Page curve from Defect extremal surface

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arXiv:2012.07612，2105.09106 with Feiyu Deng and Jinwei Chu

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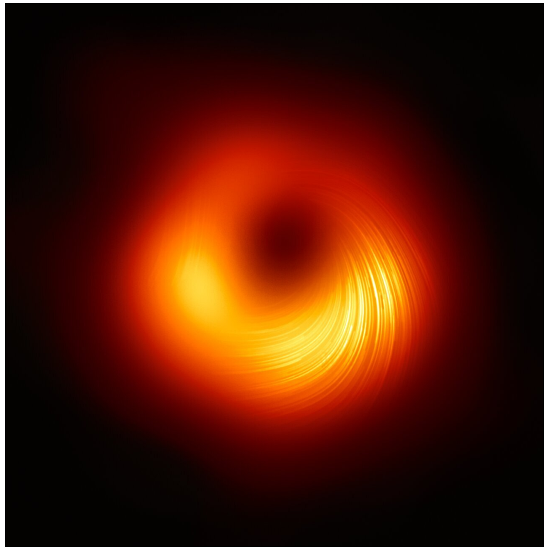
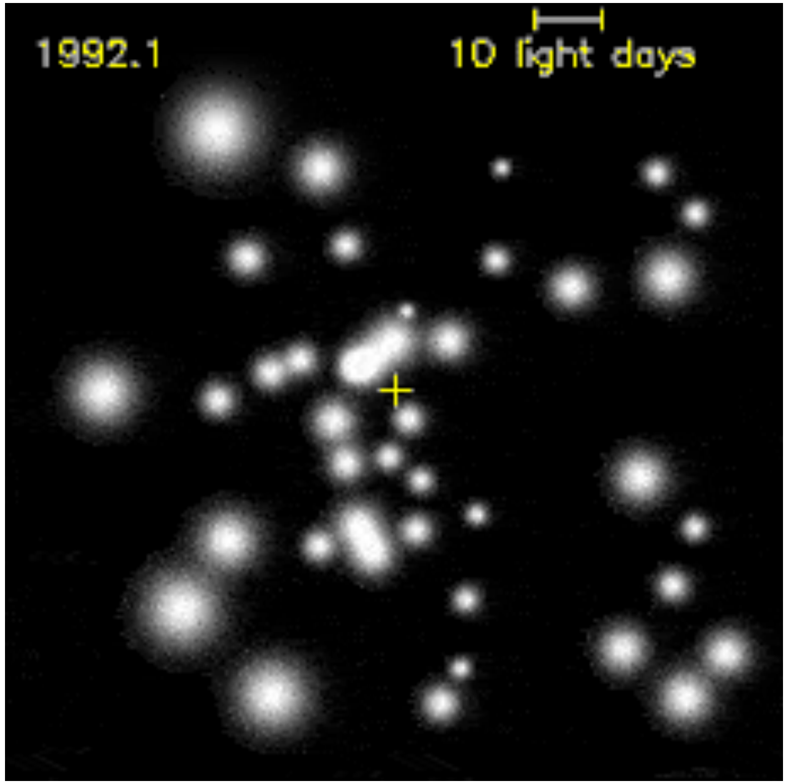
Black holes are there!!

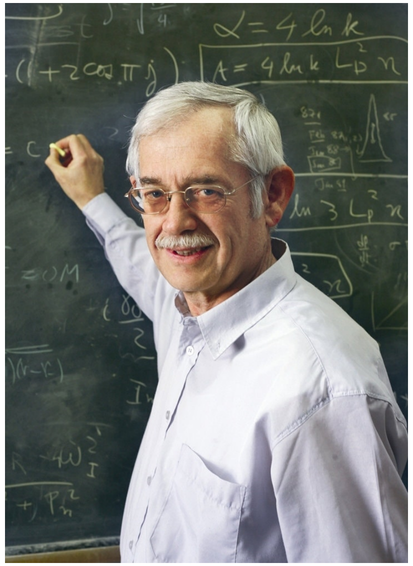


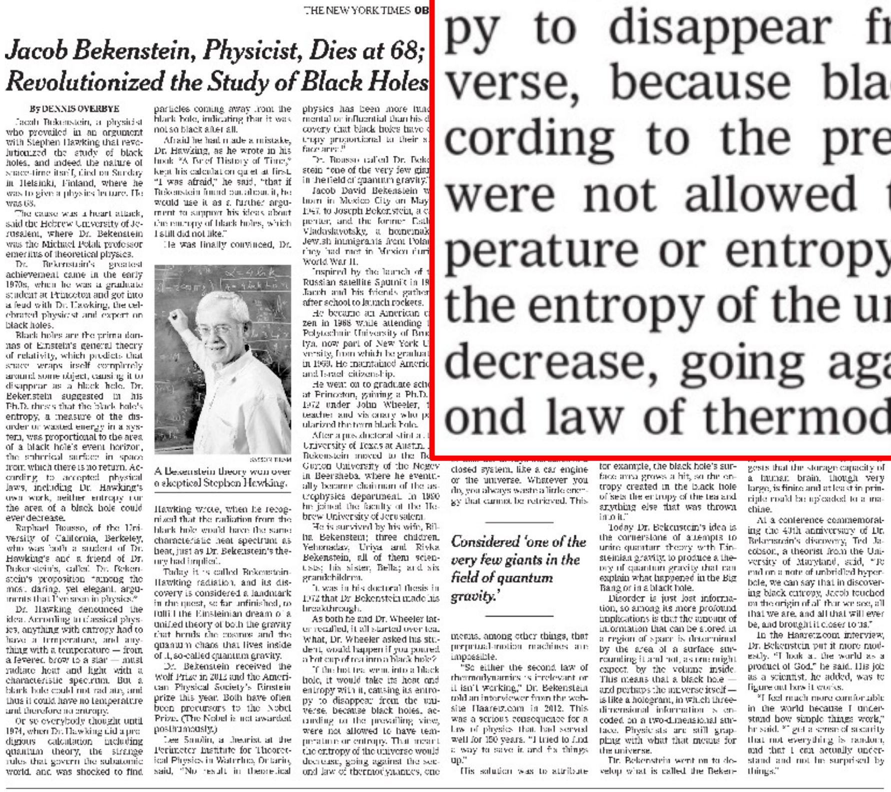


Is that all?

