S-Confinement of 3D Argyres-Douglas Theories and Seiberg-like Dualties

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- Introduction
- Part I: 3D Reduction of $D_p[SU(N)]$ Argyres-Douglas Theories and Confinement
- Part II: Revisit Dualities for Adjoint SQCD
- Conclusion

Based on CH, Sungjoon Kim, “S-confinement of 3d Argyres-Douglas theories and the Seiberg-like duality with an adjoint,” arXiv:2407.XXXX.

What kinds of theories are confining?

Confinement of Supersymmetric Models

- Example I

4d $\mathcal{N} = 1$ SQCD w/ $G = SU(2) + 4\,(Q, \tilde{Q})$

$W = 0$

↓

SCFT

Confinement of Supersymmetric Models

- Example I

4d $\mathcal{N} = 1$ SQCD w/ $G = SU(2) + 4\,(Q, \tilde{Q})$

$$W = 0$$

Seiberg 94

$SU(2) + 4\,(Q, \tilde{Q}) + M_{ij}$

$$W = M_{ij} \tilde{Q}_j Q_i$$

SCFT

Electric-magnetic duality (Seiberg)

$$SU(2) \; + \; 4\,(Q, \tilde{Q})$$

$$W \; = \; 0$$

$$\tilde{Q}_i Q_j\,, \quad Q_i Q_j\,, \quad \tilde{Q}_i \tilde{Q}_j$$

Electric-magnetic duality

$\Leftrightarrow$

$$SU(2) \; + \; 4\,(Q, \tilde{Q}) \; + \; M_{ij}$$

$$W \; = \; M_{ij} \tilde{Q}_j Q_i$$

$$M_{ij}\,, \quad \tilde{Q}_i \tilde{Q}_j\,, \quad Q_i Q_j$$

Mass deformation

$$+ \, \Delta W \; = \; m Q_3 \tilde{Q}_4$$

$\downarrow$ Confinement

Free chirals

$$+ \, \Delta W \; = \; m M_{34}$$

$\downarrow$ Higgs mechanism

Free chirals

Confinement of Supersymmetric Models

- Example II

3d $\mathcal{N} = 2$ SQCD w/ $G = U(2) + 4\,(Q, \tilde{Q})$

$W = 0$

↓

SCFT

Confinement of Supersymmetric Models

- Example II

3d $\mathcal{N} = 2$ SQCD w/ $G = U(2) + 4\,(Q, \tilde{Q})$

$$W = 0$$

Aharony 97

$U(2) + 4\,(Q, \tilde{Q}) + M_{ij} + V^{\pm}$

$$W = M_{ij}\tilde{Q}_j Q_i + V^{+}\hat{v}^{+} + V^{-}\hat{v}^{-}$$

SCFT

Aharony duality

$$U(2) \;+\; 4\,(Q, \tilde{Q})$$

$$W \;=\; 0$$

$$\tilde{Q}_i Q_j\,, \quad \hat{V}^{\pm}$$

Aharony duality

$\Longleftrightarrow$

$$U(2) \;+\; 4\,(Q, \tilde{Q}) \;+\; M_{ij} \;+\; V^{\pm}$$

$$W = M_{ij}\tilde{Q}_j Q_i + V^+ \hat{v}^+ + V^- \hat{v}^-$$

$$M_{ij}\,, \quad V^{\pm}$$

Mass deformation

$$+\,\Delta W \;=\; m Q_3 \tilde{Q}_4$$

$\downarrow$ Confinement

Free chirals

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Free chirals

Aharony duality

$$U(2) \ + \ 4\ (Q, \tilde{Q})$$

$$W \ = \ 0$$

$$\tilde{Q}_i Q_j \,, \quad \hat{V}^{\pm}$$

Aharony duality

$\Leftrightarrow$

$$U(2) \ + \ 4\ (Q, \tilde{Q}) \ + \ M_{ij} \ + \ V^{\pm}$$

$$W = M_{ij}\tilde{Q}_j Q_i + V^+ \hat{v}^+ + V^- \hat{v}^-$$

$$M_{ij} \,, \quad V^{\pm}$$

Mass deformation

Benini, Benvenuti, Pasquetti 17

$$+ \, \Delta W \ = \ \hat{V}^+ \ + \ \hat{V}^-$$

↓ (Monopole) Confinement

Free chirals

$$+ \, \Delta W \ = \ V^+ \ + \ V^-$$

↓ (Monopole)
Higgs mechanism

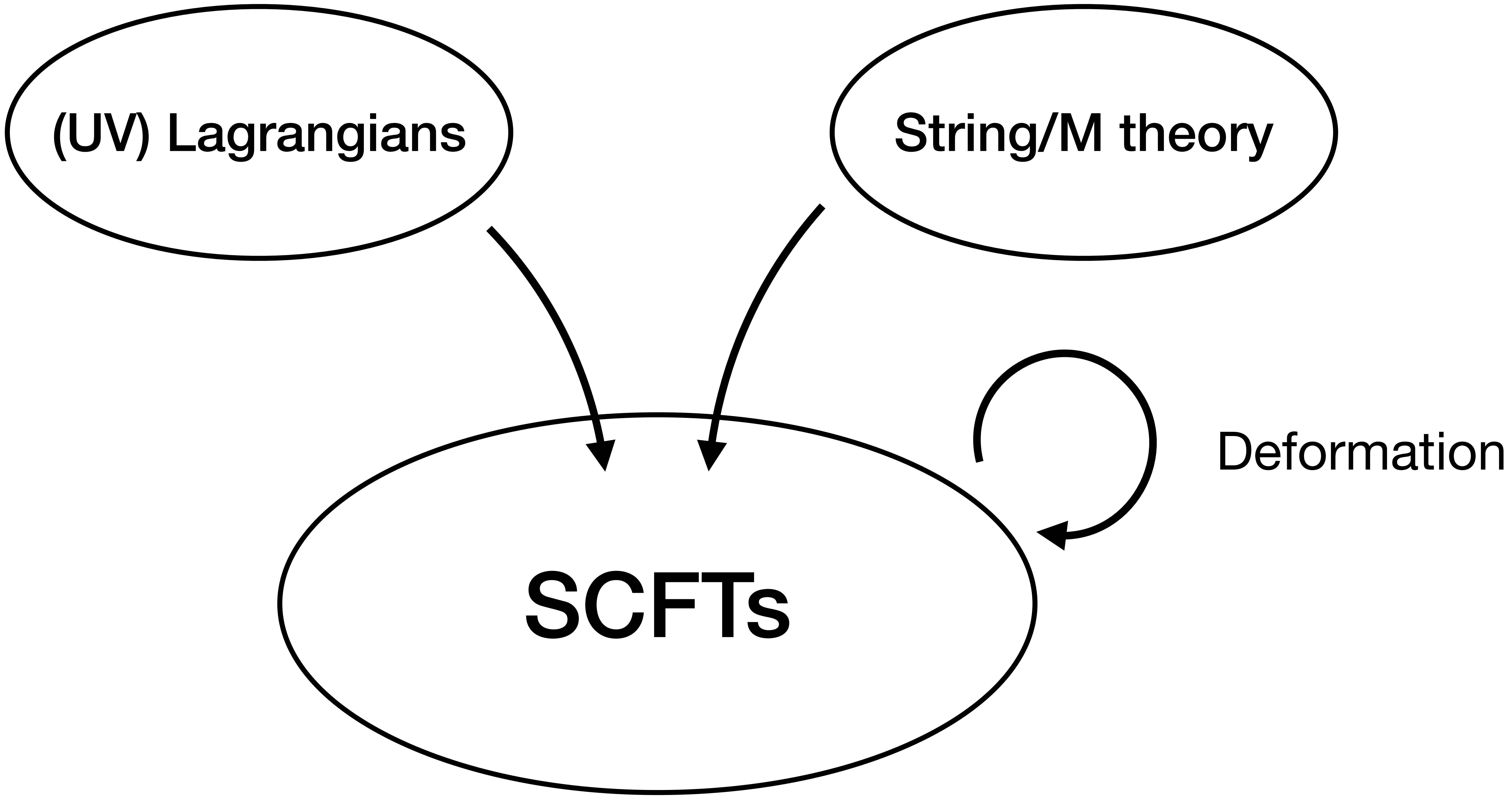
Free chirals

- SQCDs have supersymmetric deformation leading to *confinement* of the theory

- Other cases? E.g., non-Lagrangian theories?

World of Superconformal Field Theories

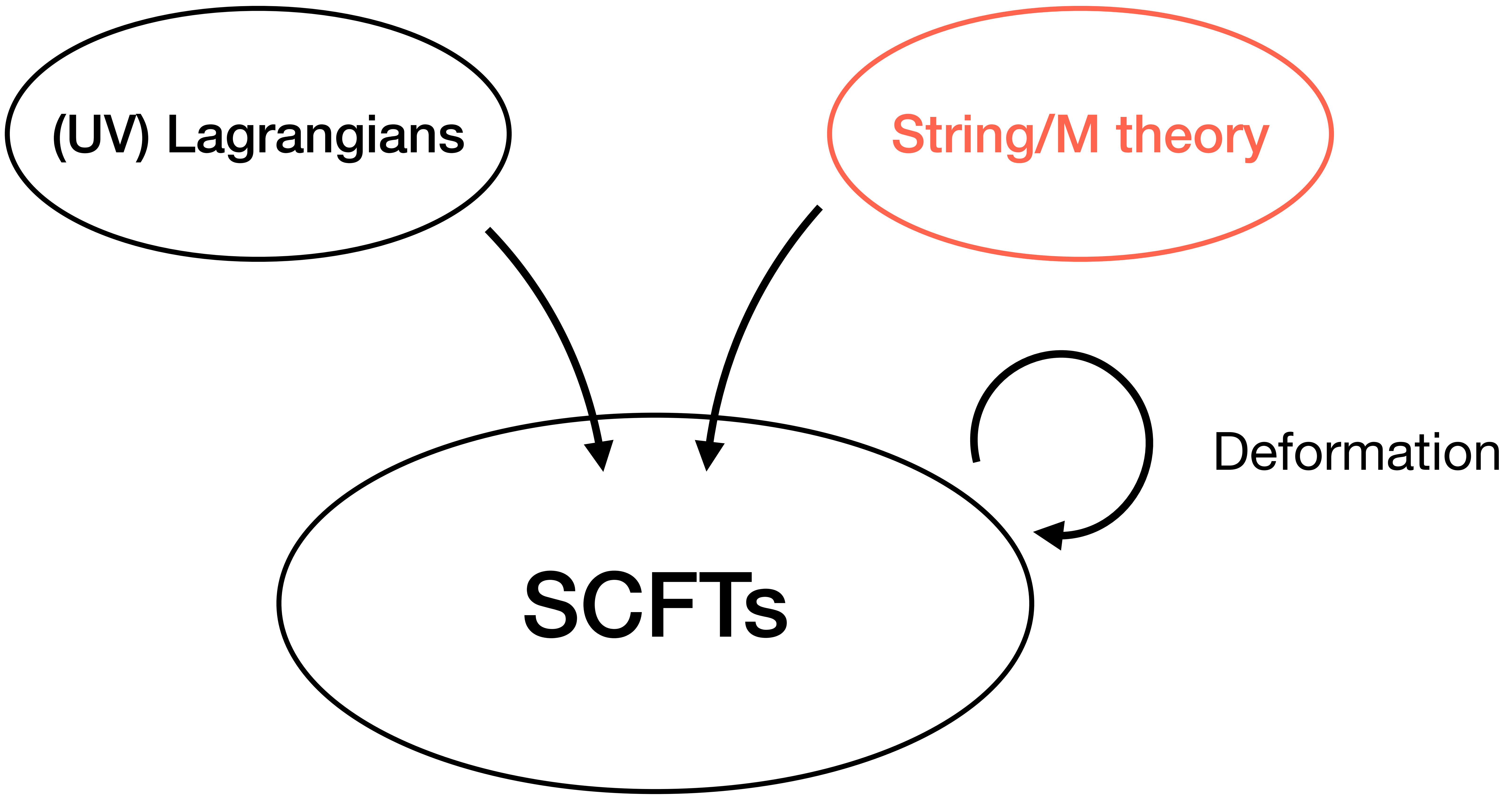
- There are many ways to construct SCFTs

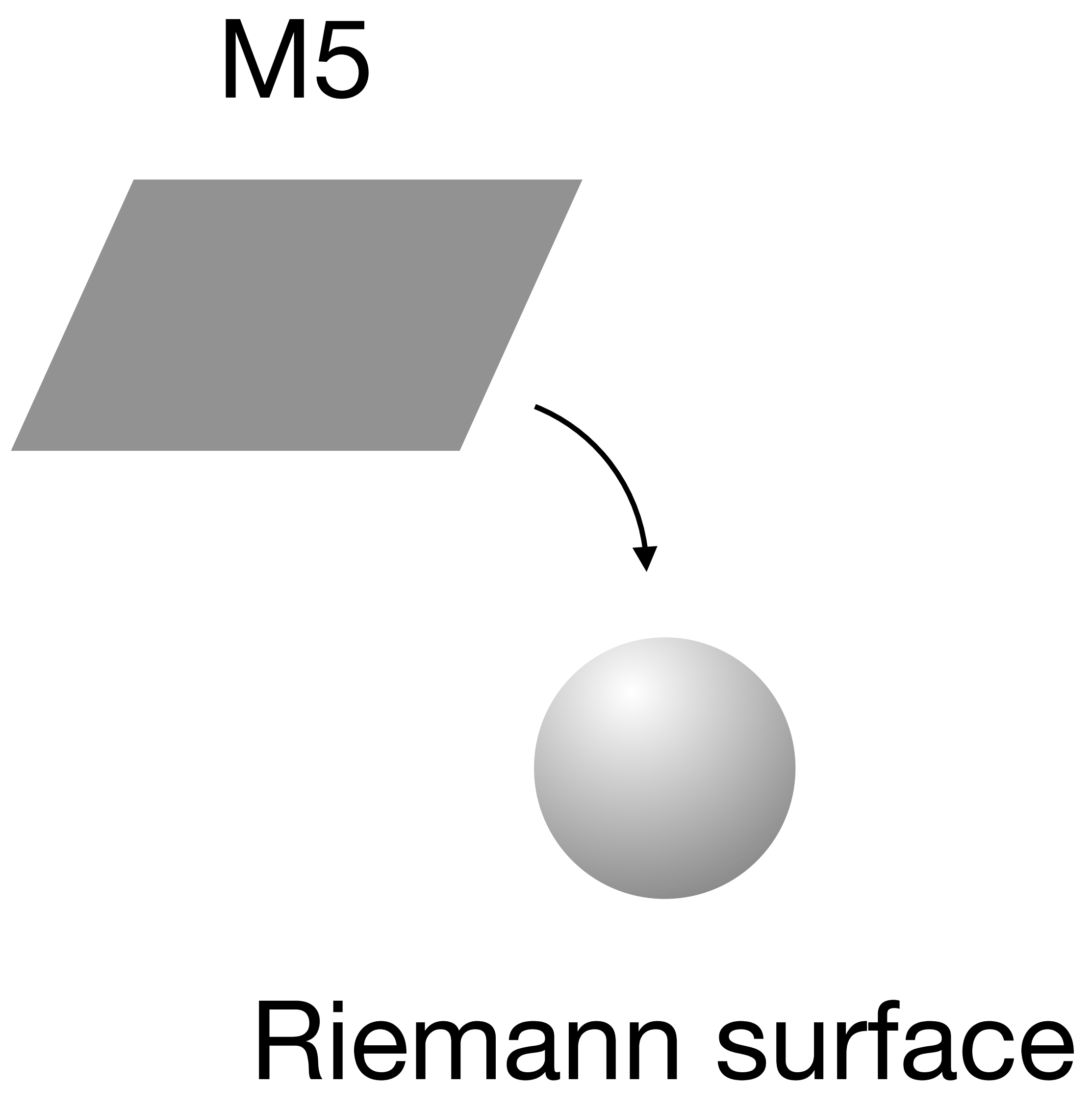


World of Superconformal Field Theories

- There are many ways to construct SCFTs

Class S, geometric engineering, …





➡ 4-dimensional superconformal field theories

For example, one compactified on a Riemann sphere with an irregular singularity: (A_1, A_k) [Dan Xie 12]

A variety of SCFTs have been constructed and

classified by data of the Riemann surface.

Deformation of Argyres-Douglas theories

- Dan Xie, Wenbin Yan 21

- Deformation of (A_1, A_k) by the Coulomb branch operator of the lowest dimension leads to free chirals in the IR.

