Geometric Framework for Supersymmetric Theories

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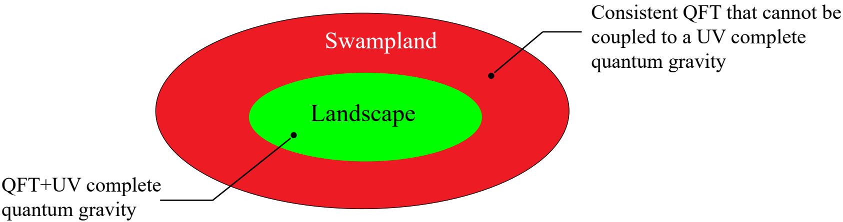
Introduction

(1) Construct non-trivial QFTs in various space-time dimensions

(2) Construct UV complete quantum gravity theories

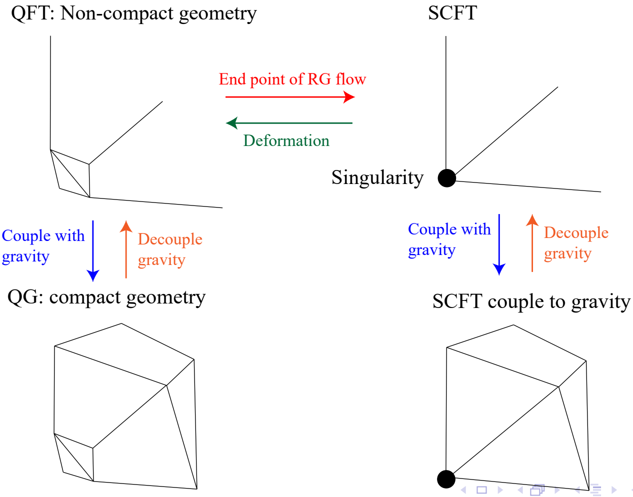
(3) Decide which low energy effective field theory can be coupled to gravity



- In this talk, assuming Minkowski space-time
- Einstein gravity in the low energy limit

Geometric framework

- Superstring/M/F-theory on a geometric space, get lower dimensional theory in $\mathbb{R}^{1,d-1}$
- Typically preserves a fraction of SUSY



Geometric framework

- Spacetime dimensions and SUSY

(1) 6d (1,0)

(2) 5d $\mathcal{N} = 1$

(3) 4d $\mathcal{N} = 1$

(4) 2d (0,2)

- 6d (1,0) supermultiplets:

(1) Tensor multiplet: self-dual tensor $B_{\mu\nu}$, 1 chiral spinor Ψ, real scalar Φ

(2) Vector multiplet: A_μ, 1 anti-chiral spinor χ

(3) Hypermultiplet: 1 chiral spinor ψ, 2 complex scalars ϕ

- Bosonic action

$$S = 2\pi \int \eta^{ij} \left(-\frac{1}{4}\partial_\mu \Phi_i \partial^\mu \Phi_j - \frac{1}{2} H_i \wedge *H_j + \frac{1}{4}\Phi_i Tr(F_j \wedge *F_j) + \frac{1}{4} B_i \wedge F_j \wedge F_j + \dots \right) \tag{1}$$

- Tensor-gauge quiver

$$\begin{array}{ccccccc} & \mathfrak{g}_1 & \mathfrak{g}_2 & & \mathfrak{g}_n & & \\ [G_L] & -\eta^{11}- & \eta^{22} & -\cdots- & \eta^{nn} & -[G_R] \\ & [G_{F,1}] & [G_{F,2}] & & [G_{F,n}] & & \end{array} \tag{2}$$

- $\mathfrak{g}_i$: gauge group; G_L, G_R, $G_{F,j}$: flavor symmetry group

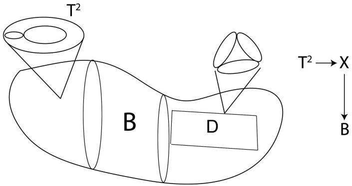
6d (1,0) theory

- Geometric classification of 6d (1,0) SCFTs in F-theory framework

(Heckman, Morrison, Vafa 13', Heckman, Morrison, Rudelius, Vafa 15')(Bhardwaj 19')

- F-theory: IIB superstring theory with a varying axiodilaton

$\tau = C_0 + i g_s^{-1}$



- Elliptic threefold X over a non-Ricci-flat complex surface B
- $\tau \cong$ modulus of the T^2 fiber
- Degenerate $T^2 \to$ location of 7-branes in IIB superstring theory

