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New progress of color-kinematics duality

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Based on arXiv: 2106.05280 and in progress with Guanda Lin (林冠达) and Siyuan Zhang (张思源)

Quantum field theory

Standard model of particle physics

Higgs particle finally discovered in 2012

Physics 2013

Photo: Pnicolet via Wikimedia Commons François Englert

Photo: G-M Greuel via Wikimedia Commons Peter W. Higgs

Unifying Gravity?

Gravity is not renormalizable.

QED is renormalizable -> we know the full theory

$$
\mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} (i\gamma^{\mu} D_{\mu} - m)\psi
$$

Gravity: needs infinitely many "counter terms":

$$
\mathcal{L} = \sqrt{g}(R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \cdots)
$$

This may be understood from the dimension of the coupling: Gravity coupling has mass dimension -2. High-dimensional local operators (as counter terms) appear at high orders.

An analogy: "four-fermion effective theory", coupling dimension is -2

Four-fermion theory is an "effective theory" of the more fundamental electroweak theory.

An analogy: "four-fermion effective theory", coupling dimension is -2

Gravity is also an effective theory of some fundamental theory

An analogy: "four-fermion effective theory", coupling dimension is -2

Gravity is also an effective theory of some fundamental theory

String theory: String states (?)

Effective theory

Effective theory only works up to certain energy scale. Above such energy scale, the theory is meaningless.

Quantum gravity

Effective theory only works up to certain energy scale. Above such energy scale, the theory is meaningless.

A fundamental quantum gravity theory is necessary:

Toward a quantum theory of gravity

String theory, Loop quantum gravity, Asymptotic safety,

…

Toward a quantum theory of gravity

Outline

Motivation

Color-kinematics duality

New form factor results

Summary and discussion

Gauge-gravity duality: holography

AdS/CFT correspondence

Gauge-gravity duality: double copy

KLT relation

CLNS - 85/667 September 1985

A Relation Between Tree Amplitudes of Closed and Open **Strings**

H. Kawai, D.C. Lewellen, and S.-H.H. Tye

Newman Laboratory of Nuclear Studies **Cornell University** Ithaca, New York 14853

ABSTRACT

 t rue in field theories of four- and field theories of four- and five-particle scattering scatterin amplitudes, in the field theory limit theory limit theory limit theory limit theory limit the KLT reduce to: strings. In particular, we demonstrate its use by showing how to write down, without any direct calculation, all four-point heterotic string tree amplitudes with massless external particles.

 $M_4^{\text{tree}}(1,2,3,4) = -i s_{12} A_4^{\text{tree}}(1,2,3,4) A_4^{\text{tree}}(1,2,4,3),$

 $M_5^{\text{tree}}(1,2,3,4,5) = i s_{12} s_{34} A_5^{\text{tree}}(1,2,3,4,5) A_5^{\text{tree}}(2,1,4,3,5)$ $+is_{13}s_{24}A_5^{\rm tree}(1,3,2,4,5)\, A_5^{\rm tree}(3,1,4,2,5)\,$

Gauge-gravity duality: double copy

$\overline{\mathcal{L}}$ COIOI-KINema **NI** Color-kinematics duality

In 2008 Bern, Carrasco and Johansson n zooo Deni, Ganasco and Jonansso
proposed a duality between color and kinematics factors: α \overline{d}

Tarn Carrasco Johansson 2008] **[Bern, Carrasco, Johansson 2008]**

Gauge symmetry Spacetime symmetry

Example: 4-pt amplitude r t 2 3 2 3 3.3 1.3

$$
A_4(1,2,3,4) = \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}
$$

$$
c_s = \tilde{f}^{a_1 a_2 b} \tilde{f}^{b a_3 a_4}, \quad c_t = \tilde{f}^{a_2 a_3 b} \tilde{f}^{b a_4 a_1}, \quad c_u = \tilde{f}^{a_1 a_3 b} \tilde{f}^{b a_2 a_4}
$$

Example: 4-pt amplitude r t 2 3 2 3 Evamnla: 1-nt amnlitu \mathbf{y} amplitude

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$$

For more general tree-level amplitudes, the existence of such a representation has

$$
c_s = c_t + c_u
$$
Jacobi identity

Example: 4-pt amplitude r t 2 3 2 3 Evamnla: 1-nt amnlitu \mathbf{y} amplitude

$$
A_4(1,2,3,4) = \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}
$$

 $\begin{array}{ccccccccc}\nJ & \gamma & \sim_L & J & J & \gamma & \sim_L & J\n\end{array}$ $c_s = \tilde f^{a_1a_2b} \tilde f^{ba_3a_4}, \quad c_t = \tilde f^{a_2a_3b} \tilde f^{ba_4a_1}, \quad c_u = \tilde f^{a_1a_3b} \tilde f^{ba_2a_4}$

$$
c_s = c_t + c_u \Rightarrow n_s = n_t + n_u
$$

Jacobi identity
dual Jacobi relation

c^s = ˜f ^a1a2^b ˜fba3a⁴ , c^t = ˜f ^a2a3^b ˜fba4a¹ , c^u = ˜f ^a1a3^b ˜fba2a⁴ . (2.2) For more general tree-level amplitudes, the existence of such a representation has **Not trivial !**

If the gauge amplitude satisfies CK duality, one can directly construct gravity amplitude:

$$
A_4(1,2,3,4) = \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}
$$

$$
M_4(1,2,3,4) = \frac{n_s n_s}{s} + \frac{n_t n_t}{t} + \frac{n_u n_u}{u}
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$$

Gauge invariance, via double copy, implies the diffeomorphism invariance in gravity:

$$
n_i \to n_i + \delta_i, \qquad \sum_i \frac{c_i \delta_i}{D_i} = 0 \qquad \frac{c_i = c_j + c_k}{n_i = n_j + n_k} \qquad \sum_i \frac{n_i \delta_i}{D_i} = 0
$$

