

From gauge theory to gravity

— Recent progress on color-kinematics duality and double copy

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Based on: Zeyu Li and GY, arXiv:2312.04319; Zeyu Li, GY, Guorui Zhu in preparation

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Outline

- **e** Introduction
- Constructing CK-dual numerators
- New strategy of deformation
- Summary and outlook

Gauge and gravity theories

What is "quantum gravity"?

Feynman and quantum gravity

OUANTUM THEORY OF GRAVITATION*

BY R. P. FEYNMAN

(Received July 3, 1963)

My subject is the quantum theory of gravitation. My interest in it is primarily in the relation of one part of nature to another. There's a certain irrationality to any work in gravitation, so it's hard to explain why you do any of it; for example, as far as quantum effects are concerned let us consider the effect of the gravitational attraction between an electron and a proton in a hydrogen atom; it changes the energy a little bit. Changing the energy of a quantum system means that the phase of the wave function is slowly shifted relative to what it would have been were no perturbation present. The effect of gravitation on the hydrogen atom is to shift the phase by 43 seconds of phase in every hundred times the lifetime of the universe! An atom made purely by gravitation, let us say two neutrons held together by gravitation, has a Bohr orbit of 10⁸ light years. The energy of this system is 10-70 rydbergs. I wish to discuss here the possibility of calculating the Lamb correction to this thing, an energy, of the order 10⁻¹²⁰. This irrationality is shown also in the strange gadgets of Prof. Weber, in the absurd creations of Prof. Wheeler and other such things, because the dimensions are so peculiar. It is therefore clear that the problem we are working on is not the correct problem; the correct problem is what determines the size of gravitation? But since I am among equally irrational men I won't be criticized I hope for the fact that there is no possible, practical reason for making these calculations.

I am limiting myself to not discussing the questions of quantum geometry nor what happens when the fields are of very short wave length. I am not trying to discuss any problems which we don't already have in present quantum field theory of other fields, not that I believe that gravitation is incapable of solving the problems that we have in the present theory, but because I wish to limit my subject. I suppose that no wave lengths are shorter than one-millionth of the Compton wave length of a proton, and therefore it is legitimate to analyze everything in perturbation approximation; and I will carry out the perturbation approximation as far as I can in every direction, so that we can have as many terms as we want, which means that we can go to ten to the minus two-hundred and something rydbergs.

I am investigating this subject despite the real difficulty that there are no experiments. Therefore there is so real challenge to compute true, physical situations. And so I made

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Feynman and quantum gravity

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- Feynman's tree theorem
- The idea of Faddeev-Popov quantization and ghost

Higher loop gravity \prime \prime \prime \prime \blacktriangle

feynRuleSlides page 2

 σ

1

2

 τ

High-loop gravity can be very difficult using Feynman diagrani $(\eta^{\sigma\rho} + (k_3^{\sigma} - k_1^{\sigma})\eta^{\rho\mu})$ α r α , α ⁺, α \overline{u} gravity can be very difficult using Feynman diagrams