(1) Geometric gauge group

(2) Matter fields

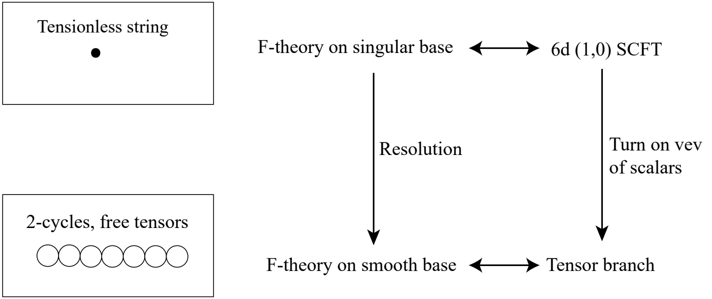
6d (1,0) theory

- In F-theory, consider 2-cycles $C_i \subset B$.
- D3 brane wrapping $C_i \to$ String that coupled to $B_{\mu\nu,i}$ with tension

$$T_i \propto \mathrm{Vol}(C_i) \propto |\langle \Phi_i \rangle|. \tag{3}$$

(1) All $T_i > 0$: IR "tensor branch" description of the 6d theory.

(2) All $T_i = 0$: SCFT with tensionless strings.



Examples

• 6d (1,0) SCFTs are classified by their tensor branch (shrinkable geometry)

(1) Rank-1 E-string

$$[E_8] - 1 \tag{4}$$

(2) Minimal (E_8, E_8) Conformal Matter (Del Zotto, Heckman, Tomasiello, Vafa 14')

$$[E_8] - 1 - 2 - \overset{\mathfrak{su}(2)}{2} - \overset{\mathfrak{g}_2}{3} - 1 - \overset{\mathfrak{f}_4}{5} - 1 - \overset{\mathfrak{g}_2}{3} - \overset{\mathfrak{su}(2)}{2} - 2 - 1 - [E_8]\,. \tag{5}$$

(3) Non-minimal (E_8, E_8) Conformal Matter with order N

• Glue N copies of these chains by gauging $[E_8]$

Couple to gravity

- Question: what is the 6d (1,0) SCFT with the longest tensor branch that can be coupled to supergravity?
- F-theory on a compact elliptic CY3 X_3 with $(h^{1,1}, h^{2,1}) = (491, 11)$

(Morrison, Taylor 12')(Taylor 12')

- Base geometry (toric):

$$(-12//-11//(-12//)^{13}, -11//-12, 0)\,. \tag{6}$$

$$// \equiv -1, -2, -2, -3, -1, -5, -1, -3, -2, -2, -1\,. \tag{7}$$

- Gravity coupled to the tensor branch of non-minimal (E_8, E_8) conformal matter with order 16.
- # Tensor multiplets = 193.
- How to prove this physically?

5d $\mathcal{N} = 1$ theories

- 5d $\mathcal{N} = 1$ SUSY gauge theory, supermultiplets:

(1) Vector multiplet with gauge group G: A_μ, λ, ϕ

(2) Hypermultiplets in rep. R: ψ, $h \oplus h^c$

$$S = \int d^5x \left[\frac{1}{g_{\mathrm{YM}}^2} \mathrm{Tr}(F^{\mu\nu} F_{\mu\nu}) + i\bar{\psi} \not{D} \psi + \dots \right] \quad (8)$$

- Action is always non-renormalizable!
- For some 5d $\mathcal{N} = 1$ gauge theories, it can be UV completed to a strongly coupled SCFT when $g_{\mathrm{YM}} \to \infty$ (Seiberg 96')(Intriligator, Morrison, Seiberg 96').
- The UV flavor symmetry is enhanced from IR flavor symmetry $G_{F,IR}$ to

$$G_F \supset G_{F,IR} \,. \quad (9)$$

- Seiberg E_{N_f+1} theories: $SU(2) + N_f \boldsymbol{F}$ ($N_f \leq 7$),

$$G_F = E_{N_f+1} \supset SO(2N_f) = G_{F,IR} \,. \quad (10)$$

5d $\mathcal{N} = 1$ theories

(1) What if $N_f = 8$?

- KK reduction of rank-1 E-string theory on S^1!
- Known as the "marginal theory", has a 6d UV completion.

(2) What if $N_f > 8$?

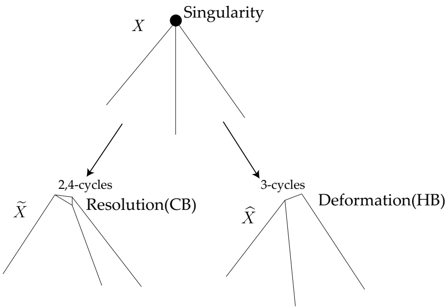
- No UV completion by itself.