- As a classical solution of Einstein's equation, BH is described by 3 parameters
- A black hole will eat entropy, which will violate 2^{nd} law
- Consider two black holes form a bigger one, the horizon area increases
- It is likely black hole has entropy, proportional to its horizon area

Bekenstein's discovery




THE NEW YORK TIMES

Jacob Bekenstein, Physicist, Dies at 68; Revolutionized the Study of Black Holes

By DENNIS OVERBYE

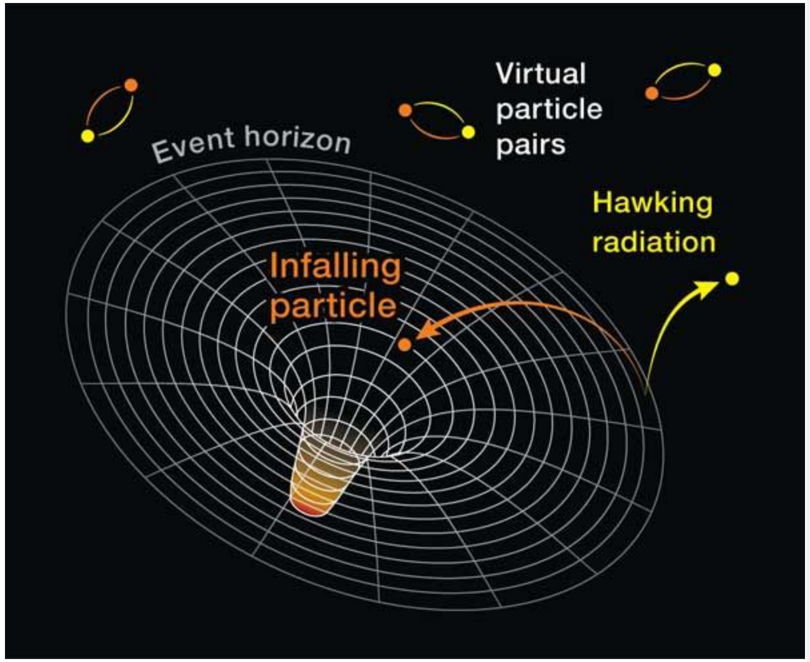
A Bekenstein theory won over a skeptical Stephen Hawking.

Considered 'one of the very few giants in the field of quantum gravity.'

As both he and Dr. Wheeler later recalled, it all started over tea. What, Dr. Wheeler asked his student, would happen if you poured a hot cup of tea into a black hole?

If the hot tea went into a black hole, it would take its heat and entropy with it, causing its entropy to disappear from the universe, because black holes, according to the prevailing view, were not allowed to have temperature or entropy. That meant the entropy of the universe would decrease, going against the second law of thermodynamics, one

Hawking radiation



$$S_{\mathrm{BH}} = \frac{Ac^3}{4\hbar G}$$

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G k_{\mathrm{B}} M}$$

Introduction

- Quantum gravity is the key to understand the origin of our universe

- A simpler object involving quantum gravity is black hole. They have a temperature that leads to Hawking radiation.

- Black holes also have entropy, given by the Area of the horizons.

- The question is whether black holes behave like ordinary quantum systems. People believe they do (string theory, AdS/CFT) but do not know how.

- Importantly, there is a paradox if they do: consider a black hole formed by a pure state, after evaporation it becomes a thermal state (according to Hawking)-> information is lost

- You may argue that strange things can happen at the end of the evaporation. But the paradox already shows up near the middle age of BH.

- To understand this, we first introduce 2 different notions of entropy: fine-grained entropy and coarse-grained entropy.

Fine-grained < coarse-grained [Review: arXiv:2006.06872]

- 1st : Fine-grained entropy is simply the von Neumann entropy. It is Shannon's entropy with distribution replaced by density matrix. It is invariant under unitary time evolution.

- 2nd: Coarse-grained entropy is defined as follows. We only measure simple observables A_i. And consider all possible density matrices which give the same result as our system.

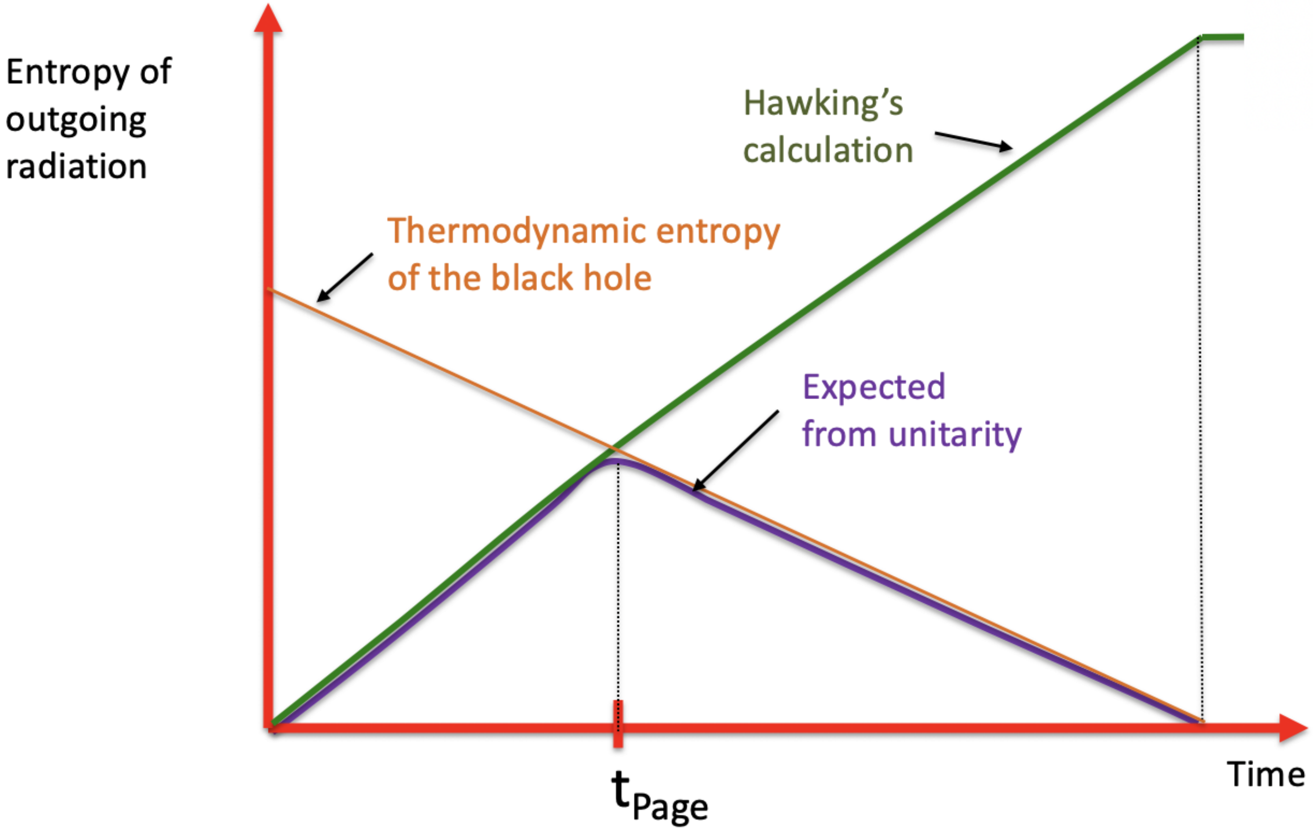
$$Tr[\tilde{\rho} A_i] = Tr[\rho A_i]$$

We then choose the maximal von Neumann entropy over all possible density matrices $S(\tilde{\rho})$. It increases under unitary time evolution. -> entropy in thermodynamics.

