MULTI-WAY JUNCTION CONDITIONS

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with Jia-yin Shen and Li-xin Li, arXiv:2402.00694

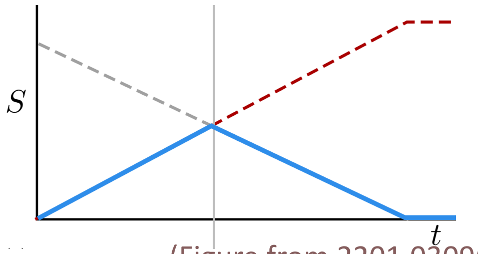
BACKGROUND

BH EVAPORATION AND ENSEMBLE AVERAGE

Penginton; Almheiri, Engelhardt, Marolf, Maxfield;Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini; Penington, Shenker, Stanford, Yang ...

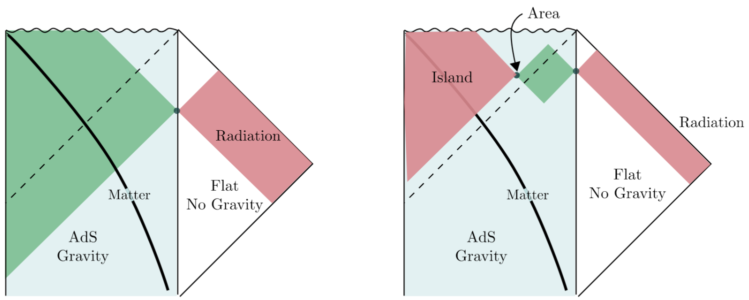
A "solvable" incarnation of the information paradox

- The information paradox:
  Are Hawking radiations from Blackholes thermal or informative?

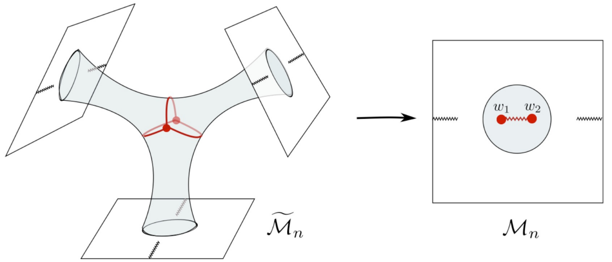


(Figure from 2201.03096)

- Recent breakthroughs in this puzzle in low-dimensional solvable toy models
  - New quantum extremal surface in an evaporating black hole
  - Alternatively, the necessity of including the spacetime wormholes in the gravitational path integral

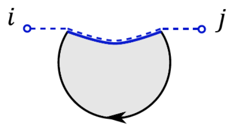


(Figure from 1911.12333)

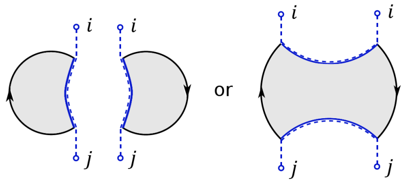


BH EVAPORATION AND ENSEMBLE AVERAGE

- Spacetime wormholes are tied with ensemble averages of theories (Coleman; Giddings Strominger; Maldacena Maoz)
- Evidence including e.g.



$$\overline{\langle\psi_i|\psi_j\rangle} = \delta_{ij},$$



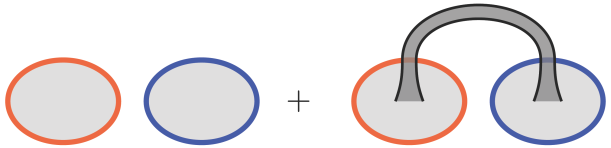
$$\overline{|\langle\psi_i|\psi_j\rangle|^2} = \delta_{ij} + \frac{Z_2}{Z_1^2}$$

$$\Leftrightarrow \quad \langle\psi_i|\psi_j\rangle = \delta_{ij} + x_{ij}, \quad \overline{x_{ij}} = 0, \quad \overline{x_{ij}^2} = \frac{Z_2}{Z_1^2}$$

(Penington, Shenker, Stanford, Yang 2020)

$$\left\langle Z[J_1]\cdots Z[J_n]\right\rangle := \int_{\Phi\sim J}\mathcal{D}\Phi\, e^{-S[\Phi]}$$

$$\left\langle Z[J_1]Z[J_2]\right\rangle =$$



$$\left\langle Z^n\right\rangle = \sum_{p\perp\{1,2,\ldots n\}} \lambda^{|p|} = B_n(\lambda) = \sum_{d=0}^{\infty} d^n p_d(\lambda) = \left\langle x^n\right\rangle_{\text{Pois}}, \qquad p_d(\lambda) = e^{-\lambda}\frac{\lambda^d}{d!}$$

(Marolf, Maxfield, 2020
CP, Tian, Yu 2021, **CP**, Tian, Yang 2022)

- Disordered models are special cases of the "ensemble average theories"

NICE STORIES, AND PAGE CURVES CAN BE REPRODUCED.

