

On-Shell Simplicity and Off-Shell Dynamics

The Two Faces of Quantum Field Theory in Gravity

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Perturbative Gravity: Quantum and Classical

General relativity is a nonlinear field theory,

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R,$$

Expanding around flat spacetime,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad \kappa^2 = 32\pi G,$$

gives perturbative gravity.

Quantum Gravity

- scattering amplitudes
- loop corrections
- ultraviolet structure

Classical Gravity

- effective potentials
- impulse & radiation
- gravitational waves

The same perturbative field-theory language successfully describes both.

DeWitt, Phys. Rev. **162** (1967); Bern et al., Phys. Rept. **873** (2020); Kosower, Maybee & O'Connell, JHEP **02** (2019) 137....

Why On Shell?

Perturbative gravity quickly becomes complicated.

$$S_{\text{EH}} = S_2 + \kappa S_3 + \kappa^2 S_4 + \dots$$

Feynman expansion

- infinitely many vertices
- gauge redundancy
- rapidly growing diagrams
- cumbersome intermediate expressions

On-shell methods

- physical external states
- gauge-invariant observables
- hidden structures
- remarkable simplification

The final physical answer is often much simpler than the intermediate calculation. Modern on-shell methods aim to exploit the hidden simplicity of perturbative gravity.

Elvang & Huang, *Scattering Amplitudes in Gauge Theory and Gravity* (2015); Bern et al., *Phys. Rept.* **873** (2020).

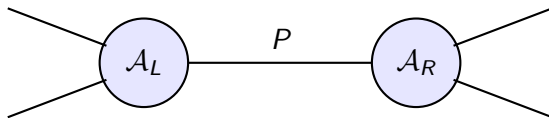
Factorization: complicated processes split into simpler ones

When an intermediate momentum approaches its mass shell,

$$p^2 \longrightarrow 0,$$

an n -point amplitude develops a pole and factorizes:

$$\mathcal{A}_n \longrightarrow \sum_{\text{states}} \mathcal{A}_L \frac{1}{p^2} \mathcal{A}_R.$$



locality + unitarity

The amplitude reconstructs itself from lower-point physical data.

Weinberg, *Phys. Rev.* **140** (1965); Britto–Cachazo–Feng–Witten, *PRL* **94** (2005); Arkani-Hamed–Kaplan, *JHEP* **04** (2008) 076.

Gauge-theory organization and color–kinematics duality

A gauge-theory amplitude can be organized as

$$\mathcal{A}_{\text{YM}} = g^{n-2} \sum_i \frac{c_i n_i}{D_i}.$$

$$c_i$$

color factors

- Lie algebra
- structure constants
- Jacobi identities

$$n_i$$

kinematics

- momenta
- polarizations
- spin information

$$D_i$$

propagators

- graph topology
- physical poles
- loop denominators

Color–kinematics duality asks whether

$$c_i + c_j + c_k = 0 \quad \Longrightarrow \quad n_i + n_j + n_k = 0.$$

Kinematics may be organized to satisfy the same algebra as color.

The double copy

Once the kinematic numerators satisfy the same Jacobi relations as color,

$$\mathcal{A}_{\text{YM}} = g^{n-2} \sum_i \frac{c_i n_i}{D_i},$$

one may replace the color factor by a second kinematic numerator:

$$c_i \longrightarrow \tilde{n}_i.$$

This gives a gravity amplitude,

$$\mathcal{M}_{\text{grav}} = \left(\frac{\kappa}{2}\right)^{n-2} \sum_i \frac{n_i \tilde{n}_i}{D_i}.$$

$\text{gravity} \sim \text{gauge theory} \times \text{gauge theory}$

Gravity is reconstructed from simpler gauge-theory building blocks.

Kawai–Lewellen–Tye, NPB **269** (1986) 1; Bern–Carrasco–Johansson, PRL **105** (2010) 061602; Bern et al., Phys. Rept. **873** (2020).

