

AdS Virasoro-Shapiro near Flat-space Limit an Evolution of Bootstrap Computation

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w/ 法肖羽 (Xiaoyu Fa), 王波 (Bo Wang), to appear

21th Century
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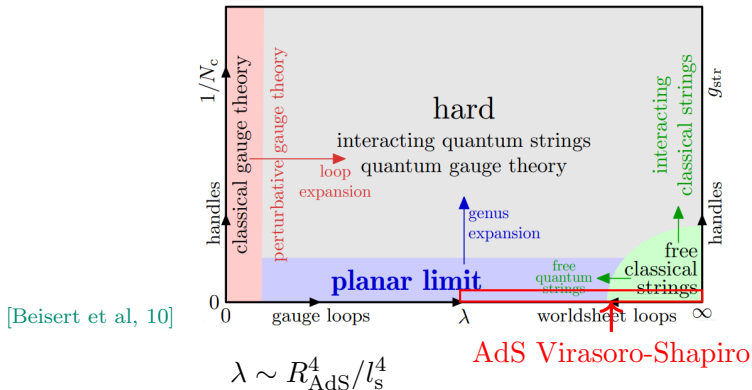
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$\mathcal{N} = 4$ SYM



□ Probe stringy physics using scattering of ordinary gravitons.

$\mathcal{N} = 4$ SYM

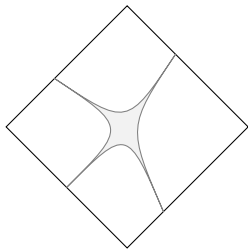
Motivation:

- Effective description of (perturbative) string on AdS.
- Data for planar $\mathcal{N} = 4$ SYM in the strongly-coupled region (especially OPE coefficients).
- Novel structure in amplitudes? (Not obvious from “first principles”.)

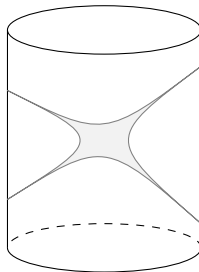
Objects under study $\langle \mathcal{O}_{p_1} \mathcal{O}_{p_2} \mathcal{O}_{p_3} \mathcal{O}_{p_4} \rangle \equiv \langle p_1 p_2 p_3 p_4 \rangle$.

$$\mathcal{O}_p \propto \text{tr}(\phi^{I_1} \phi^{I_2} \cdots \phi^{I_p}) y_{I_1} y_{I_2} \cdots y_{I_p}, \quad y : \text{R-sym polarization}$$

Target



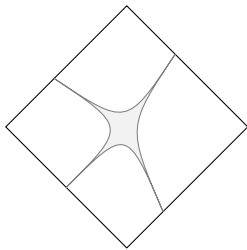
Virasoro-Shapiro
(finite l_s)



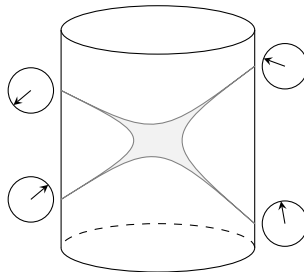
AdS Virasoro-Shapiro
 $N \gg 1$ but finite λ (finite l_s and R_{AdS})

VS + curvature correction
 $N \gg \lambda \gg 1$ (finite l_s but large R_{AdS})

Target



Virasoro-Shapiro
(finite l_s)



AdS Virasoro-Shapiro
 $N \gg 1$ but finite λ (finite l_s and R_{AdS})

VS + curvature correction
 $N \gg \lambda \gg 1$ (finite l_s but large R_{AdS} , all KK)

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$$

Virasoro-Shapiro ($A^{(0)}$)

$$-\frac{\Gamma(-S)\Gamma(-T)\Gamma(-U)}{\Gamma(S+1)\Gamma(T+1)\Gamma(U+1)} \\ (S+T+U=0)$$

AdS Virasoro-Shapiro

$$\langle 2222 \rangle_{\text{int}} = \int [d\gamma] M(s, t) \prod_{i < j} \frac{\Gamma(\gamma_{ij})}{(x_{ij}^2)^{\gamma_{ij}}}$$

[Penedones, '10]

$$R_{\text{AdS}} \rightarrow \infty$$

$$A = \boxed{A^{(0)}} + \lambda^{-\frac{1}{2}} A^{(1)} + \dots$$

- ☹ Sufficient knowledge on the structure at fixed curvature order.
- 😊 Study the low energy expansion.

