A Differential Representation for Holographic Correlators

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some preliminary observations on structures of loop scattering in AdS

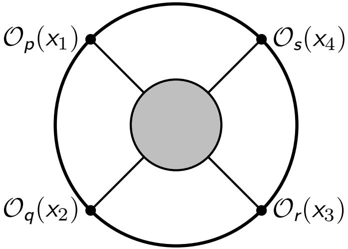
arxiv:2403.10607

+ work to appear next week

黄中杰

王波

Scattering half-BPS operators in AdS



typical examples: **supergravitons** in $\mathrm{AdS}_5 \times \mathrm{S}^5$,
supergluons in $\mathrm{AdS}_5 \times \mathrm{S}^3$, $\cdots$

[NUMEROUS literature][Bissi, Sinha, Zhou '22][Heslop '22]

<u>focus</u>: weakly-coupled dynamics in the bulk

Scattering half-BPS operators in AdS

- $\mathrm{AdS}_5 \times \mathrm{S}^5 \Leftrightarrow \mathcal{N} = 4$ super Yang–Mills
  supergravitons + Kaluza–Klein tower:

$$\mathcal{O}_p(x; y) \equiv \mathcal{O}_p^{I_1 I_2 \ldots I_p}(x) y^{I_1} y^{I_2} \cdots y^{I_p}.$$

  p - dimension (KK charge). y - $\mathrm{SU_R}(4) \approx \mathrm{SO}(6)$.

- $\mathrm{AdS}_5 \times \mathrm{S}^3 \Leftrightarrow$ some $\mathcal{N} = \in$ SCFTs (gluon sector only)
  supergluons + Kaluza–Klein tower:

$$\mathcal{O}_p^I(x; v, \bar{v}) \equiv \mathcal{O}_p^{I;\alpha_1 \ldots \alpha_p;\beta_1 \ldots \beta_{p-1}} v_{\alpha_1} \cdots v_{\alpha_p} \bar{v}_{\beta_1} \cdots \bar{v}_{\beta_{p-2}}.$$

  I - flavor (boundary) / gauge (bulk). v - $\mathrm{SU_R}(2)$. $\bar{v}$ - $\mathrm{SU_L}(2)$.

Basic structure of the correlator

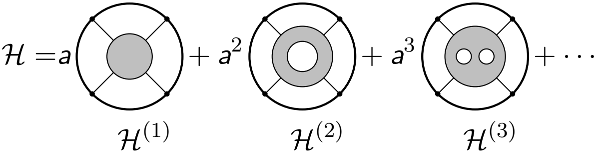
- Superconformal Ward identity [Nirschl, Osborn, '04]

$$\langle pqrs \rangle = (\text{protected}) + (\text{kinematics}) \times \underbrace{\mathcal{H}(U, V; \epsilon)}_{\text{reduced correlator}}$$

$U \equiv z\bar{z}, \ V \equiv (1-z)(1-\bar{z})$: conformal cross ratios.

ϵ: polarizations for internal symmetries.

- Bulk perturbation expansion $(N \gg 1, \ \lambda \gg 1)$



$\mathrm{AdS}_5 \times \mathrm{S}^5$: $a \propto 1/N^2$. $\mathrm{AdS}_5 \times \mathrm{S}^3$: $a \propto 1/N$.

$1/\lambda$ corrections (stringy effects) omitted.

A tale of two representations

position space $\Longleftrightarrow$ **Mellin space**

$\mathcal{H}(U, V; \epsilon)$ $\qquad$ $\mathcal{M}(s, t; \epsilon)$

Example: supergraviton $\langle 2222 \rangle$

$$\mathcal{H}_{2222} = \int \frac{\mathrm{d}s\mathrm{d}t}{(2\pi i)^2} U^{\frac{s+4}{2}} V^{\frac{t-4}{2}} \Gamma^2(\frac{4-s}{2})\Gamma^2(\frac{4-t}{2})\Gamma^2(\frac{4-\tilde{u}}{2}) \mathcal{M}_{2222}$$

A tale of two representations

position space ⟺ **Mellin space**

$$\mathcal{H}(U, V; \epsilon) \qquad\qquad \mathcal{M}(s, t; \epsilon)$$

Example: supergraviton $\langle 2222 \rangle$ @ tree

$$\mathcal{H}^{(1)}_{2222} = \frac{P_0(z,\bar{z})}{(z-\bar{z})^4} + \frac{P_1(z,\bar{z})}{(z-\bar{z})^6}\log U + \frac{P_2(z,\bar{z})}{(z-\bar{z})^6}\log V + \frac{P_3(z,\bar{z})}{(z-\bar{z})^7}\underbrace{\left[2\mathrm{Li}_2(z) - 2\mathrm{Li}_2(\bar{z}) + \log(z\bar{z})\log\left(\frac{1-z}{1-\bar{z}}\right)\right]}_{W_2(z,\bar{z}) \quad \text{Bloch–Wigner}}$$

