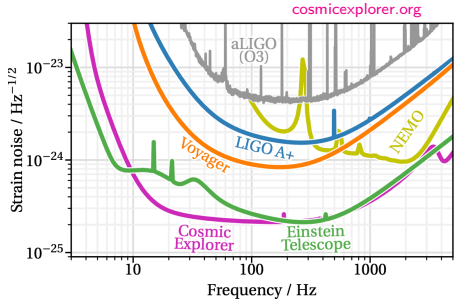
Future ground based observatories

- ▶ Einstein Telescope
- ▶ Cosmic Explorer

Future space based observatories

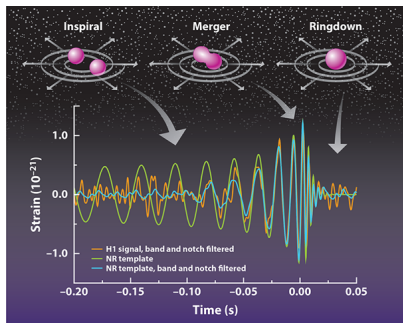
- ▶ LISA
- ▶ TaiJi
- ▶ TianQin

Require accurate theoretical prediction



GW from binary system

Numerical relativity provides the truth but computationally expensive



Accurate theoretical prediction of the GW production puts challenges on controlling higher order perturbative expansions

$$H = \frac{\mathbf{p}_1^2}{2m_1} + \frac{\mathbf{p}_2^2}{2m_2} - \frac{Gm_1m_2}{|\mathbf{r}|} + (\text{corrections from general relativity})$$

How to organize perturbations?

- ▶ Post-Newtonian (PN) expansion

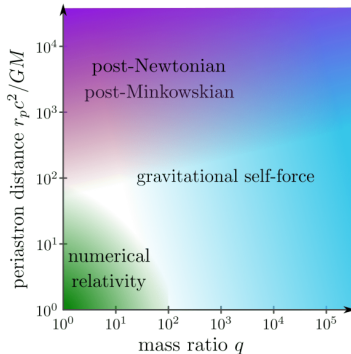
$$v^2 \sim \frac{Gm}{r} \ll 1$$

- ▶ Post-Minkowskian (PM) expansion

$$\frac{Gm}{r} \ll v^2 \sim 1$$

- ▶ Self-force expansion

$$\frac{Gm}{r} \sim v^2 \sim 1, \quad \frac{m_1}{m_2} \ll 1$$



Khalil, Buonanno, Steinhoff, Vines, 2204.05047

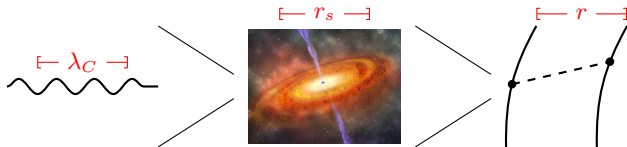
Field-theory based methods naturally lead to PM expansion

Why field theory and on-shell method?

- ▶ **Efficient on-shell techniques:** generalized unitarity, on-shell recursion, double copy, etc
- ▶ **Powerful and versatile EFT construction:** systematic handling of physics across scales
- ▶ **Advanced integration techniques:** IBP reduction, differential equation, etc

EFT perspective

From quantum to classical two-body problem



$$\lambda_C \ll r_s \implies Gm^2 \gg 1$$

Classical contribution is encoded in every loop order

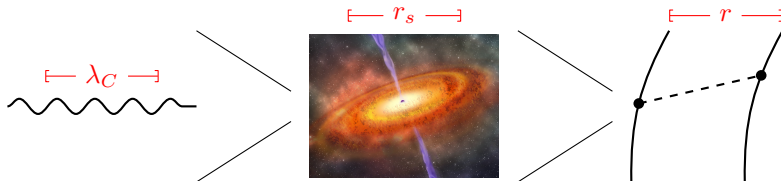
$$r_s \ll r \implies Gmq \ll 1$$

Gmq is the small parameter in the classical perturbative expansion

$$\lambda_C \ll r \implies q/m \ll 1 \text{ or } J \gg 1 \text{ [also } S \gg 1 \text{ in the presence of spin]}$$

Keep only the leading order in the small momentum-transfer expansion
(Bohr's correspondence principle)

EFT perspective



Full theory:
$$S_{\text{full}} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} R - \frac{1}{2} \int d^4x \sum_{i=1,2} (g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i - m_i^2 \phi_i^2) + \mathcal{O}(R^2 \phi^2)$$

[graviton: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$]

Classical limit $(q, \ell, G) \rightarrow (\hbar q, \hbar \ell, \hbar^{-1} G)$
Integrate out soft gravitons

method of regions
Beneke, Smirnov, hep-ph/9711391

KMOC

Observables
(classical)

Eikonal formula

$$\mathcal{M}_{\text{QFT}} = \mathcal{M}_{\text{EFT}}$$

Solve V_{PM} by matching amplitudes

EOM

Effective theory:
$$S_{\text{eff}} = - \int dt \left[m_1 \sqrt{1 - \mathbf{v}_1^2} + m_2 \sqrt{1 - \mathbf{v}_2^2} + V_{\text{PM}} \right]$$

V_{PM} given by ansatz

Latest progress

order	deflection & spin kick					waveform				Integration complexity
	plain	spin-orbit	spin-spin	spin ²	tidal	plain	spin-orbit spin-spin	spin ²	tidal	
1PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	X	trivial	trivial	trivial	trivial	~ tree-level
2PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	WQFT WEFT Amps	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps	WQFT WEFT Amps	~ 1-loop
3PM cons	WQFT WEFT Amps HEFT	WQFT Amps	WQFT Amps	Amps	WQFT WEFT	Amps HEFT	Amps			~ 2-loop
3PM diss	WQFT WEFT Amps HEFT	WQFT	WQFT		WQFT WEFT					
4PM cons	WQFT WEFT Amps	WQFT			WQFT					~ 3-loop
4PM diss	WQFT WEFT Amps	WQFT			WQFT					
5PM cons (2SF)	WQFT Amps									~ 4-loop

WQFT: see Jakobsen, Mogull, Plefka, Steinhoff

WEFT: see K  lin, Liu, Porto

HEFT: see Brandhuber, Chen, Travaglini, Wen; Aoude, Haddad, Helset, Damgaard

Updated from Jan Plefka's talk at MIAPbP 2024

Current frontier: partial results at **5PM-2SF**

Bern, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng, 2509.17412

Bern, Herrmann, Roiban, Ruf, Smirnov, Smith, Zeng, 2512.23654

Driesse, Jakobsen, Mogull, Nega, Plefka, Sauer, Usovitsch, 2601.16256

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Classical potential and field theory

- Pioneering work by Iwasaki Yoichi (岩崎洋一)

Fourth-Order Gravitational Potential Based on Quantum Field Theory.

Y. IWASAKI

Research Institute for Fundamental Physics, Kyoto University - Kyoto

(ricevuto l'1 Marzo 1971)

Loop is classical!

