EXCITATION TRANSMISSION THROUGH A NON-HERMITIAN TRAVERSABLE WORMHOLE

XIAN-HUI GE 葛先辉

上海大学物理系

S.Z. Cao, XHG, 2024, 2404.11436 [hep-th]
W.H. Cai, S. Z. Cao, XHG, PRD 2022 arXiv: 2208.10800
J. Low, S. Z. Cao, XHG, PRD 2023 arXiv: 2305.05394

MOTIVATIONS

- Why SYK?
- Non-Hermitian systems: One of the primary reasons is due to the probability in nature effectively becomes non-conserving due to the presence of energy, particles, and information regarding the external degrees of freedom that are out of the Hilbert space.
- A huge class of non-conservative Hamiltonians can exhibit entirely real spectra as long as they commute with the parity-time (PT) operator.
- The similarity transformations can also enable one to construct non-Hermitian Hamiltonian with real spectrum.
- Thus, considering the non-Hermiticity and two coupled SYK models, we develop novel non-Hermitian two-site SYK models.

Introduction to SYK model

- In 1993, Sachdev and Ye proposed a quantum Heisenberg model with random exchanges

$$H_g = \frac{1}{\sqrt{M}} \sum_g J_{ij} \vec{S}_i \cdot \vec{S}_j \qquad P[J_{ij}] \sim e^{-NJ_{ij}^2/2J^2}$$

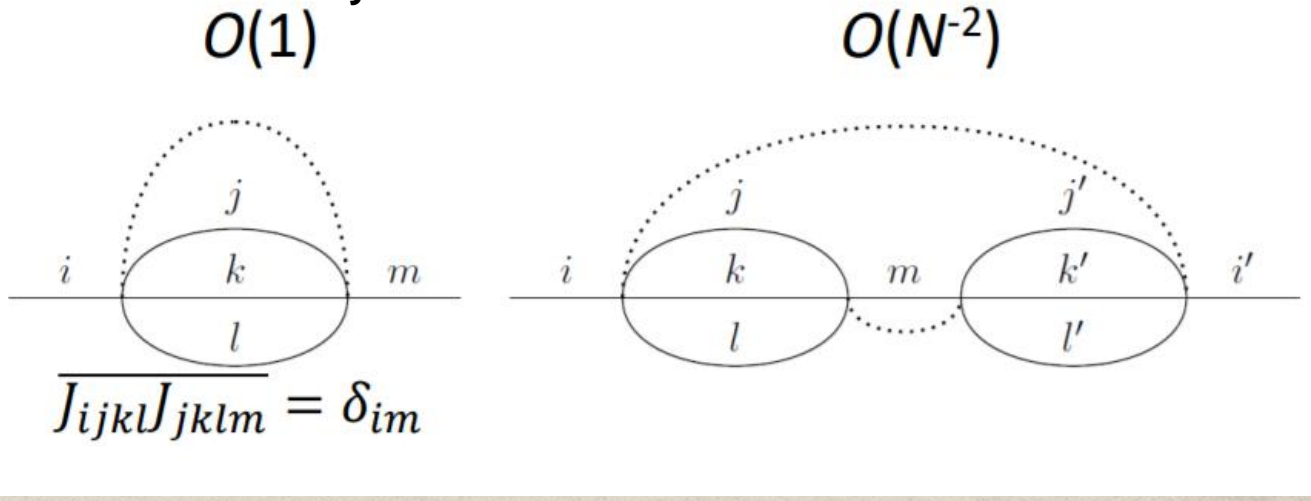
- Sachdev-Ye model

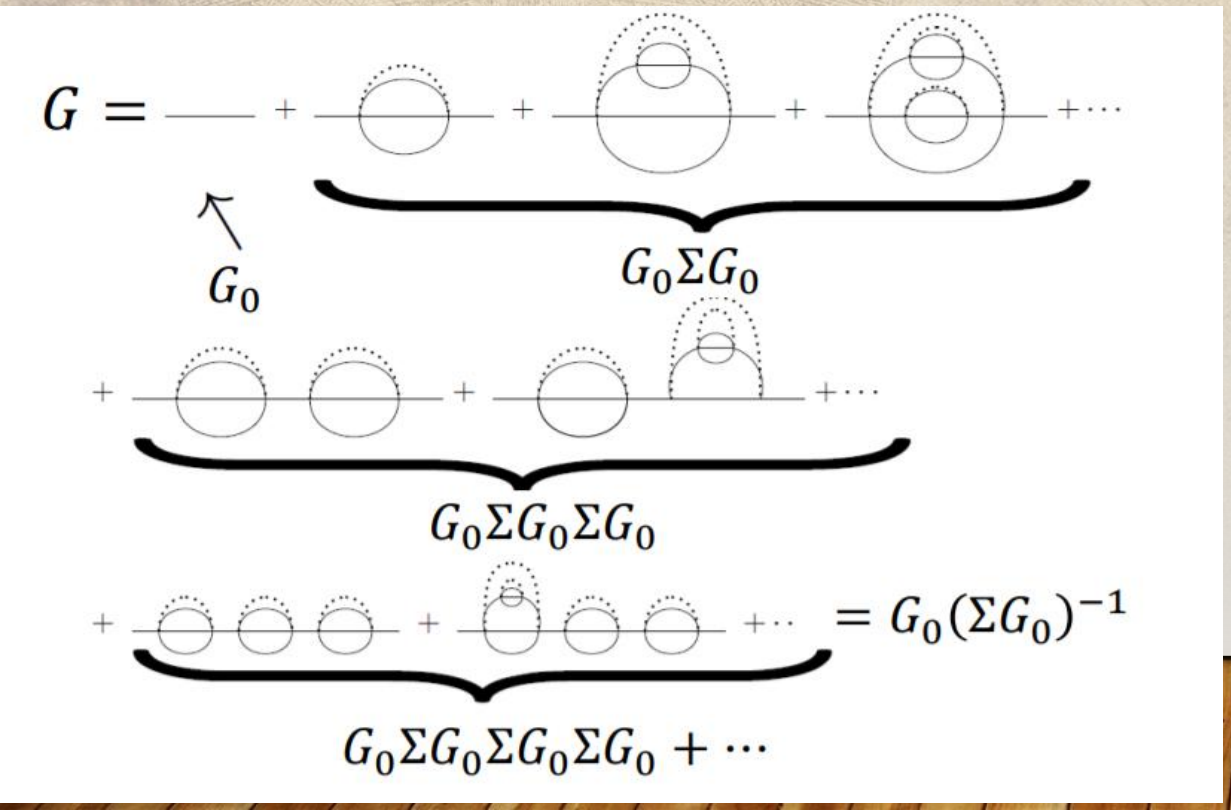
$$H_{SY} = \frac{1}{\sqrt{M}} \sum_{ij} J_{ij} c^+_{i\mu} c^+_{j\nu} c_{i\nu} c_{j\mu}$$

S. Sachdev and J. Ye, PRL, 70, 1993

- Kitaev's contribution: $H_{SYK} = (i)^{\frac{q}{2}} \sum_{ijkl} J_{ijkl} \chi_i \chi_j \chi_k \chi_l$
- Properties of SYK model 1:
- Analytically solvable in $N \gg 1$ limi

J. Maldacena and D. Stanford, arXiv:1604.07818





Only "melonic" diagram contributes

$$G(i\omega)^{-1} = i\omega - \Sigma(i\omega) \qquad \Sigma = J^2 G^3$$

Properties of SYK model

- @ Low energy limit, Ignore $i\omega$ $(\omega, T \ll J)$

$$G(i\omega)^{-1} = i\omega - \Sigma(i\omega) \qquad \Sigma = J^2 G^3$$

We have

$$\int dt G(t_1, t)\Sigma(t, t_2) = -\delta(t_1, t_2) \qquad J^2 \int dt G(t_1, t)G(t, t_2)^3 = -\delta(t_1, t_2)$$

- $$G(t) = -\left(\frac{1}{4\pi J^2}\right)^{1/4} \frac{\mathrm{sgn}(t)}{\sqrt{t}}$$

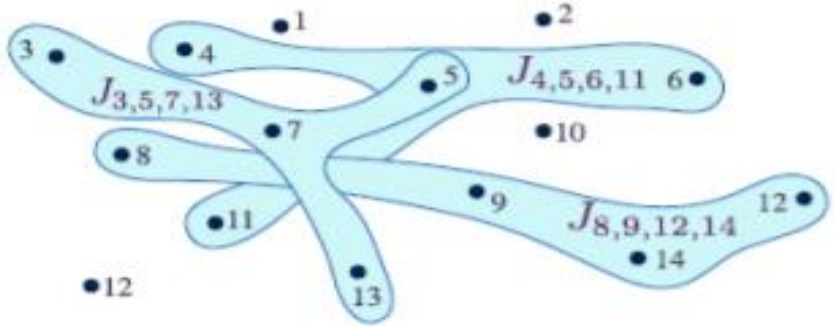
@Emergent conformal symmetry

@ Satisfies the chaos bound $\lambda_L \leqslant \frac{2\pi k_B T}{\hbar}$

Holographic duality

Global AdS2 ?