- The phenomenon persists for other examples; e.g.,

$$(A_1, D_{2k+1}) = D_{2k+1}[SU(2)] \longrightarrow \text{Three free chirals}$$

Compactification on a Riemann sphere with one irregular & one regular singularities

Deformation of Argyres-Douglas theories

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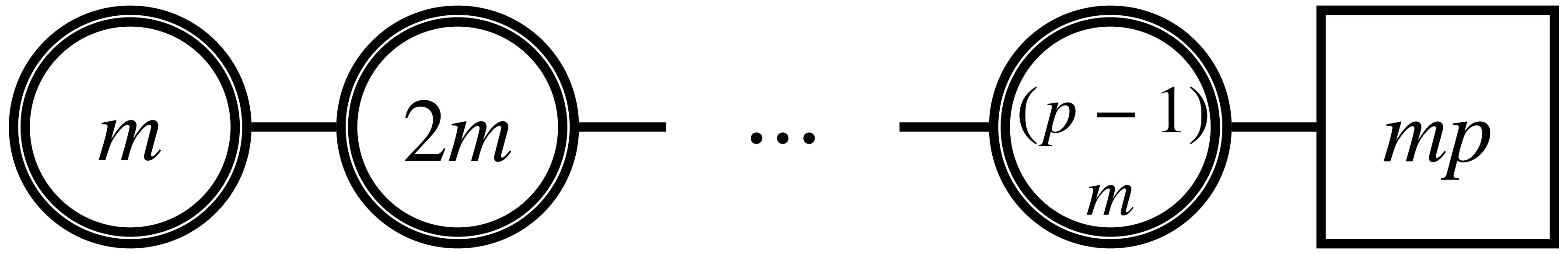
- Deformation of (A_1, A_k) by the Coulomb branch operator of the lowest dimension leads to free chirals in the IR.
- The phenomenon persists for other examples; e.g.,

Multiple M5s → $D_p[SU(N)]$

$$(A_1, D_{2k+1}) = D_{2k+1}[SU(2)] \longrightarrow \text{Three free chirals}$$

Compactification on a Riemann sphere with one irregular & one regular singularities

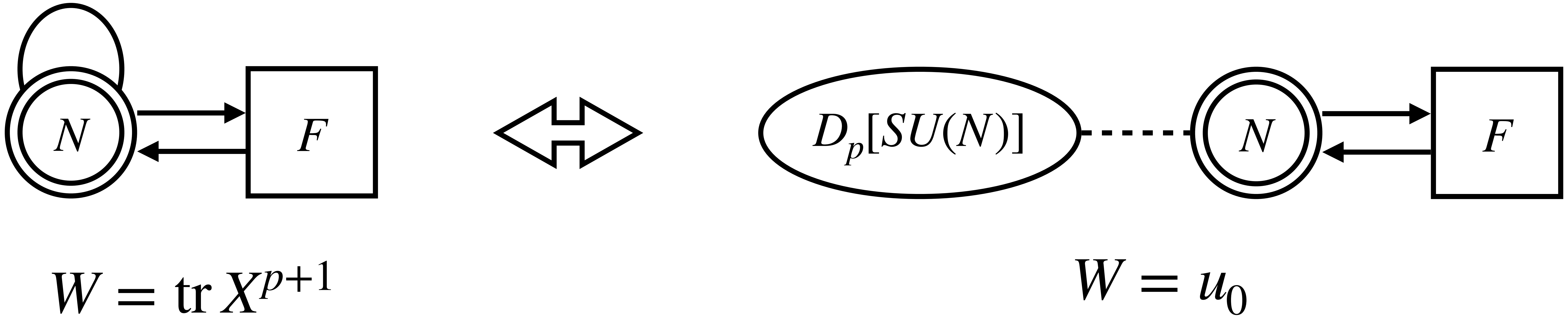
- The $D_p[SU(N)]$ theories allow Lagrangian dual descriptions when $N = mp$ [Cecotti, Del Zotto, Giacomelli 13].



- On the other hand, the $D_p[SU(N)]$ theories are non-Lagrangian when $\gcd(p, N) = 1$.

The Maruyoshi-Nardoni-Song Duality

- Recently, an interesting 4d $\mathcal{N} = 1$ duality involving $D_p[SU(N)]$ has been proposed for $p < N$ satisfying $\gcd(p, N) = 1$ [Maruyoshi, Nardoni, Song 23]:



- Replace an adjoint by a $D_p[SU(N)]$ tail; i.e., $D_p[SU(N)]$ is confined into a (gauged) chiral fields.

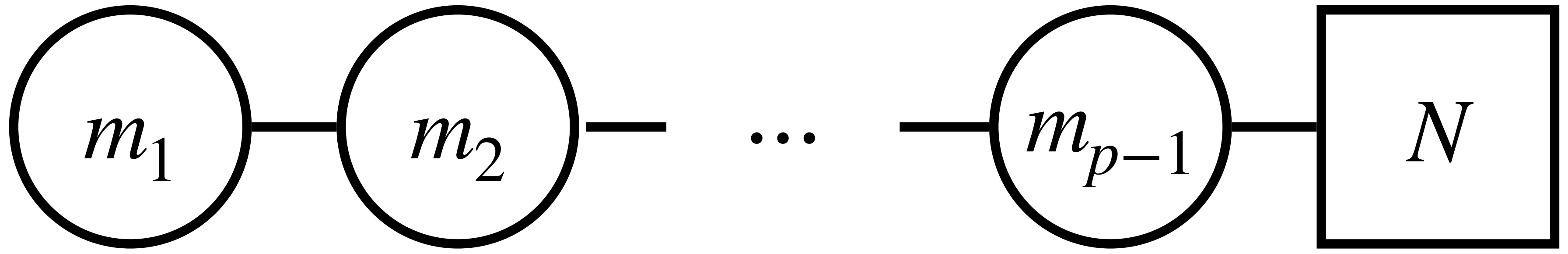
- Pass many nontrivial tests

Part I: 3D Reduction of $D_p[SU(N)]$ Argyres-Douglas Theories and Confinement

3D Reduction of $D_p[SU(N)]$ Theories

$\mathbb{D}_p[SU(N)]$

- Interestingly, the 3d reduction of 4d $D_p[SU(N)]$ theories always has UV *Lagrangian* descriptions [Closset, Giacomelli, Schafer-Nameki, Wang 12]; e.g., if $\gcd(p, N) = 1$,



$$m_j = \lfloor jN/p \rfloor\,, \qquad j = 1, \ldots, p-1$$

$$W = \sum_{i=1}^{p-1} \mathrm{Tr}_i\, \Phi^{(i)} Q_i \tilde{Q}_i + \sum_{i=1}^{p-2} \mathrm{Tr}_{i+1}\, \Phi^{(i+1)} \tilde{Q}_i Q_i$$

- If $\gcd(p, N) \neq 1$, it includes SU gauge nodes.

- **Confining deformation?**

Confinement of 3D $\mathbb{D}_p[SU(N)]$

- Let's assume some simplifying conditions.

- 3d $\mathbb{D}_p[SU(N)]$ theories are either good or ugly in Gaiotto-Witten's sense.

- Focus on the good case, where each node satisfies

$$m_{j-1} + m_{j+1} - 2m_j \geq 0$$

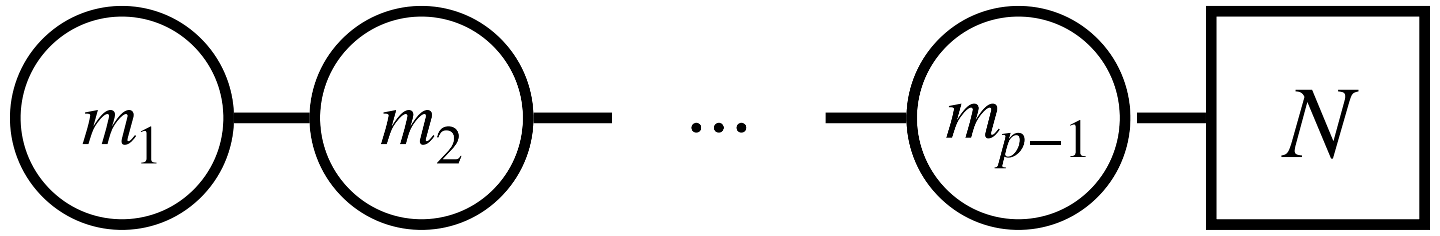
$$\Updownarrow$$

$$N = \pm 1 \mod p$$

- Also assume $p < N$, simplifying the formulas.

- **Proposal:** The 3d $\mathbb{D}_p[SU(N)]$ theory with deformation ΔW is confining.

$\mathbb{D}_p[SU(N)]$:



$$m_j = \lfloor jN/p \rfloor\,, \qquad j = 1,\ldots,p-1$$

$\mathbb{D}_p[SU(N)]$ with

$$\Delta W = \eta \sum_{i=1}^{p-1} \mathrm{Tr}\,\Phi^{(i)} + \sum_{i=1}^{p-1} \hat{v}^{(i),+} + \hat{v}^{(1,p-1),-}$$

$$\hat{v}^{(i),\pm} = \left(0^{i-1}\,,\ \pm 1\,, 0^{p-i-1}\right)$$

$$\hat{v}^{(1,p-1),\pm} = (\pm 1\,,\ \ldots\ ,\ \pm 1)$$

$\Leftrightarrow$

A matrix-valued chiral field X with

$$W = \mathrm{Tr}\,X^{p+1}$$

Evidence I

- Superconformal index

$$I \;=\; \mathrm{tr}\,(-1)^F x^{R+2j}$$

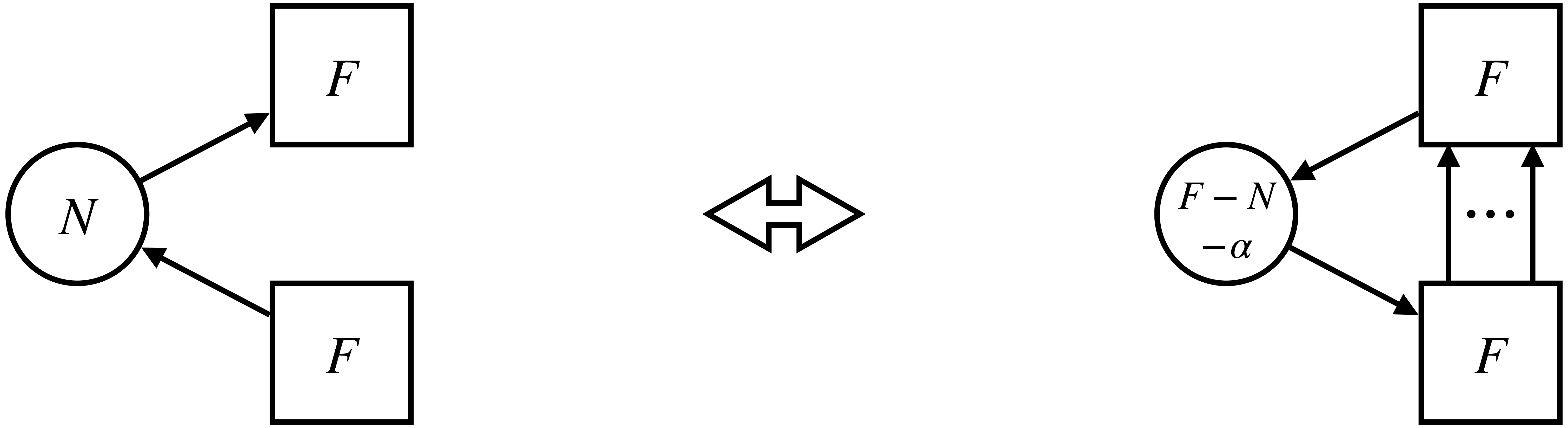
$$\Downarrow$$

$$I_{\mathbb{D}_p[SU(N)]+\Delta W} \quad = \quad PE\left[\frac{N^2\left(x^{\frac{2}{p+1}} - x^{\frac{2p}{p+1}}\right)}{1-x^2}\right] \quad = \quad I_{WZ}$$

- Precisely matching the spectrum of BPS states! (Tested for some N & p)

Evidence II

- More powerfully, one can prove the confinement only assuming the **Aharony-BBP** dualities [Aharony 97, Benini, Benvenuti, Pasquetti 17]:



$$W_A = \begin{cases} 0 \\ \hat{V}^+ \\ \hat{V}^+ + \hat{V}^- \end{cases} \qquad W_B = \begin{cases} V^+\hat{v}^+ + V^-\hat{v}^- + M\tilde{q}q \\ \hat{v}^+ + V^-\hat{v}^- + M\tilde{q}q \\ \hat{v}^+ + \hat{v}^- + M\tilde{q}q \end{cases} \qquad \alpha = \begin{cases} 0 \\ 1 \\ 2 \end{cases}$$

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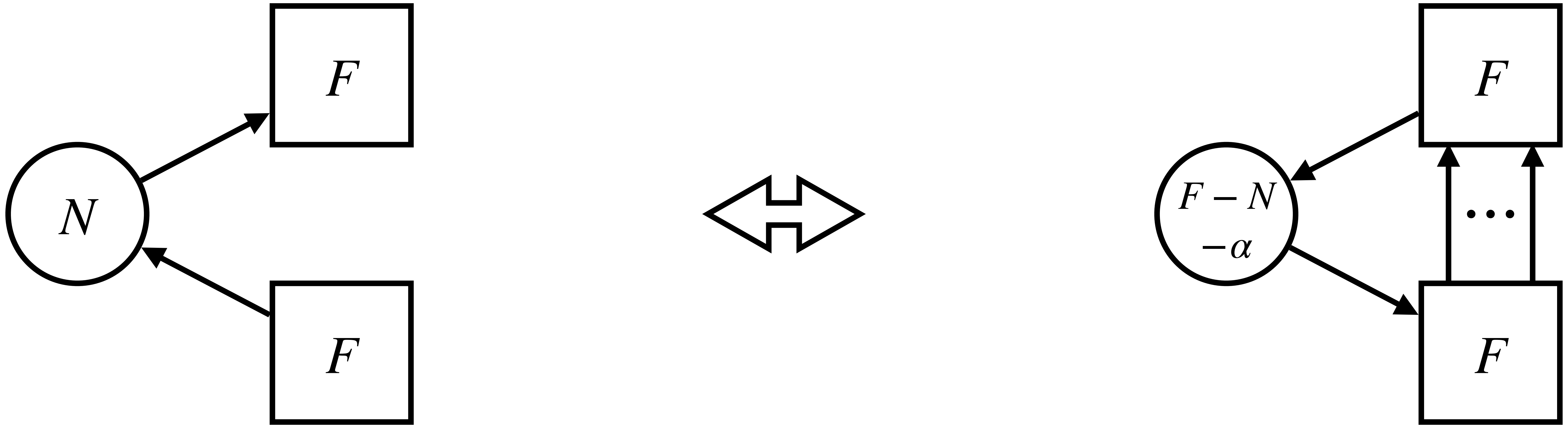

Aharony

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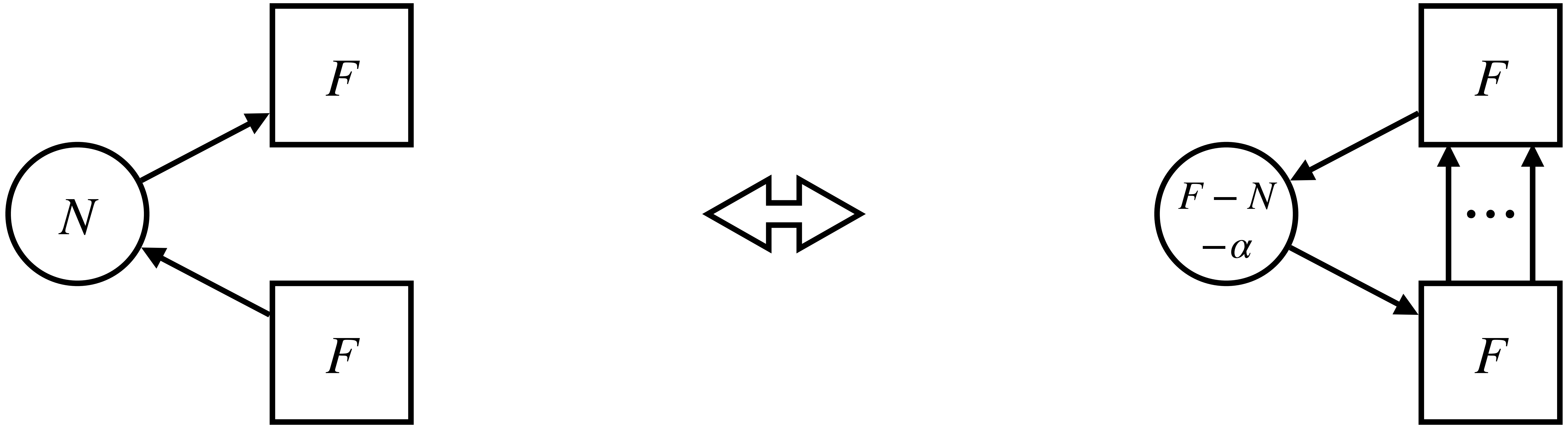

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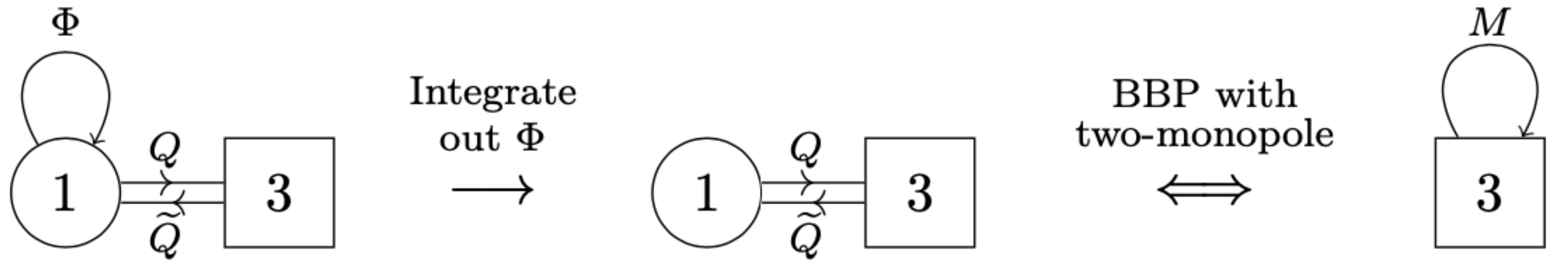
Mass def.