(point out the mistake in earlier literature)

- Effective field theory: $\exp[S_{\text{eff}}] = \int D\phi_{\text{short}} \exp[S_{\text{full}}]$

Goldberger, Rothstein, hep-th/0409156

Porto, gr-qc/0511061 Levi, Steinhoff, 1501.04956

and many more

- Classical interaction from amplitudes through “universality”

Vaidya, 1304.7475

Bjerrum-Bohr, Donoghue, Vanhove, 1309.0804

Bjerrum-Bohr, Donoghue, Holstein, Plante, Vanhove, 1410.7590

We want more synergy between EFT and on-shell amplitudes

Why amplitudeologists are working on GW?

It all starts with an enticing invitation



PHYSICAL REVIEW D **97**, 044038 (2018)

High-energy gravitational scattering and the general relativistic two-body problem

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(Received 29 October 2017; published 26 February 2018)

A technique for translating the classical scattering function of two gravitationally interacting bodies into a corresponding (effective one-body) Hamiltonian description has been recently introduced [Phys. Rev. D **94**, 104015 (2016)]. Using this technique, we derive, for the first time, to second-order in Newton's constant (i.e. one classical loop) the Hamiltonian of two point masses having an arbitrary (possibly relativistic) relative velocity. The resulting (second post-Minkowskian) Hamiltonian is found to have a tame high-energy structure which we relate both to gravitational self-force studies of large mass-ratio binary systems, and to the ultra high-energy quantum scattering results of Amati, Ciafaloni and Veneziano. We derive several consequences of our second post-Minkowskian Hamiltonian: (i) the need to use special phase-space gauges to get a tame high-energy limit; and (ii) predictions about a (rest-mass independent) linear Regge trajectory behavior of high-angular-momenta, high-energy circular orbits. Ways of testing these predictions by dedicated numerical simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the two-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian.

DOI: [10.1103/PhysRevD.97.044038](https://doi.org/10.1103/PhysRevD.97.044038)

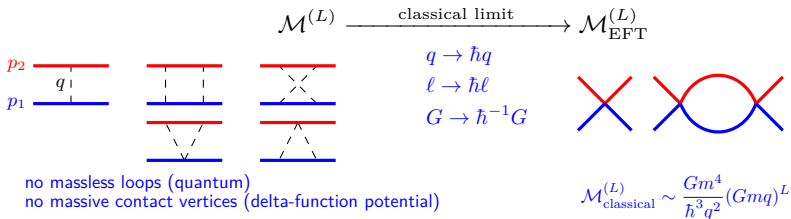
- Full theory: Schwarzschild black hole $\Rightarrow$ scalar field ϕ

$$\mathcal{L} = \sqrt{-g} \left[-\frac{1}{16\pi G} R + \frac{1}{2} \sum_{i=1,2} (g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i - m_i^2 \phi_i^2) \right] + \mathcal{O}(R^2 \phi^2)$$

- Effective theory: potential $V(\mathbf{k}, \mathbf{k}')$ given by an ansatz

$$L = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \left[\sum_{i=1,2} a_i^\dagger(-\mathbf{k}) \left(i\partial_t + \sqrt{\mathbf{k}^2 + m_i^2} \right) a_i(\mathbf{k}) - \int \frac{d^3 \mathbf{k}'}{(2\pi)^3} V(\mathbf{k}, \mathbf{k}') a_1^\dagger(\mathbf{k}') a_1(\mathbf{k}) a_2^\dagger(-\mathbf{k}') a_2(\mathbf{k}) \right]$$

- Solve the EFT potential by matching the full theory and EFT amplitudes order-by-order in G in the classical limit



Hamiltonian: $H = E_1 + E_2 + \sum_{n=1}^{\infty} \frac{G^n}{|\mathbf{r}|^n} c_{n\text{PM}}(\mathbf{p}^2)$

$$c_{1\text{PM}} = -\frac{\nu^2(m_1 + m_2)^2}{\gamma^2\xi}(2\sigma^2 - 1)$$

Westpfahl and Goller 1979

$$c_{2\text{PM}} = -\frac{\nu^2(m_1 + m_2)^3}{\gamma^2\xi} \left[\frac{3(5\sigma^2 - 1)}{4} - \frac{4\nu\sigma(2\sigma^2 - 1)}{\gamma\xi} + \frac{\nu^2(1 - \xi)(2\sigma^2 - 1)^2}{2\gamma^3\xi^2} \right]$$

Bel, Damour, Deruelle, Ibanez, Martin, 1981
Westpfahl 1985

$$c_{3\text{PM}} = \frac{\nu^2 m^4}{\gamma^2\xi} \left[\frac{3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3}{12} - \frac{4\nu(3 + 12\sigma^2 - 4\sigma^4) \operatorname{arcsinh} \sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \right. \\ \left. - \frac{3\nu\gamma(2\sigma^2 - 1)(5\sigma^2 - 1)}{2(1 + \gamma)(1 + \sigma)} + \frac{3\nu\sigma(20\sigma^2 - 7)}{2\gamma\xi} + \frac{\nu^2(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2)(2\sigma^2 - 1)}{4\gamma^3\xi^2} \right. \\ \left. + \frac{2\nu^3(3 - 4\xi)\sigma(2\sigma^2 - 1)^2}{\gamma^4\xi^3} - \frac{\nu^4(1 - 2\xi)(2\sigma^2 - 1)^3}{2\gamma^6\xi^4} \right]$$

Bern, Cheung, Roiban, Solon, Shen, Zeng, 1901.04424
(This is what Damour asked for)

State-of-the-art: partial results at $c_{5\text{PM}}^{\text{hyp } 2\text{SF}}$

Bern, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng, 2509.17412
Bern, Herrmann, Roiban, Ruf, Smirnov, Smith, Zeng, 2512.23654
Driesse, Jakobsen, Mogull, Nega, Plefka, Sauer, Usovitsch, 2601.16256

Including spin and finite size effect

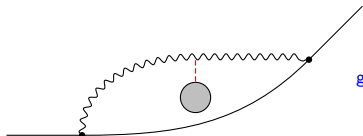
Bern, Luna, Roiban, Shen, Zeng, 2005.03071
Bern, Kosmopoulos, Luna, Roiban, **FT**, 2203.06202
Bern, Kosmopoulos, Luna, Roiban, Scheopner, **FT**, Vines, 2308.14176
Alaverdian, Bern, Kosmopoulos, Luna, Roiban, Scheopner, **FT**, 2407.10928, 2503.03739

$$E_1 = \sqrt{\mathbf{p}^2 + m_1^2} \quad E_2 = \sqrt{\mathbf{p}^2 + m_2^2}$$

$$\gamma = \frac{E_1 + E_2}{m_1 + m_2} \quad \xi = \frac{E_1 E_2}{(E_1 + E_2)^2}$$

$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2} \quad \sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

