天津大学 量子交叉研究中心
Center for Joint Quantum Studies, Tianjin University
http://cjqs.tju.edu.cn

# Degenerated Black Rings in $D = 5$ Supergravity

H. Lü (吕宏)

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## Strings and Related Topics

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arxiv: 2106.12632 with Shi-Fa Guo (郭世发) and Yi Pang (庞毅)

# Outline

- Introduction
- $D = 5$ supergravity
- Degenerated black rings
- Related to extremal black holes
- Related to smooth solitons and time machines
- Conclusions

# Introduction

If black holes are particles of GR, its spectrum is remarkably simple, typically specified by $(M, J, Q)$.

In $D = 4$ Einstein-Maxwell gravity, any black object that is asymptotic to Minkowski spacetime (with no direction singled out) is necessarily a black **hole**, i.e. its horizon topology is an $S^2$.

However, the spacetime structure can be far richer in higher dimensions.

# Theoretical reasons

In Einstein theory, only the Ricci tensor $R_{\mu\nu}$ is constrained by his eponymous equations of motion, but the spacetime is characterized by its Riemann tensor, or by Ricci and Weyl tensors together.

However,

| Dimensions | $R_{\mu\nu}$ | $W_{\mu\nu\rho\sigma}$ | $R_{\mu\nu\rho\sigma}$ |
|---|---|---|---|
| 3 | 6 | 0 | 6 |
| 4 | 10 | 10 | 20 |
| 5 | 15 | 35 | 50 |
| 6 | 21 | 84 | 105 |
| $d$ | $\frac{d(d+1)}{2}$ | $\frac{(d-3)d(d+1)(d+2)}{12}$ | $\frac{d^2(d^2-1)}{12}$ |

Matter's ability to constrain the spacetime via the Einstein equation becomes weaker in higher dimensions.

But how can new topology arise?

# An explicit realization in $D=5$

In $D=5$ Minkowski spacetimes $ds_5^2 = -dt^2 + dr^2 + r^2 d\Omega_3^2$, the round 3-sphere metric can be expressed in terms of three Euler angles $(\theta, \psi, \phi)$, with $\Delta\psi = 4\pi$:

$$d\Omega_3^2 = \frac{1}{4}\Big(\sigma_3^2 + (d\theta^2 + \sin^2\theta d\phi^2)\Big)\,, \qquad \sigma_3 = d\psi + \cos\theta d\phi\,.$$

The black hole horizon remains a 3-sphere, albeit it can be squashed, e.g.

$$ds_5^2 = \rho^n d\tau^2 + \frac{d\rho^2}{\rho^n} + \frac{1}{4} r_0^2 \Big(a(\sigma_3 + \omega\rho d\tau)^2 + (d\theta^2 + \sin^2\theta d\phi^2)\Big)\,,$$

where $n = 1, 2$. However, there can be a new possibility where the $\psi$ untwists with the $S^2$, e.g.

$$ds_5^2 = \rho^n d\tau^2 + \frac{d\rho^2}{\rho^n} + a(d\psi + \omega\rho d\tau)^2 + b(d\theta^2 + \sin^2\theta d\phi^2)\,.$$

The horizon is then $S^2 \times S^1$, a ring rather than a hole. (In this talk, we consider $a \to 0$.)

Black rings [Emparan, Reall hep-th/0110260; Pomeransky and R.A. Senkov hep-th/ 0612005] are actually rather complicated.

# Two equal angular momenta

Obviously, few new things can arise from spherically-symmetric spacetimes, but general rotations can be complicated in higher dimensions.

In $D = 5$, there are four space dimensions, which can support two orthogonal rotations. When two rotations are equal, the metric becomes significantly simpler:

$$ds_5^2 = -\frac{h(r)}{W(r)}dt^2 + \frac{dr^2}{f(r)} + \frac{1}{4}r^2 W(r)(\sigma_3 + \omega(r)dt)^2 + \frac{1}{4}r^2(d\theta^2 + \sin^2\theta d\phi^2)\,.$$

It is cohomogeneity-one metric, with four metric functions depending on $r$ only.

The most general ansatz with isometry of $SU(2) \times U(1) \times$time translation invariance.

