Building Quantum Space-times with BCFT Legos

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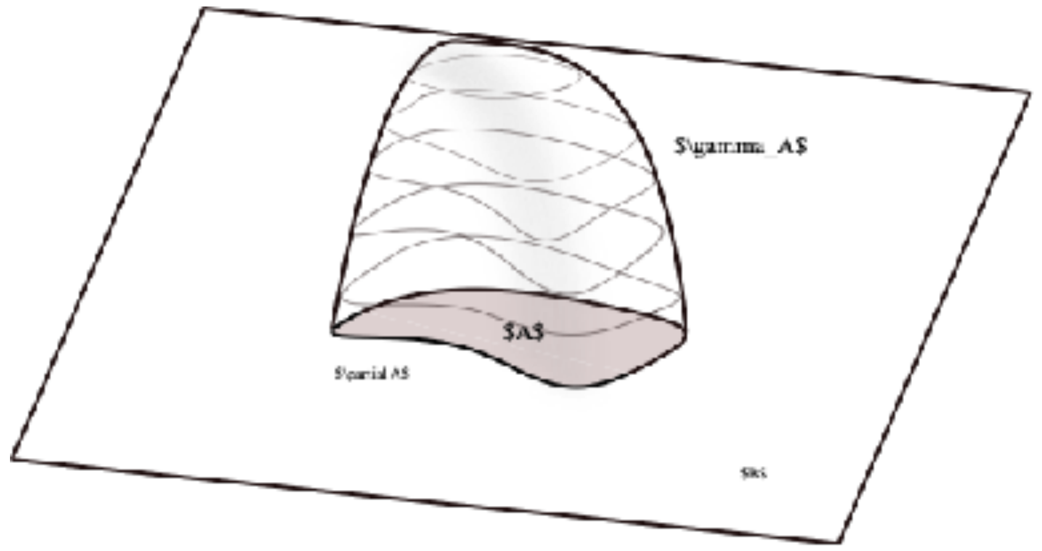
Based on works with :
Yikun Jiang, Lin Chen, Bingxin Lao
2404.00877 , 2403.03179
Gong Cheng, Lin Chen, Zhengcheng Gu
2311.18005
Lin Chen, Haochen Zhang, Kaixin Ji, Ce Shen, Ruoshui Wang
2210.12127

清华大学丘成桐数学科学中心
Yau Mathematical Sciences Center, Tsinghua University

Many body entanglement and holographic theories

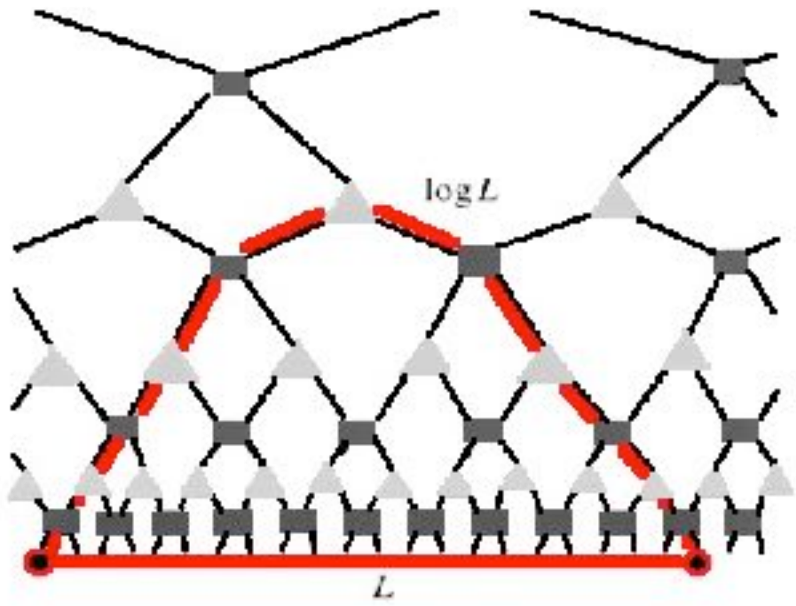
AdS/CFT says entanglement is geometry

Ryu-Takayanagi Formula:
(2006)



$$S_{EE} = \frac{A}{4G}$$

Tensor network is a framework to construct models that realise these ideas



Picture courtesy Orus

Brian Swingle (2012)

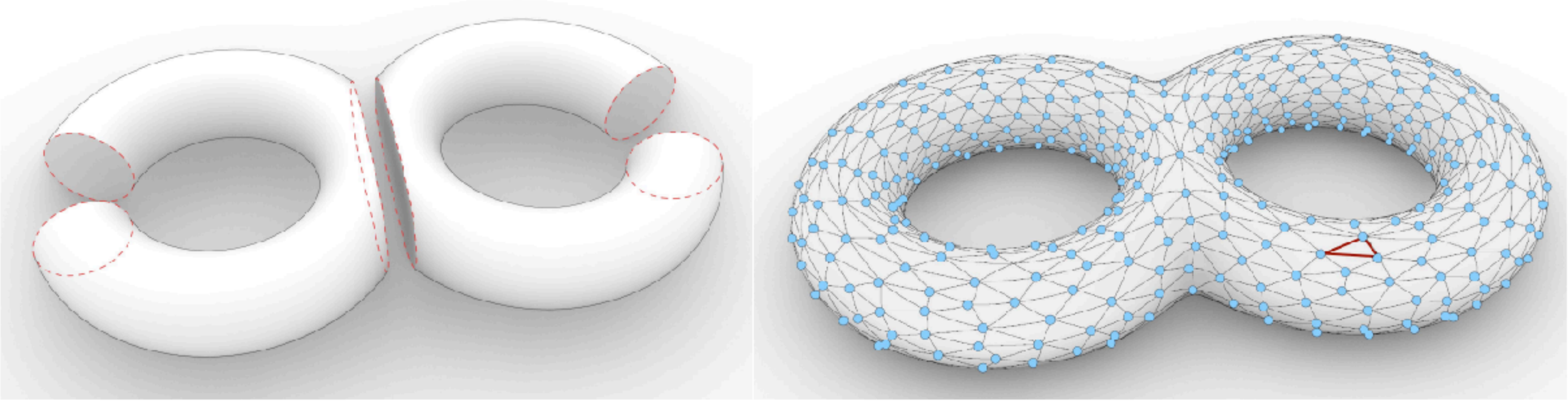
Tensor network is a geometrization of entanglement. It is explicitly local.