If the gauge amplitude satisfies CK duality, one can directly construct gravity amplitude:

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$$

$$
M_4(1,2,3,4) = \frac{n_s n_s}{s} + \frac{n_t n_t}{t} + \frac{n_u n_u}{u}
$$

"double-copy" can be used also at **high loops**:

$$
A^{(\ell)} \sim \sum_{i} \int \frac{C_i \times N_i}{\prod D}
$$
 \longrightarrow $M^{(\ell)} \sim \sum_{i} \int \frac{N_i \times N_i}{\prod D}$
\n**Gauge** X **Gauge**

High-loop graviton amplitudes 1 May 10 Ma \blacksquare PSU(2, 2|4) α, α˙ |A A = 1, 2, 3, 4 ! → 0 **Feynman Diagrams for Gravity Suppose we want to check UV properties of gravity theories:**

CK-duality v.s. Double-copy

By studying the simpler gauge theory, one may understand the far more complicated gravity theory.

Proved at tree-level:

- String Monodromy relation **Bjerrum-Bohr et.al 2009; Stieberger 2009**
- BCFW recursion **Feng, Huang, Jia 2010**

Still a conjecture at loop level, relying on explicit constructions:

- \cdot 4-loop 4-point amplitudes in N=4 **Bern, et.al, 2012**
- 5-loop Sudakov form factor in N=4 **G. Yang, 2016**
- 2-loop 5-point amplitudes in pure YM **O'Connell and Mogull 2015**

It is usually non-trivial to find CK dual solution at high loops.

New 3-loop solutions for form factors

with 24 free parameters!

arXiv: 2106.05280 with Guanda Lin, Siyuan Zhang

Three-loop form factors 4 *s* for *h s* $\overline{\blacktriangle}$ $\mathbf l$ 2*p*² *T s* $\overline{}$ *,* (117) **Solution 140 April 140 Concert 140 April 140 Concert 140 April 140 Concert 140 Concert 140 Concert 140 Concert**

We consider three-loop three-point form factor in N=4 SYM:

$$
\mathcal{F}_{\mathcal{O}_i, n} = \int d^4x \, e^{-iq \cdot x} \langle p_1, p_2, p_3 | \operatorname{tr}(F^2)(x) | 0 \rangle
$$

. Is a TV-4 version of Figgs τ u there is only a single scale (*mh/mt*) at LO. The expansion in *^m^h mt* 4 version of Higg gs+3-giuon amplitudes in QCD: $\cup\cup\cup$ It is a N=4 version of Higgs+3-gluon amplitudes in QCD: 24 version om riggsto-giuon e

Three-loop form factors

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$$

It is a N=4 version of Higgs+3-gluon amplitudes in QCD:

N=4 result provides the maximally transcendental part in QCD

CK-duality

Ansatz of the loop integrand

Main challenge: it is not clear whether the solution exists

Ansatz

3-loop integral topologies:

Ansatz

Ansatz

3-loop integral topologies:

Solving ansatz

3-loop integral topologies:

*p*1 $p_3 \leftarrow$ p_2 ℓ_c ℓ_b ℓ_a x_2 ℓ_c ℓ_a *p*3 *p*1 *p*2 ℓ_b *xc xb xa x*0 $\overline{}$ *x*2 *x*3 *x*0 *x*1 x_3 \cdot x_a \overline{xy} *xc* $(3) - Master 1$ (4) $- Master 2$

Solve ansatz: Symmetry constraints and Unitarity cuts

Most complicated cuts

Final solution with 24 free parameters for the N*k*MHV amplitudes and form factors, they can be computed via BCFW on-shell T inal solution with 24 free narameters

Integration and checks ll llogration d

All free parameters cancel at integrand level. ployed to reduce the number of parameters to only 26. $\frac{1}{\sqrt{2}}$ grands are suitable for further integral reductions or nu-suitable for further integral reductions or nu-suita
Integral reductions or nu-suitable for the further integral reductions of the further integral reductions of t ree parameters cancer at integrand lever.

 \overline{a} α do performation. us to check the IR divergences and also extract the in-Color decomposition:

$$
F_{\mathcal{O}_2,3}^{(3)} = \mathcal{F}_{\mathcal{O}_2,3}^{(0)} \sum_{\sigma_3} \sum_i \int \prod_{j=1}^3 d^D \ell_j \frac{1}{S_i} \frac{C_i N_i}{\prod_{\alpha_i} P_{\alpha_i}^2}
$$

$$
F_{\mathcal{O}_2,3}^{(3)} = \mathcal{F}_{\mathcal{O}_2,3}^{(0)} f_{123} (N_c^3 \mathcal{I}_{\mathcal{O}_2}^{(3)} + 12 N_c \mathcal{I}_{\mathcal{O}_2,\text{NP}}^{(3)})
$$

Integration and checks \blacksquare only planar contribution, while that of tr(²) contains

$$
\boldsymbol{F}_{\mathcal{O}_2,3}^{(3)} = \mathcal{F}_{\mathcal{O}_2,3}^{(0)} f_{123} (N_c^3 \mathcal{I}_{\mathcal{O}_2}^{(3)} + 12 N_c \mathcal{I}_{\mathcal{O}_2,NP}^{(3)})
$$

 \overline{D} $\overline{$ Three-loop IR divergences provide important check, (which are also a research frontier).