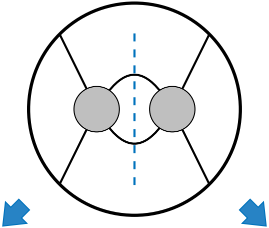
[Arutyunov, Frolov, '00], [Dolan, et al, '06]

$$\mathcal{M}^{(1)}_{2222} = \frac{2}{(s-2)(t-2)(\tilde{u}-2)}, \qquad s + t + \tilde{u} = 4$$

[Rastelli, Zhou '16]

“Boundary conditions” for loop-level bootstrap

unitarity recursion [Aharony et al ‘16]



position space

- max power of $\log U$
- explicit function in front of $\log^2 U$
- function type of the entire $\mathcal{H}^{(2)}$

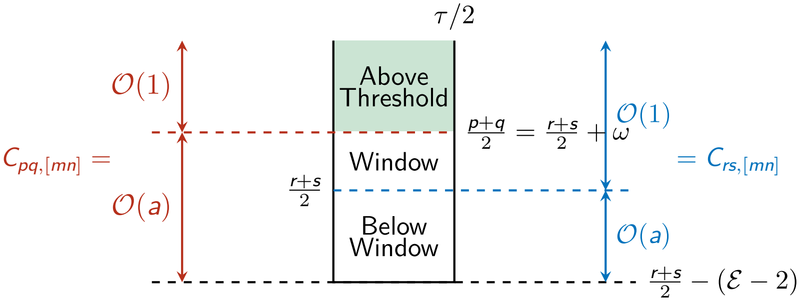
Mellin space

- max power of S pole
- explicit residues of $\log^2 U$
- pole structure of the entire $\mathcal{M}^{(2)}$

Position vs Mellin

position space	⟺	Mellin space
$\sum_i \frac{P_i(z,\bar{z})}{(z-\bar{z})^{\#}} \left\{ \begin{matrix} 1 \\ \log U,\ \log V \\ W_2 \end{matrix} \right\}$	tree any KK 😀	$\sum_{m,n,l=2}^{\text{truncate}} \frac{R_{mnl}}{(s-m)(t-n)(\tilde{u}-l)}$
$\sum_{\text{weight}\leq 4} \frac{P_i(z,\bar{z})}{(z-\bar{z})^{\#}} G_i^{\text{SV}}(z,\bar{z})$	one loop $\langle 2222 \rangle$ 🙂	$\sum_{m,n=4}^{\infty} \frac{a_{mn}}{(s-m)(t-n)} + (\text{crossing})$
$\sum_{\text{weight}\leq 4} \frac{P_i(z,\bar{z})}{(z-\bar{z})^{\#}} G_i^{\text{SV}}(z,\bar{z})$	one loop higher KK 😐	$\sum_{m,n=4}^{\infty} \frac{a_{mn}}{(s-m)(t-n)} + (\text{crossing})$
$\sum_{\text{weight}\leq 6} \frac{P_i(z,\bar{z})}{(z-\bar{z})^{\#}} G_i^{\text{SV}}(z,\bar{z})$	two loops $\langle 2222 \rangle$ ☹	🚫

Various contributions in the OPE



[Aprile, Drummond, Heslop, Paul, '19] [Huang, Wang, EYY, Zhou, '23]

ω: size of the window
$\mathcal{E}$: extremality (amount of R structures)

Above threshold (leading logarithmic) data can be easily determined by a hidden higher dimensional conformal symmetry.

[Caron-Huot, Trinh '18]

Back to tree level

position space ⟺ **Mellin space**

$$\mathcal{W}_2 = \frac{W_2}{(z-\bar{z})} \equiv \bar{D}_{1111}$$

tree
$\langle 2222 \rangle$

?