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

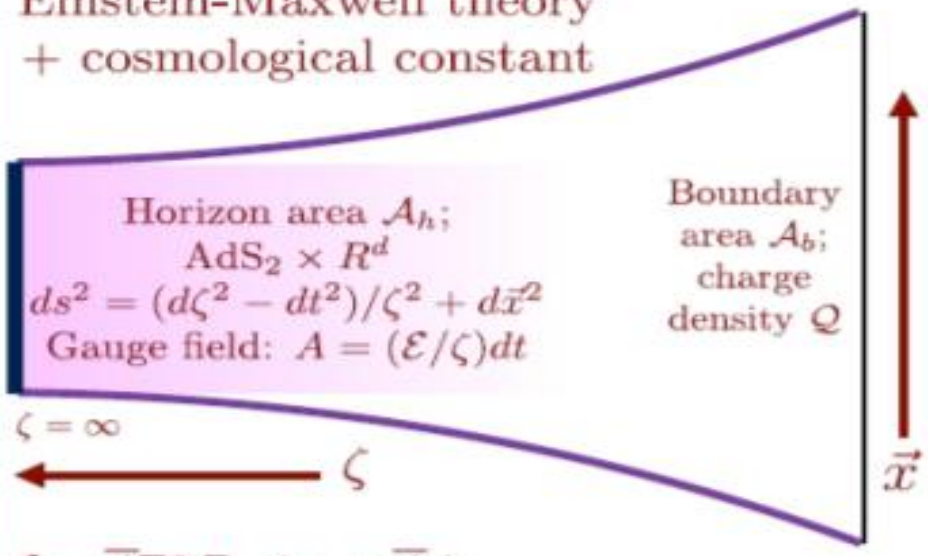
$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \tau > 0 \\ e^{-2\pi\mathcal{E}} |\tau|^{-1/2} & , \tau < 0. \end{cases}$$

Known "equation of state" determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density $\mathcal{S}$ obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory + cosmological constant



$$\mathcal{L} = \overline{\psi}\Gamma^a D_a \psi + m\overline{\psi}\psi$$

$$-\left\langle \psi(\tau)\overline{\psi}(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \tau > 0 \\ e^{-2\pi\mathcal{E}} |\tau|^{-1/2} & , \tau < 0. \end{cases}$$

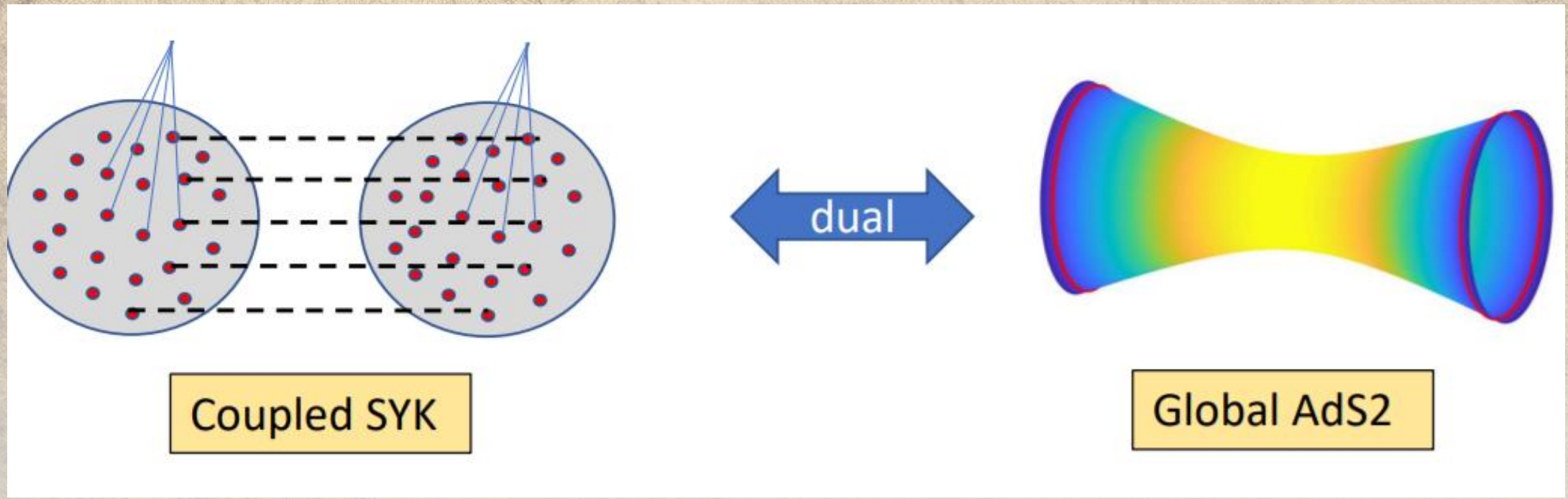
"Equation of state" relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS_2

Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

[S. Sachdev, Phys. Rev. X **5**, 041025 (2015)]

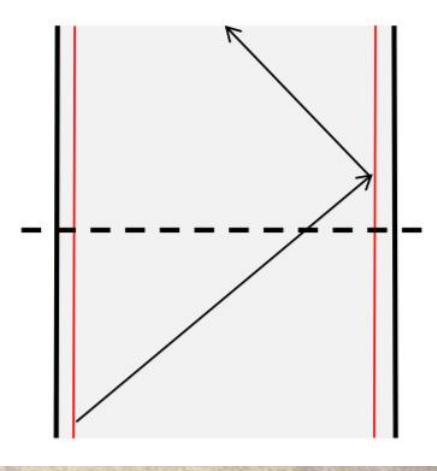
Eternal traversable wormhole and Hawking-Page phase transition

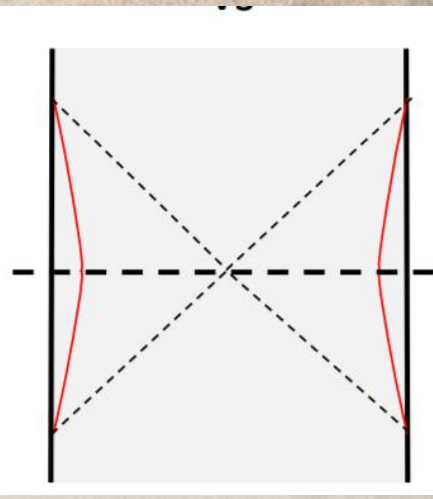


J. Maldecana and X. L. Qi, 1804.00491

Two coupled SYK ⟷ Global AdS2

Global AdS2





- Two causally connected boundaries
- Negative averaged null energy from matter field
- Eternal versions of the traversable wormhole
- The same state as $|\mathrm{TFD}\,(t=0)\rangle$ but with different time evolution
- Finding dual of global AdS2 ~ Finding a Hamiltonian with as ground state