Information paradox

- Bekenstein-Hawking entropy is coarse-grained entropy.
- Hawking radiation comes from separating entangled outgoing Hawking quanta and interior partner.
- As the entropy of radiation gets bigger and bigger, we run into trouble because, the entangled partners in black hole should have the same entropy (for an initial pure state), which exceeds the horizon entropy.
- In fact, the constantly increasing result was made by Hawking. Page suggested that the outgoing radiation entropy should follow Page curve

Page curve



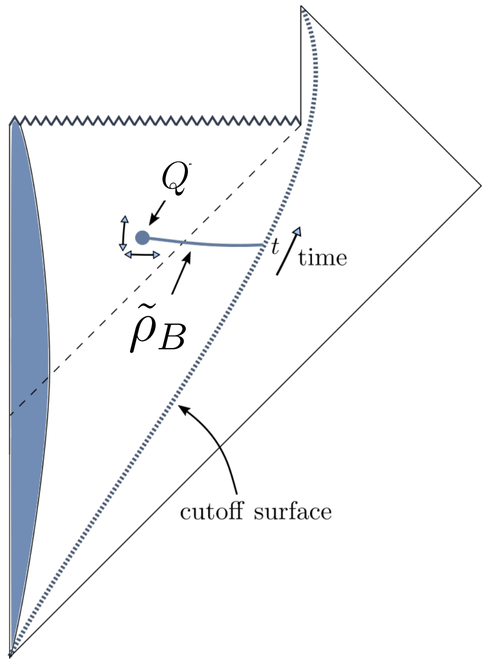
[Fig from arXiv:2006.06872]

How to derive Page curve?

Entropy formula for BH [Penington; Almheiri-Engelhardt-Marolf-Maxfield, 2019]

- The fine-grained entropy of black hole surround by quantum fields is given in terms of semiclassical entropy by

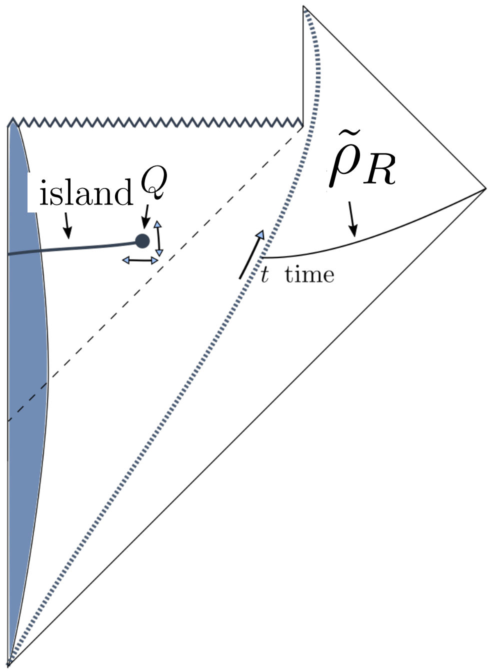
$$S_B = \mathrm{ext}_Q \left\{ \frac{\mathrm{Area}(Q)}{4G_N} + S(\tilde{\rho}_B) \right\} ,$$



Island formula for radiation [Almheiri-Mahajan-Maldacena-Zhao, 2019]

- Similarly, the fine-grained entropy of radiation is given in terms of semiclassical entropy by

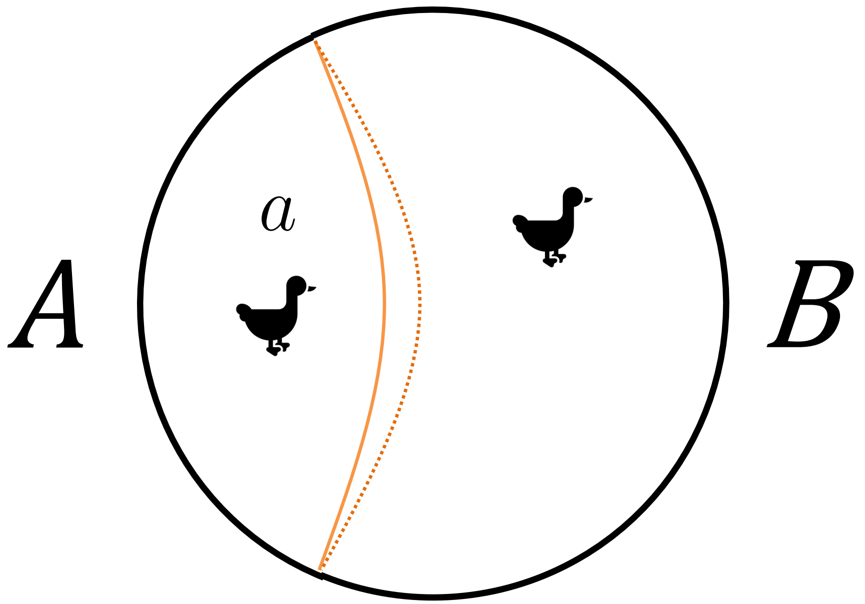
$$S(\rho_R) = \text{ext}_I \left\{ \frac{\text{Area}(\partial I = Q)}{4G_N} + S(\tilde{\rho}_{R\cup I}) \right\}$$



Entanglement entropy [Engelhardt-Wall,RT,HRT]

- QES origins from holographic entanglement entropy in AdS/CFT with bulk matter

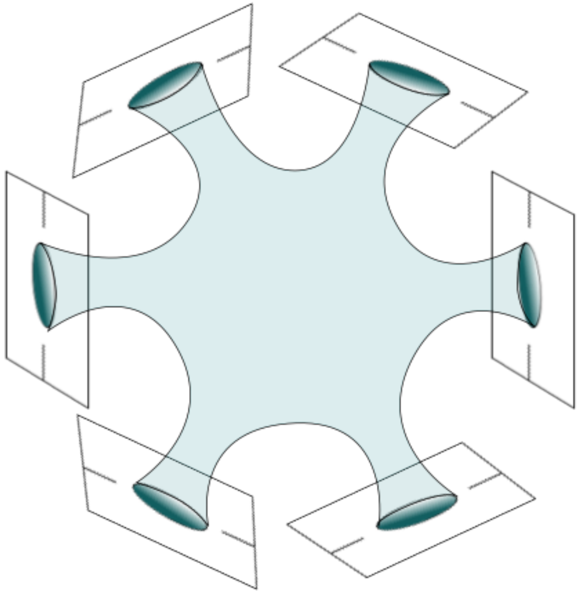
$$S(A) = \text{ext}_Q \left\{ \frac{\text{Area}(Q)}{4G_N} + S^{\text{bulk}}(a) \right\}$$

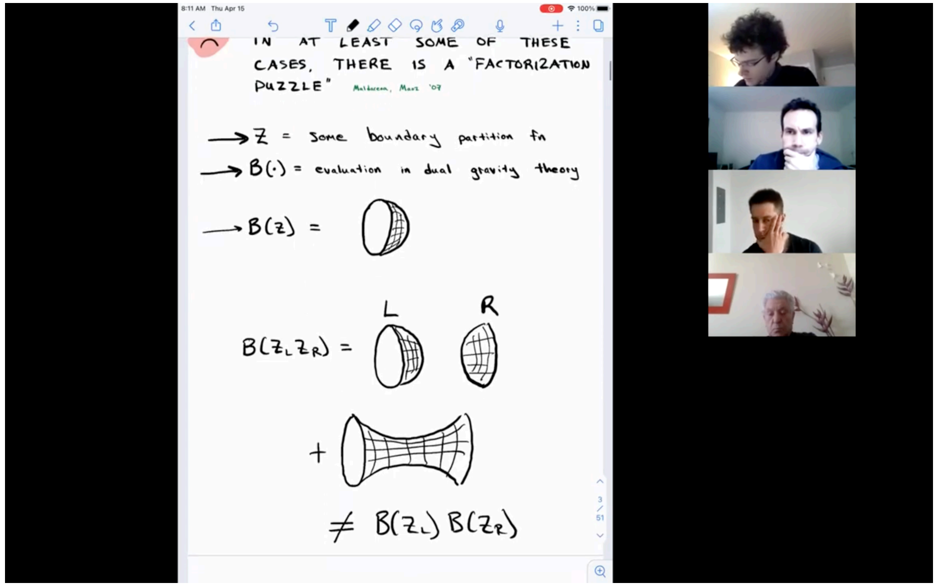


Motivations

- How to justify island formula?