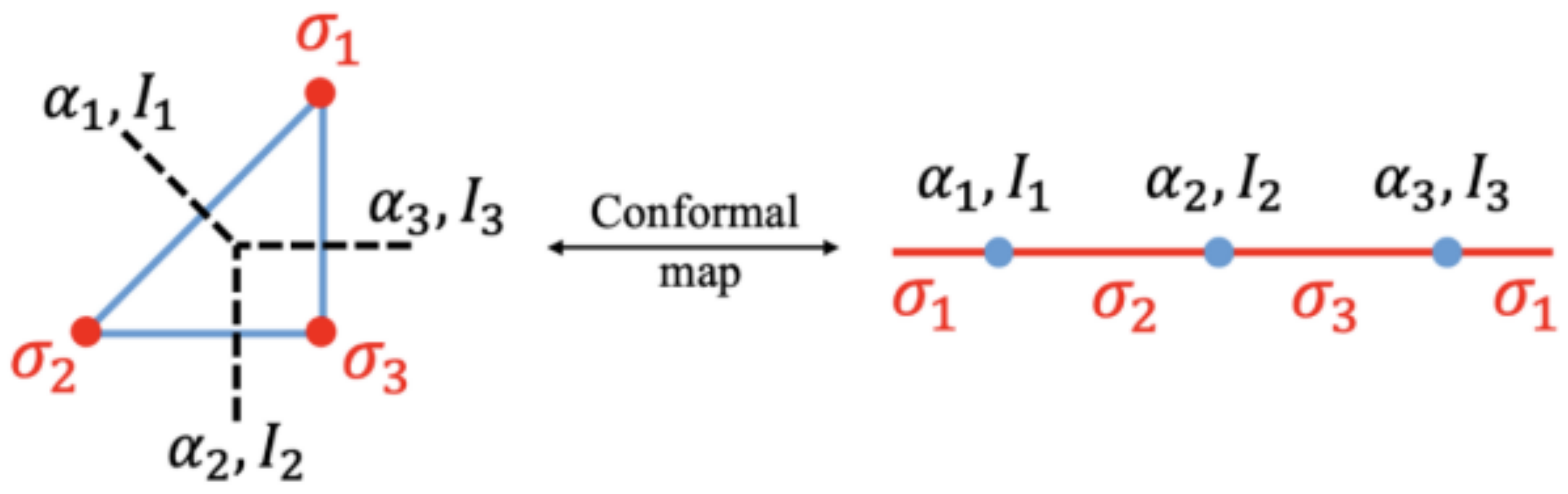
Overview

- Step 1 Cut 2D CFT path-integral into pieces
- Step 2 Read off a 3D Bulk
- Step 3 We solved an RG fixed point problem!
- Step 4 Try Liouville CFT — it spits out a *quantum sum* of hyperbolic geometries weighted by the Einstein-Hilbert action (in the large c limit)

