第五届全国场论与弦论学术研讨会

From gauge theory to gravity

— Recent progress on color-kinematics duality and double copy

Gang Yang

ITP-CAS

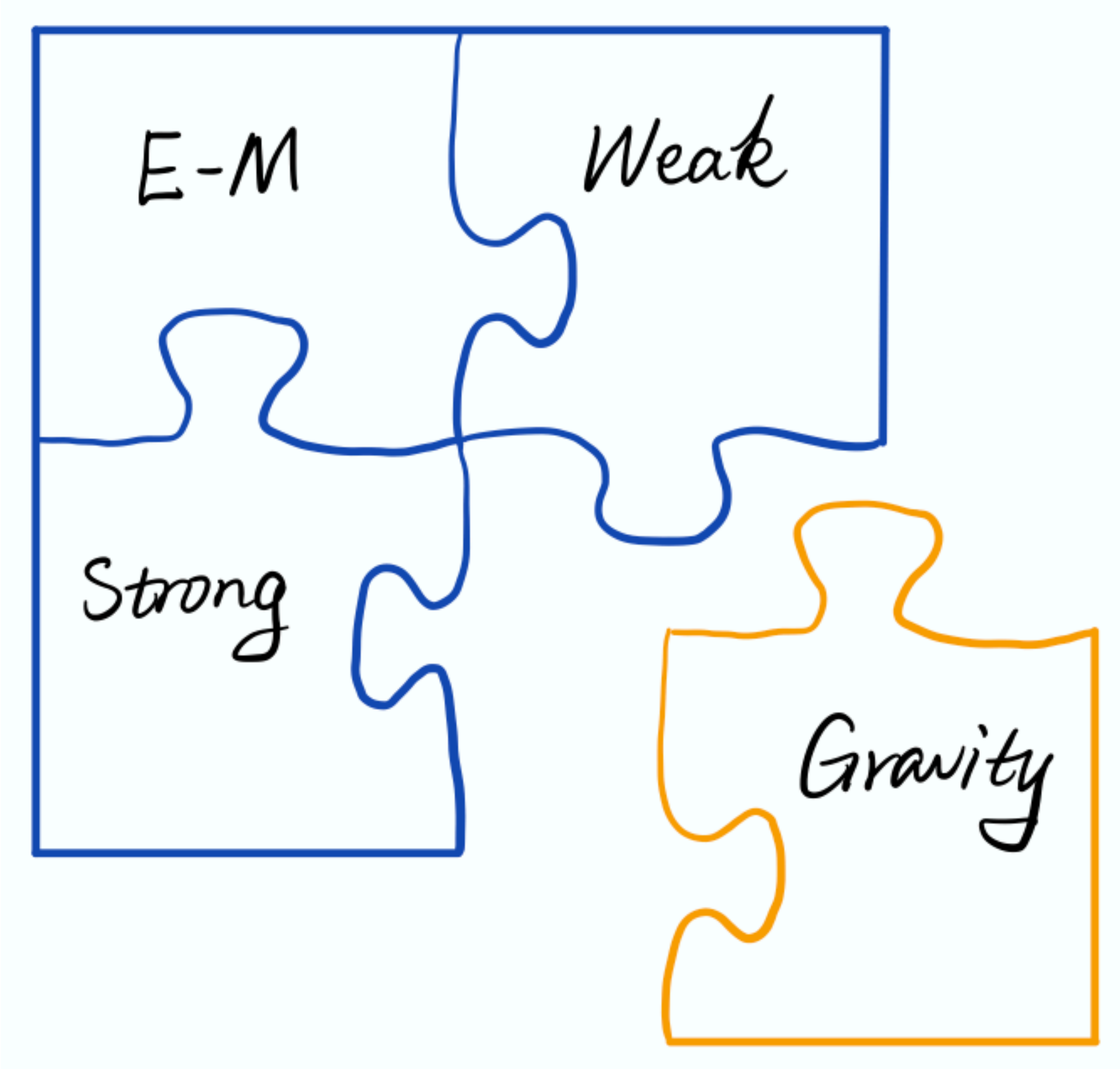
Based on: Zeyu Li and GY, arXiv:2312.04319; Zeyu Li, GY, Guorui Zhu in preparation

中国科学技术大学 2024.6.23-28

Outline

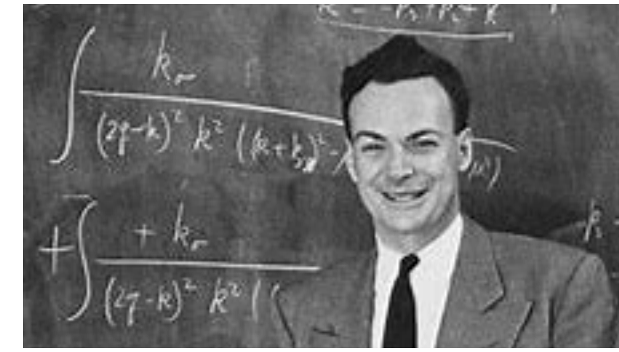
- Introduction
- Constructing CK-dual numerators
- New strategy of deformation
- Summary and outlook

Gauge and gravity theories



What is "quantum gravity"?

Feynman and quantum gravity



QUANTUM THEORY OF GRAVITATION*

By R. P. Feynman

(*Received July 3, 1963*)

My subject is the quantum theory of gravitation. My interest in it is primarily in the relation of one part of nature to another. There's a certain irrationality to any work in gravitation, so it's hard to explain why you do any of it; for example, as far as quantum effects are concerned let us consider the effect of the gravitational attraction between an electron and a proton in a hydrogen atom; it changes the energy a little bit. Changing the energy of a quantum system means that the phase of the wave function is slowly shifted relative to what it would have been were no perturbation present. The effect of gravitation on the hydrogen atom is to shift the phase by 43 seconds of phase in every hundred times the lifetime of the universe! An atom made purely by gravitation, let us say two neutrons held together by gravitation, has a Bohr orbit of 10^8 light years. The energy of this system is 10^{-70} rydbergs. I wish to discuss here the possibility of calculating the Lamb correction to this thing, an energy, of the order 10^{-120}. This irrationality is shown also in the strange gadgets of Prof. Weber, in the absurd creations of Prof. Wheeler and other such things, because the dimensions are so peculiar. It is therefore clear that the problem we are working on is not the correct problem; the correct problem is what determines the size of gravitation? But since I am among equally irrational men I won't be criticized I hope for the fact that there is no possible, practical reason for making these calculations.

I am limiting myself to not discussing the questions of quantum geometry nor what happens when the fields are of very short wave length. I am not trying to discuss any problems which we don't already have in present quantum field theory of other fields, not that I believe that gravitation is incapable of solving the problems that we have in the present theory, but because I wish to limit my subject. I suppose that no wave lengths are shorter than one-millionth of the Compton wave length of a proton, and therefore it is legitimate to analyze everything in perturbation approximation; and I will carry out the perturbation approximation as far as I can in every direction, so that we can have as many terms as we want, which means that we can go to ten to the minus two-hundred and something rydbergs.

I am investigating this subject despite the real difficulty that there are no experiments. Therefore there is so real challenge to compute true, physical situations. And so I made

"There's a certain irrationality to any work in gravitation."

"This irrationality is shown in the strange gadgets of Prof. Weber, in the absurd creation of Prof. Wheeler … "

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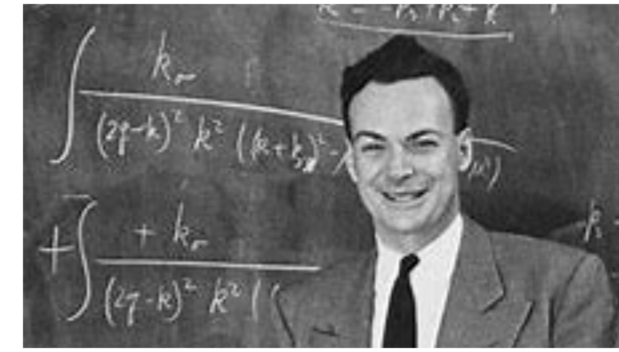
Feynman and quantum gravity



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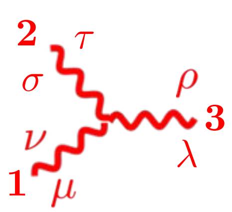
By studying loop diagrams, Feynman made discoveries that are important for gauge theory:

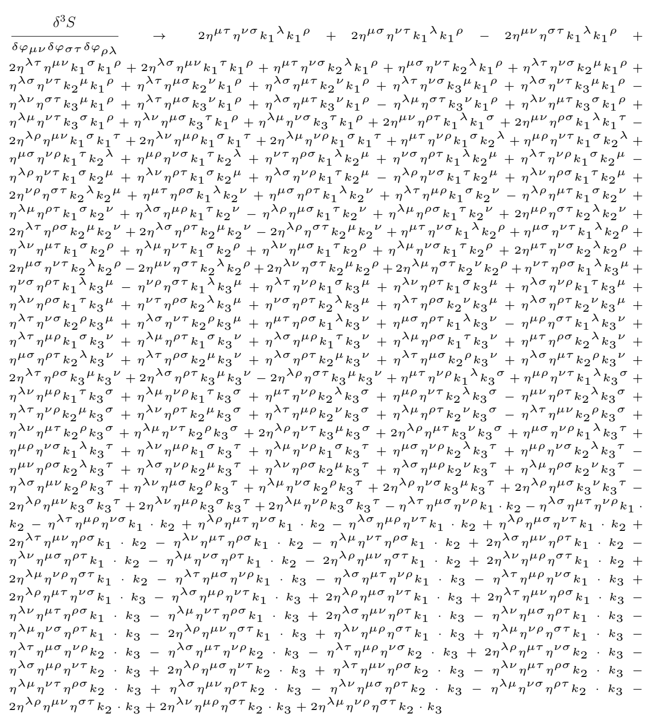
- Feynman's tree theorem
- The idea of Faddeev-Popov quantization and ghost

Higher loop gravity

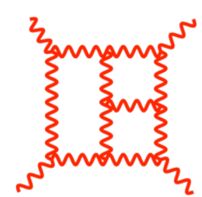
High-loop gravity can be very difficult using Feynman diagram:



