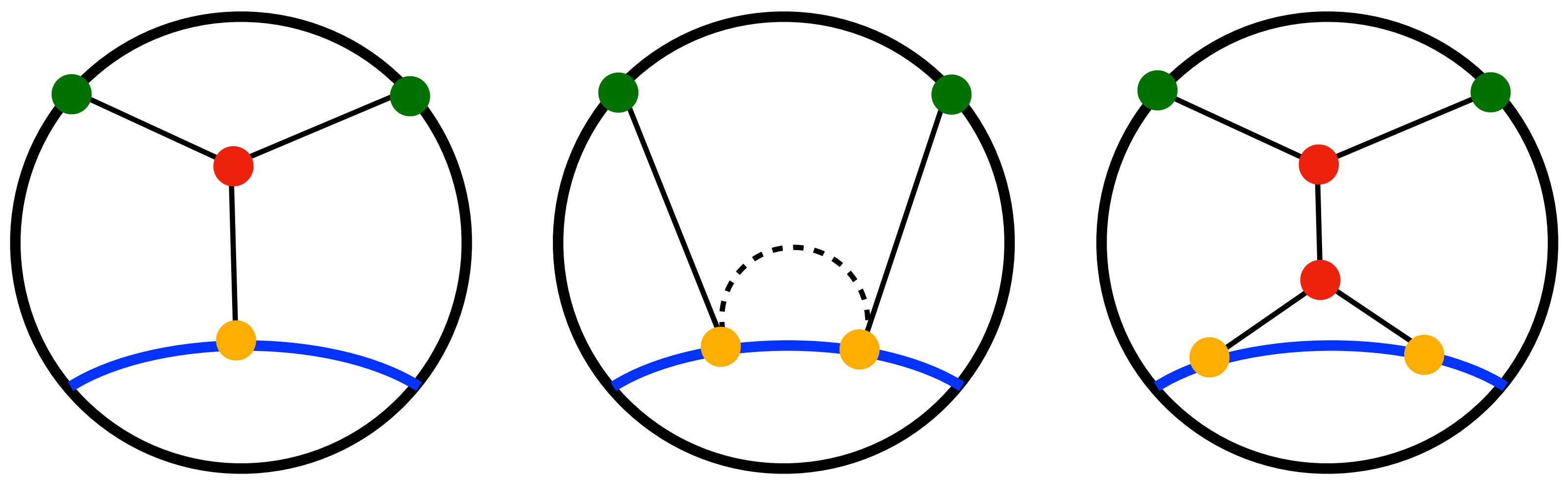
Holographic Defect Dynamics from Analytic Bootstrap

Based on: 2310.19230 and 2406.13287
(with Junding Chen, Aleix Gimenez-Grau, Hynek Paul)



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Motivations

- Defects are extended objects which lead to enrichment of theories.
- A wide range of applications both experimental (impurities, domain walls, boundary effects…) and formal (Wilson loops, D-branes, symmetry generators…).
- Also interesting objects to study in the conformal bootstrap program

- The insertion of (planar) defects breaks part of conformal symmetry.
- Still shares a lot in common with CFTs without defects (OPE, conformal blocks). Described by an enlarged set of CFT data.

Defect CFT data: $\{\Delta_i, C_{ijk}, \hat{\Delta}_a, \hat{C}_{abc}, \hat{\mu}_{ia}\}$

$$\langle \hat{O}_a(x_1)\hat{O}_a(x_2)\rangle = \frac{1}{x_{12}^{2\hat{\Delta}_a}} \qquad \langle \hat{O}_a(x_1)\hat{O}_b(x_2)\hat{O}_c(x_3)\rangle = \frac{\hat{C}_{abc}}{x_{12}^{\hat{\Delta}_{abc}}x_{13}^{\hat{\Delta}_{acb}}x_{23}^{\hat{\Delta}_{bca}}} \qquad \langle O_i(x_1)\hat{O}_a(x_2)\rangle = \frac{\hat{\mu}_{ia}}{(|x_1^{\perp}|^2 + |x_{12}^{\parallel}|^2)^{\hat{\Delta}_a}|x_1^{\perp}|^{\Delta_i - \hat{\Delta}_a}}$$

Motivations

- Simpler kinematics: an ideal testing ground for developing new bootstrap techniques. The simplest kinematically nontrivial case is 2pt function of bulk operators.
- On the other hand, many existing tools (Lorentzian inversion formula, dispersion relation) generalize easily to the defect case, giving us good control of the CFT data.

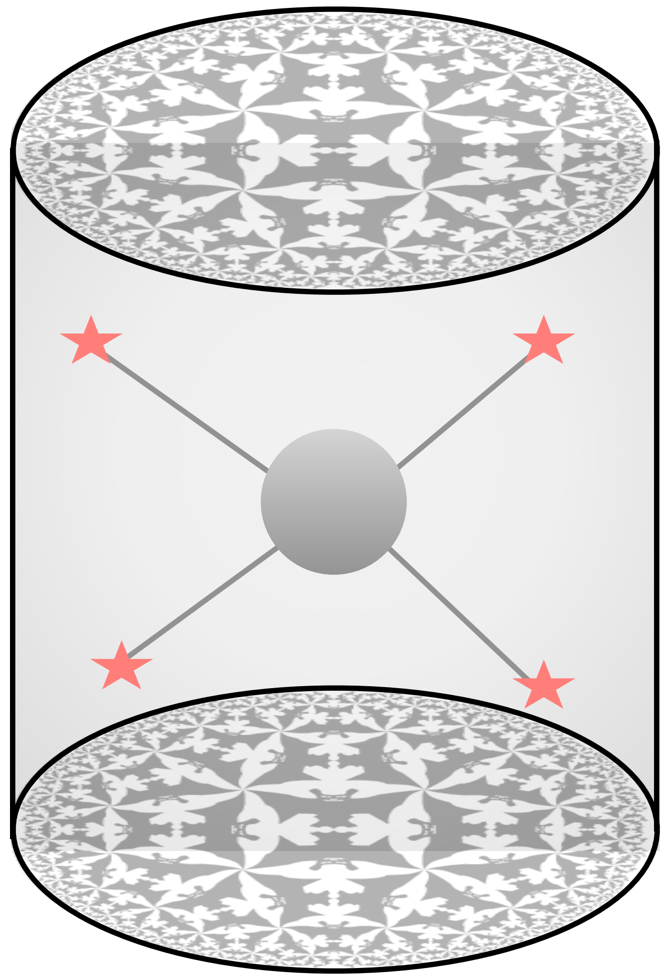
• By contrast, we know much less about correlators

  - Given a theory it is also important to know how to compute the correlators (equivalent to knowing infinitely many CFT data).
  - So far most progress is in the weakly coupled regime where standard techniques such as Feynman diagrams, ϵ expansion, large N expansion apply.
  - In the strongly coupled regime where AdS/CFT is a useful description, almost nothing is known about holographic correlators in the presence of a defect.

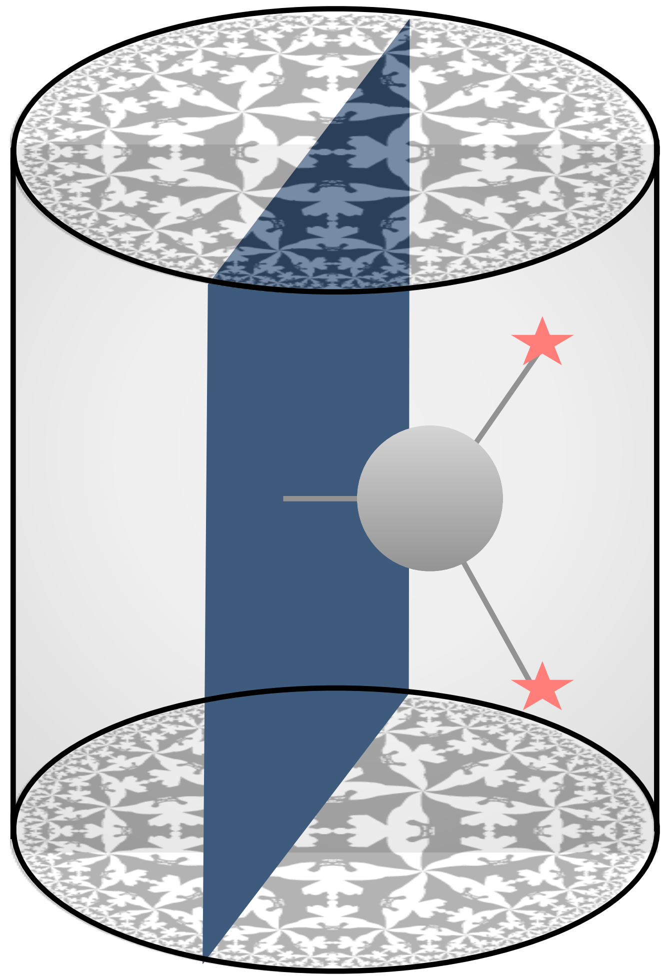
Motivations

- This should be contrasted with the significant recent progress in the case without defects (higher-point, higher loops, stringy corrections…).

- New observables



On-shell scattering amplitudes



Form factors with extended objects

- In this talk I will report bootstrap methods for computing holographic defect correlators, both at tree and loop levels. As a concrete example, I will focus on 6d (2,0) theory with half-BPS surface defects.

Surface defect in 6d (2,0) theory

- The defect system can be realized using N coincident M5 branes and a probe M2 brane. The dual geometry is

$$AdS_3 \subset AdS_7 \times S^4$$

and we consider the large N limit.

- In the $AdS_7 \times S^4$ bulk: we have 11d SUGRA (dual to 6d (2,0) theory).

  d.o.f.: KK modes of 11d SUGRA $\rightarrow$ 1/2-BPS multiplets labelled by $k = 2, 3, \ldots$.

