

Applications of Topological Holographic Principle

LING YAN HUNG,
YMSC, TSINGHUA UNIVERSITY

3rd, August, 2026

第七届全国“场论与弦论”学术研讨会, 合肥

Based on works with : **2021-2025**

Kaixin Ji, Ce Shen, Yidun Wan, Yu Zhao 2506.05324 (Accepted in principle, Nature Physics)

Gong Cheng, Lin Chen, Zheng-Cheng Gu 2311.18005 Phys.Rev.X 15 (2025) 1, 011073

Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang

2210.12127 Physical Review X 14 (4), 041033

Hao Geng, Yikun Jiang 2505.20385 JHEP 262 (2026)

Yikun Jiang, Bingxin Lao, 2504.21660

Lin Chen, Yikun Jiang, Bingxin Lao

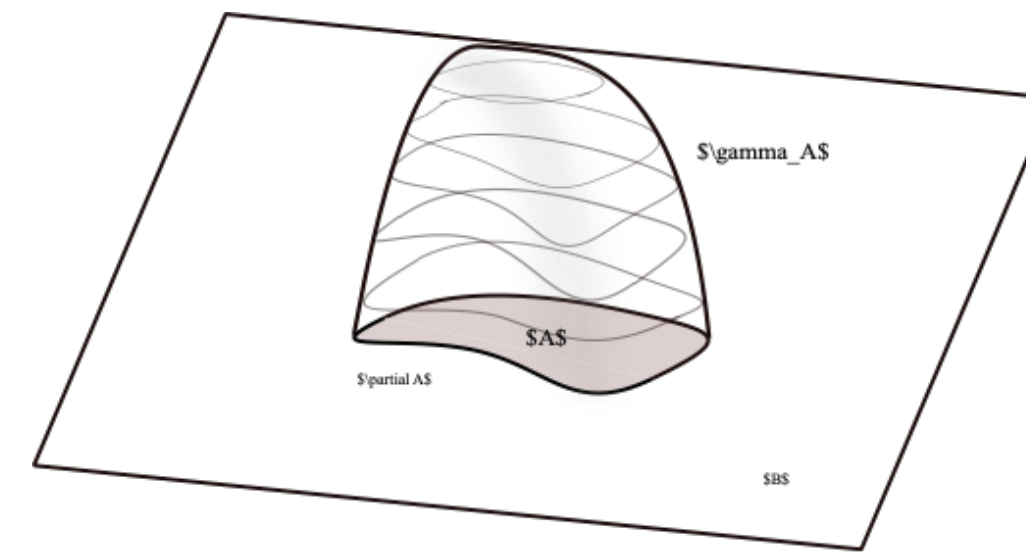
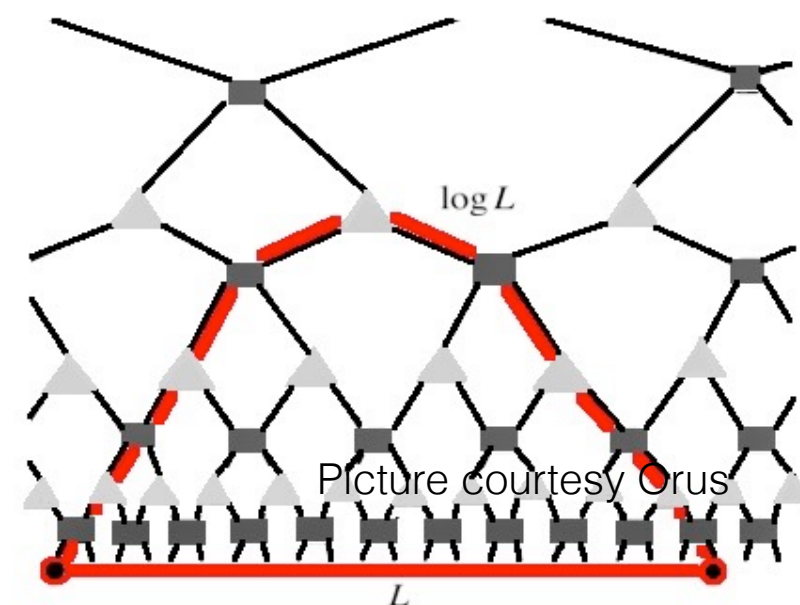
2404.00877, 2403.03179 SciPost Phys. 19 (2025) 6, 163



清华大学 丘成桐数学科学中心
Yau Mathematical Sciences Center, Tsinghua University

Why did I care about holographic tensor network?

- Microscopic local map between CFT and AdS dofs - making the blackbox transparent!
- But continuous construction it was not possible to explicitly deal with a non-Gaussian path-integral in general.



To construct these tensor networks, entanglement is not enough — it is only a number , but CFT has way more structures!!!!

2016-2020

Work done in collaboration
with : Arpan Bhattacharya,
Yang Lei, Wei Li, Charles
Melby-Thompson,

JHEP 04 (2019) 170,

JHEP 05 (2019) 118,

JHEP 01 (2018) 139,

JHEP 08 (2016) 086

Lin Chen, Xirong Liu

PRL 127 (22) 221602,

JHEP (2021) 06,

JHEP (2021) 097

In 2018, Frank Verstrate told
me this construction
looks like something called
“strange correlators” he is
working on - where 2D lattice
models are expressible as
boundaries to 3D TQFT

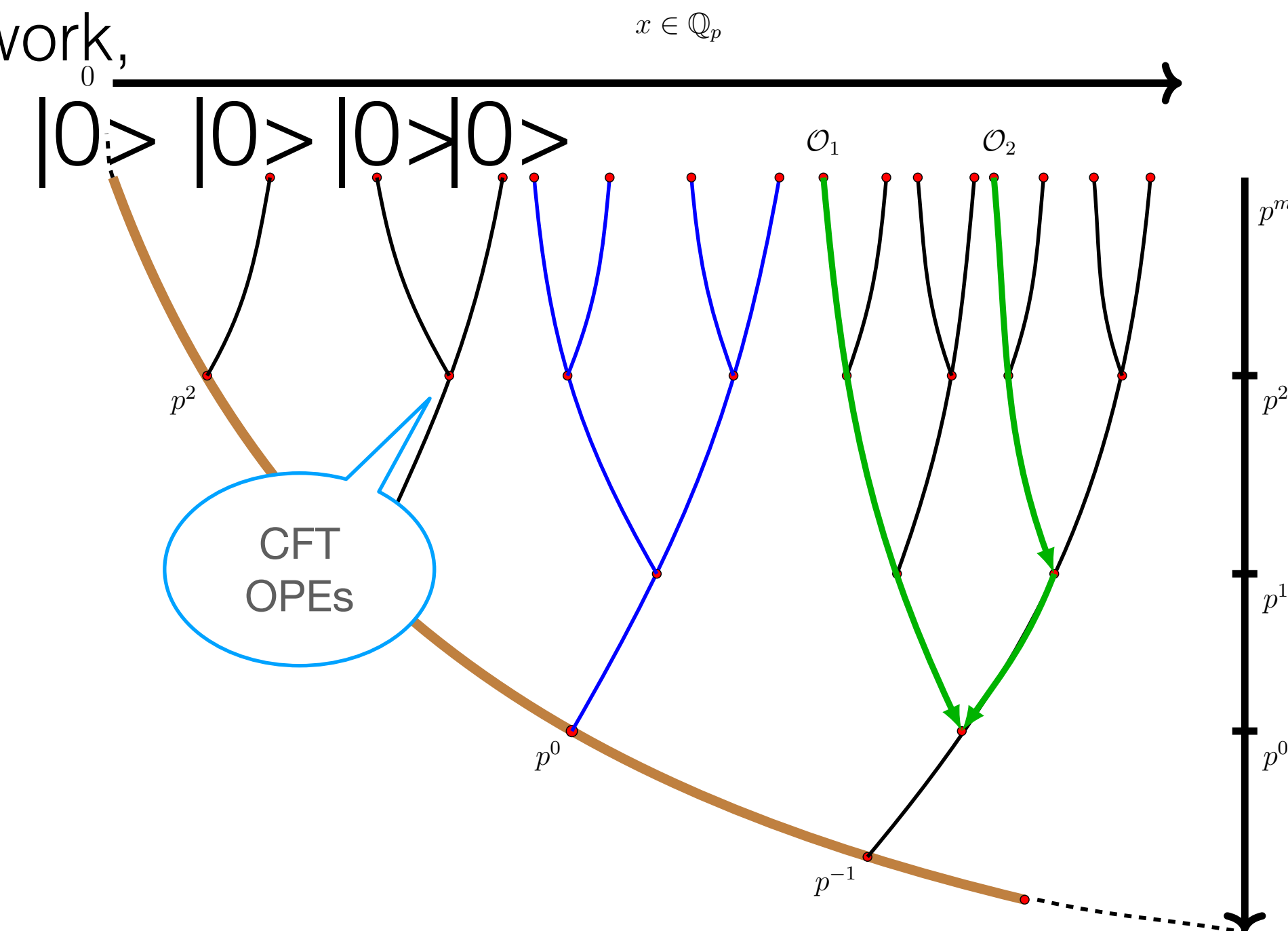
Towards building a holographic network describing realistic CFTs??

Some lessons learned... You need CFT data

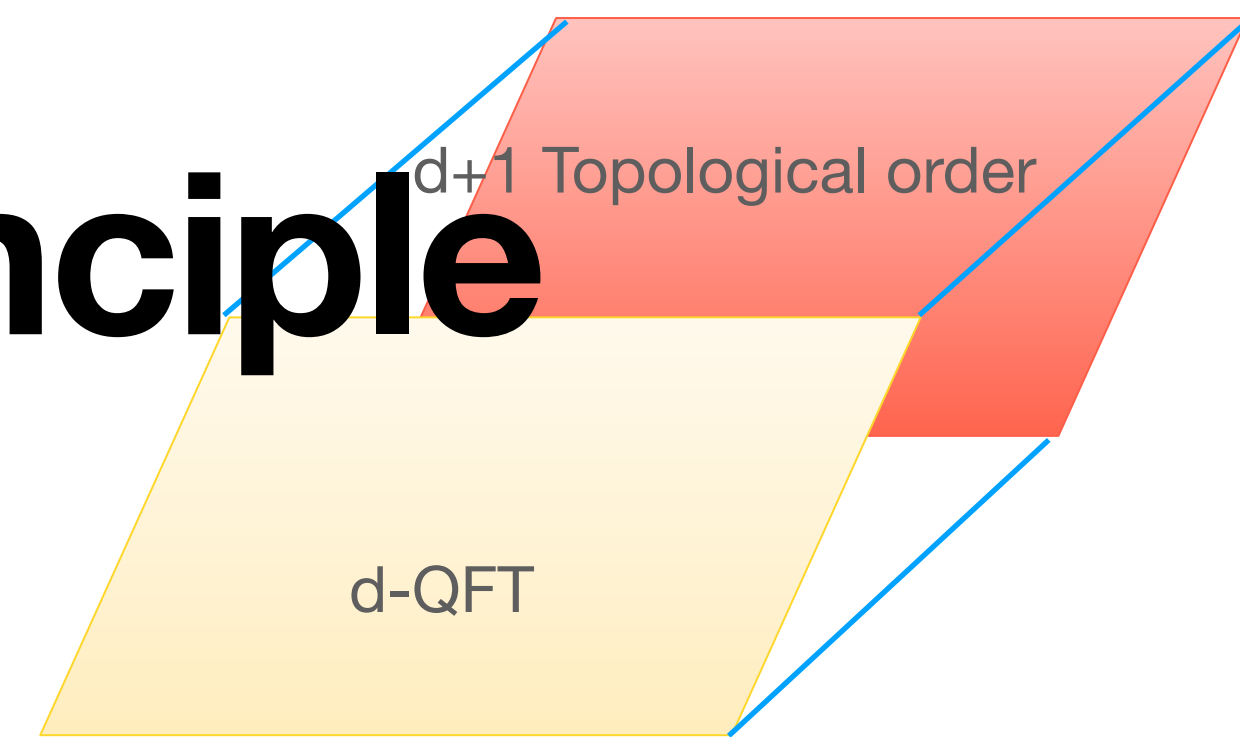
- The tensor network describing the partition function. The bulk is a p-adic CS Wilson line network,

with some simple bc.

- Our partition function has a 2D TQFT bulk and 1D boundary !



Topological Holographic Principle



- Topological Holographic Principle states that a symmetric d -dimensional model can be expressed as a $d+1$ dimensional TQFT path-integral over a “sandwich”
- Bulk imposes the “generalised symmetries” of the boundary.
- “Generalised symmetries” refers to the collection of all the topological defects of different dimensions .
- The collection of these operators form a mathematical structure: a (higher) fusion category — we will need only 1-(spherical) fusion category in the following to discuss 1d topological defects of 2D CFT.
- Topological defects in the d -QFT corresponds to topological excitations (e.g. anyon lines) in the $d+1$ TQFT (symTFT/symTO).
- e.g. Verlinde lines in 2D CFT $\longrightarrow$ Wilson lines/anyon lines in a corresponding 3D Chern-Simons theory. — this is in fact well-known since the 80s.

Sandwich and SymTFT:

Kong, Wen, Zheng 2015, 2017;
Kong, Zheng 2019;
Ji, Wen 2019;
Kong, Lan, Wen, Zhang, Zheng 2020;
Gaiotto, Kulp 2020;
Apruzzi, Bonetti, Garcia Etxebarria, Hosseini, Schafer-Nameki 2021;
Freed, Moore, Teleman 2022;

Overview - Applications of the Topological Holographic Principles

- $D \longrightarrow D+1$
Using CFT data to derive higher dimensional dual
- symmetry preserving RG algorithms - the bridge between continuity and discreteness
- $D \longleftarrow D+1$
Predict new critical lattice models and discover new CFTs
- Is this an opportunity for more general strongly interacting field theories?

