Probing the Supersymmetric Grand Unified Theories at the Future Proton-Proton Colliders and Hyper-Kamiokande Experiment

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The Nature

The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful.
If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living.

Jules H. Poincare

The Standard Model

The particle physics Standard Model (SM) is a model that describes the elementary particles in the nature and the fundamental interactions between them.

Fundamental Interactions

Interactions	Invariant	Symmetry	Fields	Spin
Gravity	Diffeomorphism		Graviton	2
Strong	Gauge	$SU(3)_C$	Gluon	1
Weak	Gauge	$SU(2)_L$	$W^{\pm}$, W^0	1
Hypercharge	Gauge	$U(1)_Y$	B^0	1

Elementary Particles

- Three families of SM fermions:

$$\textbf{Quarks}: Q_1 = \begin{pmatrix} U & U & U \\ D & D & D \end{pmatrix}_{\rm L}, \; (U \;\; U \;\; U)_R, \; (D \;\; D \;\; D)_R \; .$$

$$\textbf{Leptons}: \qquad L_1 = \begin{pmatrix} \nu \\ E \end{pmatrix}_{\rm L}, \quad E_R \; .$$

- One Higgs doublet

Breaking $SU(2)_L \times U(1)_Y$ down to the $U(1)_{\rm EM}$.

$$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix} .$$

The convincing evidence for new physics beyond the SM:

- Dark energy
- Dark matter
- Neutrino masses and mixings
- Baryon asymmetry
- Inflation

The SM is incomplete!

Major Theoretical Problems in the SM

- Fine-tuning problems

  Cosmological constant problem, gauge hierarchy problem, strong CP probelm, and the SM fermion mass hierarchies.

- Aesthetic problems:

  Interaction unification, fermion unification, charge quantization, gauge coupling unification.

- Stability problem.

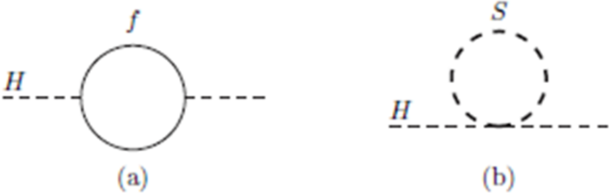
The aesthetic problems can be solved in the Grand Unified Theories (GUTs) if the SM gauge couplings are unified.

Gauge Hierarchy Problem

$$-\mathcal{L} = \lambda_f H \bar{f} f + \lambda_S |H|^2 |S|^2 \ .$$

$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2} \Lambda_{\rm UV}^2 + \frac{\lambda_S}{16\pi^2} \Lambda_{\rm UV}^2 \ .$$

Gauge Hierarchy Problem



Supersymmetry

- A supersymmetry is a space-time symmetry, which turns a bosonic state into a fermionic state, and vice versa.

$$Q|\mathrm{Boson}\rangle = |\mathrm{Fermion}\rangle, \qquad Q|\mathrm{Fermion}\rangle = |\mathrm{Boson}\rangle.$$

- Algebra: supersymmetry generator Q is a fermionic operator with spin-1/2.

$$\begin{aligned} \{Q, Q^\dagger\} &= P^\mu, \\ \{Q, Q\} &= \{Q^\dagger, Q^\dagger\} = 0, \\ [P^\mu, Q] &= [P^\mu, Q^\dagger] = 0. \end{aligned}$$

- Each supermultiplet contains an equal number of fermion and boson degrees of freedom.

The Supersymmetry Standard Model

- Four-dimesional $N = 1$ supersymmetry: Kähler potential, superpotential, gauge kinetic function.
- A chiral SM fermion has a complex scalar partner.
- Gauge bosons and Higgs fields have a spin $1/2$ partner.
- Graviton has a spin $3/2$ partner.