(factorization problem for spacetime wormholes)





Randall-Sundrum

+

AdS/CFT

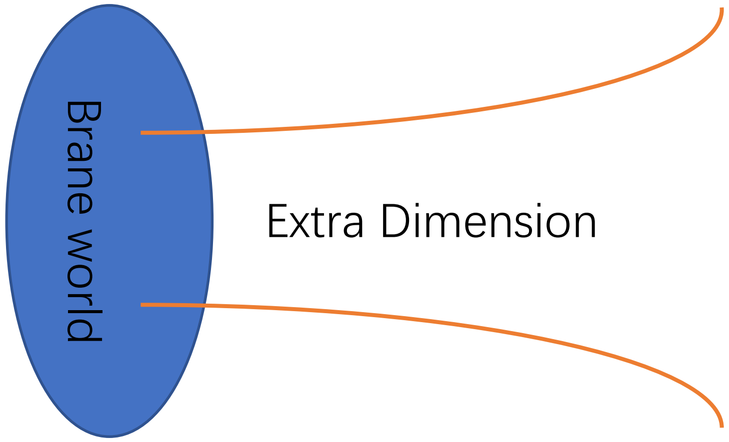
额外维部
分约化

[Deng-Chu-YZ, 2020] [Chu-Deng-YZ, 2021]

Outline

- Randall-Sundrum brane world
- Defect extremal surface => island formula
- Derive Page curve

Randall Sundrum II



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An Alternative to Compactification

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Conventional wisdom states that Newton's force law implies only four noncompact dimensions. We demonstrate that this is not necessarily true in the presence of a nonfactorizable background geometry. The specific example we study is a single 3-brane embedded in five dimensions. We show that even without a gap in the Kaluza-Klein spectrum, four-dimensional Newtonian and general relativistic gravity is reproduced to more than adequate precision.

[Citation：6904]

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2,$$

$$S_{\rm eff} \supset \int d^4x \int_0^{\pi r_c} dy\, 2M^3 r_c e^{-2k|y|}\overline{R}\,.$$

$$M_{\rm Pl}^2 = 2M^3 \int_0^{\pi r_c} dy\, e^{-2k|y|} = \frac{M^3}{k}[1 - e^{-2kr_c\pi}]$$

AdS/BCFT

[Karch-Randall,Takayanagi]

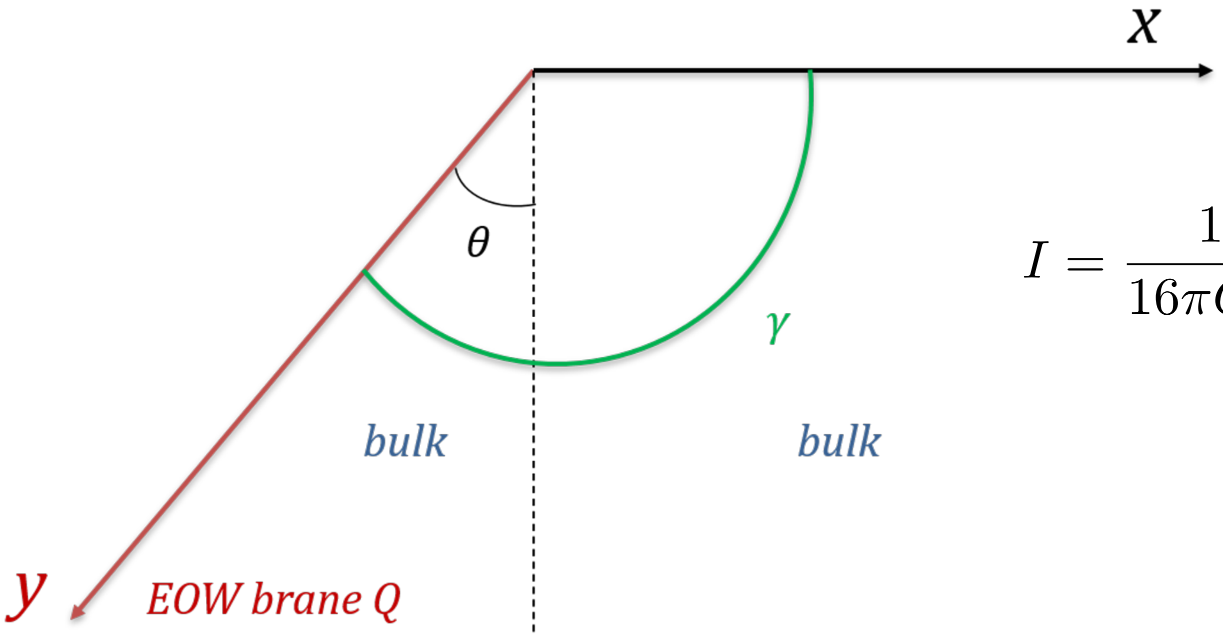


Figure 1. Holographic dual of a BCFT_2 defined on half space $(x > 0)$.

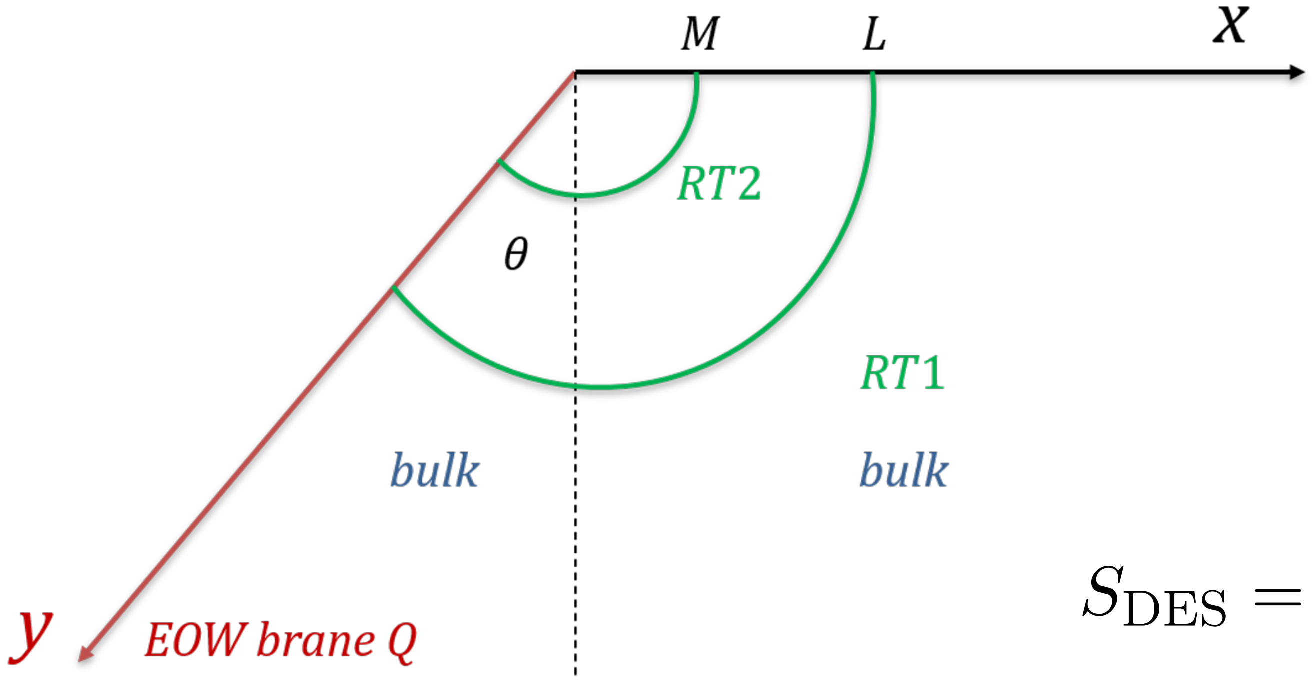
$$I = \frac{1}{16\pi G_N}\int_N \sqrt{-g}(R-2\Lambda) + \frac{1}{8\pi G_N}\int_Q \sqrt{-h}(K-T)$$