Planar part is given by the BDS ansatz: Bern, Dixon, Smirnov 2005

$$
\mathcal{I}^{(3)}(\epsilon) = -\frac{1}{3} \left(\mathcal{I}^{(1)}(\epsilon) \right)^3 + \mathcal{I}^{(2)}(\epsilon) \mathcal{I}^{(1)}(\epsilon) + f^{(3)}(\epsilon) \mathcal{I}^{(1)}(3\epsilon) + \mathcal{R}^{(3)} + C^{(3)} + O(\epsilon)
$$

Integration and checks

Numerical result for planar part:

Integration and checks \blacksquare only planar contribution, while that of tr(²) contains For *O*3, it is tricky to define the collinear limit of the an *i*ntegration of more, the planar remainders *R* can be obtained by the gluon amplitudes which have phenomenological interests. IN CHACKS CK-dual representations contain a large number of free more, the planar remainders *R* can be obtained by the OPE bookstrap method is a result, one can predict the planer form σ θ the planar form factors from θ $\Delta \Gamma$ K Γ *s*¹² = *s*²³ = *s*¹³ at the finite order is to be corrected:

$$
\boldsymbol{F}^{(3)}_{\mathcal{O}_2,3} = \mathcal{F}^{(0)}_{\mathcal{O}_2,3} f_{123} \big(N_c^3 \mathcal{I}^{(3)}_{\mathcal{O}_2} + 12 N_c \mathcal{I}^{(3)}_{\mathcal{O}_2,\text{NP}} \big)
$$

As for the non-planar correction, the IR subtraction, the IR Full color IR structure:

ipo

 $\overline{}$

Non-dipole term |

<i>Z Non-d

$$
\boldsymbol{F}(p_i, a_i, \epsilon) = \boldsymbol{Z}(p_i, \epsilon) \boldsymbol{F}^{\text{fin}}(p_i, a_i, \epsilon) \qquad \qquad \boldsymbol{Z} = \mathcal{P} \exp \left[-\sum_{\ell=1}^{\infty} g^{2\ell} \left(\text{dipole terms} + \underbrace{\left(\frac{1}{\ell \epsilon} \boldsymbol{\Delta}^{(\ell)} \right)}_{\ell \epsilon} \right) \right]
$$

 Δ simple to consider Δ form factor. A review of the construction shows that t where the dipole terms can be completely fixed by cusper terms can be completely fixed by cusper terms can be c
Settlement of the dipole terms completely fixed by cusper terms completely fixed by cusper terms of the cusper and collinear and collinear and contribute only and contribute Non-dipole term needs to consider the consider to consider the consider to consider the consider to consider the consider to c
In the consider to consider the consider to consider the consider to consider the consider to consider the con

factor comes from the insertion of color-singlet operator.

$$
\boldsymbol{\Delta}_{3}^{(3)} = \alpha \sum_{i} \sum_{j < k, j, k \neq i} \tilde{f}_{abe} \tilde{f}_{cde} (\mathbf{T}^a_i \mathbf{T}^d_i + \mathbf{T}^d_i \mathbf{T}^a_i) \mathbf{T}^b_j \mathbf{T}^c_k
$$

Δ lmolid Dubr Gardi 2015 *factor Called Barn*, *Cardi 2010* loop order $\mathbf{41}$ and result on $\mathbf{42}$ butions in our problems. For the Almelid, Duhr, Gardi 2015 which leaves the integrand unchanged unchanged unchanged unchanged unchanged unchanged unchanged unchanged unch

Leaves the integrand unchanged unchanged unchanged unchanged unchanged unchanged unchanged unchanged unchange

- Starting at 3-loop and collections in No (non pl to the planer three-point form for α beyond a conductions of the small number of the small number of \mathcal{L} is a contract of external lines, one only \mathcal{L} • Sub-leading in Nc (non-planar)
- \bullet IR starting at 1/ens \ldots over \ldots gives \ldots and \ldots • IR starting at 1/eps

Summary and discussion

CK-duality v.s. Double-copy

By studying the simpler gauge theory, one may understand the far more complicated gravity theory.

Summary

We obtain 3-loop form factors via CK duality and unitarity cut:

$$
\boldsymbol{F}_{\mathcal{O}_2,3}^{(3)} = \mathcal{F}_{\mathcal{O}_2,3}^{(0)} \sum_{\sigma_3} \sum_{i} \int \prod_{j=1}^{3} d^D \ell_j \frac{1}{S_i} \underbrace{\binom{C_i}{N_i}}_{\prod_{\alpha_i} P_{\alpha_i}} \underbrace{P_{\alpha_i}^2}_{\Gamma_{\alpha_i} P_{\alpha_i}} + N_t + N_u = 0
$$

I-dual numera
2 *Iors contain a large number of free parameter*

² The CK-dual numerators contain a large number of free parameters:

 $\frac{1}{2}$ ⁺ *^R*(3) ⁺ *^C*(3) ⁺ *^O*(✏)*,* (9) tr(ϕ^2) : 24 parameters tr(ϕ^3) : 10 parameters

We evaluate the integrals numerically and find consistent IR and $f(3)$ planar finite results.

Free parameters

It is usually non-trivial to find CK dual solution at high loops.

Large number of free parameters -> form factor is a nice arena for applying CK duality, and probably the duality can be realized at higher loops.

These deformation correspond to "CK-duality preserving generalized GT". Is there any deep interpretation?

Does the double-copy of form factor have gravity correspondence?

Gauge-gravity duality

Extra slides

Generalized gauge transformation

New relations in form factors est. error ⁸ ⇥ ¹⁰¹⁰ ² ⇥ ¹⁰⁴ ⁰*.*⁰⁰¹ ⁰*.*⁰⁰⁶ ⁰*.*⁰³ ⁰*.*² ¹*.*⁷ **1** form factors **I** *s*¹² = *s*²³ = *s*¹³ at the finite order is to be corrected: = 160*.*308 *±* 0*.*006 *.* (12) λ σ ialiuli σ ili tulti

The result does not change since $C + C = 0$ because of the small number of external lines, one only The result does not change since $C_a + C_b = 0$

*x*2

 $\mathbf{1}_{7}^{(0)}$

Unitarity

$$
\sum_{n} |S_{m+n}|^2 = 1
$$
 $S^{\dagger} S = 1$

Generalized optical theorem $S = 1 + iT$ $\xrightarrow{S^{\dagger}S=1} i(T^{\dagger} - T) = T^{\dagger}T$ $\langle f|T|i\rangle = A(i\rightarrow f)$ $\Sigma |X\rangle\langle X| = 1$ $\mathcal{I}_m[A(i\rightarrow f)]=\sum_{x}A^*(f\rightarrow x)A(i\rightarrow x)$