Gravitational tail



gravitational tails depend on the trajectory

$$c_4^{\text{hyp}} = \frac{M^7 \nu^2}{4\xi E^2} \left[\mathcal{M}_4^{\text{p}} + \nu \left(4\mathcal{M}_4^{\text{t}} \log\left(\frac{p_\infty}{2}\right) + \mathcal{M}_4^{\pi^2} + \mathcal{M}_4^{\text{rem}} \right) \right] + \mathcal{D}^3 \left[\frac{E^3 \xi^3}{3} c_1^4 \right] + \mathcal{D}^2 \left[\left(\frac{E^3 \xi^3}{p^2} + \frac{E\xi(3\xi-1)}{2} \right) c_1^4 - 2E^2 \xi^2 c_1^2 c_2 \right] \\ + \left(\mathcal{D} + \frac{1}{p^2} \right) \left[E\xi(2c_1 c_3 + c_2^2) + \left(\frac{4\xi-1}{4E} + \frac{2E^3 \xi^3}{p^4} + \frac{E\xi(3\xi-1)}{p^2} \right) c_1^4 + \left((1-3\xi) - \frac{4E^2 \xi^2}{p^2} \right) c_1^2 c_2 \right], \quad (8)$$

where $\mathcal{D} = \frac{d}{dp^2}$

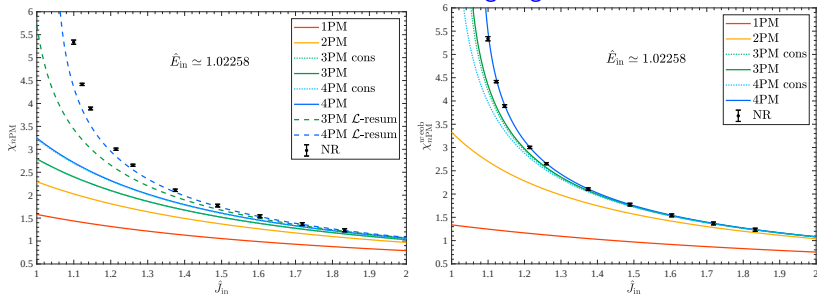
Bern, Parra-Martinez, Roiban, Ruf, Solon, Shen, Zeng, 2112.10750

Comparison with numerical relativity

Scattering process

Effective-one-body (EOB) resummation [Buonanno, Damour, gr-qc/9811091] significantly improves the agreement with NR result!

Observable: scattering angle

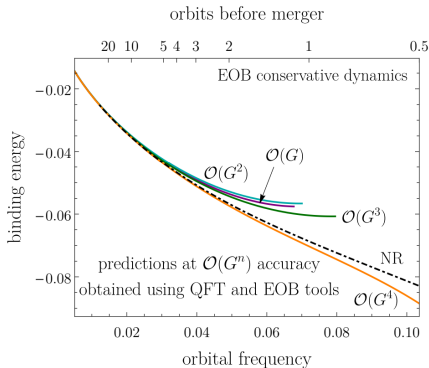


Comparison with numerical relativity

Bound system

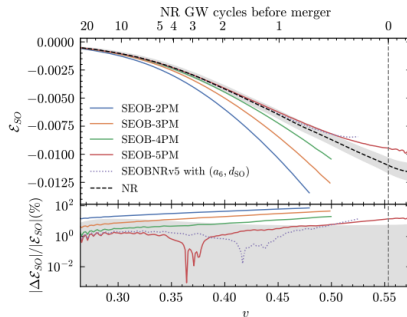
PM-informed two-body effective Hamiltonian

- ▶ Local contribution: direct analytic continuation
- ▶ Nonlocal (tail) contribution: supplement PN results from GR computations



Khalil, Buonanno, Steinhoff, Vines, 2205.05047

Snowmass white paper, 2204.05194



Buonanno, Mogull, Patil, Pompili, 2405.19181

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KMOC formalism Kosower, Maybee, O'Connell, 1811.10950

Observable in a scattering process as an in-in correlator

$$\begin{aligned}\Delta O &= \langle \psi_{\text{out}} | \mathbb{O} | \psi_{\text{out}} \rangle - \langle \psi_{\text{in}} | \mathbb{O} | \psi_{\text{in}} \rangle \\ &= \langle \psi_{\text{in}} | \hat{S}^\dagger \mathbb{O} \hat{S} | \psi_{\text{in}} \rangle - \langle \psi_{\text{in}} | \mathbb{O} | \psi_{\text{in}} \rangle \\ &= i \langle \psi_{\text{in}} | [\mathbb{O}, \hat{T}] | \psi_{\text{in}} \rangle + \langle \psi_{\text{in}} | \hat{T}^\dagger [\mathbb{O}, \hat{T}] | \psi_{\text{in}} \rangle\end{aligned}$$
$$\begin{aligned}\hat{S} &= 1 + i\hat{T} \\ \hat{T} - \hat{T}^\dagger &= i\hat{T}^\dagger \hat{T}\end{aligned}$$

Incoming state: $|\psi_{\text{in}}\rangle = \int d\Phi[p_1] d\Phi[p_2] \phi(p_1) \phi(p_2) e^{ip_1 \cdot b_1 + ip_2 \cdot b_2} |p_1 p_2\rangle$

Single particle phase-space and wavefunction:

$$d\Phi[p] = \frac{d^4 p}{(2\pi)^4} 2\pi \Theta(p^0) \delta(p^2 - m^2) \quad \int d\Phi[p] |\phi(p)|^2 = 1$$

We can compute observables by dressing amplitudes and cuts with the corresponding operators

$$\begin{aligned}\Delta O &= \int d\Phi[p_1] d\Phi[p_2] d\Phi[p'_1] d\Phi[p'_2] \phi(p_1) \phi(p_2) \phi(p'_1) \phi(p'_2) e^{i(p_1 - p'_1) \cdot b_1 + i(p_2 - p'_2) \cdot b_2} \\ &\quad \times \left[i \langle p'_1 p'_2 | [\mathbb{O}, \hat{T}] | p_1 p_2 \rangle + \langle p'_1 p'_2 | \hat{T}^\dagger [\mathbb{O}, \hat{T}] | p_1 p_2 \rangle \right]\end{aligned}$$

We can choose $\mathbb{O}$ to be P^μ , K^μ , $h_{\mu\nu}$, $\mathcal{E}(n)$, etc
impulse energy loss waveform ANEC

Classical observables Kosower, Maybee, O'Connell, 1811.10950

The wave packets are highly localized, while q_i is much smaller than their spread

$$\begin{aligned} & \int d\Phi[p_1] d\Phi[p_2] d\Phi[p'_1] d\Phi[p'_2] \phi(p_1) \phi(p_2) \phi^*(p'_1) \phi^*(p'_2) e^{i(p_1 - p'_1) \cdot b_1 + i(p_2 - p'_2) \cdot b_2} \\ & \simeq \int \frac{d^d q_1}{(2\pi)^d} \frac{d^d q_2}{(2\pi)^d} \hat{\delta}(2\bar{p}_1 \cdot q_1) \hat{\delta}(2\bar{p}_2 \cdot q_2) e^{iq_1 \cdot b_1 + iq_2 \cdot b_2} \quad \hat{\delta}(x) = 2\pi \delta(x) \end{aligned}$$

Resolution of identity in the two-massive-particle subspace

$$\mathbb{I} = \sum_X \int d\Phi[r_1] d\Phi[r_2] |r_1 r_2 X\rangle \langle r_1 r_2 X|$$

Expansion in the soft region $(q_i, \ell, k, G) \rightarrow (\hbar q_i, \hbar \ell, \hbar k, \hbar^{-1} G)$