$$D \longrightarrow D+1$$

Discretise CFTs and Predict Higher dimensional dual

Cut 2D CFT path-integral into pieces

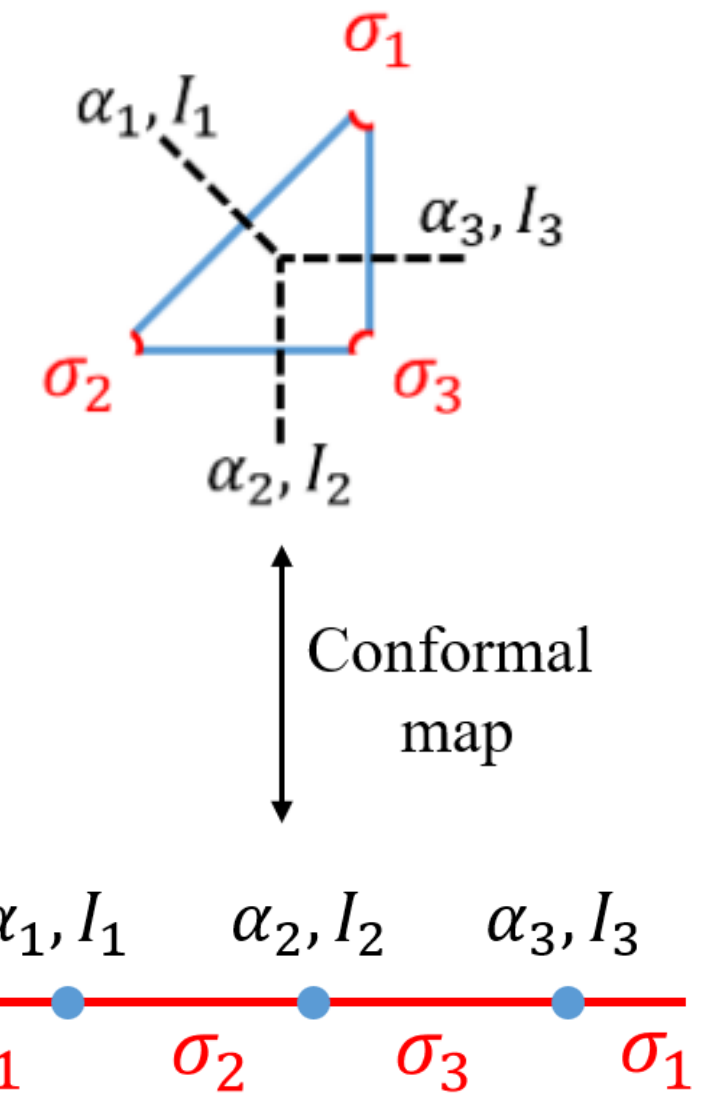
Gabriel Wong, LYH 2020

Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang, LYH 2022

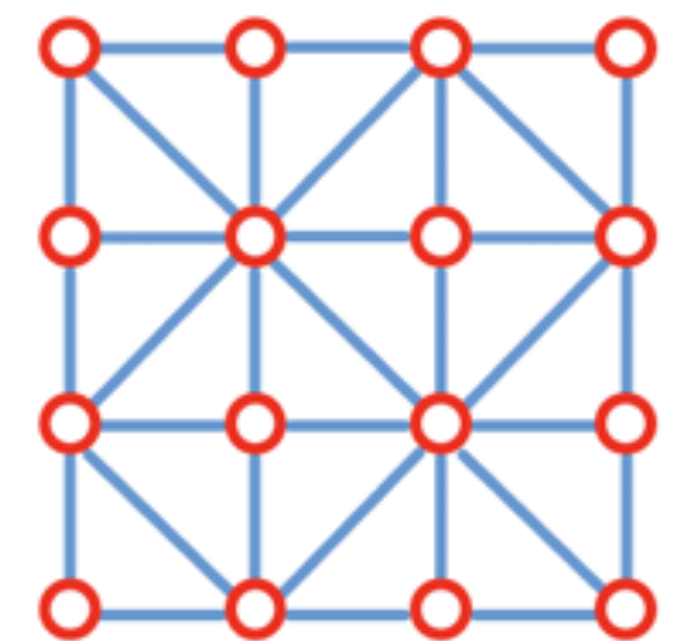
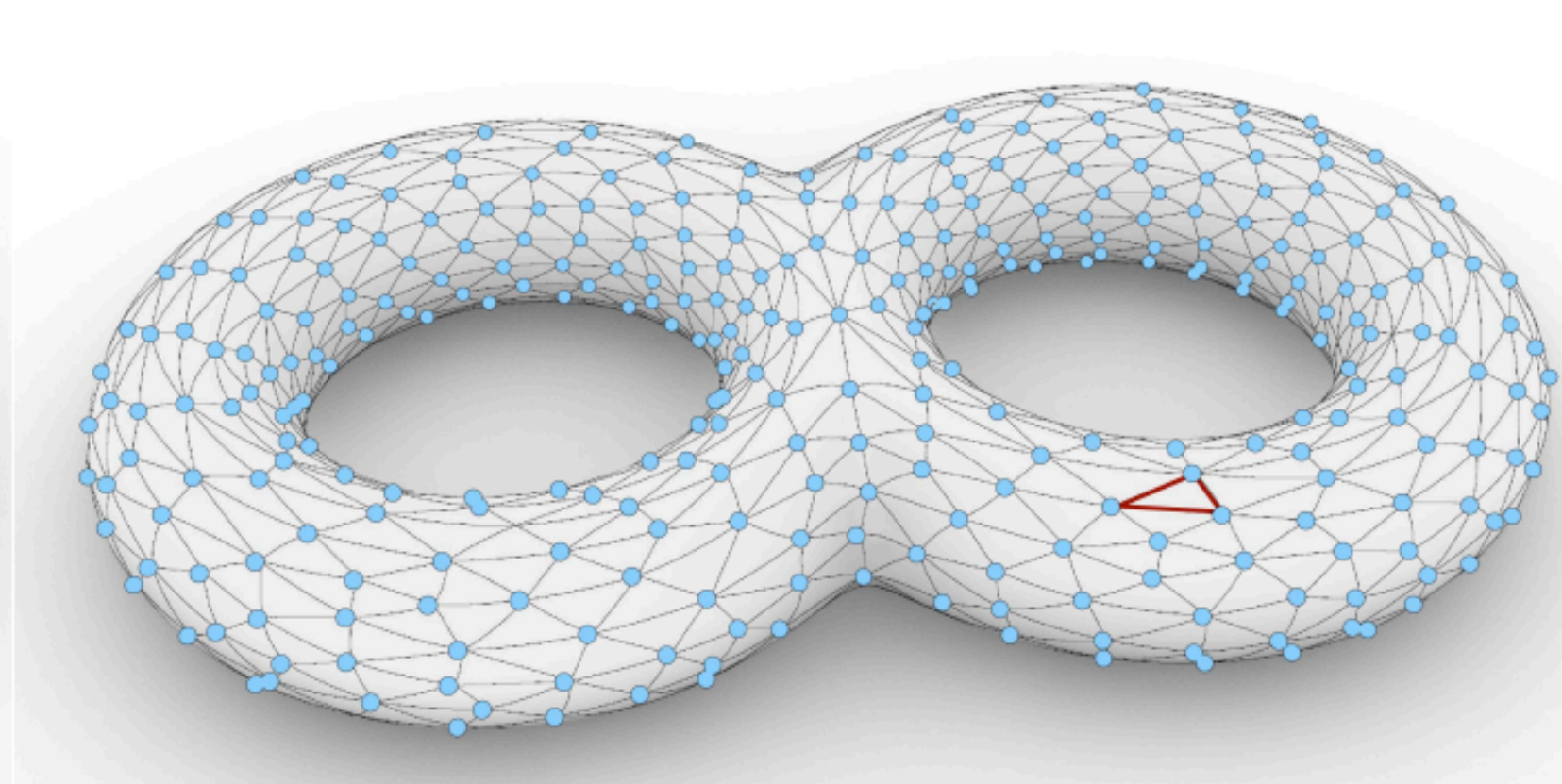
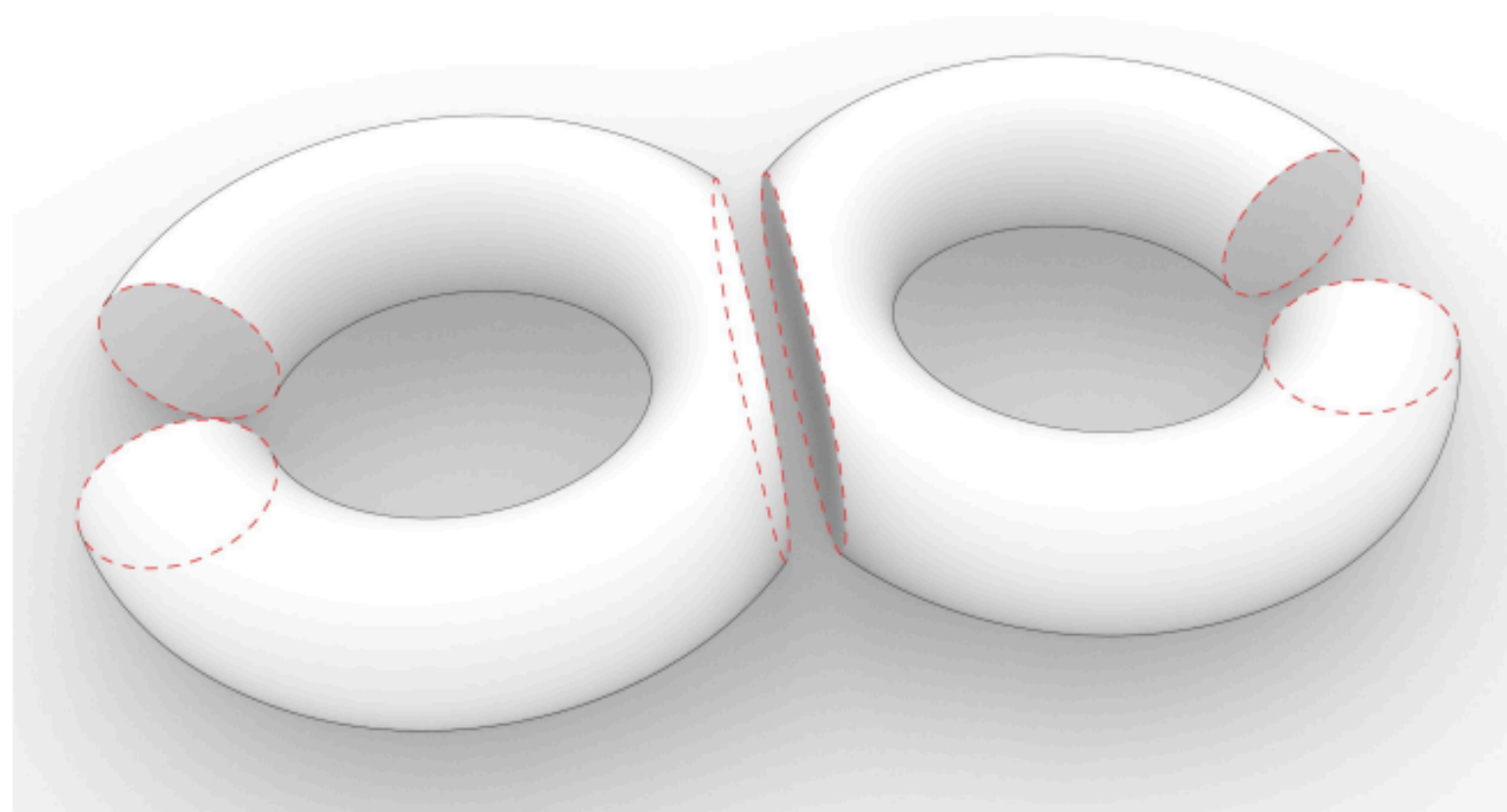
Brehm, Runkel 2022

- Cut 2D manifold into triangles.
- Each triangle = three-point function of boundary-changing operators.
- Glue triangles: summing over states on the edge (primaries + descendants).
- Leaving holes (radius R) at each vertex, labeled by conformal boundary conditions.

$$\mathcal{T}_{(\alpha_1, I_1)(\alpha_2, I_2)(\alpha_3, I_3)}^{\sigma_1 \sigma_2 \sigma_3}(\Delta, R) =$$



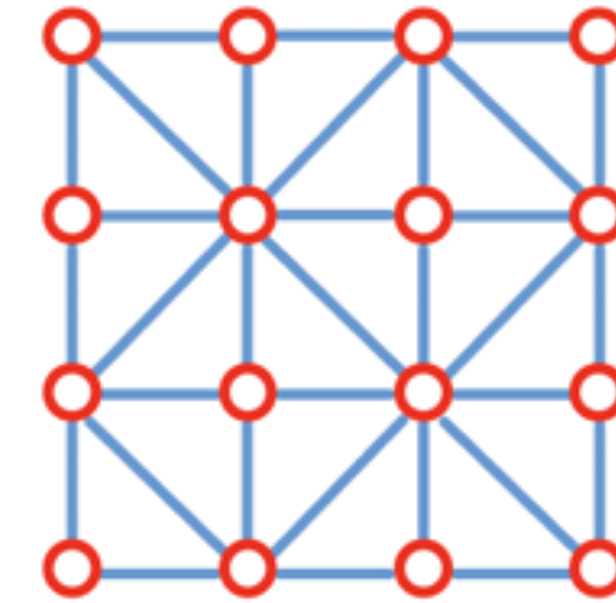
$$\mathcal{Z}(R, \sigma_a) = \sum_{\{\alpha_i\}} \sum_{\{I_i\}} \prod_{\Delta} \mathcal{T}_{(\alpha_i, I_i)(\alpha_j, I_j)(\alpha_k, I_k)}^{\sigma_a \sigma_b \sigma_c}(\Delta, R)$$



Cut a CFT into triangles

$$\mathcal{Z}(R, \sigma_a) = \sum_{\{\alpha_i\}} \sum_{\{I_i\}} \prod_{\Delta} \mathcal{T}_{(\alpha_i, I_i)(\alpha_j, I_j)(\alpha_k, I_k)}^{\sigma_a \sigma_b \sigma_c}(\Delta, R)$$

- Shrink the holes.



Shrinkable Boundary Condition =
“Entanglement Brane Boundary Condition” in RCFT
 Wong, LYH 2020,
 Brehm, Runkel 2022

Vacuum Ishibashi state $|e\rangle\rangle = \sum_a \omega_{\sigma_a} |\sigma_a\rangle$

“Quantum dimensions”

Shrinkable boundary designed to preserve **all** the CFT generalised symmetries.

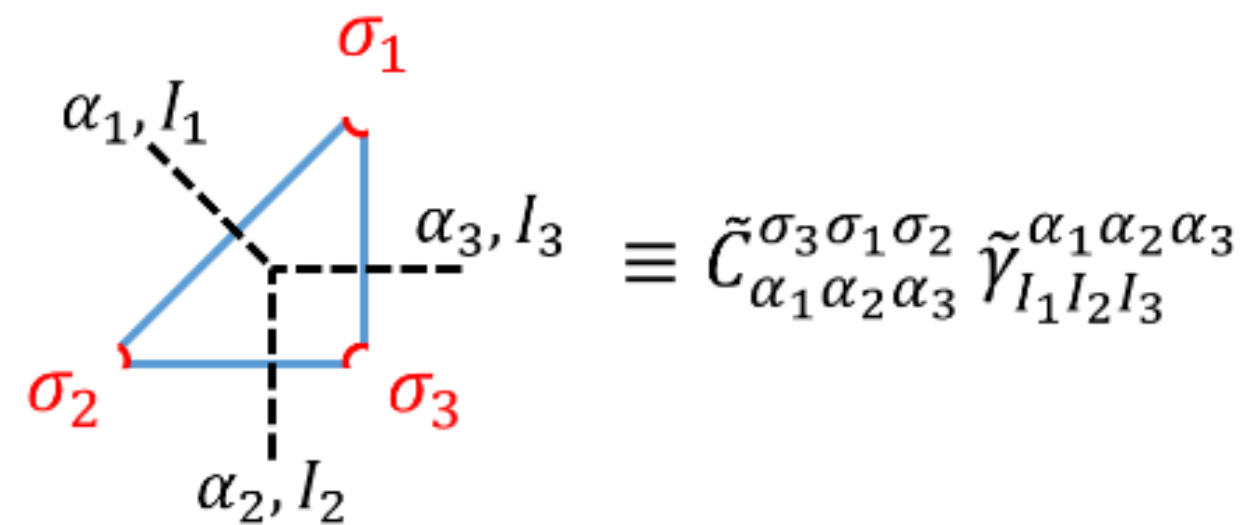
$$\mathcal{Z}(R) = \sum_{\{\sigma_a\}} \prod_v \omega_{\sigma_a} \sum_{\{\alpha_i\}} \sum_{\{I_i\}} \prod_{\Delta} \mathcal{T}_{(\alpha_i, I_i)(\alpha_j, I_j)(\alpha_k, I_k)}^{\sigma_a \sigma_b \sigma_c}(\Delta, R) .$$

$$Z_{\text{CFT}} = \lim_{R \rightarrow 0} \prod_v e^{-c/6 \ln R} \mathcal{Z}(R)$$

Emergence of 3D Bulk from BCFT OPE

Lin Chen, Kaixin Ji, Haochen Zhang, Ce Shen, Ruoshui Wang, LYH 2022

$$\mathcal{Z}(R) = \sum_{\{\sigma_a\}} \prod_v \omega_{\sigma_a} \sum_{\{\alpha_i\}} \sum_{\{I_i\}} \prod_{\Delta} \mathcal{T}_{(\alpha_i, I_i)(\alpha_j, I_j)(\alpha_k, I_k)}^{\sigma_a \sigma_b \sigma_c}(\Delta, R) .$$