5d $\mathcal{N} = 1$ theories

- Natural realization: 11d M-theory on canonical threefold singularity
- Classification of canonical threefold singularities $X \to$ partial
classification of 5d $\mathcal{N} = 1$ SCFT $\mathcal{T}_{\mathbf{X}}^{5\mathrm{d}}$! (Xie, Yau 15')
- Goal: read off information of $\mathcal{T}_{\mathbf{X}}^{5\mathrm{d}}$ using geometric data of X.
- G_F, IR gauge theory limits...

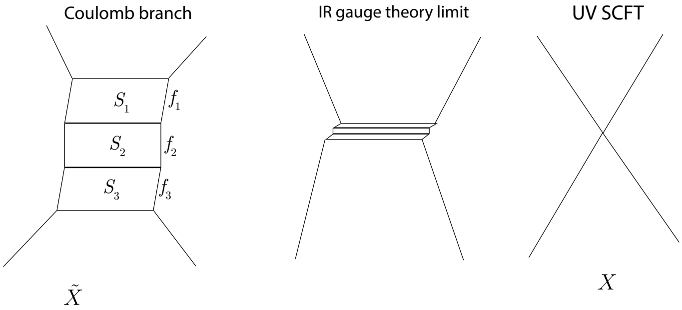
(1) $\langle\phi\rangle \neq 0$: Coulomb branch (CB)

(2) $\langle h\rangle \neq 0$: Higgs branch (HB)



Coulomb branches

- Effective theory: $U(1)^r$ gauge theory+matter fields
- Real dimension of CB r: *rank* of 5d $\mathcal{N}=1$ SCFT
- Resolution of X: $\tilde{X}$, a non-compact Calabi-Yau threefold (Ricci-flat)
- $r = \#$ of compact 4-cycles S_i
- Gauge boson and matter hypermultiplets arise from M2-brane wrapping 2-cycles in $\tilde{X}$
- IR non-Abelian gauge theory description: ruling structure of S_i



Geometric Classifications

(1) Partial classification of 5d SCFTs based on the geometries of S_i

Bhardwaj, Jefferson, Katz, Kim, Tarazi, Vafa, Zafrir...

- Up to rank $r = 3$, or IR gauge theory with a simple gauge group.

(2) Resolution of non-isolated singularities

- Apruzzi, Lawrie, Lin, Schafer-Nameki, YNW, "5d Superconformal Field Theories and Graphs", Physics Letters B 800, (2020) 135077
- Apruzzi, Lawrie, Lin, Schafer-Nameki, YNW, "Fibers add Flavor, Part I: Classification of 5d SCFTs, Flavor Symmetries and BPS States", JHEP 11 (2019) 068
- Apruzzi, Lawrie, Lin, Schafer-Nameki, YNW, "Fibers add Flavor, Part II: 5d SCFTs, Gauge Theories, and Dualities", JHEP 03 (2020) 052
- Apruzzi, Schafer-Nameki, YNW, "5d SCFTs from Decoupling and Gluing", JHEP 08 (2020) 153

(3) Resolution and deformation of isolated singularities

- Closset, Schafer-Nameki, YNW, "Coulomb and Higgs Branches from Canonical Singularities: Part 0", JHEP 02 (2021) 003
- Closset, Giacomelli, Schafer-Nameki, YNW, "5d and 4d SCFTs: Canonical Singularities, Trinions and S-Dualities", JHEP 05 (2021) 274

Non-isolated singularities

- Resolutions of non-minimal Weierstrass models in 6d F-theory(Apruzzi, Lawrie, Ling, Schafer-Nameki, YNW 19')
- Example: rank-1 E-string