 x_{k_1} ρ +

⇢ ⁺ ⌘⌧ ⌘⌫*k*³

⇢ ⌘*µ*⌘⌧ *^k*³

⇢ + 2⌘*µ*⌫ ⌘⇢⌧ *^k*¹

^µ ⁺ ⌘⌫ ⌘⇢⌧ *^k*¹

^µ ⌘⇢⌘⌫*^k*¹

 $\overline{1}$

⇢ ⁺ ⌘*µ*⌘⌫*k*¹

^µ ⁺ ⌘⌫ ⌘⇢⌧ *^k*¹

^µ ⁺ ⌘⌧ ⌘⇢ *^k*²

⌫ ⁺ ⌘*µ* ⌘⇢⌧ *^k*¹

 $\frac{1}{2}$

 $\overline{\mathbf{2}}$

⁺ ⌘*^µ*⇢⌘⌫⌧ *^k*²

⁺ ⌘*µ*⌘⇢⌧ *^k*²

⌧ ⁺ ⌘*µ* ⌘⌫⇢*k*²

⌧ ⁺ ⌘ ⌘*µ*⇢*k*²

⌧ + 2⌘⇢⌘⌫*^k*³

 $\theta = 2\eta^{\mu\nu} \eta^{\sigma\tau} k_1$ ⇢ ⁺ ⌘*µ* ⌘⌫⌧ *^k*² x_{k_1} $\theta + \eta^{\lambda \tau} \eta^{\nu \sigma} k_2$ μ_{k_1} $^{\rho}$ + μ_{k_1} $\theta + \eta^{\lambda\sigma} \eta^{\nu\tau} k_3$ μ_{k_1} ρ $=$ $\begin{array}{c} \nu_{k_1} \\ \end{array}$ $\theta + \eta^{\lambda\nu} \eta^{\mu\tau} k_3$ $\frac{\sigma}{\cdot}$ k_1 $^{\rho}$ + x_{k_1} $\sigma + 2\eta^{\mu\nu}\eta^{\rho\sigma}k_1$ x_{k_1} $\frac{\tau}{\tau}$ – $\frac{1}{2}$ σ_{k_2} $\lambda + \eta^{\mu\rho} \eta^{\nu\tau} k_1$ σ_{k_2} $^{\lambda}$ + λ_{k_2} μ + $\eta^{\lambda\tau} \eta^{\nu\rho} k_1$ σ_{k_2} *^µ* $\frac{\tau}{k_2}$ $\mu + \eta^{\lambda\nu} \eta^{\rho\sigma} k_1$ $\frac{\tau}{k_2}$ $^{\mu}$ + σ_{k_2} $\mu = \eta^{\lambda\rho} \eta^{\mu\tau} k_1$ σ_{k_2} $^{\nu}$ + τ_{k_2} $\mu^{\nu} + 2\eta^{\mu\rho}\eta^{\sigma\tau}k_2$ λ_{k_2} \sim + $k₁$ λ_{k_2} $\theta + \eta^{\mu\sigma} \eta^{\nu\tau} k_1$ $\frac{\lambda}{k_2}$ $^{\rho}$ + $\frac{\tau}{k_2}$ $\mu^{\mu} + 2\eta^{\mu\tau} \eta^{\nu\sigma} k_2$ $\frac{\lambda}{k_2}$ $^{\rho}$ + k_2 $\frac{\nu}{k_2}$ $\theta + \eta^{\nu\tau} \eta^{\rho\sigma} k_1$ λ_{k_3} $^{\mu}$ + σ_{k_3} $\mu + \eta^{\lambda\sigma} \eta^{\nu\rho} k_1$ τ_{k_3} $^{\mu}$ + $\frac{\nu}{1}k_3$ μ + $\eta^{\lambda\sigma}\eta^{\rho\tau}k_2$ $\frac{\nu}{1}k_3$ $^{\mu}$ + λ_{k_3} $\mu = \eta^{\mu \rho} \eta^{\sigma \tau} k_1$ $\frac{\lambda}{k_3}$ \sim + τ_{k_3} μ^{ν} + $\eta^{\mu\tau} \eta^{\rho\sigma} k_2$ λ_{k_3} $^{\nu}$ + $\frac{\rho_{k3}}{k}$ $\mu + \eta^{\lambda\sigma} \eta^{\mu\tau} k_2$ $\frac{\rho_{k_3}}{k_3}$ \sim + $\frac{1}{2}$ λ_{k_3} σ + $\eta^{\mu\rho} \eta^{\nu\tau} k_1$ λ_{k_3} σ + λ_{k_3} σ - $\eta^{\mu\nu} \eta^{\rho\tau} k_2$ λ_{k_3} σ + $\frac{\nu}{k_3}$ σ - $\eta^{\lambda\tau} \eta^{\mu\nu} k_2$ $\frac{\rho_{k_3}}{k_3}$ σ ₊ 33 $\frac{\nu}{k_3}$ σ + $\eta^{\mu\sigma} \eta^{\nu\rho} k_1$ λ_{k_3} τ + λ_{k_3} τ + $\eta^{\mu\rho}\eta^{\nu\sigma}k_2$ λ_{k_3} τ – $\frac{\nu}{k_3}$ τ + $\eta^{\lambda\mu}\eta^{\rho\sigma}k_2$ $\frac{\nu}{k_3}$ τ – μ_{k_3} $\sigma + 2\eta^{\lambda\rho}\eta^{\mu\sigma}k_3$ $\frac{\nu}{k_3}$ τ – $\eta^{\nu\rho} k_1 \cdot k_2 - \eta^{\lambda\sigma} \eta^{\mu\tau} \eta^{\nu\rho} k_1$ k_2 + $\eta^{\lambda \rho} \eta^{\mu \sigma} \eta^{\nu \tau} k_1 \cdot k_2$ + $k_1 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1 \cdot k_2$ k_2 + $2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1 \cdot k_2$ + $k_3 - \eta^{\lambda \tau} \eta^{\mu \rho} \eta^{\nu \sigma} k_1 \cdot k_3 +$ $x_3 + 2\eta^{\lambda\tau} \eta^{\mu\nu} \eta^{\rho\sigma} k_1 \cdot k_3$ $k_3 - \eta^{\lambda \nu} \eta^{\mu \sigma} \eta^{\rho \tau} k_1 \cdot k_3$ $k_3 + \eta^{\lambda \mu} \eta^{\nu \rho} \eta^{\sigma \tau} k_1 \cdot k_3$ $k_1 + 2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_2 \cdot k_3$ $k_3 - \eta^{\lambda \nu} \eta^{\mu \tau} \eta^{\rho \sigma} k_2 \cdot k_3 B_3 - \eta^{\lambda \mu} \eta^{\nu \sigma} \eta^{\rho \tau} k_2 \cdot k_3 -$ 100 terms 3-vertex more than