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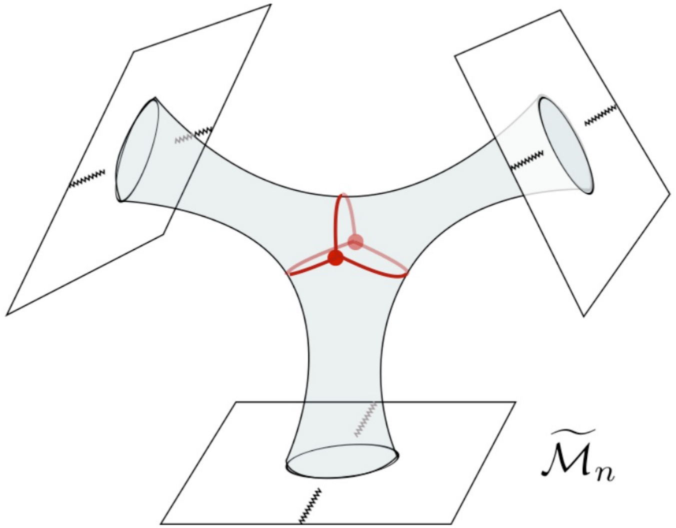
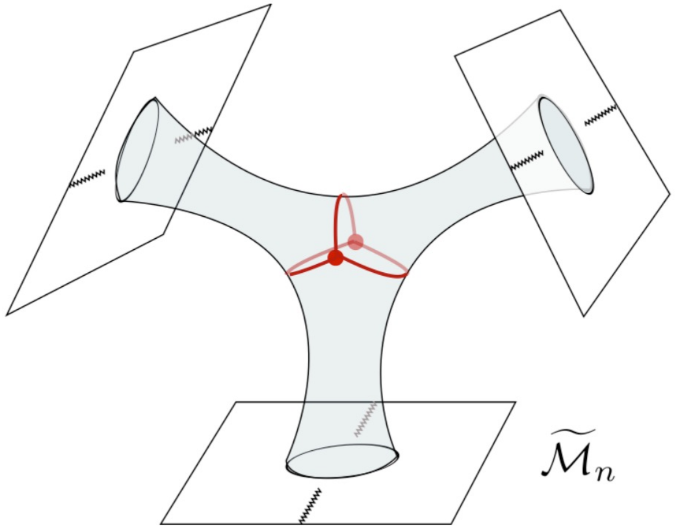
BUT THIS IS NOT THE END OF THE STORY, IT IS RATHER THE TIP OF AN ICEBURG…



Ensemble average theories: physics described by not a single Hamiltonian, but an ensemble of them

Multi-boundary geometries with Lorentzian signature



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HIGH-DIMENSIONAL DISORDERED MODELS

CP, *2018*, Chang, Colin-Ellerin, **CP**, Rangamani, 2021, 2022, and W.I.P.

1. If there exist high dimensional covariant disordered models ?

   2D and 3D models with different numbers of SUSY and tunable parameters

2. Do they share similar nice features as their low dimensional counterparts ?

   Solvable in the large-N limit, analytically in the IR and numerically in general

3. Do they fulfill the usual requirements obeyed by conventional QFTs ?

   Consistent with various **bootstrap** bounds, hence compatible with many requirements

4. If there are clear connections with other well-known conventional QFTs ?

   Observe **higher-spin limits** in different models, which sets up clear connections with higher-spin theories and probably string theory

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UV “free” fixed point

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UV “free” fixed point + disorder

