Black holes and quantum gravity

Seok Kim

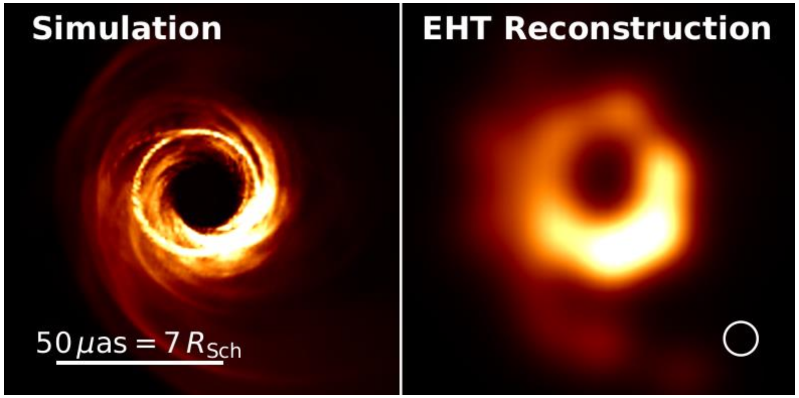
(Seoul National University)

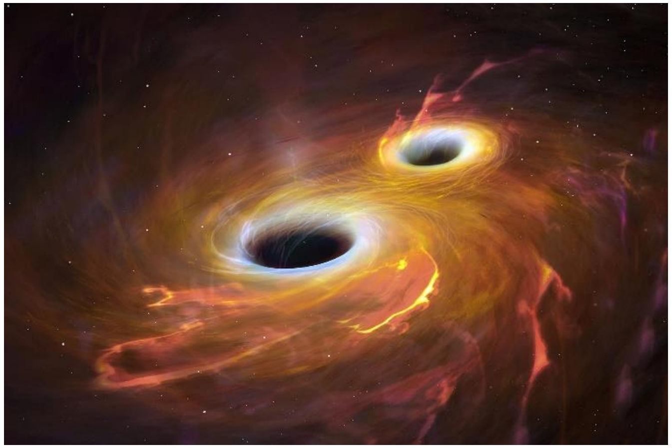
ICTS/PCFT colloquium, USTC

May 17, 2024

Exotic properties of black holes

Golden era of observational black hole physics:





The conceptual novelties that make black holes fascinating have been around for a long while.

- Singularity
- Event horizon & information problems

Guidance to the physics of quantum gravity

- Probes the fundamental aspects of Nature

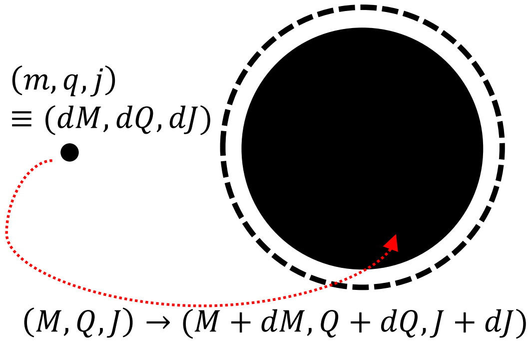
Today's topic: interplay of the two themes quantum gravity & black holes

- Quantum gravity (say string theory) explains black hole physics.
- Black holes constrain/predict novel "quantum gravity" physics in extreme conditions

No hair, missing information

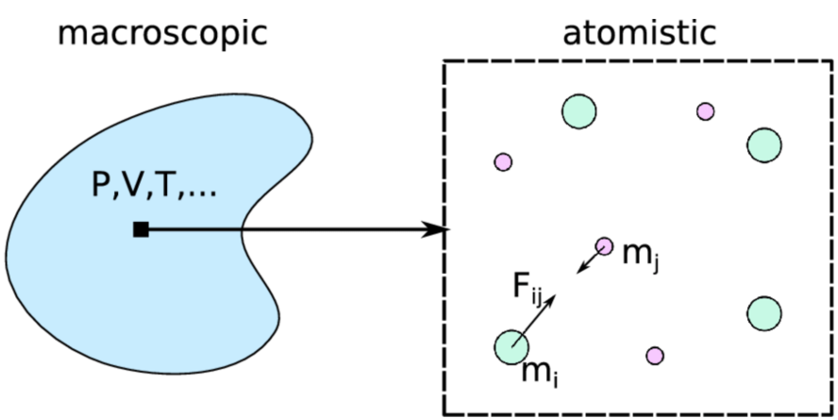
"No hair theorem"