 λ

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 δ^3S $\delta\varphi_{\mu\nu}\delta\varphi_{\sigma\tau}\delta\varphi_{\rho\lambda}$ $\rightarrow \qquad 2 \eta^{\mu\tau} \eta^{\nu\sigma} k_1$ x_{k_1} θ + $2\eta^{\mu\sigma}\eta^{\nu\tau}k_1$ x_{k_1} $\theta = 2\eta^{\mu\nu}\eta^{\sigma\tau}k_1$ x_{k_1} $\rho +$ $2\eta^{\lambda\tau}\eta^{\mu\nu}k_1$ σ_{k_1} $\mu^{\rho} + 2\eta^{\lambda\sigma} \eta^{\mu\nu} k_1$ \mathbf{r}_{k_1} $\theta + \eta^{\mu\tau} \eta^{\nu\sigma} k_2$ x_{k_1} $\theta + \eta^{\mu\sigma} \eta^{\nu\tau} k_2$ x_{k_1} $\theta + \eta^{\lambda \tau} \eta^{\nu \sigma} k_2$ $^{\mu}$ _{k₁} $^{\rho}$ + $\eta^{\lambda\sigma}\eta^{\nu\tau}k_2$ μ_{k_1} $\theta + \eta^{\lambda \tau} \eta^{\mu \sigma} k_2$ $\begin{array}{c} \nu \\ k_1 \end{array}$ $\theta + \eta^{\lambda\sigma} \eta^{\mu\tau} k_2$ $\begin{array}{c} \nu \\ k_1 \end{array}$ $\theta + \eta^{\lambda \tau} \eta^{\nu \sigma} k_3$ μ_{k_1} $\theta + \eta^{\lambda \sigma} \eta^{\nu \tau} k_3$ μ_{k_1} $\rho =$ $n^{\lambda\nu} n^{\sigma\tau} k_3$ μ_{k_1} $\theta + \eta^{\lambda \tau} \eta^{\mu \sigma} k_3$ $\begin{array}{c} \nu k_1 \end{array}$ $\theta + \eta^{\lambda\sigma} \eta^{\mu\tau} k_3$ $\begin{array}{c} \nu k_1 \end{array}$ $\theta = \eta^{\lambda\mu} \eta^{\sigma\tau} k_3$ $\begin{array}{c} \nu k_1 \end{array}$ θ + $\eta^{\lambda\nu}\eta^{\mu\tau}k_3$ $\frac{\sigma}{\cdot}$ k_1 $^{\rho}$ + $\eta^{\lambda\mu}\eta^{\nu\tau}k_3$ σ_{k_1} $\theta + \eta^{\lambda\nu} \eta^{\mu\sigma} k_3$ τ_{k_1} $\theta + \eta^{\lambda\mu} \eta^{\nu\sigma} k_3$ \mathbf{r}_{k_1} $\int^{\rho} + 2\eta^{\mu\nu}\eta^{\rho\tau} k_1$ x_{k_1} $\sigma + 2\eta^{\mu\nu}\eta^{\rho\sigma}k_1$ x_{k_1} $\frac{\tau}{\cdot}$ – $2\eta^{\lambda\rho}\eta^{\mu\nu}k_1$ σ_{k_1} $\sigma + 2\eta^{\lambda\nu} \eta^{\mu\rho} k_1$ σ_{k_1} $\tau + 2\eta^{\lambda\mu} \eta^{\nu\rho} k_1$ σ_{k_1} τ + $\eta^{\mu\tau} \eta^{\nu\rho} k_1$ σ_{k_2} $\lambda + \eta^{\mu\rho} \eta^{\nu\tau} k_1$ σ_{k_2} $^{\lambda}$ + $\eta^{\mu\sigma}\eta^{\nu\rho}k_1$ $\frac{\tau}{k_2}$ $\lambda + 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σ_{k_3} $\tau = \eta^{\lambda \tau} \eta^{\mu \sigma} \eta^{\nu \rho} k_1 \cdot k_2 = \eta^{\lambda \sigma} \eta^{\mu \tau} \eta^{\nu \rho} k_1 \cdot k_2$ k_2 - $\eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1 \cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1 \cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1 \cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1 \cdot k_2 +$ $2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_{1}$ \cdot k_{2} = $\eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_{1}$ \cdot k_{2} + $2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_{1}$ \cdot k_{2} = $\int \eta^{\lambda\nu} \eta^{\mu\sigma} \eta^{\rho\tau} k_1 \cdot k_2 = \eta^{\lambda\mu} \eta^{\nu\sigma} \eta^{\rho\tau} k_1 \cdot k_2 = 2 \eta^{\lambda\rho} \eta^{\mu\nu} \eta^{\sigma\tau} k_1 \cdot k_2 + 2 \eta^{\lambda\nu} \eta^{\mu\rho} \eta^{\sigma\tau} k_1 \cdot k_2 +$ $2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1$ $\cdot k_2$ $=$ $\eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1$ $\cdot k_3$ $=$ $\eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1$ $\cdot k_3$ $+$ $2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1 \cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1 \cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1 \cdot k_3 + 2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1 \cdot k_3 \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1 \cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1 \cdot k_3 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1 \cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1 \cdot k_3 - \eta^{\mu\nu}\eta^{\mu\nu}\eta^{\rho\tau}k_1 \cdot k_3$ $\eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1$ *·* $k_3 = 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1$ *·* $k_3 + \eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1$ *·* $k_3 = 0$ $\eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_2 \cdot k_3 = \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_2 \cdot k_3 = \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_2 \cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_2 \cdot k_3 =$ $\eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_2 \cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_2 \cdot k_3 + \eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_2 \cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_2 \cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\nu}\eta^{\rho\sigma}k_3$ $\eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_2 \cdot k_3 + \eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_2 \cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_2 \cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_2 \cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_3$ $2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_2 \cdot k_3 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_2 \cdot k_3 + 2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_2 \cdot k_3$ σ τ 2 1 δ^3S \overline{a}