The Minimal Supersymmetry Standard Model (MSSM)

- Two Higgs doublets H_u and H_d: holomorphic superpotential and anomaly cancellation.
- Unlike the SM, proton can decay at the renormalizable level in the SSMs. To forbid the proton decay operaors, we introduce a Z_2 R symmetry: $R = (-1)^{3B-L+2s}$.
- The SM particle are even while the supersymmetric particles are odd.
- Dark matter: neutralino, sneutrino, gravitino, etc.

Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks	Q	$(\widetilde{u}_L \quad \widetilde{d}_L)$	$(u_L \quad d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
quarks	$\overline{u}$	$\widetilde{u}_R^*$	$u_R^\dagger$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}_R^*$	$d_R^\dagger$	$(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons	L	$(\widetilde{\nu} \ \widetilde{e}_L)$	$(\nu \quad e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
leptons	$\overline{e}$	$\widetilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs	H_u	$(H_u^+ \quad H_u^0)$	$(\widetilde{H}_u^+ \quad \widetilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$
Higgsinos	H_d	$(H_d^0 \quad H_d^-)$	$(\widetilde{H}_d^0 \quad \widetilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

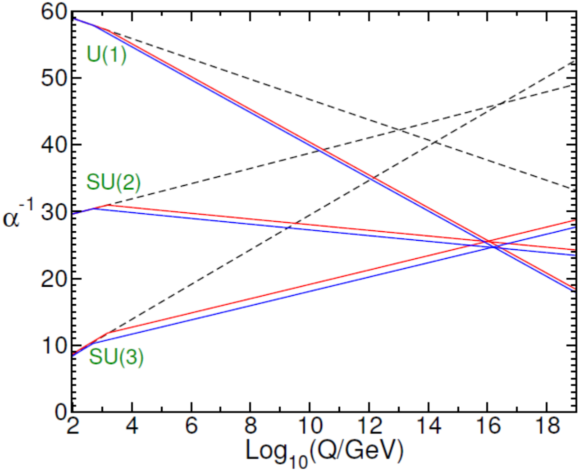
Table : Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions.

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\ \mathbf{8},\ \mathbf{1}\ ,\ 0)$
Winos, W bosons	$\widetilde{W}^{\pm}\quad \widetilde{W}^{0}$	$W^{\pm}\quad W^{0}$	$(\ \mathbf{1},\ \mathbf{3}\ ,\ 0)$
Bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\ \mathbf{1},\ \mathbf{1}\ ,\ 0)$

Table : Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

Neutralinos: neutral Higgsinos, Wino and Bino.
Chargino: charged Higgsinos and Wino.

Gauge Coupling Unification for the SM and MSSM



The Supersymmetric Standard Models

- A natural solution to the gauge hierarchy problem in the SM.
- Gauge coupling unification can be achieved.
- The Lightest Supersymmetric Particle (LSP) such as the LSP neutralino etc can be a dark matter candidate.
- The electroweak gauge symmetry can be broken radiatively due to the large top quark Yukawa coupling.
- Generating baryon asymmetry via the electroweak baryogenesis.
- Electroweak precision: R parity.

Problems in the MSSM

- μ problem: $\mu H_u H_d$
- Little hierarchy problem
- CP violation and EDMs
- FCNC
- Dimension-5 proton decays

Supersymmetry Breakings and Mediations

- The supersymmetry breaking is broken in the hidden sector via F-term and/or D-term.
- The supersymmetry breaking mediations: gravity mediation, gauge mediation, and anomaly mediation, etc.

The Grand Unified Theories

- The Unification Conjecture: all the fundamental interactions have the same origin!!!
- Gauge coupling unification in the SSMs strongly suggests the GUTs.

The Road to the Unification

- The First Unification by Newton

  The Celestial and Terretrial Gravity are the same!

- The Second Unification by James C. Maxwell:

  The Electricity and Magnetism are different manifestations of the same phenomenon!

- The Kaluza-Klein Theory

  The unification of gravity and $U(1)_{\rm EM}$!

- The Third Unification by Glashow, Salam, and Weinberg

  The Electroweak Theory for the Weak and Electromagnetic Interaction.

