# Page time as transition of information channels: Information retrieval from radiating black holes

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# Foreword

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This talk is focused the information recoverability based on the final-state projection and non-isometric mapping proposals and show the emergence of Page time as a transition of information channels in this setup.

# Background

The von Neumann entropy of a system $A$ is

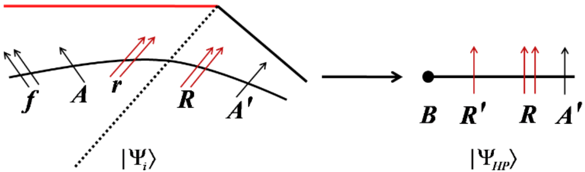
$$S_A = -\mathrm{tr}\rho_A \log \rho_A = -\mathrm{tr}\left((\mathcal{U}\rho_A\mathcal{U}^\dagger)\log(\mathcal{U}\rho_A\mathcal{U}^\dagger)\right) , \qquad (1)$$

which is invariant under the unitary transformation $\mathcal{U}$ on A.

- This shows that the **fine-grained entropy** is conserved for a closed system. Conservation of Information.
- This is the fundamental law of QM, which means that the difference between states always exists and cannot disappear (Liouville Thrm).

# Background

- Two descriptions of black holes.

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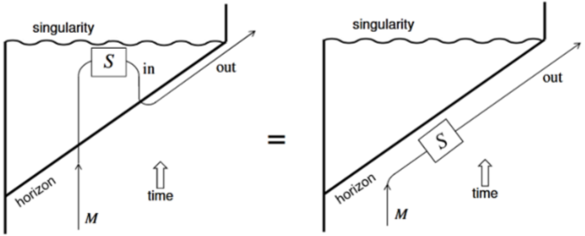
- At late times, the number of modes inside the black hole exceeds the holographic bounds.
- To be consistent with the fundamental picture, the modes in the interior of a black hole has to be reduced in the effective field theory description.

## Dynamical process: Horowitz-Maldacena model.

Black hole in pure state $|\psi\rangle_M \in H_M$, initial Unruh vacuum fluctuation $|U\rangle_{in\times out} = \sum_i \frac{1}{\sqrt{N}}|i\rangle_{in}|i\rangle_{out} \in H_{in}\otimes H_{out}$.

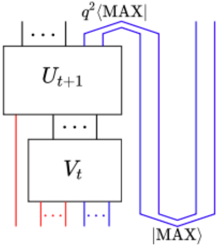
The final state boundary condition is the maximally entangled state in $H_M \otimes H_{in}$ at the singularity, namely $\langle BH| =_{M\otimes in} \langle U|(S\otimes I) = \frac{1}{N}\sum_{m,i} S_{m,i}\langle m|_M\langle i|_{in}$, where $S$ is unitary. This supports information escape.




However, information can escape only if $\langle BH|$ is a maximally entangled state. Information will be completely lost if it is a product state. The interactions between the infalling states with matters will cause departure from unitarity and information loss [Gottesman,Preskill;Loyd;Lee,Yoem].

## Non-isometric encoding: AEHPV model.

The mapping from the fundamental (code space) to the effective field theory descriptions is $V : \mathcal{H}_l \otimes \mathcal{H}_r \to \mathcal{H}_B$.




The mapping is nonunitary and non-isometric. However, it can be shown that the map $V$ on average preserves the inner product [Akers,Engelhardt,Harlow,Penington,Vardhan 2022]

$$\int dU \langle\psi_2|V^\dagger V|\psi_1\rangle = \langle\psi_1|\psi_1\rangle$$

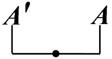
The dynamical mapping realized to the HM model as a quantum teleportation. This mapping gives the island formula for black hole entropy.