3-vertex
more than
100 terms

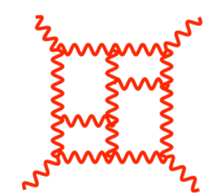


3-loop



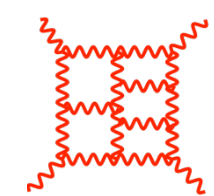
$\sim 10^{20}$ terms

4-loop



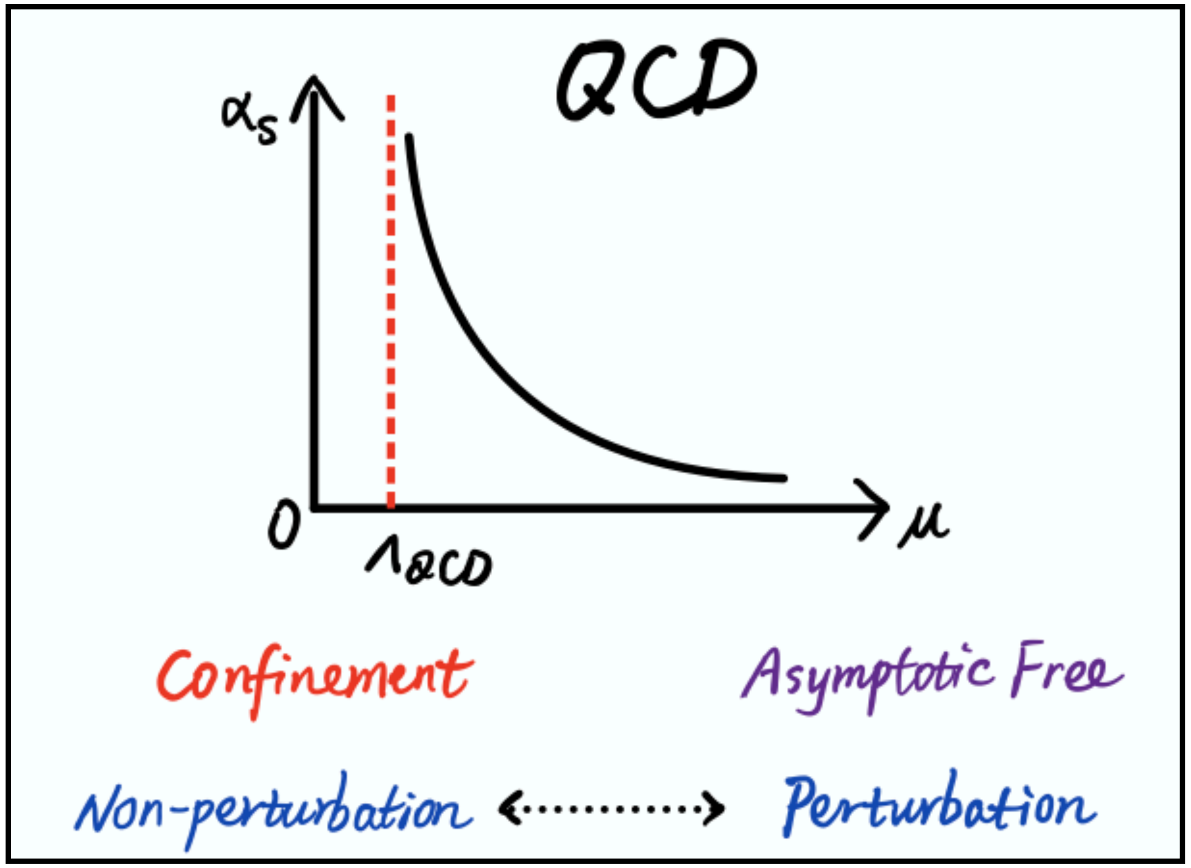
$\sim 10^{26}$ terms

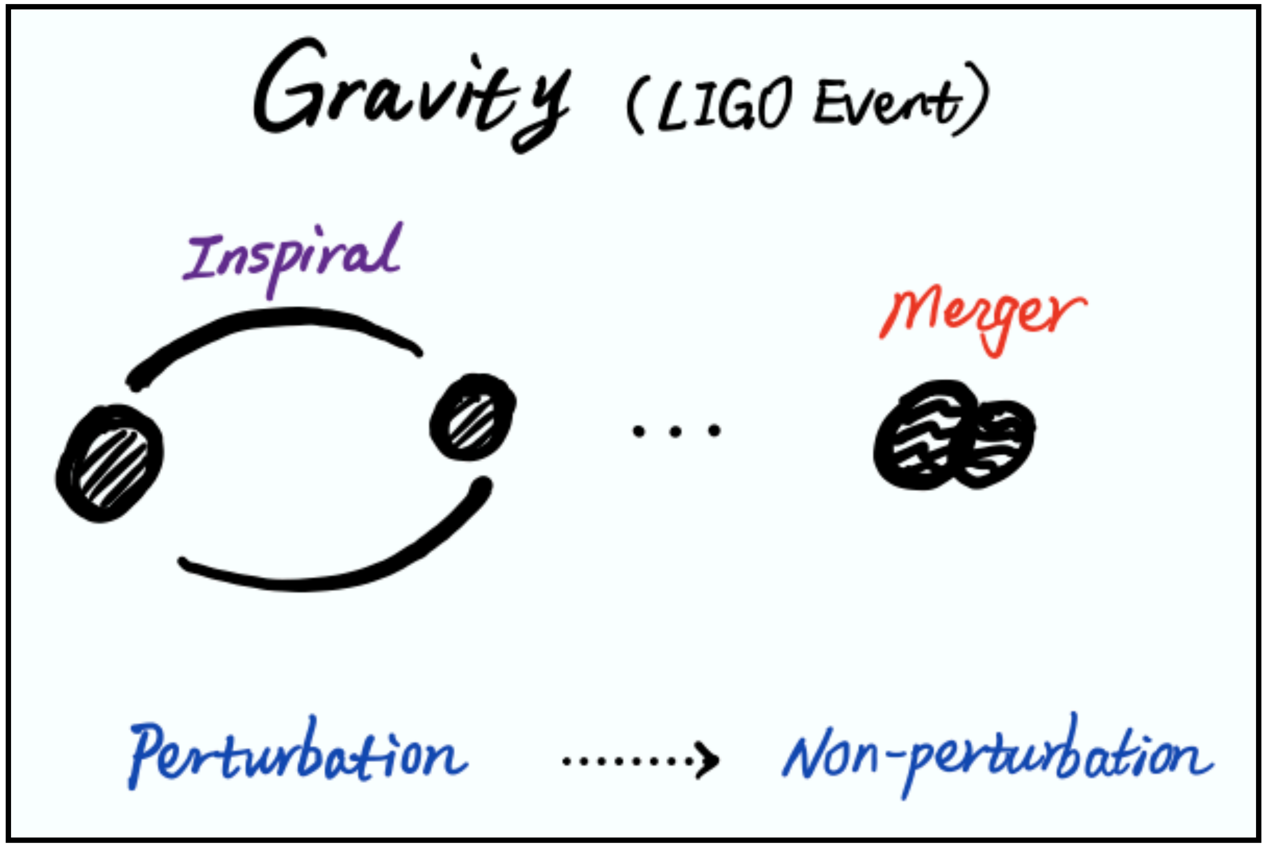
5-loop



$\sim 10^{31}$ terms

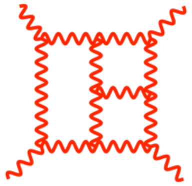
Higher loop perturbation is important



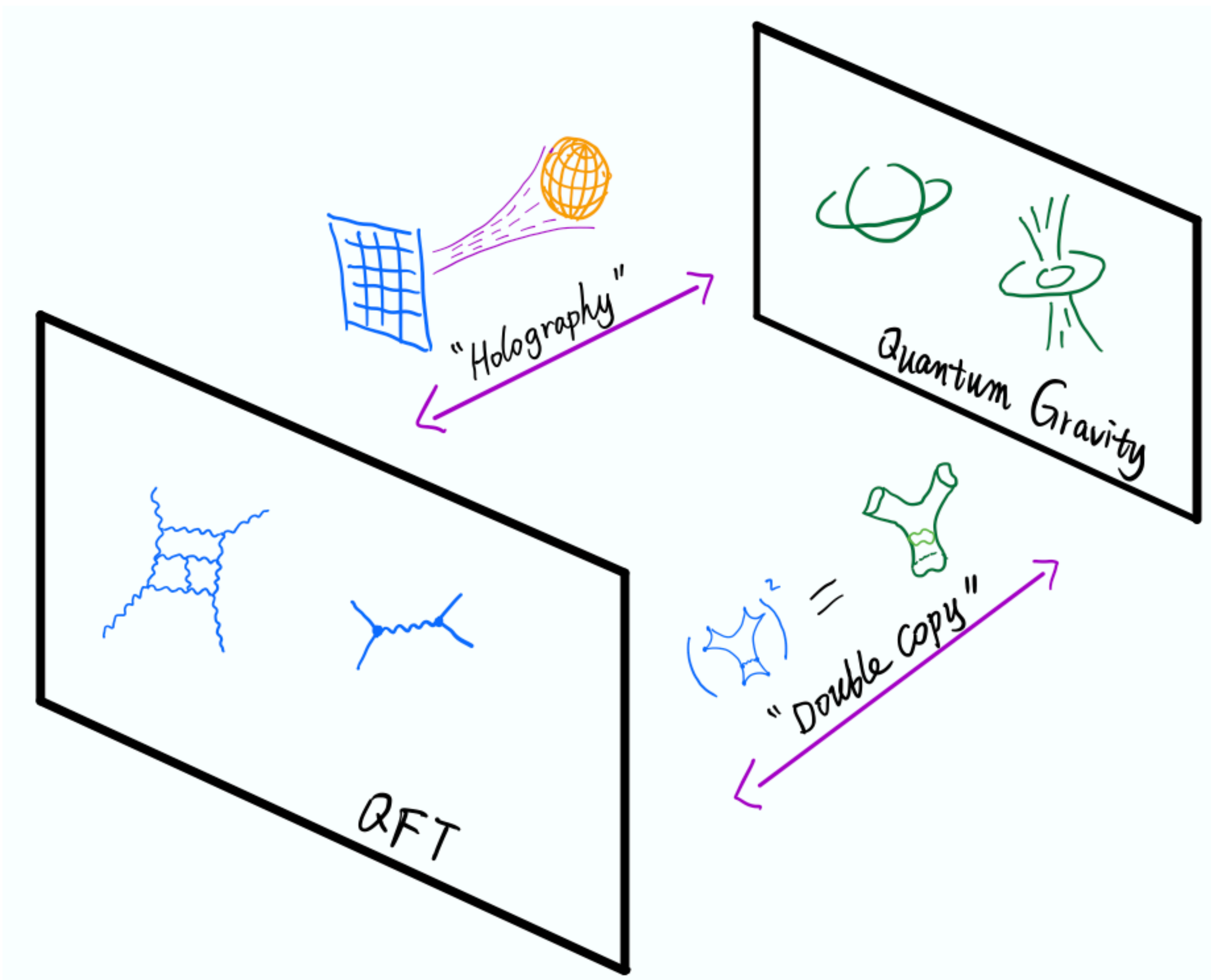


Understanding the structure of ultraviolet divergences in gravity:

$$\mathscr{L} = \sqrt{-g}(R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \cdots)$$



From gauge theory to gravity



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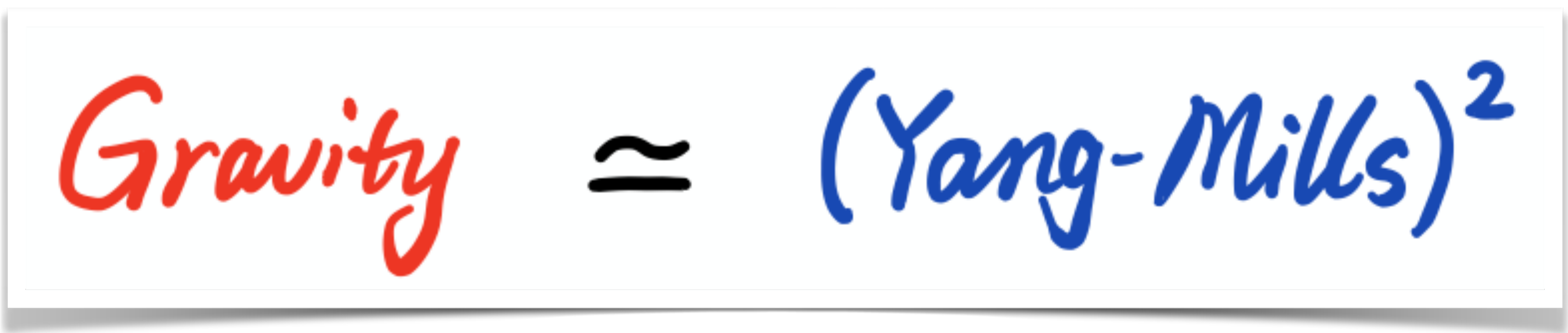
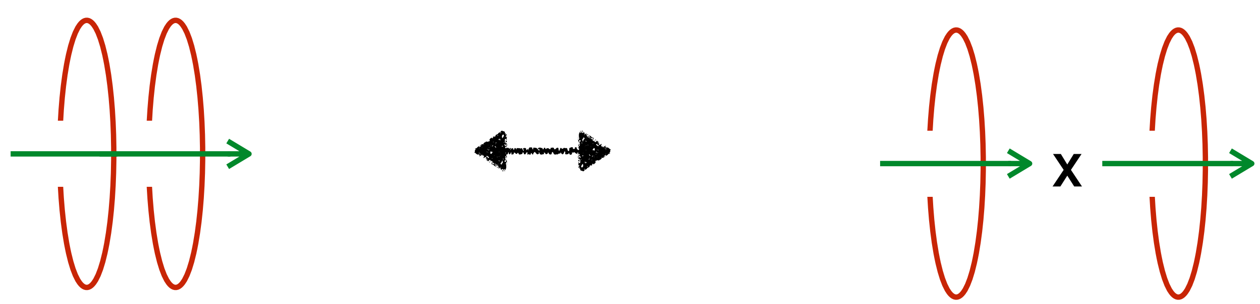
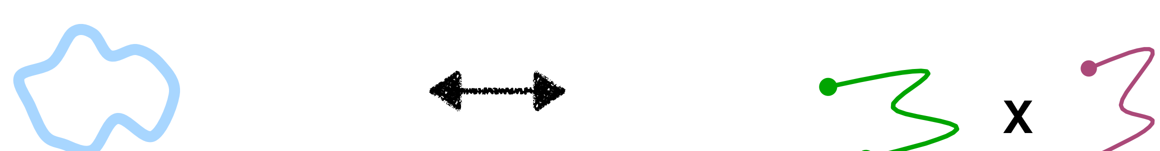