(Later we will return to the direct analysis on the structure.)

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$$

Virasoro-Shapiro ($A^{(0)}$)

AdS Virasoro-Shapiro

stringy
pole

$$-\frac{\Gamma(-S)\Gamma(-T)\Gamma(-U)}{\Gamma(S+1)\Gamma(T+1)\Gamma(U+1)} \\ (S+T+U=0)$$

$$\langle 2222 \rangle_{\text{int}} = \int [d\gamma] M(s, t) \prod_{i < j} \frac{\Gamma(\gamma_{ij})}{(x_{ij}^2)^{\gamma_{ij}}}$$

low energy

effective
vertices

$$\frac{1}{STU} + 2\zeta_3 + 2\hat{\sigma}_2\zeta_5 + 2\hat{\sigma}_3\zeta_3^2 + \dots \\ (\hat{\sigma}_2 = S^2 + T^2 + U^2, \quad \hat{\sigma}_3 = STU)$$

$$M = M^{\text{SG}} + \lambda^{-\frac{3}{2}} M^3 + \dots$$

$$R_{\text{AdS}} \rightarrow \infty$$

$$A = \boxed{A^{(0)}} + \lambda^{-\frac{1}{2}} A^{(1)} + \dots$$

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$$

□ Low-energy expansion of Mellin (Fixed R_{AdS} , small ℓ_s)

$$M(s, t) = \frac{8}{(s-2)(t-2)(u-2)} + \sum_{a,b=0}^{\infty} \frac{\#_{a,b} \sigma_2^a \sigma_3^b}{\lambda^{\frac{3}{2}+a+\frac{3}{2}b}} \left(\alpha_{a,b}^{(0)} + \underbrace{\frac{\alpha_{a,b}^{(1)}}{\sqrt{\lambda}} + \frac{\alpha_{a,b}^{(2)}}{\lambda} + \dots}_{\text{curvature effects}} \right)$$

$$\sum p = 2\Sigma, s + t + u = 2\Sigma - 4, \sigma_2 = s^2 + t^2 + u^2, \sigma_3 = stu.$$

Asymptotic series. Cannot directly compare with Virasoro-Shapiro.

□ Flat space limit via Borel transformed Mellin (Fixed ℓ_s , large R_{AdS})

$$\begin{aligned} A(S, T) &= \lambda^{\frac{3}{2}} \Gamma(\Sigma - 1) \int_{-i\infty}^{+i\infty} \frac{d\beta}{2\pi i} \frac{e^\beta}{\beta^{\Sigma+2}} M\left(\frac{2\sqrt{\lambda}S}{\beta} + \frac{2\Sigma-4}{3}, \frac{2\sqrt{\lambda}T}{\beta} + \frac{2\Sigma-4}{3}\right) \\ &= \sum_{k=0}^{\infty} \lambda^{-\frac{k}{2}} A^{(k)}(S, T) \end{aligned}$$

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$$

- Low-energy expansion of Mellin

$$M(s, t) = \frac{8}{(s-2)(t-2)(u-2)} + \sum_{a,b=0}^{\infty} \frac{\#_{a,b} \sigma_2^a \sigma_3^b}{\lambda^{\frac{3}{2}+a+\frac{3}{2}b}} \left(\alpha_{a,b}^{(0)} + \frac{\alpha_{a,b}^{(1)}}{\sqrt{\lambda}} + \frac{\alpha_{a,b}^{(2)}}{\lambda} + \dots \right)$$

- Virasoro-Shapiro (determined by [flat space result](#))

$$A^{(0)} = \frac{1}{STU} + 2 \sum_{a,b=0}^{\infty} \alpha_{a,b}^{(0)} \hat{\sigma}_2^a \hat{\sigma}_3^b$$