$$-\partial_U \partial_V (1 + U\partial_U + V\partial_V)$$

$$\sum_i \frac{P_i(z,\bar{z})}{(z-\bar{z})^{\#}} \left\{ \begin{matrix} 1 \\ \log U,\ \log V \\ W_2 \end{matrix} \right\}$$

$$\frac{2}{(s-2)(t-2)(\tilde{u}-2)}$$

Back to tree level

standard form of Mellin transform

$$\mathcal{H} = \int \frac{\mathrm{d}S\mathrm{d}T}{(2\pi i)^2} U^S V^T \underbrace{\Gamma(-S)^2\Gamma(-T)^2\Gamma(1+S+T)^2}_{\widehat{\Gamma}(S,T)} \widehat{\mathcal{M}}$$

position space ⟷ **Mellin space**

$$\mathcal{W}_2 = \frac{W_2}{(z-\bar{z})} \equiv \bar{D}_{1111}$$

tree

$\langle 2222 \rangle$

$$\widehat{\mathcal{M}}_2 \equiv 1$$

$$-\partial_U\partial_V(1+U\partial_U+V\partial_V)$$

$$\sum_i \frac{P_i(z,\bar{z})}{(z-\bar{z})^{\#}} \left\{ \begin{matrix} 1 \\ \log U,\ \log V \\ W_2 \end{matrix} \right\}$$

$$-\frac{((S+T+1)_2)^2}{(S+1)(T+1)(S+T+3)}$$

The differential operators

$$-\partial_U\partial_V(1 + U\partial_U + V\partial_V)$$

can be decomposed as polynomials of

$$\mathcal{D}_U \equiv U\partial_U, \quad \mathcal{D}_V \equiv V\partial_V, \quad U^{\pm 1}, \quad V^{\pm 1}$$

<u>first type</u>

$$\mathcal{D}_U\,\mathcal{H}(U,V) = \int \frac{\mathrm{d}S\mathrm{d}T}{(2\pi i)^2} U^S V^T \,\widehat{\Gamma}(S,T) \times S\,\widehat{\mathcal{M}}(S,T)$$

position space ⟷ **Mellin space**

position space	Mellin space
$\mathcal{D}_U^m\,\mathcal{H}(U,V)$	$S^m\,\widehat{\mathcal{M}}(S,T)$
$\mathcal{D}_V^n\,\mathcal{H}(U,V)$	$T^n\,\widehat{\mathcal{M}}(S,T)$

The differential operators

$$-\partial_U \partial_V (1 + U\partial_U + V\partial_V)$$

can be decomposed as polynomials of

$$\mathcal{D}_U \equiv U\partial_U, \quad \mathcal{D}_V \equiv V\partial_V, \quad U^{\pm 1}, \quad V^{\pm 1}$$

<u>second type</u>

$$U^a\, \mathcal{H}(U,V) = \int \frac{\mathrm{d}S \mathrm{d}T}{(2\pi i)^2} U^{S+a} V^T \,\widehat{\Gamma}(S,T) \widehat{\mathcal{M}}(S,T)$$

$$= \int \frac{\mathrm{d}S \mathrm{d}T}{(2\pi i)^2} U^S V^T \,\widehat{\Gamma}(S,T) \left[(-S)_a (1+S+T)_{-a}\right]^2 \widehat{\mathcal{M}}(S-a,T)$$

position space ⟷ **Mellin space**

position space	Mellin space
$U^a\, \mathcal{H}(U,V)$	$\left[(-S)_a (1+S+T)_{-a}\right]^2 \widehat{\mathcal{M}}(S-a,T)$
$V^b\, \mathcal{H}(U,V)$	$\left[(-T)_b (1+S+T)_{-b}\right]^2 \widehat{\mathcal{M}}(S,T-b)$

The differential operators

$$-\partial_U \partial_V (1 + U\partial_U + V\partial_V)$$

can be decomposed as polynomials of

$$\mathcal{D}_U \equiv U\partial_U, \quad \mathcal{D}_V \equiv V\partial_V, \quad U^{\pm 1}, \quad V^{\pm 1}$$

position space ⟷ **Mellin space**

position space	Mellin space
$\mathcal{D}_U^m \, \mathcal{H}(U, V)$	$S^m \, \widehat{\mathcal{M}}(S, T)$
$\mathcal{D}_V^n \, \mathcal{H}(U, V)$	$T^n \, \widehat{\mathcal{M}}(S, T)$
$U^a \, \mathcal{H}(U, V)$	$\left[(-S)_a (1 + S + T)_{-a}\right]^2 \widehat{\mathcal{M}}(S - a, T)$
$V^b \, \mathcal{H}(U, V)$	$\left[(-T)_b (1 + S + T)_{-b}\right]^2 \widehat{\mathcal{M}}(S, T - b)$

Tree-level $\mathcal{H}^{(1)}$ as differentials acting on $\mathcal{W}_2$

case of $\langle 2222 \rangle$ supergraviton

position space

$$\mathcal{H}_{2222} = (U^{-1})(V^{-1}) \underbrace{\left(-\mathcal{D}_U \mathcal{D}_V (1 + \mathcal{D}_U + \mathcal{D}_V) \right)}_{\partial^3} \mathcal{W}_2$$