+ $O_{\text{海淀家长}}$

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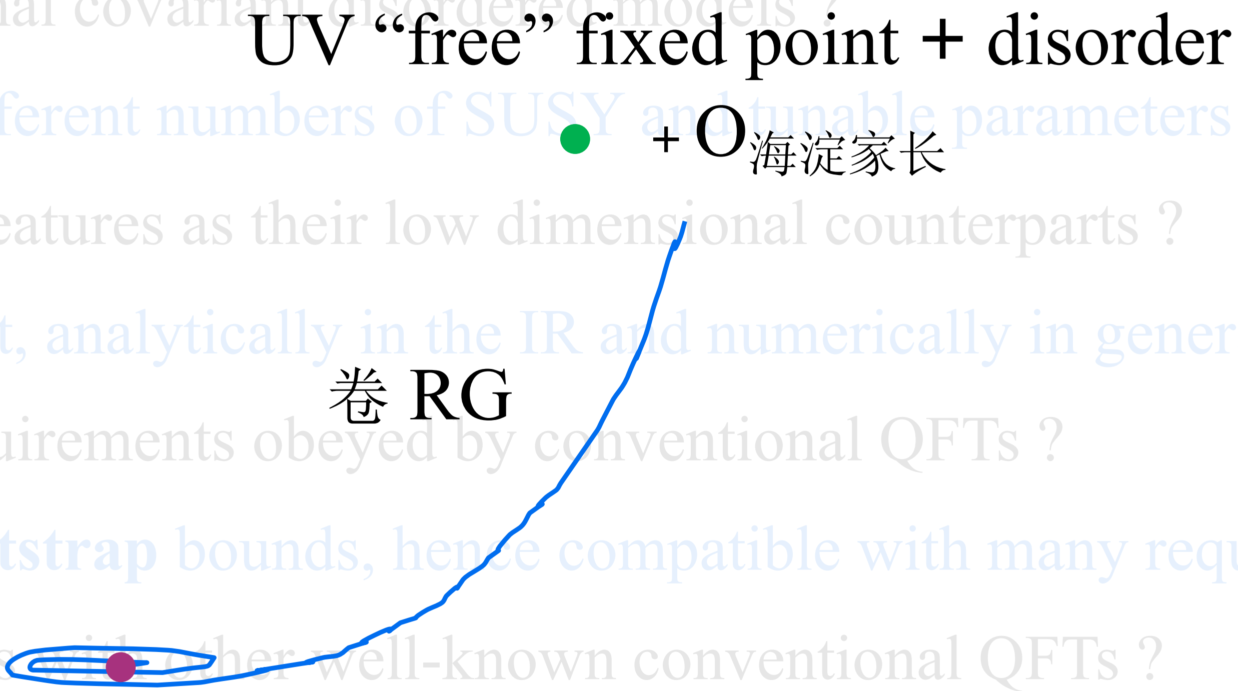

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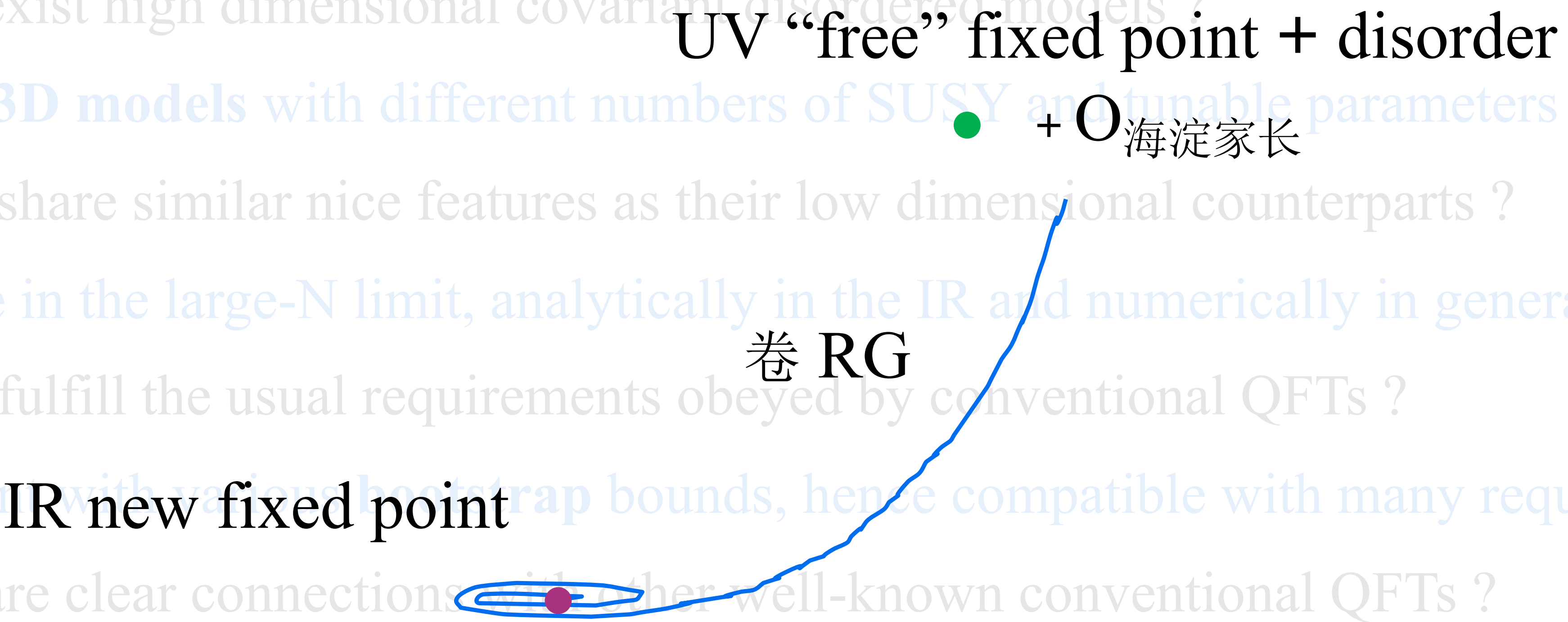

