Discussion on Firewalls

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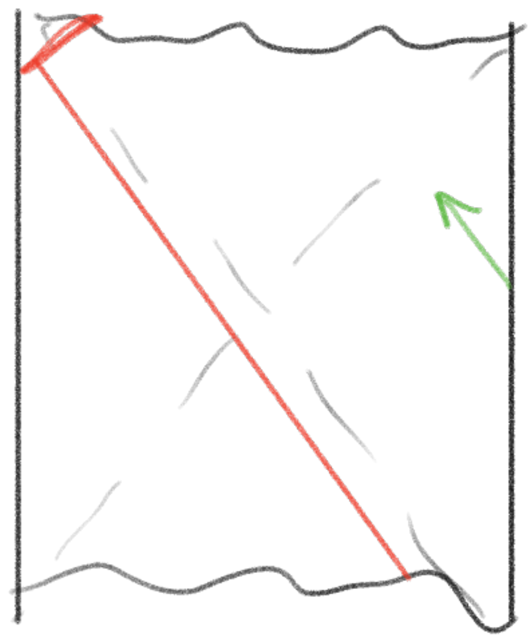
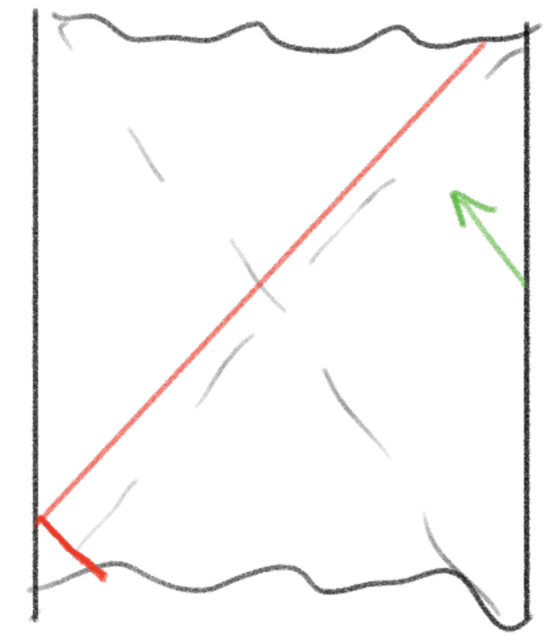
Typical State Firewall Problem



$$e^{-iHt}|\psi\rangle_E$$



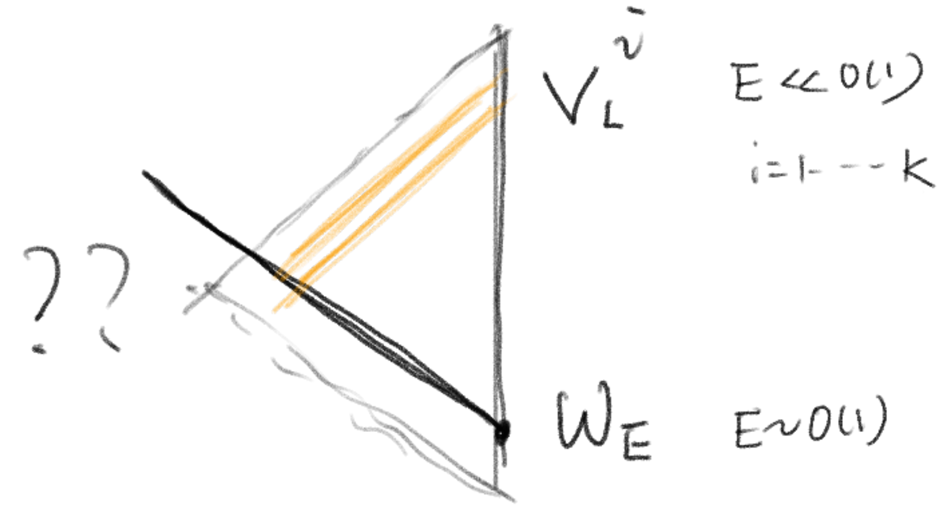
$$e^{iHt}|\psi\rangle_E$$

$t > e^{S_E}$, both states become typical in the ensemble of states w/ random phase

$$\left\{\sum_m |m\rangle C_m e^{i\theta_m}\right\}, \quad \{\theta_m\} \text{ iid random}$$

Q: If the firewall operator is a smooth function in the ensemble, then based on measure concentration, future and past states should be the same. In tension with the Penrose diagram.