- First curvature correction [[Alday, Hansen, Silva, '22](#)]
([dispersive sum rules](#) + [single-valuedness](#))

$$A^{(1)} = -\frac{2\hat{\sigma}_2}{3\hat{\sigma}_3^2} + 2 \sum_{a,b=0}^{\infty} \alpha_{a,b}^{(1)} \hat{\sigma}_2^a \hat{\sigma}_3^b$$

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$$

□ The Mellin amplitude reads

$$\begin{aligned} M(s, t) = & \frac{8}{(s-2)(t-2)(u-2)} + \lambda^{-\frac{3}{2}} 120 \zeta_3 \\ & + \lambda^{-\frac{5}{2}} (630 \sigma_2 - 1890) \zeta_5 \\ & + \lambda^{-\frac{6}{2}} (5040 \sigma_3 - 1260 \sigma_2 - 20160) \zeta_3^2 \\ & + \lambda^{-\frac{7}{2}} (\#_1 \sigma_2^2 + \#_2 \sigma_3 + \#_3 \sigma_2 + \cdots) \zeta_7 \\ & + \mathcal{O}(\lambda^{-\frac{7}{2}}). \end{aligned}$$

□ The corresponding AdS VS amplitudes are

$$\begin{aligned} A^{(0)}(S, T) &= \frac{1}{\hat{\sigma}_3} + 2\zeta_3 + \hat{\sigma}_2 \zeta_5 + \hat{\sigma}_3 \zeta_3^2 + \cdots \\ \mathcal{A}^{(1)}(S, T) &= -\frac{\hat{\sigma}_2}{3\hat{\sigma}_3^2} - \frac{22}{3} \hat{\sigma}_2 \zeta_3^2 - \frac{537}{8} \hat{\sigma}_3 \zeta_7 + \cdots \end{aligned}$$

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$$

$$A^{(0)}(S, T) = \frac{1}{\hat{\sigma}_3} + 2\zeta_3 + \hat{\sigma}_2 \zeta_5 + \hat{\sigma}_3 \zeta_3^2 + \dots$$

$$\mathcal{A}^{(1)}(S, T) = -\frac{\hat{\sigma}_2}{3\hat{\sigma}_3^2} - \frac{22}{3}\hat{\sigma}_2 \zeta_3^2 - \frac{537}{8}\hat{\sigma}_3 \zeta_7 + \dots$$

Still expressed in terms of low-energy expansion.

Can we resum them?

(Known for $A^{(0)}$: Virasoro-Shapiro.)

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$$

- The Wilson coefficients of $A^{(0)}$ and $A^{(1)}$ lie in the ring of single-valued multiple zeta values (SVMZV) [Brown, '13]. (In particular, $\zeta_{2n}^{\text{sv}} = 0$, $\zeta_{2n+1}^{\text{sv}} = 2\zeta_{2n+1}$.)
- SVMZV as boundary values of single-valued multiple polylogs (SVMPL)

$$\zeta_{s_1, \dots, s_k}^{\text{sv}} = \mathcal{L}_{0^{s_1-1} 1 0^{s_2-1} \dots 0^{s_k-1} 1}(1).$$

- SVMPL can be recursively defined [Brown, '20]. Examples:

$$\mathcal{L}_\emptyset(z) = 1, \quad \mathcal{L}_{0^n}(z) = \frac{1}{n!} \log^n |z|^2, \quad \mathcal{L}_{01}(z) = \text{Li}_2 \bar{z} - \text{Li}_2 z + \log |z|^2 \log(1 - \bar{z}).$$

- Worldsheet integral for Virasoro-Shapiro $A^{(0)}$

$$A^{(0)} = \int d^2 z |z|^{-2S-2} |1-z|^{-2T-2} \frac{1}{U^2}.$$

Worksheet at fixed curvature correction?