## Progress of questions on black hole information

- Is fine-grained entropy conserved?
  - Page curve, Island formula [Penington,Shenker,Stanford,Yang;Engelhardt,Wall,et al.]
- Can information fallen into a black hole be recovered?
  - Unitarity [Penrose,Bousso,Engelhardt, et al.]
- When can information escape?
  - “Black holes as mirrors”—Hayden-Preskill gedanken experiment (based on fundamental picture) [Hayden,Preskill 2007]
- How can it be recovered?
  - Yoshida-Kitaev decoding [Yoshida,Kitaev,Yao]

Assuming the unitary dynamics inside the black hole is known, it is clear that information dropped into a black hole can be recovered from its radiation. Decoding strategy was proposed [Yoshida,Kitaev 2017;Yoshida,Yao 2018].

# Decoding in post-selection model

- What if non-unitary processes are included inside the black hole?
- We study a dynamical post-selection model in the effective description for a radiating black hole and the viability of information retrieval. This is inspired by HM and AEHPV models.
- A dual interpretation as non-unitary dynamics in the effective picture and non-isometric map to the fundamental picture.
- Computations are done using graph representation, where 

represents the $\mathrm{EPR}$ state of $A$ and $A'$ and the black dot stands for the normalization factor $\frac{1}{\sqrt{|A|}}$. Tracing the degrees of freedom is represented by the loops.

# Model

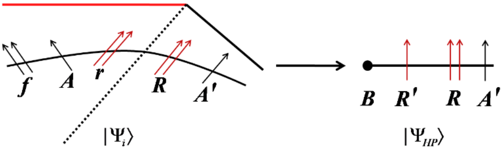
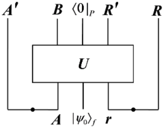



Figure: The effective modes inside the black hole are annihilated.

$$|\Psi_i\rangle = |\mathrm{EPR}\rangle_{A'A} \otimes |\psi_0\rangle_f \otimes |\mathrm{EPR}\rangle_{rR}$$

$$|\Psi_{\mathrm{HP}}\rangle = \sqrt{|P|}\langle 0|_P \left(I_{A'} \otimes U_{(Afr)(BPR')} \otimes I_R\right) |\Psi_i\rangle = \sqrt{|P|}$$

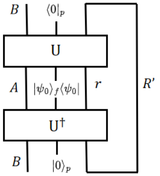


## Merits of the model: Entropy

The $n$-th Renyi entropy is related to the density matrix by

$$S_n(B) = -\frac{1}{n-1} \log \mathrm{Tr}\left(\rho_B^n\right) \, .$$

$$\rho_B \equiv |\Psi_B\rangle\langle\Psi_B| = \frac{|P|}{|R||A|} \left( \quad \right) .$$




Invoking the integration formula for Haar random unitary matrices, we can calculate the $S_n$ and take the $n = 1$ limit,

$$S_{vN} \simeq \min\left(\log|B| + \bar{S}(R') \, , \bar{S}(A'R)\right) \, .$$

The overbar means the entropy calculated in the effective description.

$\rightarrow$ It computes the black hole entropy consistent with the island formula.

# Decoding from Hawking radiation

- Decoupling condition:

  the entangled partner $A'$ of the in-fallen qudits $A$ decouples from the black hole $B$.

$$\left(\int dU \|\rho_{A'B} - \frac{1}{|A'||B|} I_{A'} \otimes I_B\|_1\right)^2 \ll 1$$

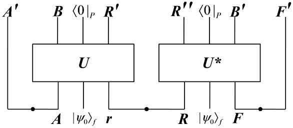
$$\Rightarrow |R'| \gg \sqrt{\frac{|f|}{|P|}}|A| \Leftrightarrow |R'| \gg \frac{|B|}{|R|}|A|$$

The Page time is approximately $|B| = |R|$. Significantly more qudits than $A$ is needed before Page time.