Classical observables

$$\Delta O_{\text{cl}} = \int \frac{d^d q_1}{(2\pi)^d} \frac{d^d q_2}{(2\pi)^d} \hat{\delta}(2\bar{p}_1 \cdot q_1) \hat{\delta}(2\bar{p}_2 \cdot q_2) e^{iq_1 \cdot b_1 + iq_2 \cdot b_2} \left[i \langle p'_1 p'_2 | [\mathbb{O}, \hat{T}] | p_1 p_2 \rangle + \langle p'_1 p'_2 | \hat{T}^\dagger [\mathbb{O}, \hat{T}] | p_1 p_2 \rangle \right]_{\text{cl}}$$

Waveform and metric perturbation

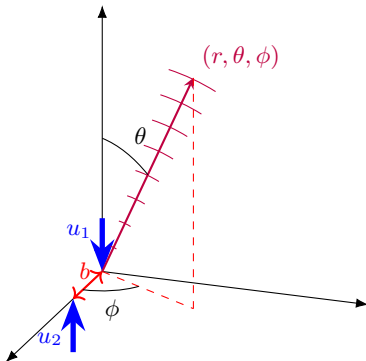
Herderschee, Roiban, **FT**, 2303.06112

Bini, Damour, De Angelis, Geralico, Herderschee, Roiban, **FT**, 2402.06604

De Angelis, Herderschee, Roiban, **FT**, 2511.10637

Metric perturbation: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

Waveform: $W(T_R, \theta, \phi) = \frac{1}{4G} \lim_{r \rightarrow \infty} r \varepsilon_+^\mu \varepsilon_+^\nu h_{\mu\nu} = \frac{1}{4G} (h_+^\infty - i h_\times^\infty)$



Operator: metric perturbation

$$\mathbb{H} = \varepsilon_+^\mu \varepsilon_+^\nu \hat{h}_{\mu\nu}(x) = \int d\Phi[k] \left[\hat{a}_{--}(k) e^{-ik \cdot x} + \text{c.c.} \right]$$

Since there is no radiation at $t \rightarrow -\infty$,

$$\Delta H(x) = \langle \psi_{\text{in}} | \hat{S}^\dagger \mathbb{H} \hat{S} | \psi_{\text{in}} \rangle = \int d\Phi[k] \left[\tilde{J}(k) e^{-ik \cdot x} - \text{c.c.} \right] \quad \tilde{J}(k) = \langle \psi_{\text{in}} | \hat{S}^\dagger \hat{a}_{--}(k) \hat{S} | \psi_{\text{in}} \rangle$$

Assuming the spatial current $J(y)$ is localized around $y = 0$,

$$\Delta H(x) \Big|_{|\mathbf{x}| \rightarrow \infty} = \frac{1}{4\pi|\mathbf{x}|} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} W(\omega, \mathbf{n}) e^{-i\omega\tau} \quad (\tau = x^0 - |\mathbf{x}| : \text{retarded time})$$

$$\mathcal{W}(\varepsilon, \omega, \mathbf{n}, p_1, p_2, b_1, b_2) = (-i) \langle \psi_{\text{in}} | \hat{S}^\dagger \hat{a}_{--}(k) \hat{S} | \psi_{\text{in}} \rangle \Big|_{k=(\omega, \omega\mathbf{n})}^{\text{IR finite}} \quad (\text{frequency domain waveform})$$

(The IR divergence can be absorbed in the definition of τ)

Relevant matrix elements for waveform

In the space of momentum transfer:

$$(-i)\langle\psi_{\text{in}}|\hat{S}^\dagger\hat{a}(k)\hat{S}|\psi_{\text{in}}\rangle = \int \frac{d^d q_1}{(2\pi)^d} \frac{d^d q_2}{(2\pi)^d} \hat{\delta}(2p_1 \cdot q_1) \hat{\delta}(2p_2 \cdot q_2) e^{iq_1 \cdot b_1 + iq_2 \cdot b_2} \\ \times \left[\langle p'_1 p'_2 k | \hat{T} | p_1 p_2 \rangle - i \langle p'_1 p'_2 | \hat{T}^\dagger \hat{a}(k) \hat{T} | p_1 p_2 \rangle \right]$$

The matrix elements can be classified as follows:

$$\langle p'_1 p'_2 k | \hat{T} | p_1 p_2 \rangle \Big|_{\text{conn}} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{1 \text{ loop}} \qquad i \langle p'_1 p'_2 | \hat{T}^\dagger \hat{a}(k) \hat{T} | p_1 p_2 \rangle \Big|_{\text{conn}} = \mathcal{S}^{1 \text{ loop}}$$



The full connected contribution to waveform:

$$\mathcal{M}_{\text{conn}} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{1 \text{ loop}} - \mathcal{S}^{1 \text{ loop}}$$

Contribution from disconnected T matrix elements:

$$\langle p'_1 p'_2 k | \hat{T} | p_1 p_2 \rangle \Big|_{\text{disc}} = \mathcal{M}_{\text{const}} \qquad i \langle p'_1 p'_2 | \hat{T}^\dagger \hat{a}(k) \hat{T} | p_1 p_2 \rangle \Big|_{\text{disc}} = \mathcal{M}_{\text{disc}}$$



Tree-level amplitudes and LO waveform

From GR: Kovacs, Thorne, Astrophys. J. 224 62, 1978

From world line QFT: Jakobsen, Mogull, Plefka, Steinhoff, 2101.12688

From KMOC: Cristofoli, Gonzo, Kosower, O'Connell, 2107.10193

For convenience, we define the Fourier transform from q -space to b -space,

$$\text{FT}[f(q_1, q_2, k)] = \int \frac{d^d q_1}{(2\pi)^d} \frac{d^d q_2}{(2\pi)^d} \hat{\delta}(2p_1 \cdot q_1) \hat{\delta}(2p_2 \cdot q_2) e^{iq_1 \cdot b_1 + iq_2 \cdot b_2} \hat{\delta}(q_1 + q_2 - k) f(q_1, q_2, k) \Big|_{k=(\omega, \omega \mathbf{n})}$$

LO waveform from KMOC

$$\mathcal{W}(\omega, \mathbf{n}) \Big|_{\text{LO}} = \text{FT}[\mathcal{M}^{\text{tree}}]$$

Classical tree-level amplitude [Luna, Nicholson, O'Connell, White, 1711.03901](#)

$$\begin{aligned} \mathcal{M}^{\text{tree}} &= -\kappa^2 m_1^2 m_2^2 \left[\frac{(P_{12} \cdot \varepsilon)^2 + \sigma(Q_{12} \cdot \varepsilon)(P_{12} \cdot \varepsilon)}{q_1^2 q_2^2} + \left(\sigma^2 - \frac{1}{d-2} \right) \left(\frac{(Q_{12} \cdot \varepsilon)^2}{4q_1^2 q_2^2} - \frac{(P_{12} \cdot \varepsilon)^2}{4w_1^2 w_2^2} \right) \right] \\ &= \mathcal{M}_{d=4}^{\text{tree}} + \epsilon \mathcal{M}_{\text{extra}}^{\text{tree}} \end{aligned}$$

Of course, at LO the $\mathcal{O}(\epsilon)$ term is irrelevant.