Spin-2

(spin-1)^2



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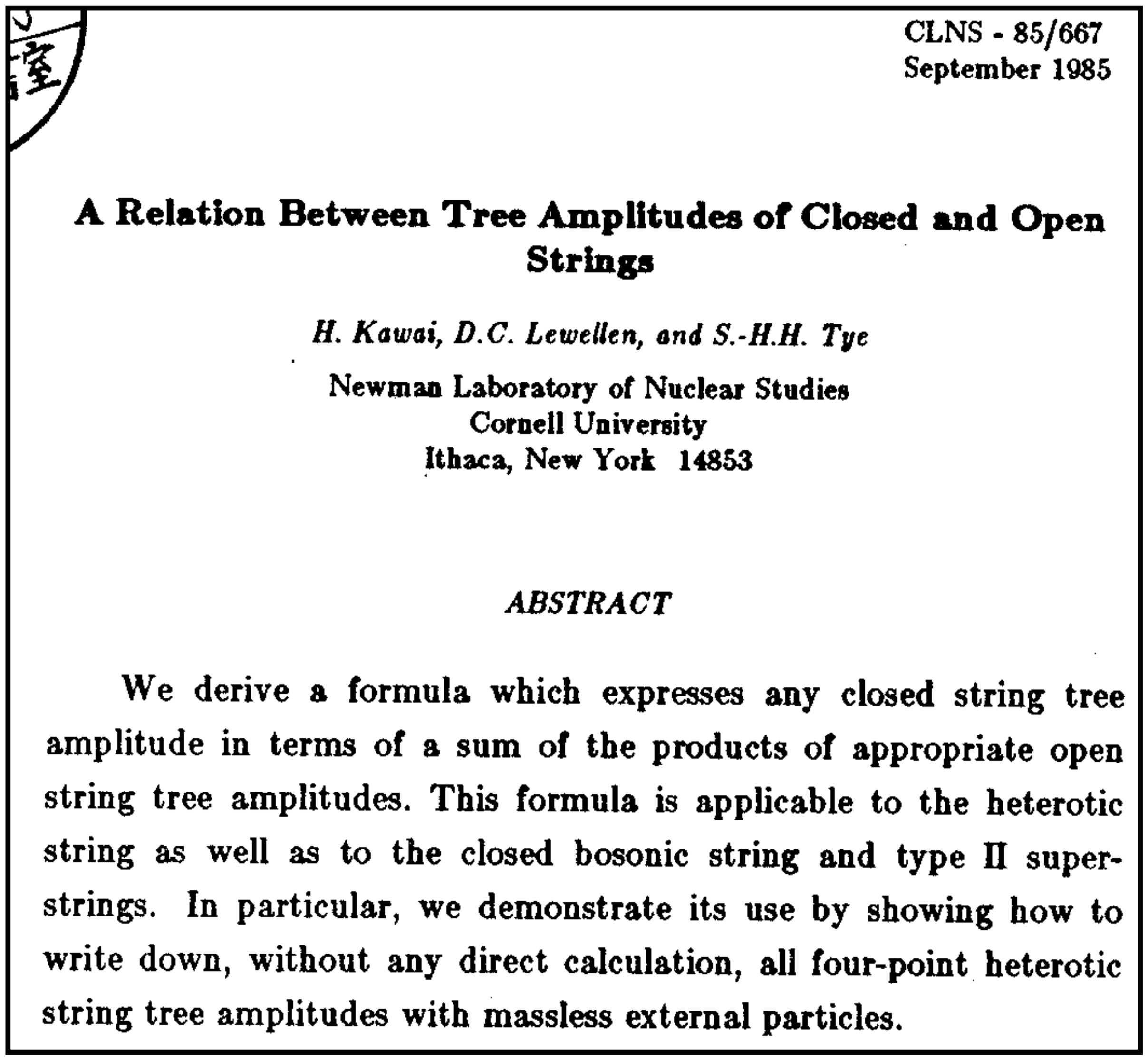

CLNS - 85/667
September 1985

A Relation Between Tree Amplitudes of Closed and Open Strings

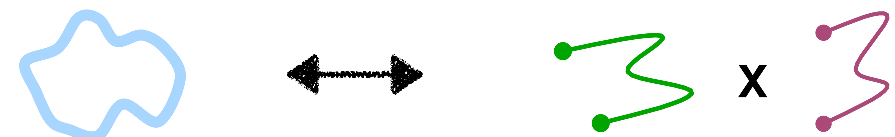
H. Kawai, D.C. Lewellen, and S.-H.H. Tye

Newman Laboratory of Nuclear Studies
Cornell University
Ithaca, New York 14853

ABSTRACT

We derive a formula which expresses any closed string tree amplitude in terms of a sum of the products of appropriate open string tree amplitudes. This formula is applicable to the heterotic string as well as to the closed bosonic string and type II superstrings. In particular, we demonstrate its use by showing how to write down, without any direct calculation, all four-point heterotic string tree amplitudes with massless external particles.

KLT relation:



$$A^{(4)}_{closed} = -\pi\kappa^2 \sin(\pi\kappa_1\cdot\kappa_2)\, A^{(4)}_{open}(s,t)\ \bar{A}^{(4)}_{open}(s,u)$$

$$\begin{aligned} A^{(5)}_{closed} &= \pi\kappa^3 A^{(5)}_{open}(12345)\bar{A}^{(5)}_{open}(21435)\sin(\pi\kappa_1\cdot\kappa_2)\sin(\pi\kappa_3\cdot\kappa_4) \\ &+ \pi\kappa^3\ A^{(5)}_{open}(13245)\bar{A}^{(5)}_{open}(31425)\sin(\pi k_1\cdot k_3)\sin(\pi k_2\cdot k_4). \end{aligned}$$

Field theory limit

$$M_4^{\text{tree}}(1,2,3,4) = -i s_{12} A_4^{\text{tree}}(1,2,3,4)\, A_4^{\text{tree}}(1,2,4,3)\,,$$

$$\begin{aligned} M_5^{\text{tree}}(1,2,3,4,5) &= i s_{12} s_{34} A_5^{\text{tree}}(1,2,3,4,5) A_5^{\text{tree}}(2,1,4,3,5) \\ &+ i s_{13} s_{24} A_5^{\text{tree}}(1,3,2,4,5)\, A_5^{\text{tree}}(3,1,4,2,5) \end{aligned}$$

New ideas are needed for loop level.

Color-kinematics duality

Color-kinematics duality was discovered by Bern-Carrasco-Johansson in 2008.

[Bern, Carrasco, Johansson 2008]

Gauge Theory

Color-Kinematics duality $\xrightarrow{\text{Double-copy}}$ Gravity

Generalizing double-copy to quantum (loop) level.

Color-kinematics duality

$$\mathcal{A}_n = i g^{n-2} \sum_{(\text{cubic graph})_i} \frac{c_i \, n_i}{\prod D_{\alpha_i}}$$

$c_i \rightarrow$ Color factor

$n_i \rightarrow$ Kinematic factor

$\prod D_{\alpha_i} \rightarrow$ Propagator

$$\boxed{c_i + c_j + c_k = 0 \Rightarrow n_i + n_j + n_k = 0}$$

$$\mathcal{M}_n = i \kappa^{n-2} \sum_{(\text{cubic graph})_i} \frac{n_i \, n_i}{\prod D_{\alpha_i}}$$

Color-kinematics duality

$$A_n = i g^{n-2} \sum_{(\text{cubic graph})_i} \frac{c_i \, n_i}{\prod D_{\alpha_i}}$$

Color factor

Kinematic factor

Propagator

Duality

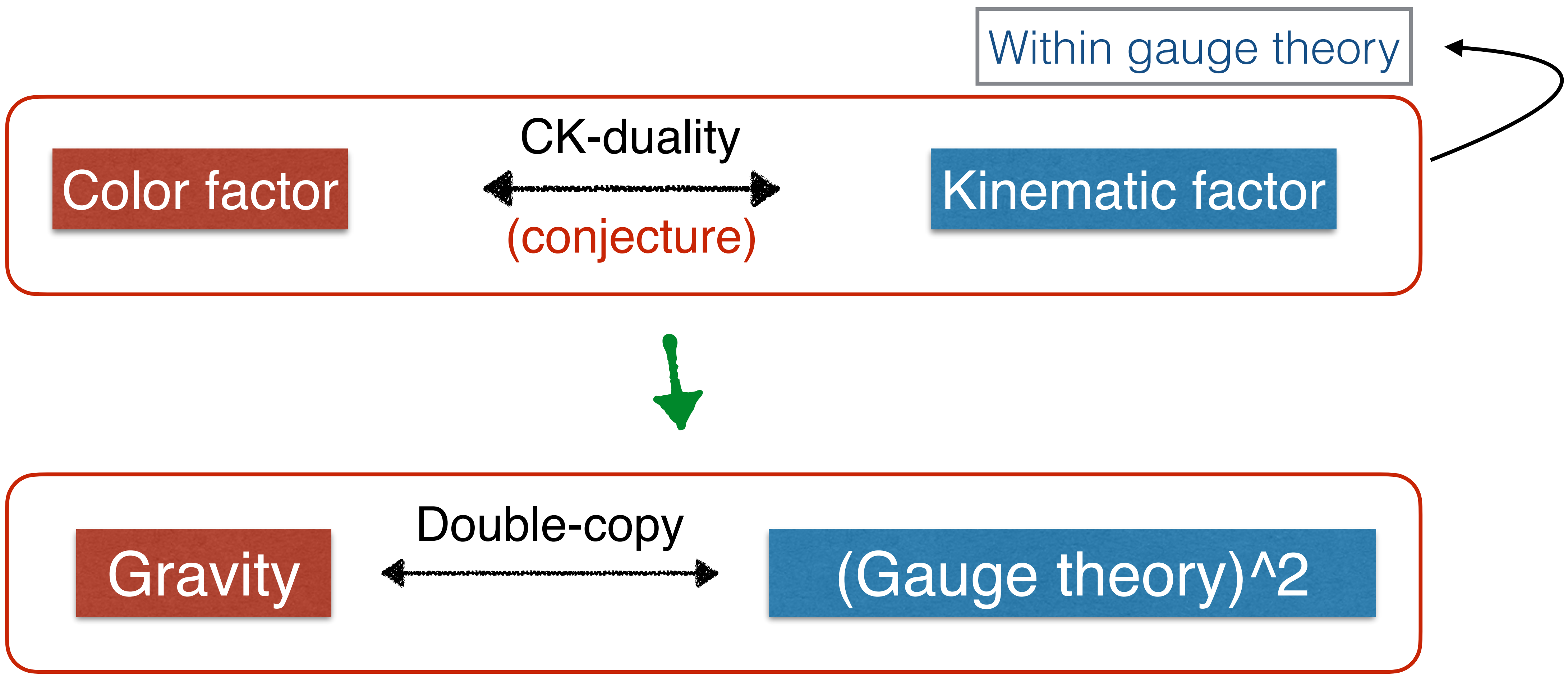
Color factor ⟷ Kinematic factor

(conjecture)

$$\tilde{f}^{abc} = \mathrm{Tr}([T^a, T^b]T^c) \qquad\qquad s_{ij} = (p_i + p_j)^2$$

Gauge symmetry **Spacetime symmetry**

CK-duality v.s. Double-copy



By studying the simpler gauge theory, one may understand the far more complicated gravity theory.