Higher loop perturbation is important

Understanding the structure of ultraviolet divergences in gravity: **Suppose we want to check UV properties of gravity theories:**

$$
\mathcal{L} = \sqrt{-g}(R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \cdots)
$$

 44.4

From gauge theory to gravity

Double copy

Gravity \simeq (Yang-Mills)²

Spin-2 $(\text{spin-1})^2$

Double copy

CLNS - 85/667 September 1985

A Relation Between Tree Amplitudes of Closed and Open **Strings**

H. Kawai, D.C. Lewellen, and S.-H.H. Tye

Newman Laboratory of Nuclear Studies **Cornell University** Ithaca, New York 14853

ABSTRACT

We derive a formula which expresses any closed string tree amplitude in terms of a sum of the products of appropriate open string tree amplitudes. This formula is applicable to the heterotic string as well as to the closed bosonic string and type II superstrings. In particular, we demonstrate its use by showing how to write down, without any direct calculation, all four-point heterotic string tree amplitudes with massless external particles.

KLT relation:

$$
A_{closed} = -\pi \kappa^2 \sin(\pi \kappa_1 \kappa_2) A_{open}^{(4)}(s,t) \overline{A}_{open}^{(4)}(s,u)
$$

 $A_{closed} = \pi \kappa^3 A_{open}^{(5)} (12345) \overline{A}_{open}^{(5)} (21435) \sin(\pi \kappa_1 \cdot \kappa_2) \sin(\pi \kappa_3 \cdot \kappa_4)$ $+\pi\kappa^3$ $A_{open}^{(5)}$ (13245) $\overline{A}_{open}^{(5)}$ (31425) $\sin(\pi k_1 \cdot k_3)\sin(\pi k_2 \cdot k_4)$.

 $M_4^{\text{tree}}(1,2,3,4) = -i s_{12} A_4^{\text{tree}}(1,2,3,4) A_4^{\text{tree}}(1,2,4,3),$

 $M_5^{\text{tree}}(1,2,3,4,5) = i s_{12} s_{34} A_5^{\text{tree}}(1,2,3,4,5) A_5^{\text{tree}}(2,1,4,3,5)$ $+is_{13}s_{24}A_5^{\rm tree}(1,3,2,4,5)\, A_5^{\rm tree}(3,1,4,2,5)\,$

 j and j is kept to distinguish the external legislation of external legislation \mathcal{L}_{max} New ideas are needed for loop level. and (13). An explicit generalization to n-point field theory gravity amplitudes theory gravity amplitudes of t

Color-kinematics duality

Color-kinematics duality was discovered by Bern-Carrasco-Johansson in 2008.

[Bern, Carrasco, Johansson 2008]

Generalizing double-copy to quantum (loop) level.

Color-kinematics duality

$$
A_n = i \int_{(a \text{bic graph})_i}^{n-2} \frac{C_i \overbrace{n_i}}{\pi D_{\alpha_i}} \xrightarrow{Rinemabic factor} (a \text{dor}
$$
\n
$$
\frac{C_i + C_j + C_k = 0 \Rightarrow n_i + n_j + n_k = 0}{n_i + n_j + n_k = 0}
$$

$$
M_n = iK^{n-2} \sum_{(cubic graph)_i} \frac{n_i n_i}{\pi D_{\alpha_i}}
$$

\mathbb{R} \mathbf{y} y
Y $\overline{}$ Color-kinematics duality

$$
A_n = i \, g^{n-2} \sum_{(cubic graph)_i} \frac{C_i \overbrace{n_i} \longrightarrow \text{Kinenatic factor}}{\pi \, D_{\alpha_i}} \text{Propagator}
$$

Gauge symmetry Spacetime symmetry

CK-duality v.s. Double-copy

By studying the simpler gauge theory, one may understand the far more complicated gravity theory.