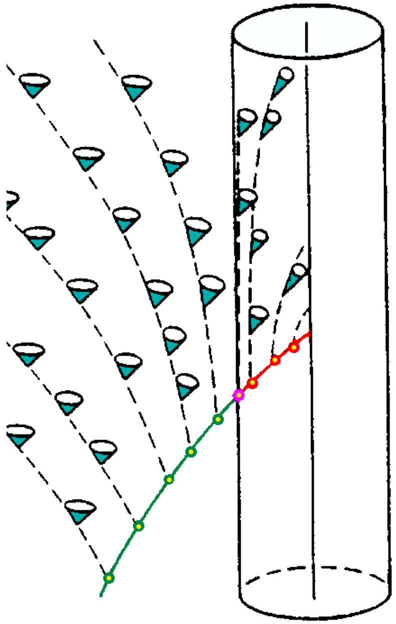
- Black holes (often) settle down to simple solutions labeled by conserved charges
- Mass (~energy), charge, angular momentum
- No information on matters which collapse to form black holes.



The "no hair" behavior, or missing information, is related to the existence of "event horizon"

A bit like macroscopic systems: Information is "hard" to access.

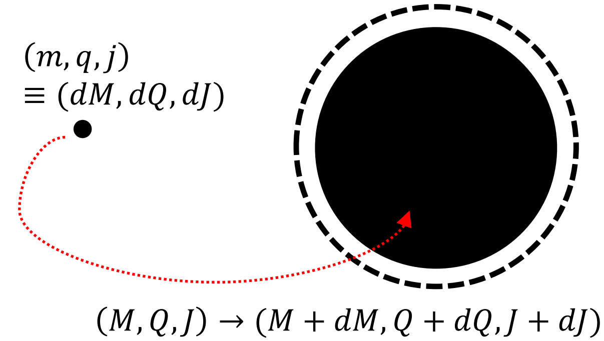




Thermodynamics

Surprisingly, the missing information of black holes induces "thermal behaviors"

- 1st law: Slowly perturb BHs: behaves as if it absorbs "heat"



$$dM = \frac{\kappa}{8\pi G} dA + \Omega dJ + \Phi dQ$$

κ: surface gravity at the event horizon
A: area of the even horizon

- 2nd law: obeys "area law" [Hawking] (1971)

$$\frac{dA}{dt} \geq 0$$

Hawking radiation strongly implies that these rules should be more than analogy.

$$\frac{c^2\kappa}{8\pi G} dA = TdS \quad \to \quad S_{BH} = \frac{A\,c^3}{4G\,\hbar}$$ Bekenstein-Hawking entropy, $T = \frac{\kappa\,\hbar}{2\pi\,c}$ Hawking temperature

Further studies over the past ~50 years:

- Establishing that this thermodynamics is based on statistical mechanics.
- Exploring the implications of the novel thermodynamics of black holes.

Entropy

Entropy is a very subtle notion in many ways.

- It is a quantum property: #(states) at given macroscopic data

$$S(E, V, \ldots) = \log(\#\, of\ states\ at\ fixed\ E, V, \ldots)$$

- Measures "possibilities" rather than a definite phenomenon.
- Measures "ignorance": Information hidden behind the event horizon

- It is a useful quantity from which direct thermodynamic observables can be easily understood, but entropy itself is a very abstract quantity.

$$\frac{1}{T} = \frac{\partial S}{\partial E} \quad , \quad \text{heat} = \Delta Q = T\Delta S \quad , \quad \ldots\ldots$$