Rewrite $\mathcal{Z}(R)$ as an overlap of two 3D states

$$\mathcal{Z}(R) = \langle \Omega(R) | \Psi \rangle = \sum_{\{\alpha_i\}} \Omega(\{\alpha_i\})(R) \Psi(\{\alpha_i\})$$

related to “strange correlator” in lattice spin models
Bal, Williamson, Vanhove, Bultinck, Haegeman, Verstraete 2018;
Aasen, Mong, Fendley 2016; 2020;

Collection of BCFT OPE (= 6j symbols= 3D tetrahedra)

Fuchs, Runkel, Schweigert 2002-2006

$$\Psi(\{\alpha_i\}) = \sum_{\{\sigma_a\}} \prod_v \omega_{\sigma_a} \prod_{\Delta} \tilde{C}_{\alpha_i \alpha_j \alpha_k}^{\sigma_a \sigma_b \sigma_c}$$

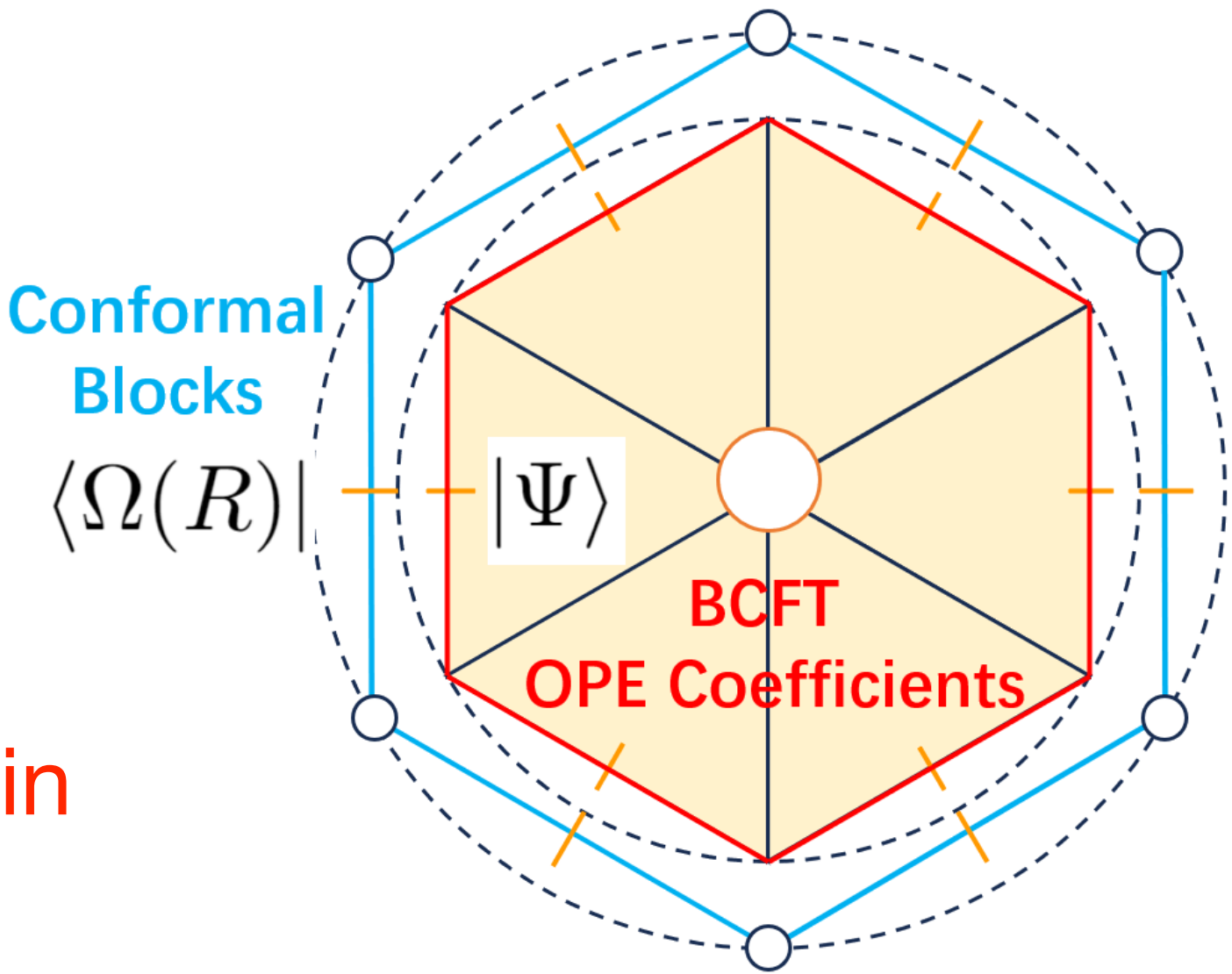
Ground state of a 3D TQFT

Turaev, Viro 1992;
Levin, Wen 2004

Collection of conformal blocks

$$\Omega(\{\alpha_i\})(R) = \sum_{\{I_i\}} \prod_{\Delta} \tilde{\gamma}_{I_i I_j I_k}^{\alpha_i \alpha_j \alpha_k}(R)$$

3D graph independence in the bulk triangulation



Apply it to Liouville CFT - Closing Holes

Lin Chen, LYH, Yikun Jiang, Bingxin Lao 2024

$$\Phi_{\alpha}^{\sigma_1 \sigma_2}(x)$$

↗ $\sigma \equiv Q/2 + iP_{\sigma}, \quad \alpha \equiv Q/2 + iP_{\alpha}$

A self-consistent collection of conformal boundary conditions

Fateev, Zamolodchikov,
Zamolodchikov 2000;
Teschner 2000;

$$Q = b + b^{-1}$$

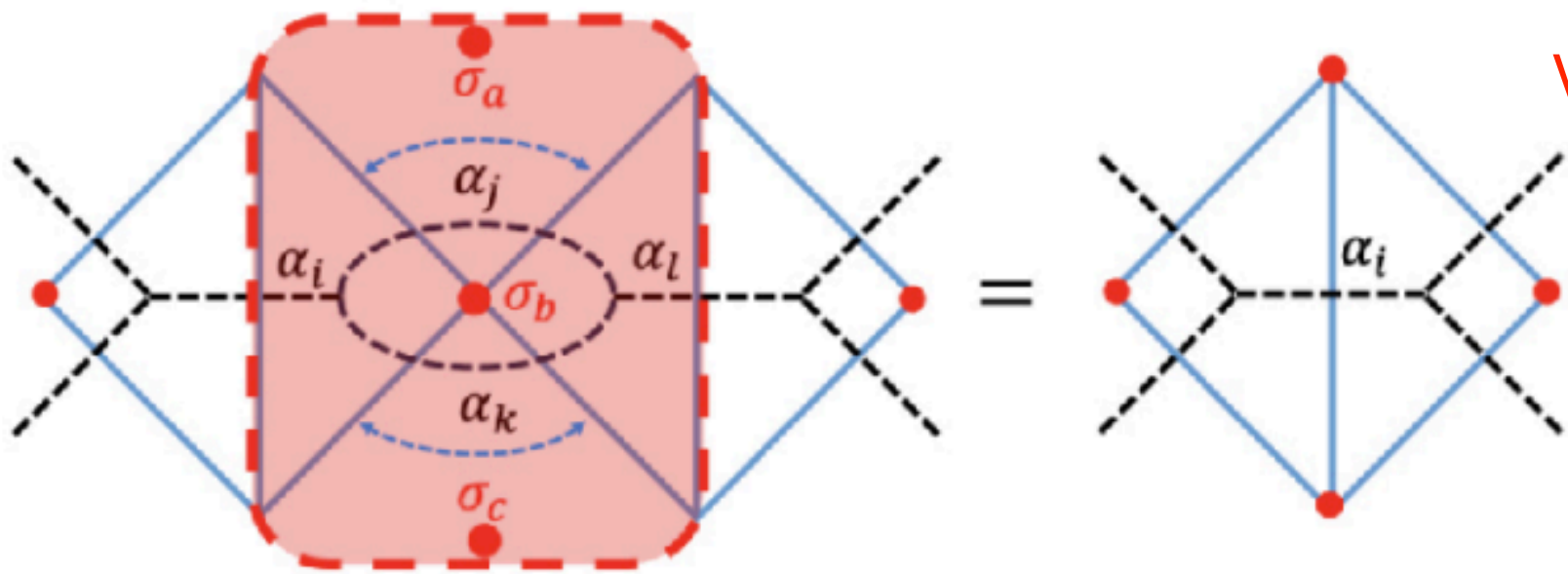
$$c = 1 + 6Q^2$$

$$C_{\alpha_1, \alpha_2, \alpha_3}^{\sigma_3, \sigma_1, \sigma_2} = (\mu(P_{\alpha_1})\mu(P_{\alpha_2})\mu(P_{\alpha_3}))^{1/4} \left\{ \begin{matrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \sigma_3 & \sigma_1 & \sigma_2 \end{matrix} \right\}_b.$$

Ponsot, Teschner 2001;
Chen, LYH, Jiang, Lao 2024;

↑
 $\mathcal{U}_q(SL(2, \mathbb{R}))$ $q = e^{\pi i b^2}$

Also Crossing Kernel of Virasoro conformal blocks



$$\mu(P) = 4\sqrt{2} \sinh(2\pi P b) \sinh\left(\frac{2\pi P}{b}\right).$$

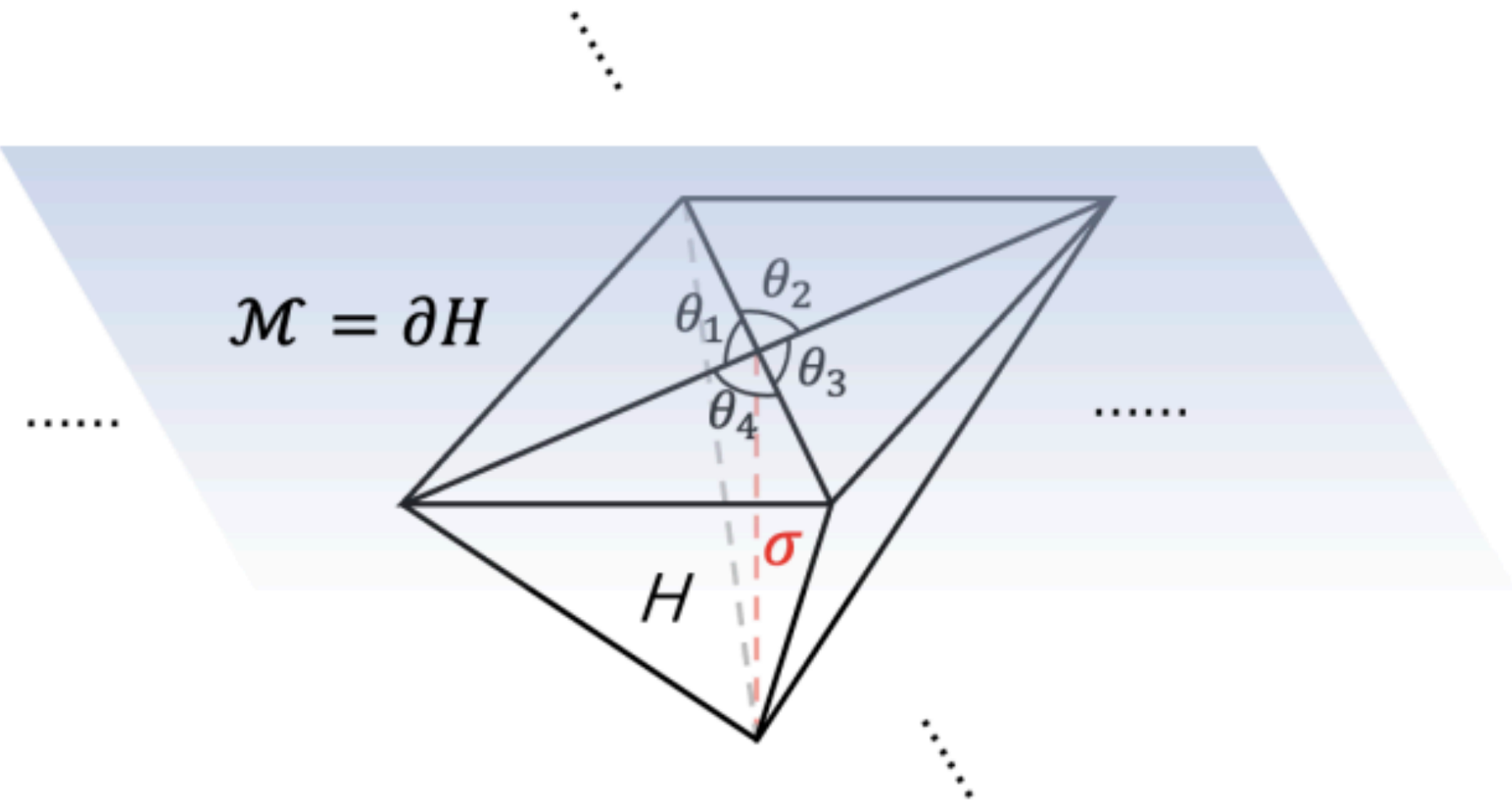
Weight the hole with the Plancherel measure (Cardy density of states)

Reading off the bulk

Lin Chen, LYH, Yikun Jiang, Bingxin Lao 2024

$$Z_{\text{Liouville}}(R) = \langle \Omega(R) | \Psi \rangle$$

$$\Psi(\{\alpha_i\}) = \prod_v \int_0^\infty dP_{\sigma_a} \mu(P_{\sigma_a}) \prod_e \sqrt{\mu(P_{\alpha_i})} \prod_{\Delta} \left\{ \begin{matrix} \alpha_i & \alpha_j & \alpha_k \\ \sigma_c & \sigma_a & \sigma_b \end{matrix} \right\}_b$$
$$\Omega(\{\alpha_i\})(R) = \prod_e \sum_{\{I_i\}} \prod_{\Delta} \left(\tilde{\gamma}_{I_i I_j I_k}^{\alpha_i \alpha_j \alpha_k}(R) \right)$$



Reading off the bulk in the large c limit

$$P = \frac{l}{2\pi b}$$

$$\left\{ \begin{array}{ccc} \frac{Q}{2} + i\frac{l_4}{2\pi b} & \frac{Q}{2} + i\frac{l_5}{2\pi b} & \frac{Q}{2} + i\frac{l_6}{2\pi b} \\ \frac{Q}{2} + i\frac{l_1}{2\pi b} & \frac{Q}{2} + i\frac{l_2}{2\pi b} & \frac{Q}{2} + i\frac{l_3}{2\pi b} \end{array} \right\}_b \xrightarrow{b \rightarrow 0} \exp \left(-\frac{\text{Vol}(T\{l_i\}) + \sum_i l_i \theta_i / 2}{\pi b^2} \right),$$

Kashaev 1997;
Teschner, Vartanov 2012;
Collier, Eberhardt, Zhang 2023;
Chen, LYH, Jiang, Lao 2024;

Large c limit reduces to on-shell Einstein-Hilbert action on hyperbolic tetrahedron.

$$S_{EH} = -\frac{1}{16\pi G_N} \int_H d^3x \sqrt{g} (R - 2\Lambda) = \frac{V(H)}{4\pi G_N} = \frac{V(H)}{\pi b^2},$$

$$\sum_i \frac{\theta_i l_i}{2\pi b^2} = \sum_i \frac{\theta_i l_i}{8\pi G_N} = \frac{1}{8\pi G_N} \sum_i \int_{\Gamma_i} \theta_i \sqrt{h},$$

**Gibbons
Hawking/
Hayward
Term**

BCFT
primary labels



Bulk
geodesic
lengths

Low dimensional AdS/CFT and topological holographic principle have a large overlap here! 3D gravity imposes an infinite number of Virasoro Lines — details of the quantum measure of geometrical sum hides in the hole closing detail!