Step 1: Cut 2D CFT path-integral into pieces

Cut a CFT into triangles



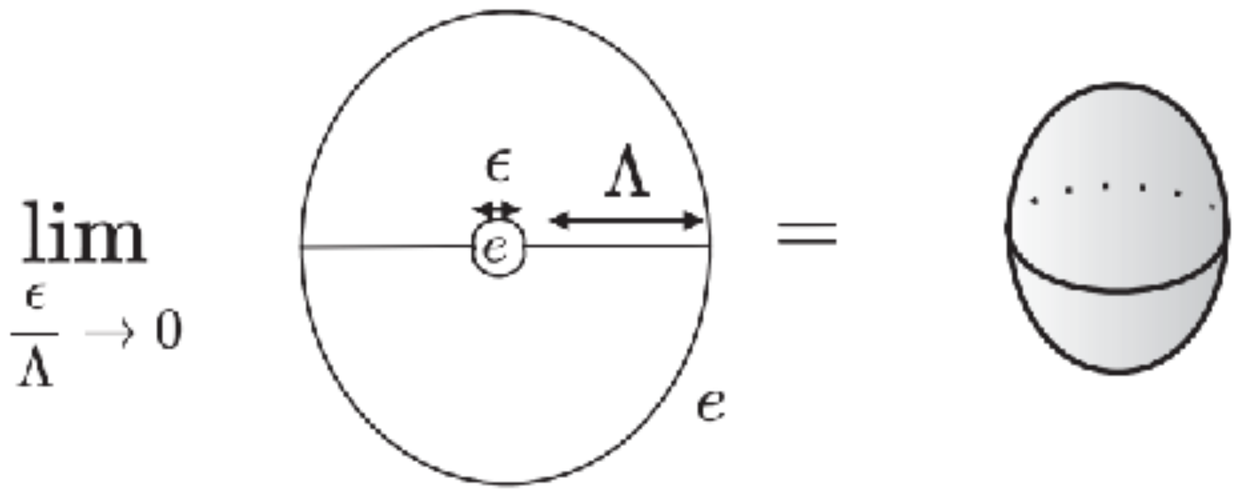


$$\mathcal{T}^{\sigma_1\sigma_2\sigma_3}_{(\alpha_1,I_1)(\alpha_2,I_2)(\alpha_3,I_3)}(\triangle) = C^{\sigma_3\sigma_1\sigma_2}_{\alpha_1\alpha_2\alpha_3}\gamma^{\alpha_1\alpha_2\alpha_3}_{I_1I_2I_3}(x_i,\epsilon)$$

What do we do with holes ? We want them to shrink to nothingness—
this gives us the original closed string partition function.
This closing a hole business is related to this question of factorisation of CFT.
Shrinkable bc =
"Entanglement Brane Boundary Condition" in RCFT
G. Wong, LYH 2020

CFT Factorisation and Entanglement Brane/Shrinkable Boundary

Wong, LYH 2020



The condition one would naively thought should be imposed on this boundary is that the holes should close. We therefore propose that the shrinkable boundary condition (in the closed string channel) to be the "0" Ishibashi state.

Ishibashi state

$$|a\rangle = \sum_b S_{ba}/\sqrt{S_{b0}}\, |b\rangle\rangle$$

$$|e\rangle\rangle = \sum_a c_a |a\rangle \qquad c_a = \sqrt{S_{00} S_{a0}}$$

quantum dimension of sector a

$$S_{a0} = d_a / D$$

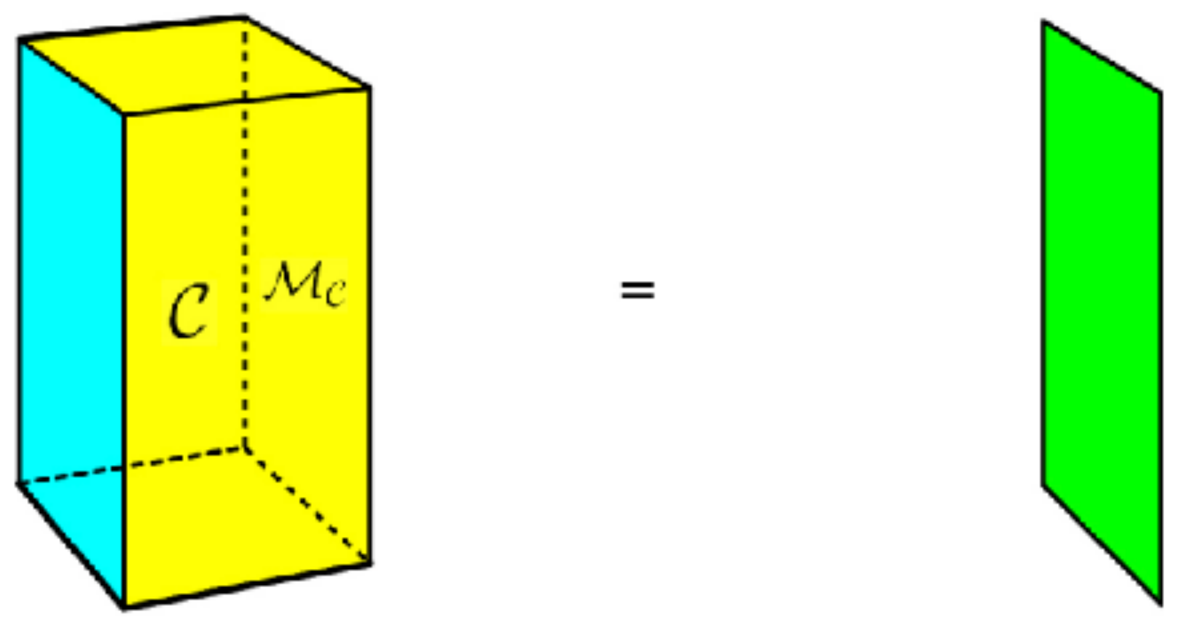
total quantum dimension

$$Z = \lim_{\epsilon \to 0} \sum_{\sigma_i} \prod_v \left(\omega_{v(\sigma_i)} e^{-\frac{\pi c}{6\epsilon}}\right) \sum_{\alpha_i} \sum_{\{I_i\}} \prod_{\triangle} T^{\sigma_1\sigma_2\sigma_3}_{(\alpha_1,I_1)(\alpha_2,I_2)(\alpha_3,I_3)}(\triangle)$$