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$$

$$A^{(0)} = \int d^2 z |z|^{-2S-2} |1-z|^{-2T-2} \frac{1}{U^2}.$$

□ Worksheet integral for $A^{(1)}$ [Alday, Hansen, Silva, '23]

$$A^{(1)}(S, T) = B^{(1)}(S, T) + B^{(1)}(U, T) + B^{(1)}(S, U),$$

$$B^{(1)}(S, T) = \int d^2 z |z|^{-2S-2} |1-z|^{-2T-2} G^{(1)}(z, \bar{z}).$$

□ Full l_s dependence. Not just low-energy expansion.

$$G^{(1)} = F_0(\mathcal{L}_{000} + \mathcal{L}_{111}) + F_1(\mathcal{L}_{010} + \mathcal{L}_{101}) + F_2(\mathcal{L}_{001} + \mathcal{L}_{110}) \\ + (\text{redundancy}).$$

$$F_0 = \frac{2S}{3U}, \quad F_1 = \frac{S+5T}{6U}, \quad F_2 = \frac{2(T-S)}{3U}.$$

Bootstrap based on the worldsheet

- # of SVMPLs with fixed weight and singularities is finite.
 $\Rightarrow$ ansatz for the worldsheet integrand.
- Applied to $A^{(2)}$ of $\langle 2222 \rangle$ [Alday, Hansen, '23] and $A^{(1)}$ of $\langle 22pp \rangle$ [Fardlli, Hansen, Silva, '23].
- Case of $\langle 22pp \rangle$

$$A^{(1)}(S, T) = B_1^{(1)}(S, T) + B_1^{(1)}(S, U) + B_1^{(1)}(U, T) + B_2^{(1)}(S, T) + B_2^{(1)}(S, U).$$

$$G_i^{(1)} = \sum_{u=1}^4 F_{i,u}^{(1)+} \mathcal{L}_u^+ + \sum_{u=1}^3 F_{i,u}^{(1)-} \mathcal{L}_u^- + (\text{redundancy}),$$

$$F_1^{(1)+} = \frac{1}{24}(-p^2, 2(p-2)p, p^2 - 2p - 6, 48), \quad F_2^{(1)+} = \frac{-p(p-2)}{24U}(3S, -2(2S+T), -2S-T, 0),$$

$$F_1^{(1)-} = \frac{p^2(T-S)}{24U}(-1, 2, 1), \quad F_2^{(1)-} = \frac{-p(p-2)}{24U}(3S, -2(2S-T), -2S+T).$$

Uplift to arbitrary Kaluza-Klein modes

Package all chiral primaries

[Caron-Huot, Trinh, '18] [Caron-Huot, Coronado, '21]

$$\mathbb{O} = \sum_{p=2}^{\infty} \#_p \mathcal{O}_p.$$

The AdS×S formalism

- AdS×S Mellin amplitude [Aprile, Vieira, '20]

$$\begin{aligned}\langle p_1 p_2 p_3 p_4 \rangle_{\text{int}} &= \int [d\gamma] M(s, t; y_{ij}) \prod_{i < j} \frac{\Gamma(\gamma_{ij})}{(x_{ij}^2)^{\gamma_{ij}}} \\ &= \sum_{n_{ij}} \int [d\gamma] \mathcal{M}(s, t; n_{ij}) \prod_{i < j} \frac{\Gamma(\gamma_{ij})}{(x_{ij}^2)^{\gamma_{ij}}} \frac{y_{ij}^{n_{ij}}}{\Gamma(n_{ij} + 1)}.\end{aligned}$$

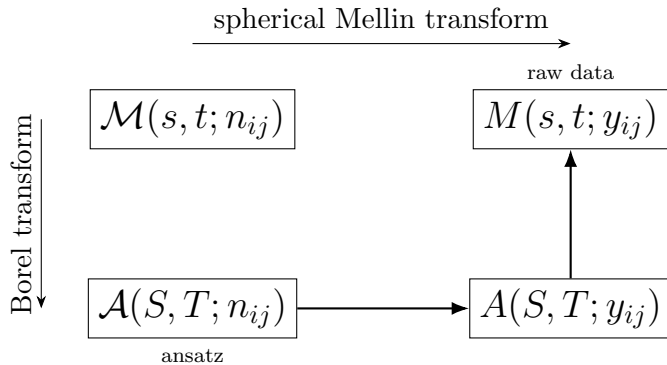
$$n_{ij} = n_{ji}, \quad \sum_{j \neq i} n_{ij} = p_i - 2.$$