J. Maldecana and X. L. Qi, 1804.00491

Two coupled SYK

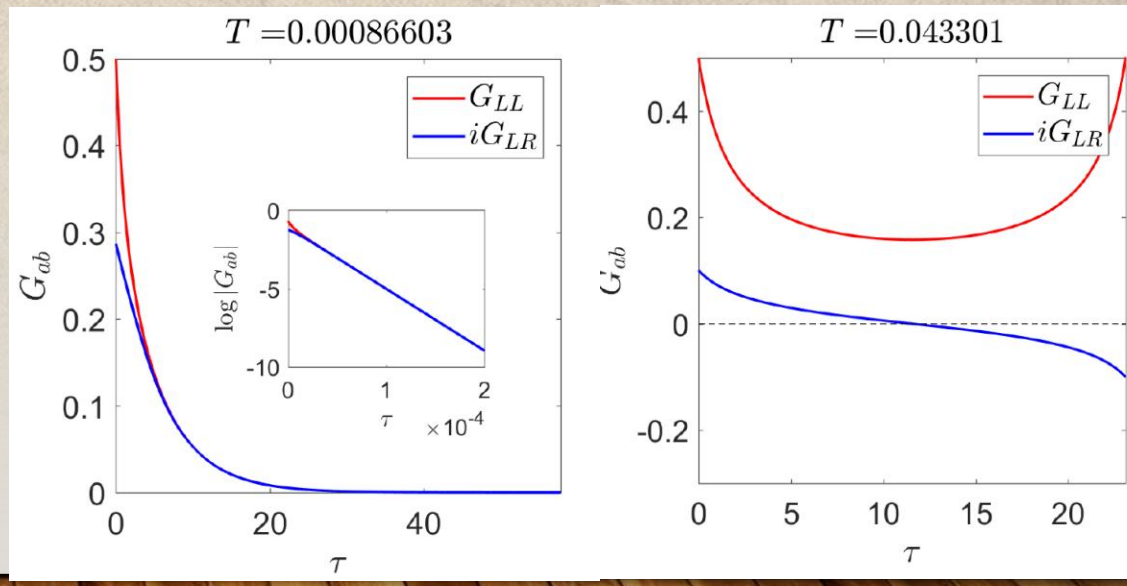
- The Hamiltonian: two SYK models with bilinear coupling

$$H = \sum J_{i_1 i_2 \ldots i_q} \left(i^{\frac{q}{2}} \chi_{i_1 L} \chi_{i_2 L} \cdots \chi_{i_q L} + i^{-\frac{q}{2}} \chi_{i_1 R} \chi_{i_2 R} \cdots \chi_{i_q R} \right) + i\mu \sum_i \chi_{iL} \chi_{iR}$$

- The Schwinger-Dyson equation

$$G(\omega_n) = (i\omega_n \mathbb{I} - \Sigma)^{-1}$$
$$\Sigma_{ab}(\tau) = J^2 G_{ab}(\tau)^{q-1} + i\mu \epsilon_{ab} \delta(\tau)$$

- Numerical solution

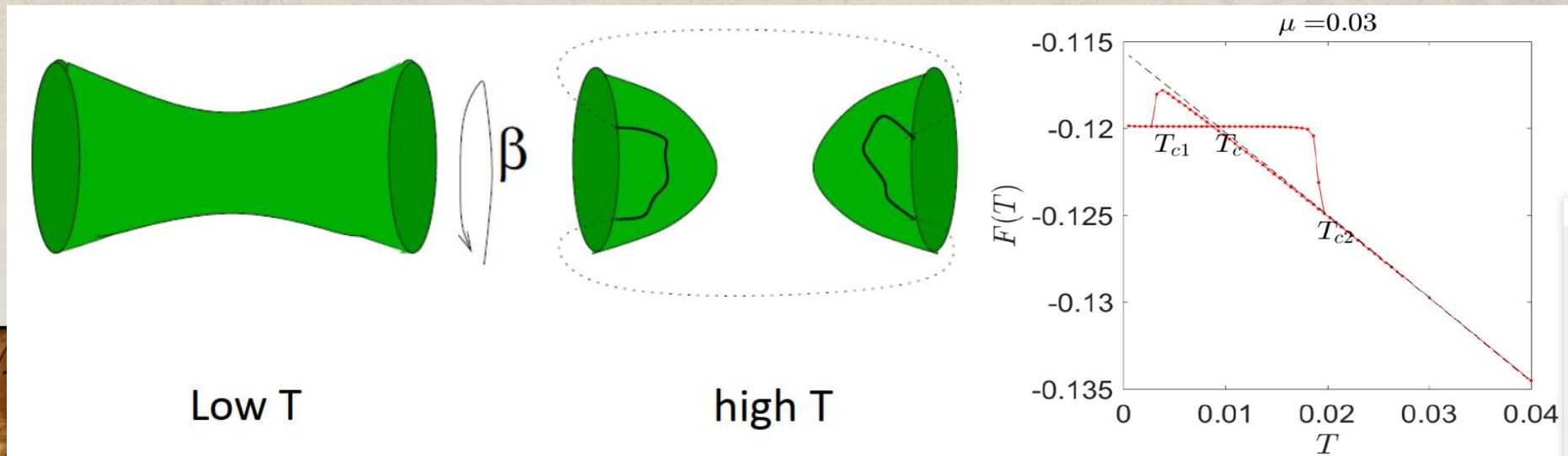


J. Maldacena and X. L. Qi, 1804.00491

Phase diagram

- Hawking-Page-like phase transition
- Low T phase: thermal gas in AdS2(wormhole)
- High T phase: two black holes, with coupled matter field
- F-T diagram: Swallow tail structure

J. Maldacena and X. L. Qi, 1804.00491



Non-Hermitian two coupled SYK model

- We consider non-Hermitian coupled SYK models.

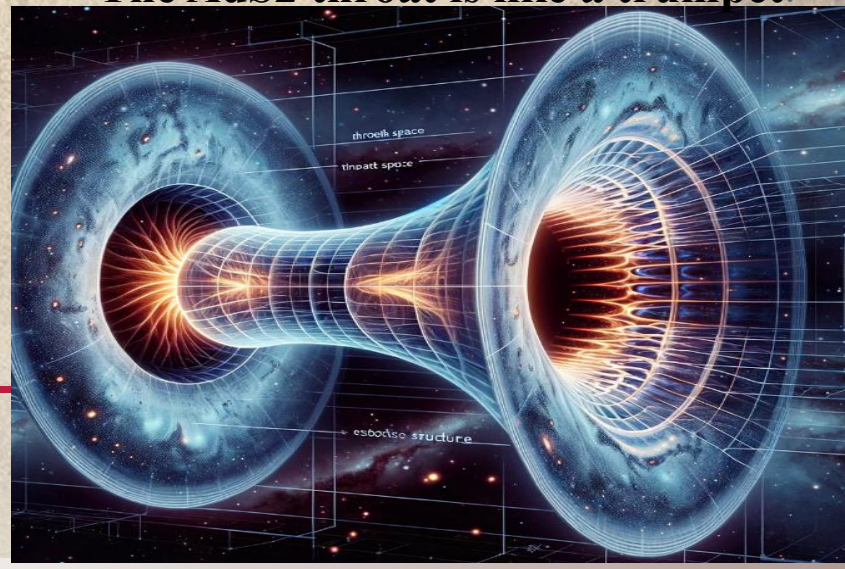
$$H = H_0 + H_{int},$$

$$H_0 = -\sum_{i,j,k,l=1}^{N} J_{ijkl} \sum_{A=L,R} \left(2C_i^{A\dagger}C_j^{A\dagger}C_k^A C_l^A + 4C_i^{A\dagger}C_j^A C_k^{A\dagger}C_l^A\right),$$

$$H_{int} = i\mu \sum_{i=1}^{N} (e^{-2\alpha} C_i^{L\dagger} C_i^R - e^{2\alpha} C_i^{R\dagger} C_i^L),$$

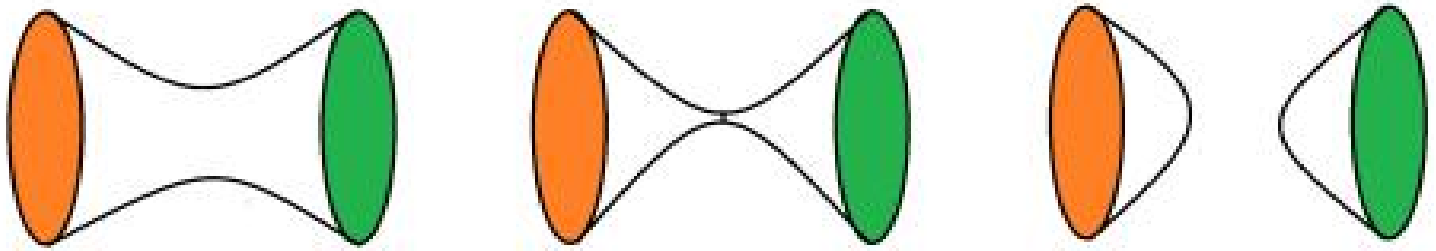
$$\{C_i^{A\dagger}, C_j^B\} = \delta_{ij}\delta_{AB}\,,\ \{C_i^{A\dagger}, C_j^{B\dagger}\} = \{C_i^A, C_j^B\} = 0\,.$$

The AdS2 throat is like a trumpet.