Derivation Using the BBP Dualities

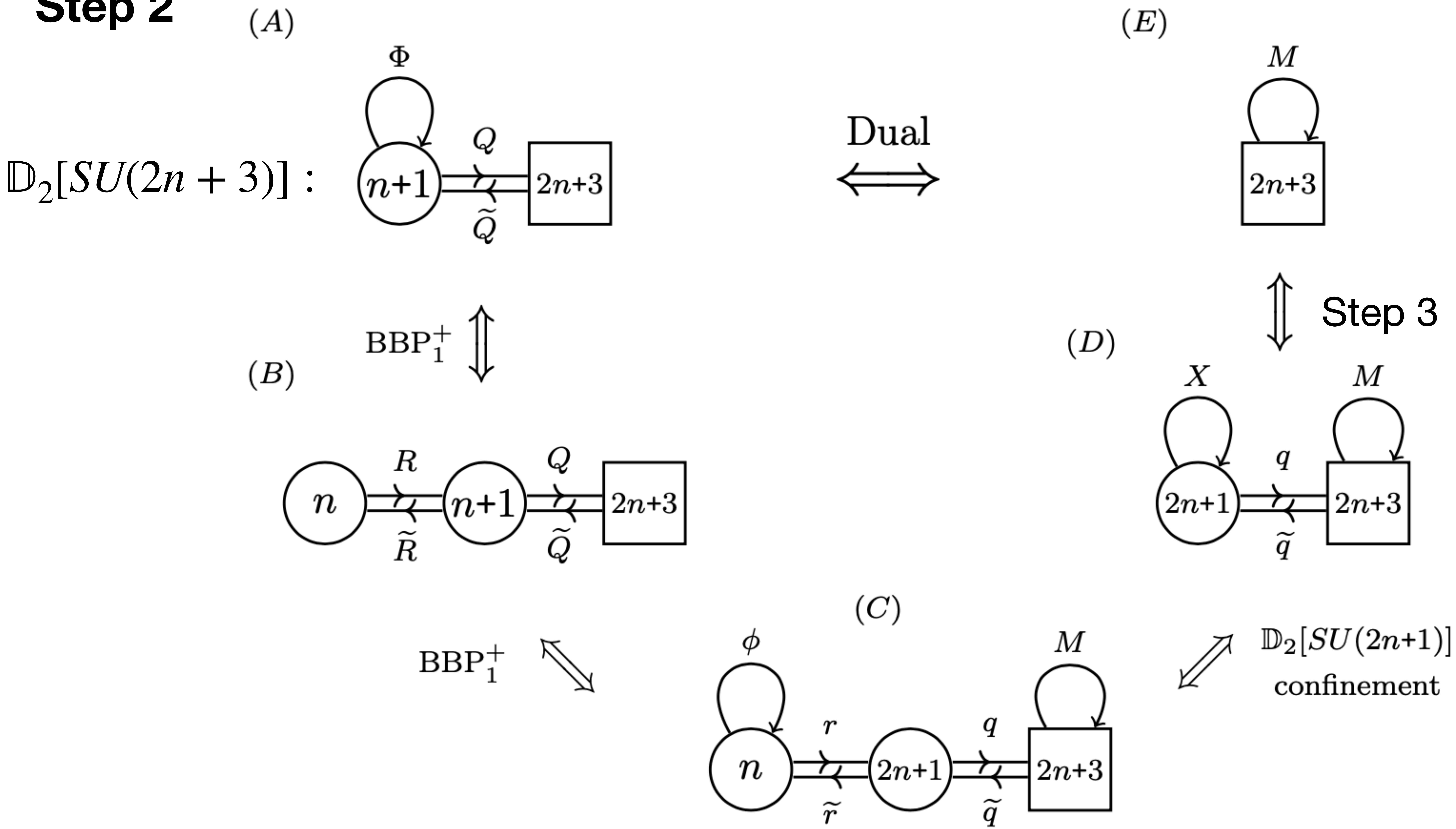
- Let's consider the $p = 2$ case. (Assume the gauge rank N is odd.)
- **Step 1**

$\mathbb{D}_2[SU(3)]$:



$$W = \Phi\,\mathrm{Tr}(Q\widetilde{Q}) + \underbrace{\eta\,\Phi + \hat{V}^{+} + \hat{V}^{-}}_{\Delta W} \qquad W = \hat{V}^{+} + \hat{V}^{-} \qquad W = \mathrm{Tr}\,M^3$$

Step 2



(A)
$\mathbb{D}_2[SU(2n+3)]$:
Φ
Q
n+1
2n+3
$\widetilde{Q}$
Dual
(E)
M
2n+3
Step 3
BBP_1^+
(B)
n
R
$\widetilde{R}$
n+1
Q
$\widetilde{Q}$
2n+3
(D)
X
M
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2n+1
2n+3
$\widetilde{q}$
BBP_1^+
(C)
ϕ
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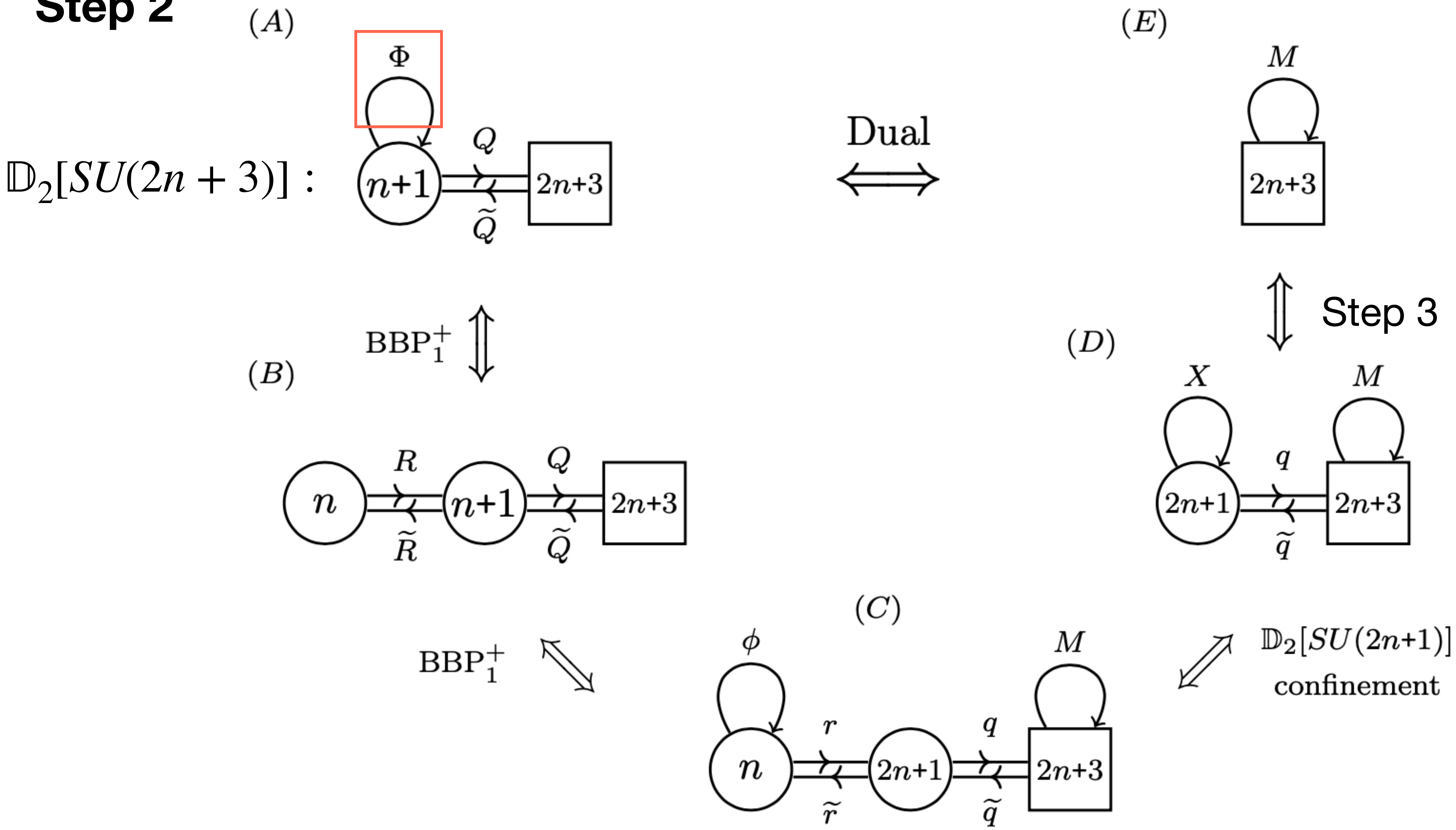
$\mathbb{D}_2[SU(2n+1)]$
confinement

Step 2



(A)
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$\mathbb{D}_2[SU(2n+3)]$:
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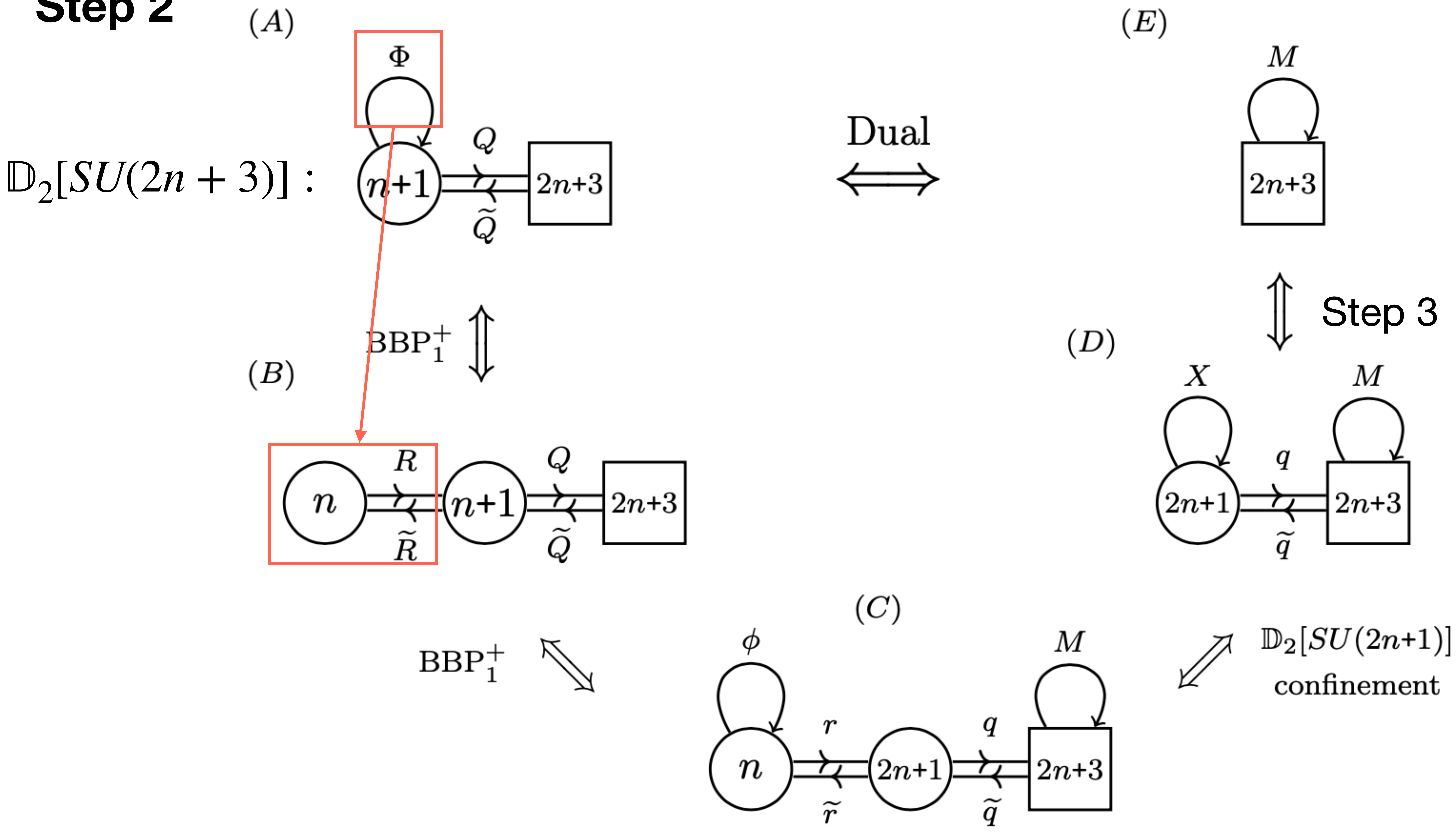

(A)
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$\mathbb{D}_2[SU(2n+3)]$:
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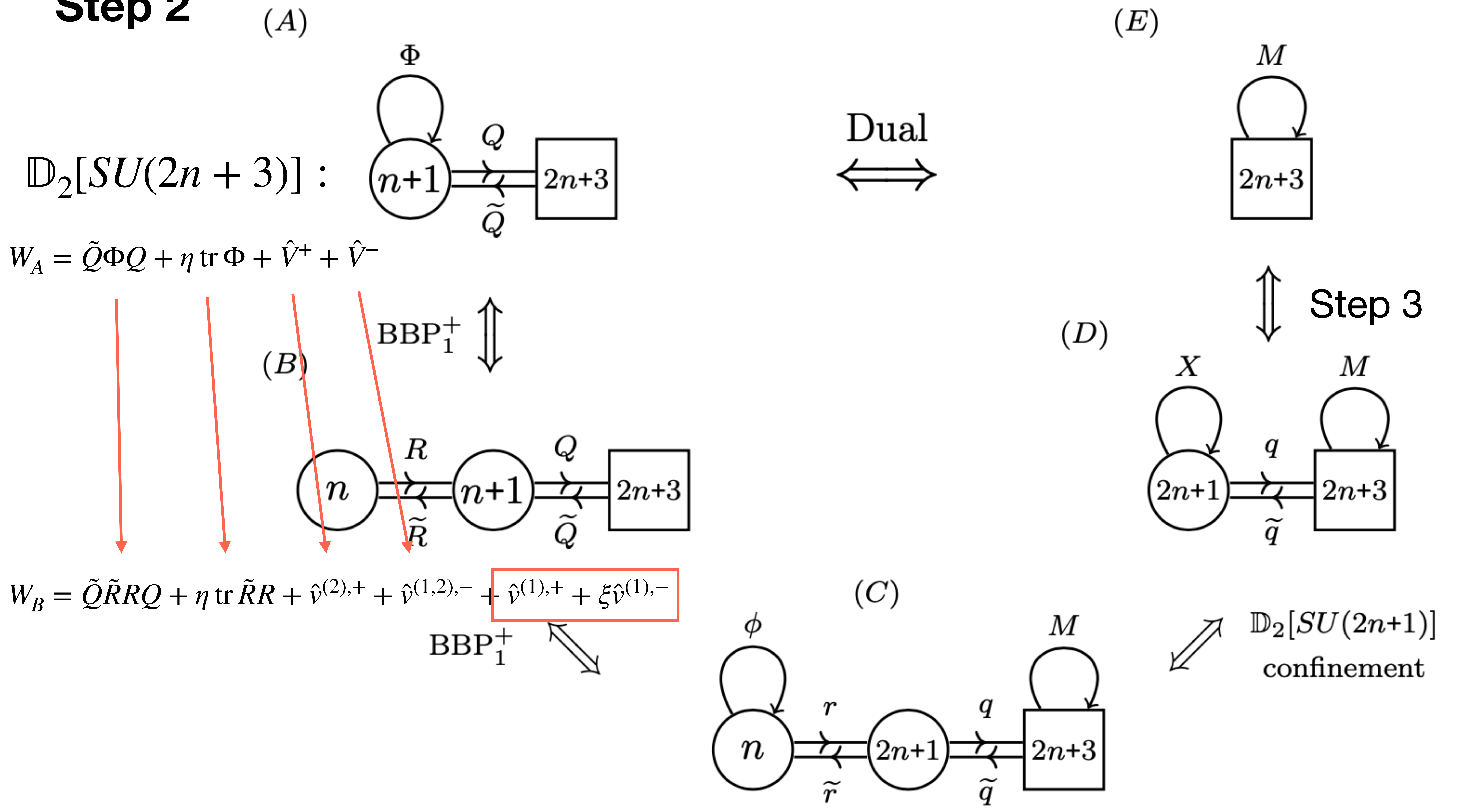
Step 2