- For $\langle \text{OOOOO} \rangle$ we simply do NOT constrain n_{ij} .
- Example: Mellin for sugra [Aprile, Vieira, '20]

$$\mathcal{M}^{\text{SG}} = -\frac{1}{(\mathbf{s} + 1)(\mathbf{t} + 1)(\mathbf{u} + 1)}.$$

$$\mathbf{s} = -\gamma_{12} + n_{12}, \quad \mathbf{t} = -\gamma_{14} + n_{14}, \quad \mathbf{s} + \mathbf{t} + \mathbf{u} = -4.$$

Strategy



Worldsheet ansatz

- A quick glimpse at curvature correction to sugra

$$\mathcal{M}_{\text{SG}} \rightarrow \mathcal{A}_{\text{SG}}^{(1)} = \frac{2 - \Sigma}{6 \hat{\sigma}_3^2} (3(n_s S^2 + n_t T^2 + n_u U^2) - (n_s + n_t + n_u - 1) \hat{\sigma}_2),$$

$$n_s = n_{12} + n_{34}, \quad n_t = n_{14} + n_{23}, \quad n_u = n_{13} + n_{24}.$$

Hints on ingredients to appear in the ansatz.

- Ansatz in terms of worldsheet integrals

$$\mathcal{A}^{(1)}(S, T) = \mathcal{B}^{(1)}(S, T; n_s, n_t) + \mathcal{B}^{(1)}(U, T; n_u, n_t) + \mathcal{B}^{(1)}(S, U; n_s, n_u),$$

$$\mathcal{B}^{(1)}(S, T; n_s, n_t) = \int d^2 z |z|^{-2S-2} |1 - z|^{-2T-2} \mathcal{G}^{(1)}(z, \bar{z}; n_s, n_t).$$

$\mathcal{B}$'s consistent with crossing.

Worldsheet ansatz

- Integrand contains SVMPLs at weight 3, with singularities only at $z = 0, 1, \infty$

$$\begin{aligned}\mathcal{L}_i^+ &= (\mathcal{L}_{000}^+, \mathcal{L}_{001}^+, \mathcal{L}_{010}^+, \zeta_3) , \\ \mathcal{L}_i^- &= (\mathcal{L}_{000}^-, \mathcal{L}_{001}^-, \mathcal{L}_{010}^-) .\end{aligned}$$

$\mathcal{L}_i^\pm$ is symmetric/anti-symmetric under $z \leftrightarrow 1 - z$, e.g.,

$$\mathcal{L}_{000}^+ = \mathcal{L}_{000} + \mathcal{L}_{111} \propto \log^3 |z|^2 + \log^3 |1 - z|^2 .$$

- Build the integrand by linear combination

$$\mathcal{G}^{(1)}(z, \bar{z}; n_s, n_t) = \sum_{i=1}^4 \mathcal{F}_i^+ \mathcal{L}_i^+ + \sum_{j=1}^3 \mathcal{F}_j^- \mathcal{L}_j^- .$$

SVMPLs guarantees that the amplitude only contains single-valued MZVs.

Worksheet ansatz

- The prefactors $\mathcal{F}_i^\pm(S, T)$ are homogeneous functions with a universal denominator U^2 and enjoy the general structure

$$\mathcal{F}_i^\pm(S, T) = \frac{f_1^\pm S^2 + f_2^\pm S T + f_3^\pm T^2}{U^2},$$

where $f_i^\pm$ are linear combinations of

$$\{\Sigma n_s, \Sigma n_t, \Sigma n_u, \Sigma, n_s, n_t, n_u\}$$

with unknown coefficients.