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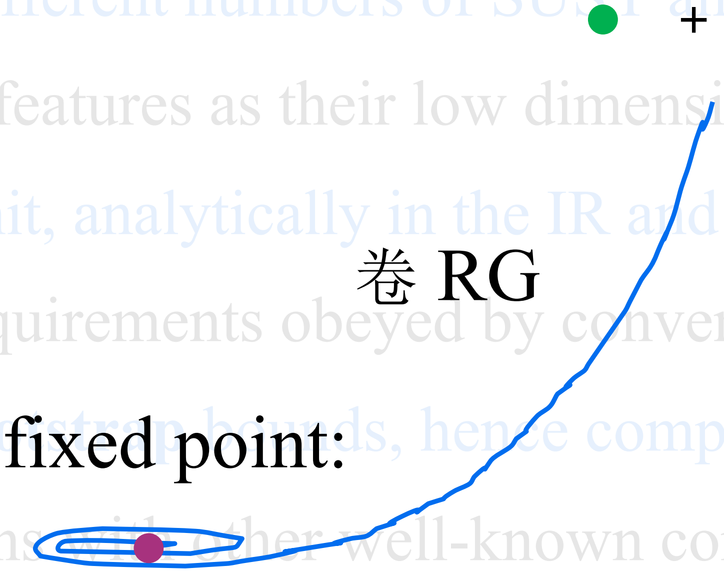
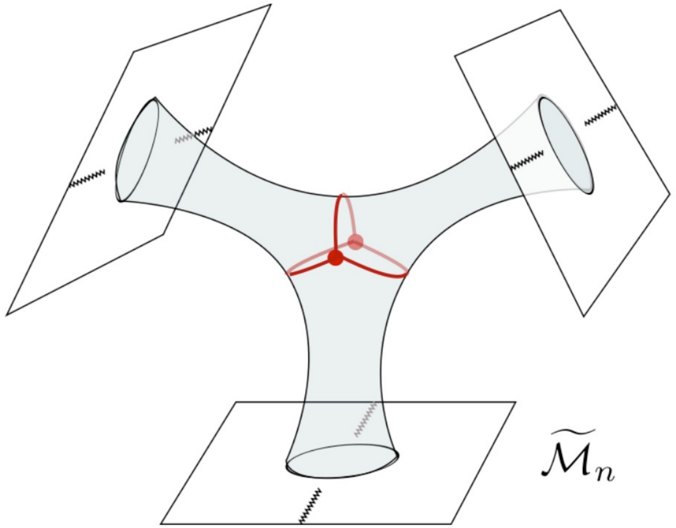




Ensemble average theories:
physics described by not a single Hamiltonian, but an ensemble of them

Multi-boundary geometries with Lorentzian signature

MOTIVATION

- We live in spacetime with Lorentzian signature.
- It is therefore interesting to construct counterparts of the multi-boundary connected Euclidean wormhole configurations in Lorentzian signature.
- However, a direct solution/continuation is not simple.
- Instead, we try to construct geometries with multiple boundaries by gluing.