Why is loop-level double copy difficult?

The double copy is remarkably powerful at tree level, but becomes much more challenging beyond tree level.

Tree level

- factorization
- color–kinematics duality
- simple recursive structures
- all multiplicity BCJ numerators

Loop level

- numerators depend on loop momenta
- loop-momentum shifts matter
- generalized gauge freedom grows
- locality and Jacobi must coexist

At L loops,

$$\mathcal{A}_{\text{YM}}^{(L)} = \sum_i \int \prod_{r=1}^L \frac{d^D \ell_r}{(2\pi)^D} \frac{c_i n_i(\ell_r)}{D_i(\ell_r)}.$$

Making color–kinematics duality explicit becomes the central challenge at loop level.

Du–Teng, JHEP **04** (2017) 033; Bern–Carrasco–Johansson, PRD **85** (2012) 105014; Bern et al., Phys. Rept. **873** (2020).

Direct bootstrap of one-loop BCJ numerators

One may directly bootstrap BCJ numerators with standard quadratic propagators,

$$\mathcal{A}_{\text{YM}}^{1\text{-loop}} = \sum_i \int \frac{d^D \ell}{(2\pi)^D} \frac{c_i n_i(\ell)}{D_i(\ell)}.$$

The construction is constrained by

unitarity + locality + graph symmetries + power counting + Jacobi relations

Theory	Published frontier
$\mathcal{N} = 4$ SYM	up to 7 points [*]
$\mathcal{N} = 2$ SYM	up to 5 points [*]
$\mathcal{N} = 0$ YM	up to 5 points [†]

Our recent work extends half-maximal SYM to six points.[‡]

The result also suggests that one-loop eight-point maximal-SUSY BCJ numerators should be within reach.

^{*} Edison, He, Johansson, Schlotterer, Teng and YZ, JHEP 02 (2023) 164; [†] Cao, He, YZ and Zhu, SCPMA. 69 (2026) 6; [‡] Jin, Li, YZ, to appear

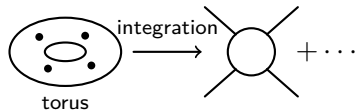
Worksheet reorganization and local Jacobi relations

A one-loop worldsheet formula schematically takes the form

$$\mathcal{A}_n^{1\text{-loop}} = \int d\mu_n^{1\text{-loop}} \mathcal{I}_{\text{color}} \mathcal{I}_{\text{kin}}.$$

The integration over punctures reorganizes field-theory graphs,

$$\text{worldsheet integrand} \Rightarrow \sum_{\text{cubic graphs}} \frac{\text{numerator}}{\text{propagators}}.$$



The resulting numerators may satisfy local Jacobi relations,

$$n_i + n_j + n_k = 0.$$

The worldsheet exposes the representation where the double copy becomes visible.

Kawai–Lewellen–Tye, NPB 269 (1986) 1; Mafra–Schlotterer, JHEP 03 (2017) 097; Geyer–Monteiro, JHEP 03 (2018) 068; Cachazo–He–Yuan, JHEP 08 (2016) 008.

Two useful representations of one-loop amplitudes

Worldsheet and forward-limit constructions naturally produce propagators linear in the loop momentum,

$$\frac{1}{2\ell \cdot K + K^2}.$$

Standard Feynman integrals use quadratic propagators,

$$\frac{1}{(\ell + K)^2}.$$

Linear representation

- natural from forward limits
- related to scattering equations
- often exposes algebraic structures
- compact at high multiplicity

Quadratic representation

- standard Feynman propagators
- manifest locality
- conventional loop integration
- direct comparison with known results

Can worldsheet simplicity survive in standard quadratic propagators?

A worldsheet basis for one-loop BCJ numerators

Another route starts from generalized one-loop Parke–Taylor factors,

$$PT^{(1)}(\alpha),$$

which provide a natural basis for one-loop worldsheet integrands.