$$\begin{aligned} P_{12}^\mu &= w_1 u_2^\mu - w_2 u_1^\mu & u_i &= p_i/m_i & w_i &= u_i \cdot k \\ Q_{12}^\mu &= (q_1 - q_2)^\mu - \frac{q_1^2}{w_1} u_1^\mu + \frac{q_2^2}{w_2} u_2^\mu \end{aligned}$$

Compact analytic result

Time domain waveform [following the conventions in De Angelis, Novichikov, Gonzo, 2309.17429]

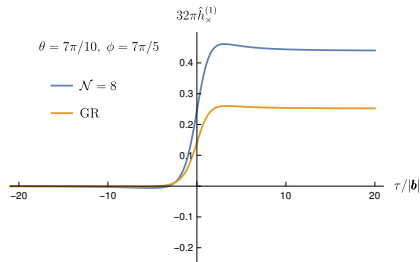
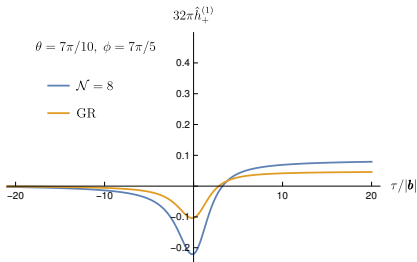
$$\begin{aligned}
 W^{(1)}(\tau, \theta, \phi) = & - \frac{Gm_1 m_2}{4\sqrt{-b^2} \hat{w}_1^2 \hat{w}_2^2 \sqrt{1+T_2^2} \left[\sigma + \sqrt{(1+T_1^2)(1+T_2^2)} + T_1 T_2 \right]} \\
 & \times \left\{ \frac{3\hat{w}_1 + 2\sigma(2T_1 T_2 \hat{w}_1 - T_2^2 \hat{w}_2 + \hat{w}_2) - (2\sigma^2 - 1)\hat{w}_1}{\sigma^2 - 1} \mathcal{F}_{1,2}^2 \right. \\
 & - \frac{4\sigma T_2 \hat{w}_2 \mathcal{F}_1 + 2(2\sigma^2 - 1) [T_1(1+T_2^2)\hat{w}_2 \mathcal{F}_1 + T_2(T_1 T_2 \hat{w}_1 + \hat{w}_2) \mathcal{F}_2]}{\sqrt{\sigma^2 - 1}} \mathcal{F}_{1,2} \\
 & \left. + 4(1+T_2^2)\hat{w}_2 \mathcal{F}_1 \mathcal{F}_2 - 4\sigma(1+T_2)^2 \hat{w}_2 (\mathcal{F}_1^2 + \mathcal{F}_2^2) + 2(2\sigma^2 - 1)(1+2T_2^2)\hat{w}_2 \mathcal{F}_1 \mathcal{F}_2 \right\} + (1 \leftrightarrow 2)
 \end{aligned}$$

where

$$\begin{aligned}
 \hat{w}_1 &= u_1 \cdot n & \hat{w}_2 &= u_2 \cdot n & T_1 &= \frac{\sqrt{\sigma^2 - 1}(\tau - b_1 \cdot n)}{\hat{w}_1 \sqrt{-b^2}} & T_2 &= \frac{\sqrt{\sigma^2 - 1}(\tau - b_2 \cdot n)}{\hat{w}_2 \sqrt{-b^2}} \\
 b^\mu &= b_1^\mu - b_2^\mu & \hat{b}^\mu &= \frac{b^\mu}{\sqrt{-b^2}}
 \end{aligned}$$

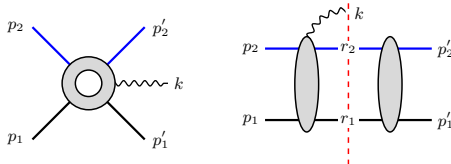
$$\mathcal{F}_{1,2} = (\varepsilon_+^* \cdot u_1) \hat{w}_2 - (\varepsilon_+^* \cdot u_2) \hat{w}_1 \quad \mathcal{F}_1 = (\hat{b} \cdot \varepsilon_+^*) \hat{w}_1 - (\hat{b} \cdot n)(\varepsilon_+^* \cdot u_1) \quad \mathcal{F}_2 = (\hat{b} \cdot \varepsilon_+^*) \hat{w}_2 - (\hat{b} \cdot n)(\varepsilon_+^* \cdot u_2)$$

LO waveform



Agree with Jakobsen, Mogull, Plefka, Steinhoff, 2101.12688
From spinning binaries: De Angelis, Novichkov, Gonzo, 2309.17429
Brandhuber, Brown, Chen, Gowdy, Travaglini, 2310.04405
Aoude, Haddad, Heissenberg, Helset, 2310.05832

One-loop matrix elements Herderschee, Roiban, FT, 2303.06112



- Use generalized unitarity to construct the integrand
- Expand the integrand in the classical limit

$$\frac{1}{(p + \ell)^2 - m^2 + i0} = \frac{1}{2p \cdot \ell + \ell^2 + i0} = \frac{1}{2p \cdot \ell + i0} \sum_{n=0}^{\infty} \left(-\frac{\ell^2}{2p \cdot \ell + i0} \right)^n$$

- Reduce the integrand to master integral basis [IBP automated by FIRE6: Smirnov, Chuharev, 1901.07808]
- Classically singular (superclassical) terms cancel between the two matrix elements
- The cut makes matter propagators retarded Caron-Huot, Giroux, Hannesdottir, Mizera, 2308.02125

$$\text{amplitude: } \text{PV} \frac{1}{2u_1 \cdot \ell} \xrightarrow{\text{add cut}} \frac{1}{2u_1 \cdot \ell - i0}$$

- Classical waveform contains only imaginary IR divergences

Equivalence to world-line formalism proved by Capatti, Zeng, 2412.10864

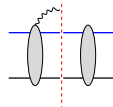
The full connected contribution

Hederschee, Roiban, **FT**, 2303.06112
 Brandhuber, Brown, Chen, De Angelis,
 Gowdy, Travaglini, 2303.06111
 Georgoudis, Heissenberg, Vazquez-Holm, 2303.07006

$$\begin{aligned}
 \mathcal{M}^{1 \text{ loop}} &= -\frac{i \kappa^2}{32\pi} (m_1 w_1 + m_2 w_2) \left[\frac{1}{\epsilon} - \log \frac{w_1 w_2}{\mu^2} \right] \mathcal{M}_{d=4}^{\text{tree}} \\
 &\quad + \kappa^4 \left[A_{\text{rat}}^R + \frac{A_1^R}{\sqrt{w_2^2 - q_1^2}} + \frac{A_2^R}{\sqrt{w_1^2 - q_2^2}} + \frac{A_3^R}{\sqrt{-q_1^2}} + \frac{A_4^R}{\sqrt{-q_2^2}} \right] \\
 &\quad + i \kappa^4 \left[A_{\text{rat}}^I + A_1^I \frac{\text{arcsinh} \frac{w_2}{\sqrt{-q_1^2}}}{\sqrt{w_2^2 - q_1^2}} + A_2^I \frac{\text{arcsinh} \frac{w_1}{\sqrt{-q_2^2}}}{\sqrt{w_1^2 - q_2^2}} + A_3^I \log \frac{q_2^2}{q_1^2} + A_4^I \log \frac{w_1}{w_2} + A_5^I \frac{\text{arccosh} \sigma}{(\sigma^2 - 1)^{3/2}} \right] \\
 &= -\frac{i \kappa^2}{32\pi\epsilon} (m_1 w_1 + m_2 w_2) \mathcal{M}_{d=4}^{\text{tree}} + \widetilde{\mathcal{M}}_{1 \text{ loop}}^{\text{fin}}
 \end{aligned}$$