- What is/are the next unification(s)?

  Grand Unified Theory (GUT) and/or String Theory.

The Grand Unified Theories: $SU(5)$ and $SO(10)$

- Gauge interaction unification.
- In $SO(10)$ model, one family of the SM fermion forms a spinor $\mathbf{16}$ representation.
- Yukawa coupling unification.
- Charge quantization.
- Weak mixing angle at weak scale M_Z.
- Neutrino masses and mixings by seesaw mechanism.
- Prediction: dimension-six proton decay via heavy gauge boson exchange.

Problems

- Gauge symmetry breaking
- Doublet-triplet splitting problem
- Proton decay problem
- Fermion mass problem

  The wrong prediction on the fermion mass ratios: $m_e/m_\mu = m_d/m_s$.

String Models

- Calabi-Yau compactification of heterotic string theory
- Orbifold compactification of heterotic string theory

  Grand Unified Theory (GUT) can be realized naturally through the elegant E_8 breaking chain:

  $E_8 \supset E_6 \supset SO(10) \supset SU(5)$
- D-brane models on Type II orientifolds

  N stacks of D-branes gives us $U(N)$ gauge symmetry: Pati-Salam Models
- Free fermionic string model builing

  Realistic models with clean particle spectra can only be constructed at the Kac-Moody level one: the Standard-like models, Pati-Salam models, and flipped $SU(5)$ models.

$\mathcal{F}$-Theory Model Building

- The models are constructed locally, and then the gravity should decoupled, *i.e.*, $M_{\rm GUT}/M_{\rm Pl}$ is a small number.
- The $SU(5)$ and $SO(10)$ gauge symmetries can be broken by the $U(1)_Y$ and $U(1)_X/U(1)_{B-L}$ fluxes.
- Gauge mediated supersymmetry breaking can be realized via instanton effects. Gravity mediated supersymmetry breaking predicts the gaugino mass relation.
- All the SM fermion Yuakwa couplings can be generated in the $SU(5)$ and $SO(10)$ models.
- The doublet-triplet splitting problem, proton decay problem, μ problem as well as the SM fermion masses and mixing problem can be solved.

Supersymmetry

- The most promising new physics beyond the Standard Model.
- Gauge coupling unification strongly suggests the Grand Unified Theories (GUTs), and the SUSY GUTs can be constructed from superstring theory.

Supersymmetry is a bridge between the low energy phenomenology and high-energy fundamental physics.

Particle Physics Paradigm

String Theory → String Models → GUTs → SSMs → SM

The Predictions of the GUTs: Proton Decays

- The dimension-6 proton decay via superheavy $(X_\mu,\ Y_\mu)$ gauge boson exchanges

$$SU(5) \ = \ \left(\begin{array}{cc} SU(3)_C & (\overline{X}_\mu, \overline{Y}_\mu) \\ (X_\mu, Y_\mu) & SU(2)_L \end{array} \right) ~.$$

- The dimension-5 proton decay via colored Higgsino exchanges in the supersymmetric GUTs.

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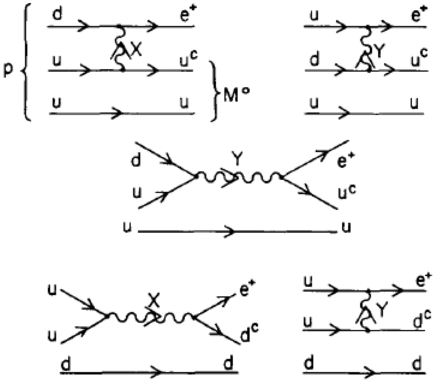
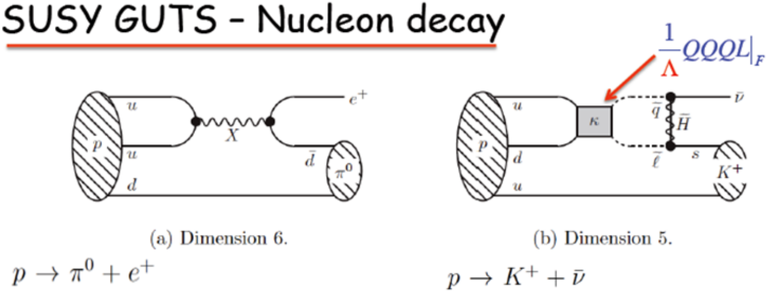
The Dimension-Six Proton Decay via $(X_\mu,\ Y_\mu)$ Exchanges