Outline

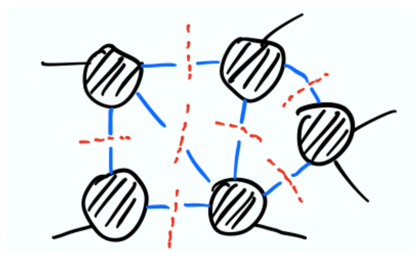
- Introduction
- Constructing CK-dual numerators
- New strategy of deformation
- Summary and outlook

A problem of linear algebra

CK-duality $c_s = c_t + c_u \quad \Rightarrow \quad n_s = n_t + n_u$

Conjecture

Compact ansatz of the loop integrand



Unitarity cuts

$$\text{Loop-ansatz}\,|_{\text{cut}} = \prod \text{tree-blocks}$$

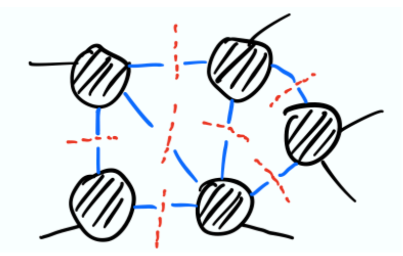
Solving linear equations

A problem of linear algebra

CK-duality $c_s = c_t + c_u \quad \Rightarrow \quad n_s = n_t + n_u$

Conjecture

Compact ansatz of the loop integrand



Unitarity cuts

$$\text{Loop-ansatz}\,|_{\text{cut}} = \prod \text{tree-blocks}$$

Solving linear equations

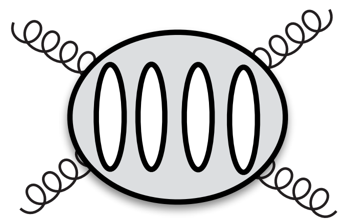
Main challenge: it is a priori not known whether the solution exists

Loop-level CK duality

For N=4 SYM, there are high loop examples that manifest **global CK-dual Jacobi relations**:

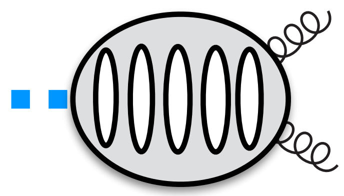
- 4-loop 4-point amplitude in N=4

  Bern, Carrasco, Dixon, Johansson, Roiban, 2012



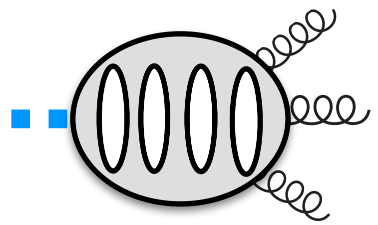
- 5-loop Sudakov form factor in N=4

  GY, 2016

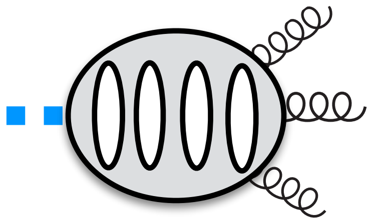


- 4-loop three-point form factor in N=4

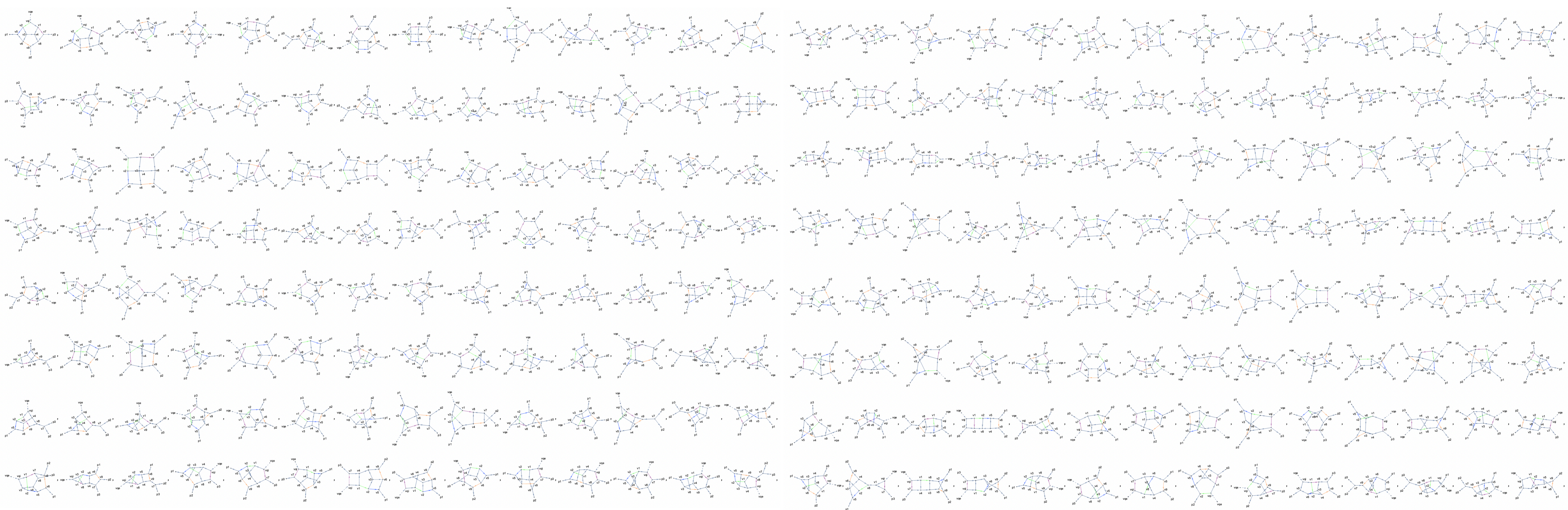
  Lin, GY, Zhang, 2021



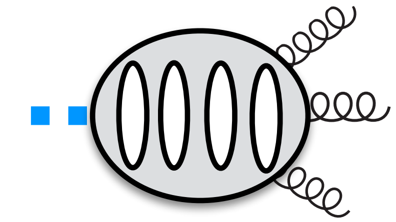
4-loop 3-point form factor



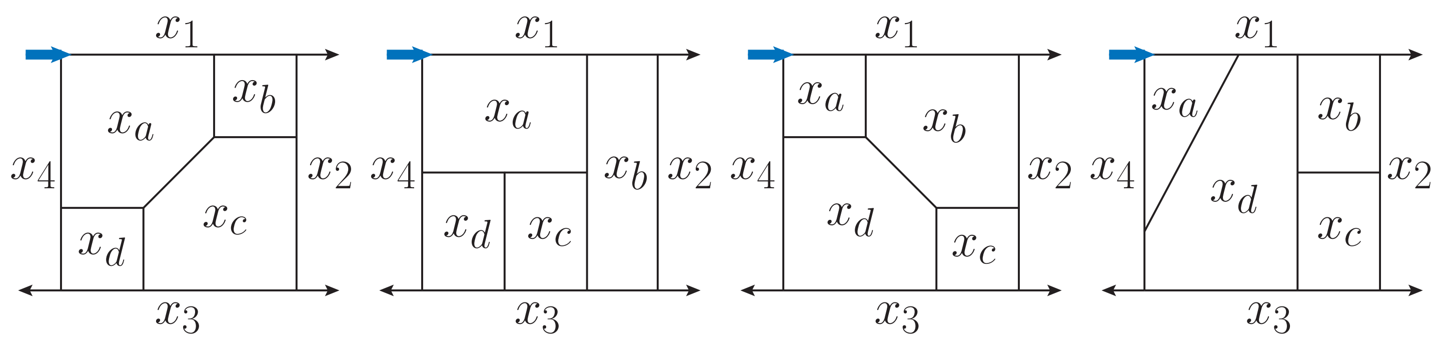
229 trivalent graphs



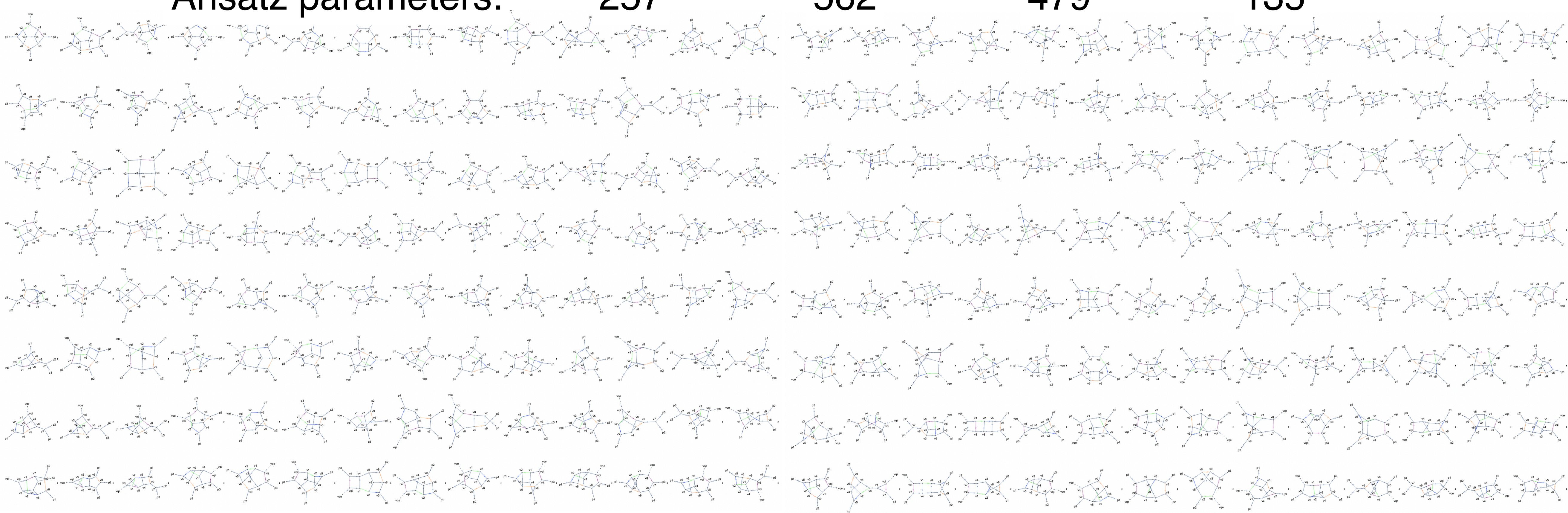
4-loop 3-point form factor



Master graphs

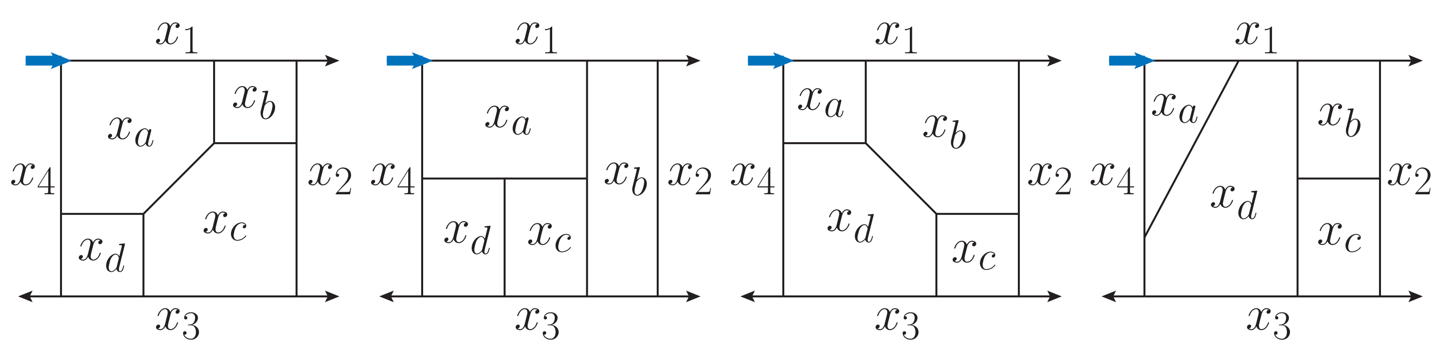


Ansatz parameters: 257 562 479 135

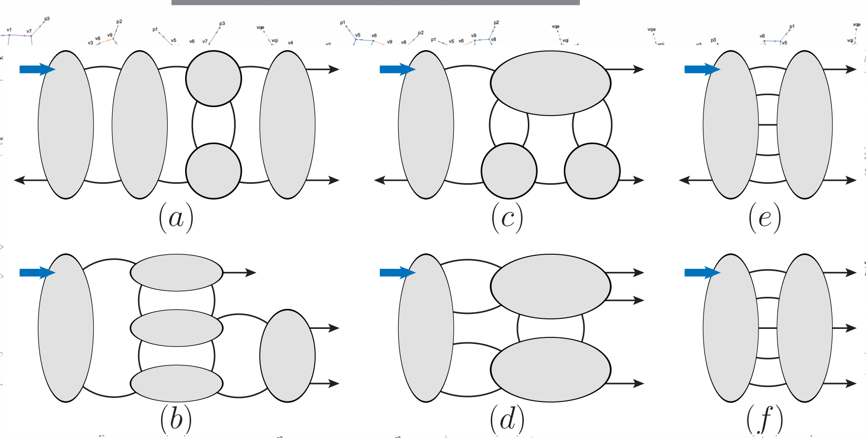


4-loop 3-point form factor

Master graphs



Unitarity cuts

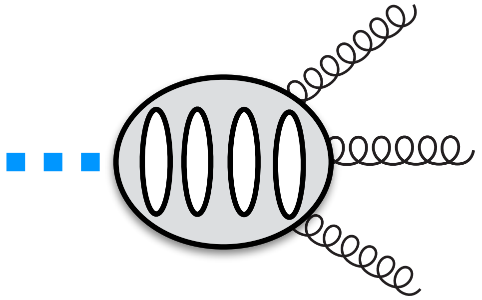


Final solution with 133 free parameters!

$$\boldsymbol{F}_3^{(4)} = \sum_{\sigma_3} \sum_{i=1}^{229} \int \prod_{j=1}^{4} d^D \ell_j \frac{1}{S_i} \sigma_3 \cdot \frac{\mathcal{F}_3^{(0)} C_i \, N_i}{\prod_{\alpha_i} P_{\alpha_i}^2}$$

Three-point form factor up to four loops

$$\mathcal{F}_3 = \int d^4x\, e^{-iq\cdot x}\langle p_1, p_2, p_3 \,|\, \mathrm{tr}(F^2)(x) \,|\, 0\rangle$$



L loops	L=1	L=2	L=3	L=4
# of cubic graphs	2	6	29	229
# of planar masters	1	2	2	4
# of free parameters	1	4	24	133

G. Lin, GY, S. Zhang, 2021

Non-supersymmetric Yang-Mills

For non-supersymmetric YM, even two-loop is challenging:

- 2-loop 4-gluon all-plus-helicity amplitude in pure YM

$$A_4^{(2)}(1^+,2^+,3^+,4^+)$$

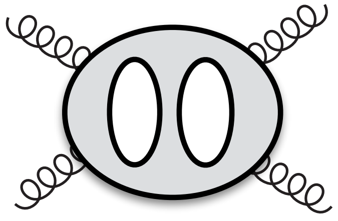
Bern, Davies, Dennen, Huang, Nohle 2013

- 2-loop 5-gluon all-plus-helicity amplitude in pure YM

$$A_5^{(2)}(1^+,2^+,3^+,4^+,5^+)$$

O'Connell and Mogull 2015

No global CK-dual solution is known for generic helicity configurations at two loops.