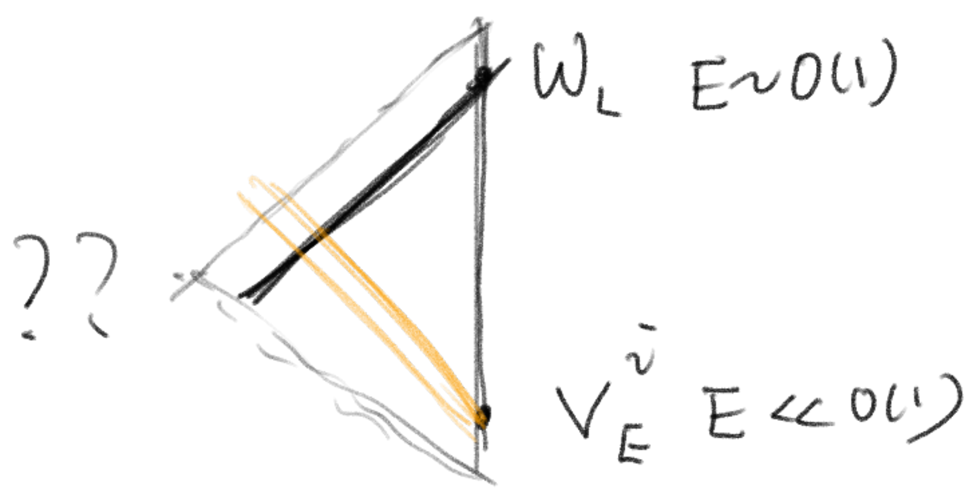
Nonlinearity of firewall operator.



$\{|\psi\rangle\}$ typical $\Rightarrow$ $\{V_L^i W_E |\psi\rangle\}_{i=1\cdots k,\ \psi}$

Each i, $\dim \mathcal{H}_i = e^{S_E}$. $\mathcal{H}_i \perp \mathcal{H}_{i'} \Rightarrow \dim(\bigoplus_{i=1}^{k} \mathcal{H}_i) \sim k e^{S_E}$.

$k e^{S_E} > \frac{1}{2} e^{S_{E+\Delta E}}$ $\Rightarrow$ more than half of the states are firewall states

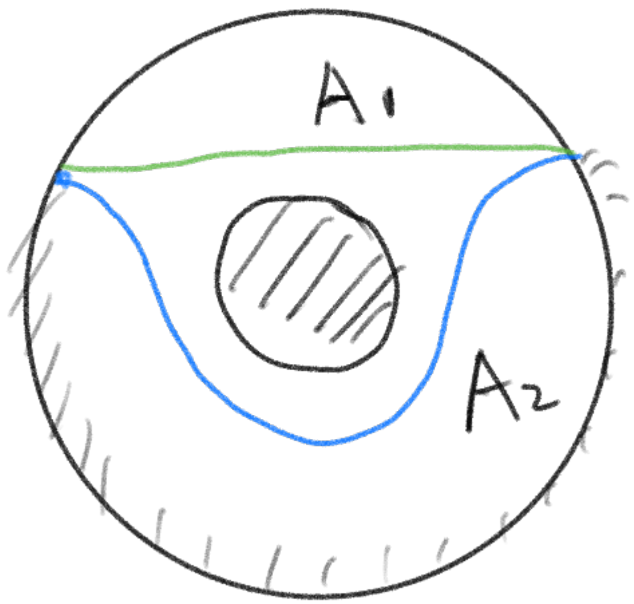


$\{V_E^i W_L |\psi\rangle\}$

$k e^{S_E} > \frac{1}{2} e^{S_{E+\Delta E}}$ $\Rightarrow$ more than half of the states are smooth states

No linear operator can work!