The Proton Decay in the Supersymmetric GUTs



SUSY GUTS - Nucleon decay

$\frac{1}{\Lambda} QQQL|_F$

(a) Dimension 6.

$p \to \pi^0 + e^+$

(b) Dimension 5.

$p \to K^+ + \bar{\nu}$

Proton Decays [1]

- The current bounds from Super-Kamiokande (SK)

$$\tau_{p\to e^+\pi^0} \geq 1.6 \times 10^{34}~\text{yrs}~, \quad \tau_{p\to \bar{\nu}K^+} \geq 5.9 \times 10^{33}~\text{yrs}~.$$

- The expected bounds from Hyper-Kamiokande (HK)

$$\tau_{p\to e^+\pi^0} \geq 1.0 \times 10^{35}~\text{yrs}~, \quad \tau_{p\to \bar{\nu}K^+} \geq 2.5 \times 10^{34}~\text{yrs}~.$$

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[1] K. Abe *et al.* [Super-Kamiokande], Phys. Rev. D **95**, no.1, 012004 (2017) [arXiv:1610.03597 [hep-ex]]; M. Yokoyama [Hyper-Kamiokande Proto Collaboration], arXiv:1705.00306 [hep-ex]; K. Abe *et al.* [Hyper-Kamiokande], [arXiv:1805.04163 [physics.ins-det]].

The LHC Supersymmetry Search Contraints

- The first two-generation squark mass low bounds are around 1.6 (1.75) TeV.
- The gluino mass low bound is around 2.25 (2.46) TeV.
- The stop and sbottom mass low bounds are around 1.16 (1.3) and 1.35 (1.45) TeV, respectively.

The SSMs are fine-tuned!!!

Supersymmetry at the Current and Future Colliders

- The wrong impression is that supersymmetry was excluded at the LHC?
- Can we rule out supersymmetry at the LHC, VLHC, FCC$_{\rm hh}$ and SppC?
  No! No!! No!!!
- Points: supersymmetry breaking soft mass scale can be pushed to be much higher than 1 TeV, while gauge coupling unification can still be realized due to the logarithmic RGE running and threshold corrections around the GUT scale.
- Conclusion: supersymmetry will defintely not die in the near future!!!

Natural Supersymmetry

The interesting question: can we rule out the natural supersymmetry at the $\mathrm{FCC_{hh}}$ and SppC? Or can we solve the supersymmetry electroweak fine-tuning problem naturally?

Fine-Tuning Definition

- Fine-tuning Definition [2]: the quantitative measure $\Delta_{\rm FT}^{\rm EENZ-BG}$ for fine-tuning is the maximum of the logarithmic derivative of M_Z with respect to all the fundamental parameters a_i at the GUT scale

$$\Delta_{\rm FT}^{\rm EENZ-BG} ~=~ {\rm Max}\{\Delta_i^{\rm GUT}\} ~, \quad \Delta_i^{\rm GUT} ~=~ \left| \frac{\partial {\rm ln}(M_Z)}{\partial {\rm ln}(a_i^{\rm GUT})}\right| ~.$$

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[2] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A **1**, 57 (1986); R. Barbieri and G. F. Giudice, Nucl. Phys. B **306**, 63 (1988).

Question: Super-Natural Supersymmetry

Can we propose a supersymmetry scenario whose the EENZ-BG fine-tuning measure is automatically 1 or order 1 ($\mathcal{O}(1)$)?

Fundamental physics principles: simplicity and naturalness.