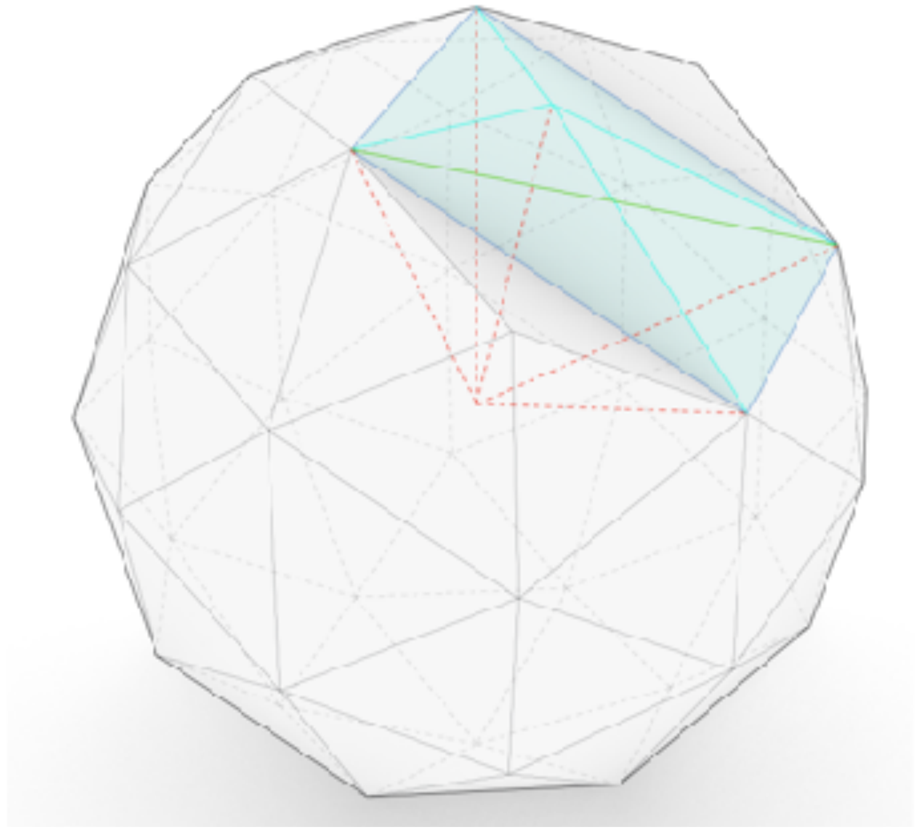
Check with a cylinder in Ising
2311.18005

Step 2: Read off a 3D Bulk

Read off a 3D Bulk



Sandwich and symmetry QFT:
Kong, Zheng;
D. Gaiotto and J. Kulp;
W. Ji and X.-G. Wen;
F. Apruzzi, F. Bonetti,
Garcıa Etxebarria, S. S. Hosseini, and
S. Schafer-Nameki;
D. S. Freed, G. W. Moore, and C. Teleman



$$Z = \lim_{\epsilon\to 0} \sum_{\sigma_i} \prod_{v} \left(\omega_{v(\sigma_i)} e^{-\frac{\pi c}{6\epsilon}}\right) \sum_{\alpha_i} \sum_{\{I_i\}} \prod_{\triangle} \mathcal{T}^{\sigma_1\sigma_2\sigma_3}_{(\alpha_1,I_1)(\alpha_2,I_2)(\alpha_3,I_3)}(\triangle)$$

$$Z = \lim_{\epsilon\to 0} \langle \Omega | \Psi \rangle$$

$$\mathcal{T}^{\sigma_1\sigma_2\sigma_3}_{(\alpha_1,I_1)(\alpha_2,I_2)(\alpha_3,I_3)}(\triangle) = C^{\sigma_3\sigma_1\sigma_2}_{\alpha_1\alpha_2\alpha_3} \gamma^{\alpha_1\alpha_2\alpha_3}_{I_1 I_2 I_3}(x_i, \epsilon)$$

$$\hat{C}^{\sigma_3\sigma_1\sigma_2;\mathrm{Racah}}_{\alpha_1\alpha_2\alpha_3} = (d_{\alpha_1} d_{\alpha_2} d_{\alpha_3})^{1/4} \begin{Bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \sigma_3 & \sigma_1 & \sigma_2 \end{Bmatrix}$$

Fuchs, Runkel Schweigert 2000s

$$\langle \Omega | = \sum_{\alpha_i, I_i} \langle \{\alpha_i\} | \prod_{\triangle} \hat{\gamma}^{\alpha_1\alpha_2\alpha_3}_{I_1 I_2 I_3}$$

$$|\Psi\rangle = \prod_v \sum_{\sigma_i,\alpha_i} \sqrt{d_{\alpha_i}} \left(\omega_{v(\sigma_i)} e^{-\frac{\pi c}{6\epsilon}}\right) \prod_{\triangle} \begin{Bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \sigma_3 & \sigma_1 & \sigma_2 \end{Bmatrix} |\{\alpha_i\}\rangle$$