Outline

e Introduction

- Constructing CK-dual numerators
- New strategy of deformation
- Summary and outlook

A problem of linear algebra structure constants \mathbf{r} abc associated to each trivalent vertex, more explicitly, more explicitly, more explicitly, \mathbf{r} $\frac{1}{2}$. $\frac{1}{2}$ and $\frac{1}{2}$, cu $\frac{1}{2}$, cu $\frac{1}{2}$

structure constants \mathbf{r} abc associated to each trivalent vertex, more explicitly, more explicitly, more explicitly, \mathbf{r} $\frac{1}{2}$. $\frac{1}{2}$ and $\frac{1}{2}$, cu $\frac{1}{2}$, cu $\frac{1}{2}$ A problem of linear algebra

Main challenge: it is a priori not known whether the solution exists Ci + Ci + Ci + Ci + Ni + Nk . (2.5) → Nk . (2.5)

Loop-level CK duality

For N=4 SYM, there are high loop examples that manifest global CK-dual Jacobi relations:

• 4-loop 4-point amplitude in $N=4$ **Bern, Carrasco, Dixon, Johansson, Roiban, 2012**

- **GY, 2016** • 5-loop Sudakov form factor in N=4
- 4-loop three-point form factor in N=4 **Lin, GY, Zhang, 2021**

4-loop 3-point form factor

229 trivalent graphs

第每次要参与一个人的人。我们的人的人的人,我们的人们的人们的人们的人们的人们的人 第一举事务委员会、教会教会、教会学、学生、学生、教会教会、教会教会、教会教会、教会 举章再举,我这就真不好。我们也不会 第一举 掷 新 我 我 这 这 这 这 这 这 部 并 这 这 这 这 这 人人 ·梅摩会学人家人家 蜂草毒毒蜂毒草 舞 海海、海洋、海岸、海岸、海洋、海岸、海洋海洋 安东东南方 医心室 海绵海绵 医骨盆 法人民 医心室 机油涂布 单一数 法一种 医白色 医大脑 基本学生的现在分词的现在分词 医中毒性骨折 医中毒性骨折

4-loop 3-point form factor

4-loop 3-point form factor

Three-point form factor up to four loops

$$
\mathcal{F}_3 = \int d^4x \, e^{-iq \cdot x} \langle p_1, p_2, p_3 | \operatorname{tr}(F^2)(x) | 0 \rangle \qquad \qquad \text{and} \qquad \text{and
$$

contributions and include all-usually one has to find a large set of the state all α **G. Lin, GY, S. Zhang, 2021**

Non-supersymmetric Yang-Mills

For non-supersymmetric YM, even two-loop is challenging:

- 2-loop 4-gluon all-plus-helicity amplitude in pure YM **Bern, Davies, Dennen, Huang, Nohle 2013** $A_4^{(2)}(1^+,2^+,3^+,4^+)$
- 2-loop 5-gluon all-plus-helicity amplitude in pure YM $A_5^{(2)}(1^+,\!2^+,\!3^+,\!4^+,\!5^+)$ O'Connell and Mogull 2015

No global CK-dual solution is known for generic helicity configurations at two loops.

• Enlarge ansatz (e.g. increasing power of loop momenta)

$$
A_5^{(2)}(1^+,2^+,3^+,4^+,5^+)
$$
 $\boxed{n^{\text{CK}} \sim \ell^{12}}$ O'Connell and Mogull 2015

 $20K$ rolotione? α of α is method to the power of loop momenta. This method has been used in α • Give up global CK relations?

thosing CK of pality on θ CK duality for pure Yang-Mills two-loop five-point amplitude with identical helicities. $\frac{3}{8}$ one can relax the constraints of $\frac{3}{8}$ by demanding that the $\frac{3}{8}$ only imposing CK+duality on cuts. Ansatz is made to all topologies and

Bern, Davies and Nohle 2015

1 2 3 ⁴ (*a*) ² 1 3 4 (*b*) 3 1 2 4 = + (*c*)

 $(n_a - n_b - n_c)|$ $\vert_{\text{cut}} = 0$

1) One can simplify the problem by considering helicity amplitudes. In a specific helicity amplitude, the physical result may be much simpler so it will be easier to realize CK duality. Hard to generalize to higher-loop/point cases. example, the (minimal type) and $\mathbf{1}$ (see all 138) contains 120904 parameters for all 14 topologies for all

Outline

e Introduction

- Constructing CK-dual numerators
- New strategy of deformation \bigcirc
- Summary and outlook