Digression (Area Operator)



$$\rho = \sum_n |E_n\rangle\langle E_n|$$

For each state $|E_n\rangle\langle E_n|$: $S = A_1$

For thermal state ρ : $S = A_2$

$\Rightarrow$ minimum area operator is not a linear operator

For the area case, we have both bulk and boundary understanding.

😀

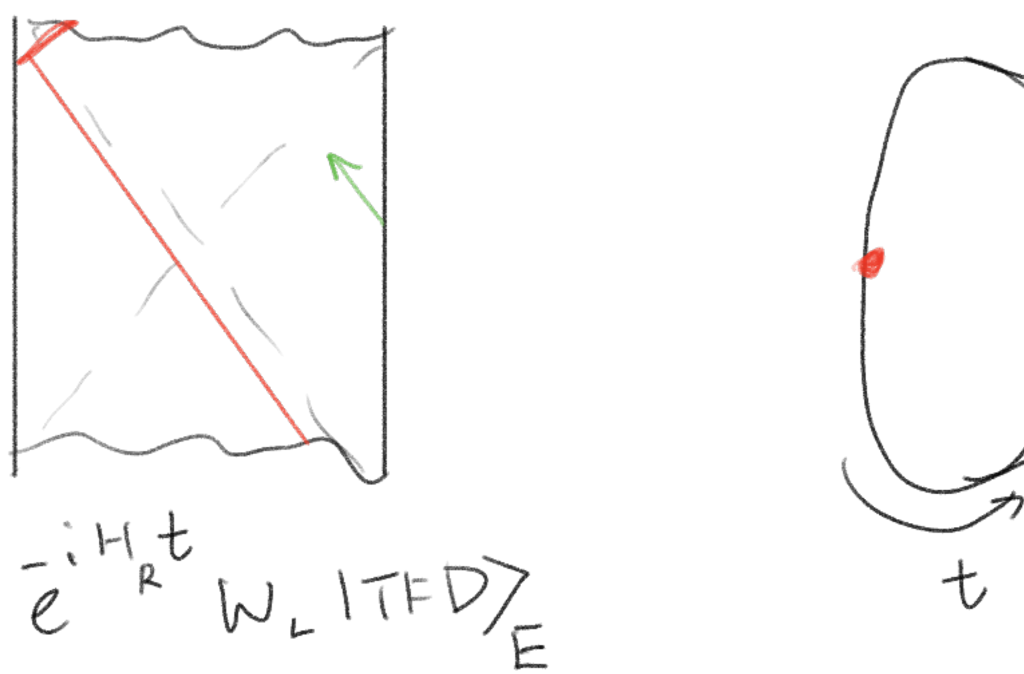
For the firewall case , we have neither.