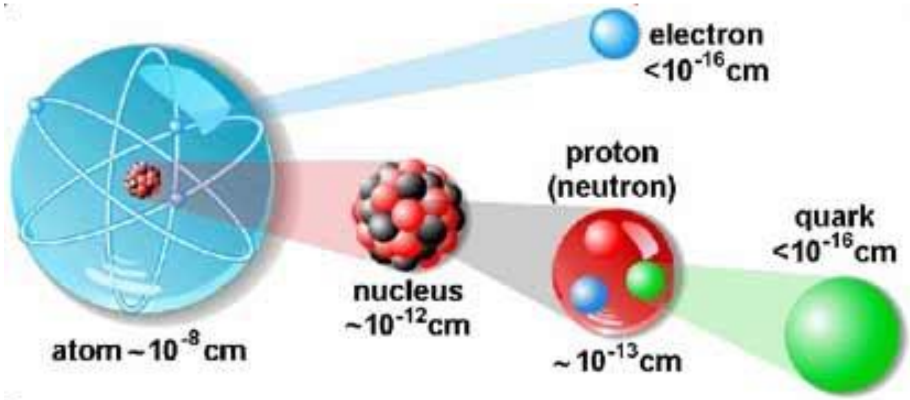
It has been a key "theoretical observable" for the quantum theory of gravity.

- Statistical account from a quantum system which makes the black hole. [Strominger, Vafa]

Bekenstein bound of entropy

Sounds strange that the information on the matter (forming BH) is definitely given.

- We rarely see short-distance information by looking at big bodies, especially in the particle physics/quantum field theory which usually have UV cutoffs.



Entropy has un upper bound: $S \leq \frac{2\pi k_B R E}{\hbar c}$. (E: energy, R: “size” of body) [Bekenstein] (1973,1981)

- The black hole entropy S_{BH} saturates this bound.
- May thus be regarded as the most fundamental, fine-grained, information of Nature.

S_{BH} is an exceptional channel to see the fundamental information of Nature.

- What are the matters made of, at the most elementary level?

Exotic entropy of black holes

Have negative specific heat: It should lose energy to be hotter.

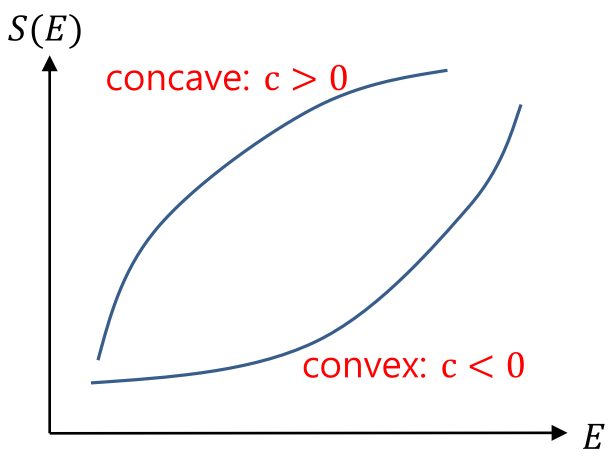
- For instance, for Schwarzschild black holess,

$D = 3 + 1$: $$T_{\rm H} = \frac{\hbar c^3}{8\pi G M k_{\rm B}} \approx 6.169 \times 10^{-8}\ {\rm K} \times \frac{M_\odot}{M}$$

$D \geq 3 + 1$: $GM \sim r^{D-3}$, $S \sim A/G \sim r^{D-2}/G$

$$\rightarrow S(M) \sim G^{\frac{1}{D-3}} M^{\frac{D-2}{D-3}} \rightarrow \frac{dS(E)}{dE} = \frac{1}{T} \sim M^{1/(D-3)}$$

- Why? Entropy grows too fast: $\frac{dS}{dE} > 0$ and $\frac{d^2S}{dE^2} > 0$.



“Unstable” in canonical ensemble, coupled to heat bath.

If we regard this entropy as representing fundamental degrees of freedom, it raises various puzzling implications.

It demands unusual structures of the “Hilbert space” (vector space of quantum states).

- Very fast growing entropy in the high energy regime (~ black hole mass).

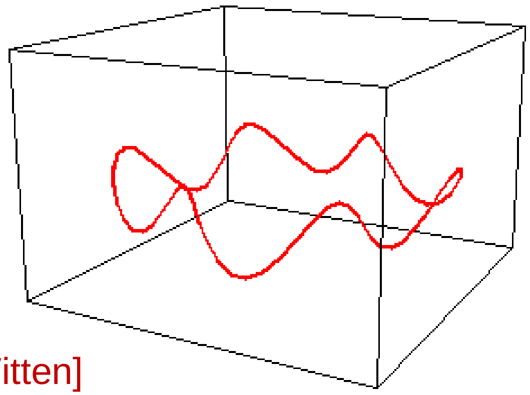
Exotic entropy of quantum gravity

Fast-growing entropy at high E is unfamiliar in standard particle physics.