The kinematic integrand is expanded as

$$\mathcal{I}_{\text{kin}} = \sum_{\alpha} N(\alpha) PT^{(1)}(\alpha).$$

After worldsheet integration,

$$PT^{(1)}(\alpha) \implies \text{quadratic-propagator diagrams,}$$

while $N(\alpha)$ become kinematic numerators.

worldsheet identities



kinematic Jacobi identities

This gives a geometric origin for one-loop BCJ organization.

Feng, He, YZ and Y.-Q. Zhang, JHEP 08 (2022) 240; Dong, Y.-Q. Zhang and YZ, Phys. Rev. D 109 (2024) L101905.

From superstring correlators to BCJ numerators

One-loop open-string correlators admit the $C \cdot E$ representation*,

$$\mathcal{K}_n = \sum C \dots E \dots$$

Our method systematically expands the worldsheet building blocks†

$$E\text{-blocks} \implies \sum_{\alpha} N(\alpha) \text{PT}^{(1)}(\alpha).$$

This leads directly to

- generalized PT expansions for all 6- and 7-point E -block topologies,
- algebraic construction of 7-point one-loop $\mathcal{N} = 4$ SYM BCJ numerators,
- standard quadratic propagators, agreement with known parity-even cuts.

The worldsheet provides BCJ numerators without a huge graph-by-graph bootstrap.

* Mafra, Schlotterer, JHEP 08 (2019) 090, JHEP 08 (2019) 091, JHEP 08 (2019) 092 † Yang, YZ, to appear

Toward eight points

The present construction naturally extends to higher multiplicity.

At seven points,

- all required E -block topologies have been expanded into generalized Parke–Taylor factors,
- the corresponding one-loop $\mathcal{N} = 4$ SYM BCJ numerators can be constructed algebraically,
- no large graph-by-graph bootstrap is required.

At eight points,

- much of the required string-theory input is already available,
- generalized unitarity provides additional constraints,
- the same strategy is expected to lead to the corresponding BCJ numerators with quadratic propagators,
- these results may in turn help constrain the remaining unknown sector of the string correlator.

Rather than introducing a larger bootstrap, we hope to extend the same algebraic construction.

Loop-level double copy from forward limits

A one-loop integrand can be reconstructed from forward limits of tree amplitudes,

$$\mathcal{A}_{n+2}^{\text{tree}}(1, \dots, n, \ell, -\ell) \implies \mathcal{A}_n^{1\text{-loop}}.$$

The key algebraic condition is

$$N_{1, \dots, n; \ell_1} = N_{i, \dots, i-1; \ell_i},$$

shifted cyclic symmetry

which removes all spurious linear propagators.

Consequences

$$\text{tree KLT / BCJ} \implies \text{loop KLT / BCJ}$$

- purely field-theoretic
- no worldsheet input
- algebraic derivation
- possible route toward higher loops

Cao, He, YZ and Zhu, SCPMA. 69 (2026) 6

Why this matters for quantum gravity

Perturbative quantum gravity is not ultraviolet complete in the ordinary sense. At increasing loop order, one encounters

new ultraviolet divergences $\longrightarrow$ higher-dimensional counterterms.

Nevertheless, gravity amplitudes often display cancellations that are invisible in individual Feynman diagrams.

Questions

- Which cancellations are accidental?
- Which follow from symmetry?
- Which follow from double copy?
- What is the true UV behavior?

On-shell advantages

- gauge invariance is manifest
- unitarity is imposed directly
- redundant off-shell terms are absent
- hidden cancellations become visible

Loop amplitudes are essential for understanding perturbative quantum gravity.

Goroff–Sagnotti, NPB **266** (1986) 709; Bern et al., PRL **98** (2007) 161303; Bern et al., Phys. Rept. **873** (2020).

Classical Information Inside Amplitudes

The same amplitudes also contain classical information.