Super-Natural Supersymmetry [3]

- Fine-Tuning Definition:

$$\Delta_{\rm FT} ~=~ {\rm Max}\{\Delta_i^{\rm GUT}\}~, \quad \Delta_i^{\rm GUT} ~=~ \left| \frac{\partial {\rm ln}(M_Z)}{\partial {\rm ln}(a_i^{\rm GUT})} \right| ~.$$

- Natural Solution:

$$M_Z^n ~=~ f_n \left(\frac{M_Z}{M_*} \right) ~ M_*^n ~.$$

$$\frac{\partial {\rm ln}(M_Z^n)}{\partial {\rm ln}(M_*^n)} ~\simeq~ \frac{M_*^n}{M_Z^n} \frac{\partial M_Z^n}{\partial M_*^n} ~\simeq~ \frac{1}{f_n} f_n \simeq \mathcal{O}(1) ~.$$

- For no-scale supergravity and M-theory on S^1/Z_2, we have $M_* = M_{1/2}$ and $M_* = M_{3/2}$, respectively.

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[3] T. Leggett, T. Li, J. A. Maxin, D. V. Nanopoulos and J. W. Walker, arXiv:1403.3099 [hep-ph]; Phys. Lett. B **740**, 66 (2015) [arXiv:1408.4459 [hep-ph]]; G. Du, T. Li, D. V. Nanopoulos and S. Raza, Phys. Rev. D **92**, no. 2, 025038 (2015) [arXiv:1502.06893 [hep-ph]]; T. Li, S. Raza and X. C. Wang, Phys. Rev. D **93**, no. 11, 115014 (2016) [arXiv:1510.06851 [hep-ph]].

The Interesting Questions?

- The SUSY electroweak fine-tuning problem can be solved by the super-natural supersymmetry.
- Can we probe supersymmetry at the future pp colliders? No?
- Can we probe the supersymmetric GUTs at the future pp colliders? Yes!!! [4]

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[4] W. Ahmed, T. Li, S. Raza and F. Z. Xu, [arXiv:2007.15059 [hep-ph]].

Future Colliders

- Lepton colliders: CEPC, CLIC, $FCC_{\rm ee}$, and ILC.
- Hadron colliders: HL-LHC, HE-LHC, $FCC_{\rm hh}$, SppC, and VLHC.

To probe the new physics beyond the SM, we do need future proton-proton colliders.

Future Proton-Proton Colliders

- Question: what is the concrete scientific goal for the future pp colliders?
- Question: what is the center-of-mass energy needed for this scientific goal?

The Scientific Goal for the Future PP Colliders

- Supersymmetry cannot be the scientific goal since the sparticles can be very heavy and then we cannot probe supersymmetry.
- The supersymmetric GUTs with grand desert hypothesis can be the scientific goal [5].

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[5] Waqas Ahmed, TL, Shabbar Raza and Fang-Zhou Xu, Phys. Lett. B **819**, 136378 (2021) [arXiv:2007.15059 [hep-ph]]; in preparation.

The Proof

- Grand desert hypothesis: no new physics between the sparticle mass scale and the GUT scale.
- For the GUTs with the GUT scale $M_{GUT} \leq 1.2 \times 10^{16}$ GeV, we can probe the dimension-six proton decay via heavy gauge boson exchange at the Hyper-Kamiokande experiment.
- For the GUTs with $M_{GUT} \geq 1.2 \times 10^{16}$ GeV, we can probe the gluino and/or squarks at the future pp colliders.
- Providing the "Concrete Scienctific Goal" for the future pp colliders and Hyper-Kamiokande experiment.

The First Part of the Proof

For the GUTs with the GUT scale $M_{GUT} \leq 1.2 \times 10^{16}$ GeV, we can probe the dimension-six proton decay via heavy gauge boson exchange at the Hyper-Kamiokande experiment.