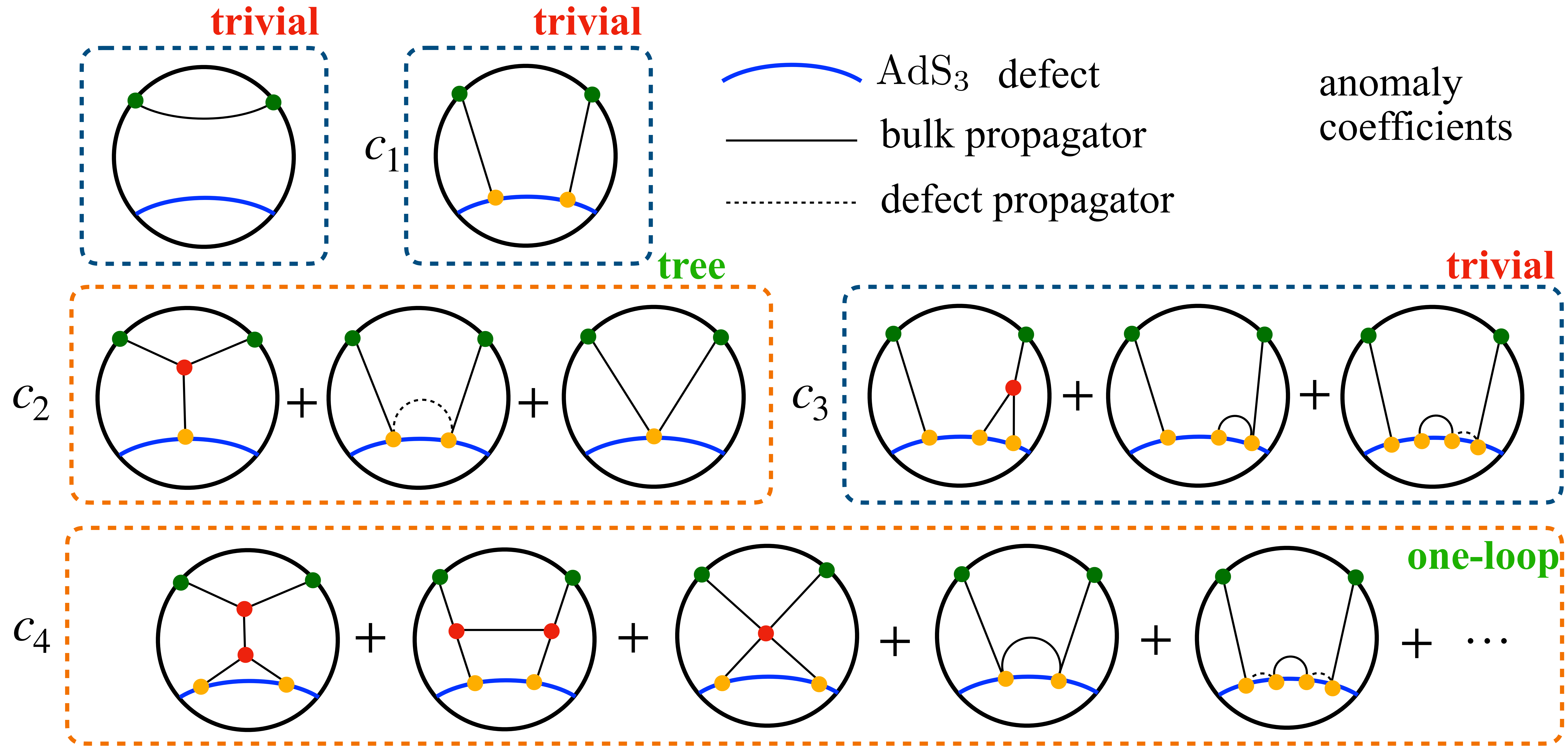
  Scf primaries S_k (super gravitons): $\Delta = 2k$, in rank-k symmetric traceless rep. of $SO(5)_R$.

- In AdS_3: this is the world-volume of M2 brane (dual to the surface defect).

  There are localized d.o.f. which can couple to the bulk.

Surface defect in 6d (2,0) theory

- Large central charge expansion



anomaly coefficients

$c \sim N^3$

$d \sim N$

$c_1 = \frac{d^2}{c}$

$c_2 = \frac{d}{c}$

$c_3 = \frac{d^3}{c^2}$

$c_4 = \frac{d^2}{c^2}$

$c_i \sim 1/N^i$

Superconformal kinematics

- We introduce R-symmetry polarizations to get rid of the indices of the super gravitons

$$S_k(x,u) = S_{I_1\ldots I_k} u^{I_1}\ldots u^{I_k}\,, \quad u\cdot u = 0$$

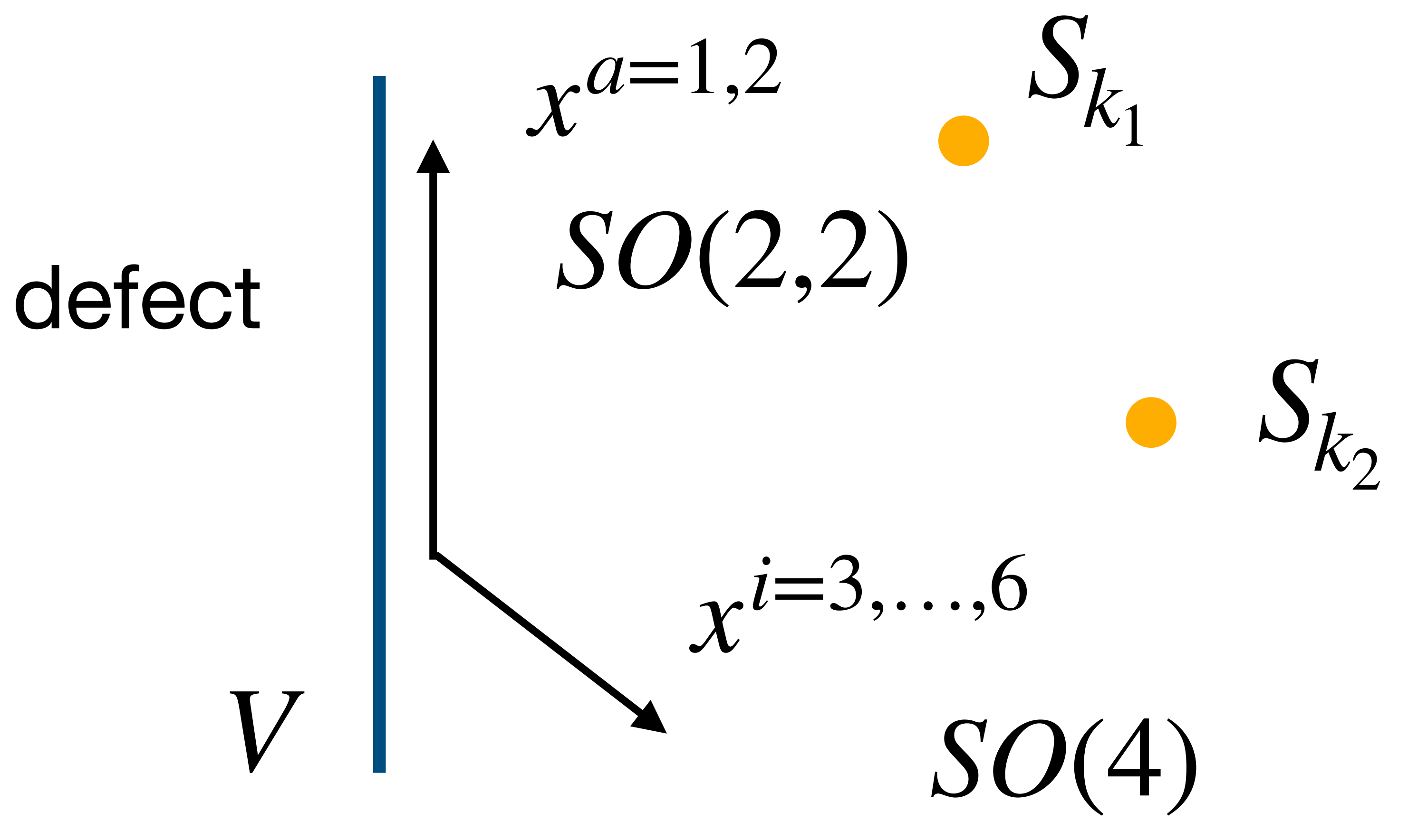
- The defect breaks half of the supersymmetry

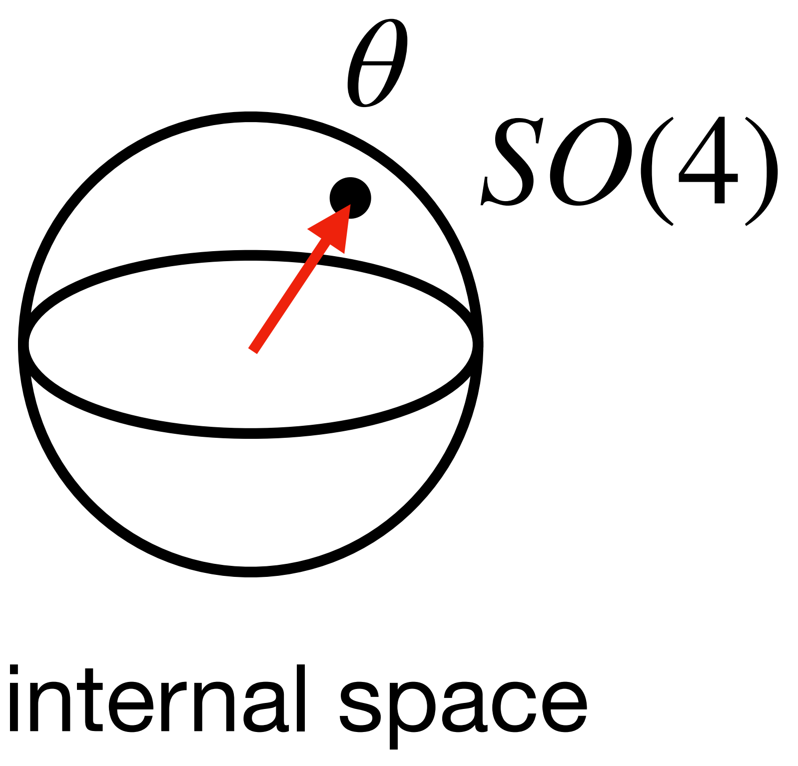
spacetime

$$SO(6,2) \to \underbrace{SO(2,2)}_{\text{defect conf. group}} \times \underbrace{SO(4)}_{\text{trans. rot.}}$$