(A)
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$\mathbb{D}_2[SU(2n+3)]$:
$W_A = \tilde{Q}\Phi Q + \eta\, \mathrm{tr}\, \Phi + \hat{V}^{+} + \hat{V}^{-}$
BBP_1^{+}
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$W_B = \tilde{Q}\tilde{R}RQ + \eta\, \mathrm{tr}\, \tilde{R}R + \hat{v}^{(2),+} + \hat{v}^{(1,2),-} + \hat{v}^{(1),+} + \xi\hat{v}^{(1),-}$
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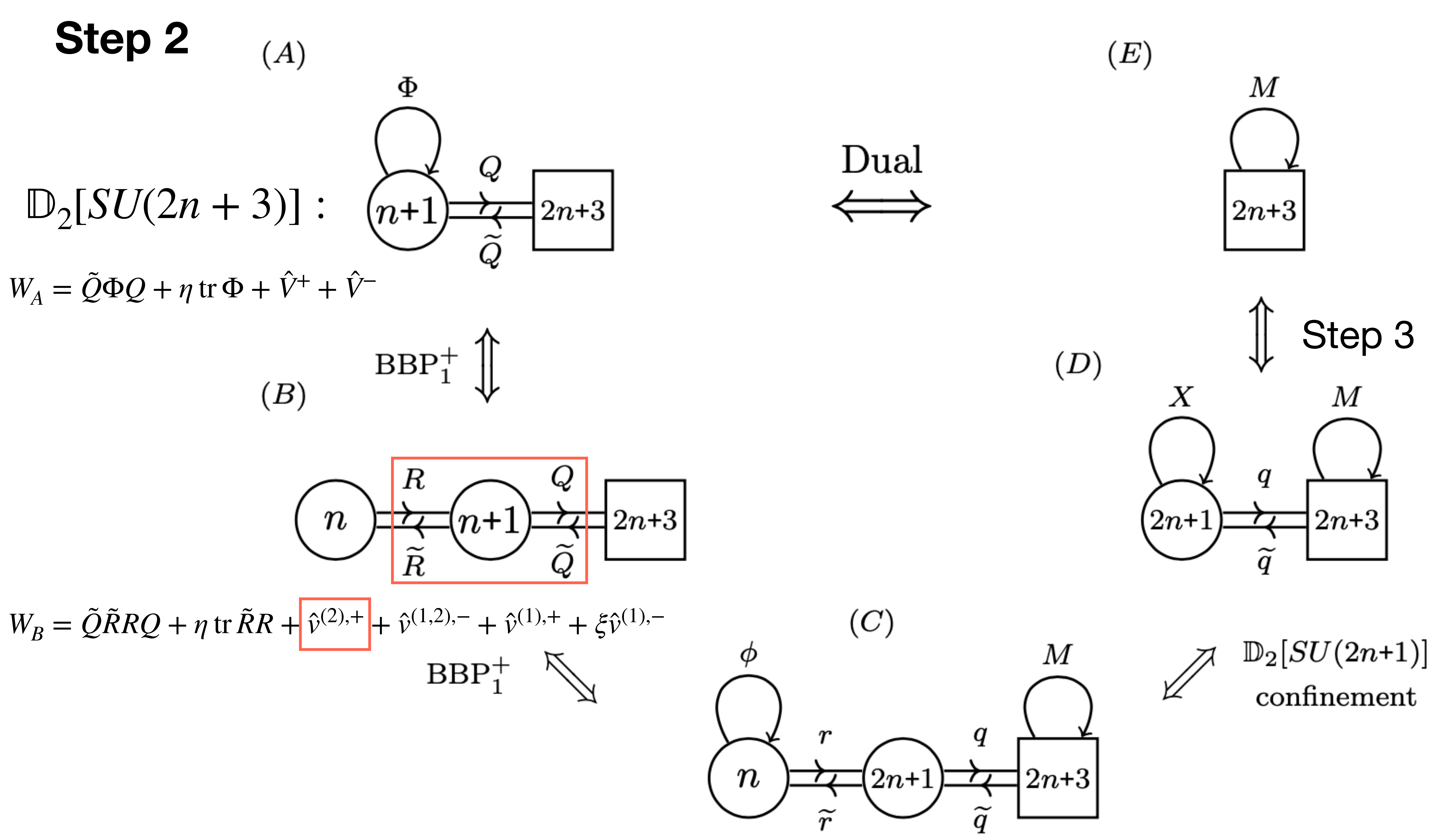
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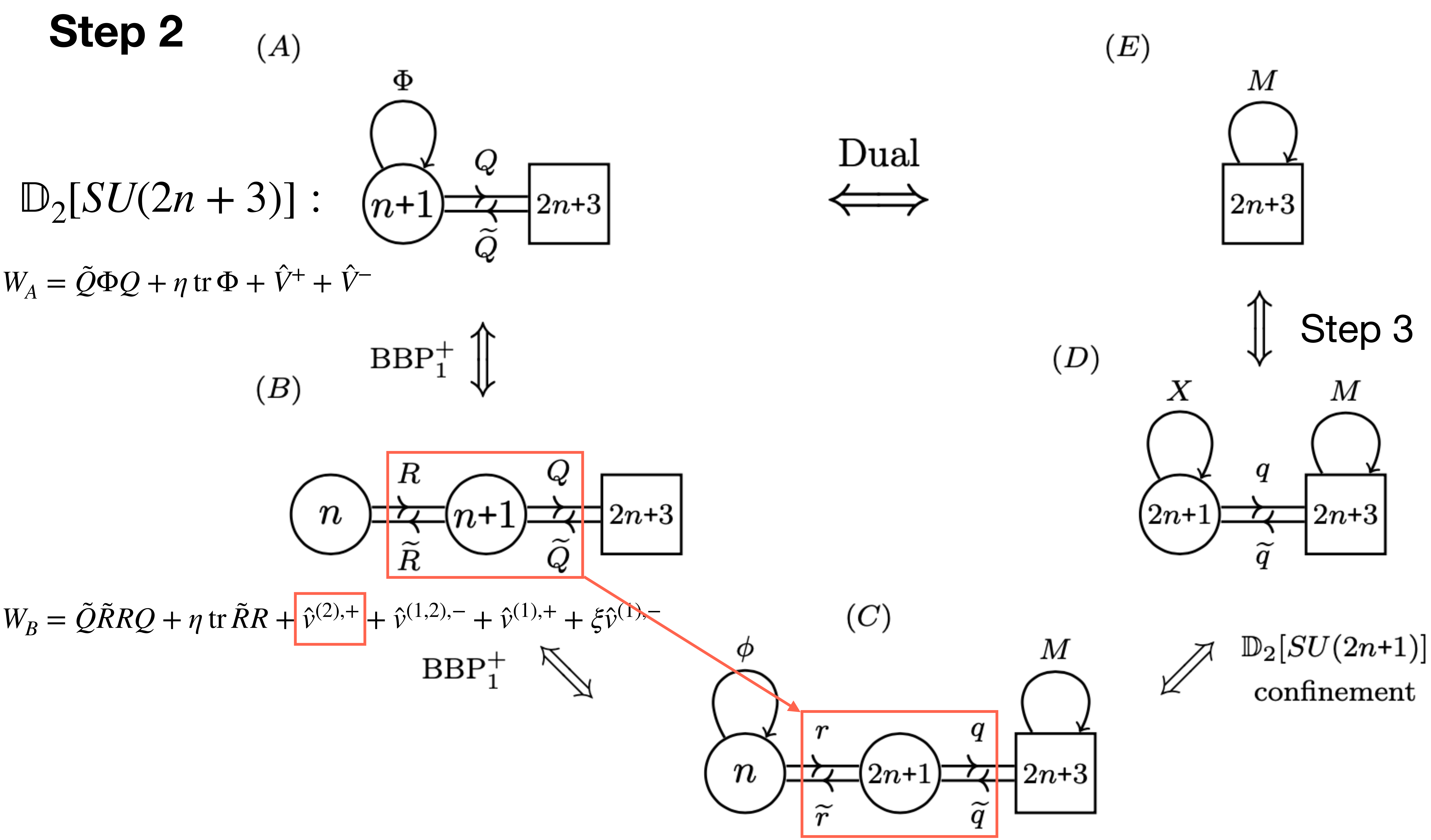
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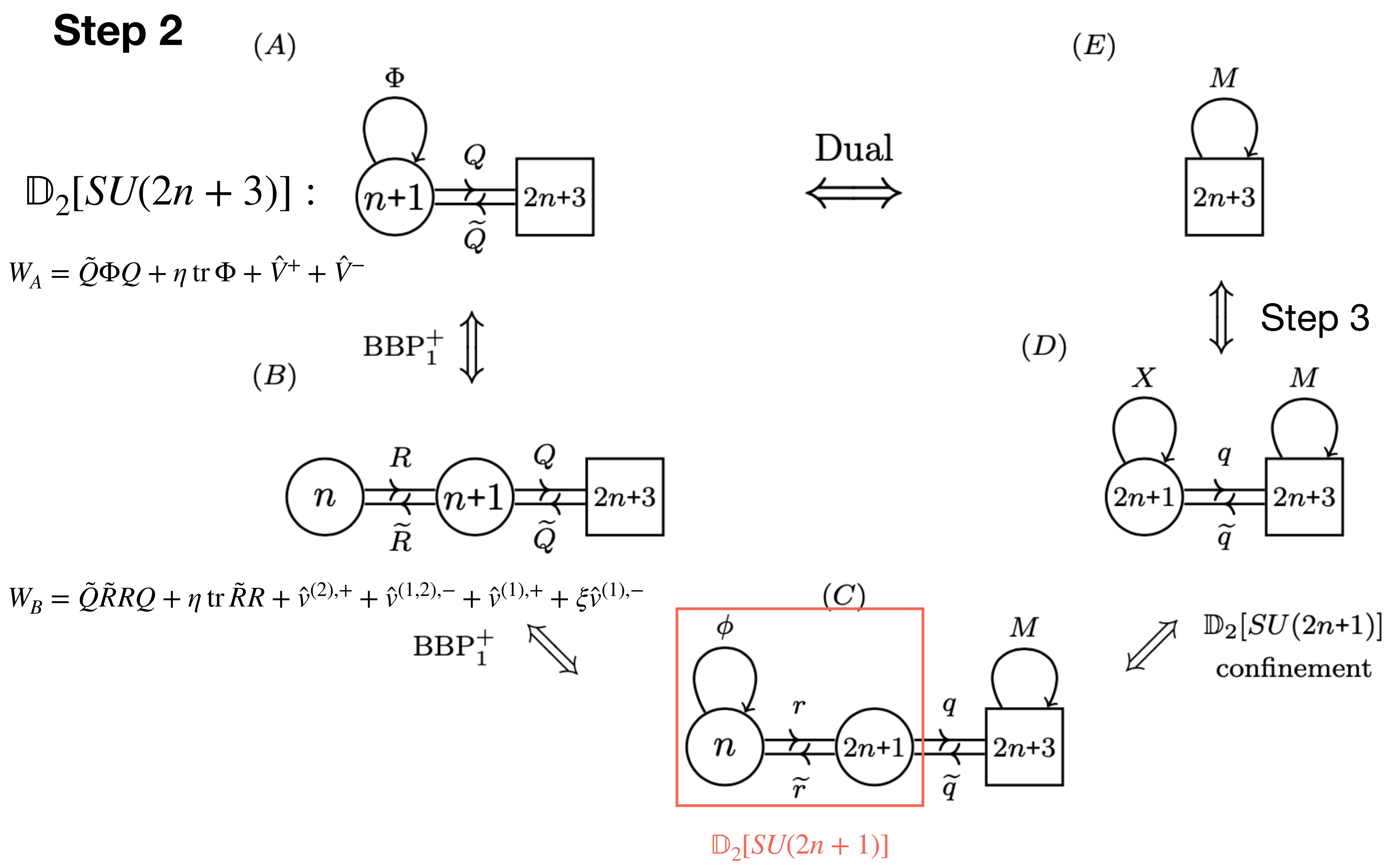
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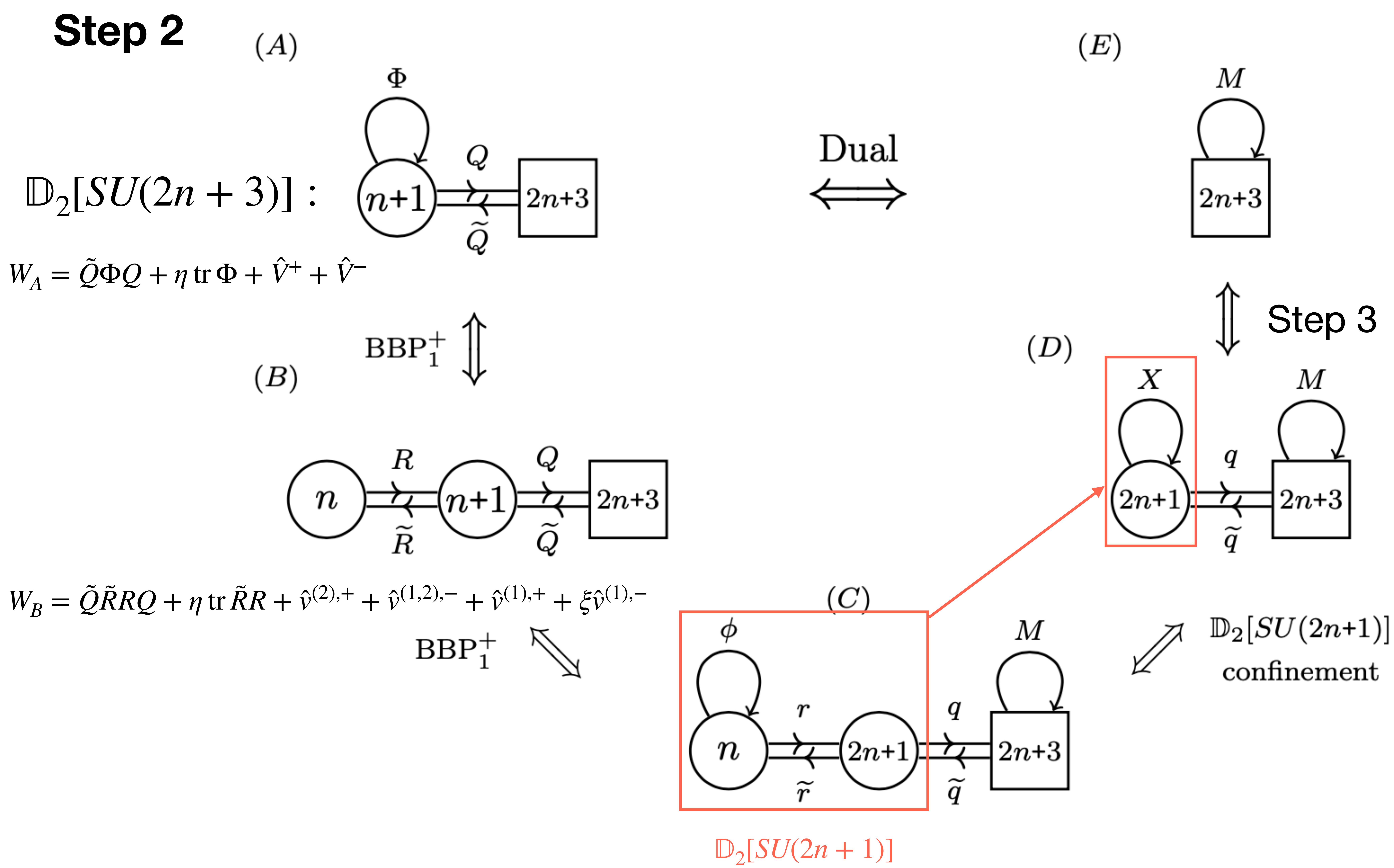

Step 2
(A)
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$\mathbb{D}_2[SU(2n+3)]$:
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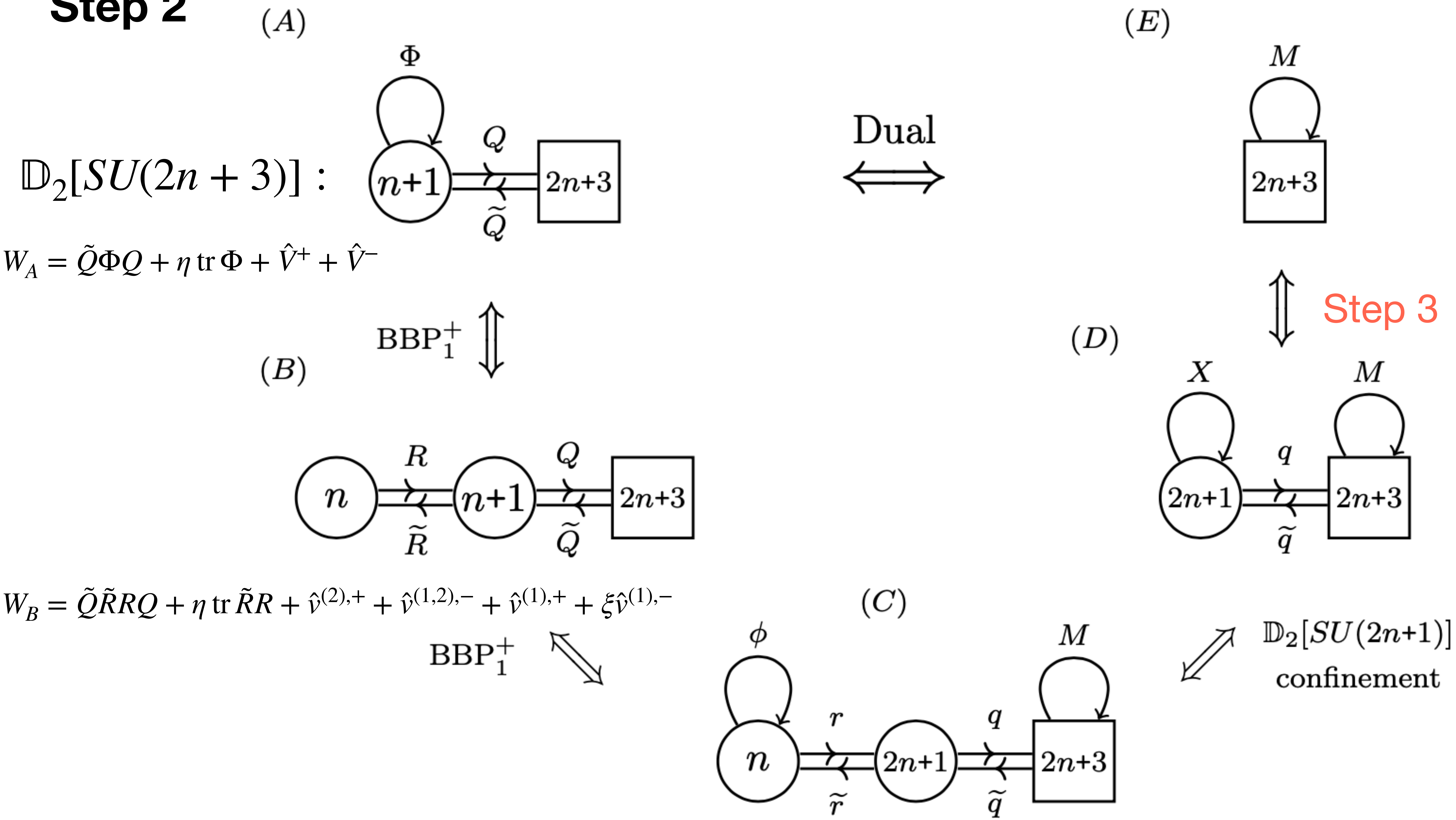
$\mathbb{D}_2[SU(2n+1)]$
confinement
$\mathbb{D}_2[SU(2n+1)]$