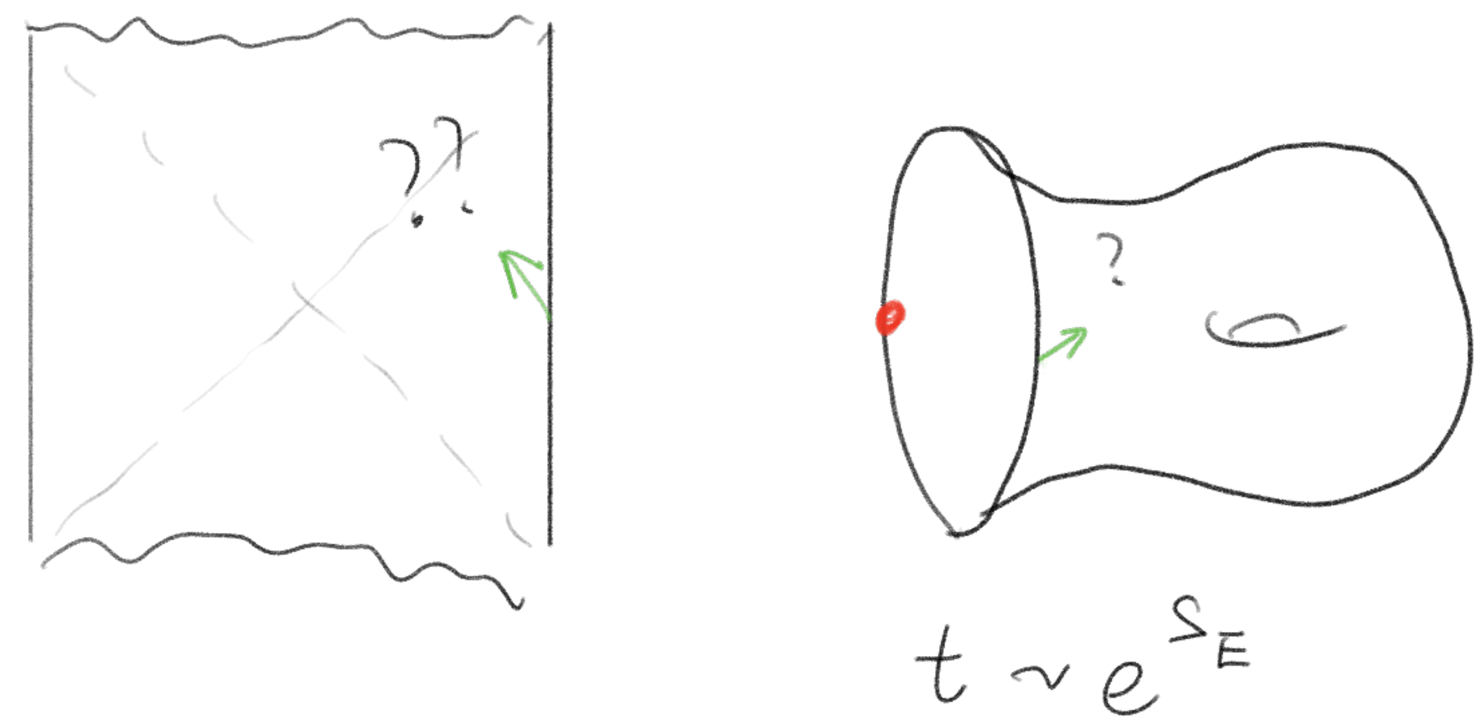
😭

Hope: a direct bulk analysis may tell us something.

Goal: Bulk analysis of the Firewall problem using spacetime Wormholes.

Challenge: Lack of first principle definition of the infalling experience.



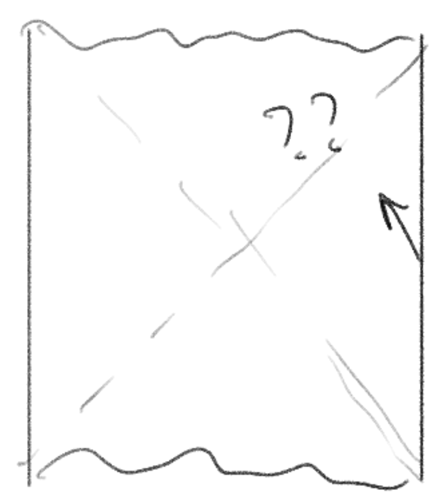


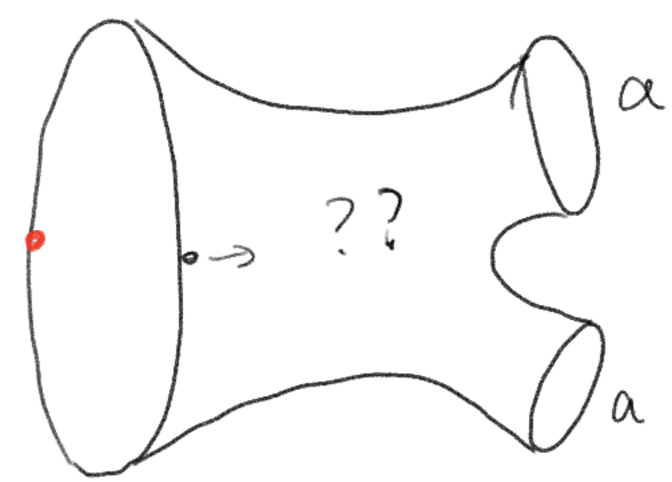
TO BE OR NOT TO BE?