- ▷ Degree and denominator motivated by $A^{(0)}$ and $A_{22pp}^{(1)}$.
- ▷ Parameters motivated by the structure of $\mathcal{A}^{(1),\text{SG}}$.

$$\begin{aligned} \mathcal{A}^{(1),\text{SG}} = \frac{2 - \Sigma}{6 \hat{\sigma}_3^2} & \left(3(n_s S^2 + n_t T^2 + n_u U^2) \right. \\ & \left. - (n_s + n_t + n_u - 1)(S^2 + T^2 + U^2) \right), \end{aligned}$$

Bootstrap & Result

- Constraints: (1) Recover SUGRA contribution $\mathcal{A}^{(1),\text{SG}}$; (2) reproduce the known $A^{(1)}$ for $\langle 22pp \rangle$.
- The result only depends on five independent SVMPLs

$$\mathcal{L}_{1/4} = \mathcal{L}_{000}^{+/-}, \quad \mathcal{L}_2 = -\mathcal{L}_{010}^+ + 4\zeta_3, \quad \mathcal{L}_{3/5} = \mathcal{L}_{000}^{+/-} - \mathcal{L}_{001}^{+/-} - \mathcal{L}_{010}^{+/-}.$$

- In terms of this basis [Wang, Wu, EYY, '25]

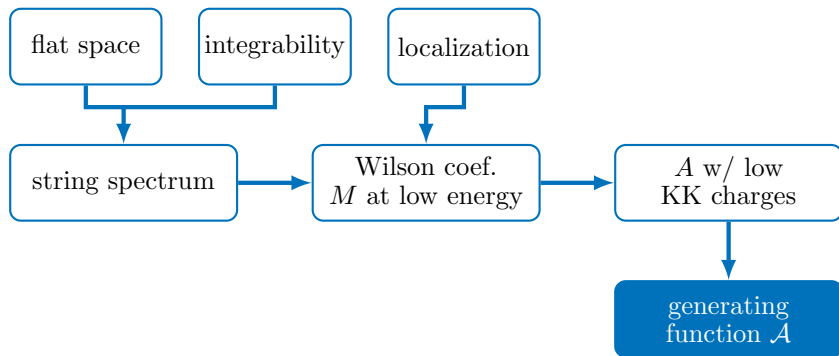
$$\mathcal{G}^{(1)}(z, \bar{z}; n_s, n_t) = (\Sigma - 2) \times \sum_{i=1}^5 \mathcal{F}_i \mathcal{L}_i,$$

$$\mathcal{F}_1 = \frac{(n_s - n_t)(S - T) + 2U}{12(S + T)}, \quad \mathcal{F}_2 = \frac{1}{2} \frac{1}{\Sigma - 2}, \quad \mathcal{F}_3 = \frac{n_s S + n_t T + n_u U}{12(S + T)},$$

$$\mathcal{F}_4 = \frac{n_s - n_t}{12}, \quad \mathcal{F}_5 = \frac{n_s S - n_t T + (n_u + 2)(T - S)}{12(S + T)}.$$

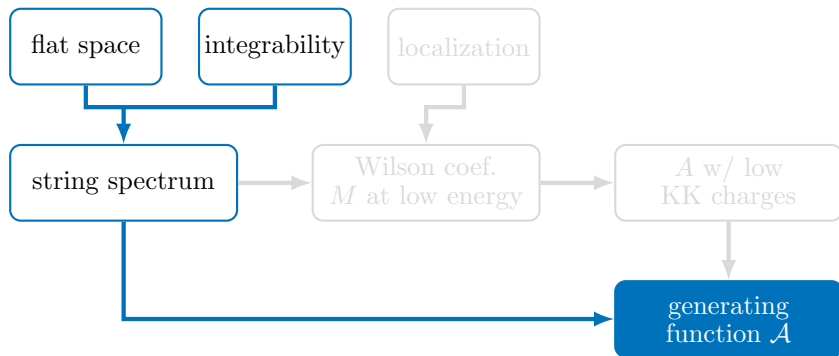
- Application of the method in other systems, e.g., [Jiang, Zhong, '25] [Wang, '25].