Mellin space

$$\widehat{\mathcal{M}}_2 \equiv 1 \underset{\partial^3}{\longrightarrow} -S\,T\,(1+S+T)$$

$$\underset{V^{-1}}{\longrightarrow} -\left(\frac{S+T+1}{T+1}\right)^2 S\,(T+1)\,(S+T+2)$$

$$\underset{U^{-1}}{\longrightarrow} -\left(\frac{S+T+1}{S+1}\right)^2 \left(\frac{S+T+2}{T+1}\right)^2 (S+1)\,(T+1)\,(S+T+3)$$

$$= -\frac{(S+T+1)^2(S+T+2)^2(S+T+3)^2}{(S+1)(T+1)(S+T+3)}$$

Tree-level $\mathcal{H}^{(1)}$ as differentials acting on $\mathcal{W}_2$

similar fact applies to ALL tree-level reduced correlators
in $\mathrm{AdS}_5 \times \mathrm{S}^5$ and $\mathrm{AdS}_5 \times \mathrm{S}^3$

$$\mathcal{H}^{(1)}_{pqrs} = \mathcal{P}_{pqrs}(U, V, U^{-1}, V^{-1}, \mathcal{D}_U, \mathcal{D}_V)\mathcal{W}_2$$

a consequence of recursion relations amoung $\bar{D}$ functions

call $\mathcal{W}_2$ or its counterpart (via standard Mellin transform) $\mathcal{M}_2$
a **seed function** in position space or Mellin space

a SINGLE seed function is sufficient at tree level

Does this continue to hold at loop level?

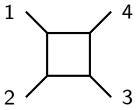
$*_*$ same type of differential operators

$*_*$ can have extra seed functions
but same set of seed functions for ALL correlators

Test on $\mathcal{H}_{2222}$ of supergluons

Mellin space

[Alday, Bissi, Zhou, '21]



color

$$\mathcal{M}^{(2)} = \ldots \underbrace{\sum_{m,n=0}^{\infty} \frac{a_{m,n}}{(S-m)(T-n)}}_{\mathcal{M}^{(2)}_{st}(S,T)} + (\text{crossing})$$

$$a_{m,n} = \frac{3m^2 n + 2m^2 + 3mn^2 + 8mn + 3m + 2n^2 + 3n}{3(m+n)(m+n+1)(m+n+2)}$$

(convention: $S = 0 \Leftrightarrow$ twist 4 in S channel)

position space

[ZH, Wang, Yuan, Zhou, '23]

(up to transcendental weight 4, too long to fit here)

Find a closed-form expression for Mellin amplitude

observation

∗ at tree level $\widehat{\mathcal{M}}_2 = 1$
poles of $\mathcal{M}^{(1)}_{pqrs}$ are created by acting with U or V

∗ this is fine since $\mathcal{M}^{(1)}_{pqrs}$ only has **finitely** many poles

∗ $\mathcal{M}^{(2)}_{pqrs}$ has **infinitely** many poles
cannot be derived in the same way

Find a closed-form expression for Mellin amplitude

intuition

we want some function that provide a grid of poles at
$S = m$ and $T = n$ $(m, n \in \mathbb{N})$

a simple choice

$$\widehat{\mathcal{M}}_3(S) \equiv \xi(S) = \psi^{(0)}(-S) + \gamma_{\mathrm{E}}$$

$$\widehat{\mathcal{M}}_4(S, T) \equiv \Phi(S, T) = -\frac{1}{2}\left((\xi(S) + \xi(T))^2 + \xi'(S) + \xi'(T) + \pi^2\right)$$

they have simple residues

$$\operatorname*{Res}_{S=m} \xi(S) = 1, \qquad \operatorname*{Res}_{S=m} \operatorname*{Res}_{T=n} \Phi(S, T) = 1, \qquad m, n \in \mathbb{N}$$

Find a closed-form expression for Mellin amplitude

$$\mathcal{M}^{(2)}_{\text{YM,st}}(S,T) =$$

$$\sum_{m,n=0}^{\infty} \underbrace{\frac{3m^2n + 2m^2 + 3mn^2 + 8mn + 3m + 2n^2 + 3n}{3(m+n)(m+n+1)(m+n+2)}} \; \underbrace{\frac{1}{(S-m)(T-n)}}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\frac{3S^2T + 2S^2 + 3ST^2 + 8ST + 3S + 2T^2 + 3T}{3(S+T)(S+T+1)(S+T+2)} \qquad \Phi(S,T)$$