 $\mathcal{I}m(i\otimes i) = \sum_{x} (i\otimes x) (x \otimes i)$

Unitarity cuts

Consider one-loop amplitudes:

Unitarity cuts

We can perform unitarity cuts:

$$
\overbrace{\bigoplus_{i=1}^{n}} = \overbrace{\bigotimes_{i=1}^{n}} \overbrace{\bigotimes_{i=1}^{n}} = \sum d_i \prod_{j=1}^{n} + \sum C_i \overbrace{\bigotimes_{j=1}^{n}} + \sum b_i \overbrace{\bigotimes_{j=1}^{n}} =
$$

and from tree products, we derive the coefficients more directly.

Cutkosky cutting rule:
$$
\frac{1}{\phi^2} = \rightarrow \Rightarrow \rightarrow \Rightarrow \Rightarrow \Rightarrow \Rightarrow \hat{r} = 2\pi i \hat{\delta}^{\dagger}(\phi^2)
$$

Unitarity cuts

We can perform unitarity cuts:

$$
\overbrace{\bigoplus_{i=1}^{n}} = \overbrace{\bigotimes_{i=1}^{n}} \overbrace{\bigotimes_{i=1}^{n}} = \sum d_i \prod_{j=1}^{n} + \sum C_i \overbrace{\bigotimes_{j=1}^{n}} + \sum b_i \overbrace{\bigotimes_{j=1}^{n}} =
$$

High-loop generalization:

$$
\frac{dy}{dx} = Integrand
$$
\n
$$
\frac{dy}{dx} = Integrand
$$
\n
$$
x^2 - y^2 = \frac{1}{2}x^2 - \frac{1}{2}x^2
$$

Interpretation and also an equation for the Lie algebra \mathcal{A} in the Lie algebra value \mathcal{A} [*L^p*¹ *, L^p*²] = *iX*(*p*1*, p*2)*L^p*1+*p*² \mathbf{a}

Does the Jacobi relation for momentum have any physical meaning? Doon the locabi relation for momentu investigate and possible structure of the fields, we choose the fields, we can invest no structure of the simplest nontrivial fields available. These are the self-dual solutions. Figure the self-dual solutions. Figure the self-d
The self-dual solutions. Figure the self-dual solutions. Figure the self-dual solutions. Figure the self-dual DOES THE JACODI FEIATION TOF MOMENTU any physical meaning? Ω meaning² scalar equation with a cubic coupling. Does the Jacobi relation for momentum have preserving di↵eomorphisms of *S*² and the Lie algebra of the generators of SU(*N*) in the planar limit *N* ! 1, in the sense that there exists an appropriate basis such that the

Self-dual YM/gravity [Monteiro. O'Connell 2011]
\n
$$
F_{\mu\nu} = \frac{i}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \qquad \Phi^a(k) = \frac{1}{2} g \int d p_1 d p_2 \frac{F_{p_1 p_2}{}^k f^{b_1 b_2 a}}{k^2} \Phi^{b_1}(p_1) \Phi^{b_2}(p_2)
$$
\n
$$
R_{\mu\nu\lambda\delta} = \frac{i}{2} \varepsilon_{\mu\nu\rho\sigma} R^{\rho\sigma}{}_{\lambda\delta} \qquad \phi(k) = \frac{1}{2} \kappa \int d p_1 d p_2 \frac{X(p_1, p_2) F_{p_1 p_2}{}^k}{k^2} \phi(p_1) \phi(p_2)
$$

 $\mathcal{L} = ik \cdot \mathcal{L}$ (2), $\mathcal{L} = \mathcal{L} + ik \cdot \mathcal{L}$ (3), $\mathcal{L} = \mathcal{L} + ik \cdot \mathcal{L}$ (3), $\mathcal{L} = i \mathcal{L}$ (3), $\mathcal{L} = i \mathcal{L}$ (3), $\mathcal{L} = -i \mathcal{L}$ $[U_{\nu} \partial_{u} + k_{u} \partial_{w}]$ $[L_{p_1}, L_{p_2}] = iX(p_1, p_2)L_{p_1+p_2} = iF_{p_1p_2}^{\mu}{}^{\kappa}L_{k}$ $\begin{array}{ccccccccccccccccc}\nI & \Omega & & I & \Omega\n\end{array}$, the solution is \mathcal{L} $L_k = e^{-ik \cdot x}(-k_w \partial_u + k_w \partial_w)$ $[L_{p_1}, L_{p_2}] = iX(p_1, p_2)L_{p_1+p_2} = iF_{p_1p_2}{}^kL_k$

*did devels really really be understood in F^p*1*^q ^k F^p*2*p*³ *q* ⌘ *d q* (*p*¹ + *q k*) *X*(*p*1*, q*) (*p*² + *p*³ *q*) *X*(*p*2*, p*3) $\frac{1}{2}$ diffeomorphism (*k*) the can be understood from the (1)(*k*) = ¹ \bigcap *d p*1*d p*² a of area-preserving diffeomorphism. *^k*² *^j*(*p*1)*j*(*p*2)*,* (57) 50 ual locabi ralation con be understoo. WE FORTUN SEEN THAT THE SEEN THAT THE KING-MILLS LIGHT AND THE KING-MILLS LIGHT AND ALGEBRA. Dual Jacobi relation can be understood from the algebra of area-preserving diffeomorphism.