The GUTs with $M_{GUT} \leq 1.2 \times 10^{16}$ GeV

- The proton lifetime from the dimension-six proton decay $p \to e^+\pi^0$ via heavy gauge boson exchange is

$$\tau_p \simeq 1.0 \times 10^{35} \times \left(\frac{2.5}{A_R}\right)^2 \times \left(\frac{0.04}{\alpha_{\rm GUT}}\right)^2 \times \left(\frac{M_{\rm GUT}}{1.0 \times 10^{16}~{\rm GeV}}\right)^4 ~{\rm years}~.$$

- At the future Hyper-Kamiokande experiment with 186 kt water, we can probe the GUTs with proton lifetime via dimension-6 proton decay at least above 1.0×10^{35} years [6]. The original Hyper-Kamiokande experimental proposal has 1,000 kt water, therefore, we can probe the GUTs with proton lifetime via dimension-6 proton decay at least above 5.37634×10^{35} years.

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[6] K. Abe *et al.* [Hyper-Kamiokande], [arXiv:1805.04163 [physics.ins-det]].

The GUTs with $M_{GUT} \leq 1.2 \times 10^{16}$ GeV

- For the original Hyper-Kamiokande experimental proposal, we can probe the GUTs with GUT scale up to 1.46726×10^{16} GeV.
- Therefore, we can probe the GUTs with GUT scale up to 1.2×10^{16} GeV.

The GUTs with $M_{GUT} \geq 1.2 \times 10^{16}$ GeV

- Gravity mediated supersymmetry breaking.
- Anomaly mediated supersymmetry breaking.
- Gauge mediated supersymmetry breaking.

The Supersymmetry Searches at the Future pp Colliders

- For the 100 TeV pp Colliders such as $\mathrm{FCC_{hh}}$ and SppC, gluino $\tilde{g}$ via heavy flavor decay, gluino via light flavor decay, and the first-two generation squarks $\tilde{q}$ can be discovered for their masses up to about 11 TeV, 17 TeV, and 14 TeV, respectively. If the gluino and first-two generation squark masses are similar, they can be probed up to 20 TeV.
- To probe the gluino $\tilde{g}$ via heavy flavor decay with mass around 15 TeV, we need the 160 TeV pp collider such as the VLHC.

Gravity Mediated Supersymmetry Breaking

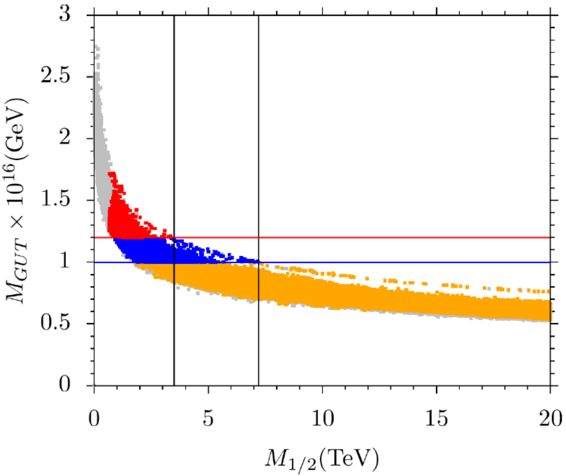
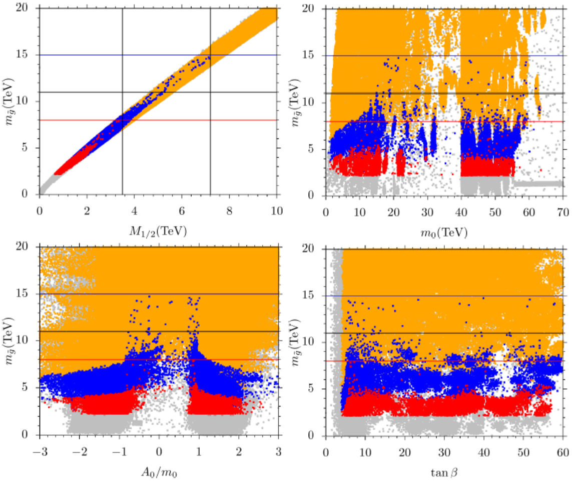
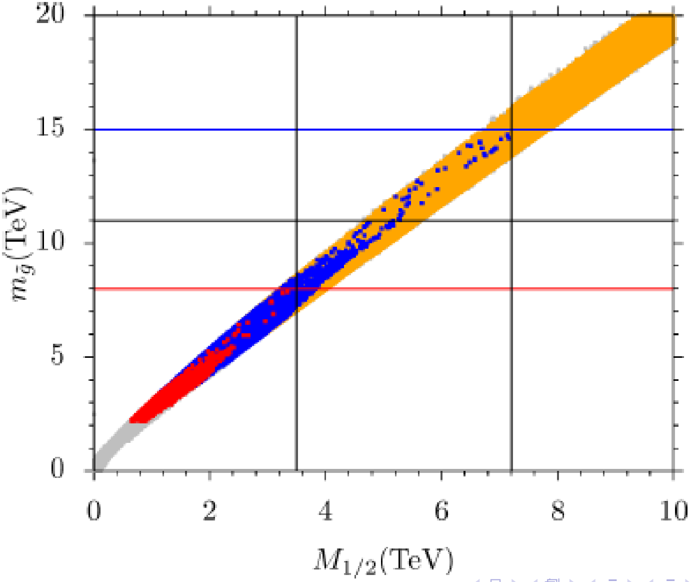