Symmetry preserving RG - The Bridge between continuity and Discreteness

Generalised Symmetry Preserving RG and Their Fixed Point

A lattice integrable model can also be written in this form:
 R. Vanhove, M. Bal, D. J. Williamson, N. Bultinck, J. Haegeman, and
 F. Verstraete;
 D. Aasen, P. Fendley, and R. S. K. Mong

$$\langle \Omega_N | \Psi_a^{LW} \rangle$$

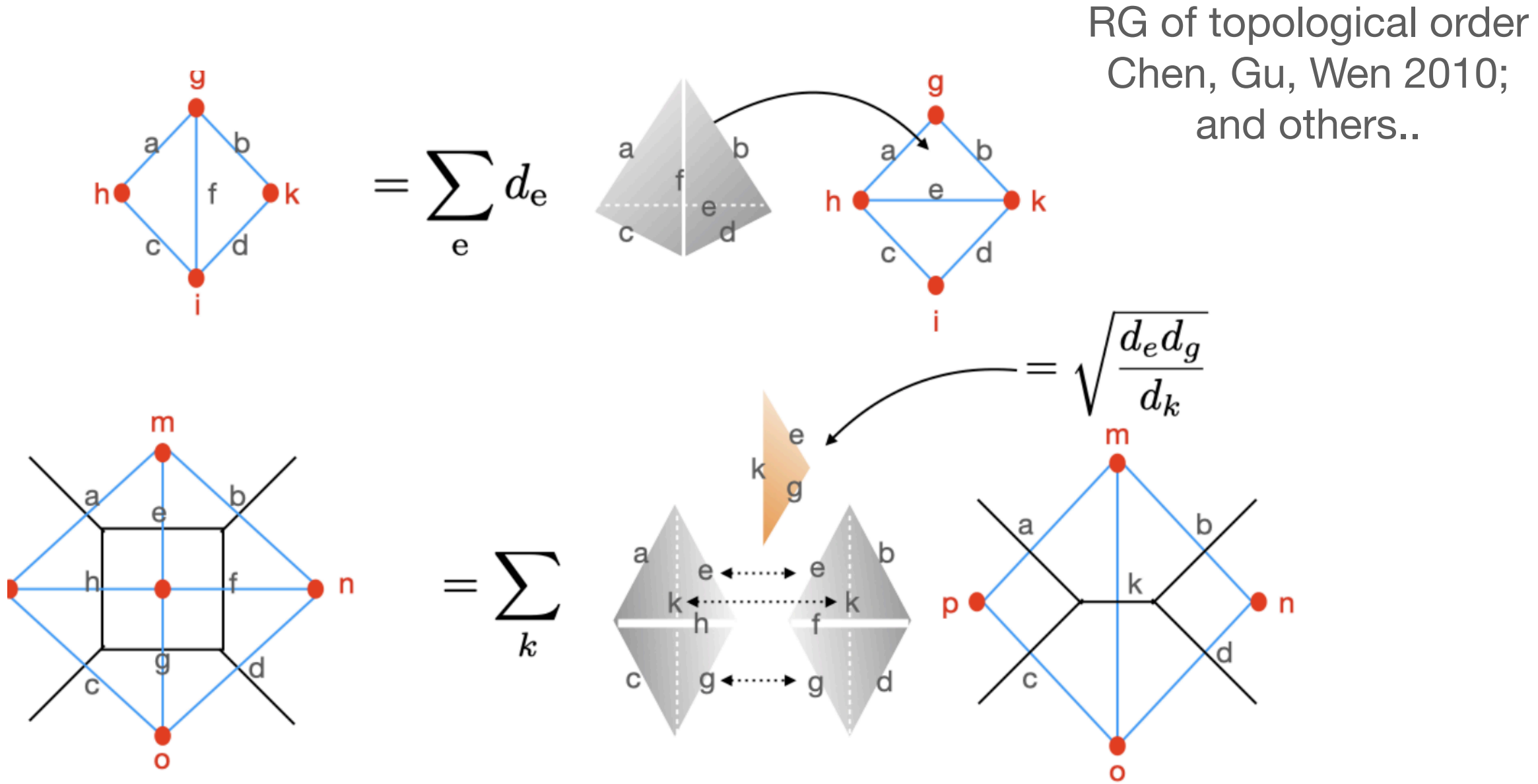
$$|\Psi\rangle_\Lambda = U|\Psi\rangle_{\sqrt{2}\Lambda}$$

$$\langle \Omega_N | FF | \Psi_{ka}^{LW} \rangle = \langle \Omega_{N-1} | \Psi_{ka}^{LW} \rangle$$

$$\sqrt{2}\Lambda \langle \Omega | =_\Lambda \langle \Omega | U$$

CFT corresponds to a
 fixed point solution!

- symmetric topological phase
 = topological eigenstate of RG
 operator from Frobenius
 algebra.
- symmetric CFT is an infinite
 bond dimension eigenstate of
 the RG operator from
 conformal blocks



**Gong Cheng, Lin Chen,
 Zheng-Cheng Gu**
[2311.18005](#)
[Phys.Rev.X 15 \(2025\) 1, 011073](#)

**Lin Chen, Kaixin Ji,
 Haochen Zhang, Ce Shen,
 Ruoshui Wang**
[2210.12127](#) [Physical Review X 14 \(4\), 041033](#)

D ← D+1

**Engineering seed boundary conditions to discover
new critical points and CFTs**

Discovering CFTs from Statistical Lattice models

Historically a treasure of CFTs

- Discretising CFT does not get you new CFTs. To predict new CFTs - we need to work backwards — Given a symmetry, construct a CFT.
- It's too hard to directly create fixed point bc — but it is possible to create bc corresponding to continuous phase transitions in a lattice — this is how historically statistical model helped discovered many CFTs!
- e.g. The Ising spin model solved by Onsagar — has a critical point which is related to the 2D Ising CFT
- The entire A-series minimal models were originally found in the A-series lattice integrable models.
- This is no longer the main-stream of producing new CFT — writing down Lagrangian has become the main-stream. Why? Can we re-live this good old times?

RSOS integrable models and Minimal models and Levin Wen models

Verstraete et al 2017; Aasen, Fendley, Mong 2016, 2020;

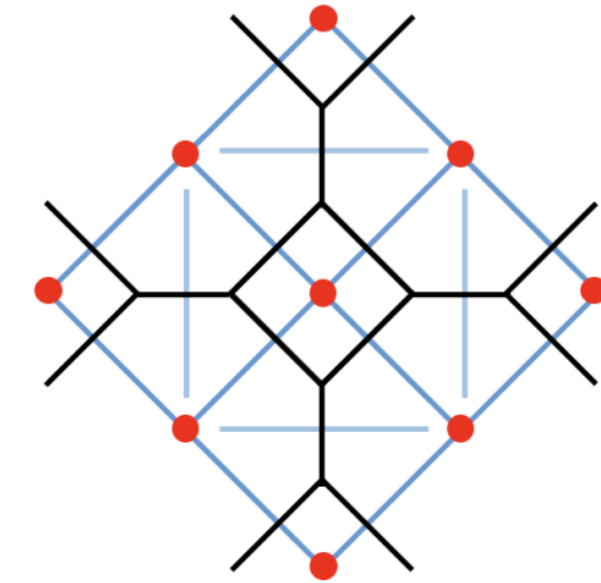
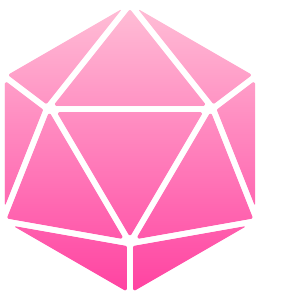
- PEPS representation of Levin-Wen models/Turaev-Viro ground state $|\Psi^{LW}\rangle$

Gu, Levin, Swingle, Wen PRB 2009; Buerhaper, Aguado, Vidal PRB 2009;

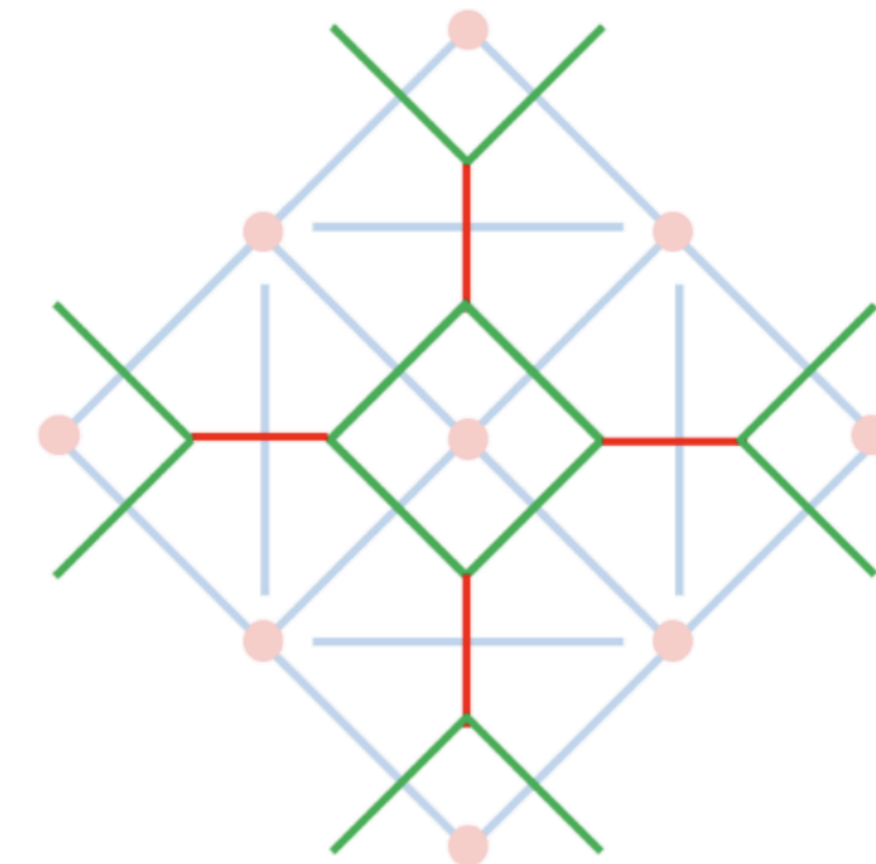
- Then pick some mysterious state $\langle\Omega_N|$ and take the overlap with $|\Psi^{LW}\rangle$ i.e. $\langle\Omega_N|\Psi^{LW}\rangle$. $\langle\Omega_N|$ is chosen such that the overlap matches exactly the partition function of well known families of integrable models — this is a realisation of the sandwich.

This observation is related to the anyon chain proposed in Feigen et al 2006

Path-integral
of a 3-ball with a two dimensional
surface



$$\begin{array}{c} z \\ a \quad b \\ y \quad x \\ c \end{array} = \begin{bmatrix} a & b & c \\ x & y & z \end{bmatrix} = \frac{1}{\sqrt{d_c d_z}} (F_y^{abx})_{cz}^*$$



— : $\langle \frac{1}{2} |$

— : $\langle 0 | + r \langle 1 |$

RSOS integrable models and Minimal models and Levin Wen models

Verstraete et al 2017; Aasen, Fendley, Mong 2016, 2020;

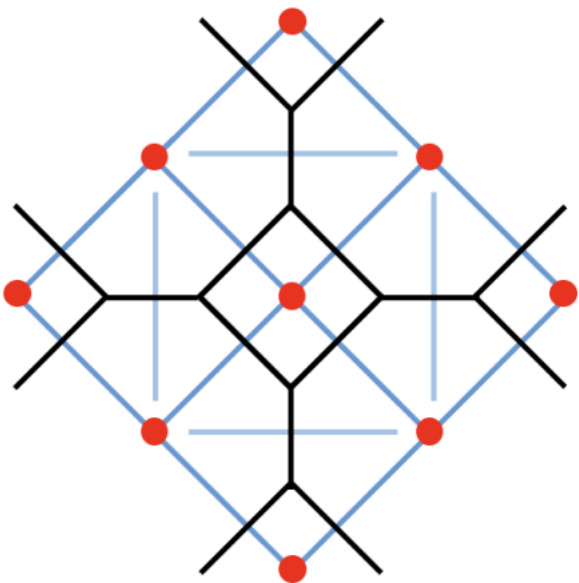
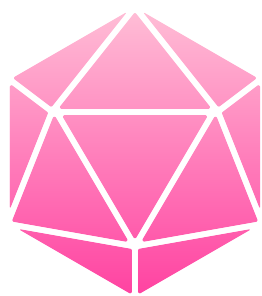
r is related to the temperature in the classical Ising spin model:

$$r = e^{-2\beta}$$

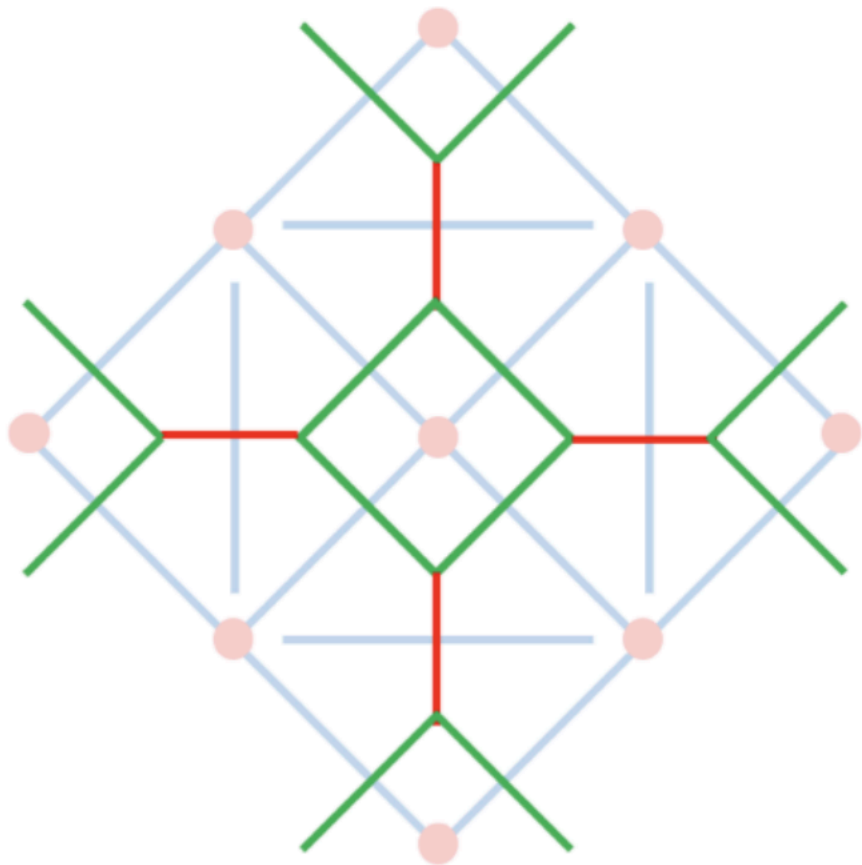
$$r_c = e^{-2\beta_c}, \qquad \beta_c = 1/2 \ln(1 + \sqrt{2})$$

The critical temperature is known for many years. Now expressed as the boundary condition of the Levin-Wen model.