 P^{ρ} - $2\eta^{\mu\nu}\eta^{\sigma\tau}k_{1}^{\lambda}k_{1}^{\rho}$ +

 \overline{a} λ_{k_1} ρ + $\eta^{\lambda \tau}$ $\eta^{\nu \sigma}$ k_2 ^{μ} k_1 ^{ρ} + ⌘⌫*k*³ $\mu_{k_1} \rho + \eta^{\lambda \sigma} \eta^{\nu \tau} k_3^{\mu} k_1^{\rho}$ *µ*⌘⌧ *^k*³ $\mu_{k_1}^{\nu} \rho + \eta^{\lambda \nu} \eta^{\mu \tau} k_3^{\sigma} k_1^{\rho} +$ $\lambda_{k_1}^{\alpha}$ + $2\eta^{\mu\nu}\eta^{\rho\sigma}$ _{k1} λ_{k_1} $\frac{\tau}{\tau}$ – ⌧ ⁺ ⌘*µ*⌧ ⌘⌫⇢*k*¹ $\sigma_{k_2}^{\lambda} + \eta^{\mu\rho}_{\mu} \eta^{\nu\tau}{}_{k_1}^{\sigma}{}_{k_2}$ $^{\lambda}$ + $\lambda_{k_2}^{\mu}$ + $\eta^{\lambda \tau} \eta^{\nu \rho} k_1^{\sigma} k_2^{\mu}$ *^µ* ⌘⇢⌘⌫*^k*¹ $\int^{\tau} k_2^{\mu} + \eta^{\lambda \nu} \eta^{\rho \sigma} {k_1}^{\tau} {k_2}^{\mu} +$ ⌘*µ*⇢*k*¹ σ_{k_2} ^{σ} $\mu - \eta^{\lambda\rho} \eta^{\mu\tau} k_1^{\sigma} k_2^{\sigma}$ $\begin{array}{c} \nu \\ + \end{array}$ $\sigma_{k_2}^{\mu}$ ^{μ} + 2 $\eta^{\mu \rho}$ $\eta^{\sigma \tau}$ _{k₂} $\frac{\lambda}{k_2}$ $\begin{array}{c} \nu \\ + \end{array}$ $\cdot^{2}1$ λ_{k_2} ^{ρ} + $\eta^{\mu\sigma}$ $\eta^{\nu\tau}$ k_1 λ_{k_2} ρ ^{μ} + $\frac{\tau}{k_2}$ μ^{ρ} + $2\eta^{\mu\tau}\eta^{\nu\sigma}k_2^{\lambda}k_2^{\rho}$ + k_2 $\int^{\nu} k_2^{\rho} + \eta^{\nu\tau} \eta^{\rho\sigma} k_1^{\ \lambda} k_3^{\ \mu} +$ $\sigma_{k_3}^{\mu}$ + $\eta^{\lambda \sigma} \eta^{\nu \rho} {k_1}^{\tau} {k_3}^{\mu}$ + ⌘⇢ *^k*² $\mu^{\nu} k_3^{\mu} + \eta^{\lambda \sigma} \eta^{\rho \tau} k_2^{\nu} k_3^{\mu} +$ λ_{k_3} $-\eta^{\mu\rho}\eta^{\sigma\tau}k_1$ λ_{k_3} ^{ν} + τ_{k_3} $+ \eta^{\mu \tau} \eta^{\rho \sigma} k_2$ λ_{k_3} ^{ν} + ⌘*µ ^k*² ${}^{\rho}k_{3}{}^{\nu}$ + $\eta^{\lambda\sigma}\eta^{\mu\tau}k_{2}{}^{\rho}k_{3}{}^{\nu}$ + ⌫ ⁺ ⌘*µ*⌧ ⌘⌫⇢*k*¹ $\lambda_{k_3}^{\vphantom{1}} \sigma + \eta^{\mu\rho} \eta^{\nu\tau} k_1^{\vphantom{1}}$ $\frac{\lambda}{k_3}$ σ + λ_{k_3} σ - $\eta^{\mu\nu} \eta^{\rho\tau} k_2^{\lambda} k_3^{\sigma}$ + $^{\nu}$ k_3^{σ} - $\eta^{\lambda\tau}$ $\eta^{\mu\nu}$ k_2^{ρ} k_3^{σ} + $\frac{1}{3}$ $\mu_{k_3}^{\nu} \sigma + \eta^{\mu \sigma} \eta^{\nu \rho} {k_1}^{\lambda} {k_3}^{\tau} +$ λ_{k_3} $\sigma + \eta^{\mu\rho} \eta^{\nu\sigma} k_2^{\lambda} k_3$ τ – ⌘*µ*⇢*k*² $v_{k_3}^{\nu}$ + $\eta^{\lambda\mu} \eta^{\rho\sigma} {k_2}^{\nu} {k_3}$ τ – $^{\mu}$ k_{3}^{τ} + $2\eta^{\lambda\rho}\eta^{\mu\sigma}$ k_{3}^{ν} k_{3}^{τ} $h^{\nu \rho} k_1 \cdot k_2 - \eta^{\lambda \sigma} \eta^{\mu \tau} \eta^{\nu \rho} k_1$. $k_2 + \eta^{\lambda\rho} \eta^{\mu\sigma} \eta^{\nu\tau} k_1 \cdot k_2 +$ $+ 2\eta^{\lambda\sigma} \eta^{\mu\nu} \eta^{\rho\tau} k_1 \cdot k_2 +$ 2 η $\lambda \nu \frac{\mu \rho}{\eta} \sigma \tau_{k_1} \cdot k_2 +$ $a_3 - \eta^{\lambda\tau} \eta^{\mu\rho} \eta^{\nu\sigma} k_1 \cdot k_3 +$ $B_3 + 2\eta^{\lambda\tau} \eta^{\mu\nu} \eta^{\rho\sigma} k_1 \cdot k_3 B_3 - \eta^{\lambda \nu} \eta^{\mu \sigma} \eta^{\rho \tau} k_1 \cdot k_3$ $a_3 + \eta^{\lambda \mu} \eta^{\nu \rho} \eta^{\sigma \tau} k_1 \cdot k_3 + 2\eta^{\lambda\rho} \eta^{\mu\tau} \eta^{\nu\sigma} k_2 \cdot k_3$ $B_3 - \eta^{\lambda \nu} \eta^{\mu \tau} \eta^{\rho \sigma} k_2 \cdot k_3$ $k_3 - \eta^{\lambda \mu} \eta^{\nu \sigma} \eta^{\rho \tau} k_2 \cdot k_3 -$