# Decoding from Hawking radiation

- Decoding strategy (similar to Hayden-Preskill)

$$|\Psi\rangle_{in} = C$$

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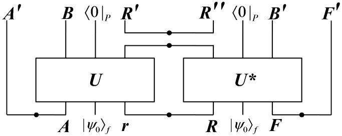
where the normalization factor $C = \min\left(1, \sqrt{\frac{|f|}{|P|}}|A|\right)|P|$

$$|\Psi\rangle_{out} = \left(I_{A'B} \otimes \Pi_{R'R''} \otimes I_{B'F'}\right)|\Psi\rangle_{in}$$

$$= \frac{C}{\sqrt{P_{\mathrm{EPR}}}}$$

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The projection probability and the fidelity of the decoding are,

$$P_{\mathrm{EPR}} \simeq \min\left(1, \frac{|P|}{|f|}\frac{1}{|A|^2}\right),$$

$$F_{\mathrm{EPR}} = \mathrm{Tr}\left(\Pi_{A'F'}|\Psi\rangle_{out\ out}\langle\Psi|\right) \simeq 1\,.$$

Remark: $\Pi_{R'R''}$ decouples the system $F'$ from $B$ and $B'$, and swap the entanglement in $AA'$ to that in $A'F'$. At late times, the decoupling condition releases the requirement for $R'$, the EPR projection of $R'$ and $R''$ has the probability of unity.

# Phase transition

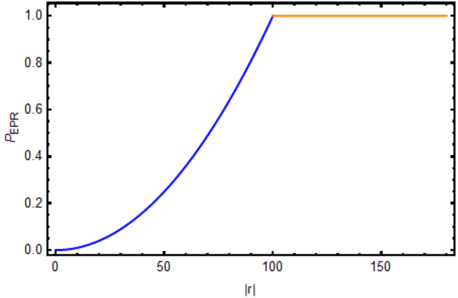



Figure: The probability of the EPR projection $P_{\mathrm{EPR}}$ varies with the dimension of early radiation $|r|$. The parameters are: $|f| = 10^4\,, |A| = 1$. As shown, the transition emerges at the Page time defined by $|r| = \sqrt{|f|} = 100$.

# Schematics: Transition of information channels

The flow of information through two different channels before and after Page time:

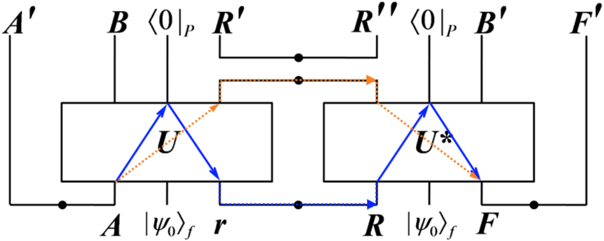



Figure: The information initially stored in the entanglement between $A$ and $A'$ is transmitted to the entanglement between $A'$ and $F'$.

# Information channels

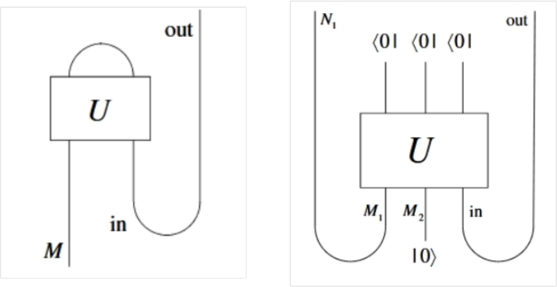



Figure: Lloyd-Preskill generic final state [Lloyd,Preskill]

$$|\mathcal{H}_{M_1}| \ll |\mathcal{H}_{M_1} \otimes \mathcal{H}_{M_2}| \qquad (2)$$

# Quantum computer simulations: EPR projection

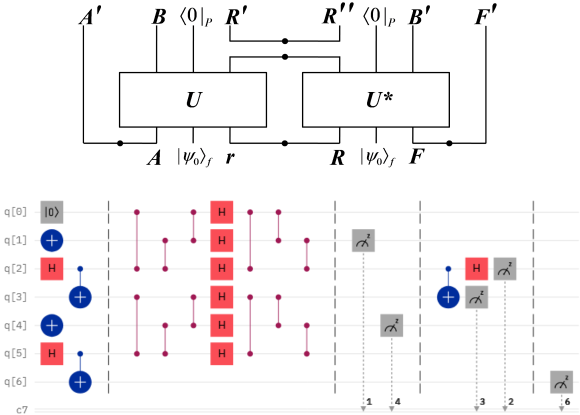



Figure: Quantum circuit for the probabilistic decoding strategy on the IBM quantum processor.