$$\begin{aligned} ds^2 &= d\rho^2 + \cosh^2\frac{\rho}{l}\cdot ds^2_{\mathrm{AdS}_2} \\ &= d\rho^2 + l^2\cosh^2\frac{\rho}{l}\cdot\frac{-dt^2+dy^2}{y^2} \end{aligned}$$

$$S_I = \frac{\mathrm{Area}\,(\gamma_I)}{4G_N} = \frac{c}{6}\log\frac{2L}{\epsilon} + \frac{c}{6}\,\mathrm{arctanh}(\sin\theta_0)$$

Defect extremal surface

[Deng-Chu-YZ,2020]



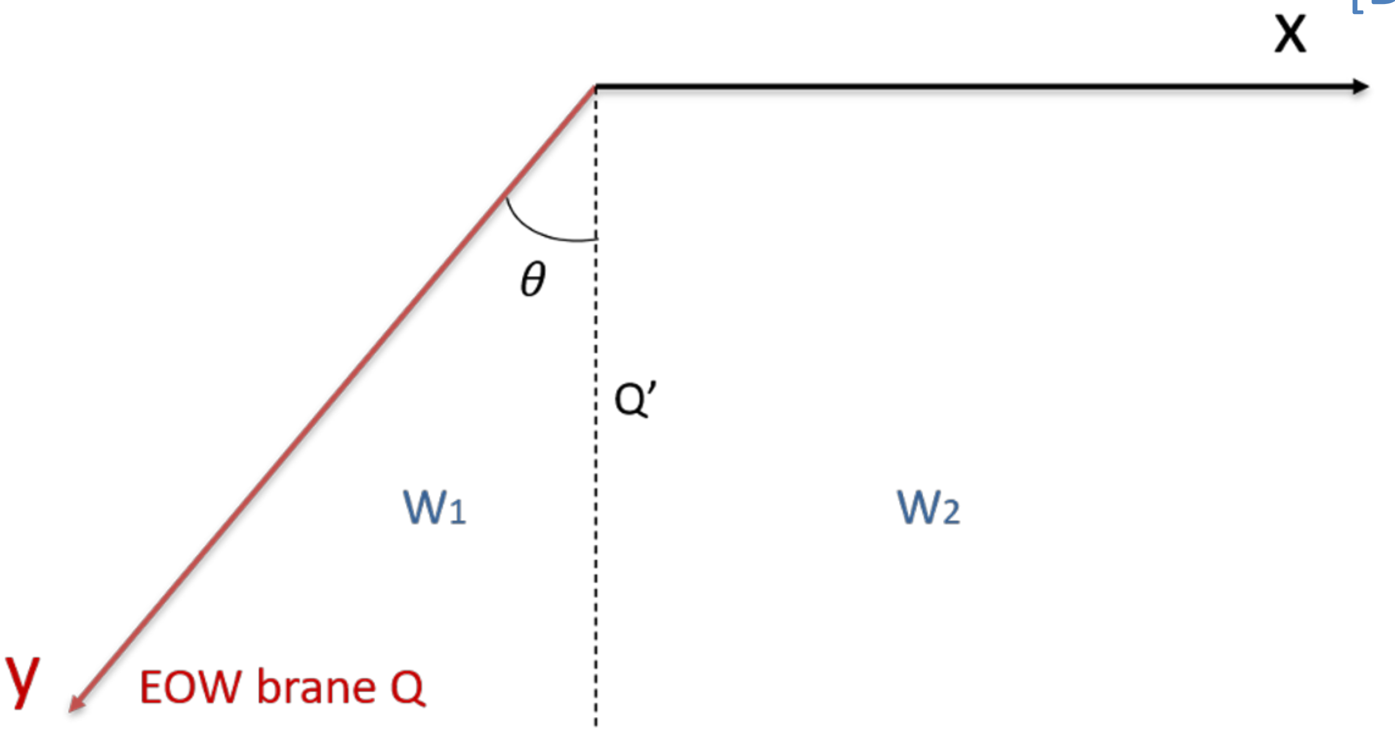
$$S_{\text{DES}} = \min_{\Gamma,X} \left\{ \text{ext}_{\Gamma,X} \left[\frac{\text{Area}(\Gamma)}{4G_N} + S_{\text{defect}}[D] \right] \right\}$$

Figure 3. Defect extremal surface in the disconnected phase.

$$S_{\text{DES}} = \frac{c}{6} \log \frac{2L}{\epsilon} + \frac{c}{6} \operatorname{arctanh}(\sin \theta_0) + \frac{c'}{6} \log \left(\frac{2l}{\epsilon_y \cos \theta_0} \right)$$

Partial reduction

[Deng-Chu-YZ,2020]