- Relativistic particles in D space dimension at high E (← almost massless particle):
- On dimensional grounds ($c = 1$, $\hbar = 1$), one finds

$$S \propto VT^D \quad , \quad E \propto VT^{D+1} \quad \rightarrow \quad S(E,V) \propto V^{\frac{1}{D+1}} E^{\frac{D}{D+1}}$$

- $S \propto E^\alpha$ with $\alpha < 1$ is not too fast: $d^2S/dE^2 \sim \alpha(\alpha-1)E^{\alpha-2} < 0$



However, familiar in quantum gravity ~ string theory

- Elementary strings: $S(E) \sim E/T_H$ at high E
- Due to ∞ tower of string oscillation modes:
- We call this "Hagedorn growth" [Hagedorn] (1965) [Sundborg] [Atick, Witten]

(Originally found in the context of hadron physics)

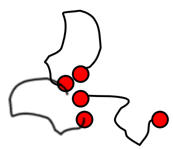
This perturbative entropy doesn't grow fast enough: $S_{BH} \propto M^{\frac{D-2}{D-3}} \gg M$.

- Non-perturbative degrees of freedom should play roles. (→ next slide)

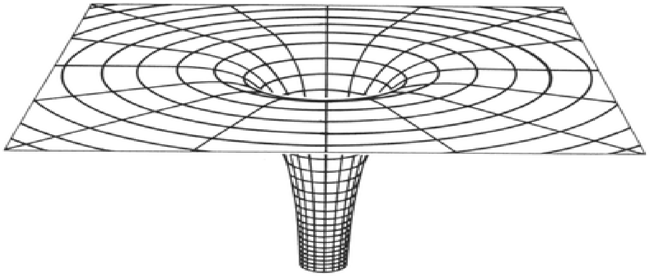
Statistical entropy of extremal black holes

To form such black holes, non-perturbative objects like “D-branes” are needed.

- New bound states of D-branes [Polchinski] (1995) & open strings → faster growth.



strong coupling →



Do honest statistical mechanics of
D-brane quantum mechanics or QFT

Agrees w/ emergent gravitational picture?

Statistical interpretation of S_{BH} justified w/ extremal “BPS” black holes:

- Carry electric charge & saturate the bound $M \geq \# Q$.
- $D = 5$: $S = 2\pi Q^{3/2}$ [Strominger, Vafa] (1996) …
- $D = 4$: $S = 2\pi Q^2$ [Maldacena,Strominger,Witten] [Vafa] (1997) …

Since Q is M, similar growth as Schwarzschild BH:

- $S_{BH} \propto M^{(D-2)/(D-3)}$
- Unstable in grand canonical ensemble
  (negative susceptibility)

Andrew Strominger & Cumrun Vafa

Black hole entropy & quantum gravity

The fast growth of the BH entropy is intimately related to the fundamental structure of the Hilbert space of quantum gravity, as least in string theory models.

- Historically, introducing so many extra d.o.f. to Einstein's relativity as strings was to avoid very technical inconsistencies of quantum gravity (UV divergences).
- Now we may also understand it as a constraint from black hole thermodynamics.

It will be interesting to review how other quantum gravity models explain black hole thermodynamics, related to the number of fundamental d.o.f. they have.

- Also, to see if the picture/scenario I presented has interesting caveats.

For instance,

- Higher spin gravities, loop quantum gravity, asymptotically free quantum gravity [Weinberg]

Currently I don't have good enough insights to answer these questions.