When difficult to find CK-dual solution

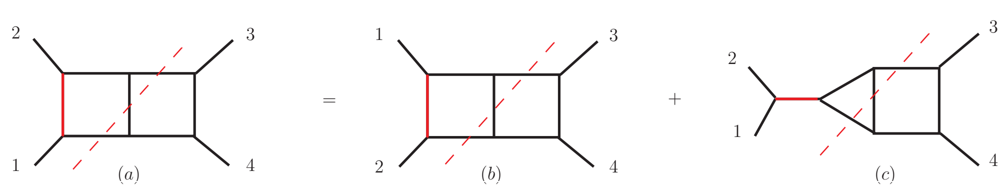
- Enlarge ansatz (e.g. increasing power of loop momenta)

$A_5^{(2)}(1^+,2^+,3^+,4^+,5^+)$ $\boxed{n^{\mathrm{CK}} \sim \ell^{12}}$ O'Connell and Mogull 2015

- Give up global CK relations?

Ansatz is made to all topologies and only imposing CK-duality on cuts.

Bern, Davies and Nohle 2015



$$(n_a - n_b - n_c)\big|_{\mathrm{cut}} = 0$$

Hard to generalize to higher-loop/point cases.

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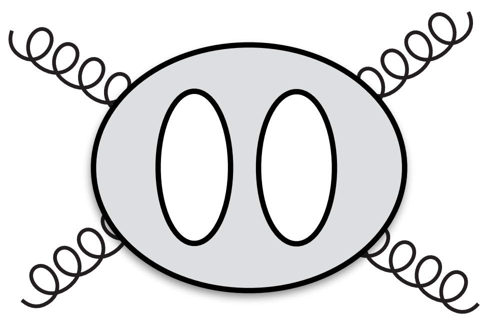
Two-loop 4-gluon amplitude

We introduce a strategy by allowing **"deformation"**.

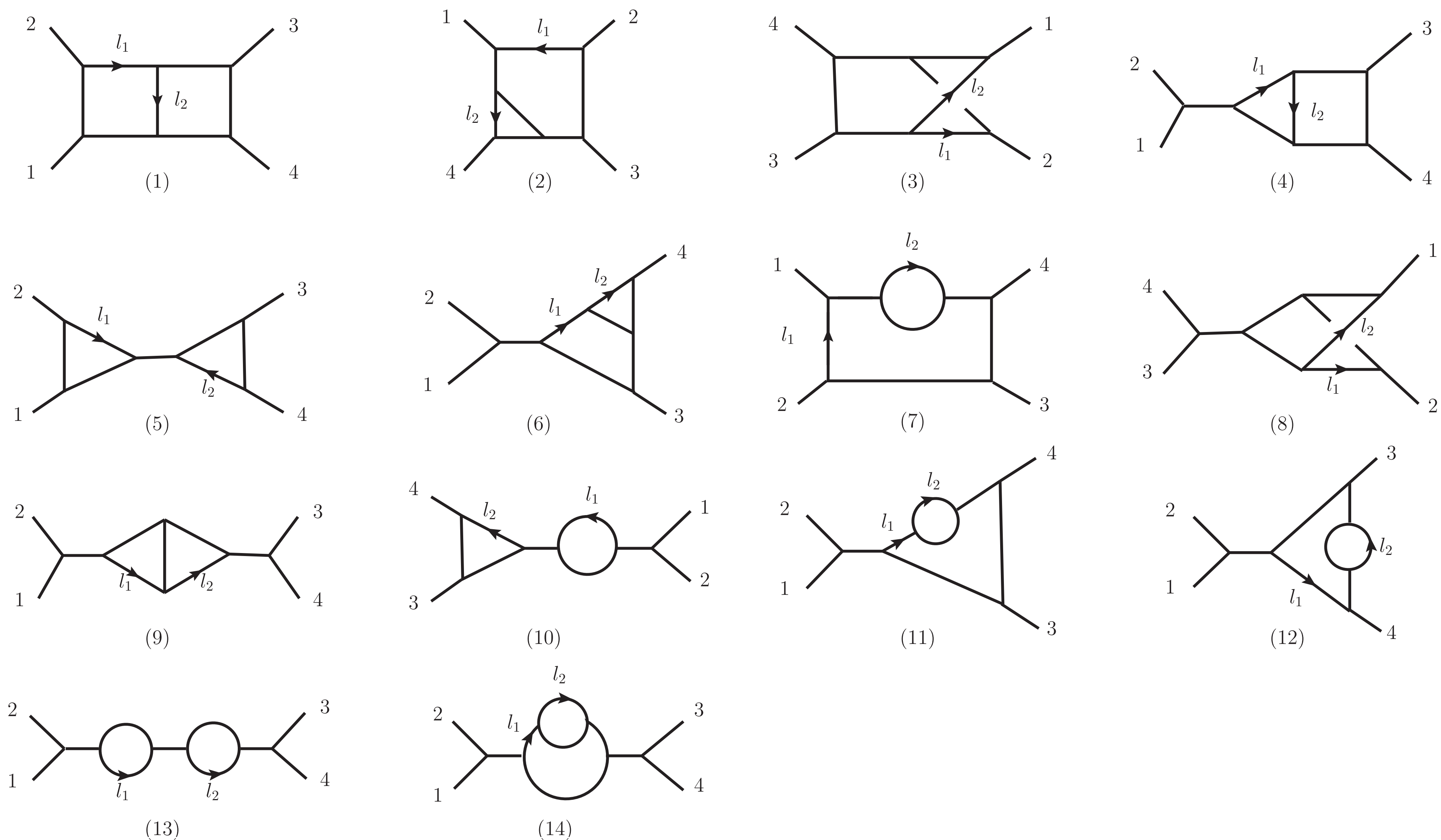


$$N_1 = n_1 + \Delta_1$$

Let us first review the standard construction.

