



中国地质大学(武汉)

数学与物理学院

School of Mathematics and Physics

$$v' = \frac{v + u}{1 + \frac{vu}{c^2}}$$
$$v'' = \frac{v + u}{1 + \frac{vu}{c^2}}$$
$$v''' = \frac{v + u}{1 + \frac{vu}{c^2}}$$

$$x = ut \cos(\alpha)$$



$$4u$$

$$E = mc^2$$



TESTING UNRUH EFFECT USING QUANTUM MANY-BODY STATES

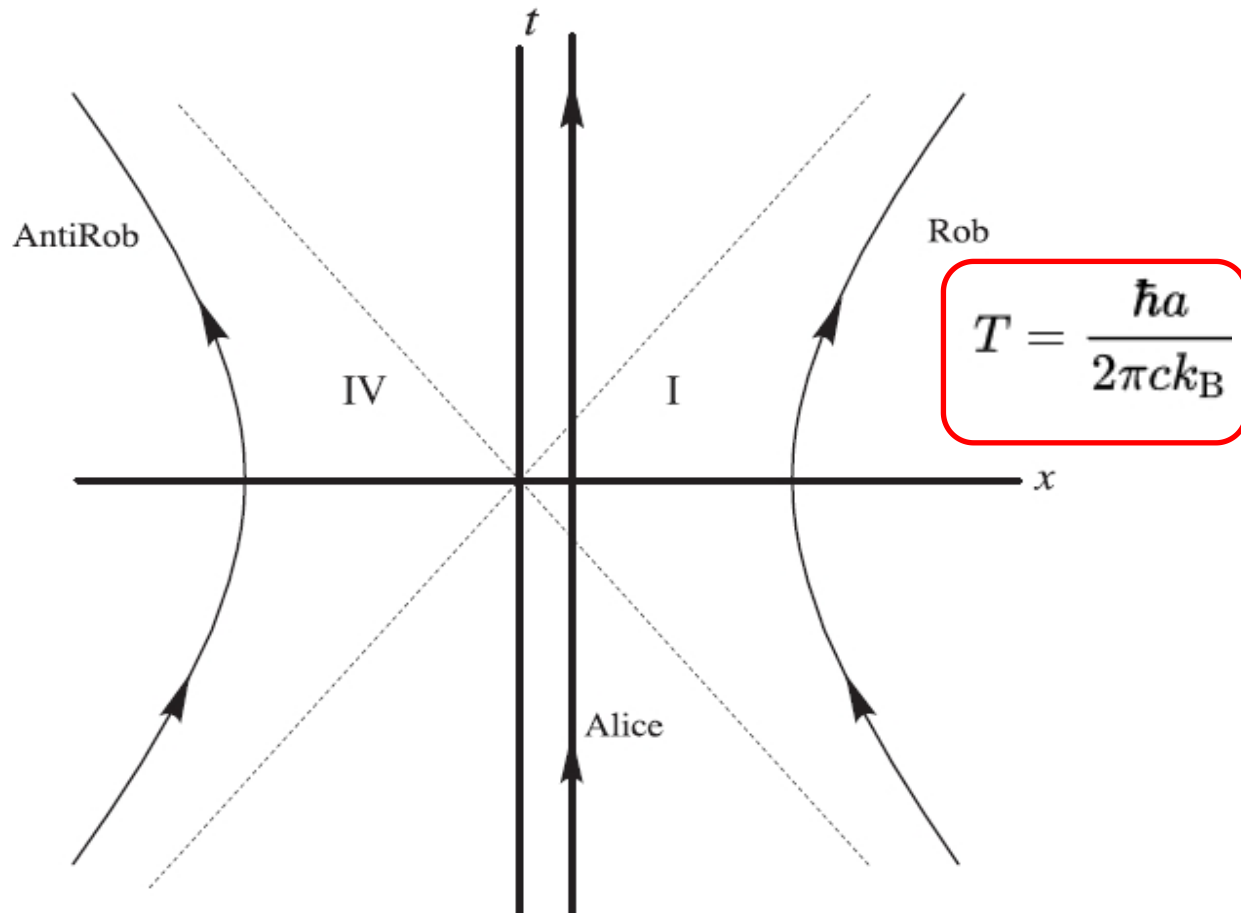
Baocheng Zhang

2021年4月 中科大彭恒武高能基础理论研究中心/交叉中心

Outline

- Unruh and Anti-Unruh effects
- Quantum many-body states
- Influence of acceleration
- Conclusion and future