Strategy: IF it was a classical saddle ⇒ read out the experience

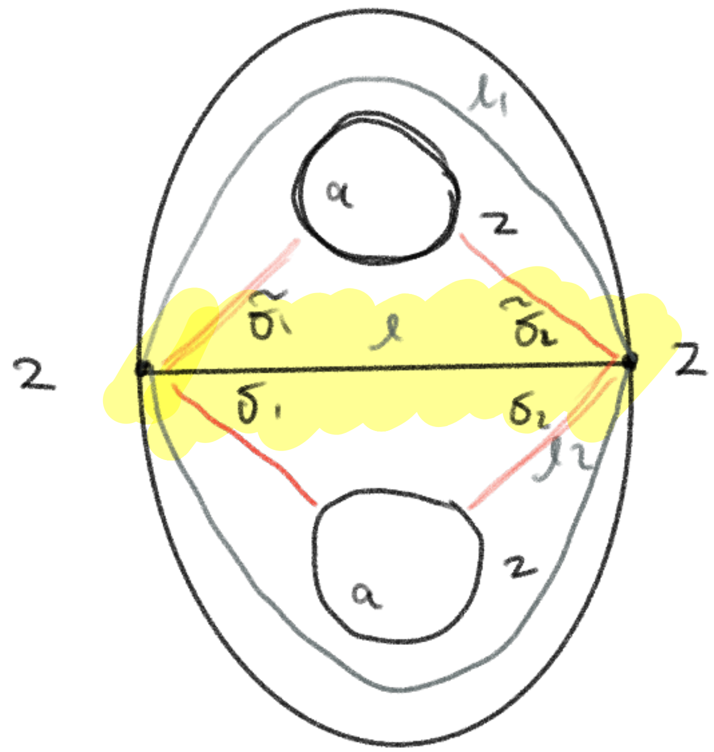
it isn't !

BUT: decompose ⇒ classical saddle + Something else.





Wormhole Path Integral



Counting variables: 8-3-1=4

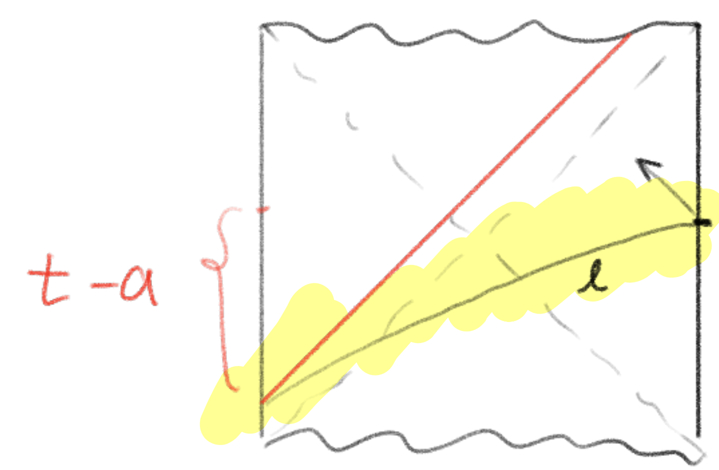
$$\int d^4\sigma \, e^{-I(\sigma)}$$

σ: Geodesic distance from boundary to baby universes.

Saddle point :

$$I(\sigma) = -i(\sigma_1+\sigma_2-\tilde{\sigma}_1-\tilde{\sigma}_2) + e^{-\sigma_1}+e^{-\sigma_2}+e^{-\tilde{\sigma}_1}+e^{-\tilde{\sigma}_2}$$