<u>problem</u>

extra poles of $S+T$

Find a closed-form expression for Mellin amplitude

$$\mathcal{M}^{(2)}_{\text{YM,st}}(S,T) = \sum_{m,n=0}^{\infty} \underbrace{\frac{3m^2n+2m^2+3mn^2+8mn+3m+2n^2+3n}{3(m+n)(m+n+1)(m+n+2)}}\ \underbrace{\frac{1}{(S-m)(T-n)}}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\frac{3S^2T+2S^2+3ST^2+8ST+3S+2T^2+3T}{3(S+T)(S+T+1)(S+T+2)} \qquad \Phi(S,T)$$

$$-\frac{\left(3S^2+3ST+5S+T\right)}{3(S+T)(S+T+2)}\xi(S)+(S\leftrightarrow T)$$

$$+\frac{2}{3(S+T)}+C$$

there seem to be three seed functions:

$$\widehat{\mathcal{M}}_4 \equiv \Phi(S,T), \quad \widehat{\mathcal{M}}_3 \equiv \xi(S), \quad \widehat{\mathcal{M}}_2 \equiv 1$$

What justifies a differential representation?

observation 1

any poles in addition to the seed functions
can ONLY come from an action of U^a or V^b

$$\Big[\frac{3S^2T+2S^2+3ST^2+8ST+3S+2T^2+3T}{3(S+T)(S+T+1)(S+T+2)}\,\Phi(S,T)$$

$$-\frac{\left(3S^2+3ST+5S+T\right)}{3(S+T)(S+T+2)}\xi(S)+(S\leftrightarrow T)$$

$$+\frac{2}{3(S+T)}+C\Big]\times\underbrace{(S+T+1)^2(S+T+2)^2}_{\Rightarrow\ \text{standard Mellin transform}}$$

What justifies a differential representation?

observation 2

the action of U^a or V^b is always accompanied
by corresponding double zeros and shifts

position space ⟷ **Mellin space**

$$V\mathcal{H}(U,V) \qquad\longleftrightarrow\qquad \frac{T^2}{(S+T)^2}\,\mathcal{M}(S,T-1)$$

double zero ↓ T^2

shift ↑ -1

these structures are uniquely fixed by the power of U (or V)

we call such phenomenon **double zero property**
a *necessary* condition for the existence of differential representation

Differential representation for $\mathcal{H}_{2222}$ of supergluons

a strategy

$$\Big[\frac{3S^2T+2S^2+3ST^2+8ST+3S+2T^2+3T}{3(S+T)(S+T+1)(S+T+2)}\Phi(S,T)$$
$$-\frac{\left(3S^2+3ST+5S+T\right)}{3(S+T)(S+T+2)}\xi(S)+(S\leftrightarrow T)$$
$$+\frac{2}{3(S+T)}+C\Big]\times(S+T+1)^2(S+T+2)^2$$

partial fraction + identities among Φ and $\xi \Longrightarrow$

$$\Big[-\frac{2T^2}{3(S+T)}\Phi(S,T-1)+\frac{\left(T^2+T+1\right)}{3(S+T+1)}\Phi(S,T)+\frac{(T+1)^2}{3(S+T+2)}\Phi(S,T+1)$$
$$-\xi(S)+C\Big]\times(S+T+1)^2(S+T+2)^2$$

goal: each piece being separately free of $S+T$ poles

Differential representation for $\mathcal{H}_{2222}$ of supergluons

two terms involving action of multiplications

$$-\frac{2T^2}{3(S+T)}(S+T+1)^2(S+T+2)^2\Phi(S,T-1)$$

$$\frac{(T+1)^2}{3(S+T+2)}(S+T+1)^2(S+T+2)^2\frac{(T+1)^2}{(T+1)^2}\Phi(S,T+1)$$

Differential representation for $\mathcal{H}_{2222}$ of supergluons

$$
\begin{aligned}
&\Big[-\frac{2}{3}(2+\mathcal{D}_U+\mathcal{D}_V)^2(1+\mathcal{D}_U+\mathcal{D}_V)^2(\mathcal{D}_U+\mathcal{D}_V)\,V \\
&\quad+\frac{1}{3}(2+\mathcal{D}_U+\mathcal{D}_V)^2(1+\mathcal{D}_U+\mathcal{D}_V)(1+\mathcal{D}_V+\mathcal{D}_V^2) \\
&\quad+\frac{1}{3}(2+\mathcal{D}_U+\mathcal{D}_V)(1+\mathcal{D}_V)^4\,V^{-1}\Big]\,\mathcal{W}_4 \\
&+(2+\mathcal{D}_U+\mathcal{D}_V)^2(1+\mathcal{D}_U+\mathcal{D}_V)^2\left(-\mathcal{W}_3+C\mathcal{W}_2\right)
\end{aligned}
$$