# Horizon and VLS

We shall further constrain the system to a subset with $h = f$. We thus consider the following metric as an example:

$$ds_5^2 = -\frac{f}{W}dt^2 + \frac{dr^2}{f} + \frac{1}{4}r^2 W(d\psi + \cos\theta d\phi + \omega dt)^2 + \frac{1}{4}r^2(d\theta^2 + \sin^2\theta d\phi^2)\,.$$

Typically, the spacetime has a curvature power-law singularity at $r = 0$, which can be shielded by a horizon at $r_+$ with $f(r_+) = 0$. We therefore assume that $r_+$ exists.

$r_+$ can be an event horizon or a Killing horizon, depending on the value of $W(r_+)$.

Consider $W(r_L) = 0$, then $r_L$ is not a coordinate singularity, but the velocity of light surface (VLS). It is a hypersurface inside which the periodic coordinate $\psi$ becomes timelike.

CGLP, hep-th/0504080.

# Classification of spacetimes

- $r_+ > r_L$: black holes
- $r_+ < r_L$: smooth time machines
- $r_+ = r_L$: solitons

These are discussed in literature e.g. CGLP, hep-th/0504080, and in this talk, we shall consider the fourth possibility

$$f(r_+) = f'(r_+) = W(r_+) = 0\,.$$

We find that the spacetime geometry can be best described as a degenerate black ring (DBR).

Time machine vs. Godel universe: both has naked CTCs, but

- Time machine: CTC is inside $r_L$;
- Godel Universe: CTC is outside $r_L$.

Analogous to the distinction between cosmologic horizon and black hole horizon.

# Why supergravities?

Static and spherically symmetric solutions are trivial to find; many of their global structures can be analysed even without exact solutions.

Rotating geometries are much more complicated and exact solutions beyond four dimensions are illusive.

The situation changes with supergravity, which has enhanced global symmetry and allows one to construct new charged black holes from the neutral ones by solution generating techniques. e.g. Sen, [hep-th/9204046]; Cvetic, Youm, [hep-th/9603100], ...

What is surprising that even though the global symmetry breaks down in gauged supergravities, rotating geometries can still be explicit constructed. e.g. [CCLP, hep-th/0506029; Wu, 1108.4159, ...]

Supersymmetry depends on dimensions and in this talk, we consider minimal (gauged) supergravity in five dimensions.

# Minimal $D = 5$ supergravity

The bosonic sector involves only the metric and graviphoton, with the Lagrangian

$$\mathcal{L} = (R + 12g^2){*\mathbf{1}} - \frac{1}{2}{*F} \wedge F + \frac{1}{3\sqrt{3}} F \wedge F \wedge A \,,$$

where $g$ is the gauge coupling of the gravitino $\Psi_\mu$. The $FFA$ term is crucial for the enhanced global symmetry.

The theory admits global AdS spacetime with radius $\ell = 1/g$:

$$ds_5^2 = -(g^2 r^2 + 1)dt^2 + \frac{dr^2}{g^2 r^2 + 1} + r^2 d\Omega_3^2 \,.$$

We are looking for solutions that is asymptotic to the global AdS.

When $g = 0$, the theory is ungauged supergravity and the vacuum is Minkowskian.

# General local solutions

Here is a general class of solutions CLP, [hep-th/0406196]

$$
\begin{aligned}
ds^2 &= -\frac{f}{W}dt^2 + \frac{dr^2}{f} + \frac{1}{4}r^2 W(\sigma_3 + \omega dt)^2 + \frac{1}{4}r^2 d\Omega_2^2\,, \\
A &= -\frac{\sqrt{3}aq}{2r^2}\left(\sigma_3 - \frac{2}{a}dt\right),
\end{aligned}
$$

where

$$
\begin{aligned}
W &= 1 + \frac{2a^2(\mu+q)}{r^4} - \frac{a^2q^2}{r^6}\,, \qquad \omega = \frac{2a\left(q^2 - qr^2 - 2\mu r^2\right)}{r^6 W}\,, \\
f &= W\left(1 - (1 - a^2g^2)\frac{r^2}{a^2}\right) + \frac{\left(r^4 + a^2q\right)^2}{a^2r^6}\,.
\end{aligned}
$$

Three parameters $(\mu, a, q)$ parameterize $(M, J, Q)$

$$
M = \frac{1}{4}\pi\left(3\mu + g^2a^2(\mu+q)\right), \quad J = \frac{1}{4}\pi a(2\mu + q)\,, \quad Q = \frac{1}{4}\sqrt{3}\pi q\,.
$$

The solutions describe black holes in general.