R-symmetry

$$SO(5) \to \underbrace{SO(4)}_{\text{fixed unit vector } \theta}$$





Supersymmetry:

$$OSp(8^*|4) \to [OSp(4^*|2)]^2$$

Superconformal kinematics

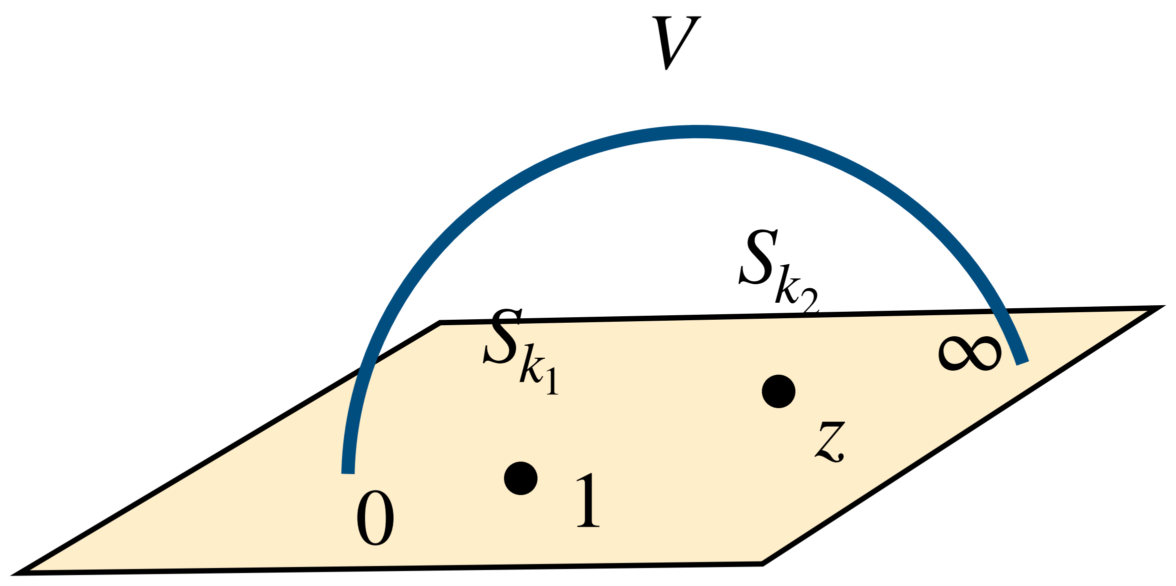
- The 2pt function can be written as a function of 3 cross ratios

$$\langle S_{k_1} S_{k_2} V \rangle = \frac{(u_1 \cdot \theta)^{k_1} (u_2 \cdot \theta)^{k_2}}{|x_1^i|^{2k_1} |x_2^i|^{2k_2}} \mathcal{F}(z, \bar{z}, \sigma)$$

Conformal:

$$\frac{x_{12}^2}{|x_1^i||x_2^i|} = \frac{(1-z)(1-\bar{z})}{\sqrt{z\bar{z}}}$$

$$\frac{x_1^j x_2^j}{|x_1^i||x_2^i|} = \frac{z+\bar{z}}{\sqrt{z\bar{z}}}$$



R-symmetry:

$$\sigma = \frac{u_1 \cdot u_2}{(u_1 \cdot \theta)(u_2 \cdot \theta)} = -\frac{(1-\omega)^2}{2\omega}$$

Fermionic generators imposes superconformal Ward identities [Meneghelli, Trepanier '22]

$$\mathcal{F}(z, \bar{z}, \omega = \bar{z}) = \zeta(z) \qquad \mathcal{F}(z, \bar{z}, \omega = z) = \zeta(\bar{z})$$

This also follows from chiral algebra [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees '13]

Superconformal kinematics

- We can easily solve these constraints as

$$\mathscr{F}(z,\bar{z},\omega) = \mathscr{F}_{\mathrm{prot}}(z,\bar{z},\omega) + \mathrm{R}\,\mathscr{H}(z,\bar{z},\omega)$$

Here

$$\mathrm{R} = \frac{(z-\omega)(\bar{z}-\omega)(z-\omega^{-1})(\bar{z}-\omega^{-1})}{z\bar{z}}$$

(determined by superconformal symmetry)

$$\mathscr{F}_{\mathrm{prot}} = \frac{(z-\omega)(z-\omega^{-1})}{(z-\bar{z})(z-\bar{z}^{-1})}\zeta(z) + (z \leftrightarrow \bar{z})$$

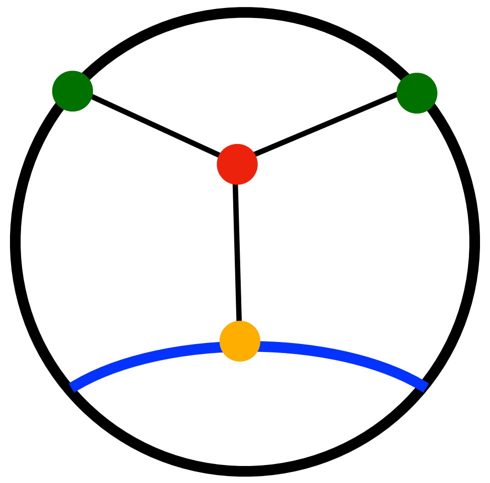
(from the meromorphic chiral correlator)

All dynamical and unprotected information is contained in the reduced correlator $\mathscr{H}$.

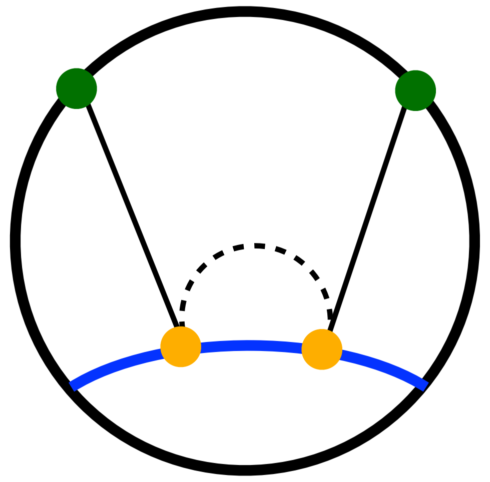
Tree-level bootstrap

- Tree level: in principle can be computed by summing up all the Witten diagrams.
- However, this requires to work out the complicated vertices and does not take advantage of the unbroken symmetries.
- A better strategy is bootstrap, similar to the defect free case [Rastelli, XZ '16]. It turns out that tree-level 2pt functions are fixed by using only symmetries and consistency conditions [Chen, Gemenez-Grau, XZ]. I will present a position space version which can be done in 3 steps.
- The starting point is to write down an ansatz

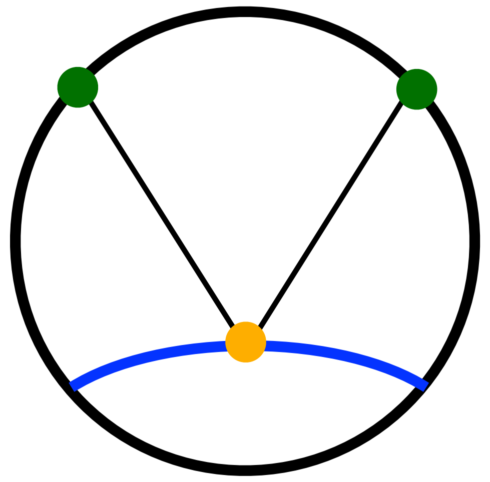
$$\langle S_{k_1} S_{k_2} V \rangle_{\text{tree}} = \sum \mu_B$$