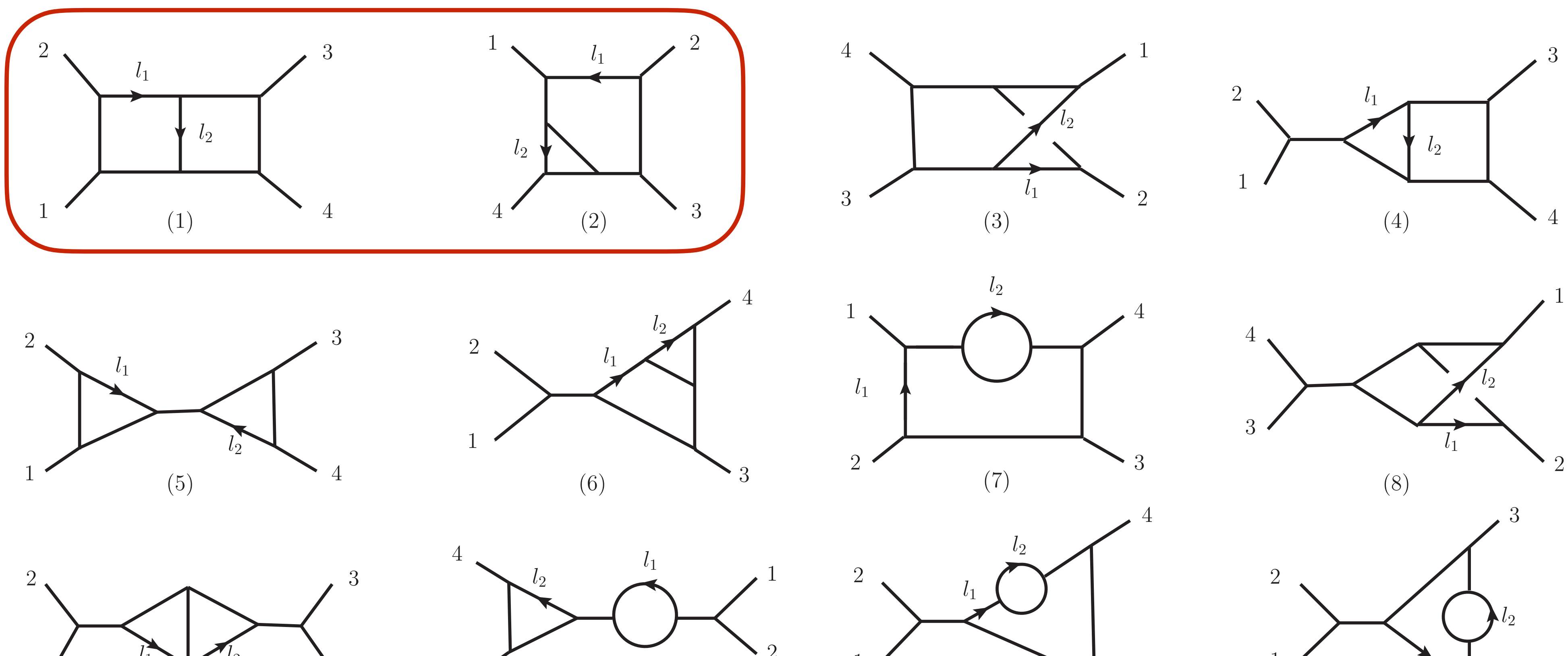
$$N_1 = n_1 + \Delta_1$$

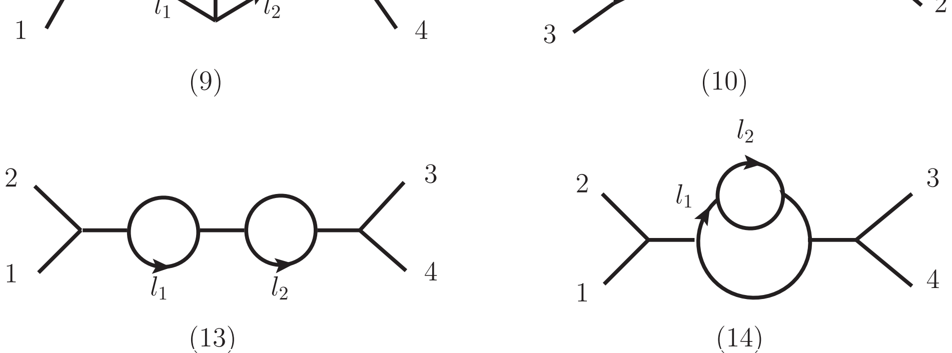
Two-loop trivalent diagrams



Two-loop trivalent diagrams

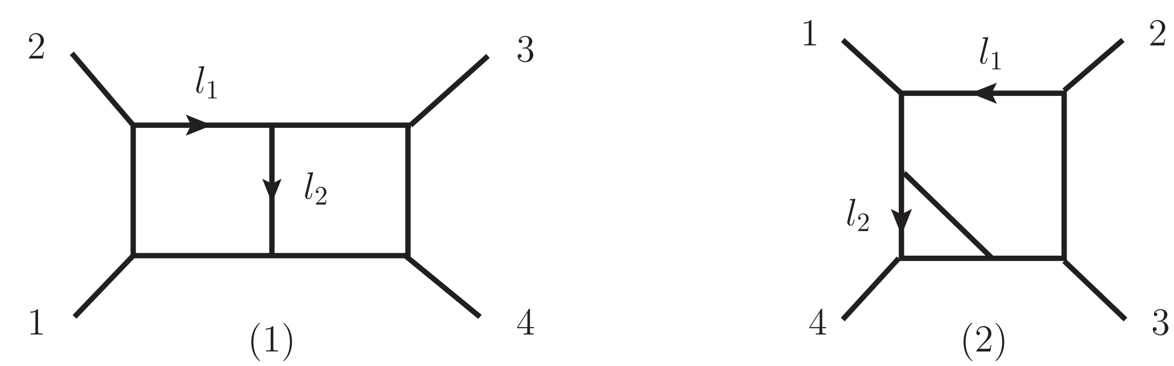
Master topologies





$$
\begin{aligned}
n_3 &= n_1[p_1,p_2,p_3,p_4,-l_2-p_2,l_1-p_2] + n_2[p_4,p_3,p_2,p_1,-l_1-l_2-p_3,-l_2+p_1] \\
n_4 &= n_1 - n_1[p_3,p_4,p_2,p_1,l_1-l_2+p_1+p_2,-l_2] \\
n_5 &= -n_1[p_1,p_2,p_3,p_4,l_1,l_1-l_2+p_1+p_2] + n_1[p_1,p_2,p_4,p_3,l_1,l_1+l_2] \\
n_6 &= n_2[p_1,p_2,p_3,p_4,l_1+p_1,l_2] - n_2[p_3,p_1,p_2,p_4,-l_1-p_1-p_2,-l_2-p_1-p_2-p_3] \\
n_7 &= -n_2[p_1,p_2,p_3,p_4,l_1,l_2] - n_2[p_1,p_2,p_3,p_4,l_1,l_1-l_2-p_1] \\
n_8 &= n_3 - n_3[p_1,p_2,p_4,p_3,l_1,l_2] \\
n_9 &= -n_4[p_1,p_2,p_3,p_4,l_1,l_1-l_2] + n_4[p_1,p_2,p_4,p_3,l_1,l_1-l_2] \\
n_{10} &= -n_4[p_1,p_2,p_3,p_4,l_1,l_1+l_2+p_1+p_2] - n_4[p_1,p_2,p_3,p_4,-l_1-p_1-p_2,-l_1+l_2] \\
n_{11} &= -n_4[p_1,p_2,p_3,p_4,-l_2-p_1-p_2,l_1-l_2] - n_4[p_1,p_2,p_3,p_4,-l_1+l_2-p_1-p_2,l_2] \\
n_{12} &= -n_6[p_1,p_2,p_3,p_4,l_1,l_1-l_2] - n_6[p_1,p_2,p_3,p_4,l_1,l_2-p_1-p_2-p_3] \\
n_{13} &= n_9 + n_9[p_1,p_2,p_3,p_4,-l_1-p_1-p_2,l_2] \\
n_{14} &= n_9[p_1,p_2,p_3,p_4,l_1-l_2,l_1] + n_9[p_1,p_2,p_3,p_4,-l_2-p_1-p_2,-l_1-p_1-p_2]\,,
\end{aligned}
$$

Ansatz for the master numerators



Polynomials in D-dim kinematics:

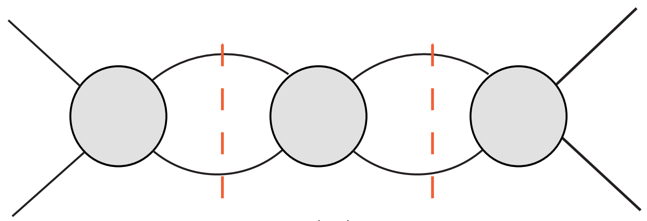
$$\boxed{n_m = \sum_k a_{mk} M_k \,, \qquad m = 1, 2\,,}$$

$$\{\varepsilon_i \cdot \varepsilon_j,\ \varepsilon_i \cdot p_j,\ \varepsilon_i \cdot l_\alpha,\ p_i \cdot l_\alpha,\ l_\alpha \cdot l_\beta\ , p_1 \cdot p_2\ , p_2 \cdot p_3\}$$

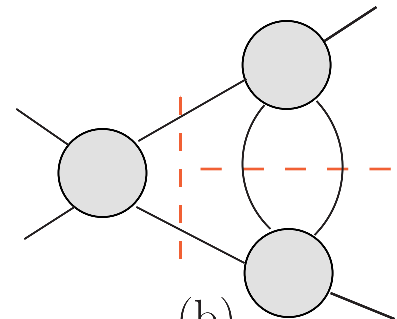
e.g.: $(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \varepsilon_4)(p_1 \cdot p_2)(p_1 \cdot l_1)(p_1 \cdot l_2)$

Parameters: $\sim$ 20,000 $\xrightarrow[\text{Symmetry}]{}$ ~1400

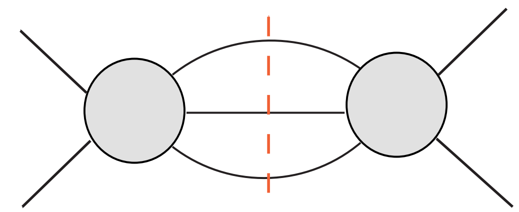
Unitarity cuts



(a)

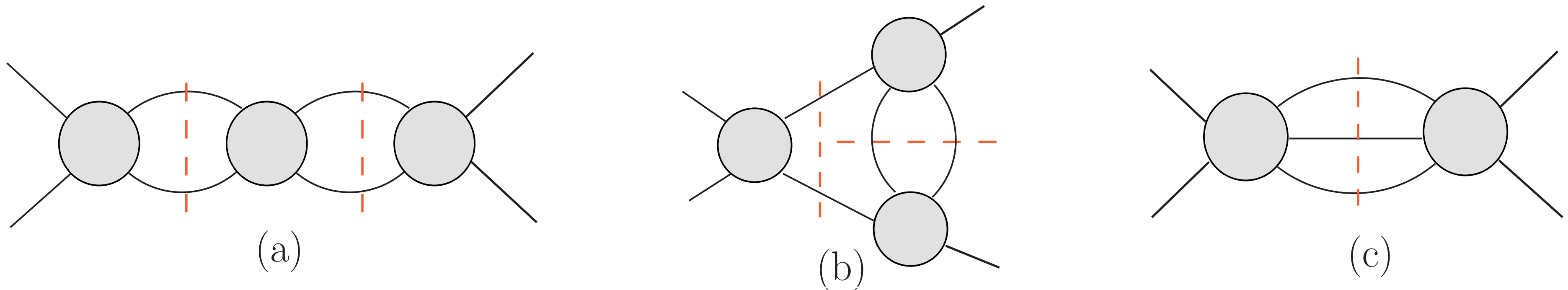


(b)



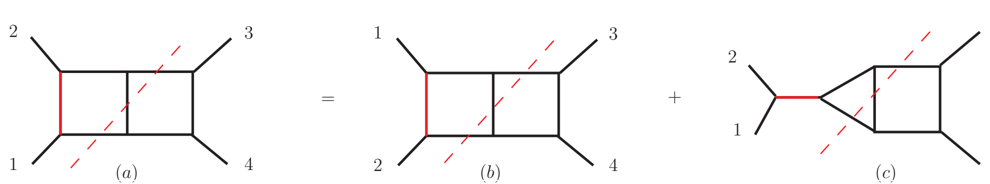
(c)

Unitarity cuts



- Require the CK-duality at cut-level

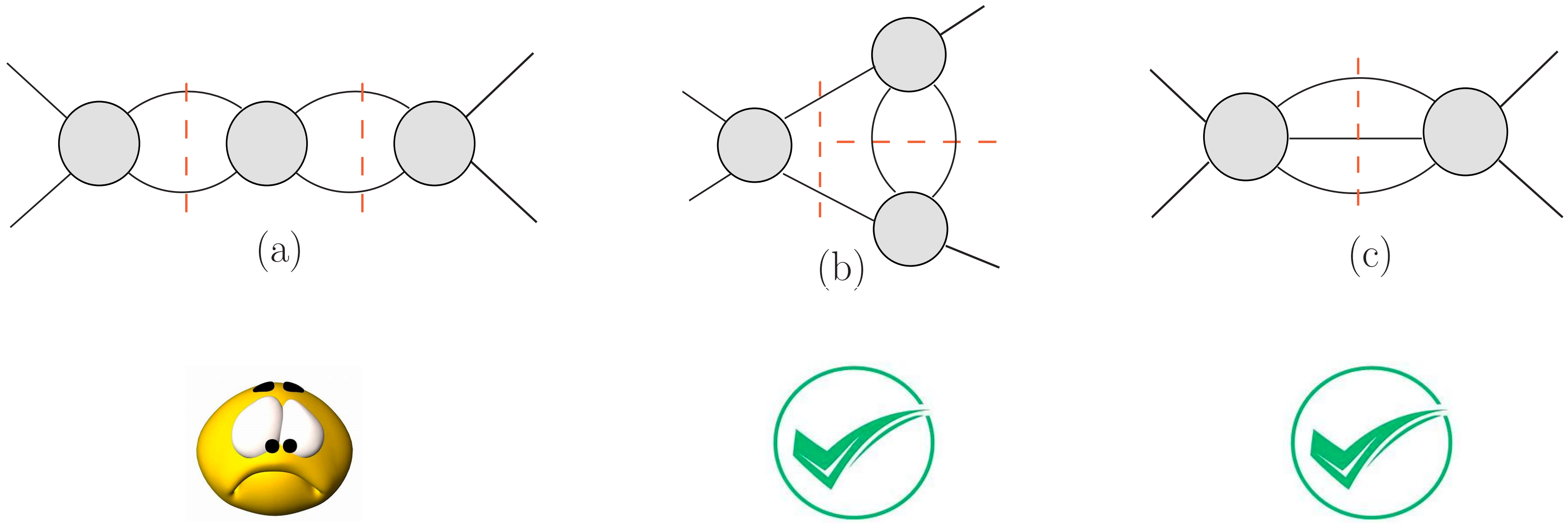
Bern, Davies and Nohle 2015



$$(n_a - n_b - n_c)\big|_{\text{cut}} = 0$$

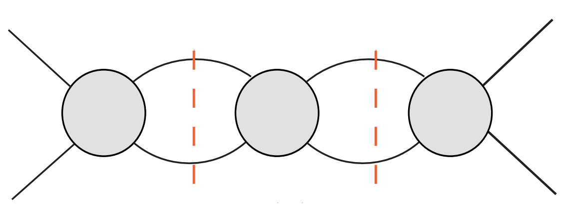
CK-duality only on cuts
initial parameters: ~120,000
after symmetry constraints: ~28,000
after cut and CK constraints: ~6,300

Unitarity cuts

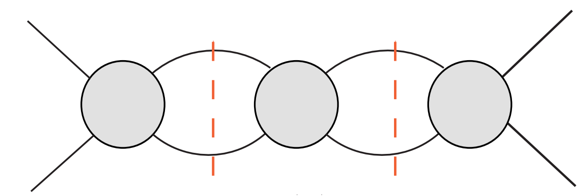


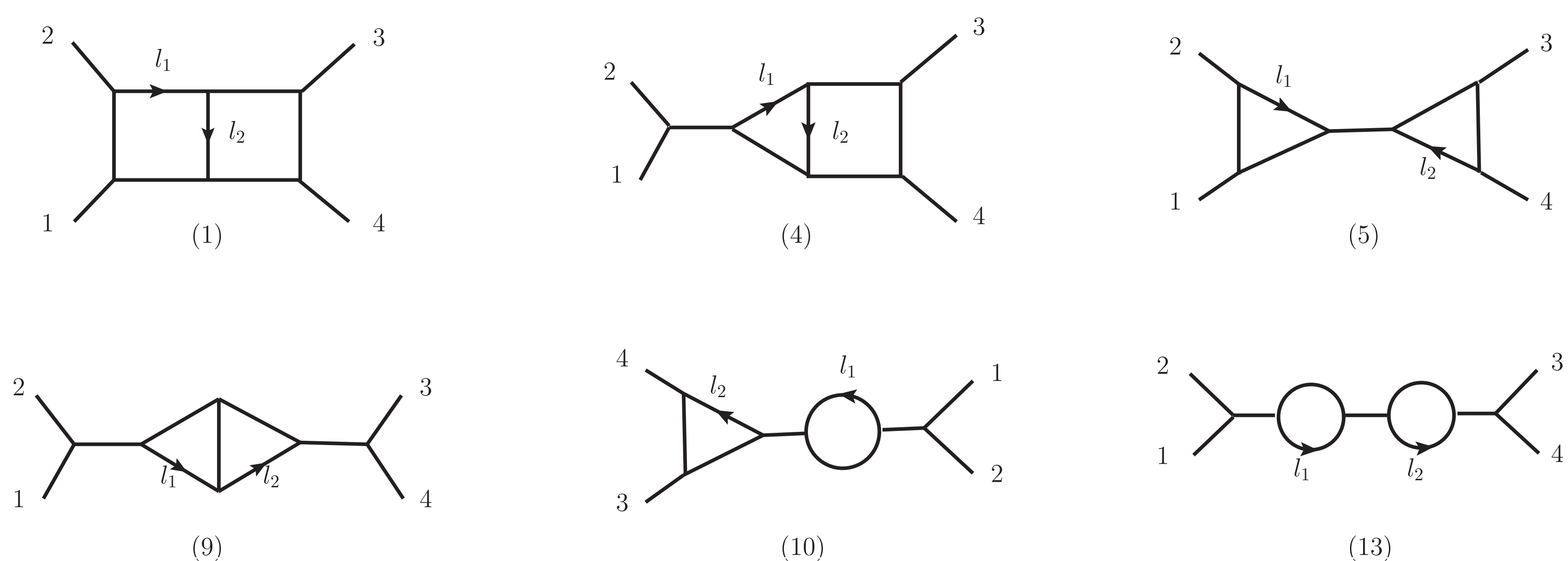
We would like to take advantage of the global CK-dual relations

$$N_i = n_i + \boxed{\Delta_i}$$ Deformation $\longrightarrow$



Deformed trivalent diagrams

Topologies that affect the ladder-double-cut: 