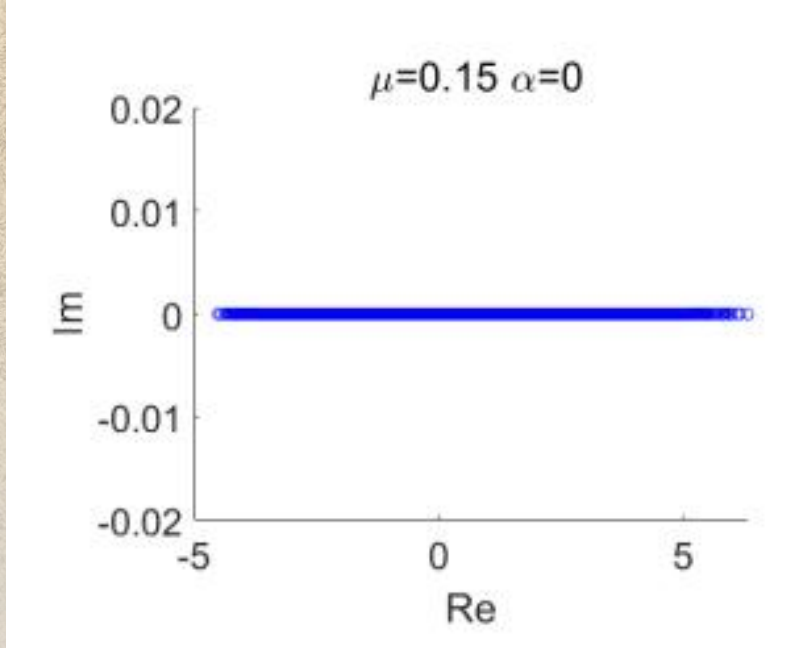
- The parameter α is a real number controlling the strength of non-Hermiticity, which is introduced by a non-Hermitian particle-hole similarity transformation.
- The coupling μ controls the coupling strength of the two SYK sites.

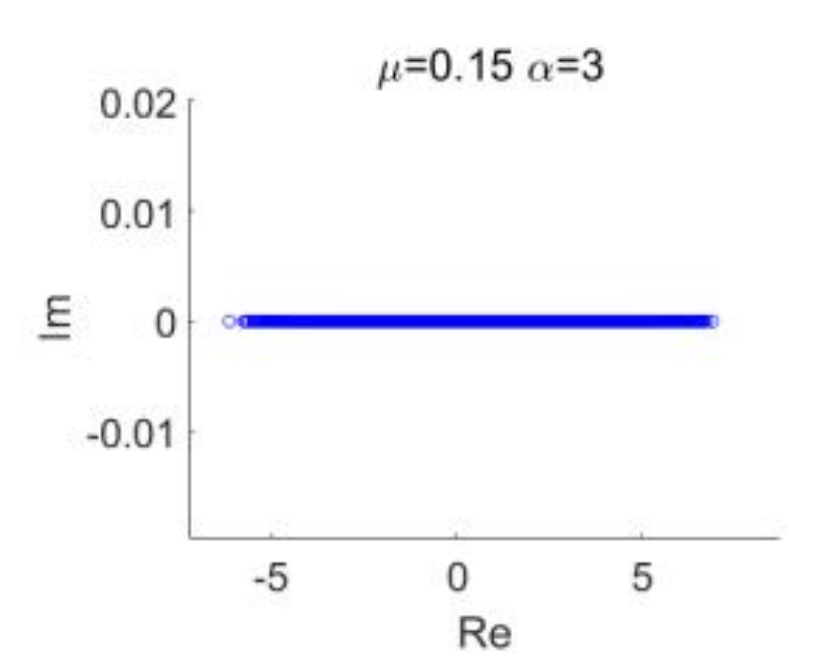
Sketched phase diagram



Left: We have a geometry connecting the two sides in the low temperature regime. The left Sachdev-Ye-Kitaev (SYK) and right SYK are in different states because of the non-Hermitian parameter.
Middle: An unstable geometry connecting two SYK sites.
Right: Two separated SYK sites at high temperature, representing the gapless two black hole phases. The non-Hermitian parameter can change the states of the left and right SYK sites, which are marked in orange and green colors. When $\alpha = 0$, the phase diagram recover that of MQ model.

Energy spectrum





Biorthogonal basis and ground states

- For this non-Hermitian systems, we introduce a biorthogonal set

$$H|\psi_n^r\rangle = E_n|\psi_n^r\rangle\,,\ H^\dagger|\psi_m^l\rangle = E_m^*|\psi_m^l\rangle$$

- The eigenstates satisfy

$$\sum_n |\psi_n^l\rangle\langle\psi_n^r| = I\,,\ \langle\psi_n^l|\psi_m^r\rangle = \delta_{nm}$$

- The corresponding ground states of coupled Hamiltonian

$$|\psi_0^r\rangle = \prod_j^N \frac{1}{\sqrt{2}}(\tilde{A}|1\rangle_{L,j}|0\rangle_{R,j} + i\tilde{B}|0\rangle_{L,j}|1\rangle_{R,j})\,,$$

$$|\psi_0^l\rangle = \prod_j^N \frac{1}{\sqrt{2}}(\tilde{C}|1\rangle_{L,j}|0\rangle_{R,j} + i\tilde{D}|0\rangle_{L,j}|1\rangle_{R,j})\,,$$

Degree of entanglement

- Constraints: the system has the ground state energy

$$H_{int}|\psi_0^r\rangle = -\mu N|\psi_0^r\rangle \qquad H_{int}^{\dagger}|\psi_0^l\rangle = -\mu N|\psi_0^l\rangle,$$

- The ground states satisfy

$$\langle\psi_0^l|\psi_0^r\rangle = 1.$$

- The coefficients $\tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}$ satisfy the relation

$$\tilde{B} = \tilde{A}e^{2\alpha}, \quad \tilde{D} = \tilde{C}e^{-2\alpha}, \quad \tilde{A}\tilde{C} = \frac{1}{2}, \quad \tilde{B}\tilde{D}$$

- The degree of entanglement of the ground states is independent of the non-Hermitian parameter α

$$P_E = -tr(\rho_A \log \rho_A) = 1$$

Low energy effective action

- The retarded Green function of non-Hermitian system is defined as

$$G_{AB}(\tau_1,\tau_2)=\frac{1}{N}\sum_n\langle\psi_n^l|TC_i^{A\dagger}(\tau_1)C_i^B(\tau_2)|\psi_n^r\rangle$$

- The Schwarzian effective action

$$\frac{S}{N}=-2\alpha_S\int d\tau\{\tan\frac{h(\tau)}{2},\tau\}+K\int d\tau\left(\phi'(\tau)^2-\varepsilon^2h'(\tau)^2\right)+\frac{\mu}{2^{2(\Delta-1)}}\int d\tau\left(h'(\tau)\right)^{2\Delta},$$

- By introducing $\phi=\log h'$, we obtain the conserved charge

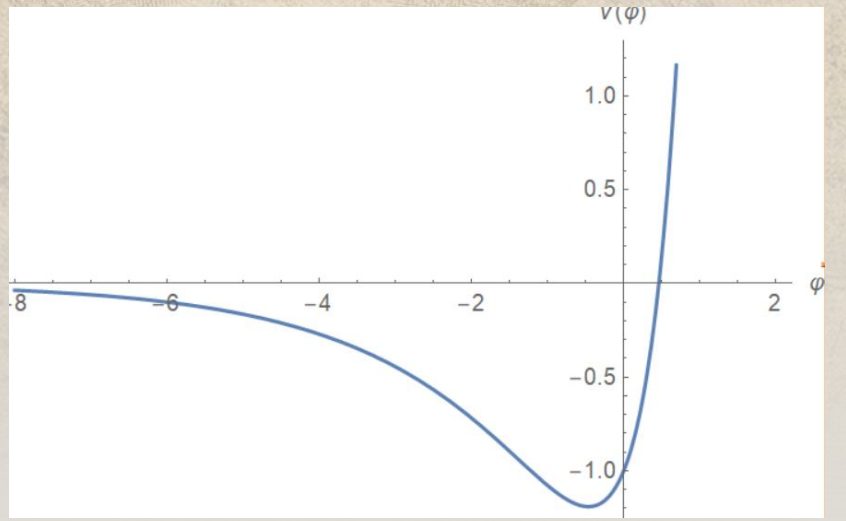
$$Q_0/N=2e^{-\phi}[-\phi''-e^{2\phi}+\Delta\mu e^{2\Delta\phi}]=0$$