Cannot be true…

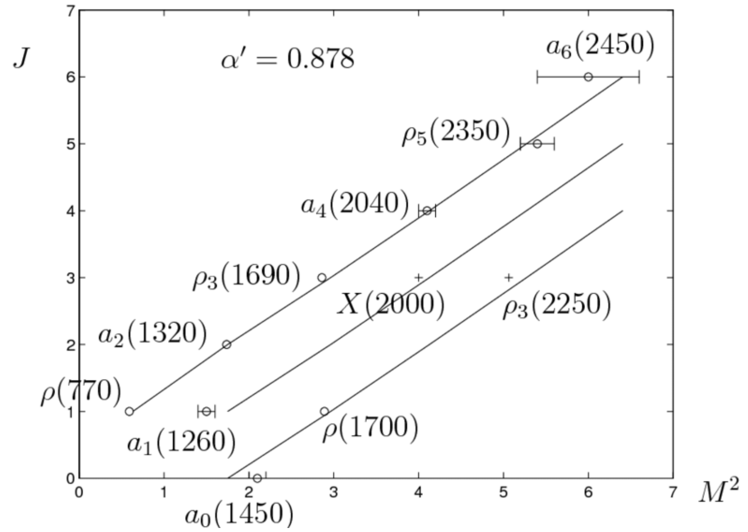
Systems with $c < 0$ are thermodynamically unstable in canonical ensemble.

- This might be an exotic feature of quantum gravity, but sounds weird.
- It is also possible that quantum gravity at high T/E is fundamentally reformulated.

Similar to QCD: tower of hadrons vs. quark-gluon

→ QCD at high T is in "quark-gluon plasma phase"

- Tempting to speculate that QG states that we just saw from black holes originate from more fundamental (& fewer) d.o.f. visible at even higher E.



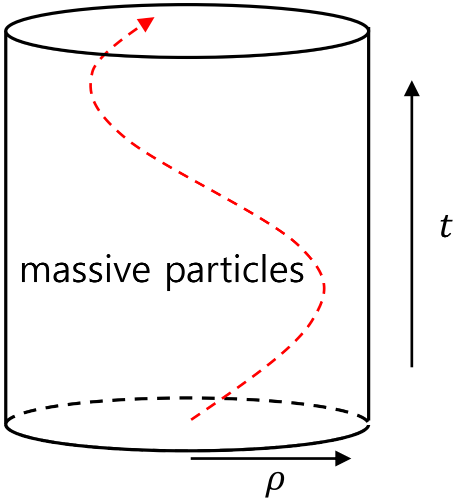
One more general subtlety of gravity at fixed T.

- Needs "finite volume" or IR regulators. (Extensive quantities ∝ "volume")
- Cannot put gravity in an artificial box: Everything is subject to equivalence principle.

Even these subtle questions can be answered with black holes.

- I'll present a model of gravity in a "box" and better phrase these questions.

Anti de Sitter (AdS)



A theoretically consistent setup for the "finite box"

- Put gravity in AdS: [Hawking, Page] (1983)

$$ds^2_{D+1} = d\rho^2 - \cosh^2\frac{\rho}{\ell}\,dt^2 + \ell^2\sinh^2\frac{\rho}{\ell}\,ds^2(S^{D-1})$$

- Technically, the above metric is for the "global AdS" spacetime.

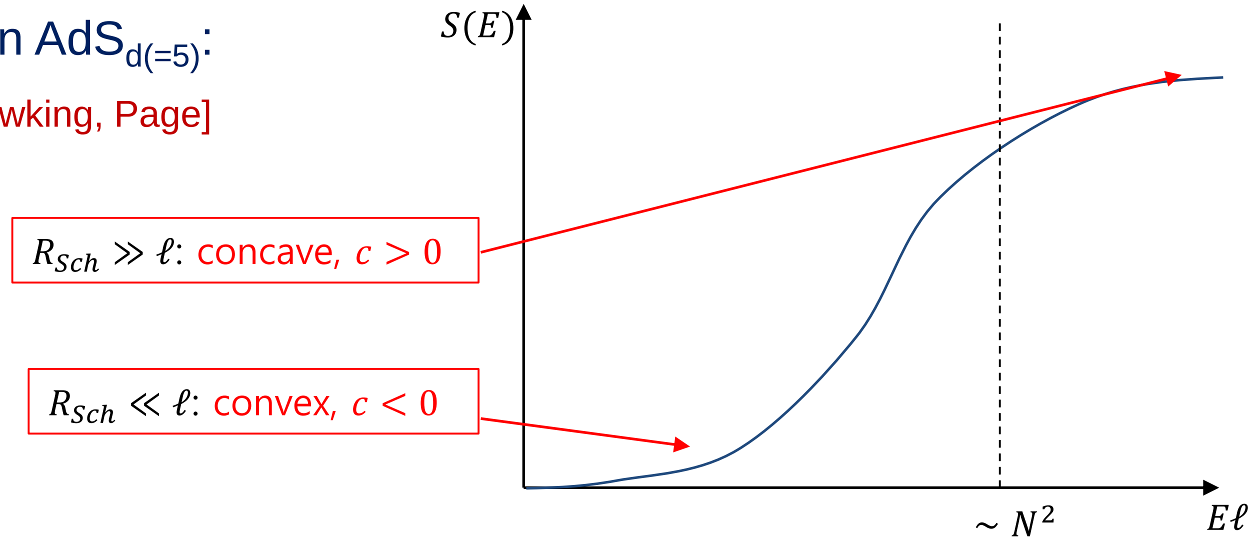
- Confines to $\rho = 0$: $\Phi \approx -g_{tt}(\rho)/2$. Massive particles (also black holes) cannot escape.
- Renders many thermal questions of QG well defined.