Path-integral
of a 3-ball with a two dimensional
surface



$$\begin{array}{c} z \\ a \quad \quad b \\ \diagdown \quad \diagup \\ y \quad \quad x \\ c \end{array} = \begin{bmatrix} a & b & c \\ x & y & z \end{bmatrix} = \frac{1}{\sqrt{d_c d_z}} (F_y^{abx})_{cz}^*$$



$$\begin{array}{l} \text{---} : \langle \frac{1}{2} | \\ \text{---} : \langle 0 | + r \langle 1 | \end{array}$$

RSOS integrable models and Minimal models and Levin Wen models

Verstraete et al 2017; Aasen, Fendley, Mong 2016, 2020; Chen et al 2022;

More general there is the A series integrable models with the same ansatz :

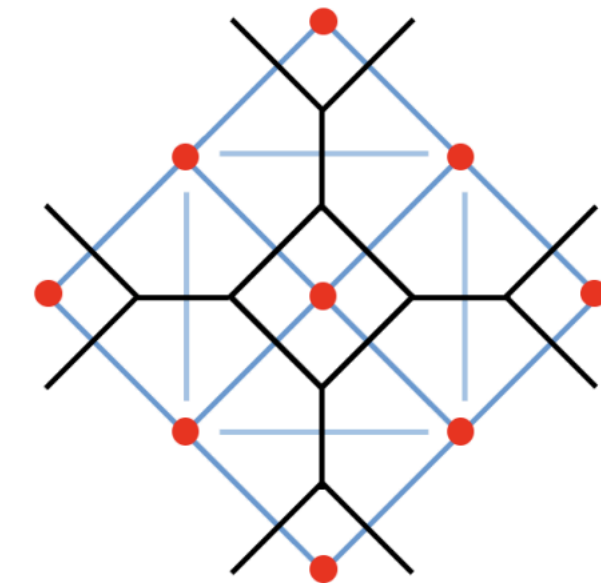
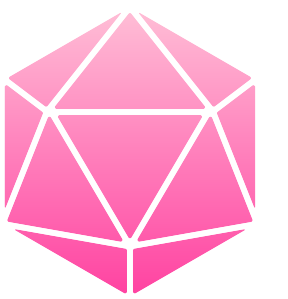
Expressed in this form, the well known critical coupling is located at :

$$r_c = \sqrt{2 \cos(2\pi/(k+2)) + 1} / (2 \cos(\pi/(k+2)) + 1)$$

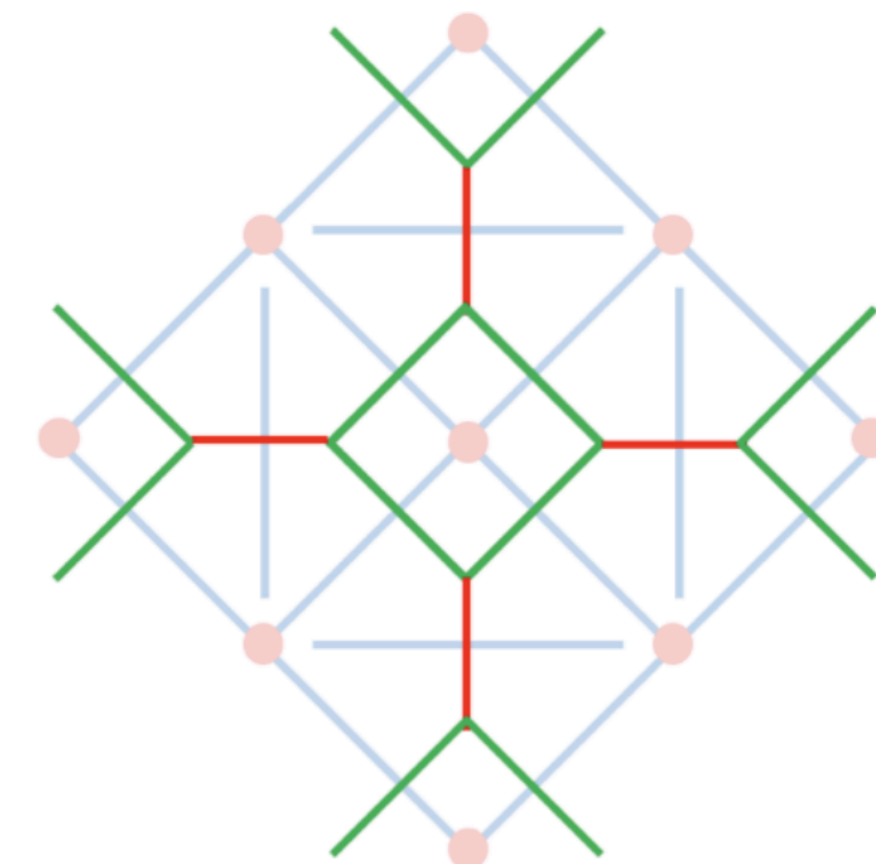
What is this? Does the Levin Wen/TV model knew about these numbers?

i.e. How exactly is this ansatz chosen in principle???? In particular, which $\langle \Omega_N |$ can produce critical points = CFTs? Is there anything special about these critical couplings???

Path-integral
of a 3-ball with a two dimensional
surface



$$\begin{array}{c} z \\ a \quad b \\ y \quad x \\ c \end{array} = \begin{bmatrix} a & b & c \\ x & y & z \end{bmatrix} = \frac{1}{\sqrt{d_c d_z}} (F_y^{abx})_{cz}^*$$

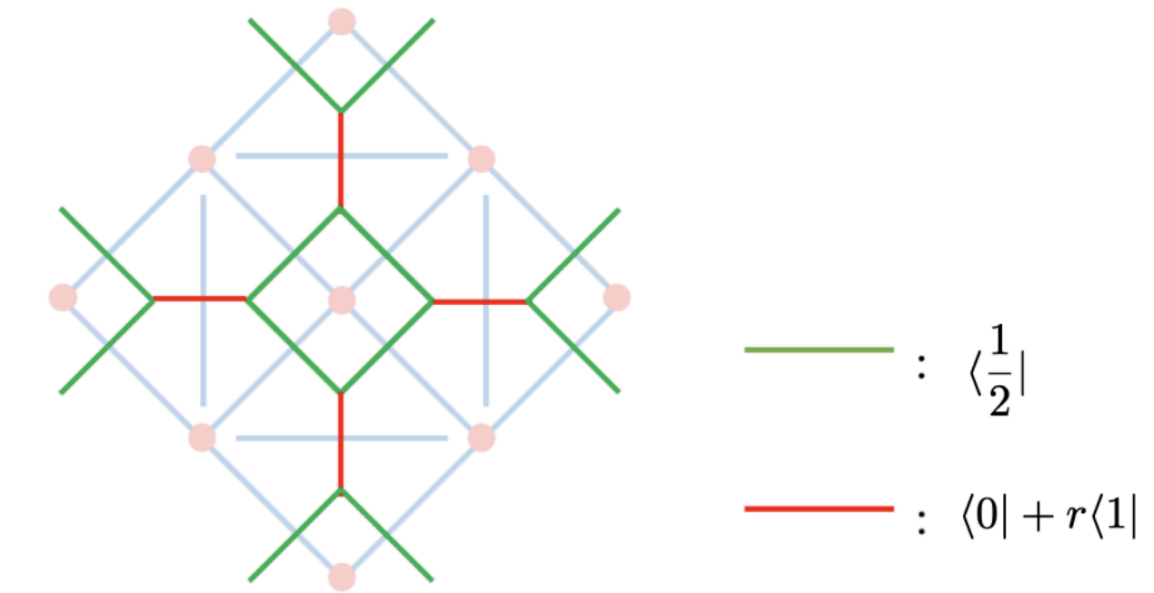


— : $\langle \frac{1}{2} |$
— : $\langle 0 | + r \langle 1 |$

**A 2D CFT Factory : an ansatz of unit
cell, and competing anyon
condensation**

Ansatz of Unit cell

Module and Frobenius algebra



Example: In the Ising example, we notice that at small beta and large beta, the red edges are coloured by one of the two Frobenius algebra of the Ising model respectively. One can check that in this case they are closely related to topological boundary condition of the 3D TV model.

There are 2 Frobenius algebra:

$$A_0 = 0 \quad \text{Electric condensate: equivalent to Lagrangian condensate} = 1 \oplus e$$

Low temperature Z2 spontaneously sym breaking

How to connect the two things? There are many ways . See Wan et al. But bottom line is to use the Ribbon operator for the Levin Wen model.

$$A_1 = 0 \oplus 1 \quad \text{Magnetic condensate: equivalent to Lagrangian condensate} = 1 \oplus m$$

High temperature dual Z2 spontaneously sym breaking

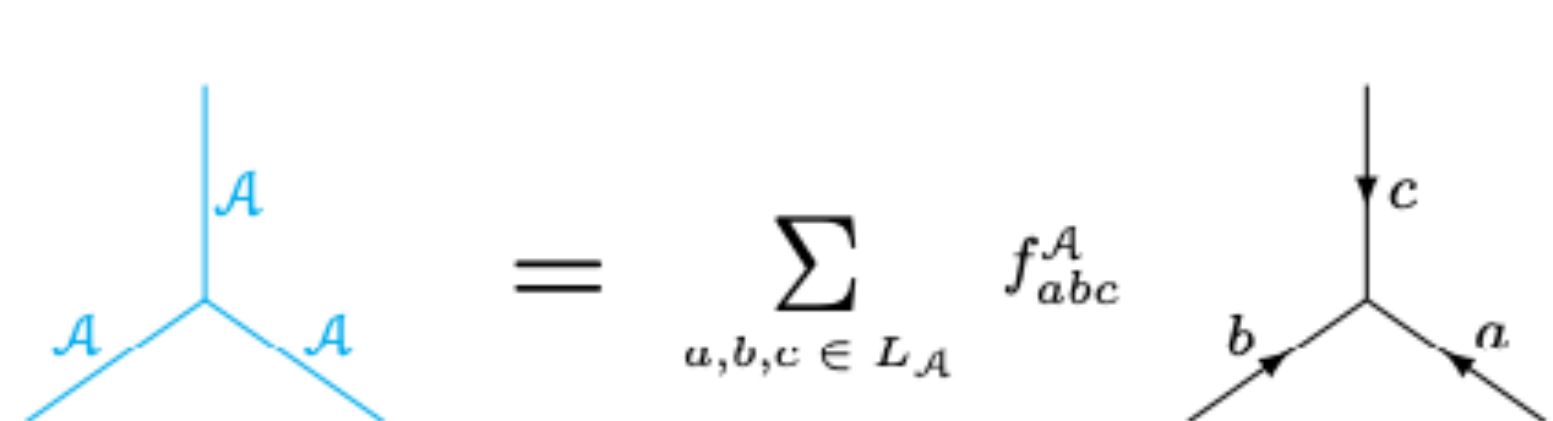
Two anyons with non-trivial braiding cannot condense together. CFT is an equilibrium point in this competition!

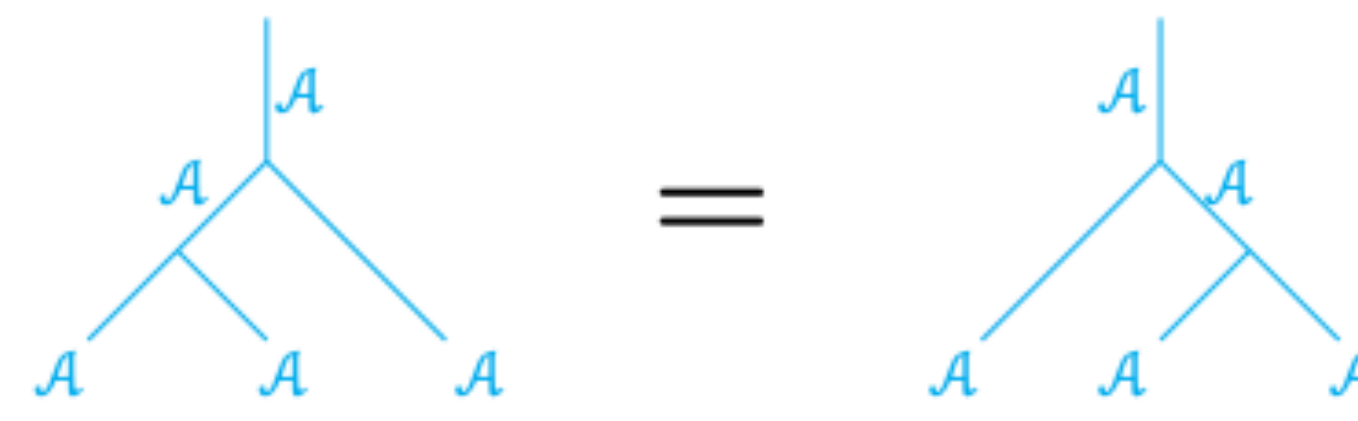
Ansatz of Unit cell

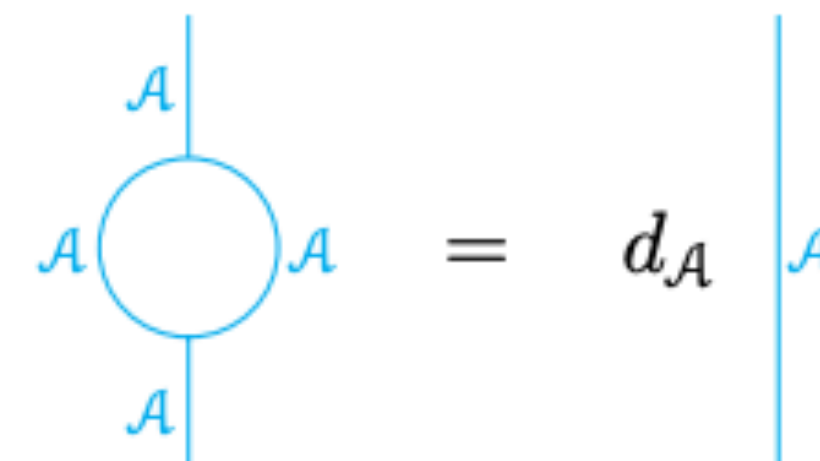
Module and Frobenius algebra

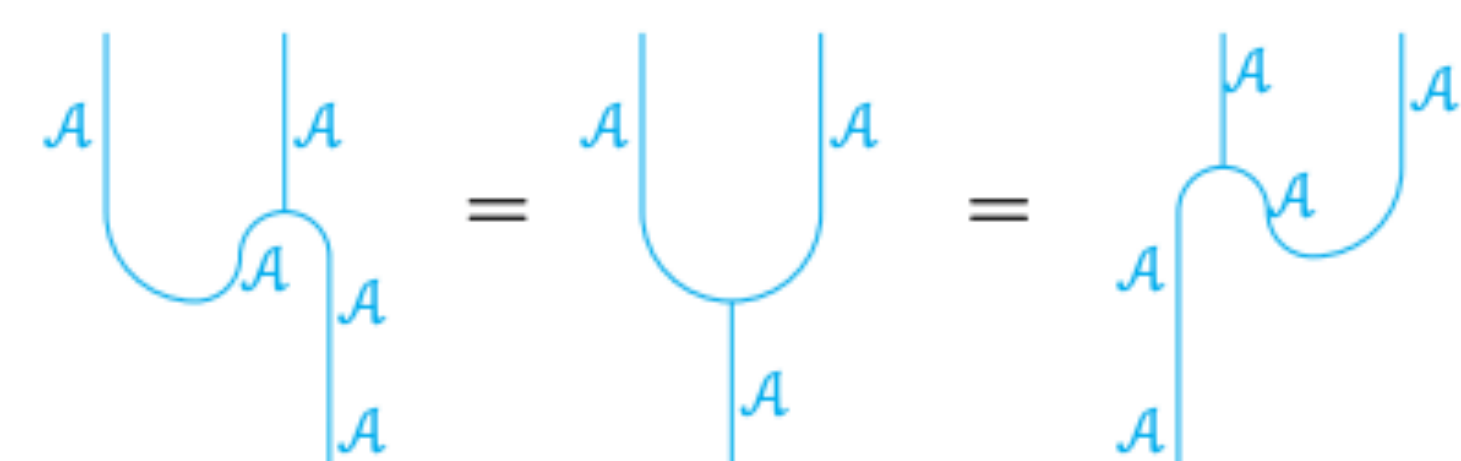
Fuchs, Runkel, Schweigert 2000s ; Hu, Wan, Wu 1706.00650;