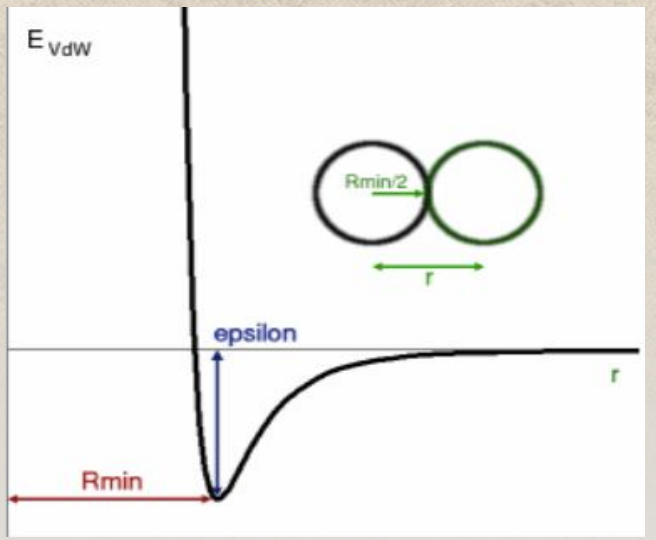
- We can derive the the action of a non-relativistic particle in a potential

$$S = N \int du \left[(\phi')^2 - \left(e^{2\phi} - \mu e^{\phi/2} \right) \right].$$

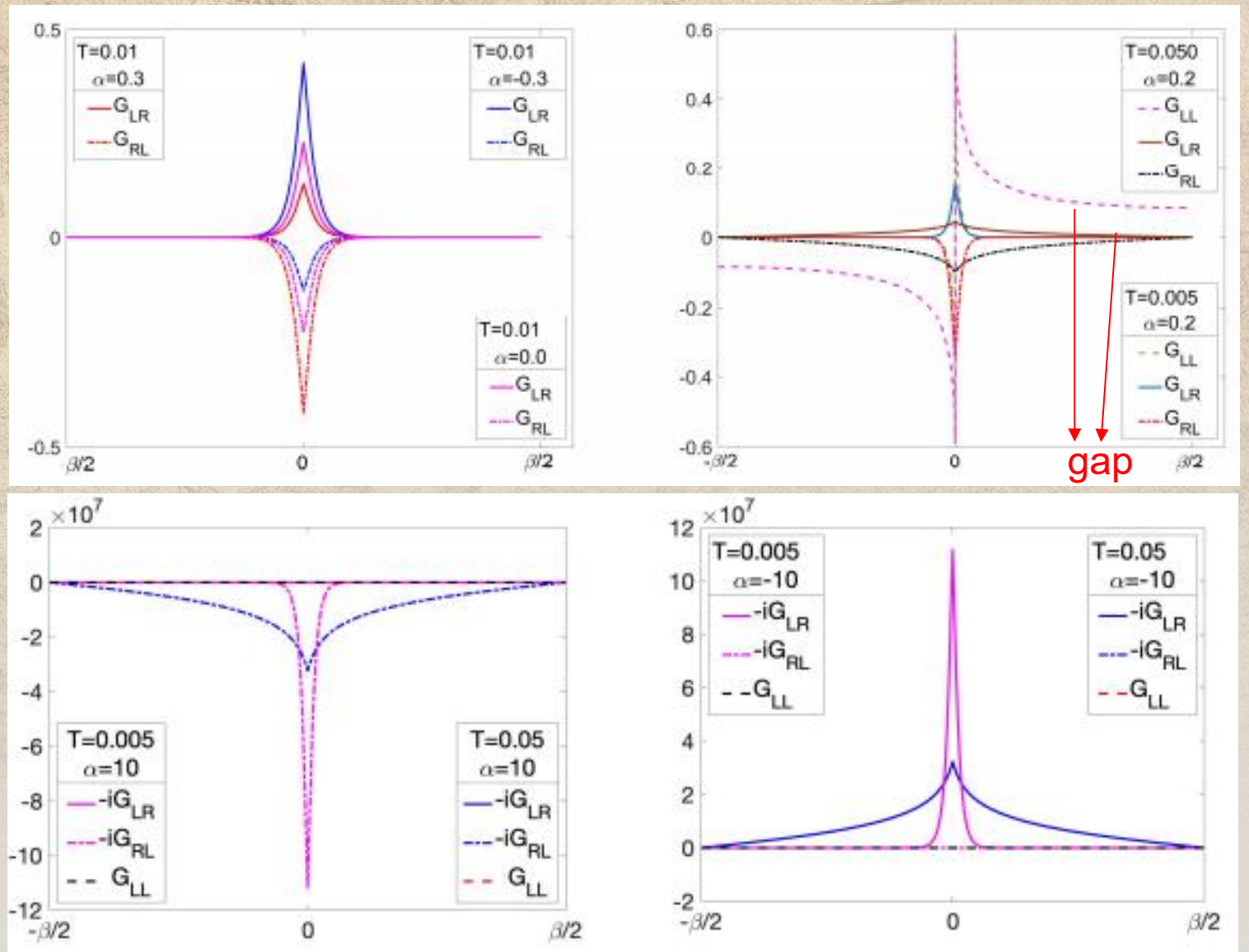
- Independent of the non-Hermitian parameter α.

- We then expect to have phase structure independent of the non-Hermitian parameter.



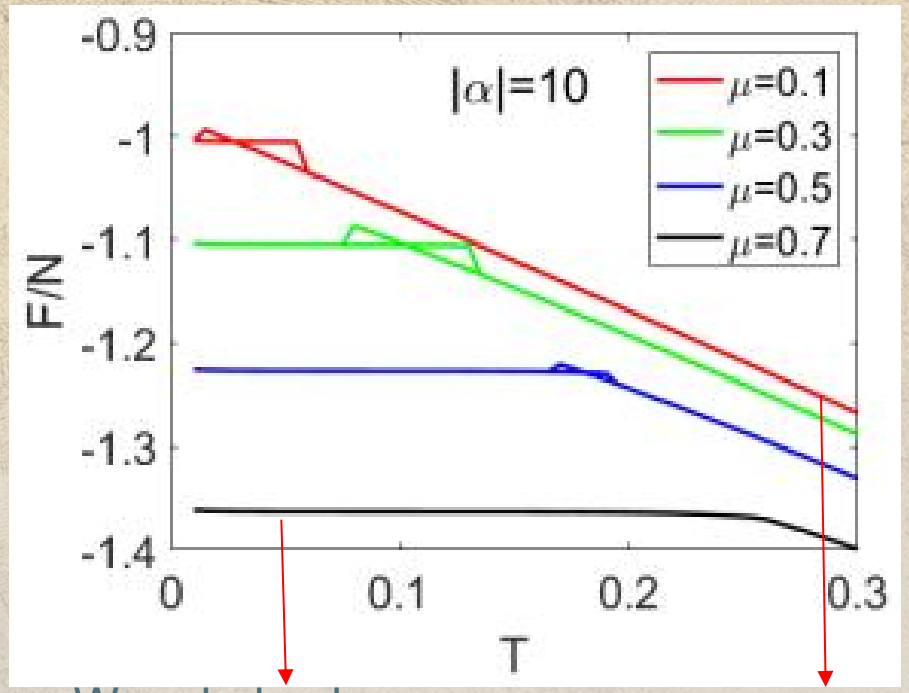


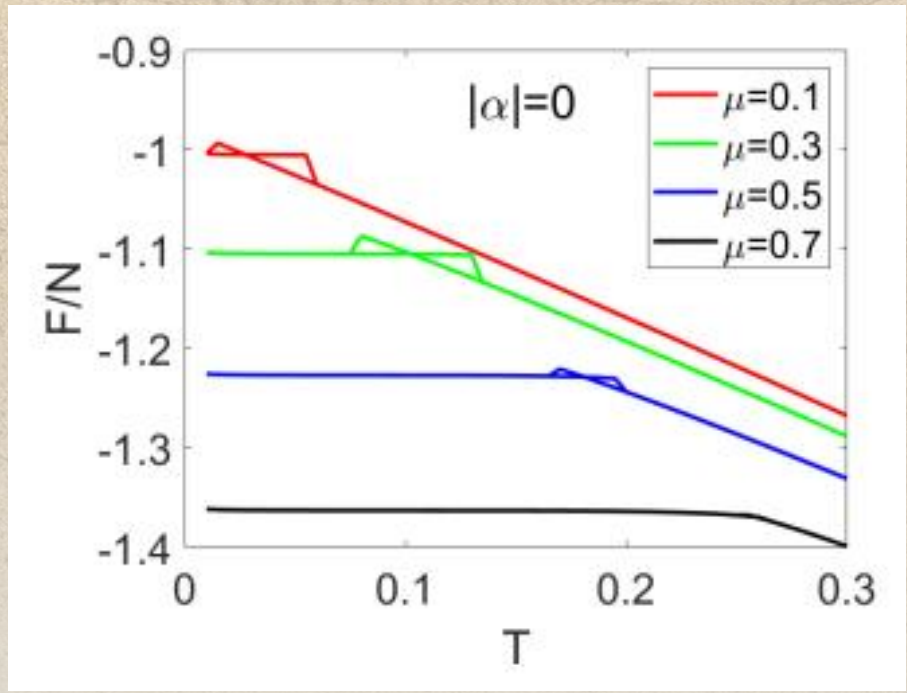
Green's functions and α-dependence



Off-diagonal Green function dominates for large α.