Two-loop 4-gluon amplitude

We introduce a strategy by allowing **"deformation"**.

$$
N_1 = n_1 + \Delta_1
$$

Let us first review the standard construction.

$$
N_1 = \bigcap_{1 \leq i \leq n} +\Delta_1
$$

Two-loop trivalent diagrams

(13)

Two-loop trivalent diagrams

Master topologies

 $n_{14} = n_9[p_1, p_2, p_3, p_4, l_1 - l_2, l_1] + n_9[p_1, p_2, p_3, p_4, -l_2 - p_1 - p_2, -l_1 - p_1 - p_2],$

Ansatz for the master numerators seur io, and the cut-cut-topologies outside the set of the set of

 \mathbf{r} Polynomials in D-dim kinematics: numerators. The ansatz is linear combination of monomials *Mk*:

$$
n_m = \sum_k a_{mk} M_k, \qquad m = 1, 2,
$$

$$
\{\varepsilon_i \cdot \varepsilon_j, \varepsilon_i \cdot p_j, \varepsilon_i \cdot l_\alpha, p_i \cdot l_\alpha, l_\alpha \cdot l_\beta, p_1 \cdot p_2, p_2 \cdot p_3\}
$$

e.g.:
$$
(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \varepsilon_4)(p_1 \cdot p_2)(p_1 \cdot l_1)(p_1 \cdot l_2)
$$

Unitarity cuts

 (a) 4 2

 (b) 4

1

 \overline{C} $\overline{$ amplitude, the physical result may be much simpler so it will be easier to realize CK duality. $\begin{bmatrix} \text{m} & \text{m} & \text{m} \\ \text{m} & \text{m} & \text{m} & \text{m} \\ \text{m} & \text{m} & \text{m} & \text{m} \end{bmatrix}$ and (α + $\overline{}$ one can try to enlarge ansatz such as introducing the non-local property to numerators to numerators to numerators $\overline{}$ CK-duality only on cuts. 1) One can simplify the problem by considering helicity amplitudes. In a specific helicity initial parameters: \sim 120,000 For two-loop four-point amplitude, helicity configuration can be (++++), (+++), (++) atter symmetry constraints: \sim 28,000 ofter cut and CK constraints: $\sim 6,300$ alter cut and \overline{C} constraints, \sim 0,000

1

Unitarity cuts

$$
N_i = n_i + \boxed{\Delta_i}
$$
 Deformation

Deformed trivalent diagrams

 $\begin{bmatrix} 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \end{bmatrix}$ integrand a strategy: we $N_t = \int u_i \cdot |\Delta_i|$ which can also be related by \mathcal{N}_i . $N_i = \left\{ \begin{array}{r} n_i \left(\begin{array}{c} \begin{array}{c} \begin{array}{c} \end{array} \\ \end{array} \right) & i \end{array} \begin{array} \begin{array}{c} \end{array} \end{array} \right. & i \end{array} \begin{array} \begin{array}{c} \end{array} \end{array} \end{array} \right. & \text{otherwise} \end{array}$ For two-loop four-point amplitude, helicity configuration can be (++++), (+++), (++) \overline{a} \int $n_i + \Delta_i, \vert i = 1, 4, 5, 9, 10, 13,$ $\overrightarrow{n_i}$, $\overrightarrow{n_i}$, others. **Deformation**

\mathbf{f} *Nⁱ* = (Deformed numerators *ⁿi,* others. (4.1)

these cuts. We ask that deformation satisfies a sub-set of dual Jacobi relations.

 $\Delta_5=-\Delta_1[p_1,p_2,p_3,p_4,l_1,l_1-l_2+p_1+p_2]+\Delta_1[p_1,p_2,p_4,p_3,l_1,l_1+l_2]$ $\Lambda_{\rm s} = -\Lambda_{\rm s}[\eta_{\rm c},\eta_{\rm s},\eta_{\rm s},l_{\rm c},l_{\rm c},l_{\rm c},l_{\rm c}], \Lambda_{\rm s}[\eta_{\rm c},\eta_{\rm s},\eta_{\rm c},l_{\rm c},l_{\rm c},l_{\rm c},l_{\rm c}]$ $\Delta_9 = -\Delta_4[p_1, p_2, p_3, p_4, l_1, l_1 - l_2] + \Delta_4[p_1, p_2, p_4, p_3, l_1, l_1 - l_2]$ $\Delta_{13} = \Delta_9 + \Delta_9[p_1, p_2, p_3, p_4, -l_1 - p_1 - p_2, l_2].$ $\Delta_4 = \Delta_1 - \Delta_1[p_3, p_4, p_2, p_1, l_1 - l_2 + p_1 + p_2, -l_2]$ $\Delta_{10} = -\Delta_4[p_1, p_2, p_3, p_4, l_1, l_1 + l_2 + p_1 + p_2] - \Delta_4[p_1, p_2, p_3, p_4, -l_1 - p_1 - p_2, -l_1 + l_2]$