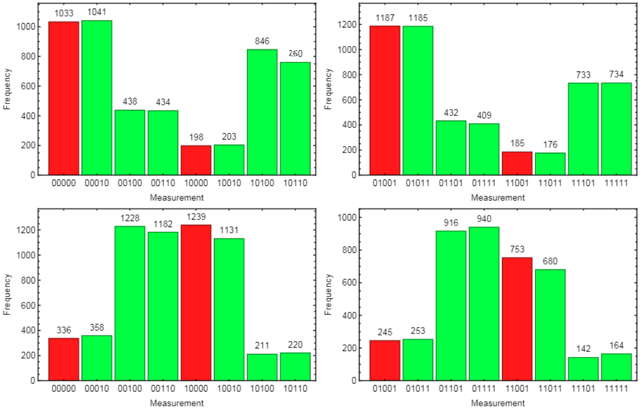
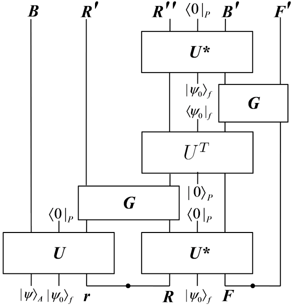
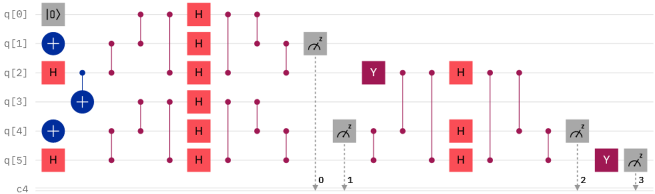


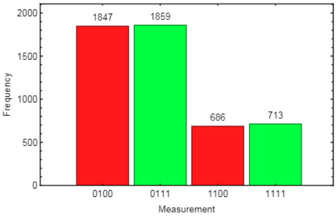

Figure: The red bars represent that the qubits $\mathrm{q}[2]\mathrm{q}[3]$ are successfully projected to the $\mathrm{EPR}$ state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, and the green bars represent that the qubits $\mathrm{q}[2]\mathrm{q}[3]$ are projected to the other incorrect $\mathrm{EPR}$ states.

# Quantum computer simulations: Grover's search

One can eliminate the EPR projection probability through Grover's search method:





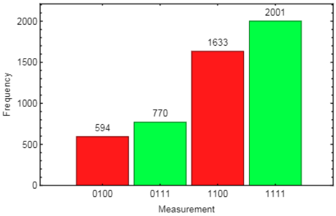




Figure: Experimental results of Grover's search decoding algorithm on the IBM-perth for 20,000 shots. Left: initial input $\mathrm{q}[0] = |0\rangle$. Right: initial input $\mathrm{q}[0] = |1\rangle$.

Correct decoding efficiency:
$\sim$ 80% for probabilistic decoding,
$\sim$70% for the Grover's search decoding.

# Summary

- We study information recoverability from Hawking radiation in the effective field theory description with local annihilation of redundant modes.
- In this model, states in the black hole interior are locally annihilated instead of through EPR projection, and the information can be recovered at a high fidelity after postselection.
- Information escapes from the black hole through two different channels before and after the Page time.
- Offers a new perspective of the Page transition.
- The viability of a toy model of our analyses is verified on 7-qubit IBM quantum computers through two decoding strategies.
- Simulating SYK models on quantum computers?

# Acknowledgement

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**Thanks for your time!**