Step 3: This is an RG fixed point

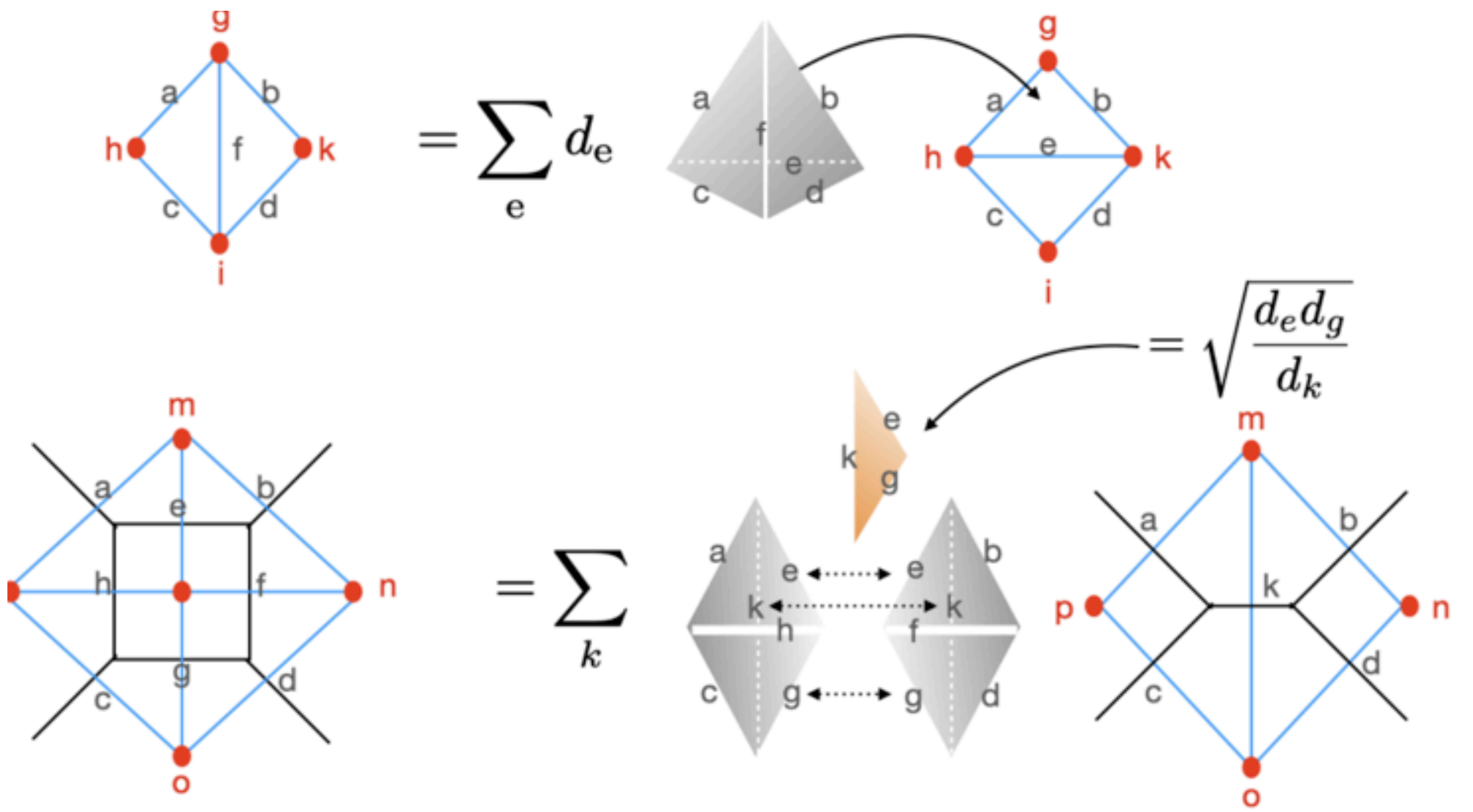
Generalised Symmetry Preserving RG and Their Fixed Point

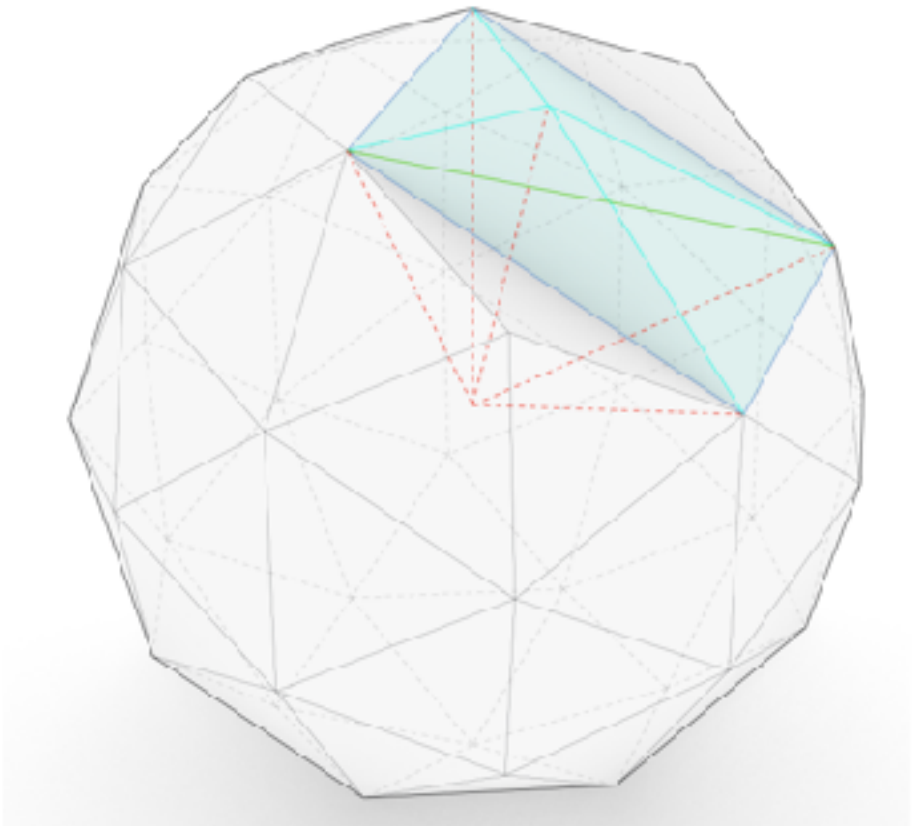
A lattice integrable model can also be written in this form:
R. Vanhove, M. Bal, D. J. Williamson, N. Bultinck, J. Haegeman, and F. Verstraete;
D. Aasen, P. Fendley, and R. S. K. Mong

$$\langle \Omega_N | \Psi_a^{LW} \rangle$$

$$|\Psi\rangle_\Lambda = U|\Psi\rangle_{\sqrt{2}\Lambda}$$

$$\langle \Omega_N | FF | \Psi_{ka}^{LW} \rangle = \langle \Omega_{N-1} | \Psi_{ka}^{LW} \rangle$$