GEOMETRY FROM GLUING

- One way of constructing new geometries is by gluing pieces together with suitable junction conditions

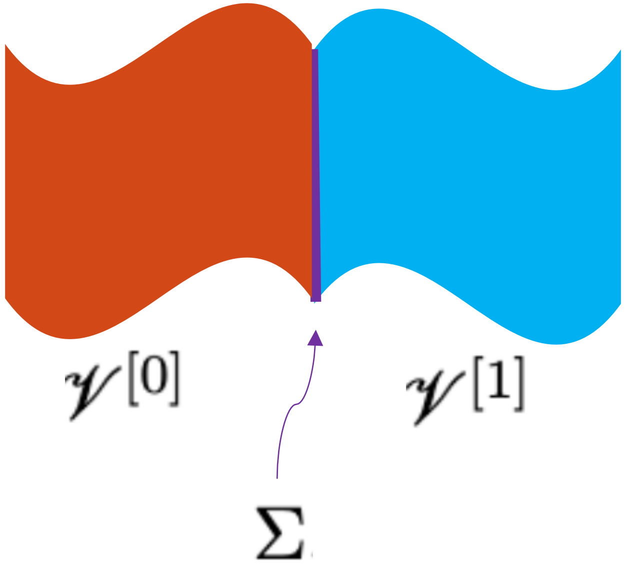
- A familiar example is the Israel junction condition in GR

Israel, Il Nuovo Cimento B, 1966

$$g^{[0]}\big|_{\Sigma} = g^{[1]}\big|_{\Sigma}$$

$$\left(\mathcal{K}^{[0]} + \mathcal{K}^{[1]}\right) - \left(\mathrm{tr}_h \mathcal{K}^{[0]} + \mathrm{tr}_h \mathcal{K}^{[1]}\right) h = -8\pi G\, \bar{\mathcal{S}}$$

$$h \coloneqq g^{[i]}\big|_{\mathscr{T}(\Sigma)}, \quad i = 0, 1$$



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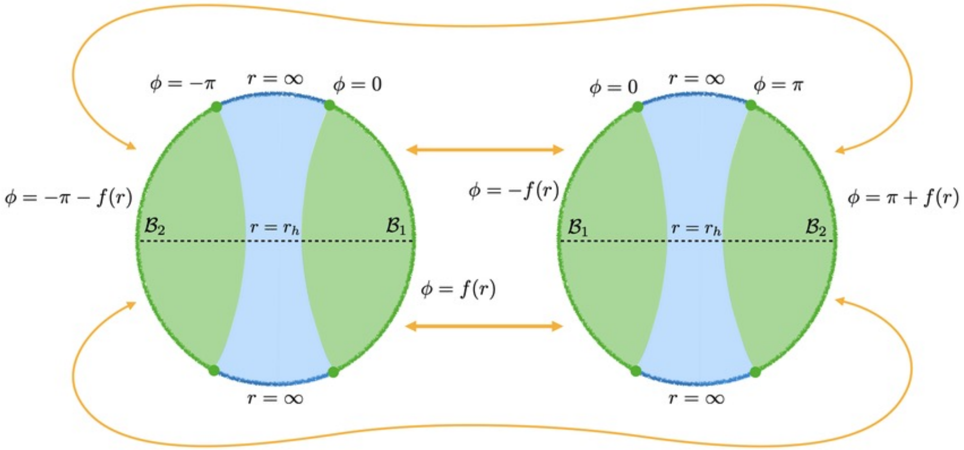
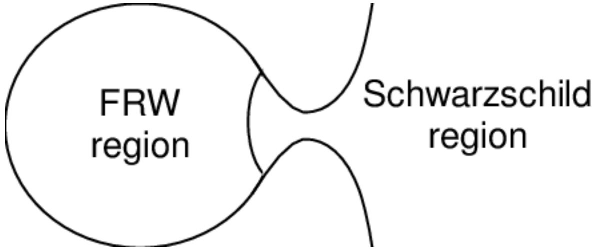




CAN WE GLUE MORE BULKS TOGETHER TO GET MULTI-BOUNDARY GEOMETRIES?