For two massive objects exchanging gravitons, $\mathcal{M}(q)$, its long-distance part determines

$$V(r), \quad \chi(b), \quad \Delta p^\mu, \quad P_{\text{rad}}^\mu$$

On-shell language

- amplitudes
- generalized unitarity
- double copy
- loop integrands

Classical observables

- effective potential
- scattering angle
- impulse
- radiation

Modern amplitude methods are no longer used only for quantum gravity, but also for classical gravity.

Cheung–Rothstein–Solon (2018); Kosower–Maybee–O’Connell (2019); Bern et al. (2019).
See talks from Zhengwen Liu, Fei Teng

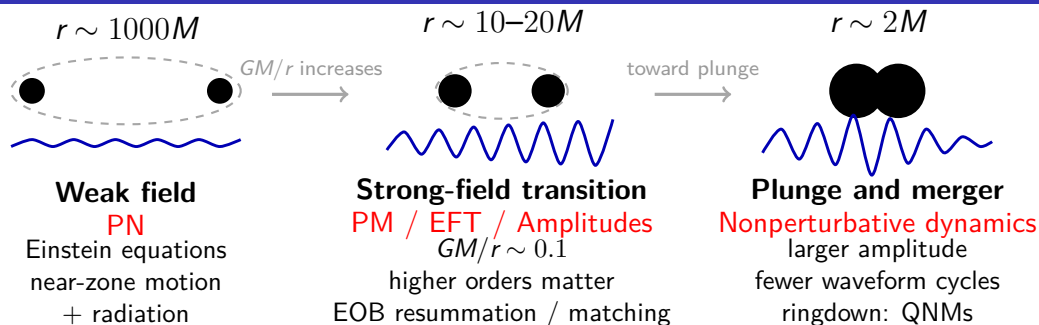
From Amplitudes to Gravitational Waves

- Modern on-shell methods already contribute to classical gravitational-wave calculations.
- To better understand their role, it is useful to zoom out and look at the entire gravitational-wave pipeline.
- At the same time, I would like to argue that on-shell methods have gradually evolved from

better calculations $\implies$ deeper field-theory structures $\implies$ better understanding of dynamics

Let's first zoom out.

From Inspiral to Merger: Where Does QFT Enter?



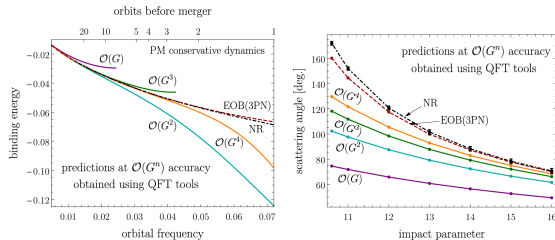
On-shell amplitudes enter where the binary dynamics is already strongly off shell.

PM $\longrightarrow$ EOB $\longrightarrow$ waveform requires a substantial translation and matching cost.

QFT methods must provide a remarkable simplification to justify such an indirect route.

Blanchet, Living Rev. Relativ. 17 (2014); Buonanno–Damour, PRD 59 (1999); Bern et al., PRL 122 (2019); Porto, Phys. Rept. 633 (2016).

Evidence: Analytical QFT Already Enters the GW Pipeline



PM information already enters the last few orbits before merger.

A. Buonanno et al., *Gravitational Waves and Scattering Amplitudes*, arXiv:2204.05194.

Pratten *et al.*, 2021). The phenomenological models use a combination of analytic post-Newtonian and EOB methods to describe the inspiral, merger and ringdown. Numerical relativity simulations are then used to calibrate EOB coefficients that could not be defined otherwise, and also free parameters associated with the merger and ringdown. These models have considered a spin-aligned

Modern waveform modeling = Analytical + Numerical.

R. Christensen and R. Meyer, Parameter estimation with gravitational waves, Rev. Mod. Phys. **94**, 025001 (2022).

Modern BBH waveform modeling is not purely numerical.