Better strategy



- Functions in the worldsheet ansatz already provide solutions to dispersion.

Skip the analysis at low energy



- Functions in the worldsheet ansatz already provide solutions to dispersion.

OPE in Mellin space

- (Super-)conformal block expansion

$$\underbrace{\mathcal{I}^{-1}\langle p_1 p_2 p_3 p_4 \rangle_{\text{int}}}_{\text{reduced correlator}} = \sum_{\mathcal{O}} f_{\mathcal{O}} G_{\tau+4,\ell}(z, \bar{z})$$

- ▷ $\mathcal{O}$: single-trace conformal primary $\leftrightarrow$ string excitations.
- ▷ $f_{\mathcal{O}}$ also captures R-symmetries.

- In Mellin space

$$M(s, t) \sim \sum_{\mathcal{O}} \sum_{m=0}^{\infty} \frac{C_{\mathcal{O}}}{s - \tau - 2m} \underbrace{Q_{\ell,m}^{\tau+4}(u - p_1 - p_4)}_{\text{Mack polynomial}}.$$

- ▷ $C_{\mathcal{O}} \propto f_{\mathcal{O}}$.
- ▷ m labels conformal descendants.



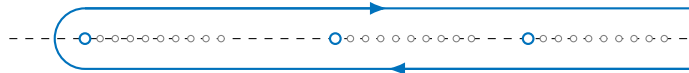
String spectrum near flat space limit

- At large λ

$$\tau = 2\sqrt{\delta} \lambda^{\frac{1}{4}} + \tau_1 + \tau_2 \lambda^{-\frac{1}{4}} + \dots, \quad \delta = 1, 2, \dots$$

$$f_{\mathcal{O}} = f_0 + f_1 \lambda^{-\frac{1}{4}} + f_2 \lambda^{-\frac{1}{2}} + \dots$$

- Borel transform



Descendant poles condensate: $\sum_{m=0}^{\infty} \mapsto \int_0^{\infty} dx \tau^2$

- In consequence

$$A(S, T) \sim \sum_{\mathcal{O}} \int_0^{\infty} dx \tau^2 \frac{e^{\beta_*}}{\beta_*^{\Sigma-2}} f_{\mathcal{O}} Q_{\ell, x\tau^2}^{\tau+2} \left(\frac{2\sqrt{\lambda}U}{\beta_*} + \frac{2\Sigma-2}{3} - p_1 - p_4 \right)$$

- At each order in the λ expansion, Q decompose into Gegenbauer C_l and its derivatives.

Stringy spectrum near flat space limit

- x integral creates poles in $(S - \delta)$ (now switch to $\mathcal{A}$)

$$\mathcal{A}^{(k)}(S, T) \sim \sum_{i=1}^{3k+1} \frac{\mathcal{R}_i^{(k)}(\delta, T)}{(S - \delta)^i} + (\text{regular})$$

- In $\mathcal{A}^{(0)}$ (VS gives complete info on $\langle f_0 \rangle$)

$$\mathcal{R}_1^{(0)} = - \sum_{l=0}^{\delta-1} \langle f_0 \rangle_{\delta, l} \frac{C_l(\frac{2T}{\delta} + 1)}{\delta^2(l+1)}$$

- In $\mathcal{A}^{(1)}$ ($\#$ are polynomials in δ and T , and $C^{(k)} = \partial_T^k C_\ell$)