- A Frobenius algebra satisfies the following conditions:

(a) 
$$= \sum_{a,b,c \in L_{\mathcal{A}}} f_{abc}^{\mathcal{A}} \text{ (vertex with legs } c, b, a \text{)}$$

(b) 
$$= \text{ (vertex with legs } A, A, A \text{)}$$

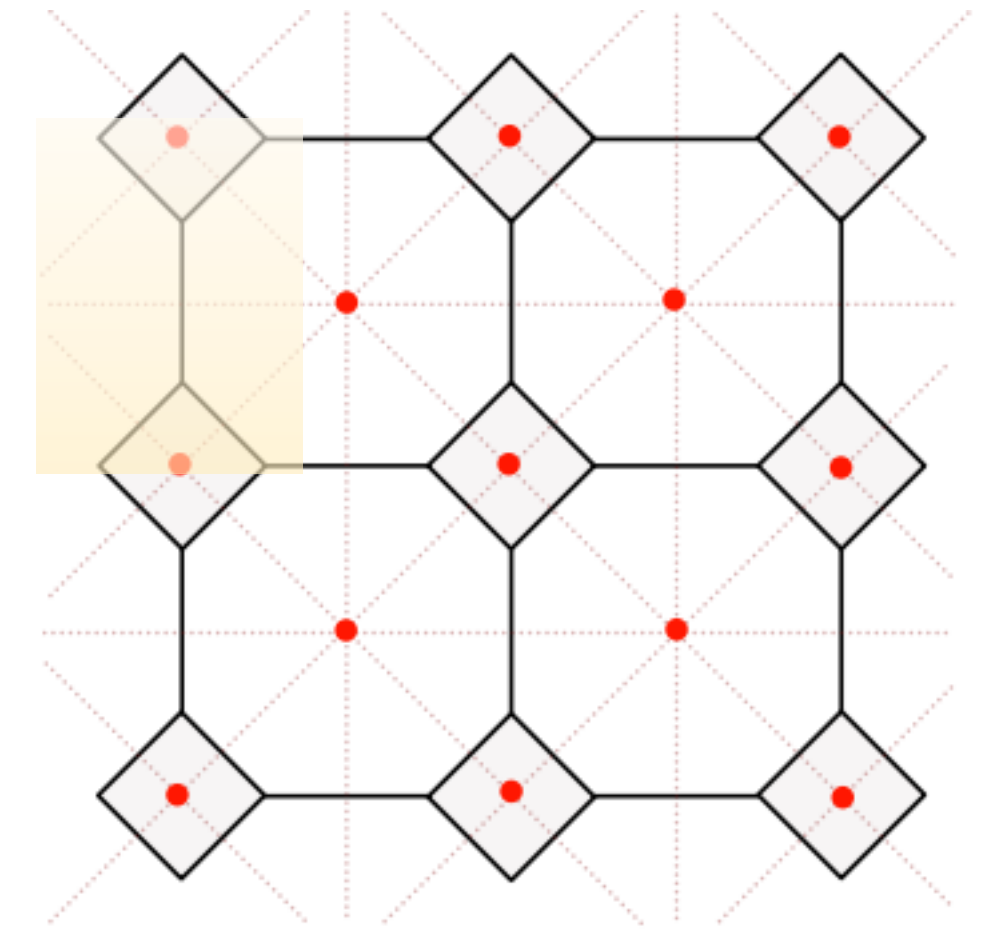
(c) 
$$= d_{\mathcal{A}} \text{ (vertical line labeled } A \text{)}$$

(d) 
$$= \text{ (vertex with legs } A, A, A \text{)}$$

Each Frobenius algebra describes a *Lagrangian algebra* which is a maximal set of anyons that condense. When a maximal set of anyons are condensed — this produces a trivial state. Each such trivial state is a topological boundary condition of the 3D TQFT.

Ansatz of Unit cell

Module and Frobenius algebra



Consider a octagon-square lattice. (Inspired by the Ising model)

We consider a unit cell in this lattice as highlighted above. We want to define a state corresponding to pouring the condensate into this cell, while it is well “isolated” from the other cells.

This is given by:

This state is a topological boundary condition of the TQFT!

$$\langle \text{X}_{\mathcal{A}}^{M_{\mathcal{A}}} | = \sum_{a \in L_{\mathcal{A}}} \sum_{x, y, u, v \in L_{M_{\mathcal{A}}}} \langle \text{X}_{u, v}^{x, y, a} | [\rho_{M_{\mathcal{A}}}]_{xy}^a ([\rho_{M_{\mathcal{A}}}]_{uv}^a)^*.$$

Here, M is a module of A. It satisfies:

$$\begin{array}{c} M \\ M \\ M \end{array} \left| \begin{array}{c} \text{blue line} \end{array} \right. \mathcal{A} = \sum_{\substack{a \in L_{\mathcal{A}} \\ x, y \in L_M}} [\rho_{M_{\mathcal{A}}}]_{xy}^a \begin{array}{c} y \\ \uparrow \\ x \end{array} \begin{array}{c} \text{diagonal line with arrow} \end{array} \mathcal{A} \quad (a)$$

$$\begin{array}{c} M \\ M \\ M \end{array} \left| \begin{array}{c} \text{blue line} \end{array} \right. \mathcal{A} = \begin{array}{c} M \\ M \\ M \end{array} \left| \begin{array}{c} \text{blue line} \end{array} \right. \mathcal{A} \quad (b)$$

Ansatz of Unit cell

Module and Frobenius algebra

- Normalising a Frobenius algebra

$$N^2_{(\mathcal{A}_i, M_{\mathcal{A}_i})} = \left\langle \begin{array}{c} M_{\mathcal{A}_i} \\ \text{---} \\ \mathcal{A}_i \\ \text{---} \\ M_{\mathcal{A}_i} \end{array} \right\rangle = \text{Diagram} = \text{Diagram} = d_{\mathcal{A}_i} = d_{\mathcal{A}_i}^0 = d_{M_{\mathcal{A}_i}}^2 d_{\mathcal{A}_i}$$

The diagrammatic equation shows the normalization of the Frobenius algebra. It starts with the square of the unit cell, represented by two identical diagrams in brackets. Each diagram consists of a central blue vertical line labeled $\mathcal{A}_i$, with red lines branching out from its top and bottom, labeled $M_{\mathcal{A}_i}$. This is followed by an equals sign and a diagram of two red loops connected by a blue vertical line. Another equals sign leads to a diagram of two red circles connected by a blue vertical line. This is followed by an equals sign and the expression $d_{\mathcal{A}_i}$. Another equals sign leads to a diagram of two red circles connected by a blue vertical line, which is then followed by an equals sign and the expression $d_{\mathcal{A}_i}^0$. Finally, an equals sign leads to the expression $d_{M_{\mathcal{A}_i}}^2 d_{\mathcal{A}_i}$.

$$d_{M_{\mathcal{A}_i}} \equiv \sum_{c \in M_{\mathcal{A}_i}} m_c d_c, \quad d_{\mathcal{A}_i} \equiv \sum_{a \in \mathcal{A}_i} n_a d_a.$$

Ansatz of Unit cell

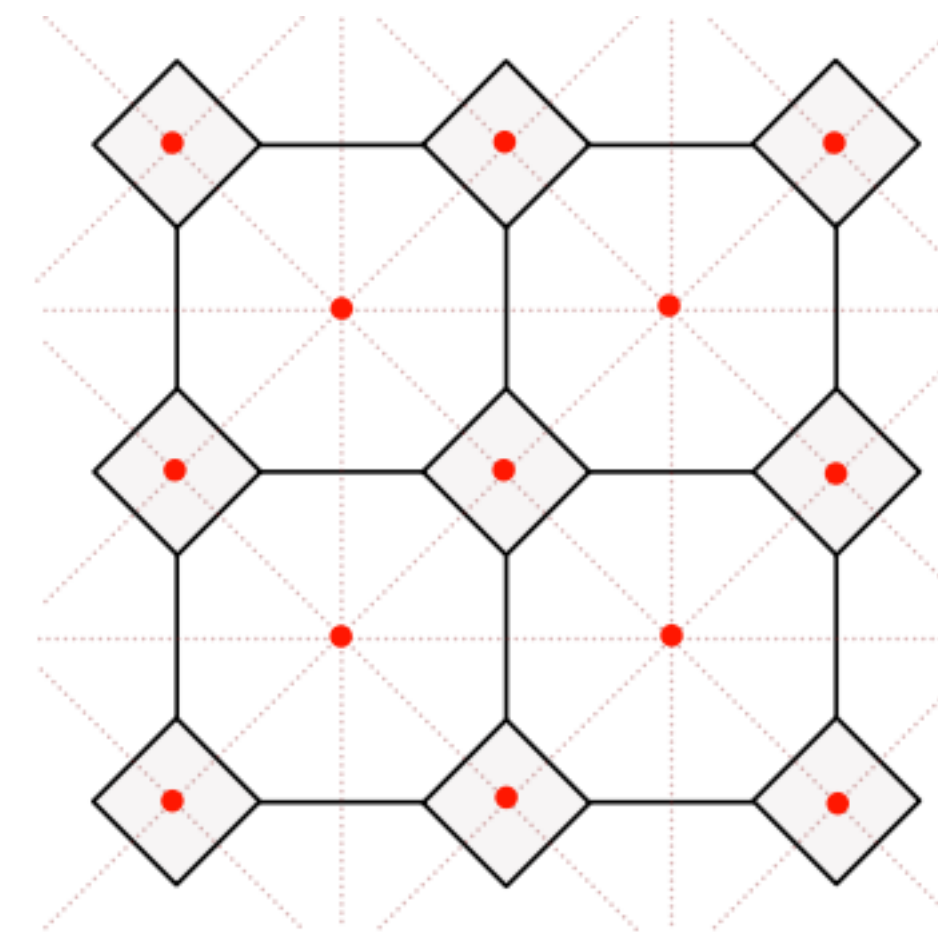
Module and Frobenius algebra

Interpolating between condensates:

$$\langle \text{Y-junction} \rangle_{(\mathcal{A}_i, M_{\mathcal{A}_i}), (\mathcal{A}_j, M_{\mathcal{A}_j})} \big|_{\text{critical}} = \langle \hat{\mathcal{A}}_i |_{M_{\mathcal{A}_i}} + \langle \hat{\mathcal{A}}_j |_{M_{\mathcal{A}_j}}$$

$$\langle \hat{\mathcal{A}}_i |_{M_{\mathcal{A}_i}} \equiv \frac{\langle \text{Y-junction with } \mathcal{A}_i \text{ and } M_{\mathcal{A}_i} \rangle}{N_{(\mathcal{A}_i, M_{\mathcal{A}_i})}}.$$

Here, we require that $\mathcal{A}_i$ and $\mathcal{A}_j$ share the module objects in M , even if the module function may not be the same. This is crucial to preserving the isolation of the condensate between unit cells in the full lattice.



Ansatz of Unit cell

Module and Frobenius algebra

Interpolating between condensates:

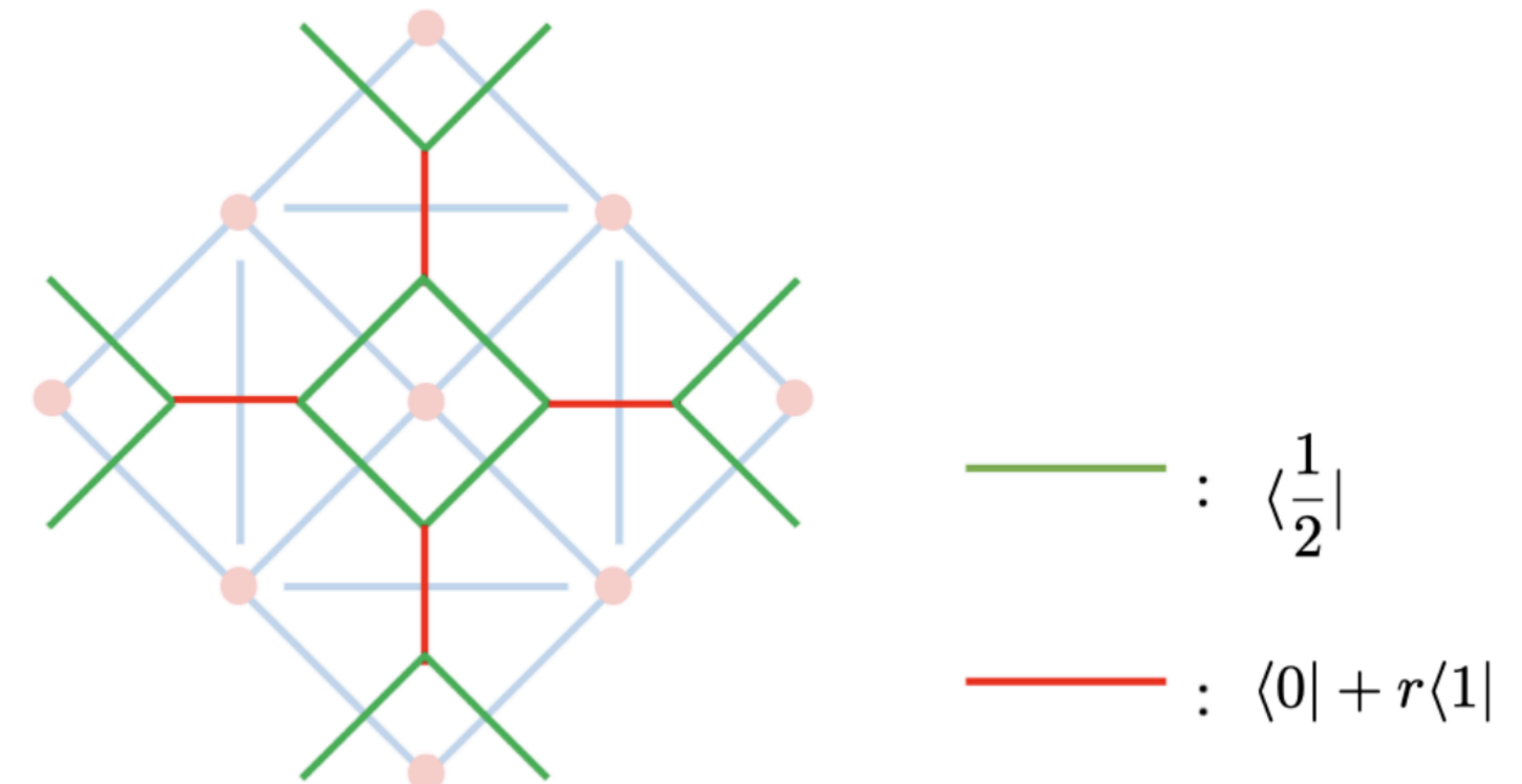
$$\langle \bigvee_{(\mathcal{A}_i, M_{\mathcal{A}_i}), (\mathcal{A}_j, M_{\mathcal{A}_j})} \big|_{\text{critical}} = \langle \hat{\mathcal{A}}_i |_{M_{\mathcal{A}_i}} + \langle \hat{\mathcal{A}}_j |_{M_{\mathcal{A}_j}}$$

$$\langle \hat{\mathcal{A}}_i |_{M_{\mathcal{A}_i}} \equiv \frac{\langle \bigvee_{\mathcal{A}_i, M_{\mathcal{A}_i}} \big|}{N_{(\mathcal{A}_i, M_{\mathcal{A}_i})}}.$$



$$r_c = \sqrt{2 \cos(2\pi/(k+2)) + 1} / (2 \cos(\pi/(k+2)) + 1)$$

Example:
Going back to
the A-series



For all k we have the following Frobenius algebra:

$$\begin{aligned} A_0 &= 0 \\ A_1 &= 0 \oplus 1 \end{aligned} \quad \mathcal{M} = \frac{1}{2}$$

Why does it work? — why is it a second order phase transition?