Unruh effect



$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \quad ds^2 = e^{2a\xi}(d\tau^2 - d\xi^2) - dx^2 - dy^2$$

$$t = a^{-1}e^{a\xi} \sinh a\tau, \quad z = a^{-1}e^{a\xi} \cosh a\tau.$$

Quantum Optics Route

Unruh acceleration radiation revisited

J. S. Ben-Benjamin,^{1,3} M. O. Scully,^{1,3,4} S. A. Fulling,¹ D. M. Lee,¹ D. N. Page,^{1,5}
A. A. Svidzinsky^{1,3} and M. S. Zubairy¹

¹*Institute for Quantum Science and Engineering,
Texas A&M University, College Station, TX 77843, USA*

M. J. Duff,^{2,6,7} R. Glauber,^{2,8} W. P. Schleich^{2,4,9} and W. G. Unruh^{2,10}

²*Hagler Institute for Advanced Studies,
Texas A&M University, College Station, TX 77843, USA*

³*Baylor University, Waco, TX 76706, USA*

⁴*Princeton University, Princeton, NJ 08544, USA*

⁵*University of Alberta, Edmonton, T6G 2R3, Canada*

⁶*Theoretical Physics, Blackett Laboratory,
Imperial College London, London SW7 2AZ, UK*

⁷*Mathematical Institute, Andrew Wiles Building,
University of Oxford, Oxford OX2 6GG, UK*

⁸*Harvard University, Cambridge, MA 02138, USA*

⁹*Universität Ulm, D-89069 Ulm, Germany*

¹⁰*University of British Columbia, Vancouver, V6T 2A6, Canada*

Unruh-DeWitt Detector

- (1+1)-dimensional model

$$H_I = \lambda \chi(\tau/\sigma) \mu(\tau) \phi(x(\tau))$$

- The time evolution is determined

$$U = I - i \int d\tau H(\tau) + O(\lambda^2).$$

- The change of the detectors is

$$U|g\rangle|0\rangle = C_0(|g\rangle|0\rangle - i\eta_0|e\rangle|1_k\rangle),$$

$$U|e\rangle|0\rangle = C_1(|e\rangle|0\rangle + i\eta_1|g\rangle|1_k\rangle)$$

$$|\eta_0|^2 = \sum_k |\langle 1_k, e | U^{(1)} | 0, g \rangle|^2 \quad |\eta_1|^2 = \sum_k |\langle 1_k, g | U^{(1)} | 0, e \rangle|^2$$

Experimental confirmation?

PRL **118**, 161102 (2017)

PHYSICAL REVIEW LETTERS

week ending
21 APRIL 2017

Proposal for Observing the Unruh Effect using Classical Electrodynamics

Gabriel Cozzella,^{1,*} André G. S. Landulfo,^{2,†} George E. A. Matsas,^{3,‡} and Daniel A. T. Vanzella^{4,§}

¹*Instituto de Física Teórica, Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271, 01140-070 São Paulo, São Paulo, Brazil*

²*Centro de Ciências Naturais e Humanas, Universidade Federal do ABC, Avenida dos Estados, 5001, 09210-580 Santo André, São Paulo, Brazil*

³*Instituto de Física Teórica, Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271, 01140-070 São Paulo, São Paulo, Brazil*

⁴*Instituto de Física de São Carlos, Universidade de São Paulo, Caixa Postal 369, 13560-970 São Carlos, São Paulo, Brazil*

(Received 10 January 2017; published 21 April 2017)