A broader question

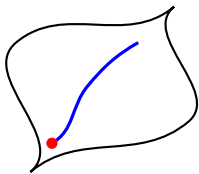
Why can an on-shell quantum-field-theory language organize strongly off-shell classical dynamics?

What does QFT really add to a classical problem?

Perhaps its main advantage is not “quantumness”, but a powerful linear representation of nonlinear dynamics.

From Field Dynamics to Hamilton–Jacobi Representation

Configuration Space



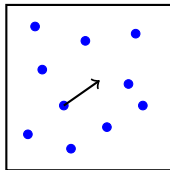
$$h_{ij}(\mathbf{x})$$

One field configuration



Trajectory in configuration
space

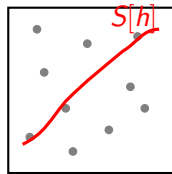
Hamiltonian



$$(h, \pi)$$

Hamiltonian flow
Ensemble evolution

Hamilton–Jacobi



$$\pi = \frac{\delta S}{\delta h}$$

Field dynamics



Functional evolution

From Hamilton–Jacobi Dynamics to a Linear Representation

Classical Ensemble Dynamics*

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \frac{\nabla S}{m} \right) = 0$$

Unique linear complex embedding†

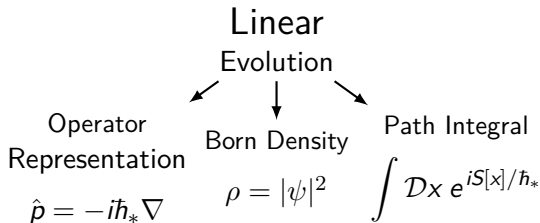
$$\psi(x, t) = \sqrt{\rho(x, t)} e^{iS(x, t)/\hbar_*}$$

$$\Rightarrow i\hbar_* \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar_*^2}{2m} \nabla^2 + V \right) \psi$$

with

$$Q = 0 \rightarrow Q = -\frac{\hbar_*^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}$$

Linear Representation



The familiar quantum machinery is largely machinery for exploiting this linear representation.

* Hall, J. Phys. A **35** (2002) 3289; Reginatto, Phys. Rev. A **58** (1998) 1775.

† YZ, 2601.22697.

A Linear Representation for a Classical Scalar Field: $x \rightarrow \phi(x)$

Lagrangian $\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad V(\phi) = \frac{1}{2}(\nabla\phi)^2 + U(\phi),$

Hamilton–Jacobi $\partial_t S[\phi, t] + \int d^3x \left[\frac{1}{2} \left(\frac{\delta S}{\delta\phi(\mathbf{x})} \right)^2 + V(\phi) \right] = 0,$

Linear embedding $\boxed{\Psi[\phi, t] = \sqrt{\rho[\phi, t]} e^{iS[\phi, t]/\hbar_*}},$

Functional Schrödinger* $i\hbar_* \partial_t \Psi[\phi, t] = \int d^3x \left[-\frac{\hbar_*^2}{2} \frac{\delta^2}{\delta\phi(\mathbf{x})^2} + V(\phi) \right] \Psi[\phi, t],$

Path integral $\mathcal{A} \sim \int \mathcal{D}\phi \exp \left[\frac{i}{\hbar_*} S[\phi] \right].$

Exactly the same linearization strategy[†], with ordinary derivatives replaced by functional derivatives.

* Hatfield, *Quantum Field Theory of Point Particles and Strings* (2019). [†] YZ, 2602.04229.