Step 2



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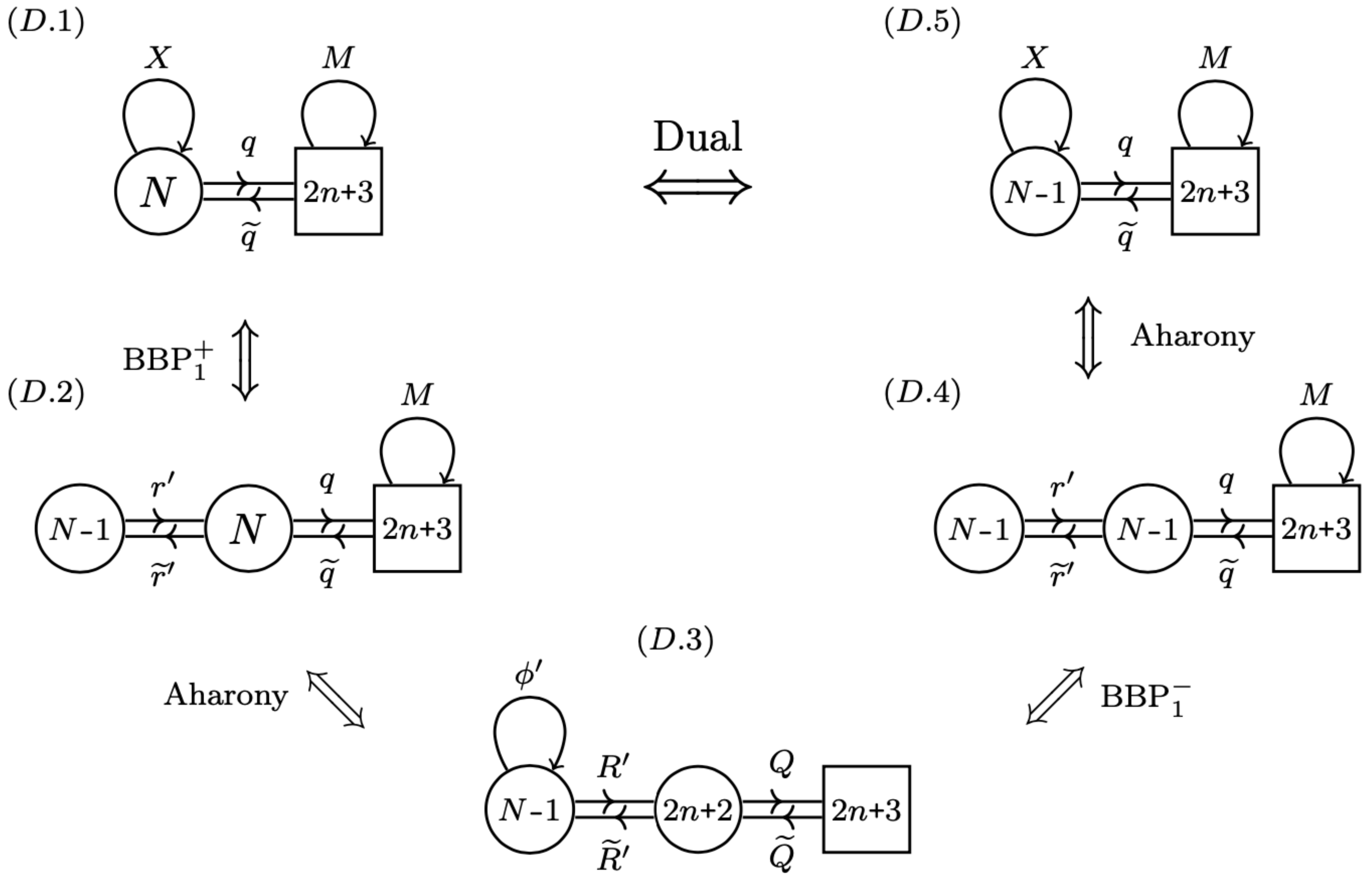
2n+3
$\mathbb{D}_2[SU(2n+1)]$
confinement

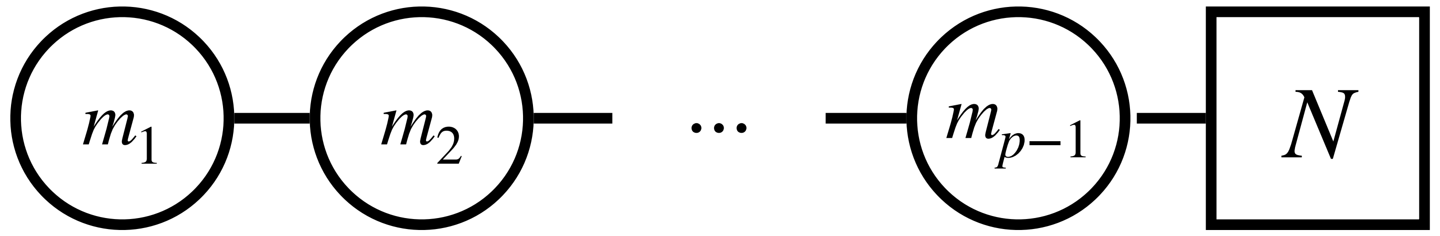
Step 3



(D.1)
X
M
q
N
2n+3
q̃
Dual
(D.5)
X
M
q
N-1
2n+3
q̃
BBP₁⁺
Aharony
(D.2)
M
r′
q
N-1
N
2n+3
r̃′
q̃
(D.4)
M
r′
q
N-1
N-1
2n+3
r̃′
q̃
(D.3)
Aharony
ϕ′
BBP₁⁻
R′
Q
N-1
2n+2
2n+3
R̃′
Q̃

Confinement of 3D $\mathbb{D}_p[SU(N)]$

- 3d $\mathbb{D}_p[SU(N)]$ theory is $\mathcal{N} = 4$ quiver gauge theory:



$$m_j = \lfloor jN/p \rfloor\,, \qquad j = 1,\ldots,p-1$$

- Confinement of $\mathbb{D}_p[SU(N)]$ triggered by

$$\Delta W = \eta \sum_{i=1}^{p-1} \mathrm{Tr}\,\Phi^{(i)} + \sum_{i=1}^{p-1} \hat{v}^{(i),+} + \hat{v}^{(1,p-1),-}$$

Confinement of 3D $\mathbb{D}_p[SU(N)]$

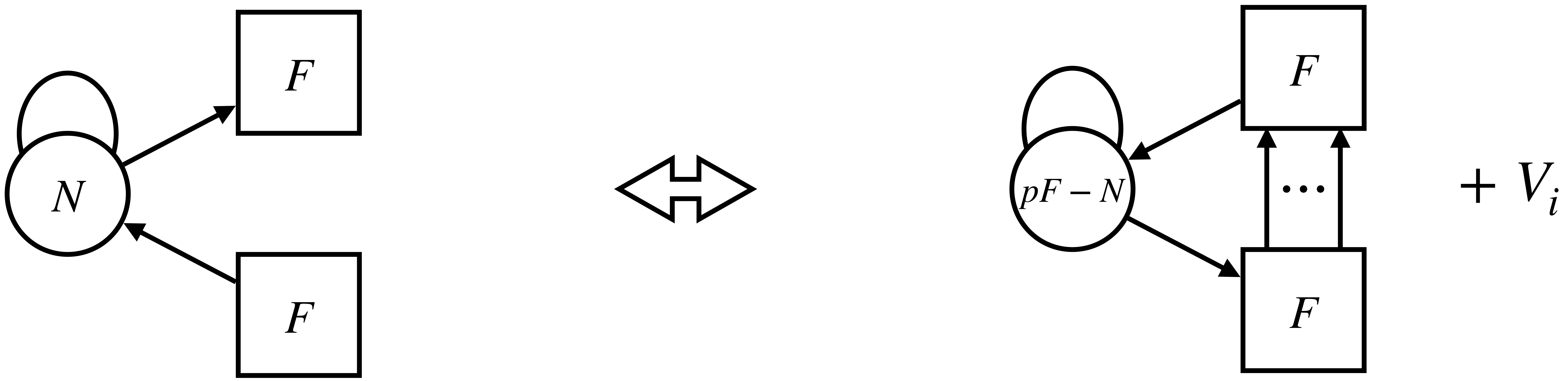
- A consequence of the BBP dualities
- An interacting theory in the IR if $p = 2$
- Support for Xie-Yan's 4-dimensional result
- *Application to Seiberg-like dualities for adjoint SQCDs*

Part II: Revisit Dualities for Adjoint SQCDs

Dualities for 3D Adjoint SQCDs

- A variety of Seiberg-like dualities for adjoint SQCDs have been studied.

- E.g., the Kim-Park duality for 3d U(N) gauge theories with a single adjoint [Kim, Park 13]:



$$W = \operatorname{Tr} X^{p+1}$$

$$W = \operatorname{Tr} x^{p+1} + \sum_{i=0}^{p-1} \left(M_{p-i-1}\, \tilde{q} x^i q + V^+_{p-i-1}\, \hat{v}^+_i + V^-_{p-i-1}\, \hat{v}^-_i \right)$$

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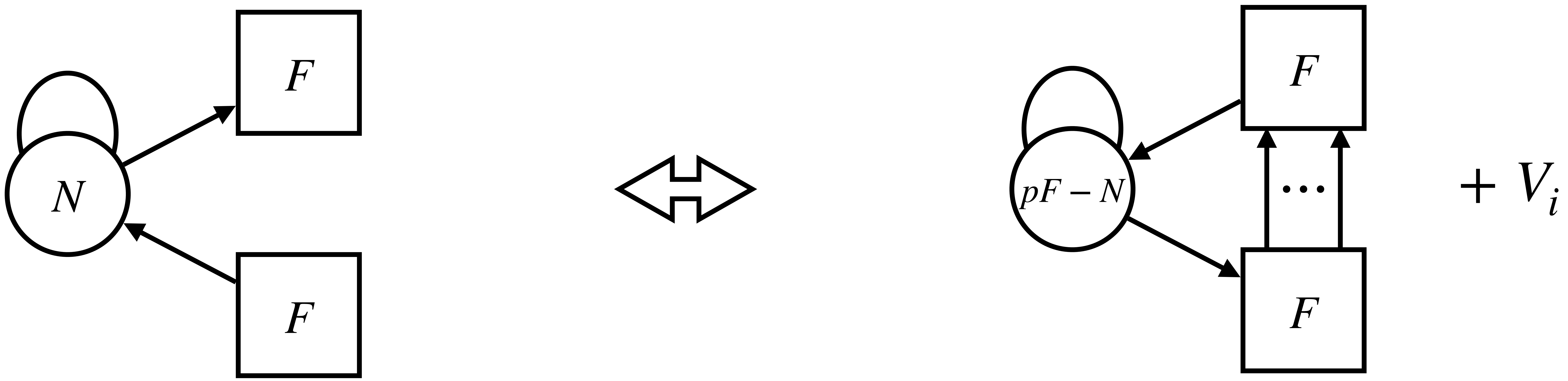

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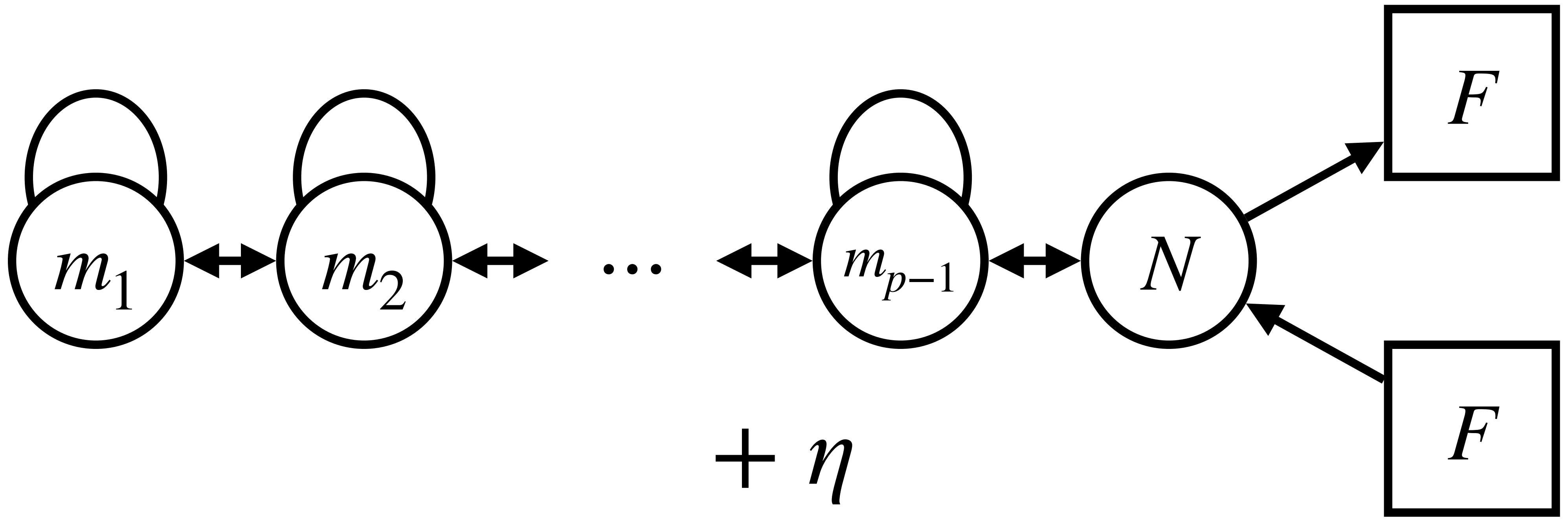
Allowing the deconfinement of the adjoint into the $\mathbb{D}_p[SU(N)]$ tail

Deconfined Kim-Park Duality

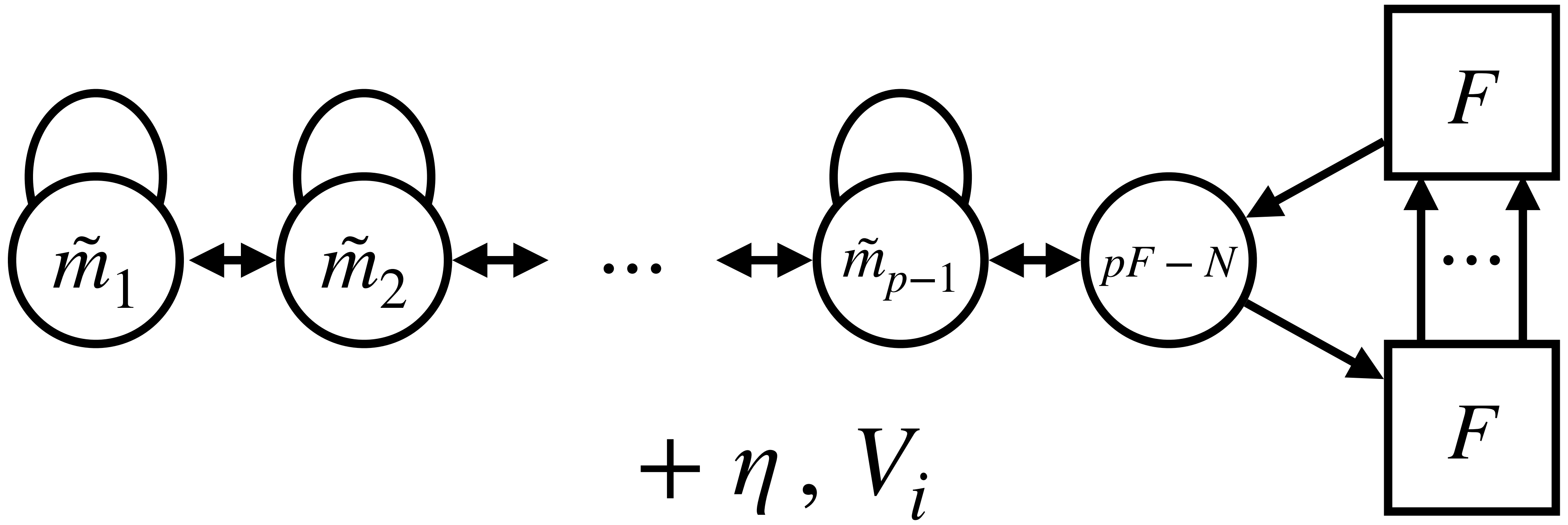


$+\,\eta$

$$m_j = \lfloor jN/p \rfloor\,, \qquad j = 1,\ldots,p-1$$

$$W_A = \sum_{i=1}^{p-1} \mathrm{Tr}_i\, \Phi^{(i)} Q_i \tilde{Q}_i + \sum_{i=1}^{p-2} \mathrm{Tr}_{i+1}\, \Phi^{(i+1)} \tilde{Q}_i Q_i + \eta \sum_{i=1}^{p-1} \mathrm{Tr}\, \Phi^{(i)} + \sum_{i=1}^{p-1} \hat{V}^{(i),+} + \hat{V}^{(1,p-1),-}$$

$$\Updownarrow$$



$+\,\eta\,,\, V_i$

$$\tilde{m}_j = \lfloor j(pF-N)/p \rfloor = jF + m_{p-j} - m_p\,, \qquad j = 1,\ldots,p-1$$

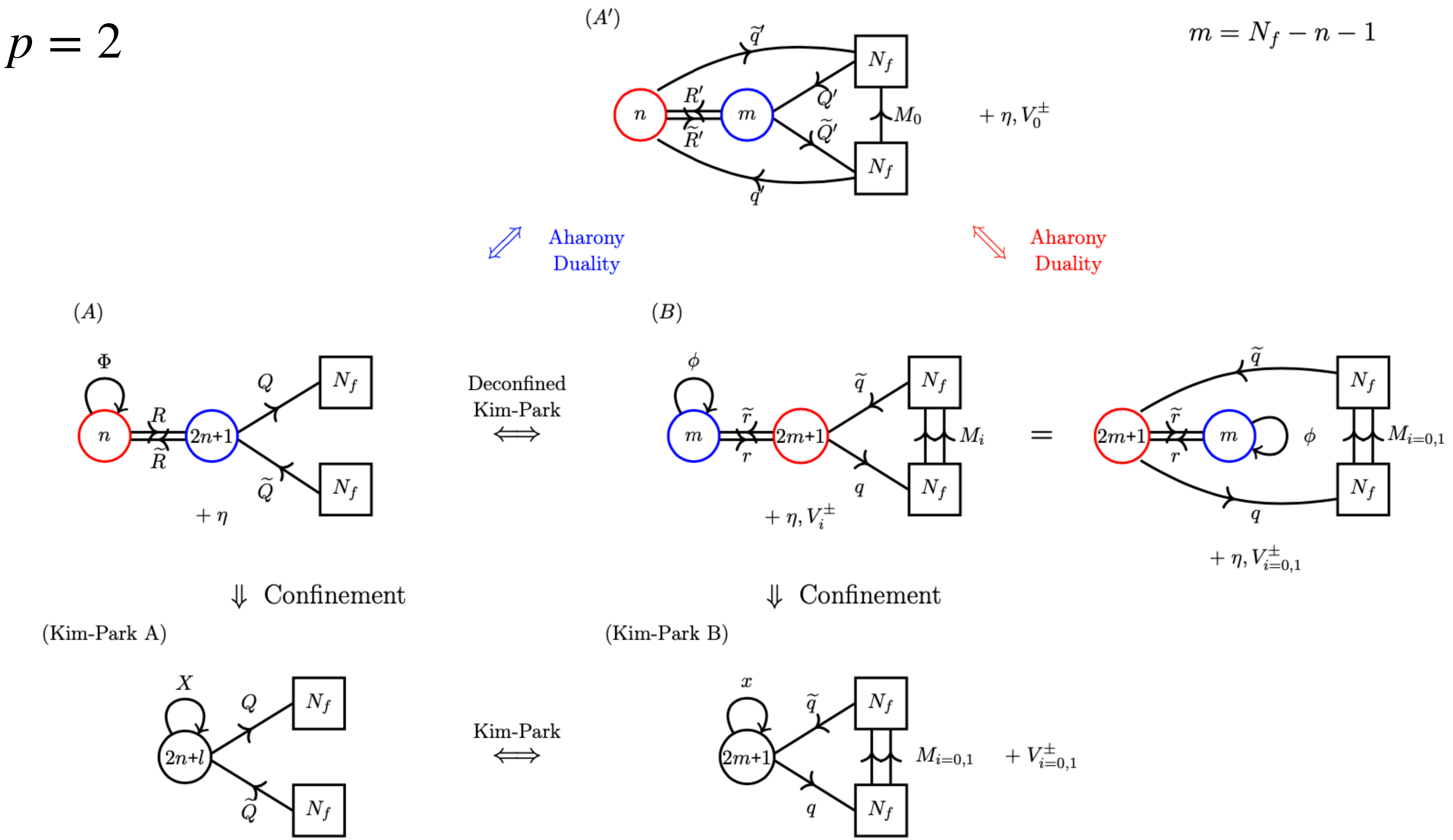
$$W_B = \sum_{i=1}^{p-1} \mathrm{Tr}_i\, \Phi^{(i)} Q_i \tilde{Q}_i + \sum_{i=1}^{p-2} \mathrm{Tr}_{i+1}\, \Phi^{(i+1)} \tilde{Q}_i Q_i + \eta \sum_{i=1}^{p-1} \mathrm{Tr}\, \Phi^{(i)} + \sum_{i=1}^{p-1} \hat{V}^{(i),+} + \hat{V}^{(1,p-1),-} + \ldots$$