The free energy





Wormhole phase

Black hole phase

The free energy and thermodynamics are not affected by α.

The real-time Green function could be α-dependent, dynamics may be affected by α.

Revival dynamics: non-Hermitian cases

- We consider non-Hermitian coupled SYK models.

$$H = H_0 + H_{int},$$

$$H_0 = -\sum_{i,j,k,l=1}^{N} J_{ijkl} \sum_{A=L,R} \left(2C_i^{A\dagger}C_j^{A\dagger}C_k^A C_l^A + 4C_i^{A\dagger}C_j^A C_k^{A\dagger}C_l^A\right),$$

$$H_{int} = i\mu \sum_{i=1}^{N} (e^{-2\alpha} C_i^{L\dagger} C_i^R - e^{2\alpha} C_i^{R\dagger} C_i^L),$$

$$\Sigma_{AB}^{>}(t) = -36J^2 G_{AB}^{>2}(t) G_{BA}^{>}(-t),$$

$$\tilde{G}_{LL}^R(\omega,\alpha) = \frac{-\omega - \tilde{\Sigma}_{RR}^R(\omega,\alpha)}{\tilde{D}(\omega,\alpha)},$$

$$\tilde{G}_{RR}^R(\omega,\alpha) = \frac{-\omega - \tilde{\Sigma}_{LL}^R(\omega,\alpha)}{\tilde{D}(\omega,\alpha)},$$

$$\tilde{G}_{LR}^R(\omega,\alpha) = \frac{-i\mu e^{-2\alpha} + \tilde{\Sigma}_{LR}^R(\omega,\alpha)}{\tilde{D}(\omega,\alpha)},$$

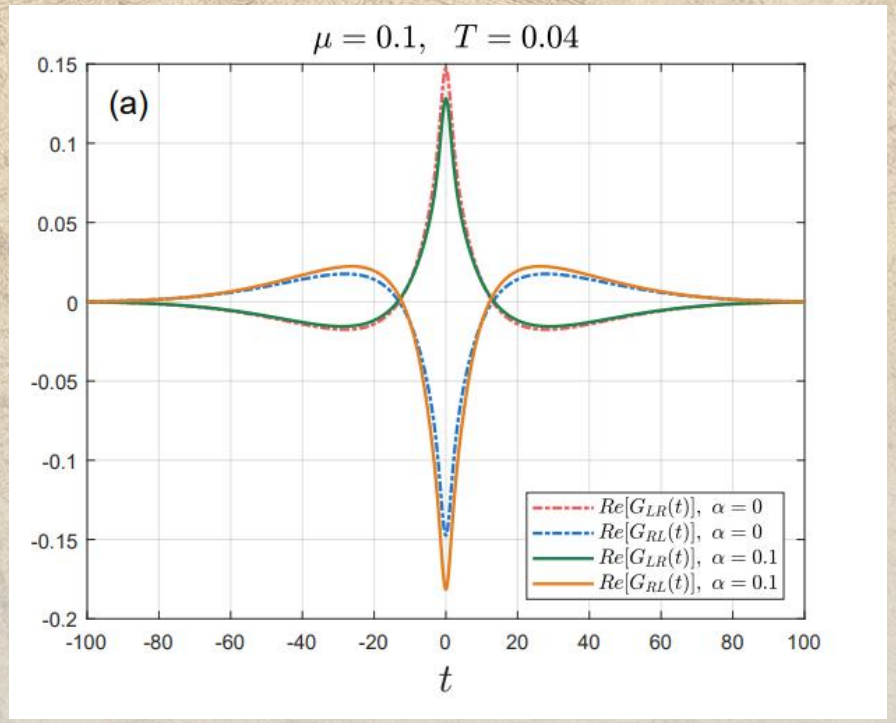
$$\tilde{G}_{RL}^R(\omega,\alpha) = \frac{i\mu e^{2\alpha} + \tilde{\Sigma}_{RL}^R(\omega,\alpha)}{\tilde{D}(\omega,\alpha)},$$

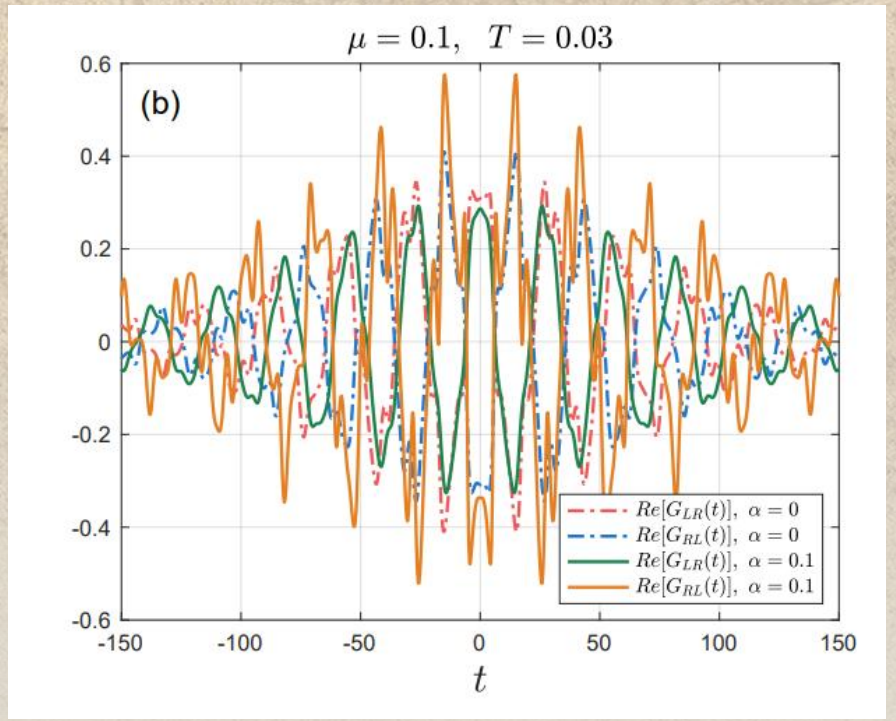
$$\tilde{D}(\omega,\alpha) = \left(-\omega - \tilde{\Sigma}_{LL}^R\right)\left(-\omega - \tilde{\Sigma}_{RR}^R\right) + \left(i\mu e^{-2\alpha} - \tilde{\Sigma}_{LR}^R\right)\left(i\mu e^{2\alpha} - \tilde{\Sigma}_{RL}^R\right).$$

Main results:

$$G_{LL}(t;\alpha) = G_{RR}(t;\alpha), \quad e^{2\alpha} G_{LR}(t;\alpha) = -e^{-2\alpha} G_{RL}(t;\alpha),$$