Ansatz of the master numerator

Figure 8: Topologies that will contribute to cut-(a) in Figure 6 thus need to be deformed. Consider different Lorentz structure separately: CK operations and they also satisfy unitarity cuts separately. Thus we can treat them inde-

$$
\Delta_{i} = \Delta_{i}^{[1]} + \Delta_{i}^{[2]} + \Delta_{i}^{[3]}.
$$
\n
$$
\Delta_{1}^{[2]} = (\varepsilon_{1} \cdot \varepsilon_{2})(\varepsilon_{3} \cdot \varepsilon_{4})(\sum_{k} c_{k}^{[1]} M_{k}^{[1]}) l_{2}^{2}
$$
\n
$$
\Delta_{1}^{[2]} = [(\varepsilon_{1} \cdot \varepsilon_{2})(\sum_{a} c_{a}^{[2]} M_{1,a}^{[2]}) + (\varepsilon_{3} \cdot \varepsilon_{4})(\sum_{b} c_{b}^{[2]} M_{2,b}^{[2]})] l_{2}^{2}
$$
\n
$$
\Delta_{1}^{[3]} = (\sum_{k} c_{k}^{[3]} M_{k}^{[3]}) l_{2}^{2}
$$

Some requirement:

2) double copy still applicable. revert to an integrand with global CK relations which makes no di↵erence from the *nⁱ* numerators. Some requirement: 1) do not affect other cuts,

Solving the master numerator *{p*¹ *· ^p*2*, p*² *· ^p*3*, p*¹ *· ^l*1*, p*² *· ^l*1*, p*³ *· ^l*1*, p*¹ *· ^l*2*, p*² *· ^l*2*, p*³ *· ^l*2*, l*¹ *· ^l*2*, l*² 1*, l*² A simple dimension analysis shows that *M*[1] *^k* has mass dimension four. In this ansatz, we **SOIVING the master numerator**

Deformation

Solution for the master numerator:

(There is a solution space with free parameters, here is a special simple choice.)

$$
\Delta \begin{bmatrix} p_2 \\ k_5 \\ p_1 \end{bmatrix} \begin{bmatrix} k_2 & k_3 \\ k_4 & k_4 \end{bmatrix} p_3 \\ + (d-2)^2 \Big\{ (\varepsilon_1 \cdot \varepsilon_2) (\varepsilon_3 \cdot \varepsilon_4) k_5^2 k_6^2 + 16(\varepsilon_1 \cdot k_5) (\varepsilon_2 \cdot k_5) (\varepsilon_3 \cdot k_6) (\varepsilon_4 \cdot k_6) \\ - 4 [(\varepsilon_1 \cdot \varepsilon_2) (\varepsilon_3 \cdot k_6) (\varepsilon_4 \cdot k_6) k_5^2 + (\varepsilon_3 \cdot \varepsilon_4) (\varepsilon_1 \cdot k_5) (\varepsilon_2 \cdot k_5) k_6^2] \Big\} \\ + (d-2) 4 \Big\{ -10 [(\varepsilon_1 \cdot k_6) (\varepsilon_2 \cdot k_6) (\varepsilon_3 \cdot k_5) (\varepsilon_4 \cdot k_5) + (\varepsilon_1 \cdot k_2) (\varepsilon_2 \cdot k_1) (\varepsilon_3 \cdot k_4) (\varepsilon_4 \cdot k_3) \\ + 20 [(\varepsilon_1 \cdot k_6) (\varepsilon_2 \cdot k_1) (\varepsilon_3 \cdot k_5) (\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2) (\varepsilon_2 \cdot k_6) (\varepsilon_3 \cdot k_4) (\varepsilon_4 \cdot k_5)] \\ + 32 [(\varepsilon_1 \cdot k_5) (\varepsilon_2 \cdot k_5) (\varepsilon_3 \cdot p_1) (\varepsilon_4 \cdot p_2) + (\varepsilon_1 \cdot p_3) (\varepsilon_2 \cdot p_4) (\varepsilon_3 \cdot k_6) (\varepsilon_4 \cdot k_6)] \\ + 47 [(\varepsilon_1 \cdot k_4) (\varepsilon_2 \cdot k_3) (\varepsilon_3 \cdot k_4) (\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2) (\varepsilon_2 \cdot k_1) (\varepsilon_3 \cdot k_2) (\varepsilon_4 \cdot k_1)] \Big\},
$$