- Matching superconformal indices (tested for some N & p)

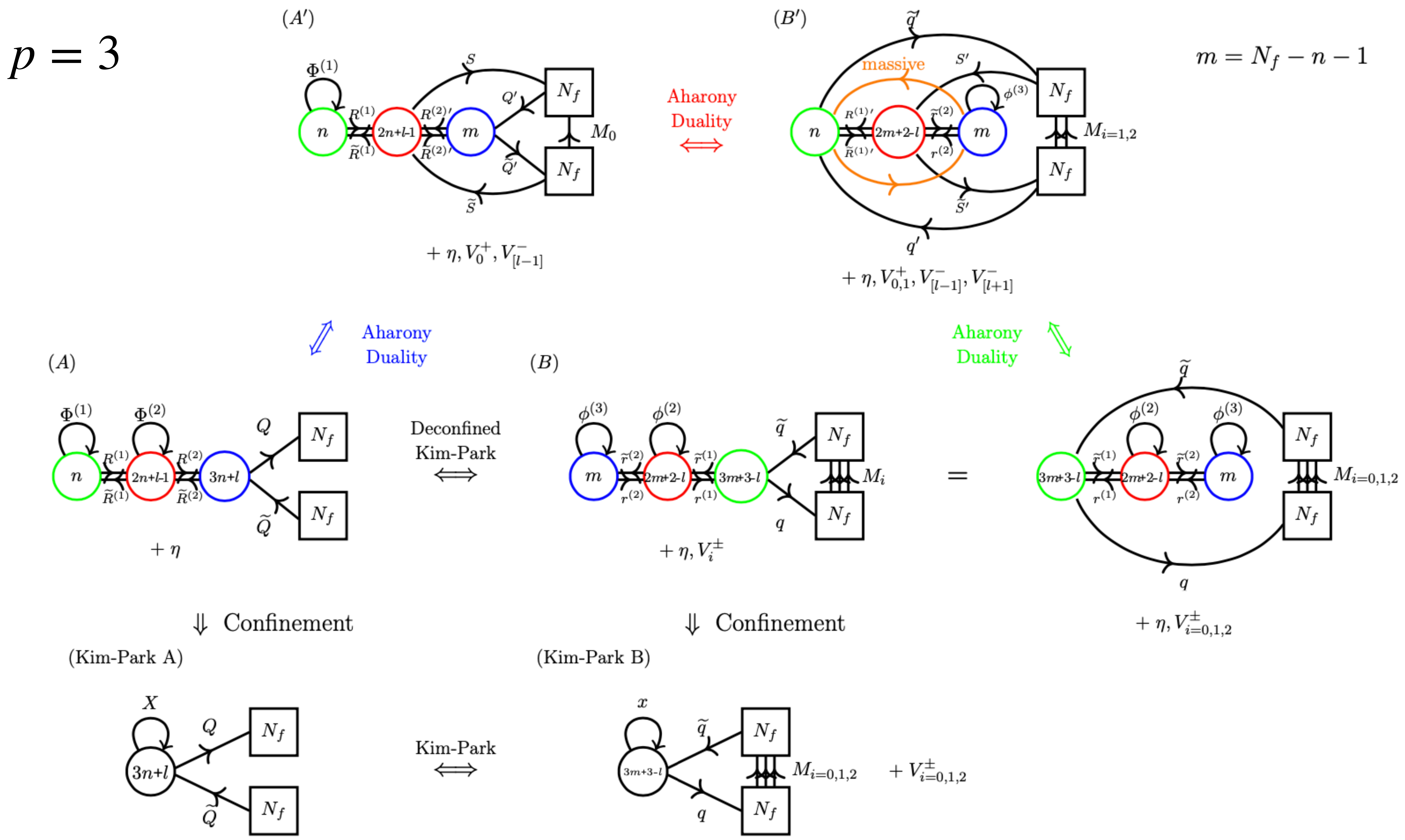
- E.g., the chiral ring generators for $p = 2$:

Kim–Park A	Theory A	Theory A$'$	Theory B	Kim–Park B
$\tilde{Q}Q$	$\tilde{Q}Q$	M_0	M_0	M_0
$\tilde{Q}XQ$	$\tilde{Q}\tilde{R}RQ$	$q'\tilde{q}'$	M_1	M_1
$\mathrm{Tr}\, X$	$\eta \sim \mathrm{Tr}\, \tilde{R}R$	η	$\eta \sim \mathrm{Tr}\, \tilde{r}r$	$\mathrm{Tr}\, x$
$\hat{V}_0^\pm$	$\hat{V}^{(2),\pm}$	$V_0^\pm$	$V_0^\pm$	$V_0^\pm$
$\hat{V}_1^\pm$	$\hat{V}^{(1,2),\pm}$	$\hat{v}^{(1),\pm}$	$V_1^\pm$	$V_1^\pm$

- Again, proved only assuming the Aharony duality



p = 2
m = N_f − n − 1
(A′)
+ η, V_0^±
Aharony Duality
Aharony Duality
(A)
(B)
+ η
Deconfined Kim-Park
+ η, V_i^±
+ η, V_{i=0,1}^±
M_i
M_{i=0,1}
⇓ Confinement
⇓ Confinement
(Kim-Park A)
(Kim-Park B)
Kim-Park
+ V_{i=0,1}^±



$p = 3$
(A')
(B')
$m = N_f - n - 1$
Aharony Duality
massive
$+\ \eta, V_0^+, V_{[l-1]}^-$
$+\ \eta, V_{0,1}^+, V_{[l-1]}^-, V_{[l+1]}^-$
Aharony Duality
Aharony Duality
(A)
(B)
Deconfined Kim-Park
$+\ \eta$
$+\ \eta, V_i^\pm$
$+\ \eta, V_{i=0,1,2}^\pm$
Confinement
Confinement
(Kim-Park A)
(Kim-Park B)
Kim-Park
$+\ V_{i=0,1,2}^\pm$

- The (deconfined) Kim-Park duality, a Seiberg-like duality for adjoint SQCDs, can be derived from the Aharany duality.
- Furthermore, such underlying relations between different supersymmetric dualities provide **new proof of various special function identities** through the localization computation of supersymmetric partition functions (Spiridonov, Rains, …)
- E.g., the superconformal index identity for the Aharony duality [CH, Yi, Yoshida 17] implies the identity for the Kim-Park duality.

Proof of the Index Identity for the Aharony Duality

- 3d superconformal index

$$I \;=\; \mathrm{tr}\,(-1)^F x^{R+2j} e^{i\mu Q}$$

⬇ SUSY localization [Kim 09, Imamura, Yokoyama 11]

$$I(x;\mu) = \sum_{\mathfrak{m}\in\mathbb{Z}^N/S^N} \frac{1}{|W_{\mathfrak{m}}|} \oint \frac{d^N a}{(2\pi)^N} Z_{cl}(x;\mu,a;\mathfrak{m})\, Z_{1-loop}(x;\mu,a;\mathfrak{m})$$

$$Z^{chiral}_{1-loop}(x;\mu,a;\mathfrak{m}) = \prod_{\rho} \left(e^{i\rho(a+\mu)}x^{-1}\right)^{-\frac{\rho(m)}{2}} \frac{\left(e^{-i\rho(a+\mu)}\,x^{2-R+|\rho(m)|};x^2\right)}{\left(e^{i\rho(a+\mu)}\,x^{R+|\rho(m)|};x^2\right)}$$

$$\vdots$$

- Factorization [CH, Kim, Park 12] (Holomorhpic blocks, Higgs-branch localization)

$$I(x;\mu) = \sum_{\mathfrak{m}\in\mathbb{Z}^N/S^N} \frac{1}{|W_{\mathfrak{m}}|} \oint \frac{d^N a}{(2\pi)^N} Z_{cl}(x;\mu,a;\mathfrak{m})\, Z_{1-loop}(x;\mu,a;\mathfrak{m})$$

⬇ Residue computation

$$I = \sum_{\text{Higgs vacua}} Z_{pert}\, Z_{vortex}\, \bar{Z}_{pert}\, \bar{Z}_{vortex}$$

- For the Aharony duality

$$Z_{pert} = \tilde{Z}_{pert}\, \tilde{Z}_M \qquad \text{Easy}$$

$$Z_{vortex} = \tilde{Z}_{vortex}\, \tilde{Z}_V \qquad \text{Difficult}$$

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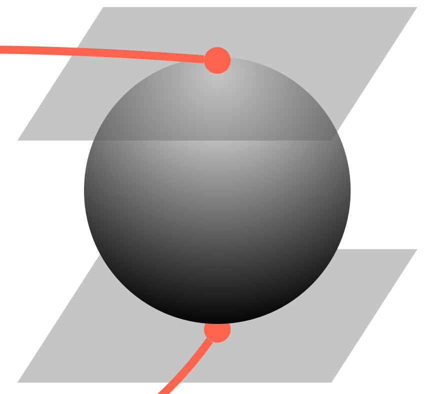

$$I = \sum_{\text{Higgs vacua}} Z_{pert} Z_{vortex} \; \bar{Z}_{pert} \bar{Z}_{vortex}$$

BPS partition functions on $\mathbf{R}^2 \times S^1$ w/

$$Z_{vortex} = \sum_n w^n Z_n$$

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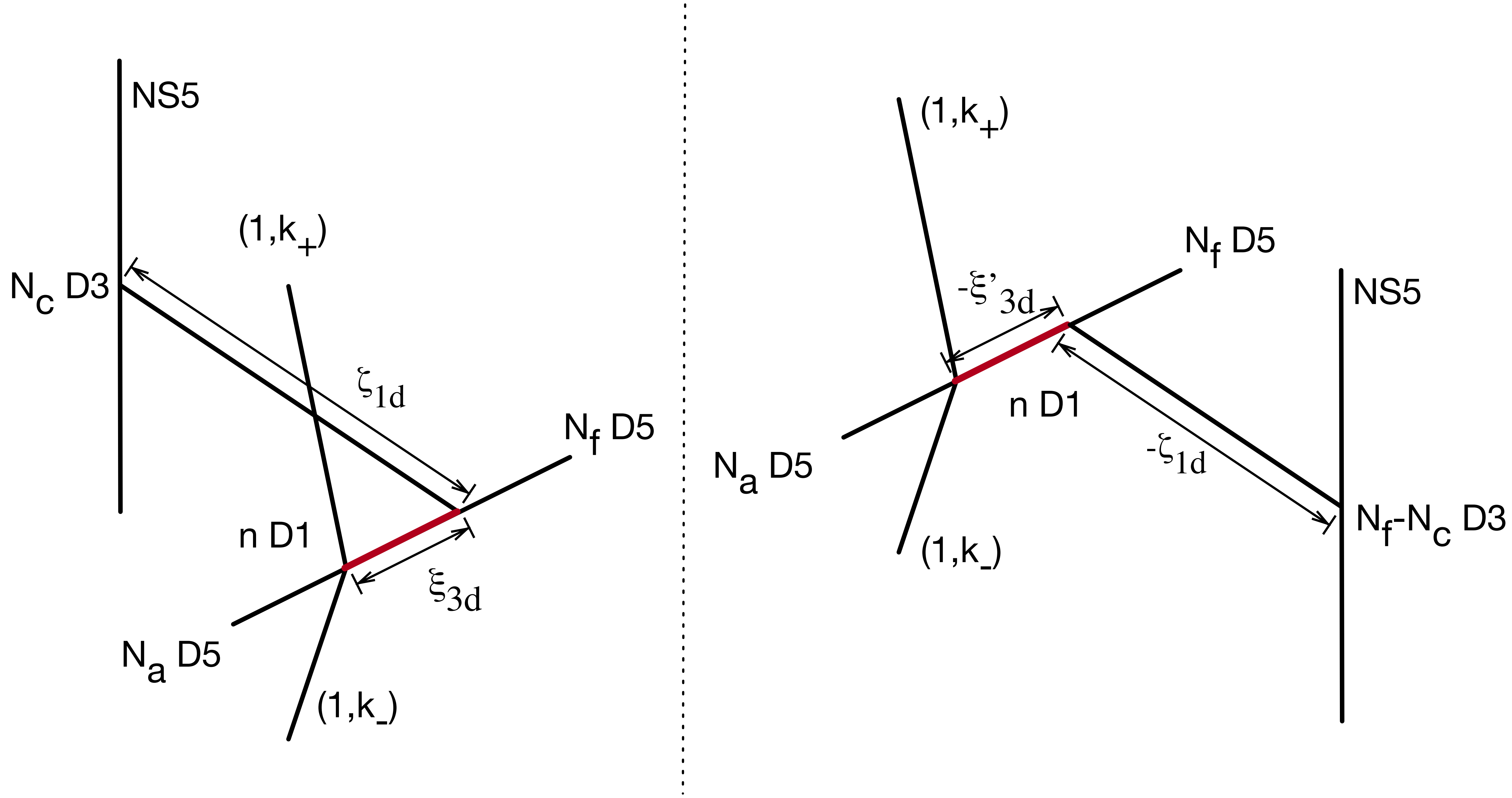
Contributions of the extra singlets on the dual side

The Aharony Duality and Vortex Wall-Crossing

- Type IIB brane picture



The Aharony Duality and Vortex Wall-Crossing

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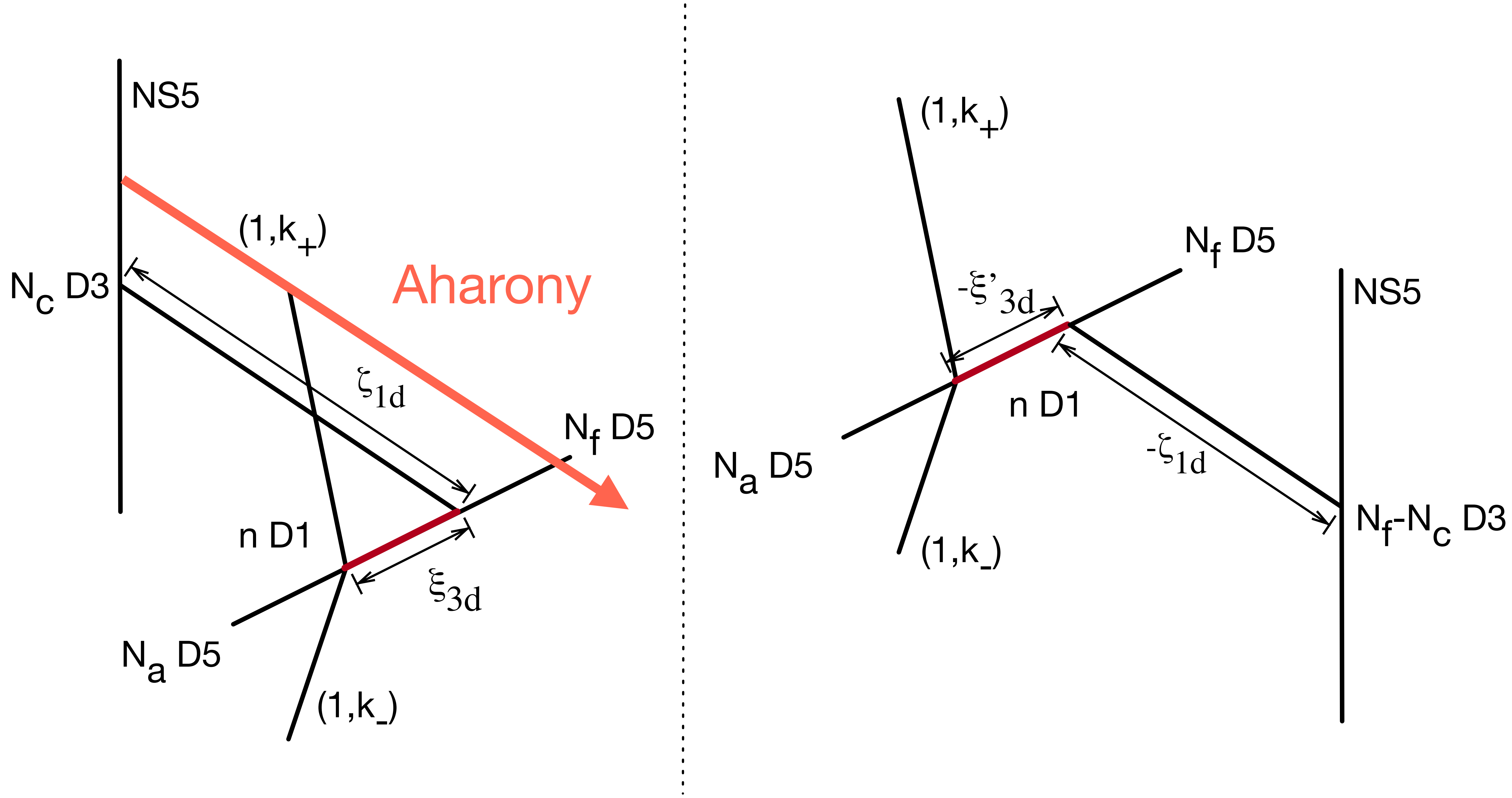