$$+ \mu_d$$



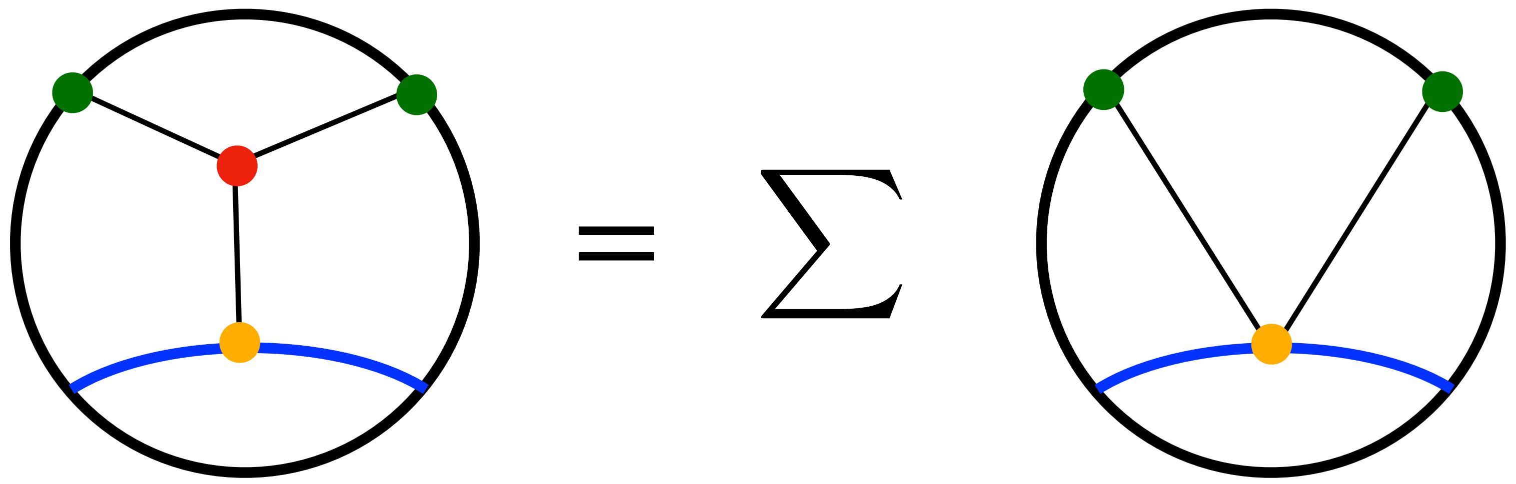
$$+ \mu_c$$

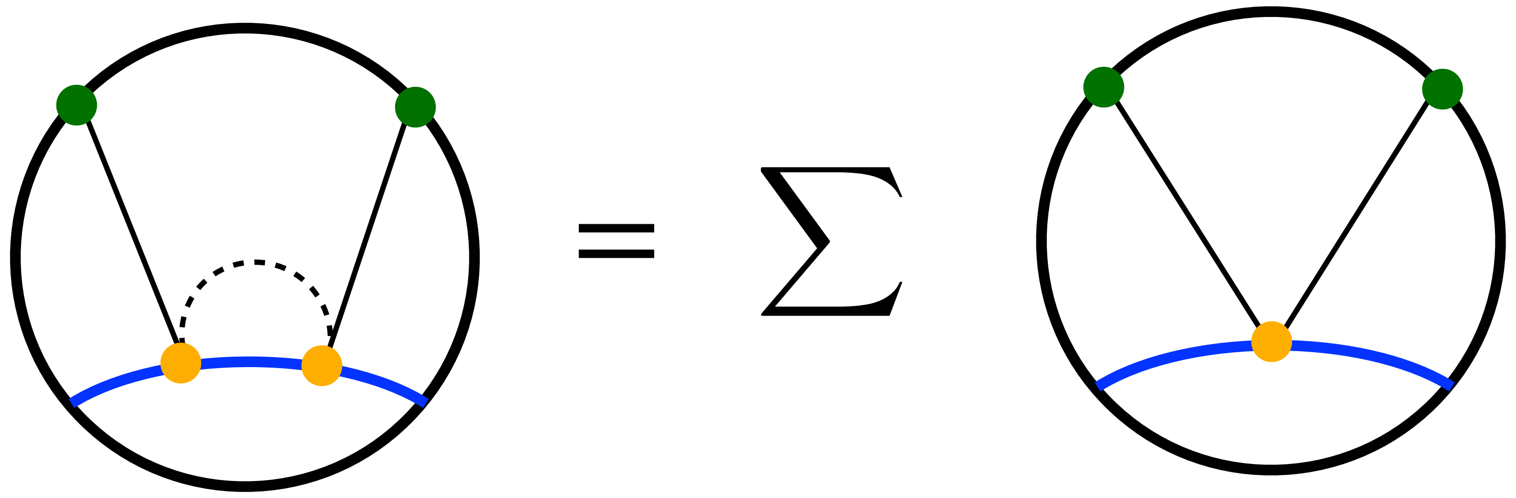


- Determined by selection rules: R-symmetry, vanishing of extremal couplings. Note fields exchanged in the defect channel has no KK modes. Moreover, we require no derivatives in the contact vertices.

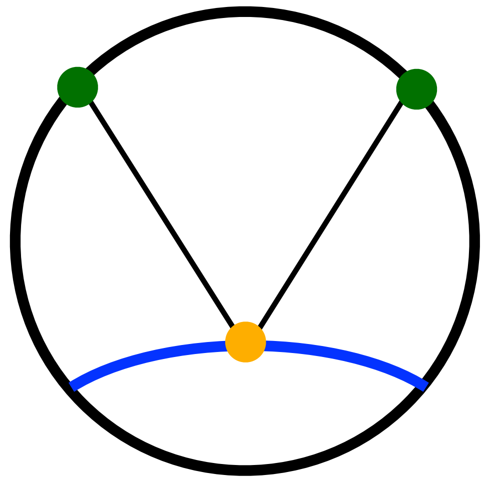
Tree-level bootstrap

- The next step is to evaluate the ansatz
  - It can be shown all bulk and defect exchange Witten diagrams can be written as a finite sum of contact Witten diagrams (a generalization of [D'Hoker, Freedman, Rastelli '99]).





  - The contact Witten diagram can be evaluated in a closed form as a 2F1 function



$$\propto \; {}_2F_1\left(\Delta_1, \Delta_2; \frac{\Delta_1+\Delta_2+1}{2}; -\frac{\xi+\chi-2}{4}\right)$$

$$\xi = \frac{(1-z)(1-\bar{z})}{\sqrt{z\bar{z}}} \qquad \chi = \frac{z+\bar{z}}{\sqrt{z\bar{z}}}$$

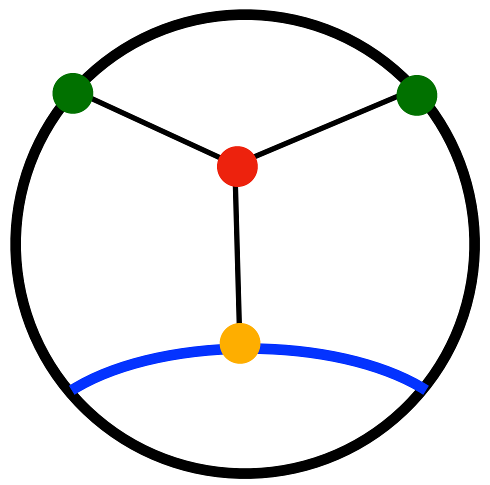
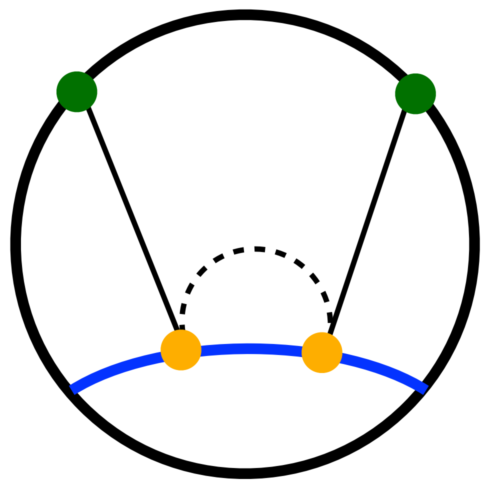
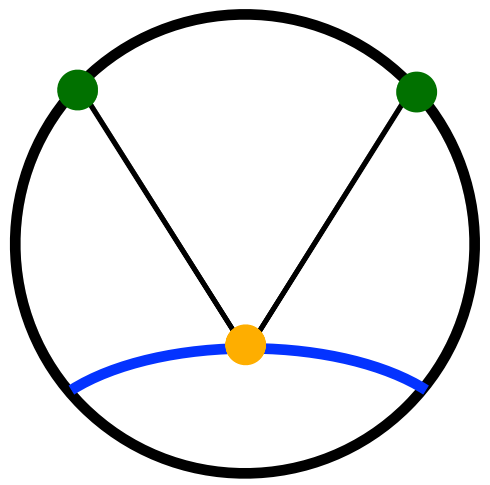
Tree-level bootstrap

- Finally, we impose the scf Ward identity

$$\partial_z \mathscr{F}(z, \bar{z}, z) = 0$$

- Remarkably, this fixes all the unknown parameters up to an overall factor!

- But this overall factor cannot be arbitrary because the unknown parameters have the interpretations as OPE data. The same data can appear in multiple correlators

$\langle S_{k_1} S_{k_2} V \rangle_{\text{tree}} = \sum \mu_B$  $+ \; \mu_d$  $+ \; \mu_c$ 

$\mu_B \rightarrow \lambda_{k_1 k_2 k} \times a_k$

- Considering all $\langle k_1 k_2 \rangle$ together allows us to reduce overall factors to that of $k_1 = k_2 = 2$, which can be fixed in terms of central charge.

Tree-level bootstrap

- The final answer takes the following form in position space

$$\mathcal{F} = \sum_k \lambda_{k_1k_2k} a_k \mathcal{P}_k + b_{k_1\mathcal{D}} b_{k_2\mathcal{D}} \widehat{\mathcal{P}} + c_{k_1} c_{k_2} (1-2\sigma) C_{2k_1,2k_2}$$

$\mathcal{P}_k$ sum of bulk exchange Witten diagrams of multiplet k $+$ contact to improve Regge

$\widehat{\mathcal{P}}$ sum of defect exchange Witten diagrams

- Note this makes the form of the contact part particularly simple.

$$\lambda_{k_1k_2k_3} = \frac{2^{\Sigma-2}\Gamma(\frac{\Sigma}{2})}{\pi^{3/2}} \prod_{i=1}^{3} \frac{\Gamma(\frac{\Sigma-2k_i+1}{2})}{\sqrt{\Gamma(2k_i-1)}}$$

(bulk 3pt function coefficients [Corrado, Florea, McNees '99, Basitanelli, Zucchini '99])

$$a_k = \frac{1}{k} b_{k\mathcal{D}} = \frac{(k-1)(2k-1)}{2^{k-\frac{1}{2}}\sqrt{\pi}c_k} = \frac{\Gamma(k)}{\sqrt{2^k\Gamma(2k-1)}}$$

new CFT data from the bootstrap

The $k_1 = k_2 = 2$ case matches [Meneghelli, Trepanier '22]

Intermezzo: Mellin space

- To better understand the analytic structure, we go to Mellin space. The standard Mellin formalism introduced by Mack and Penedones can be extended to include defects and boundaries [Rastelli, XZ '17, Goncalves, Itsios '18]. The Mellin amplitudes can be viewed as form factors with extended objects.