Revival dynamics: non-Hermitian cases

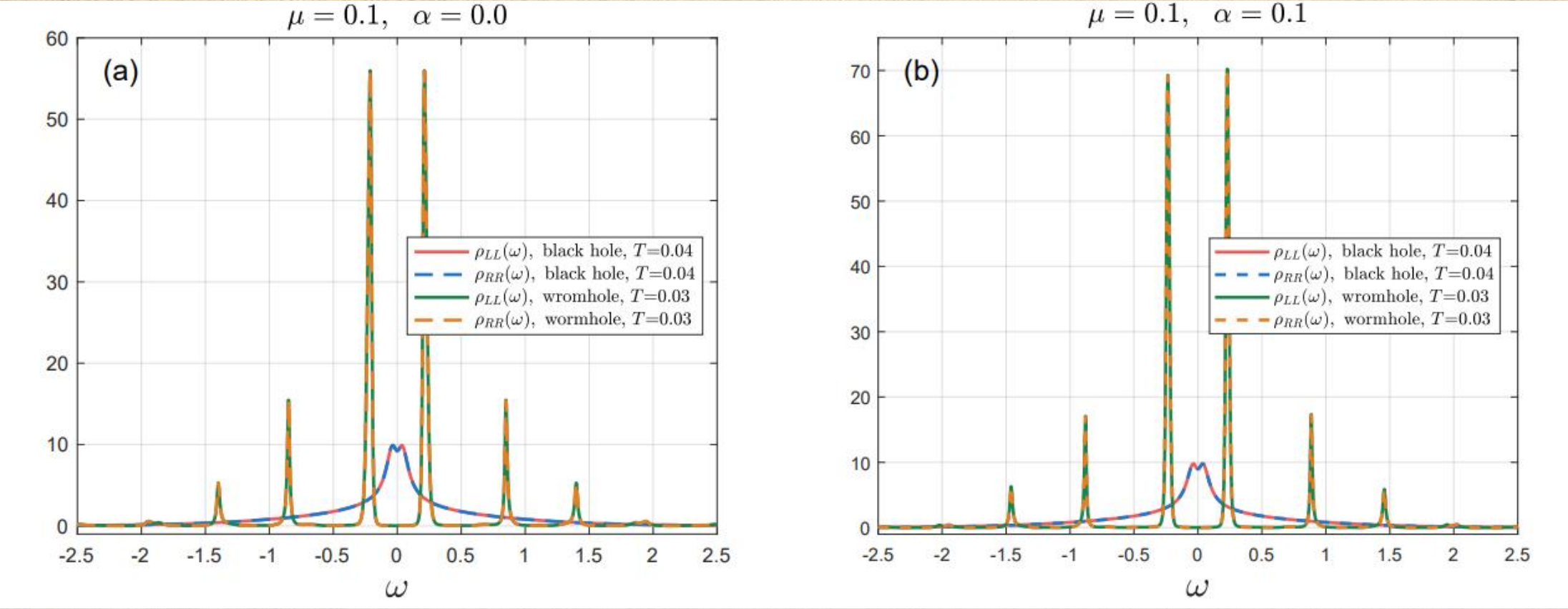




The real part of the greater Green functions with $\alpha = \pm 0.1$, the excitation created from the right side transmits more active than the excitation created from the left side for positive α.

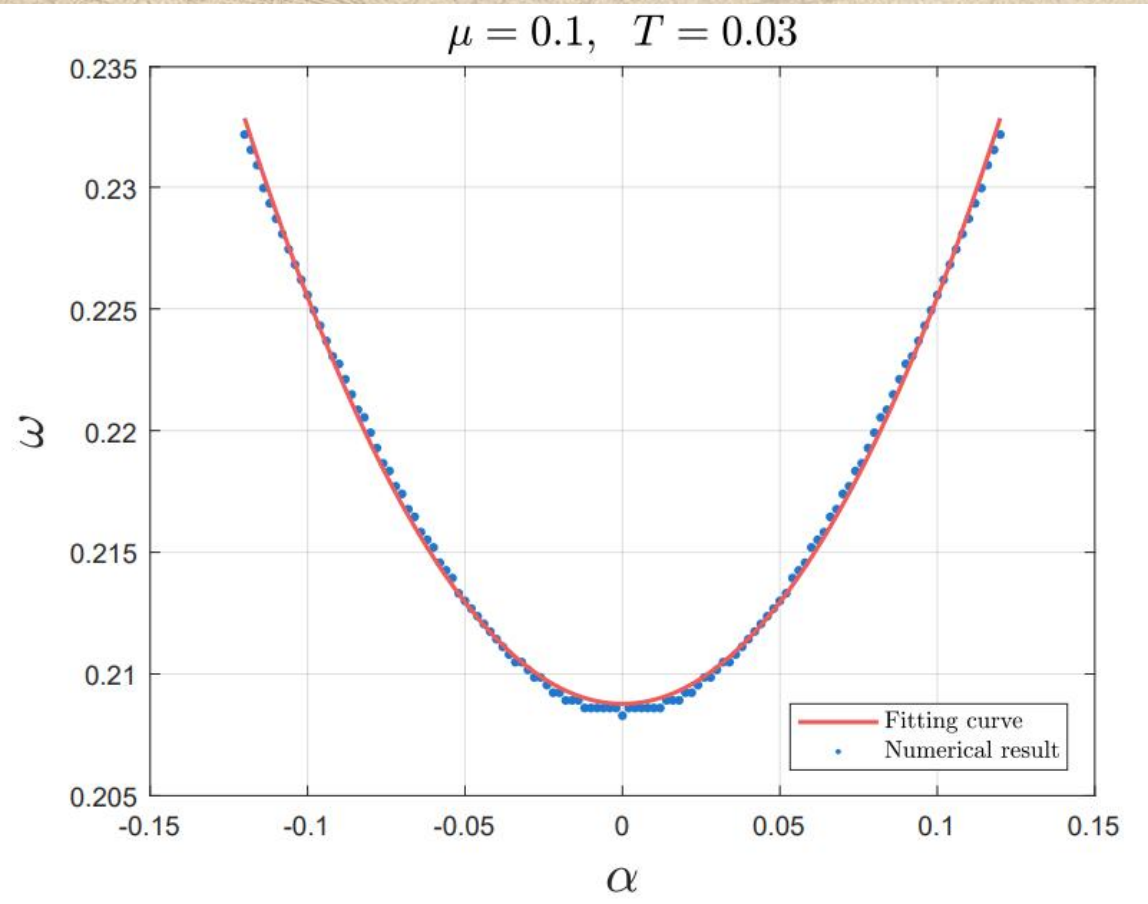
Symmetry: $G_{LL}(\tau;\alpha) = G_{RR}(\tau;\alpha), \quad e^{2\alpha} G_{LR}(\tau;\alpha) = -e^{-2\alpha} G_{RL}(\tau;\alpha),$

Revival dynamics: non-Hermitian cases



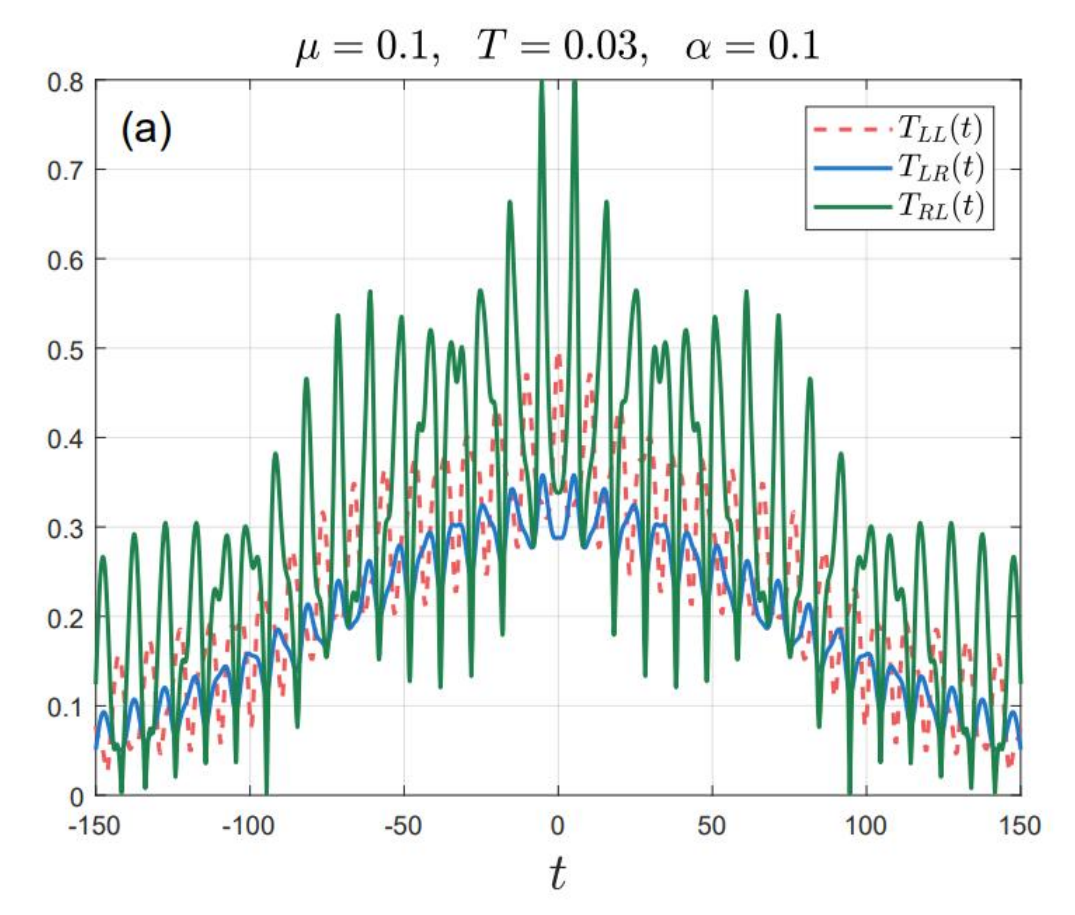
From the spectral functions of diagonal terms, the peaks distribution rely on α.