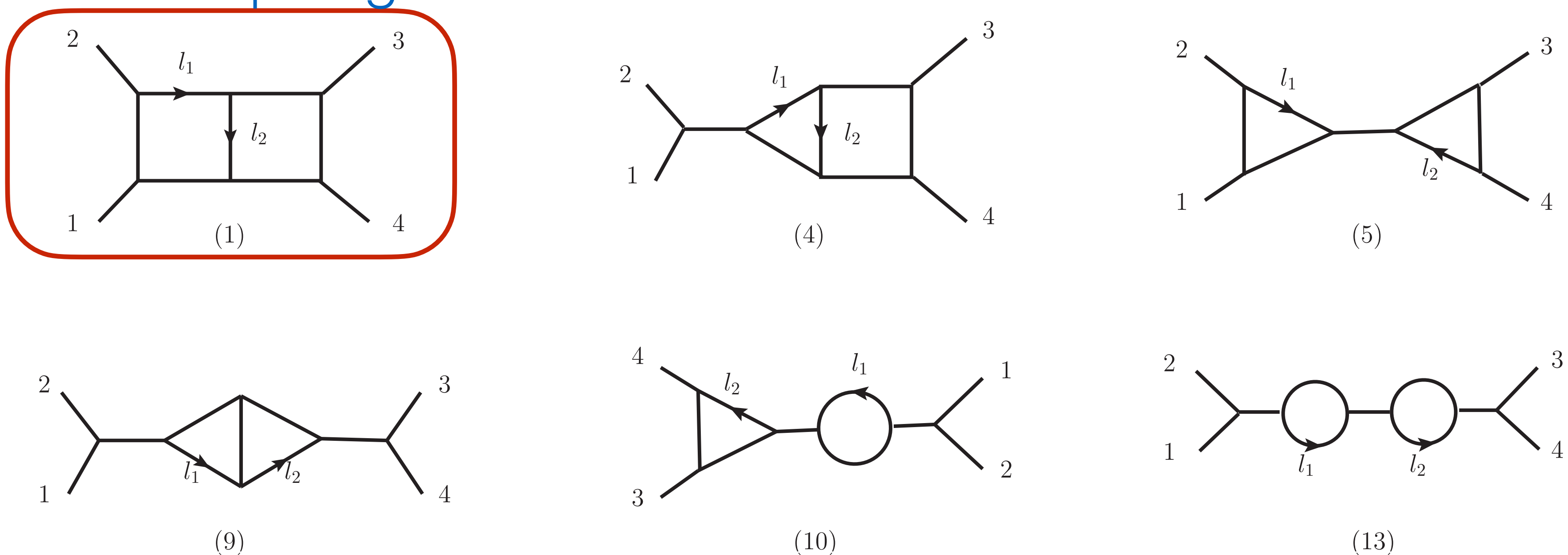
Deformation

$$N_i = \begin{cases} n_i + \boxed{\Delta_i,} & i = 1, 4, 5, 9, 10, 13, \\ n_i, & \text{others.} \end{cases}$$

Deformed numerators

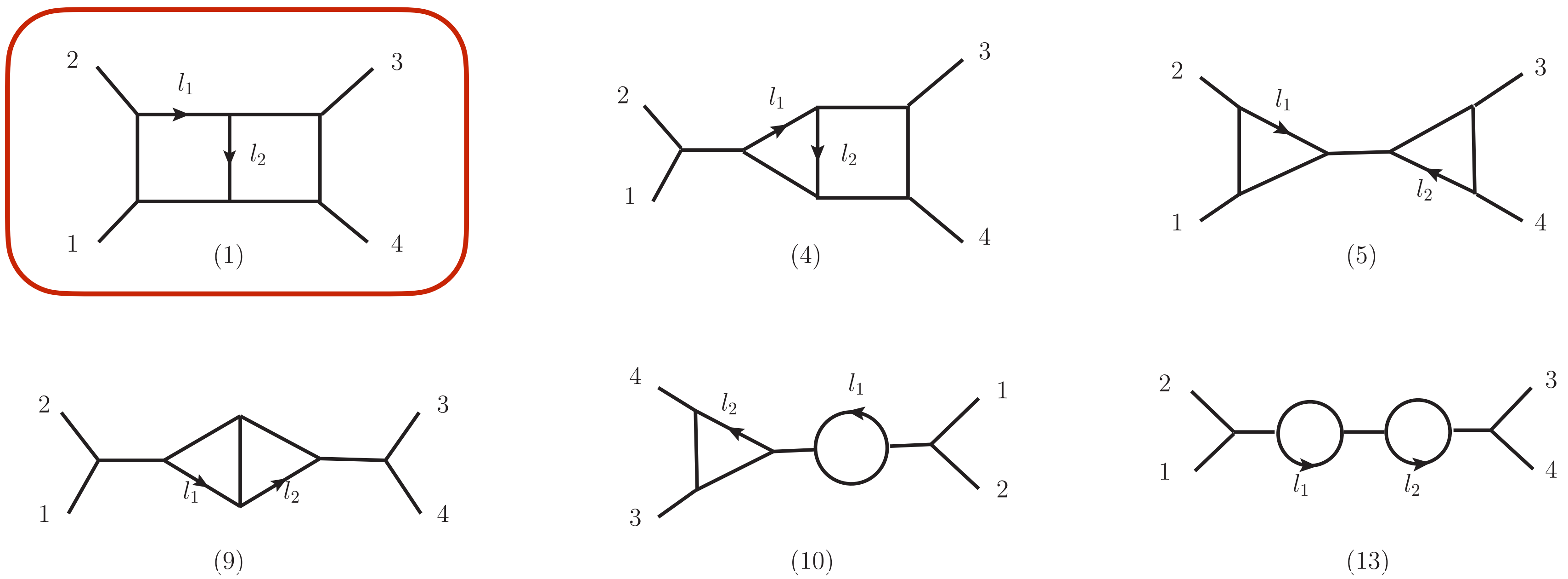
We ask that deformation satisfies a sub-set of dual Jacobi relations.

Master topologies



$$\Delta_4 = \Delta_1 - \Delta_1[p_3, p_4, p_2, p_1, l_1 - l_2 + p_1 + p_2, -l_2]$$
$$\Delta_5 = -\Delta_1[p_1, p_2, p_3, p_4, l_1, l_1 - l_2 + p_1 + p_2] + \Delta_1[p_1, p_2, p_4, p_3, l_1, l_1 + l_2]$$
$$\Delta_9 = -\Delta_4[p_1, p_2, p_3, p_4, l_1, l_1 - l_2] + \Delta_4[p_1, p_2, p_4, p_3, l_1, l_1 - l_2]$$
$$\Delta_{10} = -\Delta_4[p_1, p_2, p_3, p_4, l_1, l_1 + l_2 + p_1 + p_2] - \Delta_4[p_1, p_2, p_3, p_4, -l_1 - p_1 - p_2, -l_1 + l_2]$$
$$\Delta_{13} = \Delta_9 + \Delta_9[p_1, p_2, p_3, p_4, -l_1 - p_1 - p_2, l_2]\,.$$

Ansatz of the master numerator



Consider different Lorentz structure separately:

$$\Delta_i = \Delta_i^{[1]} + \Delta_i^{[2]} + \Delta_i^{[3]}.$$

$$\Delta_1^{[1]} = (\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \varepsilon_4)(\sum_k c_k^{[1]} M_k^{[1]})\, l_2^2$$

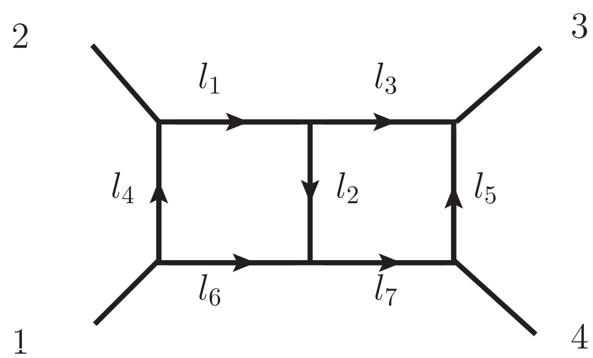
$$\Delta_1^{[2]} = \Big[(\varepsilon_1 \cdot \varepsilon_2)(\sum_a c_a^{[2]} M_{1,a}^{[2]}) + (\varepsilon_3 \cdot \varepsilon_4)(\sum_b c_b^{[2]} M_{2,b}^{[2]})\Big] l_2^2$$

$$\Delta_1^{[3]} = (\sum_k c_k^{[3]} M_k^{[3]}) l_2^2$$

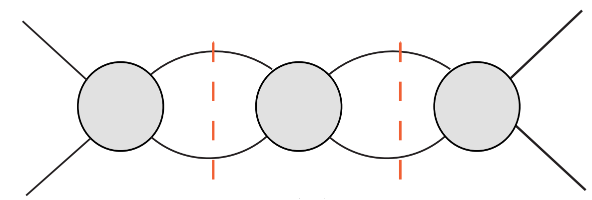
Some requirement:

1) do not affect other cuts,
2) double copy still applicable.

Solving the master numerator



$$N_i = n_i + \Delta_i$$



$$\Delta_1^{[1]} = (d-2)^2(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \varepsilon_4) l_4^2\, l_2^2\, l_5^2$$

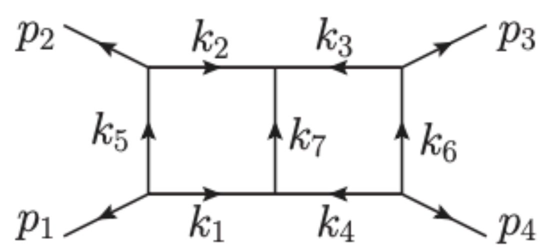
$$\Delta_1^{[2]} = -4(d-2)^2 \Big[(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot l_5)(\varepsilon_4 \cdot l_5) l_4^2 + (\varepsilon_3 \cdot \varepsilon_4)(\varepsilon_1 \cdot l_4)(\varepsilon_2 \cdot l_4) l_5^2\Big] l_2^2$$

Deformation

Solution for the master numerator:

(There is a solution space with free parameters, here is a special simple choice.)

$\Delta \Big[$



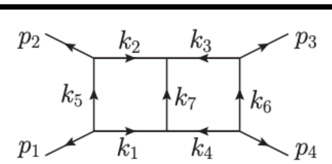
$\Big] \Big/ k_7^2 =$

$$
\begin{aligned}
&+ (d-2)^2 \Big\{ (\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \varepsilon_4) k_5^2\, k_6^2 + 16 (\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) \\
&\qquad - 4 \Big[(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) k_5^2 + (\varepsilon_3 \cdot \varepsilon_4)(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5) k_6^2 \Big] \Big\} \\
&+ (d-2) 4 \Big\{ -10 \Big[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_5) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3) \Big] \\
&\qquad + 20 \Big[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_5) \Big] \\
&\qquad + 32 \Big[(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot p_1)(\varepsilon_4 \cdot p_2) + (\varepsilon_1 \cdot p_3)(\varepsilon_2 \cdot p_4)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) \Big] \\
&\qquad + 47 \Big[(\varepsilon_1 \cdot k_4)(\varepsilon_2 \cdot k_3)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_2)(\varepsilon_4 \cdot k_1) \Big] \Big\},
\end{aligned}
$$