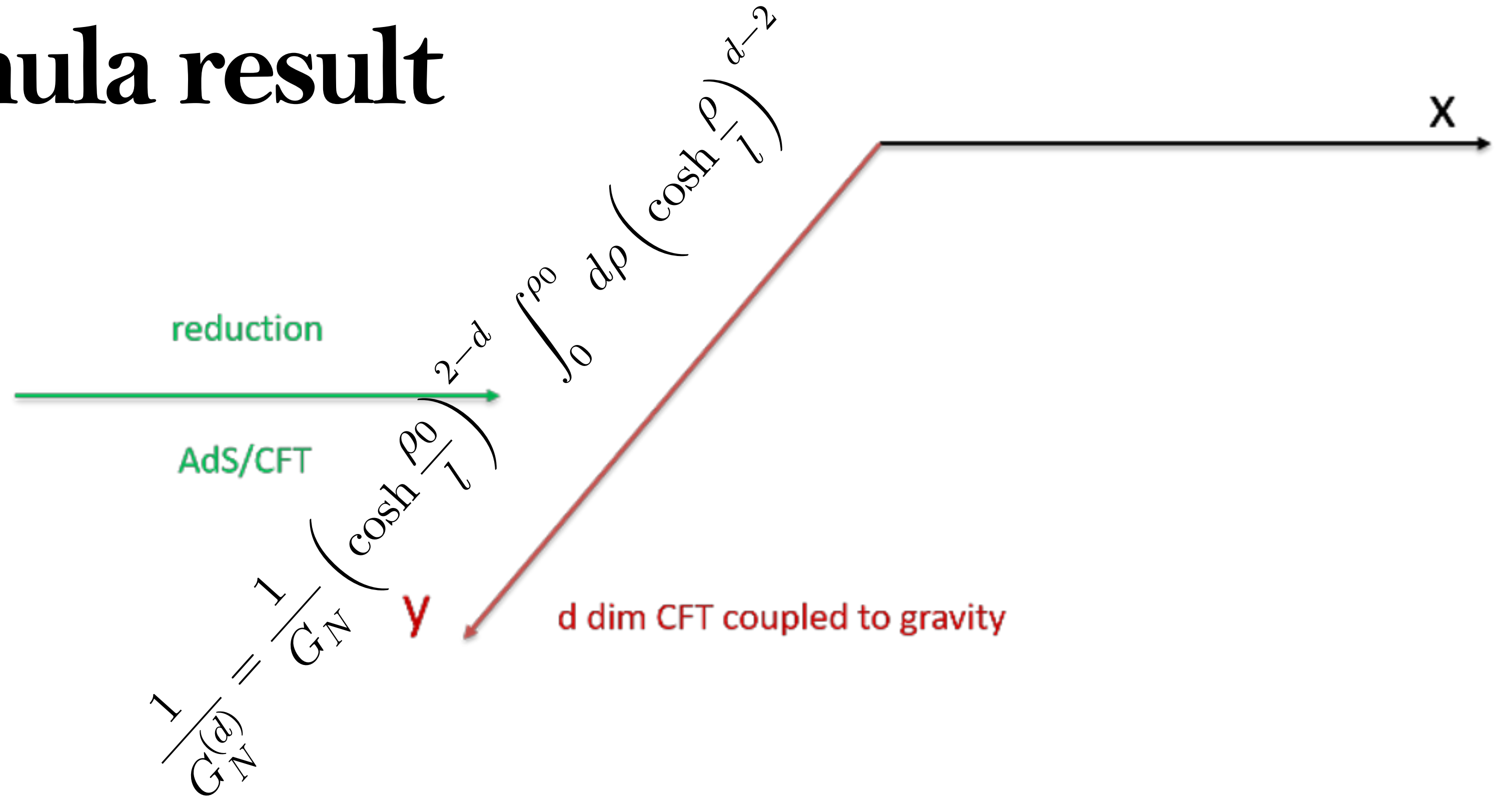


额外维部
分约化

W1: partial Randall-Sundrum
W2: AdS/CFT

Island formula result

[Deng-Chu-YZ,2020]



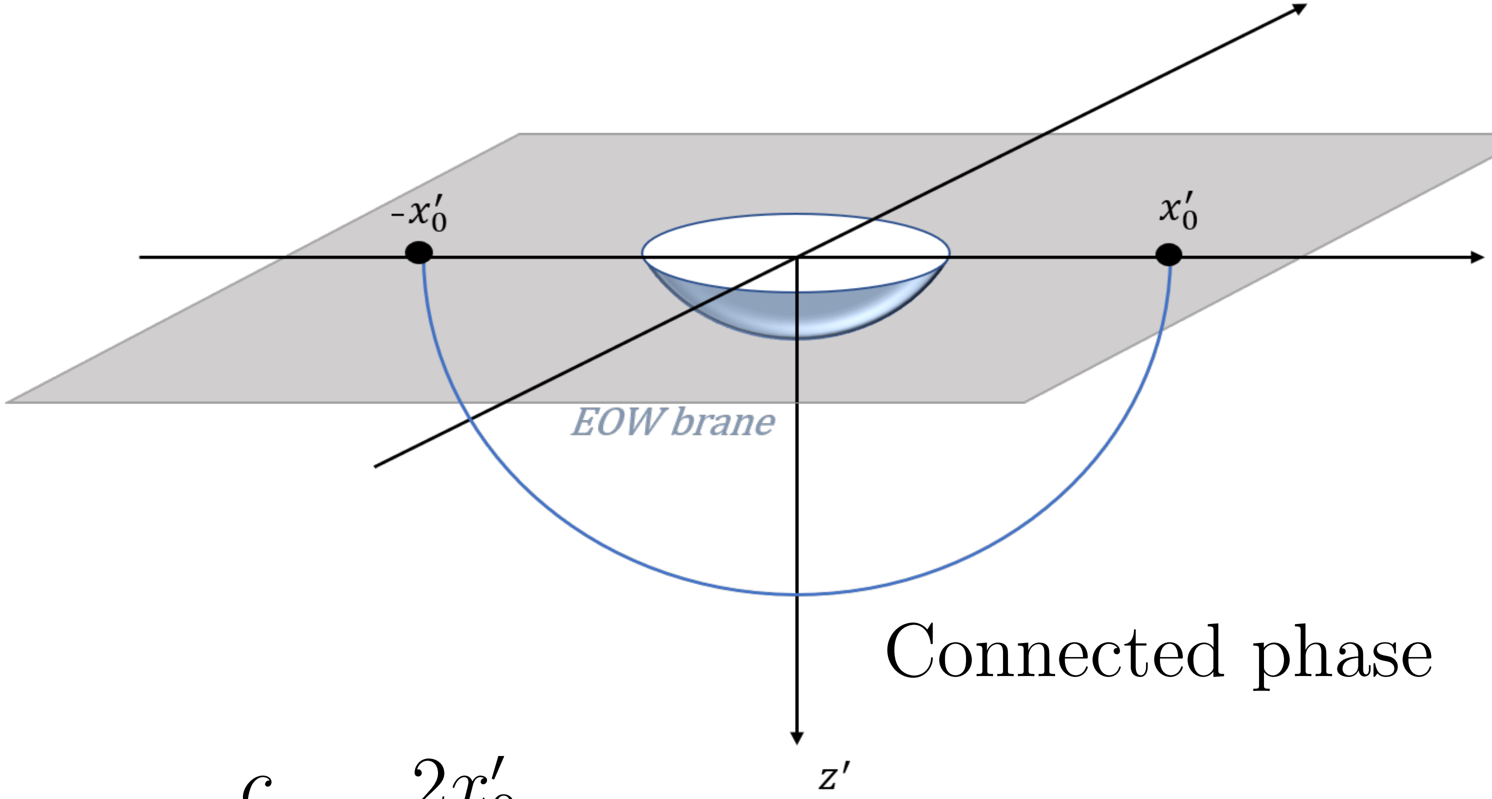
$$
\begin{aligned}
S_{\text{gen}}(a) &= S_{\text{area}}(y=-a) + S_{\text{matter}}([-a, L]) \\
&= \frac{c}{6}\,\text{arctanh}(\sin\theta_0) + \frac{c}{6}\log\frac{(L+a)^2 l}{a\cos\theta_0 \epsilon\epsilon_y}
\end{aligned}
$$

DES=QES!!