$$\begin{aligned}
 \mathcal{S}^{1 \text{ loop}} &= \frac{i \kappa^2 \Gamma}{64\pi} (m_1 w_1 + m_2 w_2) \left[\frac{1}{\epsilon} - \log \frac{w_1 w_2}{(\sigma^2 - 1) \mu^2} \right] \mathcal{M}_{d=4}^{\text{tree}} \quad \Gamma = \frac{3\sigma - 2\sigma^3}{(\sigma^2 - 1)^{3/2}} \\
 &\quad + \frac{i \kappa^4}{256\pi} \frac{(2\sigma^2 - 1)^2}{(\sigma^2 - 1)^{3/2}} (m_1 w_1 + m_2 w_2) \left[\frac{1}{\epsilon} - \log \frac{w_1 w_2}{(\sigma^2 - 1) \mu^2} \right] \frac{m_1^2 m_2^2 (u_1 \cdot f \cdot u_2)^2 (w_1^2 + \sigma w_1 w_2 + w_2^2)}{w_1^3 w_2^3} \\
 &\quad + i \kappa^4 \left[A_{\text{rat}}^{\text{cut}} + A_1^{\text{cut}} \log \frac{w_1}{w_2} + A_2^{\text{cut}} \log \frac{w_1 w_2}{-q_1^2} + A_3^{\text{cut}} \log \frac{w_1 w_2}{-q_2^2} + A_4^{\text{cut}} \frac{\text{arccosh} \sigma}{(\sigma^2 - 1)^{3/2}} \right] \\
 &= \frac{i \kappa^2 \Gamma}{64\pi\epsilon} (m_1 w_1 + m_2 w_2) \mathcal{M}_{d=4}^{\text{tree}} + \widetilde{\mathcal{S}}_{1 \text{ loop}}^{\text{fin}} + \mathcal{S}_{1 \text{ loop}}^{\text{local}}
 \end{aligned}$$



The full connected contribution

The IR divergence exponentiates Caron-Huot, Giroux, Hannesdottir, Mizera, 2308.02125

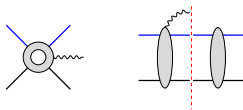
$$\mathcal{M}_{\text{conn}} = \mathcal{M}_{\text{conn}}^{\text{fin}} \exp \left[-i\omega \frac{\kappa^2 E(1 - \Gamma/2)}{32\pi\epsilon} \right] \quad (m_1 w_1 + m_2 w_2 = \omega E)$$

such that it can be absorbed by a re-definition of retarded time

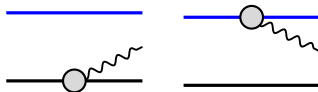
$$\tau \rightarrow \tau - \frac{\kappa^2 E(1 - \Gamma/2)}{32\pi\epsilon}$$

At one-loop level, this leads to

$$\begin{aligned} \mathcal{M}_{\text{conn}}^{\text{fin}} &= \mathcal{M}^{\text{tree}} + \mathcal{M}^{\text{1 loop}} - \mathcal{S}^{\text{1 loop}} + \frac{i\kappa^2(m_1 w_1 + m_2 w_2)(1 - \Gamma/2)}{32\pi\epsilon} \mathcal{M}^{\text{tree}} \\ &= \mathcal{M}^{\text{tree}} + \left[\widetilde{\mathcal{M}}_{\text{1 loop}}^{\text{fin}} + \frac{i\kappa^2(m_1 w_1 + m_2 w_2)\mathcal{M}_{\text{extra}}^{\text{tree}}}{32\pi} \right] - \left[\widetilde{\mathcal{S}}_{\text{1 loop}}^{\text{fin}} + \frac{i\kappa^2\Gamma(m_1 w_1 + m_2 w_2)\mathcal{M}_{\text{extra}}^{\text{tree}}}{64\pi} \right] \\ &= \mathcal{M}^{\text{tree}} + \mathcal{M}_{\text{1 loop}}^{\text{fin}} - \mathcal{S}_{\text{1 loop}}^{\text{fin}} \end{aligned}$$



Disconnected amplitude contribution



They are supported only by zero-energy gravitons

$$\mathcal{W}_{\text{const}}(\omega, \mathbf{n}) = \int d\Phi_{p_1} \mathcal{M}_3(p_1, k) e^{ik \cdot b_1} + \int d\Phi_{p_2} \mathcal{M}_3(p_2, k) e^{ik \cdot b_2} = \hat{\delta}(\omega) \left[\frac{m_1(\varepsilon \cdot u_1)^2}{u_1 \cdot n} + \frac{m_2(\varepsilon \cdot u_2)^2}{u_2 \cdot n} \right]$$

The constant background agrees with the MPM calculation

$$\mathcal{W}_{\text{const}}(\tau, \mathbf{n}) = \frac{m_1(\varepsilon \cdot u_1)^2}{u_1 \cdot n} + \frac{m_2(\varepsilon \cdot u_2)^2}{u_2 \cdot n} \quad n = (1, \mathbf{n})$$

It can be removed by a BMS supertranslation [Veneziano, Vilkovisky, 2201.11607](#)

$$\delta_T \left[\frac{h_{ab}^\infty}{r} \right] = \frac{1}{r} (2D_a D_b - \gamma_{ab} \Delta) T(n) - T(n) \partial_\tau \left[\frac{h_{ab}^\infty}{r} \right] \quad T(n) = 2G \left[m_1 \frac{w_1}{\omega} \log \frac{w_1}{\omega} + m_2 \frac{w_2}{\omega} \log \frac{w_2}{\omega} \right]$$

The MPM waveform is in the **intrinsic BMS frame**. If we assume $\mathcal{M}_3 = 0$, then we are in the **canonical BMS frame**. They are related by the above BMS supertranslation.