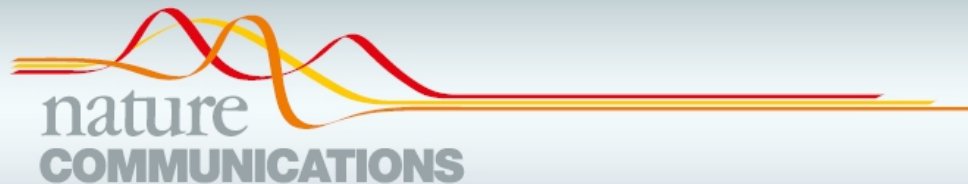
Although the Unruh effect can be rigorously considered as well tested as free quantum field theory itself, it would be nice to provide experimental evidence of its existence. This is not easy because the linear acceleration needed to reach a temperature 1 K is of order 10^{20} m/s². Here, we propose a simple experiment reachable under present technology whose result may be directly interpreted in terms of the Unruh thermal bath. Instead of waiting for experimentalists to perform it, we use standard classical electrodynamics to anticipate its output and fulfill our goal.





Question?

S. Cruz y Cruz and B. Mielnik, Non-inertial quantization: Truth or illusion, *J. Phys. Conf. Ser.* **698**, 012002 (2016).

Experimental confirmation?



Probing the Unruh effect with an accelerated extended system

Cesar A. Uliana Lima¹, Frederico Brito ¹, José A. Hoyos ¹ & Daniel A. Turolla Vanzella¹

It has been proved in the context of quantum fields in Minkowski spacetime that the vacuum state is a thermal state according to uniformly accelerated observers—a seminal result known as the Unruh effect. Recent claims, however, have challenged the validity of this result for extended systems, thus casting doubts on its physical reality. Here, we study the dynamics of an extended system, uniformly accelerated in the vacuum. We show that its reduced density matrix evolves to a Gibbs thermal state with local temperature given by the Unruh temperature $T_U = \hbar a / (2\pi c k_B)$, where a is the system's spatial-dependent proper acceleration— c is the speed of light and k_B and \hbar are the Boltzmann's and the reduced Planck's constants, respectively. This proves that the vacuum state does induce thermalization of an accelerated extended system—which is all one can expect of a legitimate thermal reservoir.

Experimental confirmation?

Quantum simulation of Unruh radiation

Jiazhong Hu ^{*}, Lei Feng , Zhendong Zhang and Cheng Chin 

Rindler transformation

$$\mathcal{H} = i\hbar \sum_k g_k (a_k^\dagger a_{-k}^\dagger - a_k a_{-k})$$

$$\begin{bmatrix} \hat{b}_k^R \\ \hat{b}_k^{\dagger L} \end{bmatrix} = \begin{bmatrix} \cosh(r_k) & \sinh(r_k) \\ \sinh(r_k) & \cosh(r_k) \end{bmatrix} \begin{bmatrix} \hat{c}_k \\ \hat{d}_k^\dagger \end{bmatrix} \quad \begin{bmatrix} a_k(\tau) \\ a_{-k}^\dagger(\tau) \end{bmatrix} = \begin{bmatrix} \cosh(g_k \tau) & \sinh(g_k \tau) \\ \sinh(g_k \tau) & \cosh(g_k \tau) \end{bmatrix} \begin{bmatrix} a_k(0) \\ a_{-k}^\dagger(0) \end{bmatrix}$$

$$\tanh r_k = e^{-\pi E_k c / \hbar A}$$

$$g_k = \frac{1}{2\tau} \ln \coth \left(\frac{\pi E_k c}{2\hbar A} \right)$$

Experimental confirmation?

PHYSICAL REVIEW A **98**, 022118 (2018)

Classical analog of the Unruh effect

Ulf Leonhardt,¹ Itay Griniasty,¹ Sander Wildeman,² Emmanuel Fort,² and Mathias Fink²

¹*Weizmann Institute of Science, Rehovot 761001, Israel*

²*Institut Langevin, ESPCI, CNRS, PSL Research University, 1 Rue Jussieu, F-75005 Paris, France*



(Received 7 September 2017; revised manuscript received 30 November 2017; published 13 August 2018)

In the Unruh effect an observer with constant acceleration perceives the quantum vacuum as thermal radiation. The Unruh effect has been believed to be a pure quantum phenomenon, but here we show theoretically how the effect arises from the correlation of noise, regardless of whether this noise is quantum or classical. We demonstrate this idea with a simple experiment on water waves where we see the first indications of a Planck spectrum in the correlation energy.


Experimental confirmation?