# Degenerated black ring

For simplicity, we consider $g = 0$, and the solution is asymptotic to $\text{Mink}_5$.

The condition $f(r_+) = f'(r_+) = W(r_+) = 0$ implies $\mu = -q = a^2 = r_+^2$. The theory depends only one parameter with

$$M = -\sqrt{3}Q = \frac{3\sqrt[3]{\pi}}{2^{2/3}} J^{\frac{2}{3}}\,, \qquad J = \frac{1}{8}\pi a^3\,.$$

The solution is remarkably simple:

$$\begin{aligned} ds^2 &= \frac{r^4 dr^2}{(r^2 - a^2)^2} + \frac{1}{4} r^2 d\Omega_2^2 \\ &\quad -\Big(1 - \frac{r^2}{a^2}\Big) dt^2 + \frac{r^4 - a^4}{a^2 r^2} dt \tilde{\sigma}_3 + \frac{r^6 - a^6}{4a^2 r^4} \tilde{\sigma}_3^2\,, \\ A &= \frac{\sqrt{3} a^3}{2r^2} \tilde{\sigma}_3\,, \qquad \tilde{\sigma}_3 = \sigma_3 - \frac{2}{a} dt\,. \end{aligned}$$

The metric has a horizon at $r = a$.

# Near horizon geometry

It is instructive to define a new radial coordinate $z$, such that $r = a(1+z^2)$. As $z$ approaches zero, the solution becomes

$$ds^2 \sim \frac{a^2 dz^2}{z^2} + z^2 \left( -\frac{2}{3} dt^2 + \frac{3}{2} a^2 \sigma_3^2 \right) + \frac{1}{4} a^2 d\Omega_2^2 \,.$$

It is locally $\mathrm{AdS}_3 \times S^2$.

As $z \to 0$, the connection in $\sigma_3 = d\psi + \cos\theta d\phi$ untwists and horizon becomes $S^2 \times S^1$ with the radius of $S^1$ shrinking to zero.

Thus we obtain degenerate black ring.

Intriguingly, an asymptotic observer will find one spatial dimension "disappears" on the horizon.

# The decoupling limit

The near-horizon geometry can be magnified and extracted as a solution on its own, under the decoupling limit

$$r = a(1+\lambda z^2)\,,\quad t = \frac{a\tau}{\sqrt{\lambda}}\,,\quad \tilde{\psi} = \frac{1}{\sqrt{\lambda}}\Big(\widehat{\psi} + \frac{2}{3}\tau\Big)\,,\quad \lambda \to 0\,.$$

In this limit, the solution becomes

$$\begin{aligned} ds^2 &= a^2\Big(\frac{dz^2}{z^2} + \frac{1}{6}z^2(-4d\tau^2 + 9d\widehat{\psi}^2) + \frac{1}{4}(d\theta^2 + \sin^2\theta d\phi^2)\Big)\,,\\ A &= \frac{1}{2}\sqrt{3}a\cos\theta d\phi\,. \end{aligned}$$

This is simply $\mathrm{AdS}_3 \times S^2$, the near-horizon geometry of magnetic string.

But decoupling limit is not the same globally as the near-horizon geometry.

DBRs, electrically charged, are different from magnetic strings.

# Almost regular

If the fibre coordinate $\psi$ is an infinite line, such as magnetic string, $AdS_3$ horizon $z \to 0$ is geodesically complete. The metric is an even function of $z$, since $r = a(1 + z^2)$. The inside of the horizon is then isomorphic to the outside.

Travelling inside the horizon arrives the same spacetime structure of the outside, with another horizon inside which is isomorphic to the outside again, and so on. The $r = 0$ singularity is totally outside the manifold. Gibbons, Horowitz, Townsend [hep-th/9410073]

However, in our DBR, $\psi$ is periodic, and hence the translational symmetry becomes singular at $z = 0$ and hence the geodesic is not complete.

Nevertheless, the $r = 0$ singularity can be avoided by introducing an orbit singularity on the horizon.

We thus have asymptotically $Mink_5$ DBR that is almost regular.