MULTI-WAY JUNCTION CONDITIONS

Shen, **CP**, Li, 2024

- We derive conditions for gluing multiple bulk geometries
  - Pure gravity

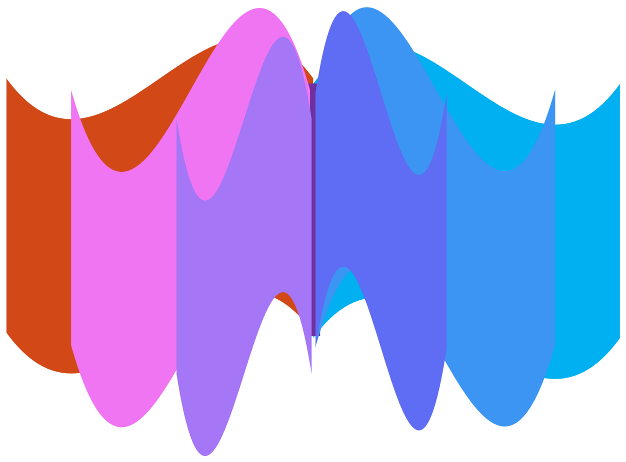
$$\sum_{i=0}^{m} \mathcal{K}^{[i]} - \mathrm{tr}_h \mathcal{K}^{[i]}\, h = -8\pi G\, \bar{\mathcal{S}}$$

  - Dilaton gravity

$$\sum_{i=1}^{m} 2\bar{\Phi}\, \mathrm{tr}_h \mathcal{K}^{[i]} + \lambda \mathfrak{L}_{n^{[i]}} \Phi^{[i]} = -4\pi G_{\mathrm{N}}\, \bar{\varrho}$$

$$\sum_{i=1}^{m} \bar{\Phi}^2 \left(\mathcal{K}^{[i]} - \mathrm{tr}_h \mathcal{K}^{[i]}\, h\right) - 2\bar{\Phi} \mathfrak{L}_{n^{[i]}} \Phi^{[i]}\, h = -8\pi G_{\mathrm{N}} \bar{\mathcal{S}}$$

where $h := g^{[i]}\big|_{\mathscr{T}(\Sigma)}$



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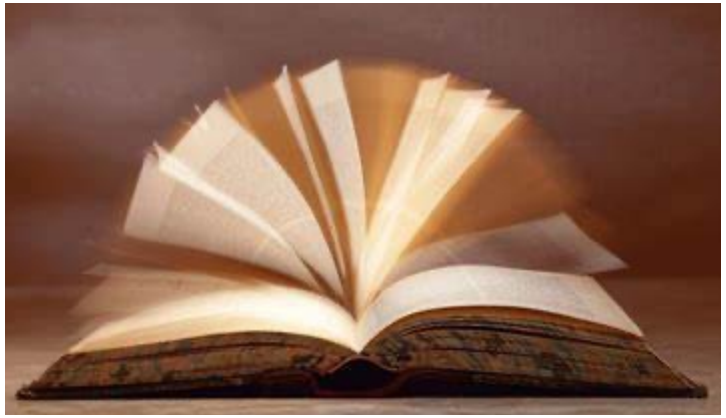


- The conditions apply to both timelike and spacelike interfaces

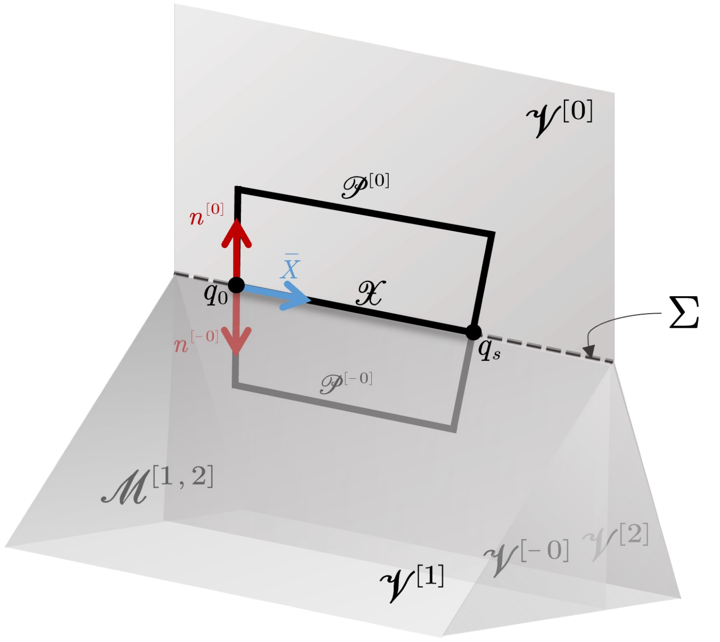
DERIVATION

- The first is geometrical in pure gravity

  Embedding in a larger spacetime

  Define various curvatures therein, e.g. $\nabla^{[i]}{}_{\bar{X}}\bar{U} = \bar{\nabla}_{\bar{X}}\bar{U} - K^{[i]}(\bar{X},\bar{U})n^{[i]}$