FIG. 1. Plot in $M_{1/2} - M_{GUT}$ plane. Gray points are consistent with REWSB and LSP neutralino. Orange points satisfy the mass bounds including $m_h = 125 \pm 3\,\text{GeV}$ and the constraints from rare $B-$ meson decays. Blue points form a subset of orange points and satisfy $1 \lesssim M_{GUT} \lesssim 1\times 10^{16}$ GeV, while red points form a subset of orange points and satisfy $M_{GUT} \gtrsim 1.2\times 10^{16}$ GeV. Two horizontal blue and red lines represent $M_{GUT} = 1 \times 10^{16}$ GeV and $M_{GUT} = 1.2 \times 10^{16}$ GeV, respectively. The first vertical line shows the upper bound on $M_{1/2}$ for red points ($M_{1/2} = 3.5$ TeV), and the second vertical line shows the upper bound on $M_{1/2}$ for blue points ($M_{1/2} = 7.2$ TeV).

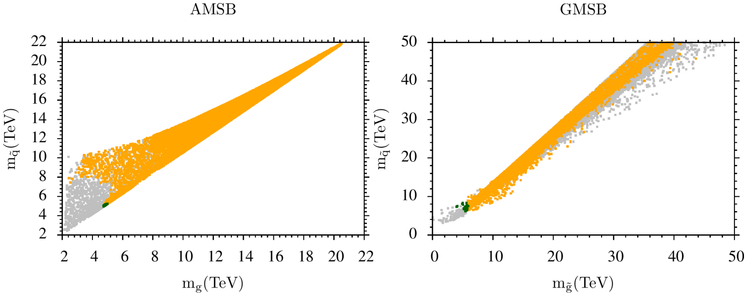
Gravity Mediated Supersymmetry Breaking



Gravity Mediated Supersymmetry Breaking



Anomaly and Gauge Mediated Supersymmetry Breakings



For anomaly and gauge mediated supersymmetry breakings, the GUTs with $M_{GUT} \geq 1.0 \times 10^{16}$ GeV are well within the reaches of the future 100 TeV pp colliders such as the $\mathrm{FCC_{hh}}$ and SppC.

Summary

- Supersymmetry is a bridge between the low energy phenomenology and high-energy fundamental physics, and thus is the promising new physics beyond the SM.
- Gauge coupling unification in the supersymmetric SM strongly implies the GUTs.
- With the grand desert hypothesis, we show that the supersymmetric GUTs can be probed at the future pp colliders and Hyper-Kamiokande experiment.

Thank You Very Much
for Your Attention!