• Incidentally, we know a microscopic description of such quantum gravity.
  - We call it "AdS/CFT correspondence" [Maldacena] (1997) ……
  - QFT at the boundary $S^{D-1} \times R$ 'holographically' describes the QG inside.
  - In simple examples, SU(N) gauge theories at $N \gg 1$: large number of "gluons"

Black holes in AdS

Schwarzschild black holes in AdS$_{d(=5)}$:

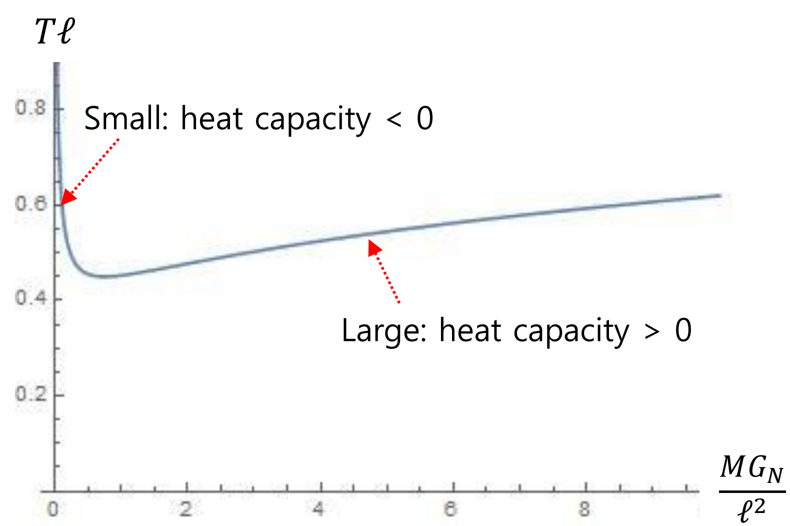
[Hawking, Page]



T-E curve:

$$T = \frac{r_+}{\pi \ell^2} + \frac{1}{2\pi r_+}$$

$$r_+^2 = -\frac{\ell^2}{2} + \ell\sqrt{\frac{\ell^2}{4} + \omega M} \qquad \omega \equiv \frac{16\pi G_N}{3\mathrm{vol}(S^3)}$$



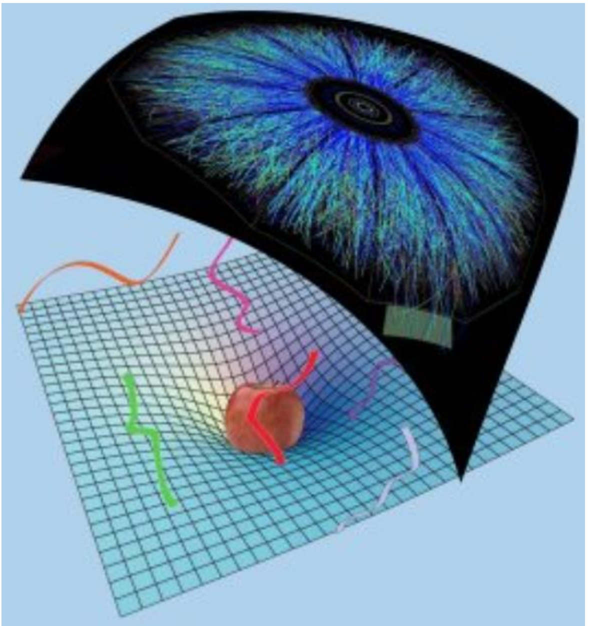
Features:

- Small BH: Like familiar BH's in the sky.
  But exotic thermodynamics.
- Large BH: Novel black holes filling the box.
  But ordinary thermodynamics.
  Determines the phase of the system.