λ σ τ 3 2 1

more than 100 terms

[DeWitt 1967]

Feynman diagram? *S*³ μ **i** alumnos anrar

 $\delta^3 S$

 \rightarrow $2\eta^{\mu\tau}\eta^{\nu\sigma}k_1^{\lambda}k_1^{\rho}$ + $2\eta^{\mu\sigma}\eta^{\nu\tau}k_1^{\lambda}k_1^{\rho}$ - $2\eta^{\mu\nu}\eta^{\sigma\tau}k_1^{\lambda}k_1^{\rho}$ +

 σ

1

2

 τ

 $\delta\varphi_{\mu\nu}\delta\varphi_{\sigma\tau}\delta\varphi_{\rho\lambda}$ $2\eta^{\lambda\tau}\eta^{\mu\nu}k_1^{\sigma}k_1^{\rho} + 2\eta^{\lambda\sigma}\eta^{\mu\nu}k_1^{\tau}k_1^{\rho} + \eta^{\mu\tau}\eta^{\nu\sigma}k_2^{\lambda}k_1^{\rho} + \eta^{\mu\sigma}\eta^{\nu\tau}k_2^{\lambda}k_1^{\rho} + \eta^{\lambda\tau}\eta^{\nu\sigma}k_2^{\mu}k_1^{\rho} +$ $\eta^{\lambda\sigma}\eta^{\nu\tau}k_2^{\mu}k_1^{\rho}+\eta^{\lambda\tau}\eta^{\mu\sigma}k_2^{\nu}k_1^{\rho}+\eta^{\lambda\sigma}\eta^{\mu\tau}k_2^{\nu}k_1^{\rho}+\eta^{\lambda\tau}\eta^{\nu\sigma}k_3^{\mu}k_1^{\rho}+\eta^{\lambda\sigma}\eta^{\nu\tau}k_3^{\mu}k_1^{\rho} \eta^{\lambda\nu}\eta^{\sigma\tau}k_{3}{}^{\mu}k_{1}{}^{\rho} + \eta^{\lambda\tau}\eta^{\mu\sigma}k_{3}{}^{\nu}k_{1}{}^{\rho} + \eta^{\lambda\sigma}\eta^{\mu\tau}k_{3}{}^{\nu}k_{1}{}^{\rho} - \eta^{\lambda\mu}\eta^{\sigma\tau}k_{3}{}^{\nu}k_{1}{}^{\rho} + \eta^{\lambda\nu}\eta^{\mu\tau}k_{3}{}^{\sigma}k_{1}{}^{\rho} +$ $\eta^{\lambda\mu}\eta^{\nu\tau}k_3^{\sigma}k_1^{\rho}+\eta^{\lambda\nu}\eta^{\mu\sigma}k_3^{\tau}k_1^{\rho}+\eta^{\lambda\mu}\eta^{\nu\sigma}k_3^{\tau}k_1^{\rho}+2\eta^{\mu\nu}\eta^{\rho\tau}k_1^{\lambda}k_1^{\sigma}+2\eta^{\mu\nu}\eta^{\rho\sigma}k_1^{\lambda}k_1^{\tau}-\eta^{\mu\nu}\eta^{\rho\tau}k_2^{\tau}k_1^{\tau}$ $2\eta^{\lambda\rho}\eta^{\mu\nu}k_1^{\ \sigma}k_1^{\ \tau}+2\eta^{\lambda\nu}\eta^{\mu\rho}k_1^{\ \sigma}k_1^{\ \tau}+2\eta^{\lambda\mu}\eta^{\nu\rho}k_1^{\ \sigma}k_1^{\ \tau}+\eta^{\mu\tau}\eta^{\nu\rho}k_1^{\ \sigma}k_2^{\ \lambda}+\eta^{\mu\rho}\eta^{\nu\tau}k_1^{\ \sigma}k_2^{\ \lambda}+$ $\eta^{\mu\sigma}\eta^{\nu\rho}k_1^{\ \tau}k_2^{\ \lambda} + \eta^{\mu\rho}\eta^{\nu\sigma}k_1^{\ \tau}k_2^{\ \lambda} + \eta^{\nu\tau}\eta^{\rho\sigma}k_1^{\ \lambda}k_2^{\ \mu} + \eta^{\nu\sigma}\eta^{\rho\tau}k_1^{\ \lambda}k_2^{\ \mu} + \eta^{\lambda\tau}\eta^{\nu\rho}k_1^{\ \sigma}k_2^{\ \mu} \eta^{\lambda\rho}\eta^{\nu\tau}k_{1}{}^{\sigma}k_{2}{}^{\mu} + \eta^{\lambda\nu}\eta^{\rho\tau}k_{1}{}^{\sigma}k_{2}{}^{\mu} + \eta^{\lambda\sigma}\eta^{\nu\rho}k_{1}{}^{\tau}k_{2}{}^{\mu} - 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2\eta^{\lambda\rho}\eta^{\sigma\tau}k_3^{\mu}k_3^{\nu} + \eta^{\mu\tau}\eta^{\nu\rho}k_1^{\lambda}k_3^{\sigma} + \eta^{\mu\rho}\eta^{\nu\tau}k_1^{\lambda}k_3^{\sigma} +$ $\eta^{\lambda\nu}\eta^{\mu\rho}k_1^{\ \tau}k_3^{\ \sigma}+\eta^{\lambda\mu}\eta^{\nu\rho}k_1^{\ \tau}k_3^{\ \sigma}+\eta^{\mu\tau}\eta^{\nu\rho}k_2^{\ \lambda}k_3^{\ \sigma}+\eta^{\mu\rho}\eta^{\nu\tau}k_2^{\ \lambda}k_3^{\ \sigma}-\eta^{\mu\nu}\eta^{\rho\tau}k_2^{\ \lambda}k_3^{\ \sigma}+\eta^{\mu\nu}\eta^{\nu\tau}k_3^{\ \nu}k_3^{\ \sigma}+\eta^{\mu\nu}\eta^{\nu\tau}k_2^{\ \lambda}k_3^{\ \sigma}+\eta^$ $\eta^{\lambda\tau}\eta^{\nu\rho}k_2^{\mu}k_3^{\sigma} + \eta^{\lambda\nu}\eta^{\rho\tau}k_2^{\mu}k_3^{\sigma} + \eta^{\lambda\tau}\eta^{\mu\rho}k_2^{\nu}k_3^{\sigma} + \eta^{\lambda\mu}\eta^{\rho\tau}k_2^{\nu}k_3^{\sigma} - \eta^{\lambda\tau}\eta^{\mu\nu}k_2^{\rho}k_3^{\sigma} +$ $\int \eta^{\lambda\nu} \eta^{\mu\tau} k_2^{\ \rho} k_3^{\ \sigma} + \eta^{\lambda\mu} \eta^{\nu\tau} k_2^{\ \rho} k_3^{\ \sigma} + 2\eta^{\lambda\rho} \eta^{\nu\tau} k_3^{\ \mu} k_3^{\ \sigma} + 2\eta^{\lambda\rho} \eta^{\mu\tau} k_3^{\ \nu} k_3^{\ \sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} k_1^{\ \lambda} k_3^{\ \tau} +$ $\eta^{\mu\rho}\eta^{\nu\sigma}k_1^{\ \lambda}k_3^{\ \tau} + \eta^{\lambda\nu}\eta^{\mu\rho}k_1^{\ \sigma}k_3^{\ \tau} + \eta^{\lambda\mu}\eta^{\nu\rho}k_1^{\ \sigma}k_3^{\ \tau} + \eta^{\mu\sigma}\eta^{\nu\rho}k_2^{\ \lambda}k_3^{\ \tau} + \eta^{\mu\rho}\eta^{\nu\sigma}k_2^{\ \lambda}k_3^{\ \tau} \eta^{\mu\nu}\eta^{\rho\sigma}k_2^{\ \lambda}k_3^{\ \tau} + \eta^{\lambda\sigma}\eta^{\nu\rho}k_2^{\ \mu}k_3^{\ \tau} + \eta^{\lambda\nu}\eta^{\rho\sigma}k_2^{\ \mu}k_3^{\ \tau} + \eta^{\lambda\sigma}\eta^{\mu\rho}k_2^{\ \nu}k_3^{\ \tau} + \eta^{\lambda\mu}\eta^{\rho\sigma}k_2^{\ \nu}k_3^{\ \tau} -$ ⌘ ⌘*µ*⌫ *^k*² ⇢*k*³ ⌧ ⁺ ⌘ ⌫ ⌘*µ ^k*² ⇢*k*³ ⌧ ⁺ ⌘ *µ*⌘⌫*k*² ⇢*k*³ ⌧ + 2⌘⇢⌘⌫*^k*³ *µk*³ ⌧ + 2⌘⇢⌘*µ ^k*³ ⌫ *^k*³ ⌧ $2\eta^{\lambda\rho}\eta^{\mu\nu}k_3^{\sigma}k_3^{\tau} + 2\eta^{\lambda\nu}\eta^{\mu\rho}k_3^{\sigma}k_3^{\tau} + 2\eta^{\lambda\mu}\eta^{\nu\rho}k_3^{\sigma}k_3^{\tau} - \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1 \cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1 \cdot k_3^{\tau}$ $k_2 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1 \cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1 \cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1 \cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1 \cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1 \cdot k_2$ $2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1$ · k_2 - $\eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1$ · k_2 - $\eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1$ · k_2 - $\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1$ · k_2 - $\int \eta^{\lambda\nu} \eta^{\mu\sigma} \eta^{\rho\tau} k_1 \cdot k_2 \ - \ \eta^{\lambda\mu} \eta^{\nu\sigma} \eta^{\rho\tau} k_1 \cdot k_2 \ - \ 2 \eta^{\lambda\rho} \eta^{\mu\nu} \eta^{\sigma\tau} k_1 \cdot k_2 \ + \ 2 \eta^{\lambda\nu} \eta^{\mu\rho} \eta^{\sigma\tau} k_1 \ \cdot \ k_2 \ + \$ $2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1$ · k_2 - $\eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1$ · k_3 - $\eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1$ · k_3 - $\eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1$ · k_3 + $2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1$ · k_3 - $\eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1$ · k_3 + $2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1$ · k_3 + $2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1$ · k_3 - $\eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1$ · k_3 - $\eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1$ · k_3 + $2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1$ · k_3 - $\eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1$ · k_3 - $\eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1$ · k_3 - $2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1$ · k_3 + $\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1$ · k_3 - $\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1$ · k_3 - $\eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_2$ · k_3 - $\eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_2$ · k_3 - $\eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_2$ · k_3 - k_3 - $\eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\sigma}k_2$ · k_3 - $\int \int \sqrt[3]{\pi} \eta^{\mu} \rho \eta^{\nu} \tau \kappa_2 + k_3 + 2 \eta^{\lambda} \rho \eta^{\mu} \sigma \eta^{\nu} \tau \kappa_2 + k_3 + \eta^{\lambda} \tau \eta^{\mu} \nu \eta^{\rho} \sigma \kappa_2 + k_3 - \eta^{\lambda \nu} \eta^{\mu} \tau \eta^{\rho} \sigma \kappa_2 + k_3 - \eta^{\lambda \nu} \eta^{\mu} \tau \eta^{\rho} \sigma \kappa_2$ $\eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_2$ · k_3 + $\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_2$ · k_3 - $\eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_2$ · k_3 - $\eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_2$ · k_3 - $2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_2\cdot k_3$

Gravitational wave

Quantum amplitudes for classical gravity:

- PN and PM correction for classical potential
- *8 CLASSICAL DOUBLE COPY* • Gravitational radiation

Gravitational wave

Quantum amplitudes for classical gravity:

- PN and PM correction for classical potential
- Gravitational radiation

Ref. [20] and discussed at length in this paper is highlighted in the shaded (red) region. The overlap gives **Z. Bern et.al, arXiv: 1908.01493** strong crosschecks on any calculations in either approach.