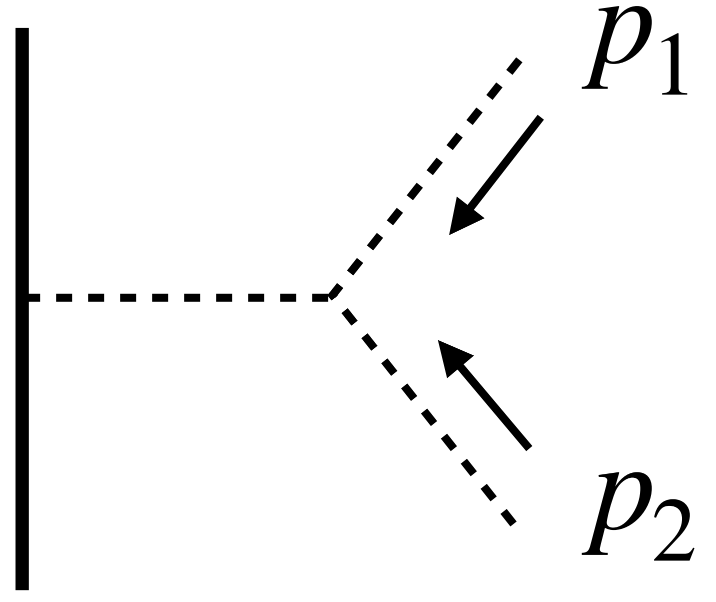
$$\mathcal{F} = \int \frac{d\delta\, d\gamma}{(2\pi i)^2} B^{-\delta} D^{\gamma} \mathcal{M}(\delta,\gamma) \Gamma_{k_1 k_2}(\delta,\gamma)$$

$$\Gamma_{k_1 k_2}(\delta,\gamma) = \Gamma(\delta)\Gamma(\gamma-\delta) \prod_{i=1}^{2} \Gamma\left(\frac{2k_i-\gamma}{2}\right)$$

$$B = \frac{\xi}{\chi} = \frac{(1-z)(1-\bar{z})}{z+\bar{z}} \qquad D = \frac{1}{\chi} = \frac{\sqrt{z\bar{z}}}{z+\bar{z}}$$

$B \to 0$: bulk OPE $\qquad$ $D \to 0$: defect OPE

- Witten diagrams are simple in Mellin space
  - Contact diagrams are just constants.
  - Exchange diagrams have poles.



$$\delta = p_1 \cdot p_2$$

$$\gamma = -p_{1,\parallel}^2 = -p_{2,\parallel}^2$$

- Translating diagram by diagram: the Mellin amplitude is a rational function.

Intermezzo: Mellin space

- An even simpler expression is given by the Mellin transform of the reduced correlator

$$\mathcal{H} = \int \frac{d\delta\, d\gamma}{(2\pi i)^2} B^{-\delta} D^{\gamma} \widetilde{\mathcal{M}}(\delta,\gamma) \widetilde{\Gamma}_{k_1k_2}(\delta,\gamma) \qquad \widetilde{\Gamma}_{k_1k_2} = \Gamma(\delta)\Gamma(\gamma-\delta)\prod_{i=1}^{2}\Gamma\left(\frac{2k_i+2-\gamma}{2}\right)$$

- The reduced Mellin amplitude is related to the Mellin amplitude by a difference operator

$$\mathcal{M}(\delta,\gamma) = \widehat{\mathrm{R}} \circ \widetilde{\mathcal{M}}$$

$$\mathrm{R} = B^2D^{-2} + 2BD^{-2}\sigma + 2D^{-2}\sigma - 8\sigma + 4\sigma^2$$

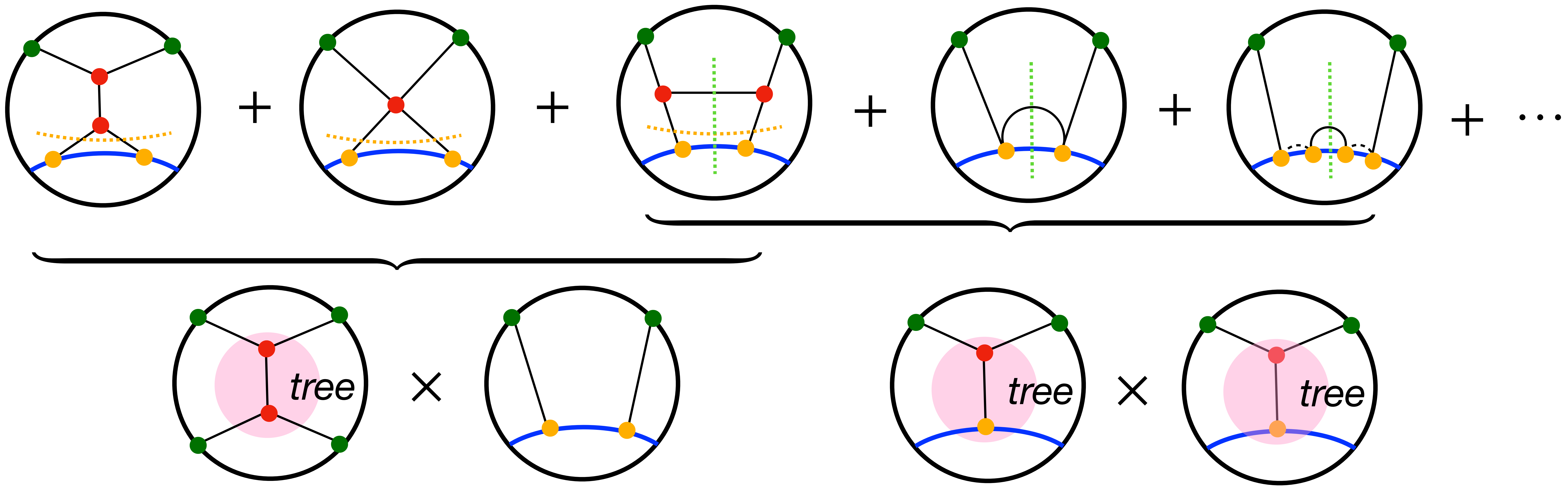
Each monomial B^mD^n can be absorbed by a shift and becomes a difference operator $\widehat{B^mD^n}$

- The reduced Mellin amplitude is a finite sum of simultaneous poles

$$\widetilde{\mathcal{M}}(\delta,\gamma,\sigma) = \sum_{i=1}^{2k_m-2}\sum_{j=2}^{k_m} \frac{\mathfrak{R}_{ij}(\sigma)}{(\delta+i)(\gamma-2j)} \qquad \mathfrak{R}_{ij}(\sigma) = \sum_{m=\lfloor \frac{i}{2} \rfloor}^{\min(i,j-1)} \frac{b_{k_1\mathscr{D}} b_{k_2\mathscr{D}} (-1)^i i \binom{m}{i-m} (2\sigma)^{m-1}}{2j!\, m!\, (k_1-j)!\, (k_2-j)!\, (j-m-1)!}$$

One-loop bootstrap

- At the next order in $1/N$, our strategy for computing these loop-level corrections is by "gluing" together tree-level correlators [Chen, Gemenez-Grau, Paul, XZ]. This generalizes the AdS unitarity method in defect free CFTs [Aharony, Alday, Bissi, Perlmutter '16]



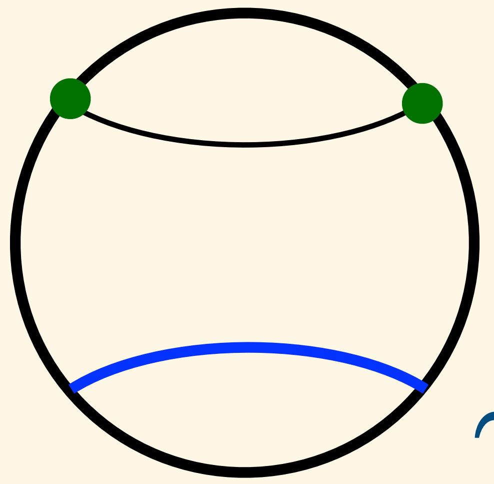
- The meaning of gluing can be made precise in terms of the leading logarithmic singularities at small cross ratios $B, D \to 0$. We will then complete the singularities into the correlator. We focus on the reduced correlator $\mathscr{H}$ and the lowest $\langle 22 \rangle$.

One-loop bootstrap

The toy version: no mixing

- We first consider a toy version where we have only S_2 and there is no operator mixing. This amounts to consistently truncating the bulk SUGRA so that there is no internal S^4.