A Linear Representation for Perturbative Gravity

Einstein gravity $S_{\text{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R, \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$

ADM Hamiltonian* $H = H_{\text{phys}}[h, \pi] + \int d^3x (n \mathcal{H} + n_i \mathcal{H}^i),$

Hamilton–Jacobi $\partial_t S[h, t] + H\left[h, \pi^{ij} = \frac{\delta S}{\delta h_{ij}}, n, n_i\right] = 0,$

Linear embedding $\boxed{\Psi[h, t] = \sqrt{\rho[h, t]} e^{iS[h, t]/\hbar_*}} \implies i\hbar_* \partial_t \Psi[h, t] = \hat{H} \Psi[h, t],$

Path integral $\mathcal{A} \sim \int \mathcal{D}h \exp\left[\frac{i}{\hbar_*} S[h]\right].$

The same strategy extends naturally to perturbative gravity, with the addition of gravitational constraints.

* Arnowitt, Deser & Misner, 1962; DeWitt, Phys. Rev. **160** (1967) 1113; Phys. Rev. **162** (1967) 1195.

What Does QFT Really Add?

- Classical dynamics is **not changed**.
- The **representation** is changed.
- Nonlinear Hamilton–Jacobi evolution

$$(\rho, S) \Longrightarrow \Psi \Longrightarrow \text{linear evolution.}$$

- The price is introducing **infinitely many linear modes**.
- Green functions, propagators, path integrals, and perturbation theory naturally become available.

Nonlinear dynamics $\Longrightarrow$ Linear representation $\Longrightarrow$ QFT machinery

QFT is not merely a quantum theory.
It is also a framework for organizing infinitely many linear modes.

When Does a Classical Problem Benefit from QFT?

- Strong nonlinearity
 - Linear representation may reveal hidden structures.
- Singular structures
 - Regularization and renormalization are already mature.
- Large redundancy
 - Gauge symmetry, constraints, and physical modes.

Poor organization $\implies$ QFT representation $\implies$ Better organization

Binary black holes satisfy all three.

Therefore,

QFT methods become remarkably effective.

Take-home Messages

- **On-shell methods simplify perturbative gravity.**

Factorization, color–kinematics duality, and the double copy expose hidden simplicity.

- **The same amplitudes also contain classical information.**

Modern amplitude methods already contribute to gravitational-wave physics.

- **QFT is more than a quantum theory.**

It also provides a powerful framework for organizing nonlinear dynamics.

On-shell simplicity



Field-theory structures



Off-shell dynamics

Future Directions: Double Copy

- **Higher multiplicity** Bern et al. (2020); Mafrá–Schlotterer (2017); Edison et al. (2023).
 - one-loop 8-point, 9-point BCJ numerators
 - from superstring integrands
- **Higher loops** Bern et al. (2020); Carrasco et al. (2015); Johansson et al. (2021); Yang et al. (2025).
 - two-loop 6-point and beyond
 - structural construction rather than explicit examples
- **Beyond Minkowski spacetime** Adamo et al. (2024); Parra-Martinez et al. (2024); Zhou (2021).
 - AdS double copy
 - dS double copy
- **Toward a deeper structure** Mafrá–Schlotterer (2017); Stieberger (2022, 2023).
 - four-dimensional supersymmetric external states
 - algebraic / string-theoretic origin of loop-level BCJ
 - loop-level KLT relations

Future Directions: Field-Theoretic Organization

- **Complete analytic waveform pipeline** Damour (2016); Buonanno et al. (2022); Damour (2024).
 - PM $\rightarrow$ EOB $\rightarrow$ Gravitational Waves
 - Clean analytic descriptions of binary dynamics
- **From amplitudes to gravitational waves**

Kosower et al. (2019); Bern et al. (2019); Di Vecchia et al. (2020); Maybee et al. (2024). @ Teng's talk

 - Translate more PM results into bound-state GW observables
 - Radiation, spin, tidal effects, waveform models
- **Beyond gravitational waves** Arkani-Hamed et al. (2026); Cheung et al. (2010).
 - Turbulence and other nonlinear classical systems
 - Field-theoretic organization beyond gravity
- **Toward a deeper understanding** Hall (2002); Reginatto (1998); YZ (2026).
 - HJ complex linearization
 - Canonical gravity and quantum gravity
 - Representation of nonlinear dynamics