PHYSICAL REVIEW X **9**, 011007 (2019)

Decoherence of the Radiation from an Accelerated Quantum Source

Daiqin Su^{*} and Timothy C. Ralph[†]

*Centre for Quantum Computation and Communication Technology, School of Mathematics and Physics,
The University of Queensland, St. Lucia, Queensland, 4072, Australia*

 (Received 21 December 2017; revised manuscript received 9 October 2018; published 11 January 2019)

Decoherence is the process via which quantum superposition states are reduced to classical mixtures. Decoherence has been predicted for relativistically accelerated quantum systems; however, examples to date have involved restricting the detected field modes to particular regions of space-time. If the global state over all space-time is measured, then unitarity returns and the decoherence is removed. Here, we study a decoherence effect associated with accelerated systems that cannot be explained in this way. In particular, we study a uniformly accelerated source of a quantum field state—a single-mode squeezer. Even though the initial state of the field is vacuum (a pure state) and the interaction with the quantum source in the accelerated frame is unitary, we find that the final state detected by inertial observers appears to be decohered, i.e., in a mixed state. This unexpected result may indicate new directions in resolving inconsistencies between relativity and quantum theory. We extend this result to a two-mode state and find that entanglement is also decohered.

Experimental confirmation?

$$T = \frac{\hbar a}{2\pi c k_B}$$

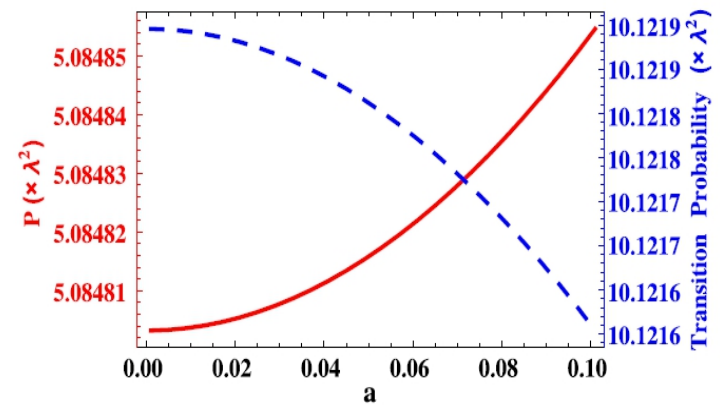
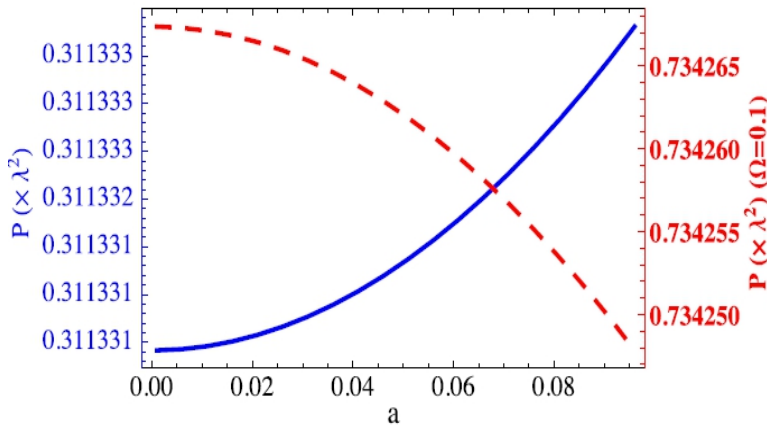
Very difficulty { $1K \sim 10^{20} m/s^2$
Thermal background

Possibility { Low temperature
New phenomena



Anti-Unruh effect

A particle detector in uniform acceleration coupled to the vacuum can cool down with increasing acceleration under certain conditions.