Revival dynamics: non-Hermitian cases



The quadratic curve fitting of the first peak of ρ_{LL}: $\omega_{\text{peak}} = 1.42\alpha^2 + 0.208$

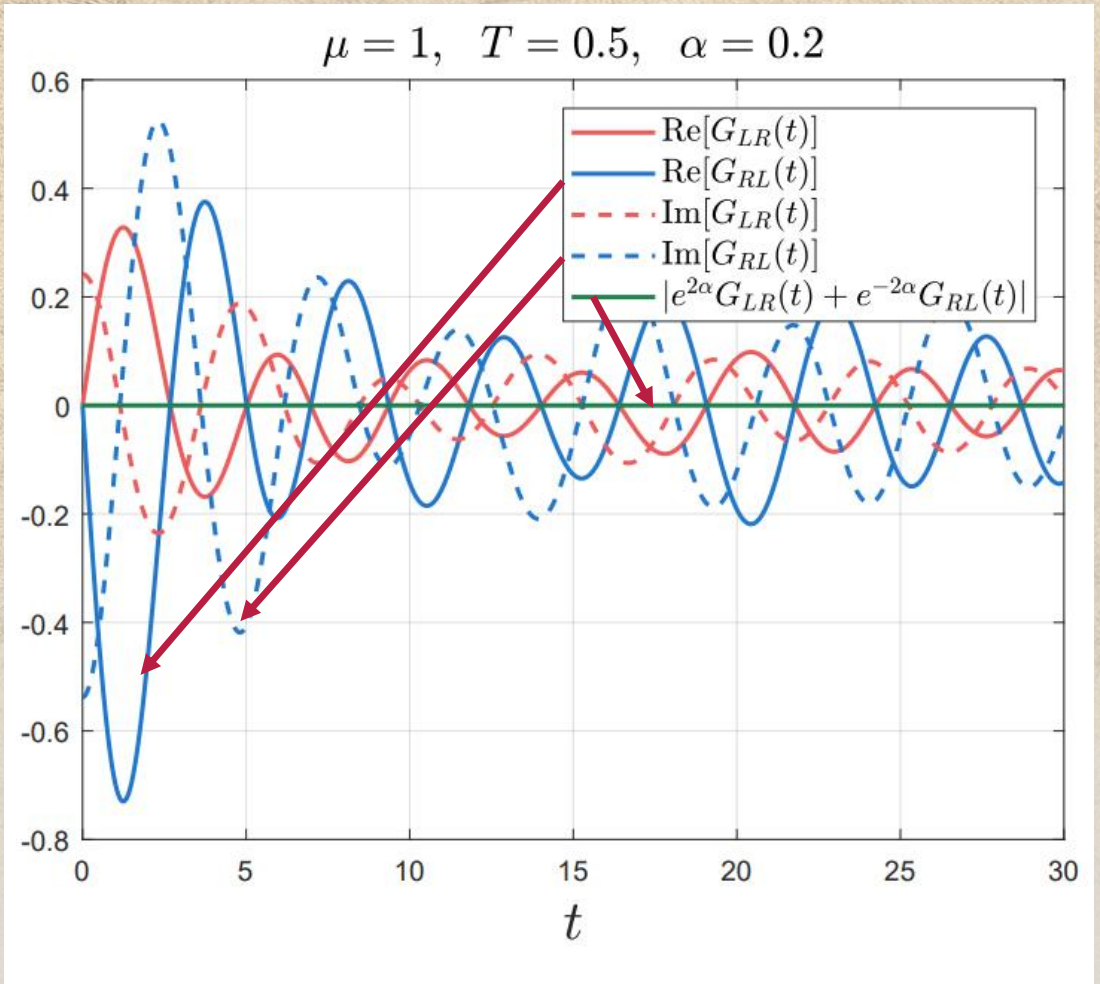
Transmission amplitude



The non-Hermitian Transmission $T_{LR}(t)$, $T_{LR}(t)$ and $T_{LR}(t)$.

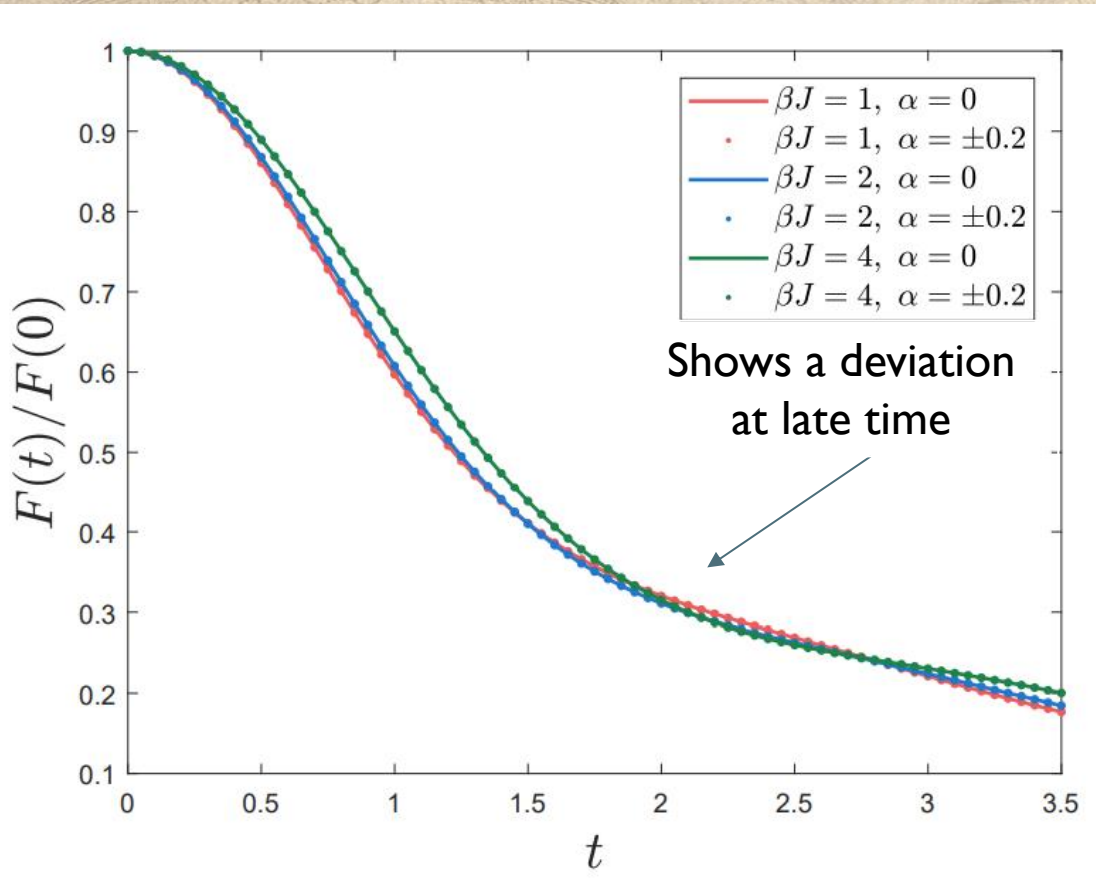
$$T_{AB}(t) = |G_{AB}(t)|$$

ED method of GFs (double check)



The $N = 6$ full diagonalization implement of Green's function.

Out-of-time-order correlators



$$F(t) = \frac{\mathrm{Tr}\left[W_i(t)\rho^{1/4}V_j(0)\rho^{1/4}W_i(t)\rho^{1/4}V_j(0)\rho^{1/4}\right]}{\mathrm{Tr}\rho}$$

These curves are realized by the exact diagonalization method at N=6 with 50 averages

IV. Conclusion and Outlook

- **Non-Hermiticity does not change the thermodynamics of coupled SYK model.**
- **Non-Hermiticity affects the dynamics of coupled SYK model:** *Asymmetric state conveyance controlled by tunable non-Hermitian parameter*

$$G_{LL}(t;\alpha) = G_{RR}(t;\alpha), \quad e^{2\alpha} G_{LR}(t;\alpha) = -e^{-2\alpha} G_{RL}(t;\alpha),$$

- **System features two phases resembling black holes and wormholes, amid directional state transfer bias.**
- **Verified off-diagonal Green's function symmetries via finite-N ED, confirming model robustness.**
- **Persistent OTOC exponential decay observed, invariant to non-Hermiticity, across Hermitian and non-Hermitian systems for all finite N cases.**

IV. Outlook

- Page curve and Island rule in SYK models
- Strange metal from deformed SYK models
- DSSYK/de Sitter duality

Thank you!