# AdS DBRs

When $g = 1/\ell$ is not equal to zero, the solution is asymptotic to AdS. The DRR condition

$$f(r_+) = f'(r_+) = W(r_+) = 0\,,$$

leads to

$$\begin{aligned} \mu &= \frac{1}{2} r_+^2 \left(2 + 3g^2 r_+^2 + g^4 r_+^4\right)\,, \\ q &= -r_+^2 \left(g^2 r_+^2 + 1\right)\,, \qquad a = \frac{r_+}{\sqrt{1 + g^2 r_+^2}}\,. \end{aligned}$$

We obtain the AdS DBR solution, whose horizon geometry is analogous.

There were no-go theorems on AdS black rings:

- No susy AdS black ring. Grover, Gutowski, Sabra, 1306.0017
- No extremal AdS black ring. Khuri, Woolgar, 1708.03627

Ours are not supersymmetric, and they are degenerate.

# DBRs from two different routes

How is our DBR sitting among the general class of solutions?

We can impose the DBR condition $f(r_+) = f'(r_+) = W(r_+) = 0$ in two steps. They lead to two pathes:

- via extremal black holes: first require $f(r_+) = f'(r_+)$ that leads to extremal black holes, followed by $W(r_+) = 0$.
- via solitons: first impose $f(r_+) = W(r_+) = 0$ that leads to smooth solitons, followed by imposing $f'(r_+) = 0$.

# Two branches of extremal black holes

The condition for extremality $f(r_+) = 0 = f'(r_+)$ implies that

$$\text{Case 1:} \qquad \mu = r_+^2\,, \qquad q = -r_+^2\,,$$
$$\text{Case 2:} \qquad \mu = r_+^2\,, \qquad q = -2a^2 + r_+^2\,.$$

- Case 1: (susy) BMPV black hole: [hep-th/9602065]

$$M = -\sqrt{3}Q = \frac{3\pi r_+^2}{4}\,, \qquad J = \frac{1}{4}\pi a r_+^2\,.$$

- Case 2: non-susy extremal black hole

$$M = \frac{3}{4}\pi r_+^2\,, \qquad J = \frac{1}{4}\pi a\left(3r_+^2 - 2a^2\right)\,, \qquad Q = \frac{1}{4}\sqrt{3}\pi\left(r_+^2 - 2a^2\right)\,.$$

or

$$8M^3 - 18MQ^2 - 6\sqrt{3}Q^3 - 27\pi J^2 = 0\,.$$

When $r_+ = a$, or $Q \sim -J^{2/3}$, the two extremal black holes become identically the same, the DBR.

# Global discontinuity

The two extremal black holes meet at $r_+ = a$, where both reduce to the same DBR, but their Gibbs' free energies are not continuous. Define Legendre transformations

$$G_1 = M - 2\Omega_+ J\,, \quad G_2 = M - 2\Omega_+ J - \Phi Q\,, \quad G_3 = M - \Phi Q\,.$$

- Case 1:

$$G_1 = \frac{3}{4}\pi r_+^2\,, \qquad G_2 = G_3 = 0\,.$$

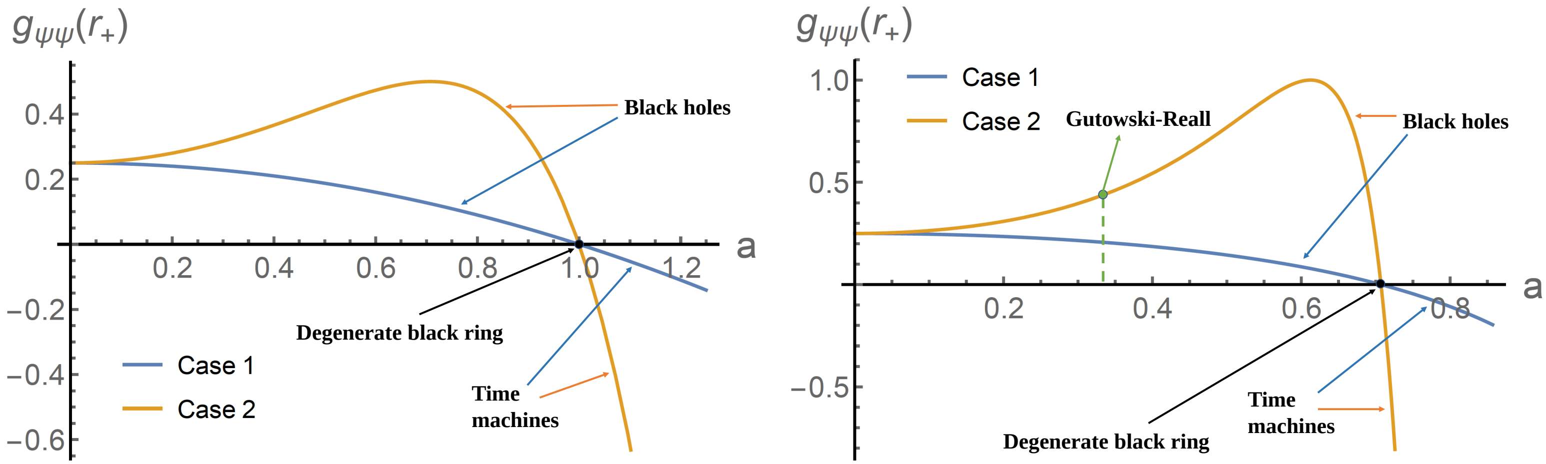
- Case 2:

$$G_1 = \frac{\pi\left(8a^4 - 6a^2 r_+^2 + 3r_+^4\right)}{4\left(r_+^2 + 2a^2\right)}, \quad G_2 = \frac{\pi a^2\left(3r_+^2 - 2a^2\right)}{2(r_+^2 + 2a^2)}\,,$$

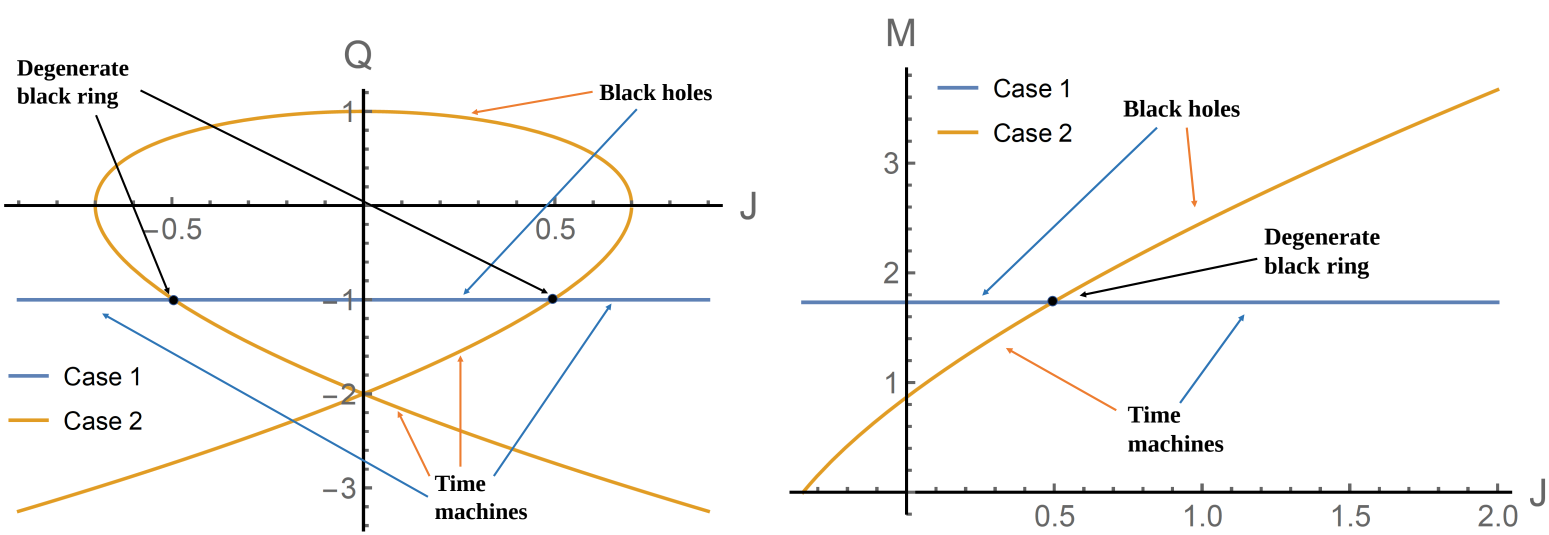
$$G_3 = \frac{3\pi a^2\left(3r_+^2 - 2a^2\right)}{2\left(r_+^2 + 2a^2\right)}\,.$$

When $r_+ = a$, the Case 2 gives $G_{1,2,3} = \left\{\frac{5\pi a^2}{12}, \frac{\pi a^2}{6}, \frac{\pi a^2}{2}\right\}$.