$\mathcal{W}_i$ the seed functions in position space

$$
\mathcal{W}_i=\int\frac{\mathrm{d}S\,\mathrm{d}T}{(2\pi i)^2}U^S V^T\underbrace{\Gamma(-S)^2\Gamma(-T)^2\Gamma(1+S+T)^2}_{\widehat{\Gamma}(S,T)}\widehat{\mathcal{M}}_i
$$

More on the position space

multiple polylogarithms (MPL)

$$G_{a_1 a_2 \cdots a_n, z} \equiv G_{a_1 a_2 \cdots a_n}(z) = \int_0^z \frac{\mathrm{d}t}{t - a_1} G_{a_2 a_3 \cdots a_n}(t)$$

$$G(z) = 1, \quad G_{\vec{0}_n}(z) = \frac{1}{n!} \log^n z$$

[Goncharov '01]

$\ast_\ast$ branch points at $z, \bar{z} = 0, 1, \infty$

$\ast_\ast$ single-valued on the Euclidean slice $\bar{z} = z^*$ (SVMPL)

# of independent SVMPLs	tree	one loop
ansatz for bootstrap	8	42
reduced correlator	4	10
seed functions	1	3

[Huang, EYY, '21]

More on the position space

Seed functions $\mathcal{W}_3,\ \mathcal{W}_4$

$$\mathcal{W}_i(z,\bar{z}) = \frac{W_i(z,\bar{z})}{z-\bar{z}}$$

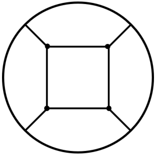
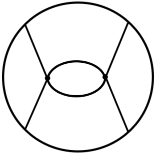
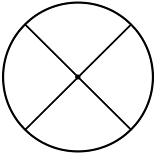
$$\begin{aligned}
W_3(z,\bar{z}) = {} & G_{00,\bar{z}}G_{1,z} - G_{1,\bar{z}}G_{00,z} + G_{01,\bar{z}}G_{0,z} + G_{0,\bar{z}}G_{01,z} - G_{10,\bar{z}}G_{0,z} \\
& - G_{10,\bar{z}}G_{1,z} + G_{0,\bar{z}}G_{10,z} - G_{1,\bar{z}}G_{10,z} + 2G_{10,\bar{z}}G_{\bar{z},z} - 2G_{01,\bar{z}}G_{\bar{z},z} \\
& + 2G_{1,\bar{z}}G_{\bar{z}0,z} - 2G_{0,\bar{z}}G_{\bar{z}1,z} + 2G_{\bar{z}01,z} - 2G_{\bar{z}10,z} - G_{001,\bar{z}} \\
& + G_{010,\bar{z}} - G_{100,\bar{z}} + G_{101,\bar{z}} - G_{001,z} + G_{010,z} + G_{100,z} - G_{101,z}
\end{aligned}$$

$$\begin{aligned}
W_4(z,\bar{z}) = {} & G_{01,\bar{z}}G_{00,z} + G_{10,\bar{z}}G_{01,z} + G_{11,\bar{z}}G_{10,z} + G_{00,\bar{z}}G_{11,z} + G_{001,\bar{z}}G_{1,z} \\
& + G_{0,\bar{z}}G_{001,z} + G_{010,\bar{z}}G_{0,z} + G_{1,\bar{z}}G_{010,z} + G_{101,\bar{z}}G_{0,z} \\
& + G_{1,\bar{z}}G_{101,z} + G_{110,\bar{z}}G_{1z} + G_{0\bar{z}}G_{110,z} + G_{0011,\bar{z}} + G_{0100,\bar{z}} \\
& + G_{1010,\bar{z}} + G_{1101,\bar{z}} + G_{0010,z} + G_{0101,z} + G_{1011,z} + G_{1100,z} \\
& - (z \leftrightarrow \bar{z})\,.
\end{aligned}$$

Loop reduction in AdS?

$$\mathcal{H}^{(2)}_{\mathrm{YM,st}}(U,V) = \mathcal{D}_4\mathcal{W}_4 + \mathcal{D}_3\mathcal{W}_3 + \mathcal{D}_2\mathcal{W}_2$$

$$\Downarrow \text{?}$$

$\mathcal{D}_4$  $+\,\mathcal{D}_3$  $+\,\mathcal{D}_2$ 

$\langle 2222 \rangle$ of supergravitons

$$\mathcal{M}^{(2)} = \mathcal{M}_{st}^{(2)}(S, T) + (\text{two other channels})$$