Disconnected cut contribution [DB, TD, SDA, AG, AH, RR, FT, 2402.06604]

The T matrices in the KMOC cut can be disconnected:

$$i \langle p'_1 p'_2 | \hat{T}^\dagger \hat{a}(k) \hat{T} | p_1 p_2 \rangle \Big|_{\text{disc.}} \supset \int d\Phi[\ell] d\Phi[r_2]$$

- ▶ The cut graviton ℓ only has support on zero energy
- ▶ We only need to consider the soft limit of the left blob

$$\mathcal{M}_6 \simeq \kappa \mathcal{M}_5 \left[\sum_a \frac{\eta_a \varepsilon_{\mu\nu}(\ell) p_a^\mu p_a^\nu}{2\ell \cdot p_a + i0\eta_a} + \frac{\varepsilon_{\mu\nu}(\ell) k^\mu k^\nu}{2\ell \cdot k + i0} \right]$$

- ▶ This loop integral requires a further regularization to **include the zero-energy graviton contribution**

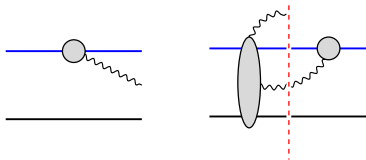
Disconnected cut contribution [DB, TD, SDA, AG, AH, RR, FT, 2402.06604]

The disconnected T matrix elements in the cut lead to

$$\mathcal{M}_{\text{disc}} = i \langle p'_1 p'_2 | \hat{T}^\dagger \hat{a}(k) \hat{T} | p_1 p_2 \rangle \Big|_{\text{disc}} = -2i\omega G \left[m_1 \frac{w_1}{\omega} \log \frac{w_1}{\omega} + m_2 \frac{w_2}{\omega} \log \frac{w_2}{\omega} \right] \mathcal{M}^{\text{tree}} = -i\omega T(n) \mathcal{M}^{\text{tree}}$$

which exactly reproduces the iteration part of the BMS supertranslation

$$\delta_T \left[\frac{h_{ab}^\infty}{r} \right] = \frac{1}{r} (2D_a D_b - \gamma_{ab} \Delta) T(n) - T(n) \partial_\tau \left[\frac{h_{ab}^\infty}{r} \right]$$



This contribution can be re-summed into a coherent zero-energy graviton dressing of asymptotic massive states [Elkhidir, O'Connell, Roiban, 2408.15961](#) and [WIP](#)

Full KMOC waveform at one-loop [DB, TD, SDA, AG, AH, RR, FT, 2402.06604]

$$\mathcal{W}^{\text{KMOC}}(\omega, \mathbf{n}) = \mathcal{W}_{\text{const}} + \mathcal{W}^{\text{tree}} + \text{FT} \left[\mathcal{M}_{1\text{ loop}}^{\text{fin}} - \mathcal{S}_{1\text{ loop}}^{\text{fin}} + \mathcal{M}_{\text{disc}} \right]$$

- Fourier transform to the impact-parameter space can only be done numerically for generic kinematic setup (see Brunello, De Angelis 2403.08009 for improvements)
- Analytic computation is available under the **soft graviton limit** and/or **small relative velocity (PN) expansion**

Soft expansion (up to the first non-universal coefficient)

$$\mathcal{W}^{\text{KMOC}}(\omega, \mathbf{n}) \sim \frac{\mathcal{A}}{\omega} + \mathcal{B} \log \omega + \mathcal{C} \omega (\log \omega)^2 + \mathcal{D} \omega \log \omega$$

All order conjecture in leading log: Alessio, Di Vecchia, Heissenberg, 2407.04128

PN expansion (up to 2.5PN)

$$\begin{aligned} \mathcal{W}^{\text{KMOC}}(\omega, \mathbf{n}) \sim & 1 + \frac{GM^2\nu}{p_\infty} (1 + p_\infty + p_\infty^2 + p_\infty^3 + p_\infty^4 + p_\infty^5 + \dots) \\ & + \frac{GM}{bp_\infty^2} \frac{GM^2\nu}{p_\infty} (1 + p_\infty + p_\infty^2 + p_\infty^3 + p_\infty^4 + p_\infty^5 + \dots) \end{aligned}$$

$$M = m_1 + m_2$$

$$\nu = m_1 m_2 / M^2$$

$$p_\infty = \sqrt{\sigma^2 - 1}$$

Full KMOC waveform at one-loop [DB, TD, SDA, AG, AH, RR, FT, 2402.06604]

$$\mathcal{W}^{\text{KMOC}}(\omega, \mathbf{n}) = \mathcal{W}_{\text{const.}} + \mathcal{W}^{\text{tree}} + \text{FT}[\mathcal{M}_{1\text{-loop}}^{\text{fin}} - \mathcal{S}_{1\text{-loop}}^{\text{fin}} + \mathcal{M}_{\text{disc}}]$$

Both in perfect agreement with
the GR based multipolar post-Minkowskian calculation

Bini, Damour, Geralico, 2309.14925

Similar comparison up to $\mathcal{O}(p_\infty^3)$ is done in Georgoudis, Heissenberg, Russo, 2402.06361

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Soft expansion (up to the first non-universal coefficient)

$$\mathcal{W}^{\text{KMOC}}(\omega, \mathbf{n}) \sim \frac{\mathcal{A}}{\omega} + \mathcal{B} \log \omega + \mathcal{C} \omega (\log \omega)^2 + \mathcal{D} \omega \log \omega$$

All order conjecture in leading log: Alessio, Di Vecchia, Heissenberg, 2407.04128

PN expansion (up to 2.5PN)

$$\begin{aligned} \mathcal{W}^{\text{KMOC}}(\omega, \mathbf{n}) \sim & 1 + \frac{GM^2\nu}{p_\infty} (1 + p_\infty + p_\infty^2 + p_\infty^3 + p_\infty^4 + p_\infty^5 + \dots) \\ & + \frac{GM}{bp_\infty^2} \frac{GM^2\nu}{p_\infty} (1 + p_\infty + p_\infty^2 + p_\infty^3 + p_\infty^4 + p_\infty^5 + \dots) \end{aligned}$$

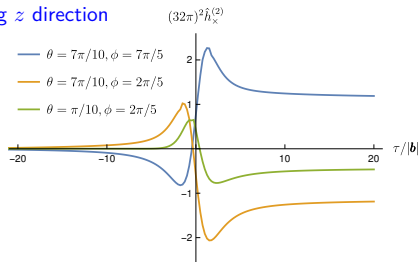
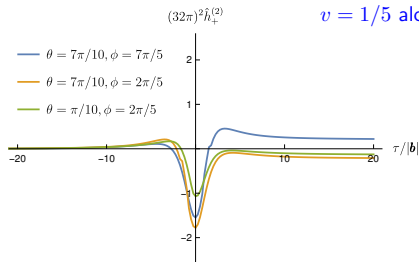
$$M = m_1 + m_2$$

$$\nu = m_1 m_2 / M^2$$

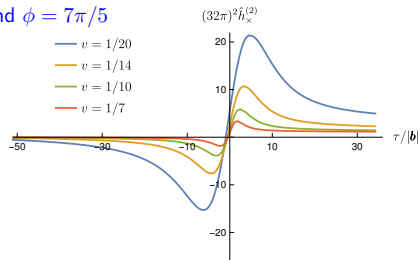
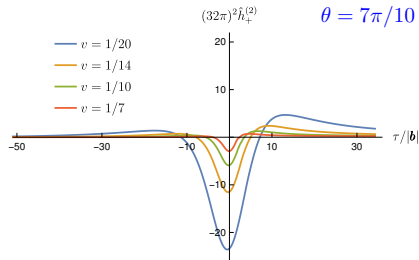
$$p_\infty = \sqrt{\sigma^2 - 1}$$

NLO waveform

$v = 1/5$ along z direction



$\theta = 7\pi/10$ and $\phi = 7\pi/5$



Current status

Comparison between MPM and EFT:

- ▶ Recently, the agreement between MPM and EFT result has been pushed to 3.5PN (or p_∞^7) [Bini, Damour, Geralico, 2604.21522](#)
- ▶ The agreement requires an additional BMS supertranslation on top of the Veneziano-Vilkovisky one

Further development:

- ▶ Improved IBP reduction: treat loop integral and Fourier transform on the same footing [Brunello, De Angelis, Kosower, 2511.05412](#)
- ▶ New integration technique using intersection theory
[Brunello, Chestnov, Crisanti, Giroux, Smith, 2510.26874](#)
- ▶ Quadratic-in-spin waveform [Bohnenblust, Ita, Kraus, Schlenk, 2505.15724, 2312.14859](#)
- ▶ 2PN at two-loop order [Bini, Damour, Geralico, 2407.02076](#)
- ▶ Universal soft coefficient to all order [Alessio, Di Vecchia, Heissenberg, 2407.04128](#)
- ▶ and more

Off-shell extension: Newman-Penrose scalar

De Angelis, Herderschee, Roiban, **FT**, 2511.10637

Peeling theorem (requiring C^4 regularity at spatial infinity): $\Psi_n \sim r^{n-5}$ [Penrose 1965]

$$\begin{aligned}\Psi_4 &= \Psi_{abcd} \lambda^a \lambda^b \lambda^c \lambda^d & \Psi_3 &= \Psi_{abcd} \lambda^a \lambda^b \lambda^c \rho^d & \Psi_2 &= \Psi_{abcd} \lambda^a \lambda^b \rho^c \rho^d \\ \Psi_1 &= \Psi_{abcd} \lambda^a \rho^b \rho^c \rho^d & \Psi_0 &= \Psi_{abcd} \rho^a \rho^b \rho^c \rho^d\end{aligned}$$

Metric perturbation at finite space-time distance

$$\begin{aligned}h^{\mu\nu}(x) &= \langle \Psi, t | \hat{h}_{\mu\nu}(t, \mathbf{x}) | \Psi, t \rangle - \langle \Psi, -\infty | \hat{h}_{\mu\nu}(-\infty, \mathbf{x}) | \Psi, -\infty \rangle \\ &= - \int \frac{d\omega}{2\pi} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{e^{-i\omega t + i\mathbf{k} \cdot \mathbf{x}}}{(\omega + i0)^2 - \mathbf{k}^2} \underbrace{T_{\alpha\beta}^{\mu\nu} J^{\alpha\beta}(\omega, \mathbf{k})}_{\bar{h}^{\mu\nu}(\omega, \mathbf{k})}\end{aligned}$$

In the soft graviton limit $k^\mu \rightarrow 0$, there are two relevant regions

- ▶ Radiation region: $k^\mu \sim r^0$
- ▶ Coulomb region: $k^\mu \sim r^{-1}$ [more scales, more challenging]

Off-shell extension: Newman-Penrose scalar

De Angelis, Herderschee, Roiban, **FT**, 2511.10637

In the soft form factor $\tilde{h}^{\mu\nu}(\omega, \mathbf{k})$:

- ▶ Analytic sector $\implies$ Penrose peeling $\Psi_n \sim r^{n-5}$
- ▶ Non-analytic sector $\implies$ reproduce known peeling violation

$$\text{tree-level NLO soft } \mathcal{B} \log \omega \implies \Psi_0 = \frac{3G^2 m_1 m_2}{4r^4} \frac{(\epsilon \cdot \mathbf{v})^2}{|\mathbf{v}|^3} + \mathcal{O}(\mathbf{v}^0) \quad \text{Coulomb region contribution}$$

Christodoulou 2002

- ▶ Tail contribution $\implies$ Novel peeling violations

$$\tilde{h}_{1\text{-loop}}^{\mu\nu} = \underbrace{-2iG(p_1 + p_2) \cdot k \log \left[\frac{\pi(-k^2)}{\mu^2} \right]}_{\text{tail}} \tilde{h}_{\text{tree}}^{\mu\nu} + (\text{analytic in } k^2)$$

leads to the novel peeling violation

$$\Psi_0 \sim G^3 r^{-4} \quad \Psi_1 \sim G^3 r^{-3} \quad \text{radiation region contribution}$$

[in addition to Damour 1985 and Christodoulou 2002]

[still need further investigation; see Boschetti, Campiglia, 2603.22681]

Table of Contents

Introduction: amplitudes meet GW

EFT matching

Observable based formalism (KMOC)

Outlook and conclusion

Recent developments: Amplitudes + X

► Re-summation:

- Perturbative orders: eikonal, Magnusian, etc Di Vecchia, Heissenberg, Russo, Veneziano, Kim, Kim, Kim, Lee, Brandhuber, Brown, Pichini, Travaglini, Vives Matasan, Alessio, Gonzo, Shi,...
- Special kinematic limit: soft, high energy, etc
Di Vecchia, Heissenberg, Alessio, Rothstein, Saavedra, Del Duca, Gonzo, Rosi, ...
- Trajectories: bound, large bend, etc
Porto, Kälín, Liu, Adamo, Gonzo, Khalaf, Telem, Shen, De Angelis, Aoki, Cristofoli, ...

► Horizon absorption

Goldberger, Rothstein, Aoude, Ochirov, Callum, Ruf, Gatica, Bautista, Huang, Kim, Chen, Hsieh, ...

► Connection with BHPT

Bautista, Bonelli, Iossa, Tanzini, Zhou, Ivanov, Li, Parra-Martinez, ...

► Connection with self-force

Cheung, Parra-Martinez, Rothstein, Shah, Wilson-Gerow, Kosmopolous, Solon, ...

► Dynamical Love number and BH analyticity

Kosmopolous, Perrone, Solon, Correia, Gopakka, Isabella, Wolz, Caron-Huot, ...

► Beyond perturbation theory

Aoki, Cristofoli, Huang, ...

► Quantum effect on curved background

Aoude, O'Connell, Sergola, White, Aoki, Cristofoli, Jeong, Yoshimura

► Beyond GR

Alviani, Falkowski, Marinellis, Wilson-Gerow, ...

Conclusion and outlook

- ▶ EFT and on-shell methods are powerful tools for solving GR two-body problems
 - Lead to results that are otherwise hard to get
 - Can be applied to a large variety of problems
- ▶ Efficient computational tools propel the rapid progress of our field
 - On-shell methods for constructing integrands
 - IBP and DE for Feynman integrals and their generalization
- ▶ Can on-shell thinking provide a new perspective for solving old problems: [Horizon absorption, superradiance, QNM, etc](#)

One of the important takeaway messages of our new results is that, after having sorted out the subtleties that were hidden in the EFT one-loop waveform, we obtained a remarkable confirmation that the classical limit of an amplitude-based waveform does correctly incorporate the many subtle classical effects that were included in the $O(G^2\eta^5)$ -accurate MPM waveform such as (i) radiation-reaction effects on the worldlines, (ii) high-multipolarity tail effects in the wave-zone, and (iii) cubically nonlinear multipole couplings in the exterior zone. The fact that the road leading to the present successful EFT/MPM comparison had some bumps, which taught us interesting lessons, is another example of the useful synergy between amplitude-based, and classical perturbation-theory-based, approaches to gravitational physics.

Thanks for listening!