# Illustrations




Geometric properties, with $a = 1$ and left: $g = 0$; right: $g = 1$




Conserved charges, with $g = 0$ and left $M = \sqrt{3}$, right $Q = -1$.

These time machines were called repulsons in [Gibbons, Herdeiro,hep-th/9906098]

# From solitons to DBR

We can now consider another route, by first consider $f(r_0) = W(r_0) = 0$. This condition implies that

$$\mu = \frac{r_0^4 \left(a^2 + r_0^2\right)}{2a^4}, \qquad q = -\frac{r_0^4}{a^2}.$$

Thus the solution appears to have two parameters:

$$Q = -\frac{\sqrt{3}\,\sqrt[3]{\pi}}{2^{2/3}} J^{2/3}, \qquad M = \frac{3J}{2a} + \frac{3\,\sqrt[3]{\pi} J^{2/3}}{2\ 2^{2/3}}.$$

Why is this a soliton? Recall

$$ds^2 = -\frac{f}{W} dt^2 + \frac{dr^2}{f} + \frac{1}{4} r^2 W (\sigma_3 + \omega dt)^2 + \frac{1}{4} r^2 d\Omega_2^2,$$

At $r = r_0$, it is not an event horizon, but a Killing horizon of bolt geometry $\mathbb{R}^2 \times S^2 \times$ time.

# Bolt geometry

Let $r = r_0 + \frac{1}{4}\rho^2$ with $\rho \to 0$, we have

$$ds^2 = -\eta dt^2 + \frac{a^2 r_0}{2\eta\left(2a^2 + r_0^2\right)}\left(d\rho^2 + \frac{1}{4}\kappa^2\rho^2\sigma_3^2\right) + \frac{1}{4}r_0^2 d\Omega_2^2\,,$$

$$\kappa = \frac{(2a^2 + r_0^2)\sqrt{\eta}}{a^2}\,, \qquad \eta \equiv 1 - \frac{r_0^2}{a^2} > 0\,.$$

For $\Delta\sigma_3 = 4\pi$, we must have either $\kappa = 0$ or $\kappa = 1$.

- $\kappa = 0$: the DBR we discussed earlier
- $\kappa = 1$: Asymptotic Mink$_5$ soliton with bolt geometry.
- $\kappa = 2$: Eguchi-Hanson instanton $\times$ time, with $Q = 0$.

For both DBR and soliton, $Q \sim -J^{2/3}$, but

$$\frac{M_{\text{soliton}}}{M_{\text{ring}}} = \cos\left(\frac{\pi}{9}\right) < 1\,.$$

# Conclusions

We gave a brief understanding why spacetime structures of black objects are much richer in higher dimensions.

We gave explicit DBR solutions in $D = 5$ minimal supergravity, where one spatial dimension (the size of the ring) literally disappears on the horizon.

These DBRs arose as the joining of two branches of extremal black holes, where there was a global discontinuity in black hole thermodynamics. (Have never seen this before. "zeroth" order phase transition?)

What are they good for? Not sure... **(Generalizing Einstein gravity so that only black objects are holes?)**

A possible application in the AdS/CFT:

$$M = c_{\frac{3}{3}} J + c_{\frac{2}{3}} J^{\frac{2}{3}} + c_{\frac{1}{3}} J^{\frac{1}{3}} + c_0 + \cdots ,$$

or

$$M = \tilde{c}_{\frac{3}{2}} Q^{\frac{3}{2}} + \tilde{c}_{\frac{2}{2}} Q + \tilde{c}_{\frac{1}{2}} Q^{\frac{1}{2}} + c_0 + \cdots .$$

These are universal structures and supergravity provides explicit coefficients; **can this be reproduced in $\mathbf{CFT_4}$?**