$$\begin{aligned} \mathcal{R}_4^{(1)} &= \sum \langle f_0 \rangle_{\delta, l} \delta \# C_\ell^{(0)}, & \mathcal{R}_3^{(1)} &= \sum \langle f_0 \rangle_{\delta, l} (\# C^{(1)} + \# C^{(0)}), \\ \mathcal{R}_2^{(1)} &= \sum \langle f_0 \rangle_{\delta, l} \delta^{-1} (\# C^{(2)} + \# C^{(1)} + \# C^{(0)}) + \sum \langle f_0 \tau_2 \rangle_{\delta, l} \delta^{-\frac{1}{2}} \# C^{(0)}, \\ \mathcal{R}_1^{(1)} &= \sum \langle f_0 \rangle_{\delta, l} \delta^{-2} (\# C^{(3)} + \# C^{(2)} + \# C^{(1)} + \# C^{(0)}) \\ &\quad + \sum \langle f_0 \tau_2 \rangle_{\delta, l} \delta^{-\frac{3}{2}} (\# C^{(1)} + \# C^{(0)}) + \sum \langle f_2 \rangle_{\delta, l} \delta^{-1} \# C^{(0)}. \end{aligned}$$

Stringy pole from the worldsheet

Bootstrap:

- Exactly the same worldsheet ansatz as before, for the generating function.
- Explore s-channel OPE, $z \rightarrow 0$: $\mathcal{L}_w \sim \sum_{\#} z^{\#} \log^{\#}(z)$

$$\int_0^1 dz z^{-S+\delta-1} \log^{k-1}(z) = -\frac{\Gamma(k+1)}{(S-\delta)^k}$$

This helps derive $\mathcal{R}_k^{(1)}$ from the ansatz.

- Matching the known $\langle f_0 \rangle_{\delta,\ell}$ completely fixes the ansatz, which further determines $\langle f_0 \tau_2 \rangle_{\delta,\ell}$ and $\langle f_2 \rangle_{\delta,\ell}$!

Bootstrapping $\mathcal{A}^{(2)}$ (work to appear)

Worksheet ansatz

- Need SVMPLs at weight 6.
- Homogeneous in $\{S, T\}$ /w degree 4.
- R-sym structures in $\{n_{ij}\}$ up to degree 4.

Residues at the stringy poles

$$\mathcal{R}_7^{(2)} : \langle f_0 \rangle,$$

$$\mathcal{R}_6^{(2)} : \langle f_0 \rangle,$$

$$\mathcal{R}_5^{(2)} : \langle f_0 \rangle, \langle f_0 \tau_2 \rangle,$$

$$\mathcal{R}_4^{(2)} : \langle f_0 \rangle, \langle f_0 \tau_2 \rangle, \langle f_2 \rangle,$$

$$\mathcal{R}_3^{(2)} : \langle f_0 \rangle, \langle f_0 \tau_2 \rangle, \langle f_2 \rangle, \langle f_0 (\tau_2)^2 \rangle,$$

$$\mathcal{R}_2^{(2)} : \langle f_0 \rangle, \langle f_0 \tau_2 \rangle, \langle f_2 \rangle, \langle f_0 (\tau_2)^2 \rangle, \langle f_0 \tau_4 + f_2 \tau_2 \rangle,$$

$$\mathcal{R}_1^{(2)} : \langle f_0 \rangle, \langle f_0 \tau_2 \rangle, \langle f_2 \rangle, \langle f_0 (\tau_2)^2 \rangle, \langle f_0 \tau_4 + f_2 \tau_2 \rangle, \langle f_4 \rangle.$$

Bootstrapping $\mathcal{A}^{(2)}$ (work to appear)

- It is not sufficient just to use existing data.
- In practice, we also set up ansatz for the three new OPE coefficients and solve with the worldsheet ansatz simultaneously.
- Bootstrap
 - ▷ Compare residues at all stringy poles (w/ proper truncation in spin level δ and spin ℓ).
 - ▷ Matching sugra contribution as well as known low-energy Wilson coefficients.
- Up to ambiguities tied to worldsheet integrand (which integrate to zero), we have completely solved $\mathcal{A}^{(2)}$.
(Temporarily not neat enough to put in the slides.)
- Crossing-check of the resulting OPE data with recent integrability result.
[Basso, Georgoudis, '25]

Outlook

- New universal pattern in the curvature expansion?
- Underlying worldsheet theory?
- Uniqueness regarding positivity + crossing?
- Higher points?

Thanks for your attention!

Questions & comments are welcome.