  Find the conditions among them



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- The second is by varying the action with appropriate boundary terms, which applies to both pure gravity and dilaton gravity

$$I_{\mathcal{V}} = \frac{1}{16\pi G_N}\sum_{i=0}^{m}\int_{V^{[i]}}\left\{(\Phi^{[i]})^2 R^{[i]} + \lambda\left(\partial\Phi^{[i]}\right)^2 - \mathcal{U}^{[i]}\left((\Phi^{[i]})^2\right)\right\}\varepsilon^{[i]} - 2\oint_{\Sigma}\bar{\Phi}^2 K^{[i]}\bar{\varepsilon}_h$$

DERIVATION

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AN APPLICATION

- Jackiw-Tetelboim gravity

$$I_{\mathscr{V}} = \frac{1}{16\pi G_{\mathrm{N}}} \sum_{i=0}^{m} \left\{ \int_{\mathscr{V}^{[i]}} \phi^{[i]} \left(R^{[i]} + \frac{2}{(L^{[i]})^2} \right) \varepsilon^{[i]} - 2 \oint_{\Sigma} \bar{\phi} \left(K^{[i]} + \frac{\chi}{m+1} \right) \bar{\varepsilon}_h \right\}$$

- General solution of the dilaton profile

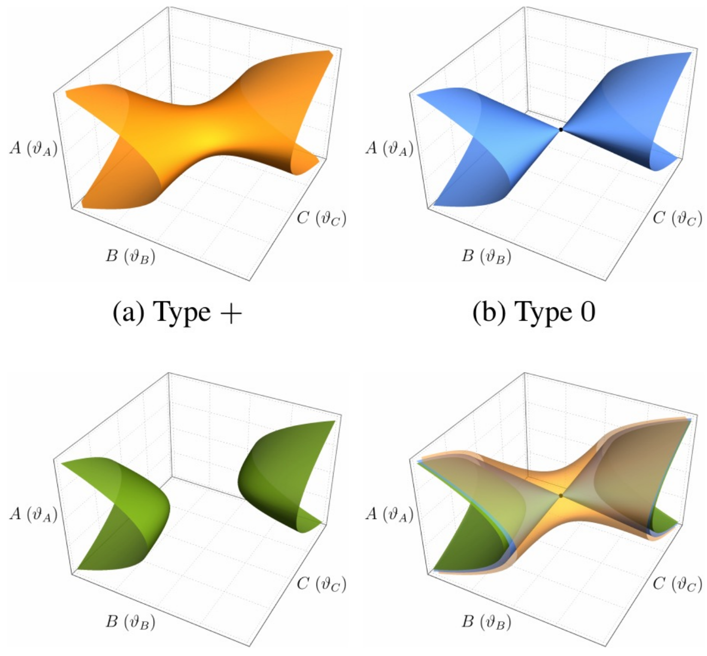
$$\phi^{[i]}(t,z) = A^{[i]} \cdot \frac{t}{z} + B^{[i]} \cdot \frac{t^2 - z^2}{z} + C^{[i]} \cdot \frac{1}{z}$$

- Classification according to the sign of $D^{[i]} = \dfrac{(A^{[i]})^2 - 4B^{[i]}C^{[i]}}{(L^{[i]})^2}$

$$\text{Type } +: \quad \phi^{[i]}(t,z) = A^{[i]} \cdot \frac{t}{z}, \quad A^{[i]} > 0;$$

$$\text{Type } 0: \quad \phi^{[i]}(t,z) = \pm (L^{[i]})^3 \cdot \frac{1}{z};$$

$$\text{Type } -: \quad \phi^{[i]}(t,z) = \pm B^{[i]} \left(\frac{t^2 - z^2}{z} + (L^{[i]})^2 \cdot \frac{1}{z} \right), \quad B^{[i]} > 0$$



(a) Type + (b) Type 0

(c) Type − (d) 3 types of dilatons

AN APPLICATION

- In this case the junction conditions are

$$\sum_{i=0}^{m} \mathrm{tr}_h K^{[i]} = -\chi, \qquad \sum_{i=0}^{m} \mathcal{L}_{n^{[i]}} \phi^{[i]} = -\bar{\phi}\chi$$

where $\phi^{[i]}$ and $\bar{\phi} = \phi^{[i]}\big|_{\Sigma}$ are the dilaton fields on $V^{[i]}$ and Σ respectively.