Derive Page Curve

Euclidean AdS/BCFT

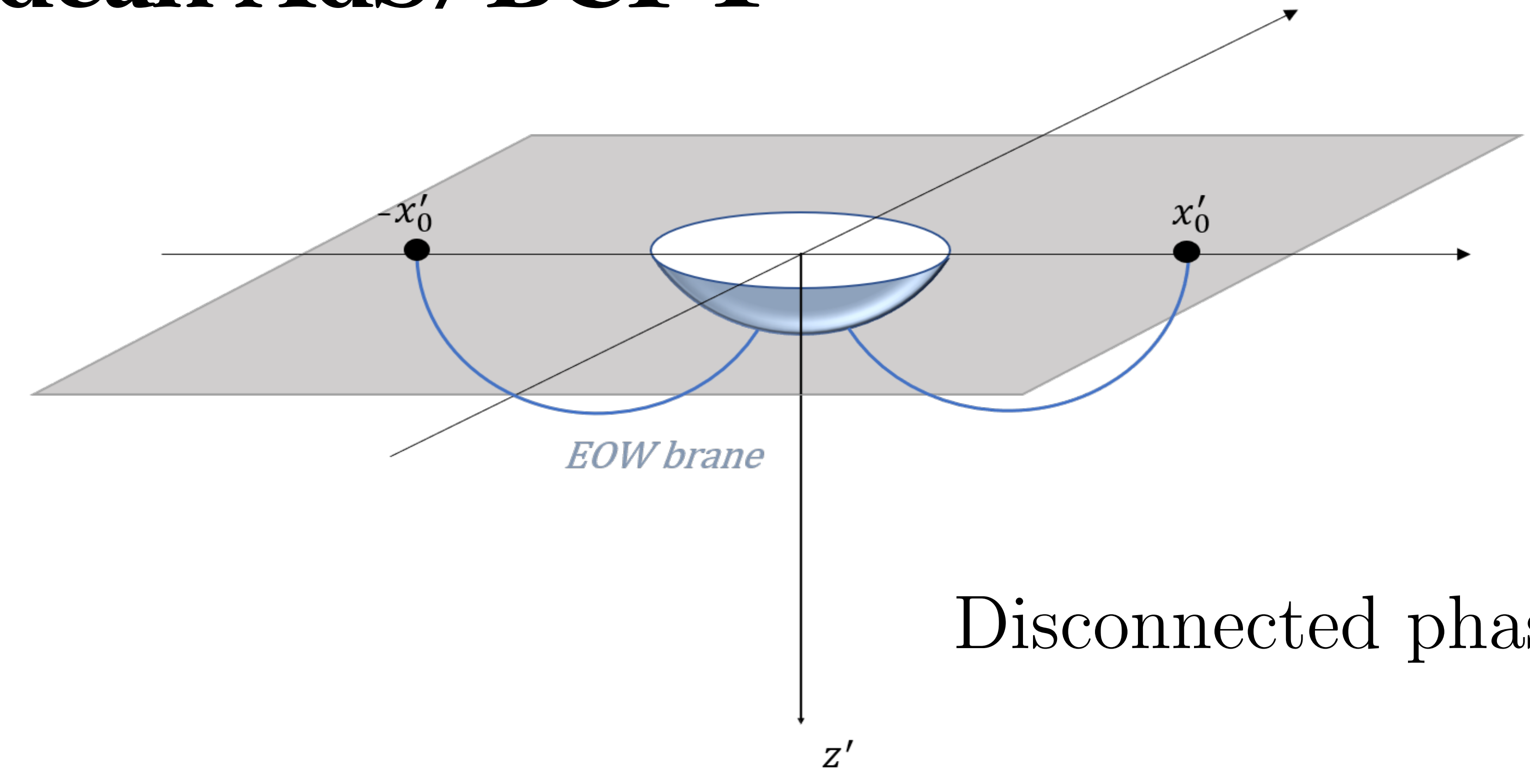
[Raamsdonk etc, 1910.12836]
[Chu-Deng-YZ,2021]



Connected phase

$$S_{\mathrm{DES}} = \frac{c}{3} \log \frac{2x_0'}{\epsilon}$$

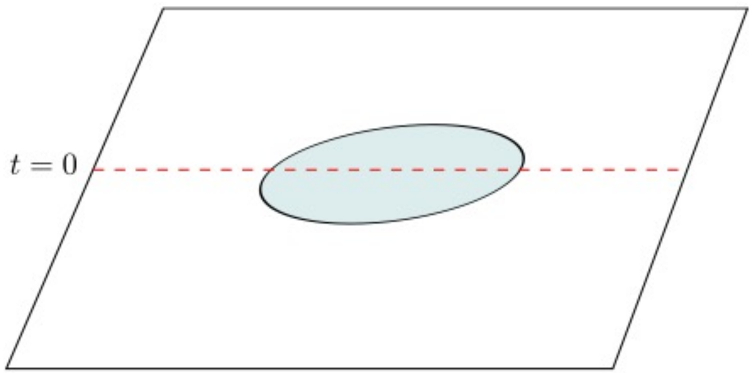
Euclidean AdS/BCFT

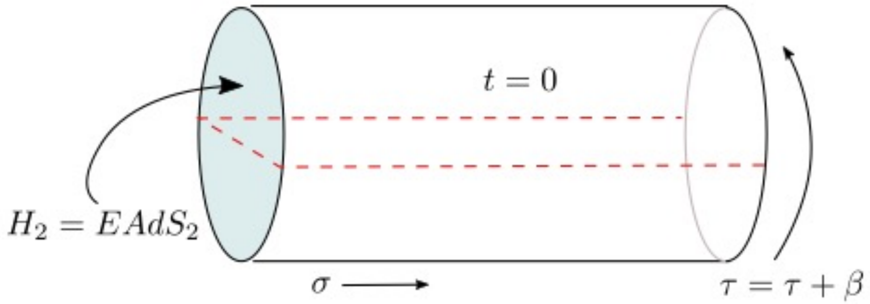


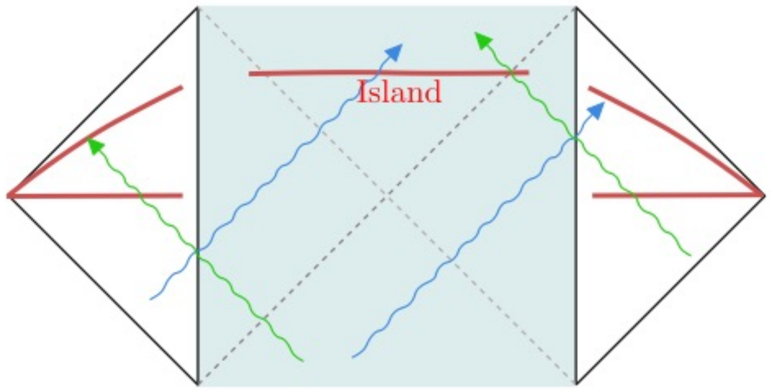
Disconnected phase

$$S_{\mathrm{DES}} = \frac{c}{3}\left(\log\frac{x_0'^2 + \tau_0'^2 - 1}{\epsilon} + \operatorname{arctanh}\sin\theta + \log\frac{2l}{\epsilon_y\cos\theta}\right)$$

Lorentzian evolution







Why eternal black hole + CFT?