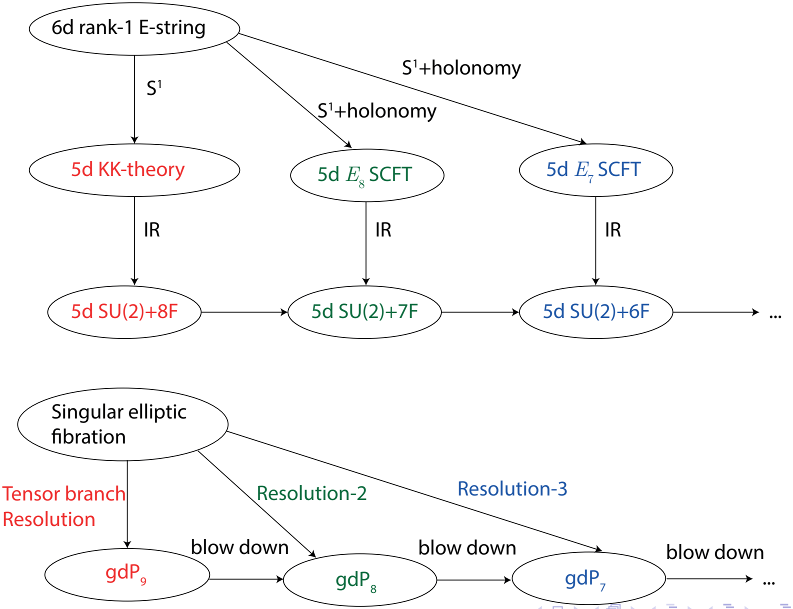
$$[E_8] - 1 \tag{11}$$

- Start with a non-isolated singularity

$$y^2 = x^3 + u^4 x + u^5 v\,. \tag{12}$$

- non-compact elliptic Calabi-Yau threefold with base $B = \mathbb{C}^2$
- After the resolution, a single compact 4-cycle S_1 and a number of new non-compact 4-cycles D_α.
- Topology of S_1 depends on the resolution sequence

Resolution of rank-1 theories



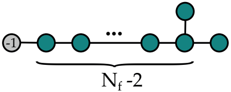
Combined Fiber Diagram

- Using the intersection numbers between non-compact and compact 4-cycles after the resolution, define a Combined Fiber Diagram (CFD)

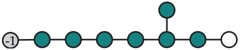
(1) The non-Abelian UV flavor symmetry $G_{F,nA}$ is given by the Dynkin diagram in green color

(2) Possible IR gauge theory descriptions are constrained by the subgraph structure of CFD;

E. g. $SU(2)/Sp(N) + N_f\mathbf{F}$: $G_{F,IR} = SO(2N_f)$

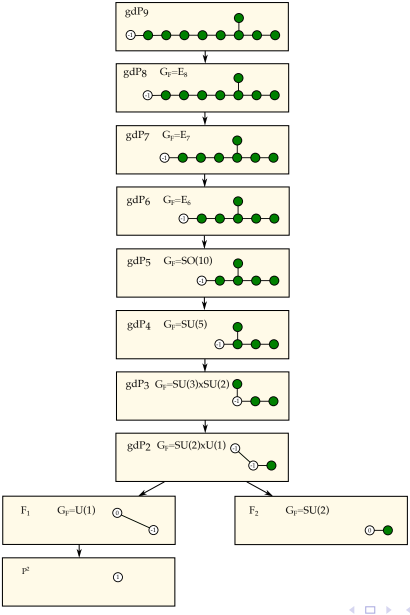


- Example: $SU(2) + 7\mathbf{F}$



(3) *CFD transition* (mass deformation of IR gauge theory): removing (-1)-vertex, transform the others

Combined Fiber Diagram for rank-1 theories

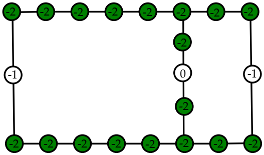


(E_8, E_8) Conformal Matter

(1) Minimal (E_8, E_8) Conformal Matter

$$[E_8] - 1 - 2 - \overset{\mathfrak{su}(2)}{2} - \overset{\mathfrak{g}_2}{3} - 1 - \overset{\mathfrak{f}_4}{5} - 1 - \overset{\mathfrak{g}_2}{3} - \overset{\mathfrak{su}(2)}{2} - 2 - 1 - [E_8] \,. \quad (13)$$

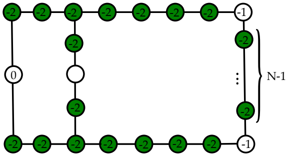
- From the topology of 21 compact surfaces $\rightarrow$ CFD for the KK theory



(2) Non-minimal (E_8, E_8) Conformal Matter with order N

- From the marginal resolution, $29N - 8$ compact surfaces.
- To get a 5d UV SCFT description, needs to decompactify $N - 1$ surfaces (Apruzzi, Schafer-Nameki, YNW 19').

- CFD for a 5d SCFT with $G_F = E_8^2 \times SU(N)$ constructed in this way:



- IR quiver gauge theory description (Ohmori, Shimizu, Tachikawa, Yonekura 19'):

$$\begin{array}{c} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad SU(3N) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad | \\ [SU(N)] - SU(2N) - SU(3N) - SU(4N) - SU(5N) - SU(6N) - SU(4N) - SU(2N) \end{array} \tag{14}$$

5d gauge theory coupled to gravity

• What is the largest non-Abelian gauge theory coupled to 5d supergravity?

• M-theory on the CY3 X_3 with $(h^{1,1}, h^{2,1}) = (491, 11)$

• X_3 also has an elliptic fibration structure, non-minimal (E_8, E_8) conformal matter with order 16!