Key: cut down on degrees of freedom in RG space

- For chosen M such that it is shared by all the interpolating algebras (for simplicity, including module function), the RG attraction basins would be reduced to these algebras.

number of sharing $A = n_M$

- This happens where $\cdots \subset \mathcal{A}_i \subset \mathcal{A}_j \subset \mathcal{A}_k$

Number of couplings in a unit cell:

$$D_{\text{unit}}(\cdots \subset \mathcal{A}_i \subset \mathcal{A}_j \subset \mathcal{A}_k) = N_{\mathcal{A}_k}. \quad D_c = D_{\text{unit}} - 1$$

This is a sufficient but not
necessary condition!

All phase transitions in a phase diagram are forced to be second order if

$$n_M > D_c.$$

i.e. all directions correspond to moving towards an incompatible phase.
No directions describe degeneracy between two phases.

Phase Diagrams and Phase Boundaries

3 phase competition: Ashkin-Teller model from A5

phase boundaries and tri-critical point

- The module: $M = 1$ $A_0 = 0 \subset A_1 = 0 \oplus 4 \subset A_3 = 0 \oplus 4 \oplus 2$

Note that this choice of module kills $A_2 = 0 \oplus 2$

$nM = 3$, $Dc = 2 \Rightarrow$ All phase transitions are second order.

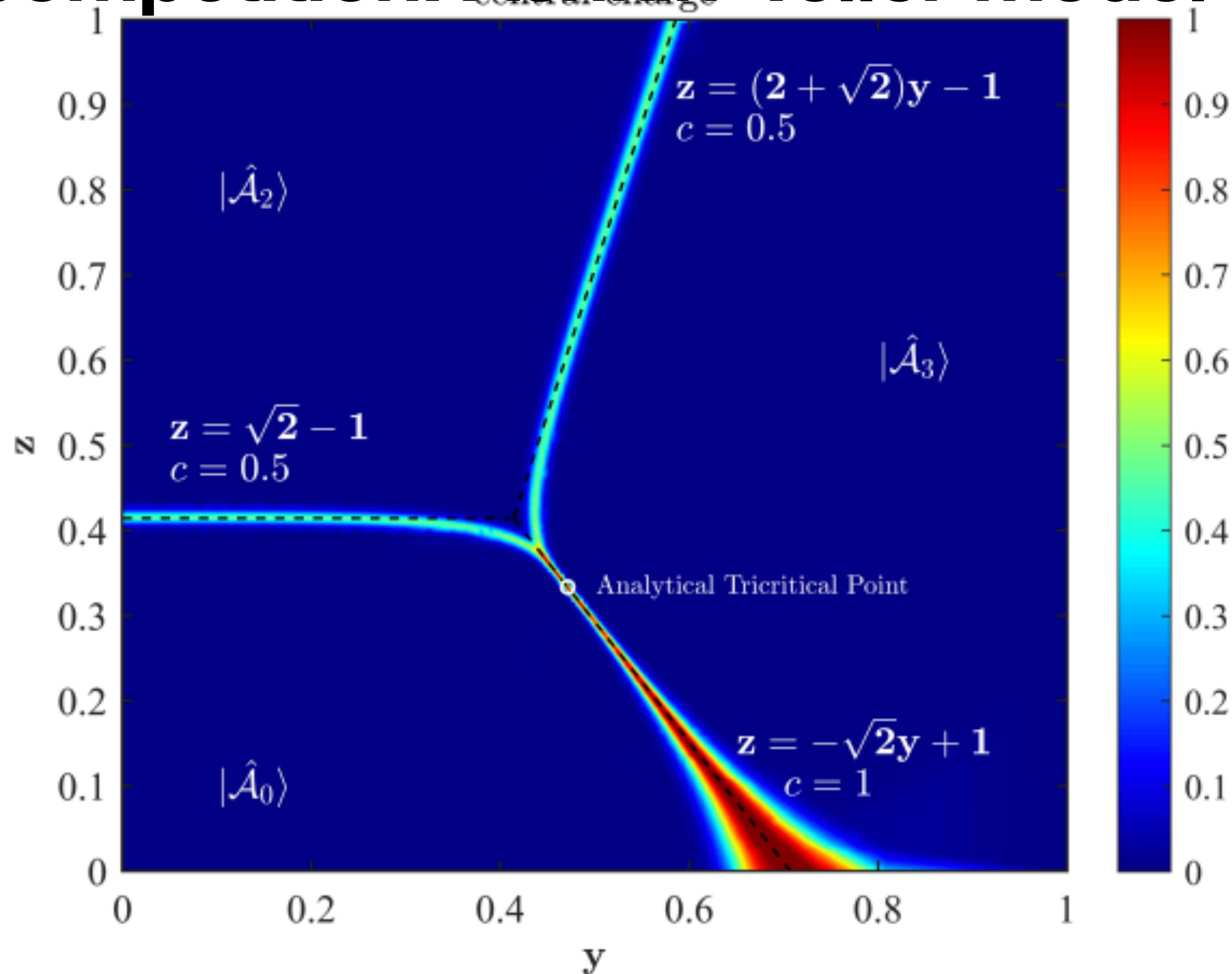
3 phase competition: Ashkin-Teller model from A5

phase boundary

- The module

Note that this

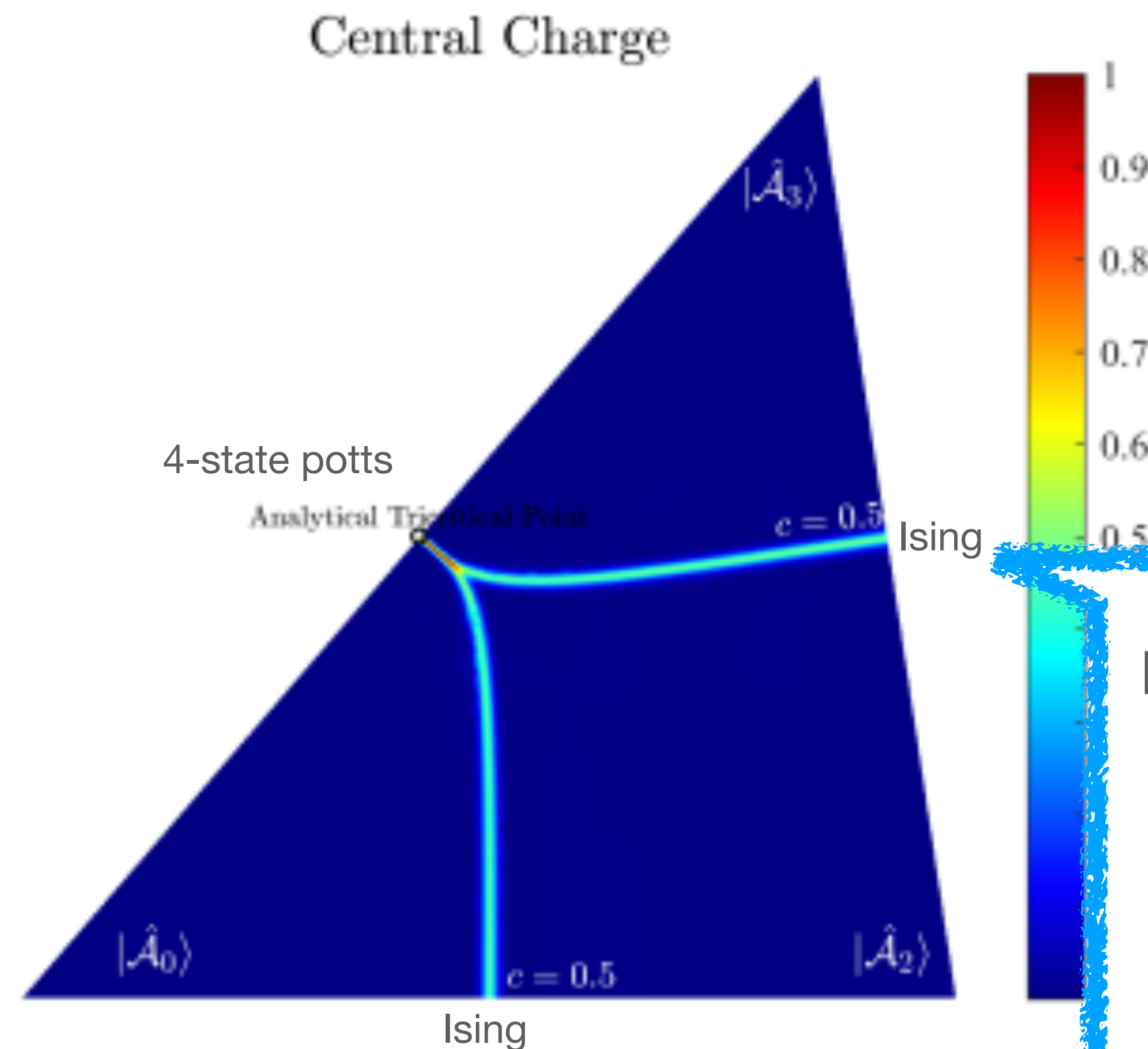
$nM = 3$, $Dc =$
second order.



3 phase competition: Ashkin-Teller model from A5

phase boundaries and tri-critical point

Phase boundaries:



$$\langle C_{ij} | = \frac{1}{2} (\langle \hat{\mathcal{A}}_i | + \langle \hat{\mathcal{A}}_j |)$$

$$\langle C_{ij}(p) | = \langle C_{ij} | + p \left(\langle \hat{\mathcal{A}}_k | - \frac{\langle \hat{\mathcal{A}}_k | B_{ij} \rangle}{\langle B_{ij} | B_{ij} \rangle} \langle B_{ij} | \right)$$

How do we understand which CFT would show up here? Particularly they seem to preserve different amount of (non)-invertible symmetries. How do we determine what is the *minimal* amount of symmetries preserved?

Novel Examples - Haagerup TQFT

Haagerup Model

$$1, \quad \alpha, \quad \alpha^2, \quad \rho, \quad \alpha\rho, \quad \alpha^2\rho,$$

The fusion rules are

1	α	α^2	ρ	$\alpha\rho$	$\alpha^2\rho$
α	α^2	1	$\alpha\rho$	$\alpha^2\rho$	ρ
α^2	1	α	$\alpha^2\rho$	ρ	$\alpha\rho$
ρ	$\alpha^2\rho$	$\alpha\rho$	$1 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$\alpha^2 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$\alpha \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$
$\alpha\rho$	ρ	$\alpha^2\rho$	$\alpha \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$1 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$\alpha^2 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$
$\alpha^2\rho$	$\alpha\rho$	ρ	$\alpha^2 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$\alpha \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$	$1 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$

$$d_1 = d_\alpha = d_{\alpha^2} = 1, \quad d_\rho = d_{\alpha\rho} = d_{\alpha^2\rho} = \frac{3 + \sqrt{13}}{2}.$$

$$L_0 = 1 \oplus \pi_1 \oplus 2\pi_2$$

$$L_1 = 1 \oplus \pi_1 \oplus 2\sigma_1$$

$$L_2 = 1 \oplus \pi_1 \oplus \pi_2 \oplus \sigma_1.$$

$$\mathcal{A}_0 = 1$$

$$\mathcal{A}_1 = \rho \otimes \mathcal{A}_0 \otimes \rho = 1 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho$$

$$\mathcal{A}_2 = 1 \oplus \alpha \oplus \alpha^2$$

$$\mathcal{A}_3 = \rho \otimes \mathcal{A}_2 \otimes \rho = (1 \oplus \rho \oplus \alpha\rho \oplus \alpha^2\rho) \otimes (1 \oplus \alpha \oplus \alpha^2).$$

$$\mathcal{A}_4 = 1 \oplus \rho \oplus \alpha\rho,$$

$$\mathcal{A}_5 = \alpha \otimes \mathcal{A}_4 \otimes \alpha^2 = 1 \oplus \alpha\rho \oplus \alpha^2\rho,$$

$$\mathcal{A}_6 = \alpha^2 \otimes \mathcal{A}_4 \otimes \alpha = 1 \oplus \rho \oplus \alpha^2\rho.$$

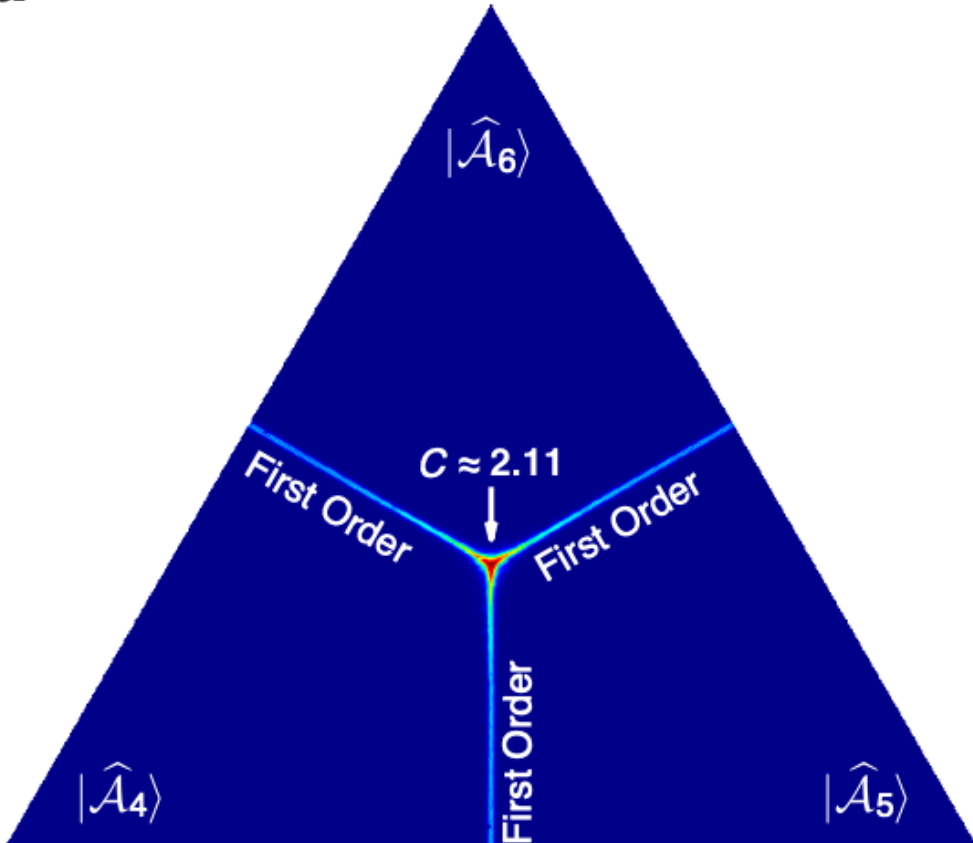
$$L_0 : \mathcal{A}_0, \mathcal{A}_1$$

$$L_1 : \mathcal{A}_2, \mathcal{A}_3$$

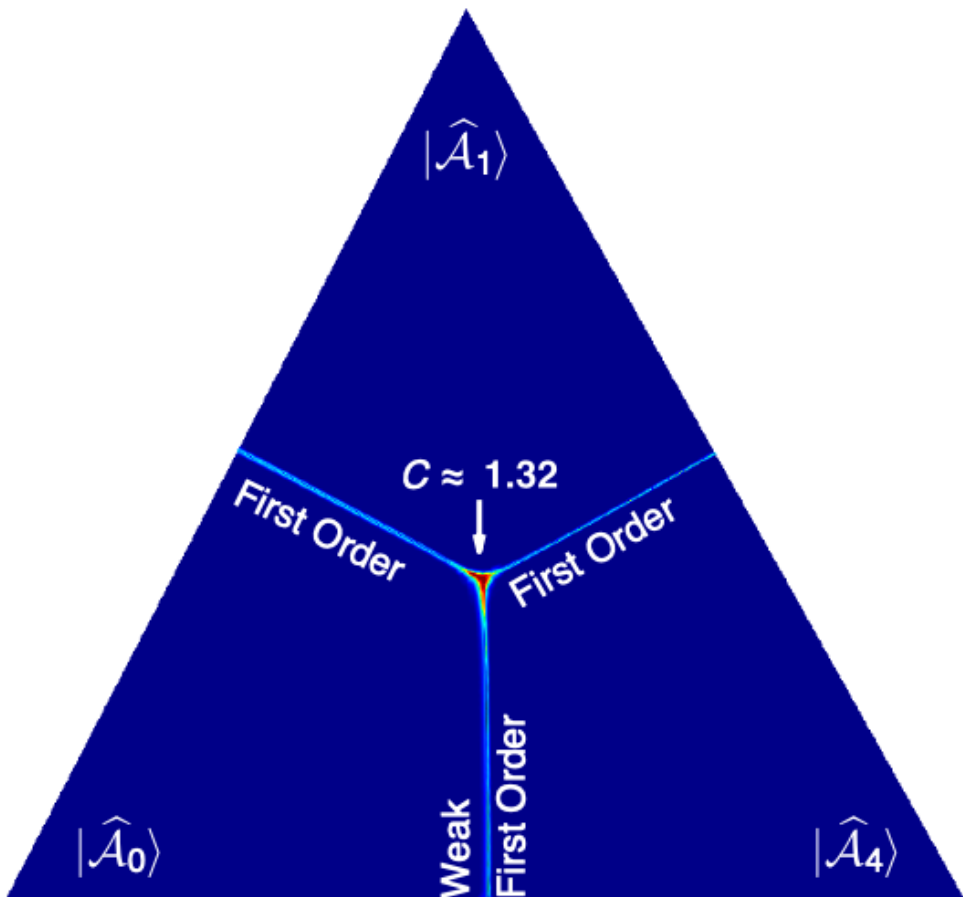
$$L_2 : \mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6.$$