- The first condition leads to

$$\chi = \frac{1}{L^{[0]}} + \frac{1}{L^{[1]}} + \cdots + \frac{1}{L^{[m]}} \quad \text{and} \quad \sum_{i=0}^{m} L^{[i]} \mathsf{Sch}\left[t^{[i]}\right] = 0$$

- The second condition is more interesting.

AN APPLICATION

- Consider gluing multiple AdS_2 geometries along an interface near the boundary of each bulk

- Choosing a parameterization such that the metric on the interface is $h = -\frac{1}{\epsilon^2}\mathrm{d}u^2$

- We then compute

$$\mathfrak{L}_{n^{[i]}}\phi^{[i]}(u) = \partial_\ell \phi^{[i]}_{(-1)}(u)\cdot\frac{1}{\epsilon} + \partial_\ell \phi^{[i]}_{(1)}(u)\cdot\epsilon + \mathcal{O}(\epsilon^2)$$

- Now we can solve the second condition order by order in ϵ

$$\overline{\phi}(u) = \overline{\phi}_{(-1)}(u)\cdot\frac{1}{\epsilon} + \overline{\phi}_{(1)}(u)\cdot\epsilon + \mathcal{O}(\epsilon^3)$$

AN APPLICATION

- To the leading order, the condition requires

$$\bar{\phi}_{(-1)}'(u)^2 = E \quad \Rightarrow \quad \bar{\phi}_{(-1)}(u) = \pm\sqrt{E}u + \alpha, \qquad \text{where} \qquad E = \frac{\sum_{i=0}^{m} D^{[i]} L^{[i]}}{\sum_{i=0}^{m} L^{[i]}}$$

- Gluing consistently multiple JT pages requires positivity of *E,* or equivalently a large enough number of type + pages

- $D^{[i]}$ can be regarded as an effective potential, in fact it is easy to compute

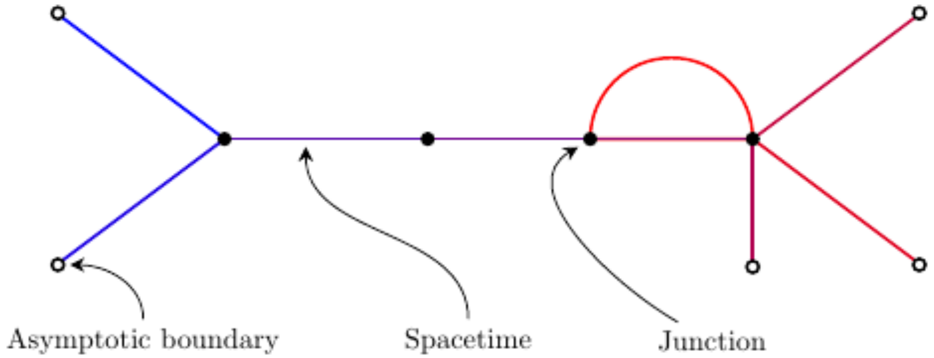
$$\mathrm{Sch}[t^{[i]}] = \frac{E - D^{[i]}}{2(\sqrt{E}u + \alpha)^2}$$

DISCUSSION

Shen, **CP**, Li, W.I.P.

- This provides tools to future study properties of connected geometries in Lorentzian signature

- In addition, we observe a resemblance with Feynman diagrams



- It is sensible since the junction conditions are in general among the extrinsic curvatures, which are classical conjugate momenta in the Hamiltonian formalism.

$$\pi^{[i]}{}_{\bar{\mu}\bar{\nu}} = \sqrt{|\det h|}\left(K^{[i]}_{\bar{\mu}\bar{\nu}} - K^{[i]} h_{\bar{\mu}\bar{\nu}}\right) \qquad \sum_i \pi^{[i]}{}_{\bar{\mu}\bar{\nu}} = -8\pi G \sqrt{|\det h|}\, \bar{S}_{\bar{\mu}\bar{\nu}}$$

THANK YOU!