$$\sigma_{1,2} = i\pi/2 \qquad \tilde{\sigma}_{1,2} = -i\pi/2 .$$

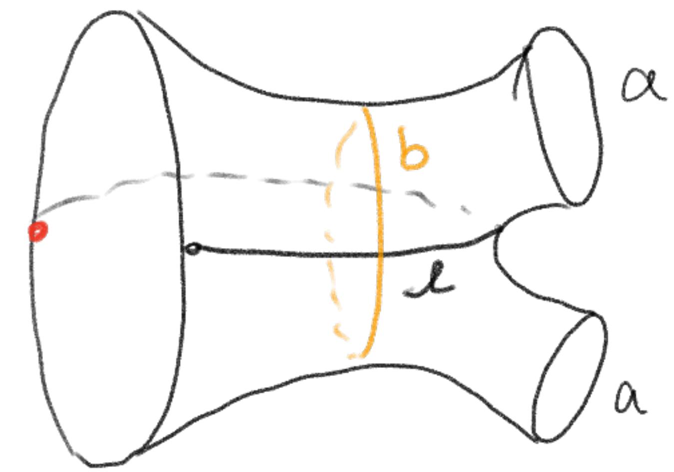


Firewall ?

Close to "saddle"

$$x = \delta\sigma + \delta\tilde{\sigma}$$
$$y = \delta\sigma - \delta\tilde{\sigma}$$

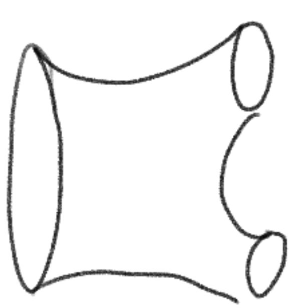
$$b = 2t + \ln|y|$$
$$\ell = a - t + x$$



$$\int dx\, dy\, e^{ixy} \overset{y \neq 0}{\Longrightarrow} 0$$

Whole partition function vanishes!

$$\int_{y\neq 0} dy\, e^{ixy} = \underbrace{\delta(x)}_{\text{saddle}} - \underbrace{\delta(x=\infty)}_{\text{something else}}$$



$= 0$

$[\, e^{-\Delta\ell} \Rightarrow e^{-\Delta X} \Rightarrow$ kills other term / just saddle $]$

Two classes of bulk observables.

For some observables $\mathcal{O}(\lambda)$, geometry appears as on-shell.

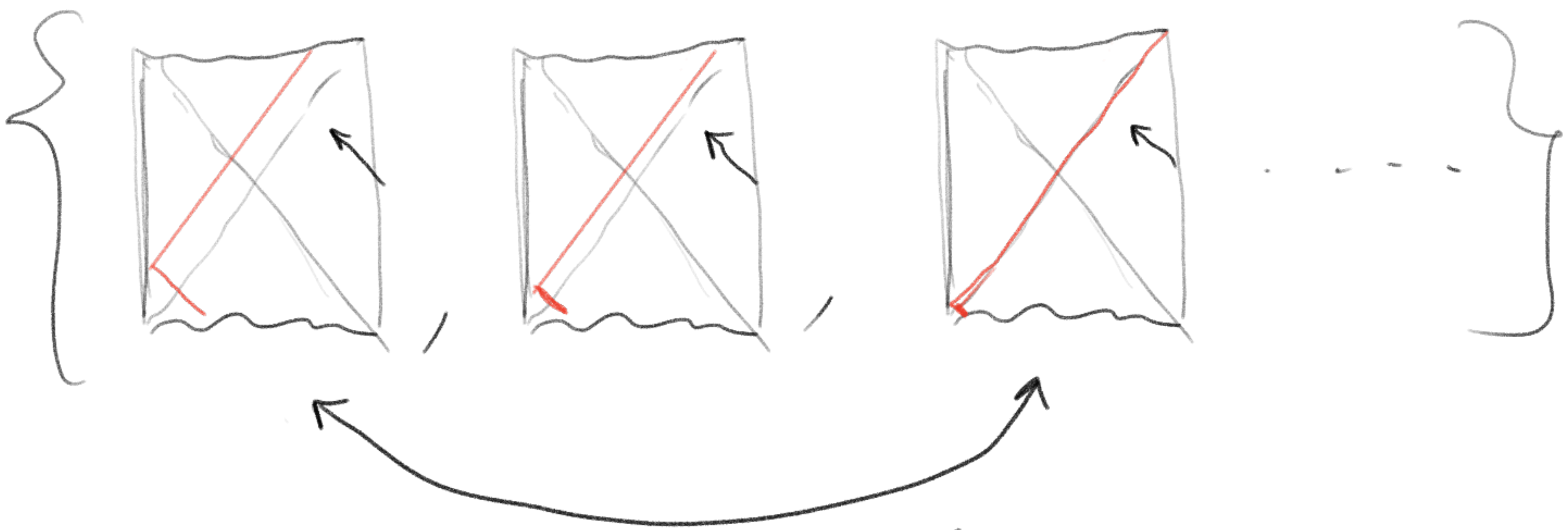
For other type observables. geometry appears as off-shell