• Non-Abelian quiver gauge theory (YNW 20'):

$$\begin{array}{ccc} & SU(48) & \\ & | & \\ SU(16) - SU(32) - SU(48) - SU(64) - SU(80) - & SU(96) & - SU(64) - SU(32)\,. \end{array} \tag{15}$$

• $SU(96)$ is the largest $SU(N)$ gauge group coupled to 5d $\mathcal{N} = 1$ supergravity?

Higgs branch from isolated singularities

- G_F of a 5d SCFT can also be encoded in the Higgs branch, computed from Magnetic quiver $\mathrm{MQ}^{(5)}$ (Ferlito, Hanany, Mekareeya, Zafrir 18').
- Consider isolated hypersurface singularity $\mathbf{X}$
- IIB on $\mathbf{X} \to$ 4d $\mathcal{N}=2$ SCFT $\mathscr{T}_{\mathbf{X}}^{\mathrm{4d}}$ was studied by (Shapere, Vafa 96')(Xie, Yau 15', 16')(Wang, Xie, Yau, Yau 16')...
- Define $\mathrm{EQ}^{(4)} :=$ reduction of $\mathscr{T}_{\mathbf{X}}^{\mathrm{4d}}$ on S^1, flow to IR
- $\mathscr{T}_{\mathbf{X}}^{\mathrm{4d}}$ and $\mathrm{EQ}^{(4)}$ have CB dimension $\widehat{r}$.
- Conjecture (Closset, Schafer-Nameki, YNW 20'):

$$\mathrm{MQ}^{(5)} \cong \mathrm{EQ}^{(4)}/U(1)^f \ , \ d_H = \widehat{r} + f \tag{16}$$

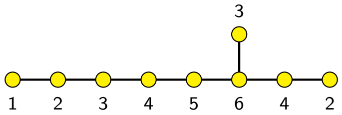
- f is the flavor rank preserved on the HB of $\mathcal{T}_{\mathbf{X}}^{\mathrm{5d}}$ and $\mathscr{T}_{\mathbf{X}}^{\mathrm{4d}}$

Higgs branch from isolated singularities

- Rank-1 Seiberg E_8 theory:

$$F(x) = x_1^2 + x_2^3 + x_3^6 + x_4^6 = 0\,. \tag{17}$$

4d $SU(N)$ quiver description of $\mathscr{T}_{\mathbf{X}}^{\mathrm{4d}}$, same as $\mathrm{EQ}^{(4)}$:



(18)

- 5d magnetic quiver $\mathrm{MQ}^{(5)} \cong \mathrm{EQ}^{(4)}/U(1)^8$, $\mathrm{MQ}^{(5)}$ is a $U(N)$ quiver with the same shape, mod out a common U(1)
- Affine E_8 Dynkin diagram $\rightarrow G_F = E_8$
- Studied a large class of non-Lagrangian 4d theories (Closset, Giacomelli, Schafer-Nameki, YNW 20')

4d $\mathcal{N} = 1$ F-theory

- What is the largest gauge group coupled to 4d $\mathcal{N} = 1$ supergravity? Largest number of axion fields?
- Current record: F-theory on elliptic Calabi-Yau fourfold with the largest $h^{1,1} = 303148$.
  - YNW, "On the Elliptic Calabi-Yau Fourfold with Maximal $h^{1,1}$", JHEP 05 (2020) 043
- Constructed the base threefold and elliptic fibration in full details
- Gauge group from 7-branes (Candelas, Perevalov, Rajesh 97'):

$$G = E_8^{2561} \times F_4^{7576} \times G_2^{20168} \times SU(2)^{30200} \tag{19}$$

- Number of axions

$$N_{\text{axion}} = 181820 \tag{20}$$

2d $(0,2)$ F-theory

- What is the largest gauge group coupled to 2d $(0,2)$ supergravity?
  - Tian, YNW, "Elliptic Calabi-Yau fivefolds and 2d (0,2) F-theory landscape", JHEP 03 (2021) 069
- Elliptic Calabi-Yau fivefold with

$$(h^{1,1}, h^{2,1}, h^{3,1}, h^{4,1}) = (247538602581, 0, 0, 151701) \tag{21}$$

- Gauge group from 7-branes:

$$G = E_8^{482\,632\,421} \times F_4^{3\,224\,195\,728} \times G_2^{11\,927\,989\,964} \times SU(2)^{25\,625\,222\,180} . \tag{22}$$

Future questions

- How to prove any of these bounds using swampland arguments, or break the records?
- Extend the program to non-geometric setups, e. g. brane webs