The Aharony Duality and Vortex Wall-Crossing

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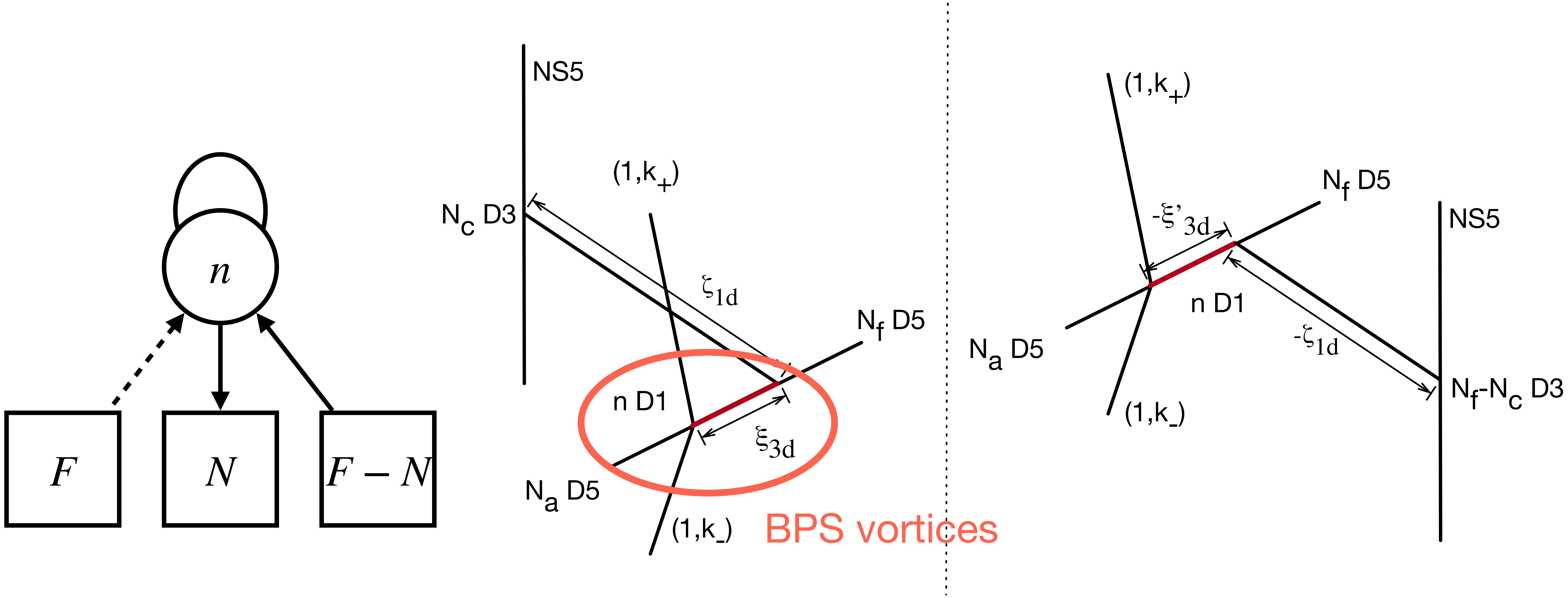

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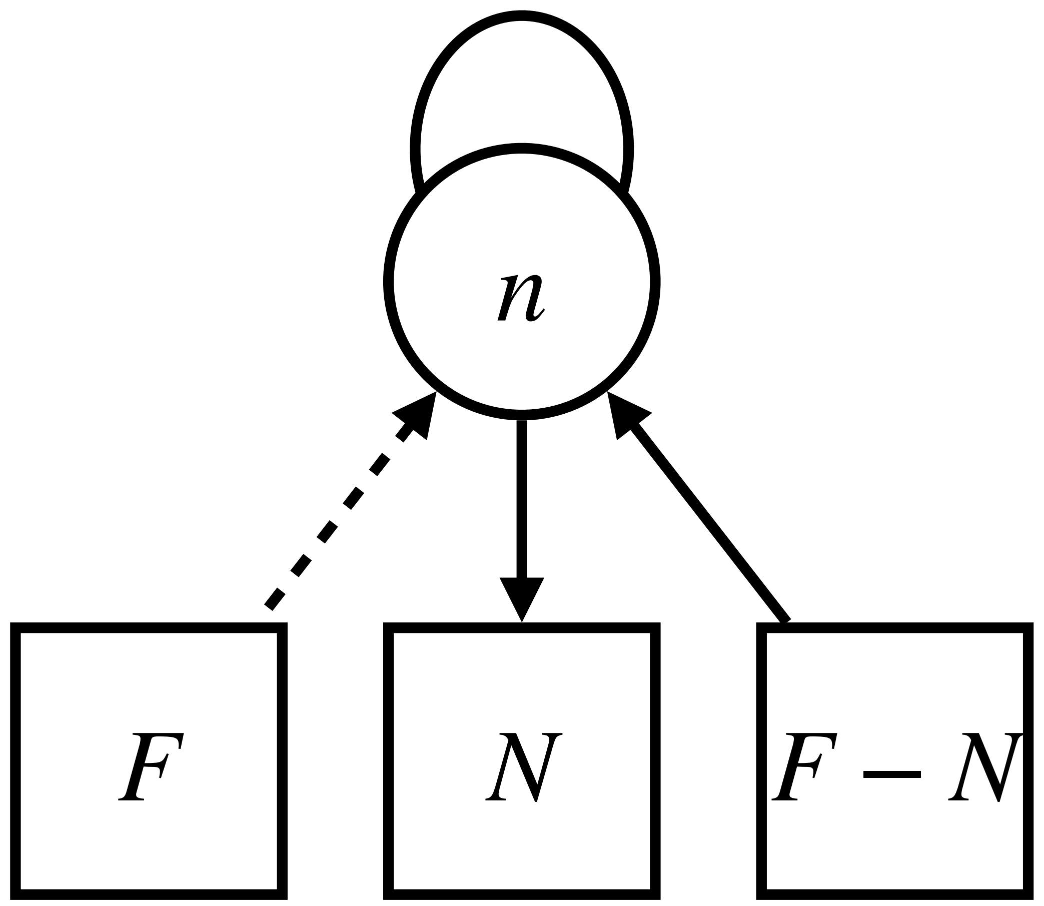

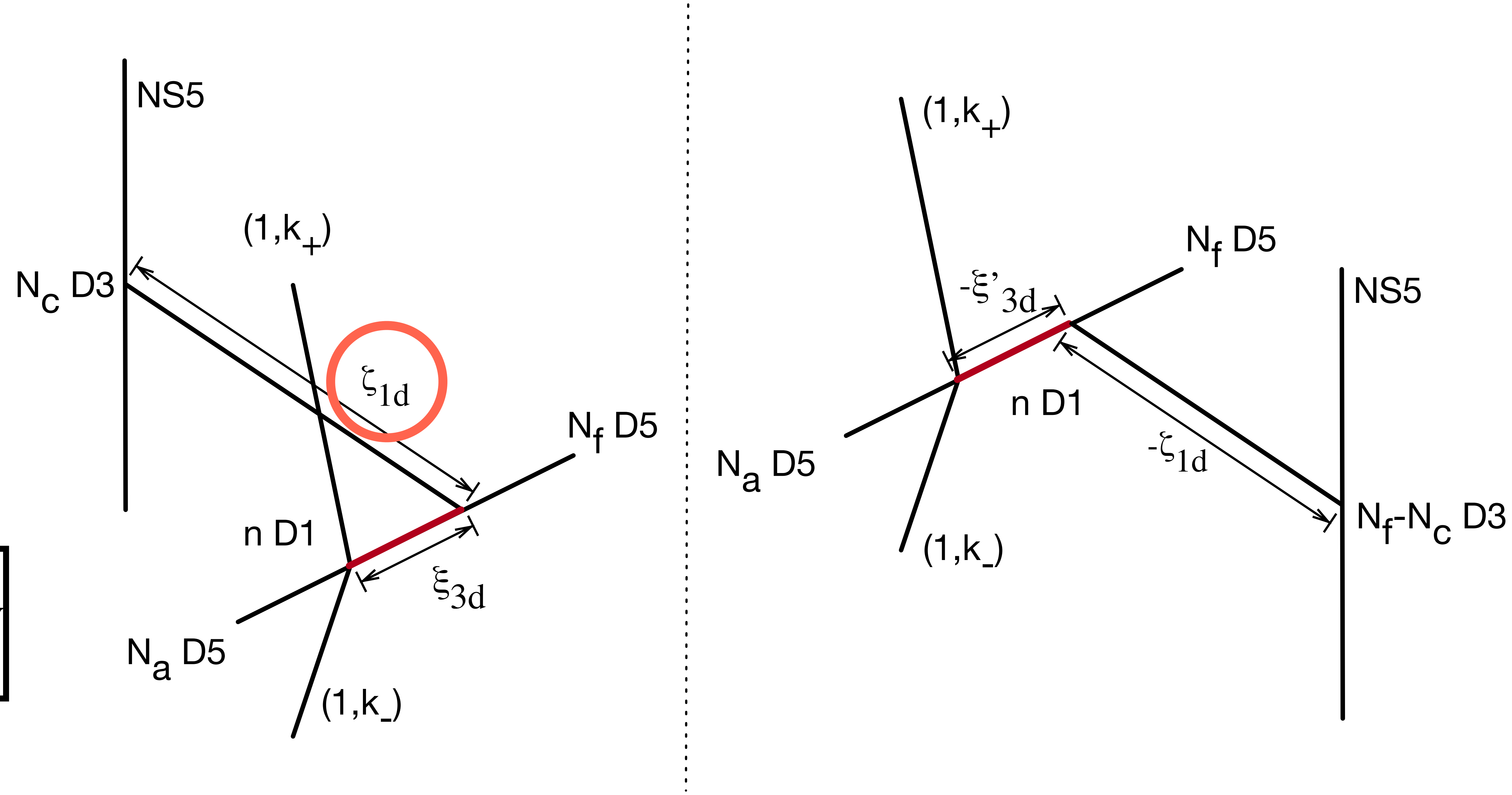

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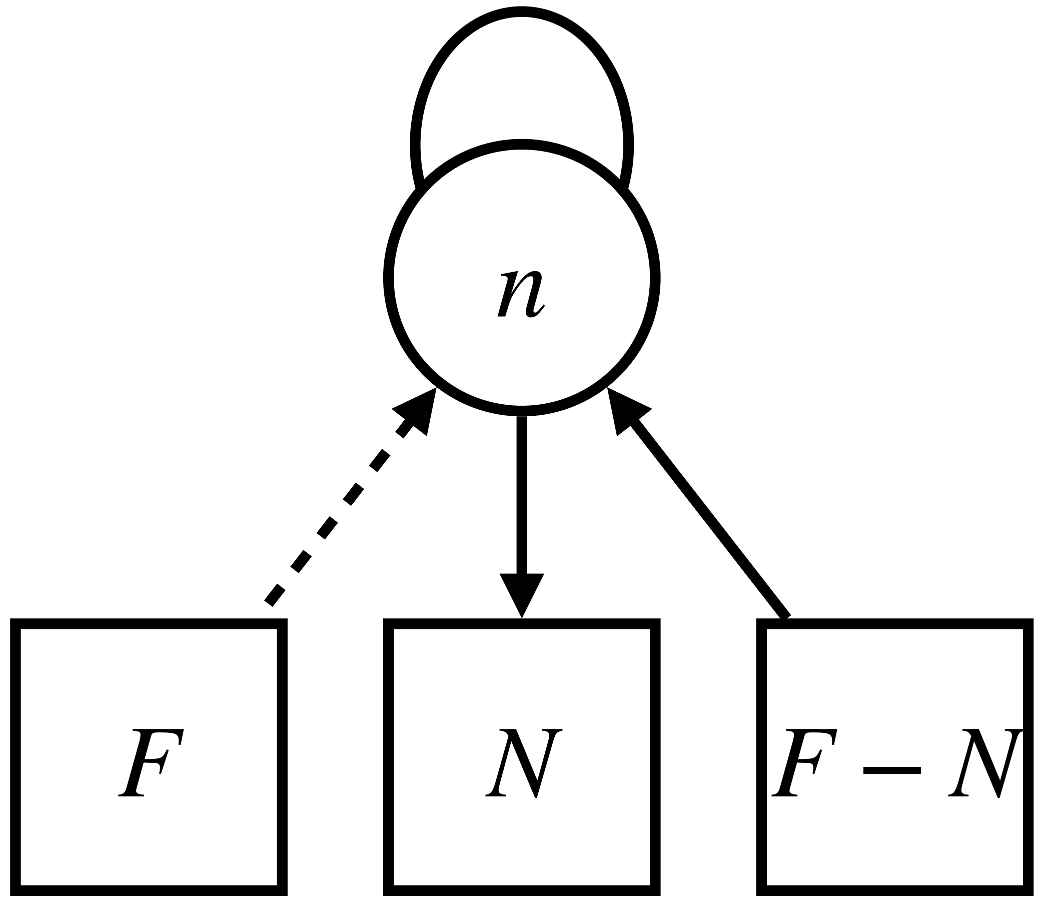