Deformation

The simplicity of the deformation:

$$
N_1 = n_1 + \Delta_1
$$

$$
\Delta \left[\begin{array}{c} p_2 \searrow k_2 & k_3 \ k_5 \end{array}\right] \left\{\begin{array}{c} k_2 & k_4 \ k_6 \end{array}\right\} / k_7^2 = (5.2) \n+ (d-2)^2 \left\{ (\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \varepsilon_4) k_5^2 k_6^2 + 16(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) \right. \\ \left. - 4 \left[(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) k_5^2 + (\varepsilon_3 \cdot \varepsilon_4)(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5) k_6^2 \right] \right\} \n+ (d-2)4 \left\{ -10 \left[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_5) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3) \right] \right. \\ \left. + 20 \left[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_5) \right] \right. \\ \left. + 32 \left[(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot p_1)(\varepsilon_4 \cdot p_2) + (\varepsilon_1 \cdot p_3)(\varepsilon_2 \cdot p_4)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) \right] \right\},
$$

Deformation Undeformed part

1-Numerators.m 2-SymmetyBasis.m n1.txt n2.txt

Plain Text Document - 1.4 MB

$$
\text{Cov}_{\text{Cov}_{\text{Cov}}} \quad N_i = \left\{ \begin{array}{l l} n_i + \Delta_i, & i = 1, 4, 5, 9, 10, 13, \\ n_i, & \text{others.} \end{array} \right.
$$

• Pass the full set of D-dimensional planar and non-planar cuts

4

1

- copy construction. To achieve this, one should at least require *Nⁱ* to satisfy CK relations Country all UN-uddititiations on Cuts, so double-copy applies • Satisfy all CK-dual relations on cuts, so double-copy applies
- relations, the deformations of deformations integrated the deformation of the deformation of the deformation of $\frac{1}{2}$ Rather than just imposing CK relations with cuts, we propose that *ⁱ* should satisfy a • Free parameters cancel after the integral IBP reduction
- Integrated result satisfies the Catani IR formula

Outline

e Introduction

- Constructing CK-dual numerators
- New strategy of deformation
- Summary and outlook \bigcirc

- Gauge and gravity theories are related by double copy.
- The key of double copy is to achieve "color-kinematics duality".
- Finding CK-dual numerators is generally difficult, and introducing "a simple deformation" may solve it.

Bern, Davies and Nohle 2015

CK-duality only on cuts initial parameters: \sim 120,000 after symmetry constraints: \sim 28,000 after cut and CK constraints: $\sim 6,300$

Duality with deformaltion initial parameters: \sim 20,000 after CK and symmetry constraints: \sim 1400 with partial cuts + deformation : ~ 500 with remaining cut: \sim 200

A new strategy to apply CK-duality and double-copy.

• Why so simple?

$$
\Delta \left[\begin{array}{c} p_2 \searrow k_2 & k_3 \ k_5 \end{array}\right] \left\{\begin{array}{c} k_2 & k_4 \ k_6 & k_7 \ k_8 & k_9 \end{array}\right\} / k_7^2 =
$$
\n
$$
+ (d-2)^2 \left\{ (\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \varepsilon_4) k_5^2 k_6^2 + 16(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) - 4 [(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) k_5^2 + (\varepsilon_3 \cdot \varepsilon_4)(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5) k_6^2] \right\}
$$
\n
$$
+ (d-2)4 \left\{ -10 [(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_5) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3)] + 20 [(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_5)]
$$
\n
$$
+ 32 [(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot p_1)(\varepsilon_4 \cdot p_2) + (\varepsilon_1 \cdot p_3)(\varepsilon_2 \cdot p_4)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6)]
$$
\n
$$
+ 47 [(\varepsilon_1 \cdot k_4)(\varepsilon_2 \cdot k_3)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_2)(\varepsilon_4 \cdot k_1)] \right\},
$$

• More examples: higher loop cases?

Zeyu Li, GY, Guorui Zhu in preparation

• Why so simple?

$$
\Delta \left[\begin{array}{c} p_2 \searrow k_2 & k_3 \ k_5 \end{array}\right] k_6 \qquad k_7 k_6
$$
\n
$$
+ (d-2)^2 \left\{ (\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \varepsilon_4) k_5^2 k_6^2 + 16(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) - 4[(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) k_5^2 + (\varepsilon_3 \cdot \varepsilon_4)(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5) k_6^2] \right\}
$$
\n
$$
+ (d-2)4 \left\{ -10[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_5) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3)] + 20[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_5)]
$$
\n
$$
+ 32[(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot p_1)(\varepsilon_4 \cdot p_2) + (\varepsilon_1 \cdot p_3)(\varepsilon_2 \cdot p_4)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6)]
$$
\n
$$
+ 47[(\varepsilon_1 \cdot k_4)(\varepsilon_2 \cdot k_3)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_2)(\varepsilon_4 \cdot k_1)] \right\},
$$

More examples: higher loop cases?

Zeyu Li, GY, Guorui Zhu in preparation

Towards 3-loop Einstein gravity?

• Are there underlying structures for the deformation?

Thank you for your attention!