Q: which class of observables does Firewall operator belong to?

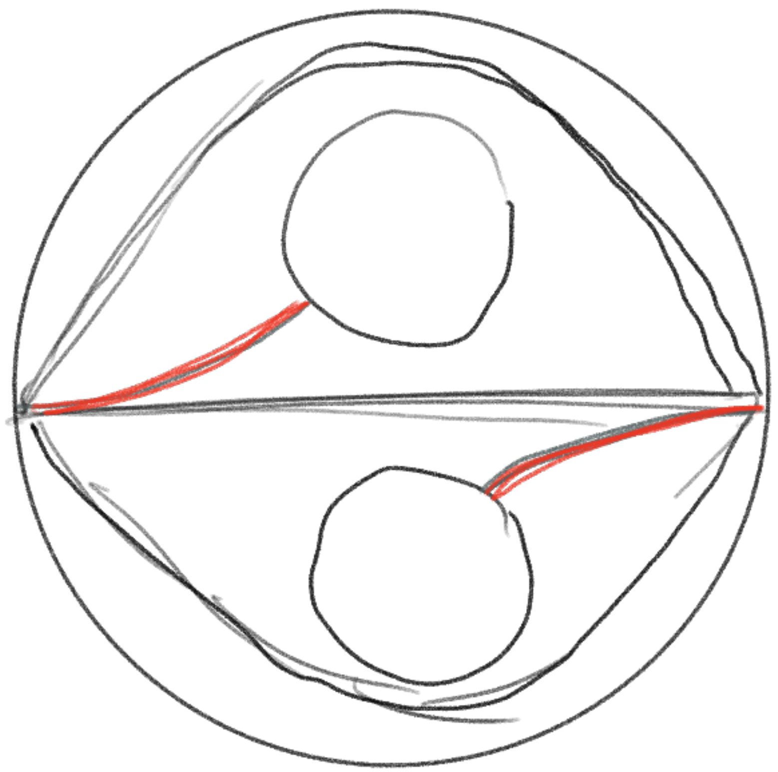
Unsolved!

Mapping Class Group

a, S



Gauge equivalent.



Overcounting.

Can be fixed by picking out the slice with minimum length among the classical saddles.

No-short cut condition

$$\Rightarrow P_{(\text{firewall})} = \frac{t^2}{2} e^{-2S_E}$$

(Stanford-Yang)

Relation w/ other works.

Blommaert-Chen-Nomura/Iliesia-Levine-Lin-Maxfield-Mezi

- All work assume the firewall operator belongs to the on-shell class operators
- The works differ in strategies of fixing the MCG
- BCN: Sum over all "On-shell" geometries -> answer is divergent, then they subtract an infinity from the divergent sum
- ILLMM: They do a partial sum over the "On-shell" geometries, the result is finite, but still has some overcounting.
- Both BCN and ILLMM: assumes the firewall operator is a linear operator!
- Both BCN and ILLMM: concludes there is 50% chance to hit a firewall in typical states.

Conclusion

- Quantum mechanics is in tension with Penrose diagram of typical states
- General argument supports the idea that firewall operator is not a linear operator
- A direct bulk analysis of the firewall problem is attempted
- The wormhole is an almost saddle, for some observables it behaves as a saddle point geometry.
- Assume we can trust the almost saddle geometry, the infaller has finite probability to hit a firewall.
- A further complication comes from overcounting due to bulk large diffeomorphism (MCG), but that can be solved for lower genus case.