Haagerup Model

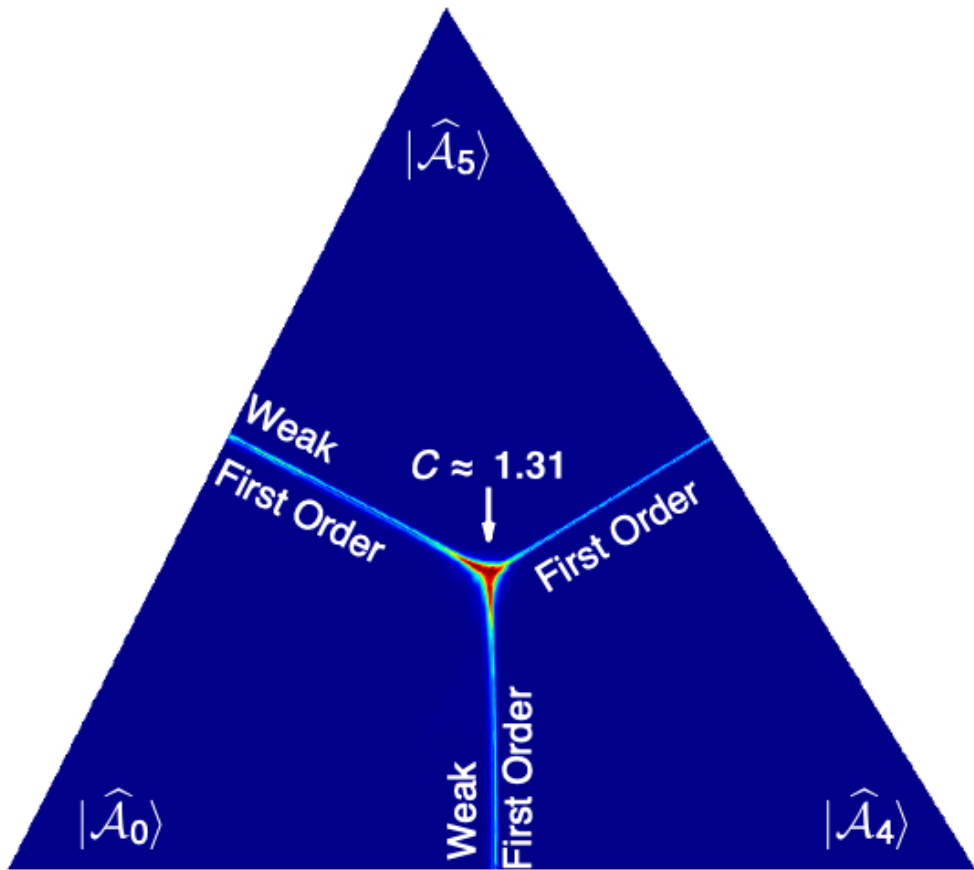
a



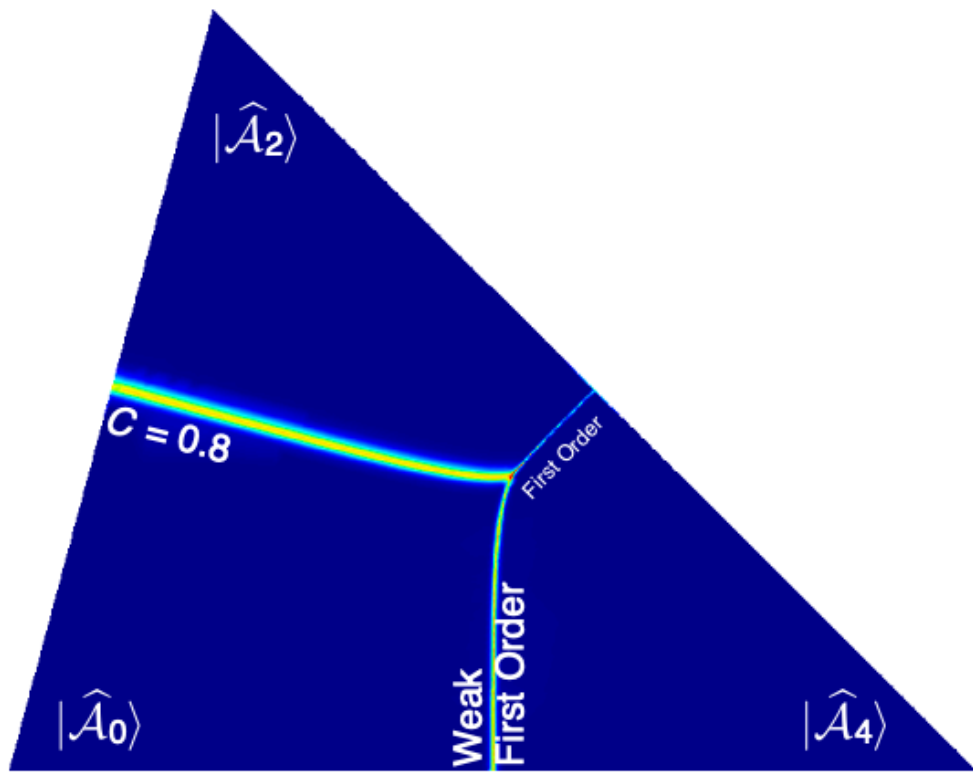
b



c



d



Haagerup Model - summary of results

H_3	$\mathcal{A}_0, \mathcal{A}_1$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	3	First	—	—	—
H_3	$\mathcal{A}_0, \mathcal{A}_2$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	2	Cont.	$\mathcal{Z}(S_3)$	0.801	0.013
H_3	$\mathcal{A}_0, \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	2	Weak	—	1.3	—
H_3	$\mathcal{A}_1, \mathcal{A}_2$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	5	First	—	—	—
H_3	$\mathcal{A}_1, \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	5	First	—	—	—
H_3	$\mathcal{A}_2, \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	4	First	—	—	—
H_3	$\mathcal{A}_{(4,5,6)}, \mathcal{A}_{(5,6,4)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	4	First	—	—	—
H_3	$\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	5	Cont.	$\mathcal{Z}(H_3)$	0.793	0.014
H_3	$\mathcal{A}_0, \mathcal{A}_2, \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	4	Cont.	$\mathcal{Z}(H_3)$	0.8047	0.0077
H_3	$\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	5	Cont.	$\mathcal{Z}(H_3)$	1.3154	0.0065
H_3	$\mathcal{A}_0, \mathcal{A}_{(4,5,6)}, \mathcal{A}_{(5,6,4)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	4	Cont.	$\mathcal{Z}(H_3)$	1.3141	0.0042
H_3	$\mathcal{A}_1, \mathcal{A}_{(4,5,6)}, \mathcal{A}_{(5,6,4)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	7	First	—	—	—
H_3	$\mathcal{A}_2, \mathcal{A}_{(4,5,6)}, \mathcal{A}_{(5,6,4)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	6	First	—	—	—
H_3	$\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_{(4,5,6)}$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	7	First	—	—	—
H_3	$\mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6$	$\rho \oplus \alpha\rho \oplus \alpha^2\rho$	6	Cont.	$\mathcal{Z}(H_3)$	2.106	0.022

Summary and Outlook

- AdS/CFT seems to coincide with the topological holographic principle, at least in low dimensions!
- Topological holographic principle — connects the discrete and the continuous world
- We produce a systematic way of constructing *critical* lattice models based on the strange correlator
- This gives the Landau paradigm for generalised symmetries a more precise handle to construct effective field theories — here, the order parameter is precisely the anyon creation operator defined on some choice of unit cells

To appear :

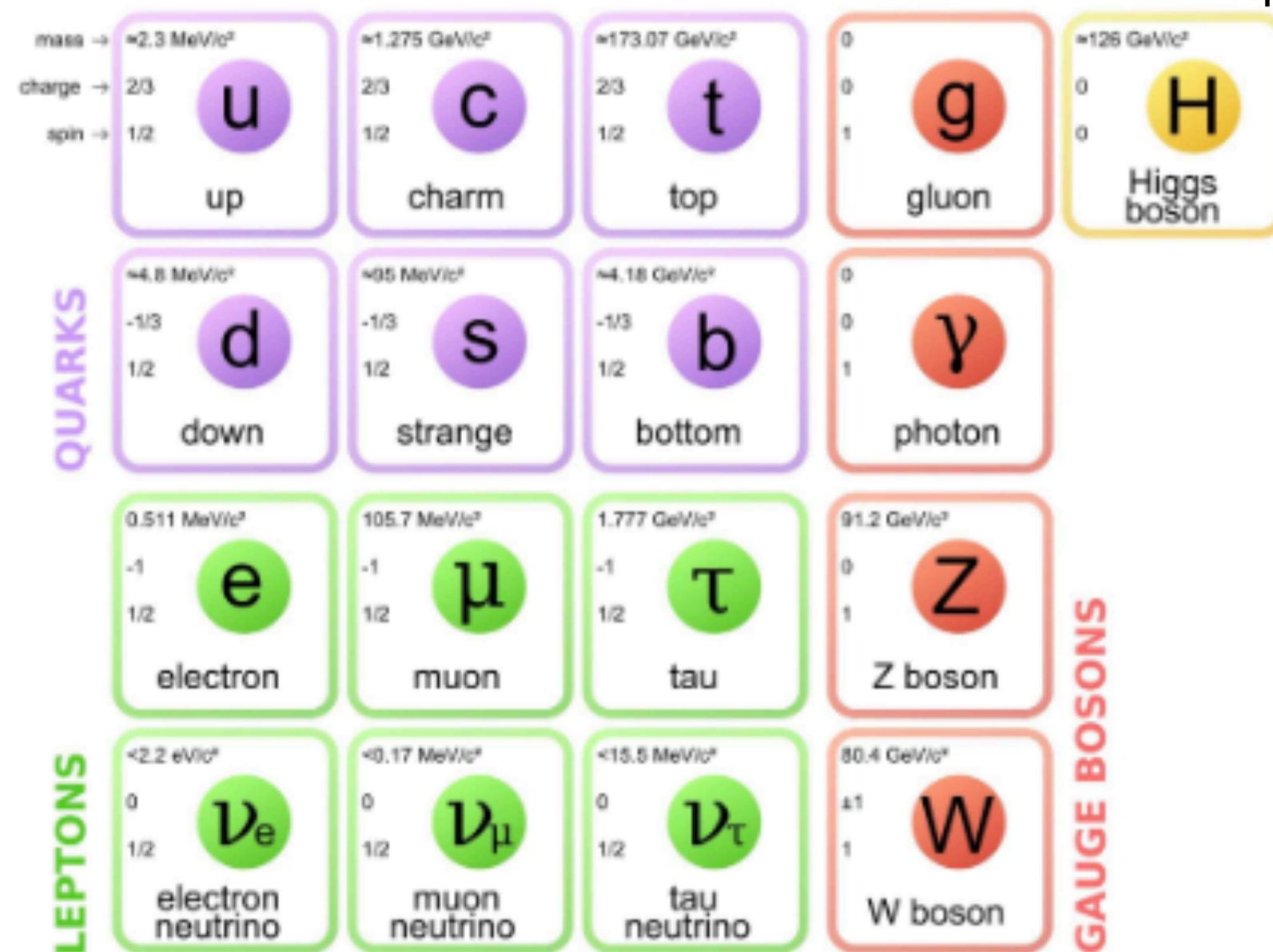
- 1) Generalisation to Fermionic CFT in 2D - recovering global spin structures from local pieces. 2026.08xxx
- 2) More generalised 2D CFT factory —
 - More general lattice. 2026.xxxxxx
 - Some new competitions between generalised topological boundaries 2026.xxxxxx

Future: Strongly Interacting Field Theory ???

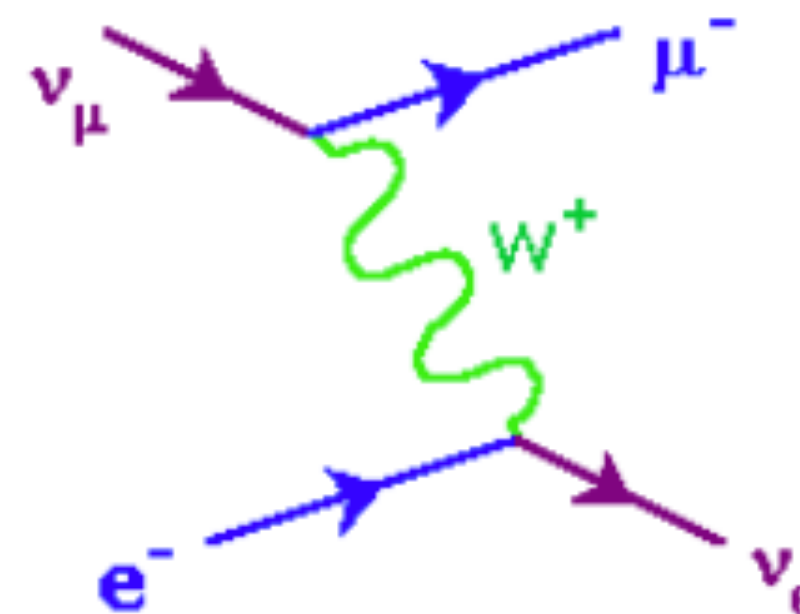
- Higher dimensions
- Include fermions and introduce chirality

Quantum Field Theory and the Electroweak Standard Model - Arbuzov, A.B. - arXiv:1801.05670

pictures courtesy Cern



Particle content of the Standard Model.
Courtesy to Wikipedia: 'Standard Model of Elementary Particles' by MissMJ ---
Own work by uploader, PBS NOVA, Fermilab, Office of Science, United States Department of Energy, Particle Data Group.



Feynman diagram for electron-neutrino scattering with W boson exchange.

But the success is mostly in the high energy limit, when the theory approaches free theory— where perturbation based on Feynman diagrams have been extremely successful.

Thank you!