CFT is an eigenstate of the RG operator.

$${}_{\sqrt{2}\Lambda}\langle \Omega | =_\Lambda \langle \Omega | U$$

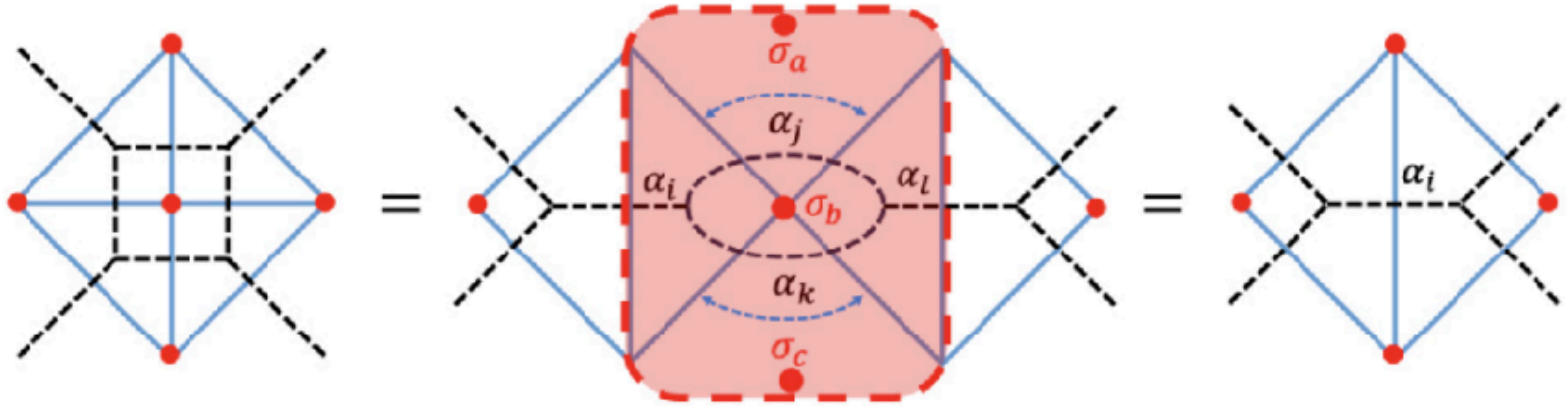
Step 4: Liouville Theory and Quantum *Gravity*

Liouville CFT - Closing Holes

Ponsot, Teschner

$$\Phi_{\alpha}^{\sigma_1\sigma_2}(x) \qquad \sigma \equiv Q/2 + iP_\sigma, \qquad \alpha \equiv Q/2 + iP_\alpha$$

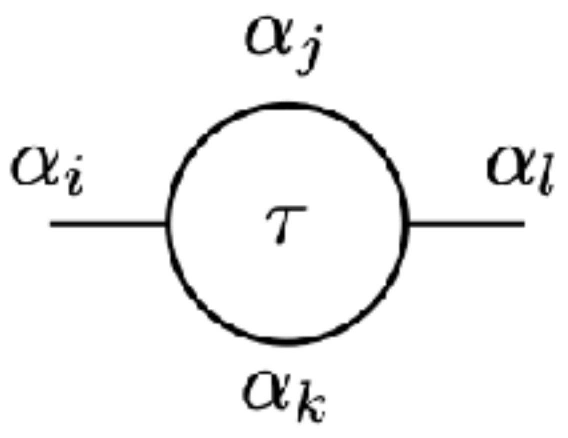
$$C^{\sigma_3,\sigma_1,\sigma_2}_{\alpha_1,\alpha_2,\alpha_3} = \frac{\left(\mu(P_{\alpha_1})\mu(P_{\alpha_2})\mu(P_{\alpha_3})\right)^{1/4}}{2^{3/8}\sqrt{\gamma_0}\,\Gamma_b(Q)} \times \sqrt{C(\alpha_1,\alpha_2,\alpha_3)} \begin{Bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \sigma_3 & \sigma_1 & \sigma_2 \end{Bmatrix}_b$$



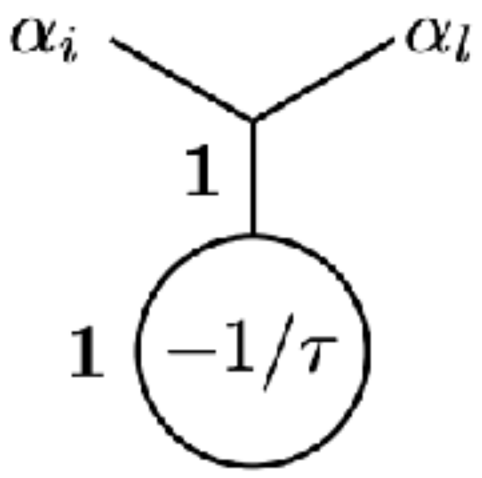
$$\mu(P) = 4\sqrt{2}\sinh(2\pi Pb)\sinh\left(\frac{2\pi P}{b}\right). \qquad \mathcal{U}_q(SL(2,\mathbb{R}))$$