Interpretation and also an equation for the Lie algebra \mathcal{A} in the Lie algebra value \mathcal{A} [*L^p*¹ *, L^p*²] = *iX*(*p*1*, p*2)*L^p*1+*p*² \mathbf{a}

Does the Jacobi relation for momentum have any physical meaning? Doon the locabi relation for momentu investigate and possible structure of the fields, we choose the fields, we can invest no structure of the simplest nontrivial fields available. These are the self-dual solutions. Figure the self-dual solutions. Figure the self-d
The self-dual solutions. Figure the self-dual solutions. Figure the self-dual solutions. Figure the self-dual DOES THE JACODI FEIATION TOF MOMENTU any physical meaning? Ω meaning² scalar equation with a cubic coupling. Does the Jacobi relation for momentum have preserving di↵eomorphisms of *S*² and the Lie algebra of the generators of SU(*N*) in the planar limit *N* ! 1, in the sense that there exists an appropriate basis such that the

Self-dual YM/gravity [Monteiro. O'Connell 2011]
\n
$$
F_{\mu\nu} = \frac{i}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \qquad \Phi^a(k) = \frac{1}{2} g \int d p_1 d p_2 \frac{F_{p_1 p_2}{}^k f^{b_1 b_2 a}}{k^2} \Phi^{b_1}(p_1) \Phi^{b_2}(p_2)
$$
\n
$$
R_{\mu\nu\lambda\delta} = \frac{i}{2} \varepsilon_{\mu\nu\rho\sigma} R^{\rho\sigma}{}_{\lambda\delta} \qquad \phi(k) = \frac{1}{2} \kappa \int d p_1 d p_2 \frac{X(p_1, p_2) F_{p_1 p_2}{}^k}{k^2} \phi(p_1) \phi(p_2)
$$

 $\mathcal{L} = ik \cdot \mathcal{L}$ (2), $\mathcal{L} = \mathcal{L} + ik \cdot \mathcal{L}$ (3), $\mathcal{L} = \mathcal{L} + ik \cdot \mathcal{L}$ (3), $\mathcal{L} = i \mathcal{L}$ (3), $\mathcal{L} = i \mathcal{L}$ (3), $\mathcal{L} = -i \mathcal{L}$ $[U_{\nu} \partial_{u} + k_{u} \partial_{w}]$ $[L_{p_1}, L_{p_2}] = iX(p_1, p_2)L_{p_1+p_2} = iF_{p_1p_2}^{\mu}{}^{\kappa}L_{k}$ $\begin{array}{ccccccccccccccccc}\nI & \Omega & & I & \Omega\n\end{array}$, the solution is \mathcal{L} $L_k = e^{-ik \cdot x}(-k_w \partial_u + k_w \partial_w)$ $[L_{p_1}, L_{p_2}] = iX(p_1, p_2)L_{p_1+p_2} = iF_{p_1p_2}{}^kL_k$

*did devels really really be understood in F^p*1*^q ^k F^p*2*p*³ *q* ⌘ *d q* (*p*¹ + *q k*) *X*(*p*1*, q*) (*p*² + *p*³ *q*) *X*(*p*2*, p*3) $\frac{1}{2}$ diffeomorphism (*k*) the can be understood from the (1)(*k*) = ¹ \bigcap *d p*1*d p*² a of area-preserving diffeomorphism. *^k*² *^j*(*p*1)*j*(*p*2)*,* (57) 50 ual locabi ralation con be understoo. WE FORTUN SEEN THAT THE SEEN THAT THE KING-MILLS LIGHT AND THE KING-MILLS LIGHT AND ALGEBRA. Dual Jacobi relation can be understood from the algebra of area-preserving diffeomorphism.

Classical solutions use the Minkowski metric diag(−, +, +, +,...) throughout, unless otherwise stated. 3 Kerr-Schild coordinates and the double copy :al solutic , (15)

Double copy for "Kerr-Schild" type solutions: Schild Coording (Karr, Schild" type colutions: \mathcal{L} specific class of solutions of the Einstein equations, namely \mathcal{L} $S_{\rm eff}$ coordinates, that will be coordinates, that will be coordinates are applicable to \sim Double copy for "Kerr-Schild" type solutions: see e.g. [63]). These have that the property that the property that the spacetime metric g μ Judie Copy for new soling type solutions.

 $w \rightarrow w \mu \nu - \nu - g \nu \nu$

$$
\begin{array}{ll}\n\text{gravity} & g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} & \text{gauge} \qquad A^a_{\mu} = k_{\mu} \phi^a \\
\\
\equiv \eta_{\mu\nu} + k_{\mu} k_{\nu} \phi & \\
\\
\eta^{\mu\nu} k_{\mu} k_{\nu} = 0 = g^{\mu\nu} k_{\mu} k_{\nu}\n\end{array}
$$

that it is null with respect to both the Minkowski and full metric. The Minkowski and full metric: the Minkows
The Minkowski and full metric: the Minkowski and full metric: the Minkowski and full metric: the Minkowski and \exists xamples: \Box Examples:

where the index on the Schwarzschild BH

Example function function function of Connell, White 2014] 555_F recent work regarding the construction of pure gravity \sim 65_F