Free propagator



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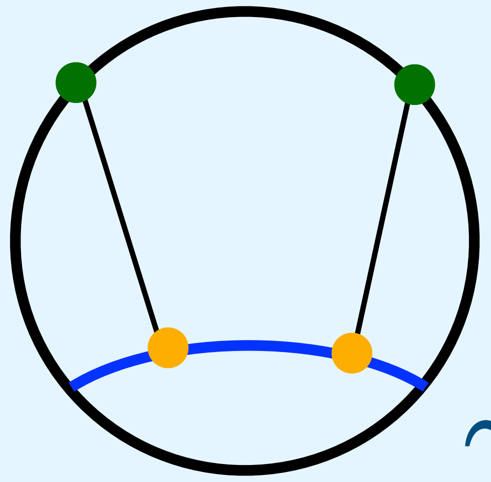
$$= \sum_{n=0}^{\infty} \sum_{s} b_{n,s}^{(0)} \, b_{n,s}^{(0)} \, \hat{g}^{d}_{\hat{\tau}_{n,s}}$$

Disconnected



$\sim 1/N$

$$= \sum_{n=0}^{\infty} \sum_{\ell \text{ even}} \lambda_{n,\ell}^{(0)} \, a_{n,\ell}^{(0)} \, g^{b}_{\tau_{n,\ell}}$$

Defect "double-trace" operators

$$\widehat{O}_{n,s} = \Box_\perp^n \, \partial_\perp^s S_2 \,|_{x^i=0}$$

Dimension: $4 + 2n + s$ $\quad \langle \widehat{O}_{n,s} S_2 \rangle \sim b_{n,s}^{(0)}$

Transverse spin: s

Bulk double-trace operators

$$O_{n,\ell} =: \Box^n S_2 \partial^\ell S_2 :$$

$\langle S_2 S_2 O_{n,\ell} \rangle \sim \lambda_{n,\ell}^{(0)}$

$\langle O_{n,\ell} \rangle \sim a_{n,\ell}^{(0)}$

Dimension: $8 + 2n + \ell$

Spin: ℓ

One-loop bootstrap

- Logs come from anomalous dimensions

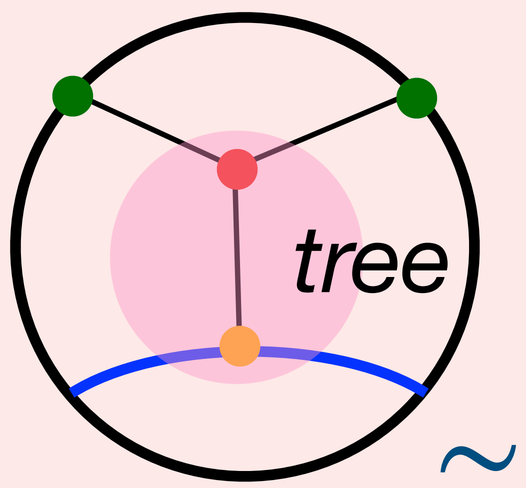
$$\hat{g}^d_{\hat{\tau},s} \sim D^{\hat{\tau}}(1+\ldots) \supset \hat{\gamma}\log D\ \hat{g}^d_{\hat{\tau},s} \qquad \hat{\tau} = \hat{\Delta} - s$$

$$g^b_{\tau,\ell} \sim B^{\frac{\tau}{2}}(1+\ldots) \supset \frac{\gamma}{2}\log B\ g^b_{\tau,\ell} \qquad \tau = \Delta - \ell$$

$$B = \frac{(1-z)(1-\bar{z})}{z+\bar{z}} \qquad D = \frac{\sqrt{z\bar{z}}}{z+\bar{z}}$$

The anomalous dimension are of order $\hat{\gamma}^{(1)}_{n,s} \sim 1/N^2$ $\quad \gamma^{(1)}_{n,\ell} \sim 1/N^3$

<u>Tree level</u> Only $\log D$, no $\log B$



$\sim 1/N^2$

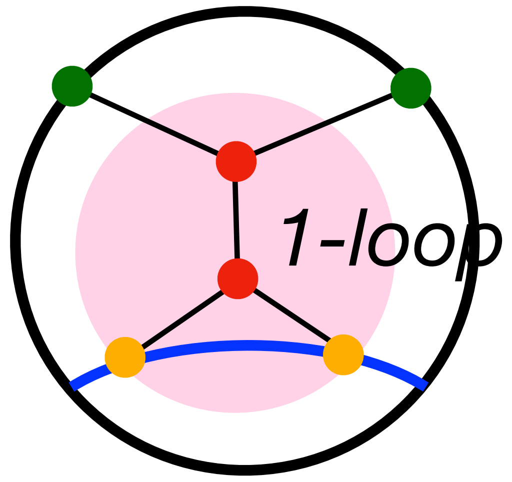
$$= \begin{cases} \sum_{n=0}^{\infty}\sum_{s} b^{(0)}_{n,s}\, \hat{\gamma}^{(1)}_{n,s}\, b^{(0)}_{n,s}\, \hat{g}^d_{\hat{\tau}_n,s} & \text{(defect channel)} \\ \sum_{n=0}^{\infty}\sum_{\ell\ \text{even}} \lambda^{(0)}_{n,\ell}\, a^{(1)}_{n,\ell}\, g^b_{\tau_n,\ell} & \text{(bulk channel)} \end{cases}$$

$\log D$ coefficient

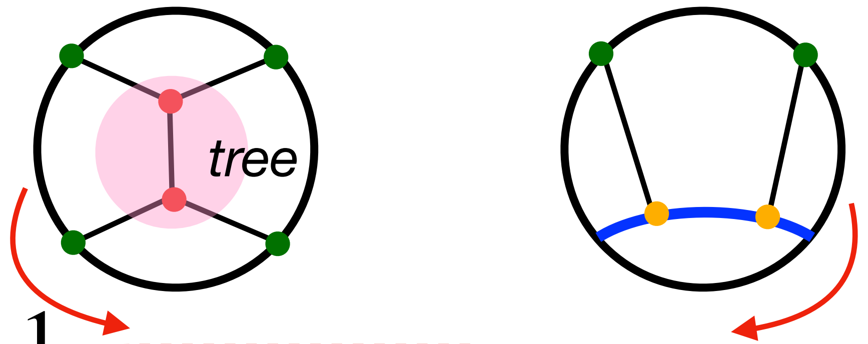
One-loop bootstrap

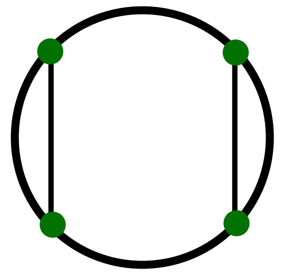
- At one-loop the leading logarithmic singularities are $\log^2 D$ and $\log B$. Their coefficients depend only on the tree-level data

Bulk channel:



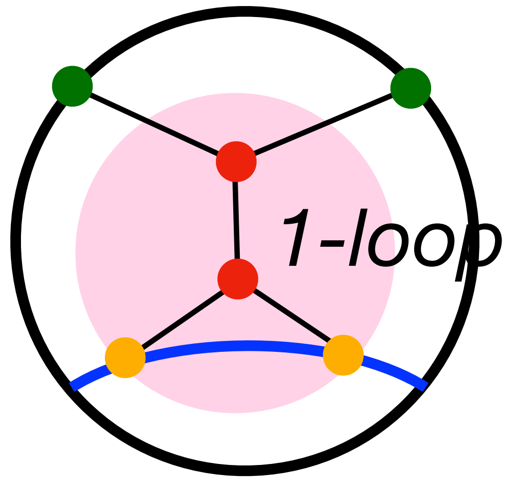
$$= \sum_{n=0}^{\infty} \sum_{\ell \text{ even}} \frac{1}{2}\lambda^{(0)}_{n,\ell}\, \gamma^{(1)}_{n,\ell}\, a^{(0)}_{n,\ell}\, g^{b}_{\tau_n,\ell} = \sum_{n=0}^{\infty} \sum_{\ell \text{ even}} \frac{(\frac{1}{2}\lambda^{(0)}_{n,\ell}\gamma^{(1)}_{n,\ell}\lambda^{(0)}_{n,\ell})(\lambda^{(0)}_{n,\ell}a^{(0)}_{n,\ell})}{(\lambda^{(0)}_{n,\ell})^2}\, g^{b}_{\tau_n,\ell}$$



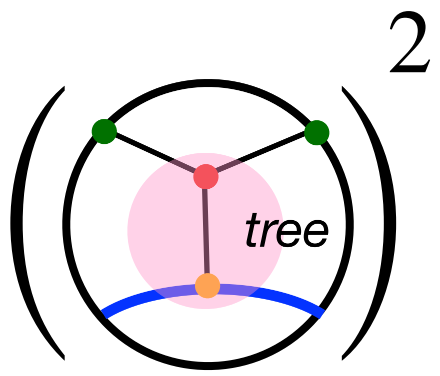


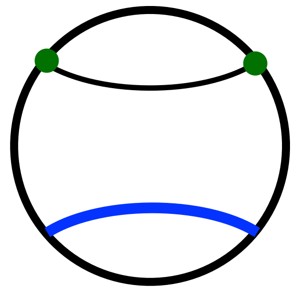
$\log B$ coefficient

Defect channel:



$$= \sum_{n=0}^{\infty} \sum_{s} \frac{1}{2} b^{(0)}_{n,s}\, (\hat{\gamma}^{(1)}_{n,s})^2\, b^{(0)}_{n,s}\, \hat{g}^{d}_{\hat{\tau}_n,s} = \sum_{n=0}^{\infty} \sum_{s} \frac{(b^{(0)}_{n,s}\,\hat{\gamma}^{(1)}_{n,s})^2}{2(b^{(0)}_{n,s})^2}\, \hat{g}^{d}_{\hat{\tau}_n,s}$$