Weight the hole with the Plancherel measure

$$\int_0^\infty dP_{\alpha_j} dP_{\alpha_k} dP_{\sigma_b} \mu(P_{\sigma_b}) C^{\sigma_b,\sigma_c,\sigma_a}_{\alpha_i,\alpha_j,\alpha_k} C^{\sigma_c,\sigma_b,\sigma_a}_{\alpha_j,\alpha_l,\alpha_k}$$



$$= \delta(P_{\alpha_i} - P_{\alpha_l}) \frac{2^{\frac{1}{4}}}{c_b} \left(\frac{1}{\sqrt{\gamma_0}\Gamma_b(Q)}\right)^2$$



Reading off the bulk

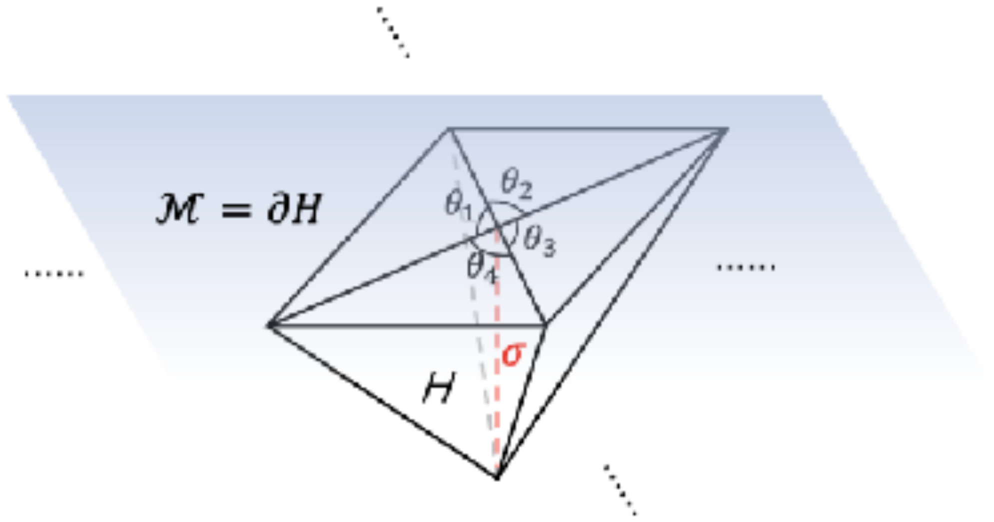
$$Z_{Liouville} = \lim_{\epsilon\to 0} \prod_v \int_0^\infty dP_{\sigma_i(v)} \left(\mathcal{N}(\epsilon)\mu(P_{\sigma_i(v)})\right)\times$$
$$\prod_i \int_0^\infty dP_{\alpha_i} \sum_{\{I_i\}} \prod_{\triangle} \mathcal{T}^{\sigma_1\sigma_2\sigma_3}_{(\alpha_1,I_1)(\alpha_2,I_2)(\alpha_3,I_3)}(\triangle),$$

$$Z_{Liouville} = \lim_{\epsilon\to 0} \langle \Omega | \Psi_{\mathcal{U}_q(SL(2,\mathbb{R}))}\rangle.$$

$$|\Psi_{\mathcal{U}_q(SL(2,\mathbb{R}))}\rangle = \prod_i \int_0^\infty dP_{\alpha_i} \sqrt{\mu(P_{\alpha_i})} \prod_v \int_0^\infty dP_{\sigma_v}$$
$$\left(\mathcal{N}(\epsilon)\mu(P_{\sigma_v})\right) \prod_{\triangle} \begin{Bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \sigma_3 & \sigma_1 & \sigma_2 \end{Bmatrix}_b |\{\alpha_i\}\rangle,$$

$$\langle\Omega| = \prod_i \int_0^\infty dP_{\alpha_i} \sum_{\{I_i\}} \langle\{\alpha_i\}| \prod_{\triangle} \left(\tilde{\gamma}^{\alpha_1\alpha_2\alpha_3}_{I_1 I_2 I_3}(\epsilon)\right).$$

Reading off the bulk



Kashaev;
Teschner;
Cotlier , Eberhardt, Zhang

$$\lim_{b\to 0}\begin{Bmatrix} \frac{\theta_1}{2\pi b} & \frac{\theta_2}{2\pi b} & \frac{\theta_3}{2\pi b} \\ \frac{\theta_4}{2\pi b} & \frac{\theta_5}{2\pi b} & \frac{\theta_6}{2\pi b} \end{Bmatrix}_b = \exp\left(-\frac{V(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6)}{\pi b^2}\right)$$

$$\lim_{b\to 0}\begin{Bmatrix} \frac{Q}{2}+i\frac{l_4}{2\pi b} & \frac{Q}{2}+i\frac{l_5}{2\pi b} & \frac{Q}{2}+i\frac{l_6}{2\pi b} \\ \frac{Q}{2}+i\frac{l_1}{2\pi b} & \frac{Q}{2}+i\frac{l_2}{2\pi b} & \frac{Q}{2}+i\frac{l_3}{2\pi b} \end{Bmatrix}_b = \exp\left(-\frac{V(\pi - il_4, \pi - il_5, \pi - il_6, \pi - il_1, \pi - il_2, \pi - il_3)}{\pi b^2}\right) = \exp\left(-\frac{\mathrm{Vol}(T\{l_i\}) + \sum_i l_i\theta_i/2}{\pi b^2}\right)$$