Deformation

The simplicity of the deformation: $N_1 = n_1 + \Delta_1$

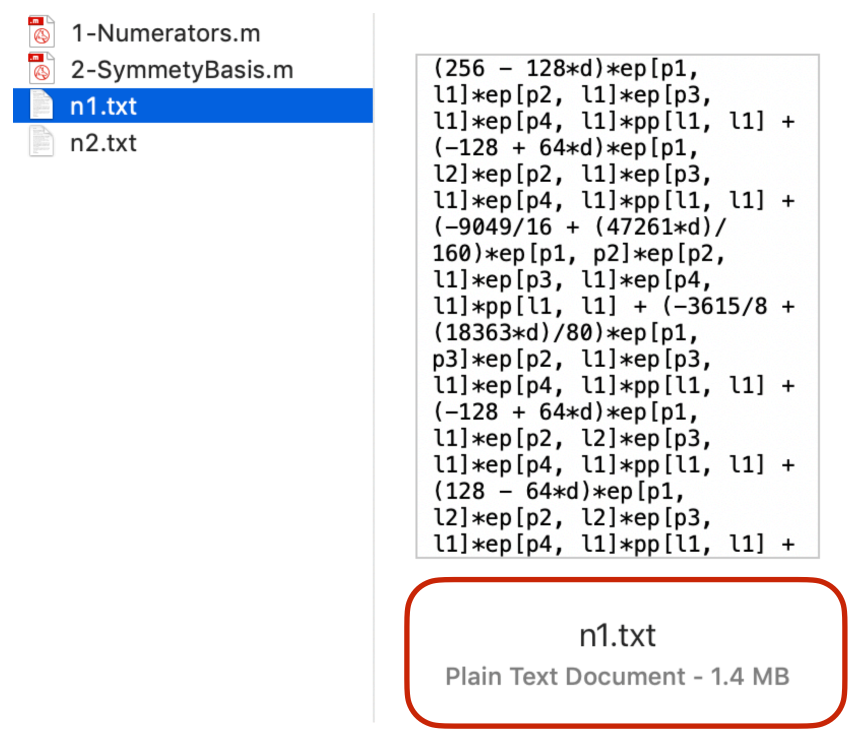
Deformation

Δ 

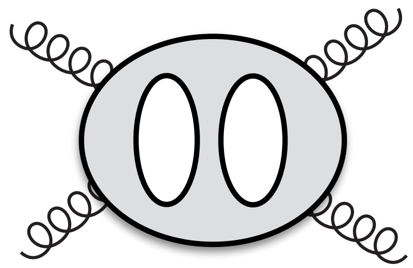
$$
\Delta\left[\ \cdots\ \right] \Big/ k_7^2 = \tag{5.2}
$$

$$
\begin{aligned}
&+ (d-2)^2 \Big\{ (\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \varepsilon_4) k_5^2\, k_6^2 + 16 (\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) \\
&\qquad - 4 \Big[(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) k_5^2 + (\varepsilon_3 \cdot \varepsilon_4)(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5) k_6^2 \Big] \Big\} \\
&+ (d-2) 4 \Big\{ - 10 \Big[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_5) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3) \Big] \\
&\qquad + 20 \Big[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_5) \Big] \\
&\qquad + 32 \Big[(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot p_1)(\varepsilon_4 \cdot p_2) + (\varepsilon_1 \cdot p_3)(\varepsilon_2 \cdot p_4)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) \Big] \\
&\qquad + 47 \Big[(\varepsilon_1 \cdot k_4)(\varepsilon_2 \cdot k_3)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_2)(\varepsilon_4 \cdot k_1) \Big] \Big\},
\end{aligned}
$$

Undeformed part

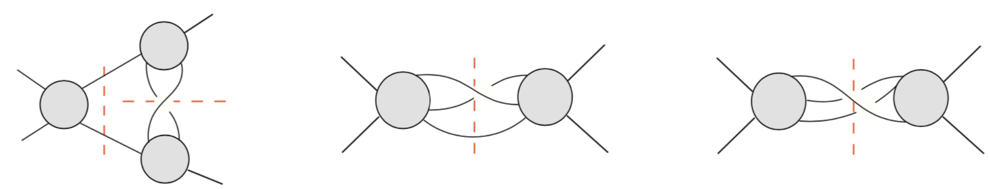


Checks of the solution



$$N_i = \begin{cases} n_i + \Delta_i, & i = 1,4,5,9,10,13, \\ n_i, & \text{others.} \end{cases}$$

- Pass the full set of D-dimensional planar and non-planar cuts



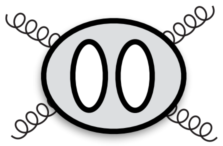
- Satisfy all CK-dual relations on cuts, so double-copy applies
- Free parameters cancel after the integral IBP reduction
- Integrated result satisfies the Catani IR formula

Outline

- Introduction
- Constructing CK-dual numerators
- New strategy of deformation
- Summary and outlook

Summary and outlook

- Gauge and gravity theories are related by double copy.
- The key of double copy is to achieve "color-kinematics duality".
- Finding CK-dual numerators is generally difficult, and introducing "a simple deformation" may solve it.



Bern, Davies and Nohle 2015

CK-duality only on cuts
initial parameters: $\sim$120,000
after symmetry constraints: $\sim$28,000
after cut and CK constraints: $\sim$6,300

$\longrightarrow$

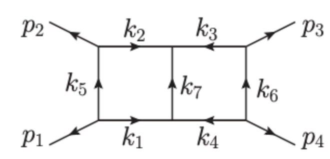
Duality with deformaltion
initial parameters: $\sim$20,000
after CK and symmetry constraints: $\sim$1400
with partial cuts + deformation : $\sim$500
with remaining cut: $\sim$200

A new strategy to apply CK-duality and double-copy.

Summary and outlook

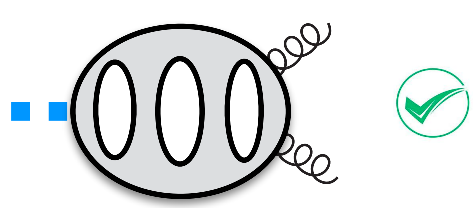
- Why so simple?

$$\Delta \left[\ \right] \Big/ k_7^2 =$$



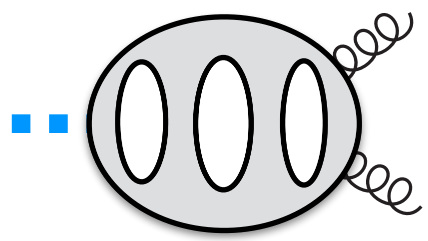
$$\begin{aligned}
&+ (d-2)^2 \Big\{ (\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \varepsilon_4) k_5^2\, k_6^2 + 16 (\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) \\
&\qquad - 4 \Big[(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) k_5^2 + (\varepsilon_3 \cdot \varepsilon_4)(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5) k_6^2 \Big] \Big\} \\
&+ (d-2) 4 \Big\{ -10 \Big[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_5) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3) \Big] \\
&\qquad + 20 \Big[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_5) \Big] \\
&\qquad + 32 \Big[(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot p_1)(\varepsilon_4 \cdot p_2) + (\varepsilon_1 \cdot p_3)(\varepsilon_2 \cdot p_4)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) \Big] \\
&\qquad + 47 \Big[(\varepsilon_1 \cdot k_4)(\varepsilon_2 \cdot k_3)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_2)(\varepsilon_4 \cdot k_1) \Big] \Big\},
\end{aligned}$$

- More examples: higher loop cases?

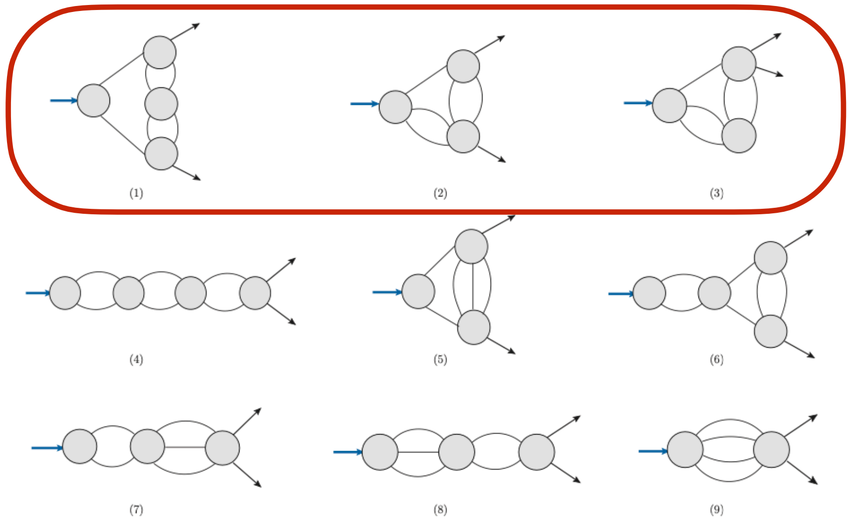


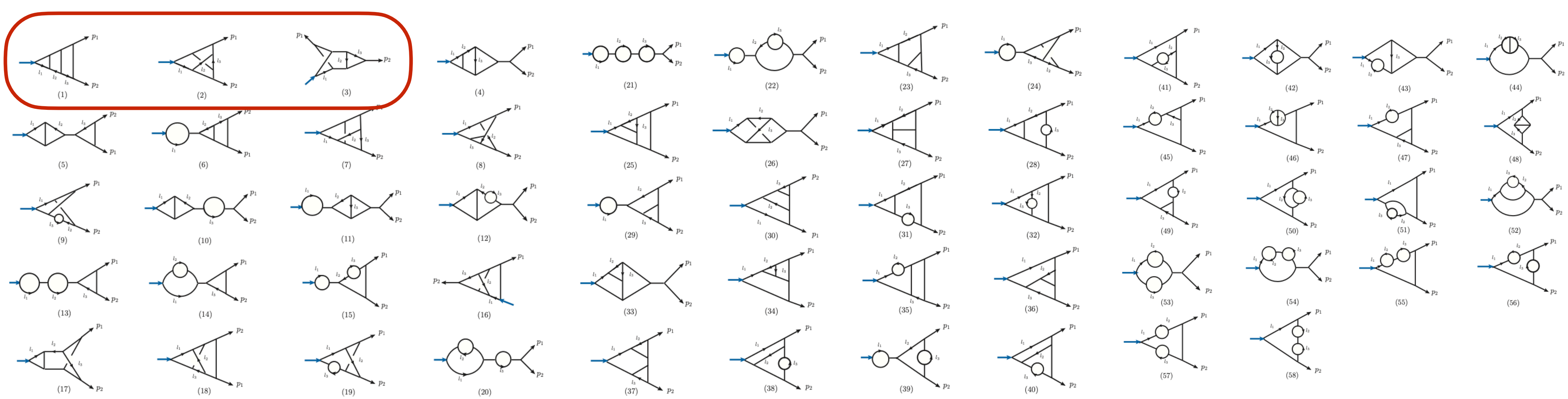
Zeyu Li, GY, Guorui Zhu in preparation

Summary and outlook



Zeyu Li, GY, Guorui Zhu in preparation

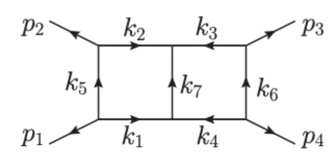




Summary and outlook

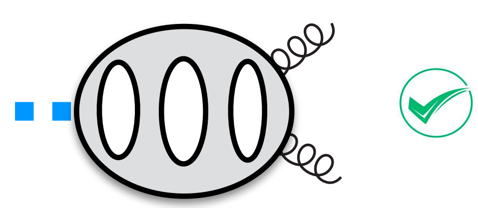
- Why so simple?

$$\Delta \Big[\quad \Big] \Big/ k_7^2 =$$

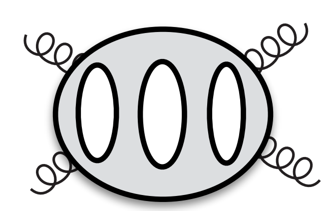


$$\begin{aligned}
&+ (d-2)^2 \Big\{ (\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \varepsilon_4) k_5^2\, k_6^2 + 16 (\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) \\
&\qquad - 4 \Big[(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) k_5^2 + (\varepsilon_3 \cdot \varepsilon_4)(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5) k_6^2 \Big] \Big\} \\
&+ (d-2) 4 \Big\{ -10 \Big[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_5) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3) \Big] \\
&\qquad + 20 \Big[(\varepsilon_1 \cdot k_6)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_5)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_6)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_5) \Big] \\
&\qquad + 32 \Big[(\varepsilon_1 \cdot k_5)(\varepsilon_2 \cdot k_5)(\varepsilon_3 \cdot p_1)(\varepsilon_4 \cdot p_2) + (\varepsilon_1 \cdot p_3)(\varepsilon_2 \cdot p_4)(\varepsilon_3 \cdot k_6)(\varepsilon_4 \cdot k_6) \Big] \\
&\qquad + 47 \Big[(\varepsilon_1 \cdot k_4)(\varepsilon_2 \cdot k_3)(\varepsilon_3 \cdot k_4)(\varepsilon_4 \cdot k_3) + (\varepsilon_1 \cdot k_2)(\varepsilon_2 \cdot k_1)(\varepsilon_3 \cdot k_2)(\varepsilon_4 \cdot k_1) \Big] \Big\},
\end{aligned}$$

- More examples: higher loop cases?



Zeyu Li, GY, Guorui Zhu in preparation



Towards 3-loop Einstein gravity?

- Are there underlying structures for the deformation?

Thank you for your attention!