$\log^2 D$ coefficient

One-loop bootstrap

The full theory: operator mixing

- In our theory there are degeneracies among operators

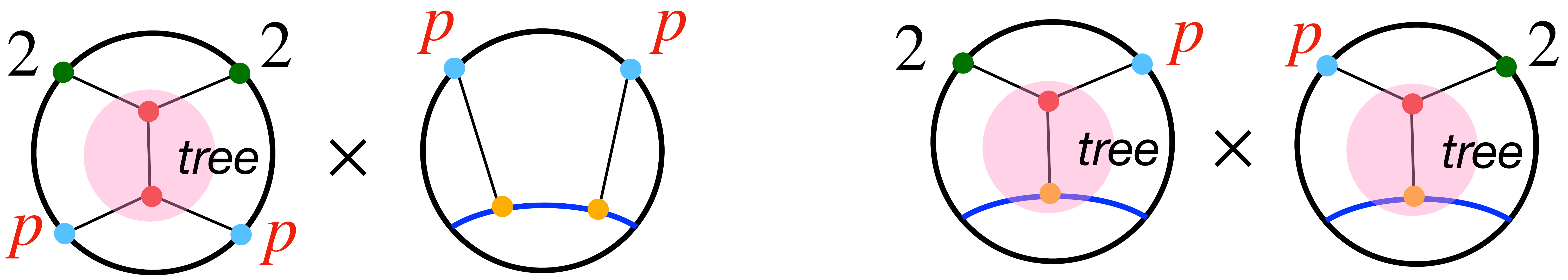
Bulk channel:	$:\Box^n S_2 \partial^\ell S_2:$	$:\Box^{n-2} S_3 \partial^\ell S_3:$	$\ldots$	$:S_{\frac{n}{2}+2}\partial^\ell S_{\frac{n}{2}+2}:$	$\tau = 8+2n$
Defect channel:	$\Box_\perp^n \partial_\perp^s S_2$	$\Box_\perp^{n-1} \partial_\perp^s S_3$	$\ldots$	$\partial_\perp^s S_{n+2}$	$\hat{\tau} = 4+2n$

- Therefore, all the coefficients in the conformal block decomposition are averages

$$\langle b_{n,s}^{(0)} (\hat{\gamma}_{n,s}^{(1)})^2 b_{n,s}^{(0)} \rangle \neq \langle (b_{n,s}^{(0)})^2 \hat{\gamma}_{n,s}^{(1)} \rangle^2 / \langle b_{n,s}^{(0)} \rangle^2$$

- In principle, we need to first unmix and get the eigenvalues. This is possible but not necessary. To compute $\langle 22 \rangle$ at one loop, there is a shortcut.

sum over p



One-loop bootstrap

- Bulk channel

Organize the CFT data into matrices

$$\Lambda^{(0)} = \begin{bmatrix} \lambda_{22\mathcal{O}_1} & \lambda_{22\mathcal{O}_2} & \cdots & \lambda_{22\mathcal{O}_{M-1}} \\ \lambda_{33\mathcal{O}_1} & \lambda_{33\mathcal{O}_2} & \cdots & \lambda_{33\mathcal{O}_{M-1}} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{MM,\mathcal{O}_1} & \lambda_{MM\mathcal{O}_2} & \cdots & \lambda_{MM\mathcal{O}_{M-1}} \end{bmatrix}$$

↓ different correlators

→ operator degeneracy

$M = \frac{n}{2} + 2$

Disconnected 4pt:

$$\Lambda^{(0)}(\Lambda^{(0)})^T = \mathbf{N}^{(0)} = \begin{bmatrix} \langle 2222\rangle^{(0)} & & & \\ & \langle 3333\rangle^{(0)} & & \\ & & \cdots & \\ & & & \langle MMMM\rangle^{(0)} \end{bmatrix}$$

Disconnected 2pt:

$$\mathbf{A}^{(0)} = (a_1^{(0)}, a_2^{(0)}, \ldots, a_{M-1}^{(0)})^T$$

$$\mathbf{A}^{(0)}\mathbf{A}^{(0)} = \begin{bmatrix} \langle 22\rangle^{(0)} & \langle 33\rangle^{(0)} & \cdots & \langle MM\rangle^{(0)} \end{bmatrix}^T$$

Tree level:

$$\Gamma^{(1)} = \mathrm{diag}(\gamma_1^{(1)}, \gamma_2^{(1)}, \ldots, \gamma_{M-1}^{(1)})$$

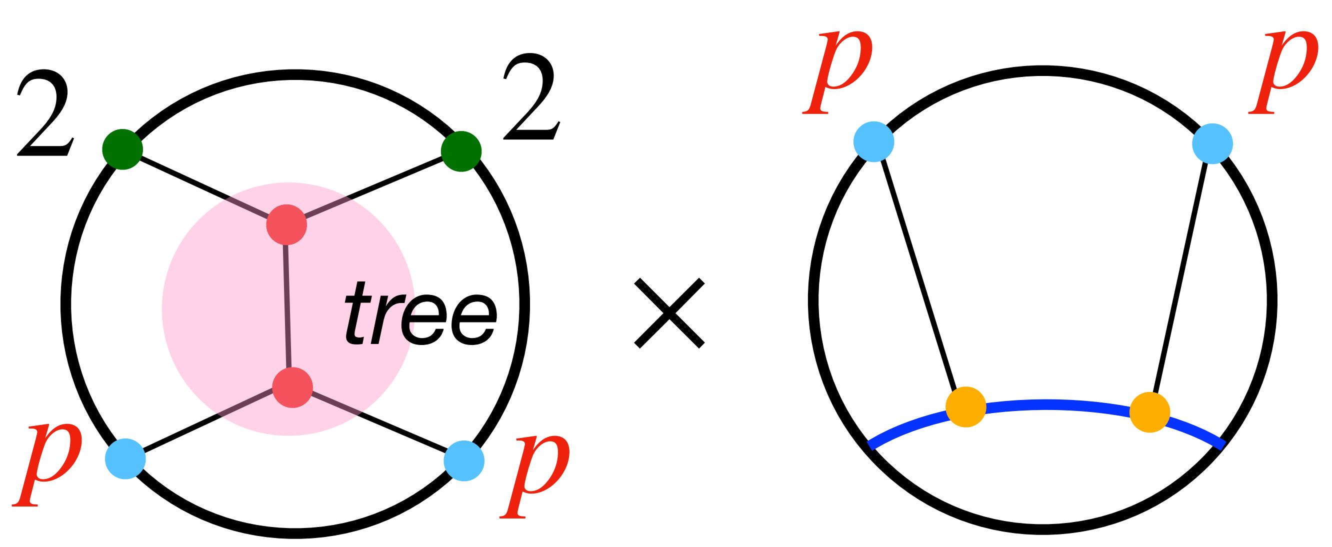
$$\Lambda^{(0)}\Gamma^{(1)}(\Lambda^{(0)})^T = \boldsymbol{\Omega}^{(1)} = \begin{bmatrix} \langle 2222\rangle^{(1)} & \langle 2233\rangle^{(1)} & \cdots & \langle 22MM\rangle^{(1)} \\ \langle 3322\rangle^{(1)} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \langle MM22\rangle^{(1)} & \cdots & \cdots & \langle MMMM\rangle^{(1)} \end{bmatrix}$$

One-loop bootstrap

1-loop $\log B$:

$$\mathbf{\Lambda}^{(0)}\mathbf{\Gamma}^{(1)}\mathbf{A}^{(0)} = \mathbf{\Lambda}^{(0)}\mathbf{\Gamma}^{(1)}(\mathbf{\Lambda}^{(0)})^T(\mathbf{N}^{(0)})^{-1}\mathbf{\Lambda}^{(0)}\mathbf{A}^{(0)}$$

$$= \mathbf{\Omega}^{(1)}(\mathbf{N}^{(0)})^{-1}(\mathbf{\Lambda}^{(0)}\mathbf{A}^{(0)})$$

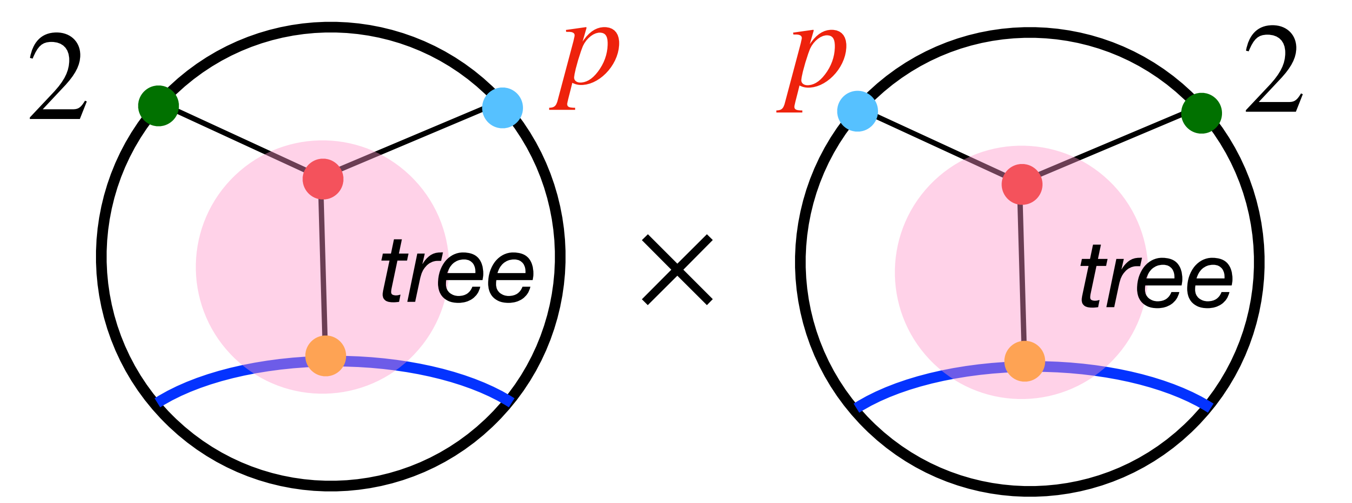


All tree-level 4pt functions are known [Alday, XZ '20]. The data $\mathbf{\Omega}^{(1)}$, $\mathbf{N}^{(0)}$ were extracted in [Alday, Chester, Raj '20]. We need only the 1st component of the vector. Therefore, we only need the tree-level $\langle 22pp\rangle^{(1)}$ and the disconnected $\langle pp\rangle^{(0)}$.