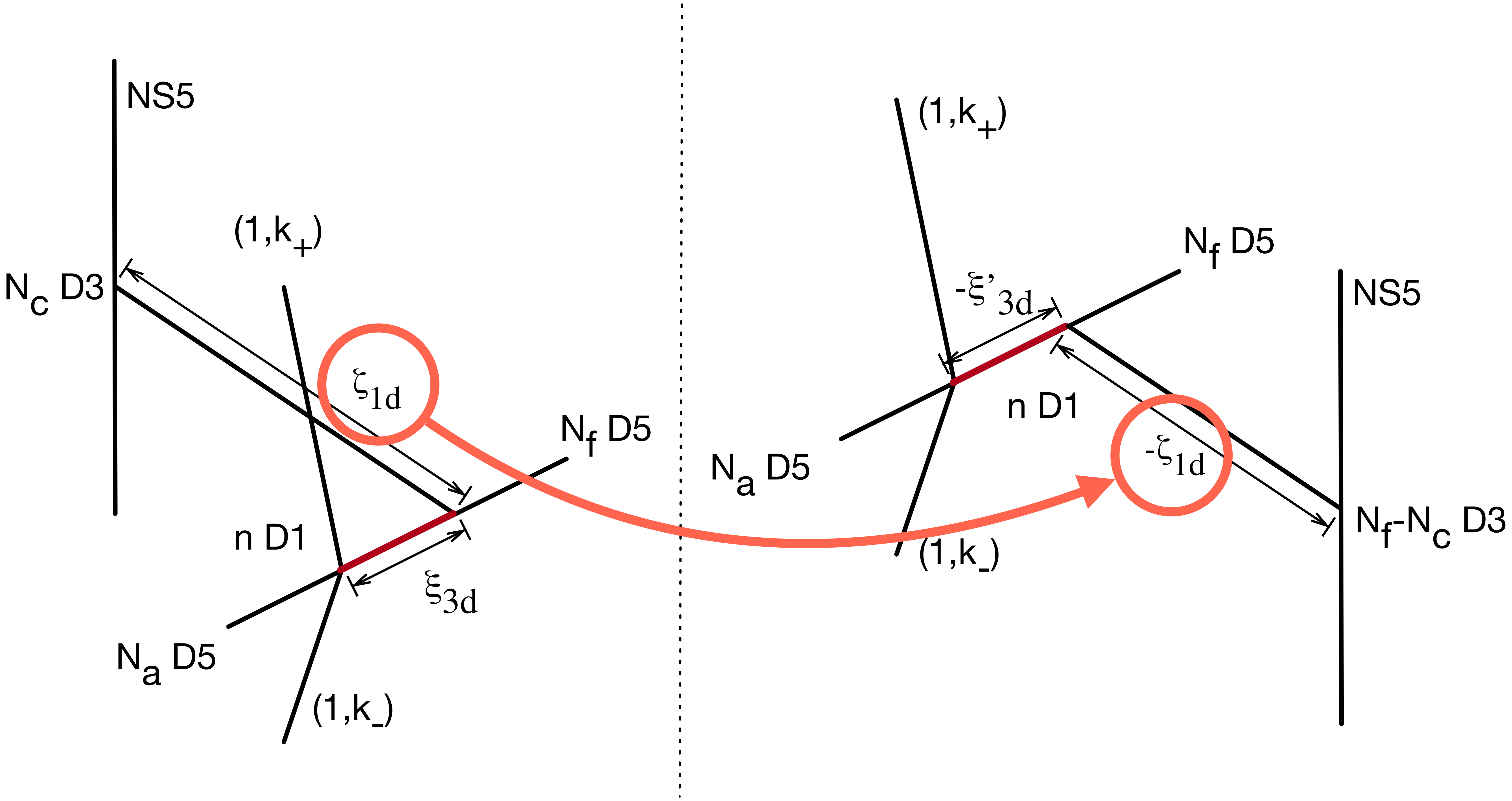

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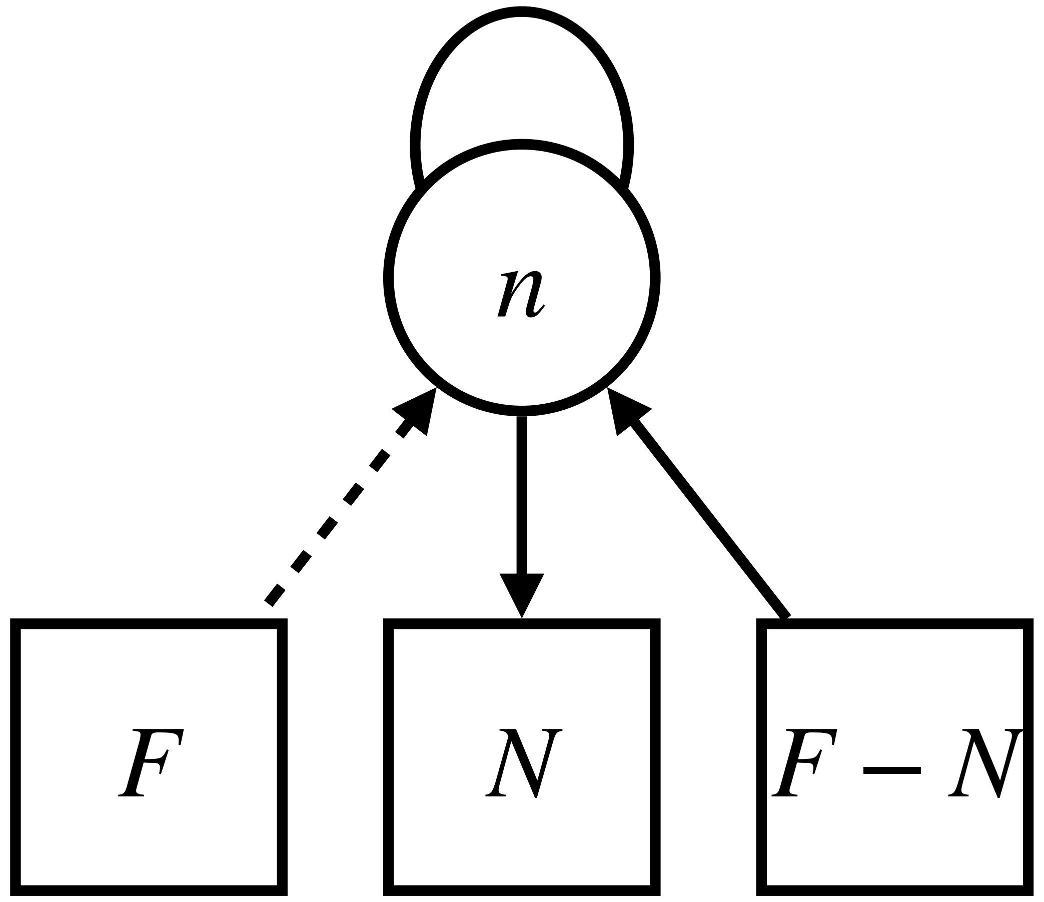
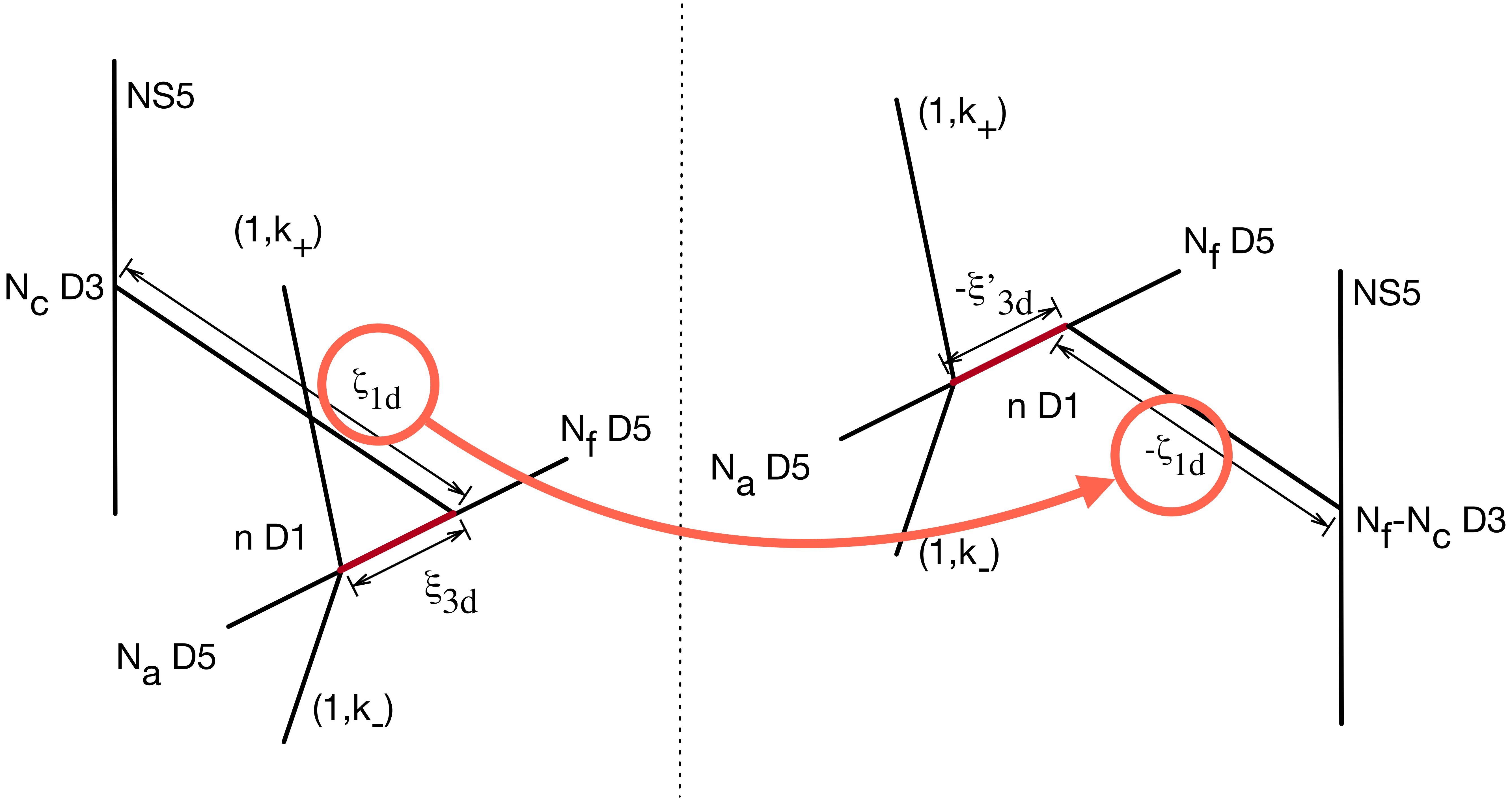




The Aharony duality of a 3d gauge theory = the wall-crossing of a 1d vortex GLSM

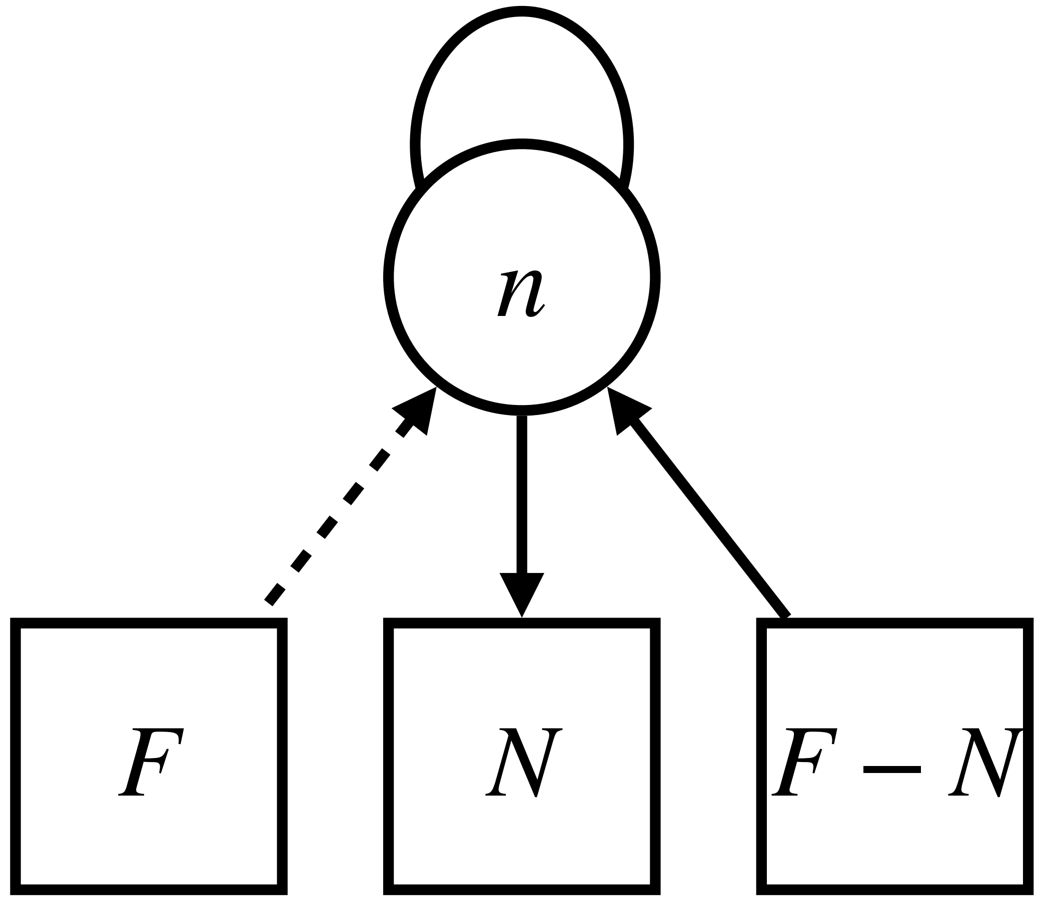
Alternative method for computing the vortex partition function Z_{vortex}

$$I = \sum_{\text{Higgs vacua}} Z_{pert}\, Z_{vortex}\, \bar{Z}_{pert}\, \bar{Z}_{vortex}$$

$$\Downarrow$$

$$Z_{vortex} = \sum_n w^n Z_n$$

$$Z_n = \frac{1}{|W|} \text{JK-Res}_{\vec{\eta}=\zeta\vec{1}} \left[g(u)\, d^n u \right]$$



$$g^n(u) = \frac{\left(\prod_{i\neq j}^{n} \sinh\frac{u_i - u_j}{2}\right)\left(\prod_{j=1}^{n}\prod_{a=1}^{F} \sinh\frac{u_j - \tilde{m}_a + \mu - \gamma}{2}\right)}{\left(\prod_{i,j}^{n} \sinh\frac{u_i - u_j - 2\gamma}{2}\right)\left(\prod_{i=1}^{n}\prod_{b=1}^{N} \sinh\frac{u_i - m_b - \mu - \gamma}{2}\right)\left(\prod_{j=1}^{n}\prod_{a=N+1}^{F} \sinh\frac{-u_j + m_a + \mu - \gamma}{2}\right)}$$

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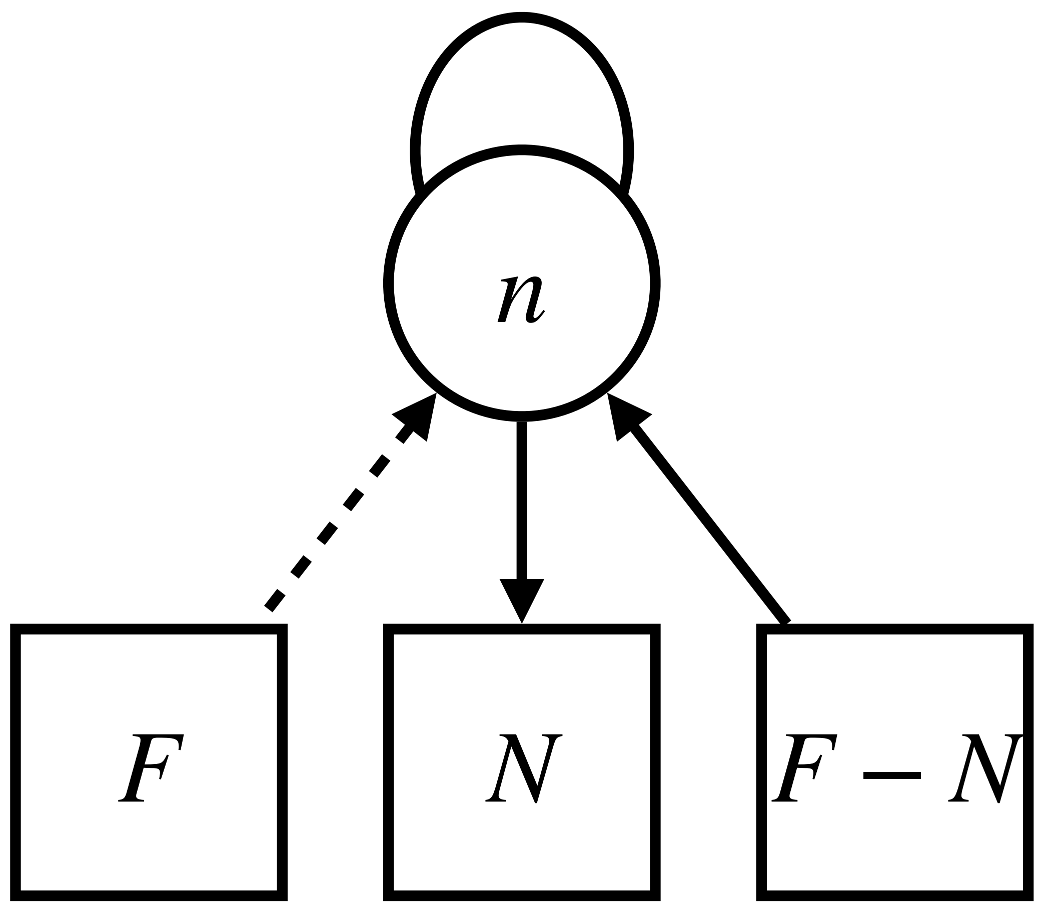
The contribution of each vortex number can be computed using the Jeffrey-Kirwan residue method [CH, Kim, Kim, Park 14, Hori, Kim, Yi 14].



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- For each vortex sector, it can be shown that

$$Z_n(\zeta) \ = \ Z_n(-\zeta) + Z_n^{wall-crossing}$$

↙ ↘

$$\sum_n w^n Z_n = Z_{vortex} \qquad \sum_n w^n \left(Z_n(-\zeta) + Z_n^{wall-crossing}\right) = \tilde{Z}_{vortex}\,\tilde{Z}_V$$

↘ ↙

$$Z_{vortex} \ = \ \tilde{Z}_{vortex}\,\tilde{Z}_V$$

⬇

$$I \ = \ \tilde{I}$$

- For each vortex sector, it can be shown that

Residues inside the integration circle $Z_n(\zeta) \;=\; Z_n(-\zeta) + Z_n^{wall-crossing}$

$$\sum_n w^n Z_n = Z_{vortex}$$

$$\sum_n w^n \left(Z_n(-\zeta) + Z_n^{wall-crossing}\right) = \tilde{Z}_{vortex}\tilde{Z}_V$$

$$Z_{vortex} \;=\; \tilde{Z}_{vortex}\tilde{Z}_V$$

⬇

$$I \;=\; \tilde{I}$$

- For each vortex sector, it can be shown that

Residues inside the integration circle $\quad Z_n(\zeta) \;=\; Z_n(-\zeta) + Z_n^{wall-crossing} \quad$ Residues outside the integration circle

$$\sum_n w^n Z_n = Z_{vortex} \qquad\qquad \sum_n w^n \left(Z_n(-\zeta) + Z_n^{wall-crossing} \right) = \tilde{Z}_{vortex} \tilde{Z}_V$$

$$Z_{vortex} \;=\; \tilde{Z}_{vortex} \tilde{Z}_V$$

$$\Downarrow$$

$$I \;=\; \tilde{I}$$

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Residues inside the integration circle $$Z_n(\zeta) \;=\; Z_n(-\zeta) + Z_n^{wall-crossing}$$ Residues outside the integration circle

$$\sum_n w^n Z_n = Z_{vortex}$$

$$\sum_n w^n \left(Z_n(-\zeta) + Z_n^{wall-crossing}\right) = \tilde{Z}_{vortex}\tilde{Z}_V$$

$$Z_{vortex} \;=\; \tilde{Z}_{vortex}\tilde{Z}_V$$

$$I \;=\; \tilde{I}$$

Provides a proof of the index identity motivated by a physical D-brane picture

Concluding Remarks

- The $\mathbb{D}_p[SU(N)]$ theories enjoy other dualities such as 3d mirror symmetry and the flip-flip duality.

- Our confining deformation can be translated into Higgsing potential by the mirror symmetry and the flip-flip duality.

- Another realization of confinement as dual Higgs mechanism.

- The Aharony duality, or its monopole deformed cousin, is a **building block** of various supersymmetric 3d dualities, such as the Seiberg-like duality with an adjoint matter and 3d mirror symmetry [CH, Pasquetti, Sacchi 21].

- Their fundamental mechanism must be universal.

Many possible generalizations

- Relaxing the conditions among the parameters
- Multiple adjoints with ADE-type superpotentials
- Non-supersymmetric counterparts?

$$W_A = \mathrm{tr}\left(X^{p+1} + Y^2\right)$$ Kim, Park 13

$$W_D = \mathrm{tr}\left(X^{p+1} + X\,Y^2\right)$$ CH, Kim, Park 13

$$W_{E_6} = \mathrm{tr}\left(Y^3 + X^4\right)$$

$$W_{E_7} = \mathrm{tr}\left(Y^3 + YX^3\right)$$

$$W_{E_8} = \mathrm{tr}\left(Y^3 + X^5\right)$$

- Many versions of 3d bosonization/particle-vortex dualities, resembling supersymmetric mirror symmetry, and generalized level-rank dualities of Chern-Simons-matter theories

- Further relations between SUSY dualities and non-SUSY dualities?

谢谢!