[Almheiri-Mahajan-Maldacena]

- AdS black holes do not evaporate

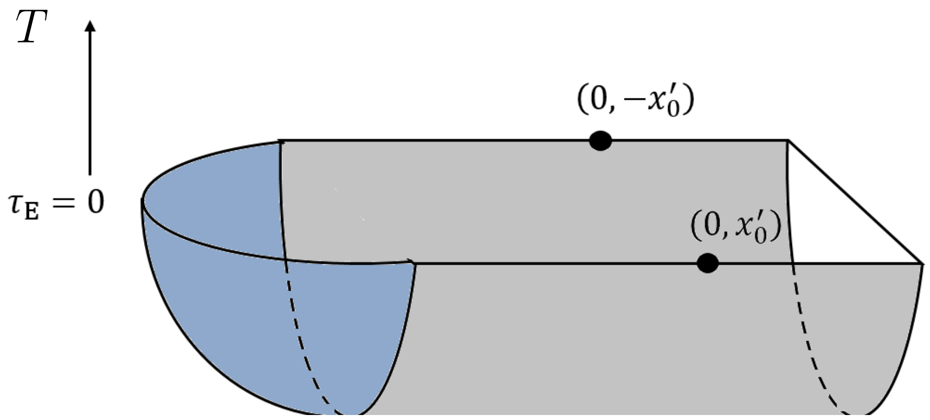
- Information paradox can be realized in AdS spacetime joined to a Minkowski region, where black hole can radiate

- 2d AdS black hole is attached to a CFT in flat region, with a transparent boundary condition

- Explicit computations can be done in this model

DES for Page curve

[Chu-Deng-YZ,2021]

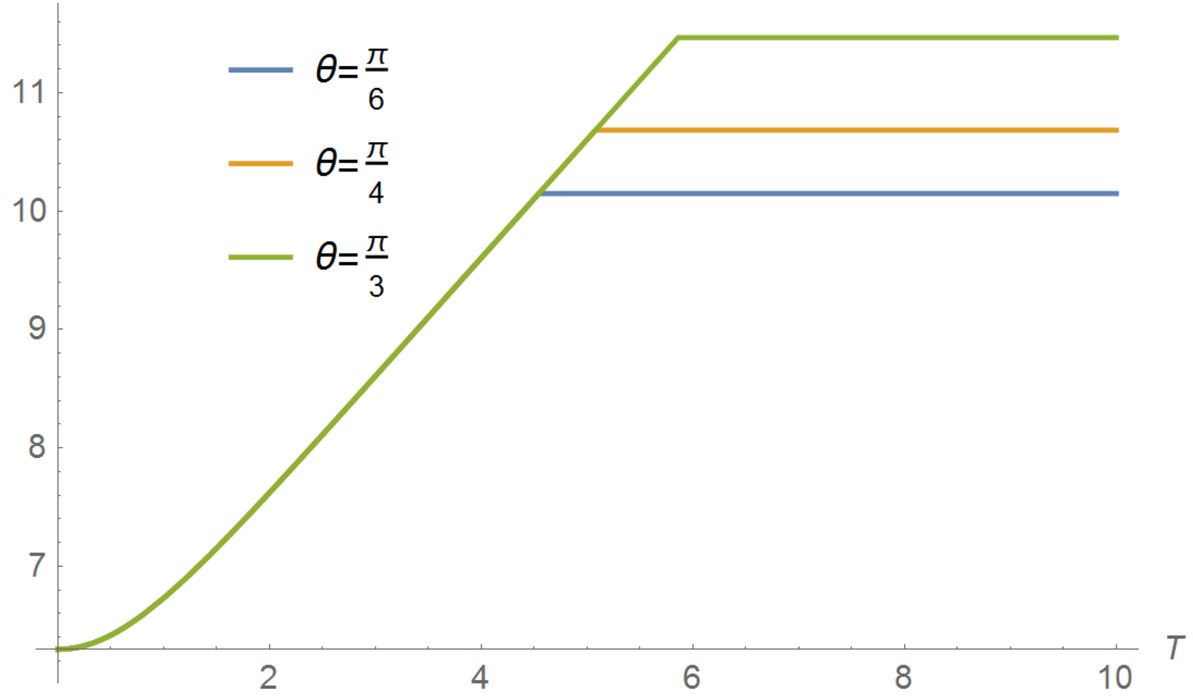


$$x' = e^X \cosh T,\ \ \tau' = i e^X \sinh T$$

$$S_{\text{DES}} = \begin{cases} \frac{c}{3}\left(\log\frac{2\cosh T}{\epsilon} + X_0\right), & T < T_P \\ \frac{c}{3}\left(\log\frac{e^{2X_0}-1}{\epsilon} + \text{arctanh}(\sin\theta) + \log\frac{2l}{\epsilon_y \cos\theta}\right), & T > T_P\ . \end{cases}$$

Island formula for Page curve

[Chu-Deng-YZ, 2021]



$$S_{\text{QES}} = \begin{cases} \frac{c}{3}\left(\log\frac{2\cosh T}{\epsilon} + X_0\right), & T < T_P \\ \frac{c}{3}\left(\log\frac{e^{2X_0-1}}{\epsilon} + \text{arctanh}\sin\theta + \log\frac{2l}{\epsilon_y\cos\theta}\right), & T > T_P \end{cases}$$

DES=QES, again!!

Higher dimensions

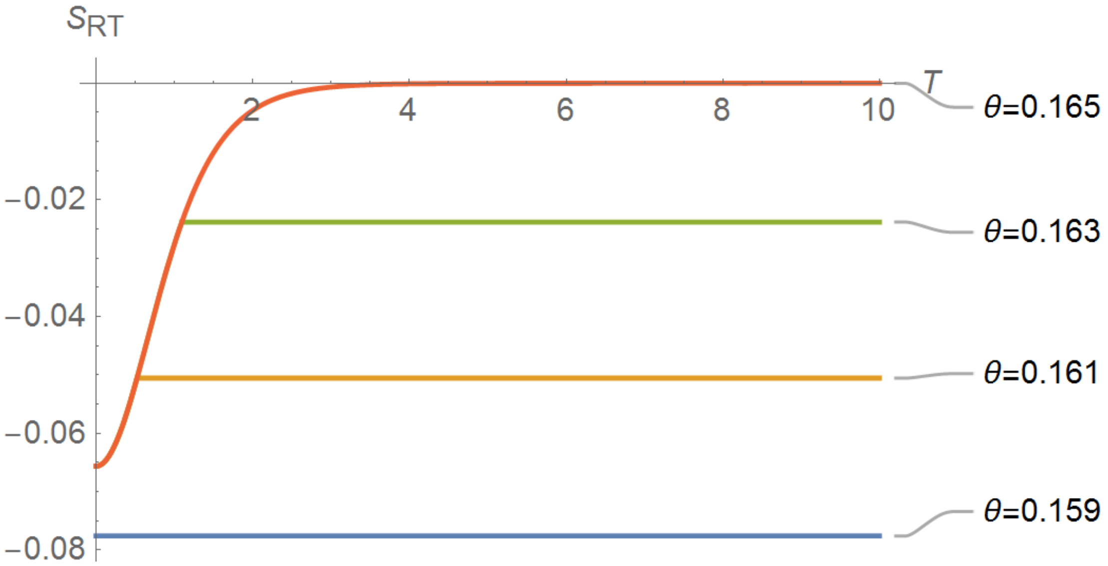


Figure 3.3: The entropy (in the unit of $\frac{l^{d-1}L^{d-2}}{4G_N^{(d+1)}}$) with respect to time T for $d = 4$, $X_0 = 0.1$ and $\theta = 0.159, 0.161, 0.163, 0.165$. We also substract the constant term $\frac{2l^{d-1}}{d-2}\frac{L^{d-2}}{\epsilon^{d-2}}$.

Summary

- Defect extremal surface gives the island formula
- Page curve can be derived from Randall-Sundrum+AdS/CFT
- Future direction: our cosmology?

Thank You!