[Alday, Zhou, '19]

$$\mathcal{M}_{st}^{(2)}(S, T) = \sum_{m,n=0}^{\infty} \frac{b_{m,n}}{(S-m)(T-n)}$$

with

$$b_{m,n} = \frac{16}{5(m+n-1)_5}(F_{m,n} + F_{n,m})$$

$$\begin{aligned} F_{m,n} = \; & 2(m-1)m(n+1)(n+2)(m+n+2)(m+n+3) \\ & + (m+1)(m+2)(n+1)(n+2)(m+n-1)(m+n) \\ & + 4m(m+1)n(n+1)(m+n+2)(m+n+3) \\ & + 8m(m+1)(n+1)(n+2)(m+n-1)(m+n+3). \end{aligned}$$

$\langle 2222 \rangle$ of supergravitons

$$
\Big[\, \frac{48}{5}\frac{(S-1)^2 S^2}{S+T-1}\Phi_{S-2,T} - \frac{8}{5}\frac{\left(4S^2+19S+20\right)S^2}{S+T}\Phi_{S-1,T}
$$

$$
-\frac{4}{5}\frac{\left(18S^4-30S^3-64S^2+5ST-44S-22\right)}{S+T+1}\Phi_{S,T}
$$

$$
+\frac{16}{5}\frac{\left(3S^2-S+1\right)(S+1)^2}{S+T+2}\Phi_{S+1,T} + \frac{8}{5}\frac{(S+1)^2(S+2)^2}{S+T+3}\Phi_{S+2,T}
$$

$$
-16(S+2T+4)\xi(S) + (S \leftrightarrow T) + C \,\Big]
$$

$$
\times (S+T+1)^2(S+T+2)^2(S+T+3)^2
$$

plus two other channels

straightforward to convert to position space

$\langle 2222 \rangle$ of supergravitons

$$
\begin{aligned}
&-16(S+2T+4)f_S \quad -16(T+2S+4)f_T \\
&-16(S+2\tilde{U}+4)f_S \qquad\qquad\qquad\qquad\qquad -16(\tilde{U}+2S+4)f_{\tilde{U}} \\
&\qquad\qquad\qquad\qquad\quad -16(T+2\tilde{U}+4)f_T \;-16(\tilde{U}+2T+4)f_{\tilde{U}}
\end{aligned}
$$

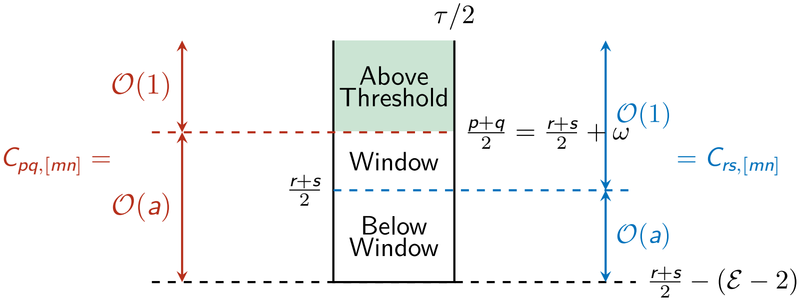
$$
f_x = \xi(x), \qquad S + T + \tilde{U} = -4
$$

ξ (and hence $\mathcal{W}_3$) drops out in the final result

$\Rightarrow$ reduces to differentials on a "box"!

Higher Kaluza–Klein charges

Various contributions in the OPE



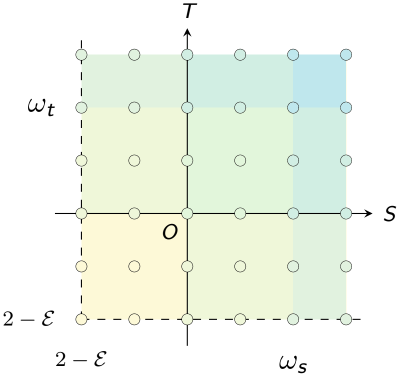
Mellin amplitudes

<u>a natural def of Mellin amplitudes</u>

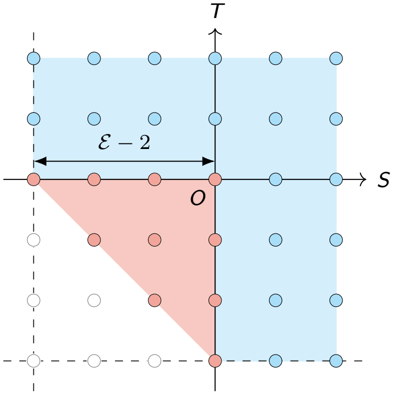
$$\mathcal{H}_{\{p_i\}}(U,V) = \int \frac{\mathrm{d}S\mathrm{d}T}{(2\pi i)^2} U^S V^T \mathcal{M}_{\{p_i\}}(S,T)$$

$$\times\, \Gamma(-S)\Gamma(-T)\Gamma(-\tilde{U})\Gamma(\omega_s - S)\Gamma(\omega_t - T)\Gamma(\omega_u - \tilde{U})$$

$$\text{with } S + T + \tilde{U} = -2 - \frac{\mathcal{N}}{2}$$



Pattern of residues in the Mellin amplitudes



Bulk + Edge Region

Corner Region

[Huang, Wang, EYY, Zhou, '23]

residues fully **analytic** in the bulk region
and can be continued to the edge region