$$\mathbf{\Omega}^{(1)} = \begin{bmatrix} \langle 2222\rangle^{(1)} & \langle 2233\rangle^{(1)} & \dots & \langle 22MM\rangle^{(1)} \\ \langle 3322\rangle^{(1)} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \langle MM22\rangle^{(1)} & \dots & \dots & \langle MMMM\rangle^{(1)} \end{bmatrix}$$

$$\mathbf{\Lambda}^{(0)}\mathbf{A}^{(0)} = \begin{bmatrix} \langle 22\rangle^{(0)} & \langle 33\rangle^{(0)} & \cdots & \langle MM\rangle^{(0)} \end{bmatrix}^T$$

One-loop bootstrap



- Defect channel

$$\hat{\mathbf{B}}^{(0)} = \begin{bmatrix} b_{2\hat{O}_1} & b_{2\hat{O}_2} & \cdots & b_{2\hat{O}_{L-1}} \\ b_{3\hat{O}_1} & b_{3\hat{O}_2} & \cdots & b_{3\hat{O}_{L-1}} \\ \cdots & \cdots & \cdots & \cdots \\ b_{L,\hat{O}_1} & b_{L,\hat{O}_2} & \cdots & b_{L,\hat{O}_{L-1}} \end{bmatrix}$$

different correlators (↓)

$L = n + 2$

operator degeneracy (→)

Free 2pt:

$$\hat{\mathbf{B}}^{(0)}(\hat{\mathbf{B}}^{(0)})^T = \hat{\mathbf{N}}^{(0)} = \begin{bmatrix} \langle 22\rangle^{(0)} & & & \\ & \langle 33\rangle^{(0)} & & \\ & & \ddots & \\ & & & \langle LL\rangle^{(0)} \end{bmatrix}$$

Tree $\log D$:

$$\hat{\Gamma}^{(1)} = \mathrm{diag}(\hat{\gamma}_1^{(1)}, \hat{\gamma}_2^{(1)}, \ldots, \hat{\gamma}_{M-1}^{(1)})$$

$$\hat{\mathbf{B}}^{(0)}\hat{\Gamma}^{(1)}(\hat{\mathbf{B}}^{(0)})^T = \hat{\Omega}^{(1)} = \begin{bmatrix} \langle 22\rangle^{(1)} & \langle 23\rangle^{(1)} & \cdots & \langle 2M\rangle^{(1)} \\ \langle 32\rangle^{(1)} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \langle M2\rangle^{(1)} & \cdots & \cdots & \langle MM\rangle^{(1)} \end{bmatrix}$$

1-loop $\log^2 D$:

$$\hat{\mathbf{B}}^{(0)}(\hat{\Gamma}^{(1)})^2(\hat{\mathbf{B}}^{(0)})^T$$

$$= \hat{\mathbf{B}}^{(0)}\hat{\Gamma}^{(1)}(\hat{\mathbf{B}}^{(0)})^T(\hat{\mathbf{N}}^{(0)})^{-1}\hat{\mathbf{B}}^{(0)}\hat{\Gamma}^{(1)}(\hat{\mathbf{B}}^{(0)})^T$$

$$= \hat{\Omega}^{(1)}(\hat{\mathbf{N}}^{(0)})^{-1}\hat{\Omega}^{(1)}$$

We need only the 11 component.

One-loop bootstrap

- To summarize, expanded in small B and D, we can compute all the leading logarithmic singularities using the tree-level data via gluing

bulk channel

$$\mathcal{H}(B,D) = \log B \log^2 D\ F_{1,2}(B,D) + \log B \log D\ F_{1,1}(B,D) + \log B\ F_{1,0}$$

defect channel

$$+\log^2 D\ F_{0,2}(B,D) \quad +\log D\ F_{0,1}(B,D) \quad +F_{0,0}(B,D)$$

- We need to complete the leading logs into the full correlator. To do this, we use Mellin space

$$\mathcal{H} = \int \frac{d\delta\, d\gamma}{(2\pi i)^2} B^{-\delta} D^{\gamma}\, \widetilde{\mathcal{M}}(\delta,\gamma)\, \widetilde{\Gamma}_{22}(\delta,\gamma) \qquad \widetilde{\Gamma}_{22} = \Gamma(\delta)\Gamma(\gamma-\delta)\Gamma^2\left(\frac{6-\gamma}{2}\right)$$

- Singularities determines pole structures:
  - $\log B \log^2 D$: the integrand has double poles at $\delta = -n$ and triple poles at $\gamma = 6+2m$
  - $\widetilde{\Gamma}_{22}$ also have poles: $\widetilde{\mathcal{M}}(\delta,\gamma)$ has simple poles.

One-loop bootstrap

- We make the assumption that the reduced Mellin amplitude has only simultaneous poles

$$\widetilde{\mathcal{M}}(\delta,\gamma)=\sum_{m,n=0}^{\infty}\frac{c_{mn}}{(\delta+n)(\gamma-6-2m)}$$

  where the numerators c_{mn} are numbers.

- Solving ansatz: we take residues and focus on the $\log B\log^2 D$ term. By matching with the known $F_{1,2}(B,D)$ in the small B, D expansion, we can extract the c_{mn}.

- For example, in the defect channel we can compute c_{mn} for fixed m

$$c_{0n}=\frac{9\left(n^4+10n^3+35n^2+50n+48\right)}{4(n+1)(n+2)(n+3)(n+4)(n+5)}\qquad c_{1n}=\frac{9\left(5n^6+81n^5+517n^4+1655n^3+2814n^2-464n+1536\right)}{4(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)(n+7)}$$

$$c_{2n}=\frac{3\left(37n^8+900n^7+9394n^6+54912n^5+196477n^4+422700n^3+959916n^2-524592n+279936\right)}{4(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)(n+7)(n+8)(n+9)}$$

One-loop bootstrap

- Checking ansatz: inserting the coefficient, we can take the $\log^2 D$ coefficient but keeping the full B dependence. We find the term without $\log B$ is also correctly reproduced

$$\mathcal{H}(B,D) = \log B \log^2 D\ F_{1,2}(B,D) + \log B \log D\ F_{1,1}(B,D) + \log B\ F_{1,0}$$
$$+\log^2 D\ F_{0,2}(B,D) + \log D\ F_{0,1}(B,D) + F_{0,0}(B,D)$$

input

check

- This implies that there are no single poles in γ!
- Similarly, in the bulk channel we find

$$c_{m,0} = \frac{9\sqrt{\pi}(5m(7m+25)+96)\Gamma(m+1)}{512\Gamma\left(m+\frac{7}{2}\right)} \qquad c_{m,1} = \frac{9\sqrt{\pi}(5m(7m+25)+96)\Gamma(m+1)}{1024\Gamma\left(m+\frac{7}{2}\right)}$$

$$c_{m,2} = \frac{9\sqrt{\pi}(m(m(329m+1865)+2830)+1024)\Gamma(m+1)}{4096\Gamma\left(m+\frac{9}{2}\right)} \qquad c_{m,3} = \frac{9\sqrt{\pi}(m(m(847m+4605)+6356)+1728)\Gamma(m+1)}{8192\Gamma\left(m+\frac{9}{2}\right)}$$

- Also we can check there are no single poles in δ.

One-loop bootstrap

- The one-loop Mellin amplitude is fixed up to regular terms. Such regular terms correspond to contact Witten diagrams and are expected as UV counter terms.

- Carry out this calculation to high orders and we can find the general form for c_{mn}

$$c_{mn} = p_m H_0 + q_{m,n} H_1 + r_{m,n} H_2 + s_m H_4$$

first one-loop defect 2pt function

$$p_m = 3(m+1)^2(m+2)^2$$

$$q_{m,n} = 7m^2n^2 + 28m^3n - 21m^2n + 64m^4 + 134m^3 + 158m^2 - 14mn^2 + 6mn + 250m + 19n^2 + 95n + 162$$

$$r_{m,n} = -7m^2n^2 - 28m^3n + 35m^2n - 102m^4 - 82m^3 - 158m^2 + 14mn^2 - 34mn - 230m - 19n^2 - 57n - 100$$

$$s_m = 35(m-1)^2m^2$$

$$H_a = \frac{\sqrt{\pi}4^m\Gamma(m+n+3)}{\Gamma(\frac{-2m-3}{2})\Gamma(2m+n+6)}\ {}_3F_2\left[\begin{matrix} -m-2\,, & \frac{-2m-n-5}{2}\,, & \frac{-2m-n-4}{2} \\ \frac{-2m-3}{2}\,, & -m-n-2 & \end{matrix}\;\middle|\; 1\right]\Bigg|_{m\to m-2}$$

Outlook

- Many things are defects (Wilson loops, giant gravitons, real projective space…)
  - The bootstrap techniques give powerful tools to study these systems.
- Flat space limit
  - A precise prescription for taking the flat-space limit of AdS amplitudes is not yet available for defect systems.
  - Useful for studying stringy corrections. Also connects to integrated correlators from localization;
- Complementary position space techniques at loop levels
  - In the defect free case, position space methods are useful for going to higher loops
  - What is the space of functions?
- Higher-point correlators.

Thank you!