Phase transition & black holes

Quarks/gluons in different phases. (like liquid/gas)

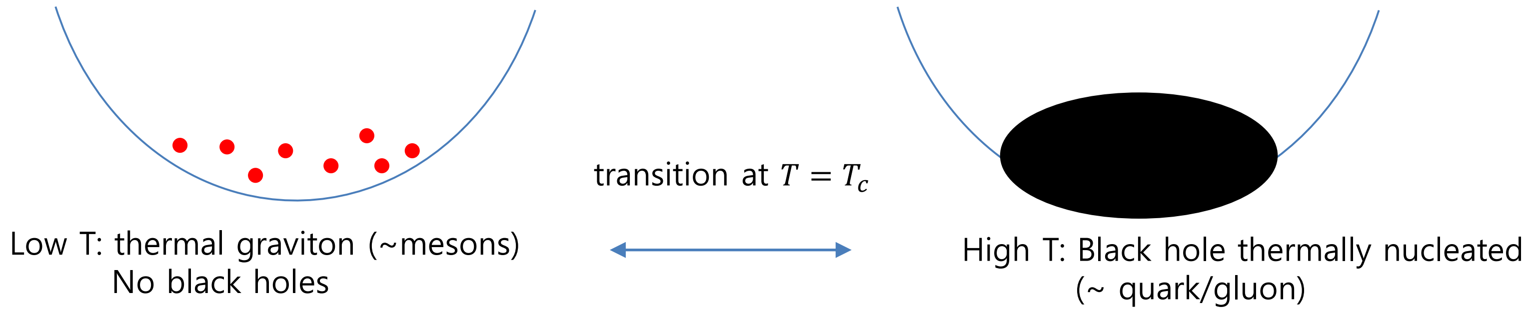
- low T : Confinement. Only see hadrons, as in nuclei.
  (like protons ~ uud, mesons ~ $\bar{u}d$, …)
- high T: Deconfinement. Liberated "quark-gluon plasma"



Phase transition at $T_c \sim 1/\ell$. ("radius of curvature")

confinement-deconfinement phase transition

In AdS: Hawking-Page phase transition (1983)



Expect: Deconfined quarks/gluons are boundary images of black holes. [Witten] (1998)

Pictures of "hot" gravity

To realize Witten's picture, we studied "exactly solvable models" & precisely saw black holes holographically. [Sunjin Choi, Joonho Kim, SK, Nahmgoong] (2018), …

- Free energy (partition function) at temperature T

$$\log Z \sim \frac{2(3c-2a)(2\pi i+2/T)^3}{27} T^2$$

[J. Kim, SK, Song] (2019)

- "central charges" $c, a \propto N^2$ represent large number of gluons.
- Reproduces S_{BH} of very hot black holes trapped in AdS:

$$S_{BH} = A_{BH}/4G = \sqrt{3}\,\pi\,[2(3c-2a)E^2]^{1/3}$$

It is indeed contributions from deconfined gluons.

The physics is basically like water vs. vapor:

- "liquid ~ gravitons" vs. "gas ~ black holes"

Large black holes represent gravity in an extreme phase at high T.

- Gravitons are not very relevant (no mesons), unlike in Einstein's general relativity.
- Should have deep implications to, say, very early universe.

Lessons from black holes

Lesson 1: quantum gravity

- Black hole entropy grows fast & suggests a large # of states of at high E.
- Quantum gravity is more delicate than naive particle physics of quantizing general relativity.
- In my mind, this is what makes string theory a more natural description.

Lesson 2: extreme phases of quantum gravity

- At even higher E, quantum gravity should undergo a phase transition.
- All objects in the traditional description of gravity/string theory, like gravitons, strings (& D-branes, etc.), lose their meanings.
- May shed lights on how our universe should behave in extreme conditions. I don’t yet know how/whether I can concretely implement these ideas to such extreme situations.