Analyticity in the differential representation

potential violation of bulk analyticity in $\widehat{\mathcal{M}}(S, T)$

type 1

$$\frac{\widehat{\delta}}{(S-a)(T-b)} \quad \text{for some } a, b \geq 0$$

but such denominator cannot be created by differentiation

type 2

$$\widehat{f}(S, T)\Phi(S-a, T-b)$$

naively, residue analytic only for $(S, T) = (m \geq a, n \geq b)$
but double zero property dictates that

$$\widehat{f}(S, T) \propto [S(S-1)\cdots(S-a+1)\, T(T-1)\cdots T(T-b+1)]^2$$

Analyticity in the differential representation

$\widehat{\mathcal{M}}(S, T)$ has to strictly meet double zero property,
with residues analytic in the bulk region.

for ordinary amplitude $\mathcal{M}(S, T)$

$$\widehat{\mathcal{M}} = (-S)_{\omega_s}(-T)_{\omega_t}(1+S+T)_{1+\frac{\mathcal{N}}{2}}(1+S+T)_{1+\frac{\mathcal{N}}{2}+2}\mathcal{M}$$

extra **zeros** when $\omega_s \neq 0$ or $\omega_t \neq 0$.

non-analyticity may come from terms such as

$$\frac{\omega_s \omega_t}{ST}$$

when residues expect to be at most LINEAR in ω's
such non-analyticity is also ABSENT

Next-next-to-extremal correlators of supergluons

$\langle pqrs \rangle$, with $s = p + q + r - 4$, and cases related by permutations

no edge/corner region; bulk region contains window in general
only a single R-symmetry structure

$$\mathcal{M}_{YM,st} = \sum_{m,n=0}^{\infty} \frac{a_{mn}}{(S-m)(T-n)}$$

residues from hidden symmetries: for $m \geq \omega_s$ or $n \geq \omega_t$

$$a_{mn} \propto (\omega_s + 1) g_{0,1} + (\omega_t + 1) g_{1,0} + (\omega_u + 1) g_{1,1}$$
$$g_{i,j} = \frac{(m+i)(n+j)}{(m+n+i+j-1)_2}$$

linearity in $\omega \Longrightarrow$ a_{mn} directly applies to the entire bulk region!

Next-next-to-extremal correlators of supergluons

note that a_{mn} itself is ambiguous at $(m, n) = (0, 0)$

after resummation the amplitude still has ambiguities of the form

$$\frac{\alpha_1 \omega_s + \alpha_2 \omega_t + \alpha_3 \omega_u + \alpha_4}{ST}$$

Next-next-to-extremal correlators of supergluons

Resummation result

$$
\begin{aligned}
\mathcal{M}^{(2)}_{\mathrm{YM},st} \propto\ & -\frac{\omega_s S(S-1)+S^2}{S+T}\Phi_{S-1,T} \\
& + \frac{2\omega_s S^2-(\omega_u-1)(S^2+S)+1}{S+T+1}\Phi_{S,T} \\
& + \frac{(\omega_u+1)(S+1)^2}{2(S+T+2)}\Phi_{S+1,T} - \frac{2\omega_t+\omega_u+3}{2}\xi_S \\
& + \frac{2\alpha_1\omega_s+2\alpha_2\omega_t+(2\alpha_3-1)\omega_u+2\alpha_4-3}{4ST} \\
& + (1\leftrightarrow 3) + C_0
\end{aligned}
$$

double zero property: the term marked red has to vanish

Next-next-to-extremal correlators of supergluons

thus we reach a closed formula for
all NNE correlators of supergluons
without ever using any data in the window

and this formula is directly valid in both
Mellin space and position space

this strategy also works for all NNE correlators of supergravitons

Outlook

Outlook

- Higher points, higher loops, higher KK modes...
- Other backgrounds
  ($\mathrm{AdS}_3 \times \mathrm{S}^3,\ \mathrm{AdS}_4 \times \mathrm{S}^7,\ \mathrm{AdS}_7 \times \mathrm{S}^4$, etc....)
- More systematic Mellin-position hybrid bootstrap
  (bootstraping the differential operator?)
- Witten diagram description?
- Weak coupling?

Thank you very much!

Questions & comments are welcome.