Murakami

$$\mathrm{Vol}(T) = V(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6) = V_l - \sum_i l_i \partial_{l_i} V_l,$$

$$V_l \equiv V(\pi - il_4, \pi - il_5, \pi - il_6, \pi - il_1, \pi - il_2, \pi - il_3).$$

$$\theta_i = 2\partial_{l_i} V_l \qquad \mathrm{mod}\ 2\pi.$$

$$S_{EH} = -\frac{1}{16\pi G_N}\int_H d^3x\sqrt{g}(R - 2\Lambda) = \frac{V(H)}{4\pi G_N} = \frac{V(H)}{\pi b^2},$$

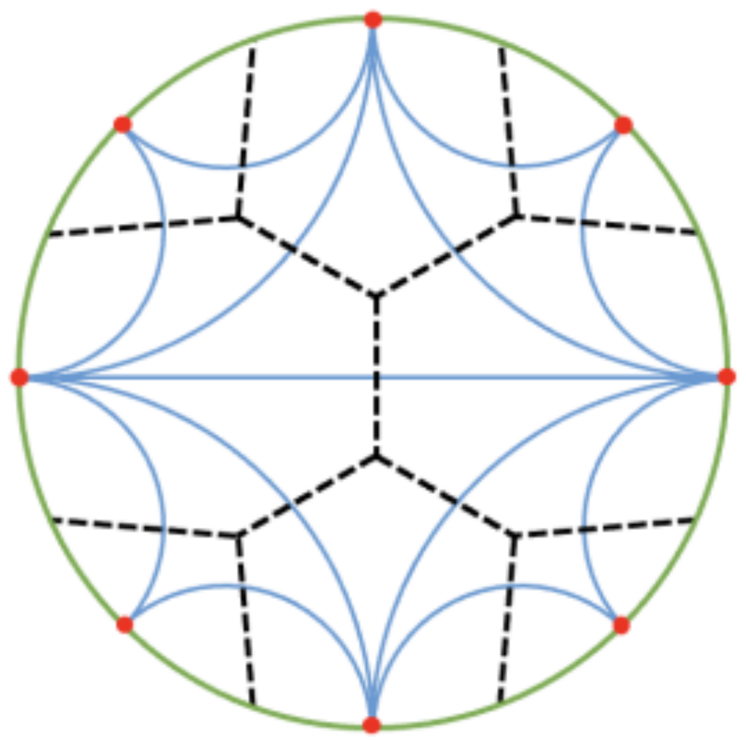
$$\sum_i \frac{\theta_i l_i}{2\pi b^2} = \sum_i \frac{\theta_i l_i}{8\pi G_N} = \frac{1}{8\pi G_N}\sum_i \int_{\Gamma_i} \theta_i \sqrt{h},$$

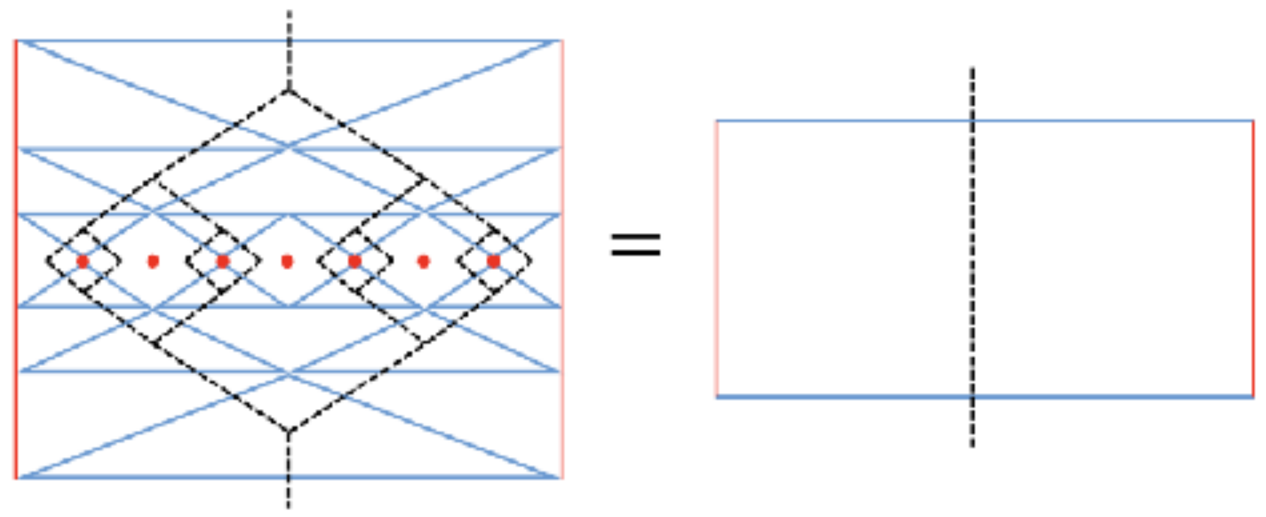
Gibbons Hawking Term

$$\exp\left(-S_{EH} + \frac{\sum_e (2\pi - \Theta_e) l_e}{8\pi G_N} + \text{surface terms on } \partial H\right)$$

Conclusion and Outlook

- UV Cutoff and TTbar? $P_k^* = \frac{1}{2b\epsilon}.$
- Wavefunctions and `` perfect tensors” ?
- JT gravity?
- Black Holes?
- Other theories — minimal models ?
- Higher Dimensions?





Thank you!