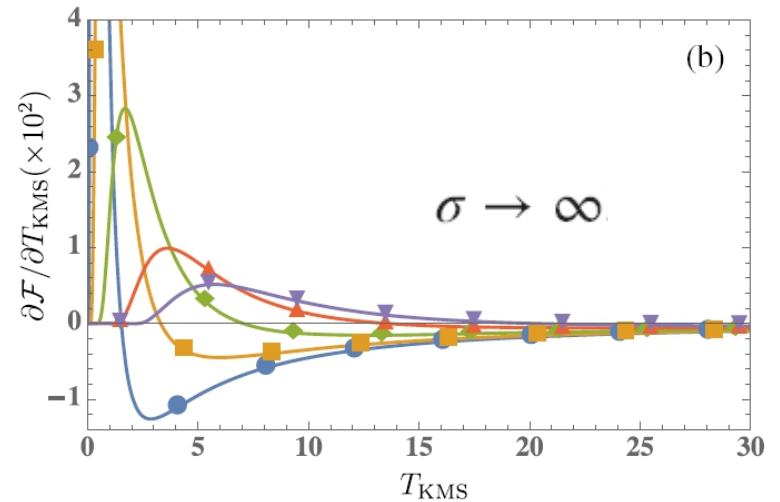
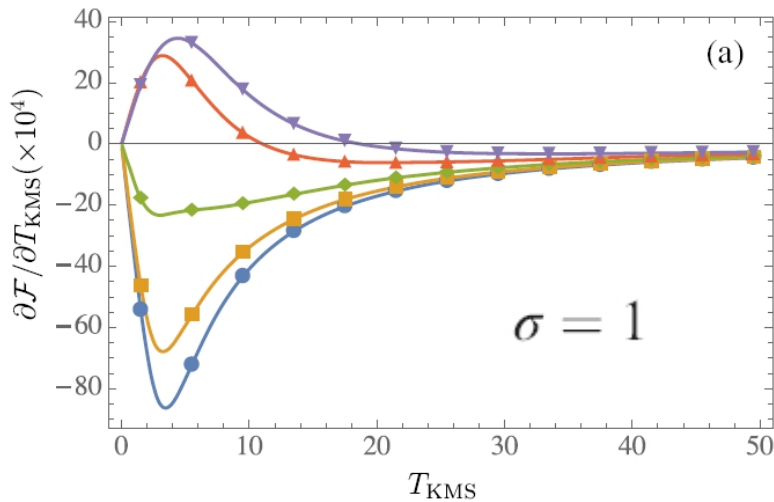


$$\mathcal{P} = \lambda^2 \sum_{n, \epsilon} \left| \int_{-\infty}^{\infty} \frac{d\tau}{\sqrt{4\pi n}} e^{i\Omega\tau + 2\pi n i \left(\frac{\epsilon}{aL} [e^{\epsilon a \tau} - 1] \right) - \tau^2 / 2\sigma^2} \right|^2$$

$$\mathcal{P} = \int_{-\infty}^{\infty} \frac{dk}{4\pi |k|} \left| \int d\tau e^{i \left[\Omega\tau - \frac{sk}{a} |k| (e^{-s_k a \tau} - 1) \right] - \frac{\tau^2}{2\sigma^2}} \right|^2,$$

Anti-Unruh effect

Is this unintuitive “cooling” effect just transient behavior?
 Can it happen under true stationarity conditions?



$$\partial_{\beta} \mathcal{F}(\Omega, \sigma, \beta) > 0 \quad T_{\text{KMS}} = 1/\beta \quad P^+ = \lambda^2 |\langle e | \mu(0) | g \rangle|^2 \sigma \mathcal{F}(\Omega, \sigma)$$

$$\mathcal{F}(\Omega, \sigma) = \frac{1}{\sigma} \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau \chi(\tau/\sigma) \chi(\tau'/\sigma) W(\tau - \tau') e^{-i\Omega(\tau - \tau')}$$

Anti-Unruh effect

The coherence for accelerated atoms

Consider an atom with the initial state

$$|\psi_i\rangle = (\alpha|g\rangle + \beta|e\rangle)|0\rangle \quad |\alpha|^2 + |\beta|^2 = 1$$

After the atom is accelerated

$$|\psi_f\rangle = \alpha|g\rangle|\psi_0\rangle + \beta|e\rangle|\psi_1\rangle$$

$$|\psi_0\rangle = C_0|0\rangle + i(\beta/\alpha)C_1\eta_1|1_k\rangle \quad |\psi_1\rangle = C_1|0\rangle - i(\alpha/\beta)C_0\eta_0|1_k\rangle$$

$$\eta_0 = \lambda \int dk I_{+,k}$$

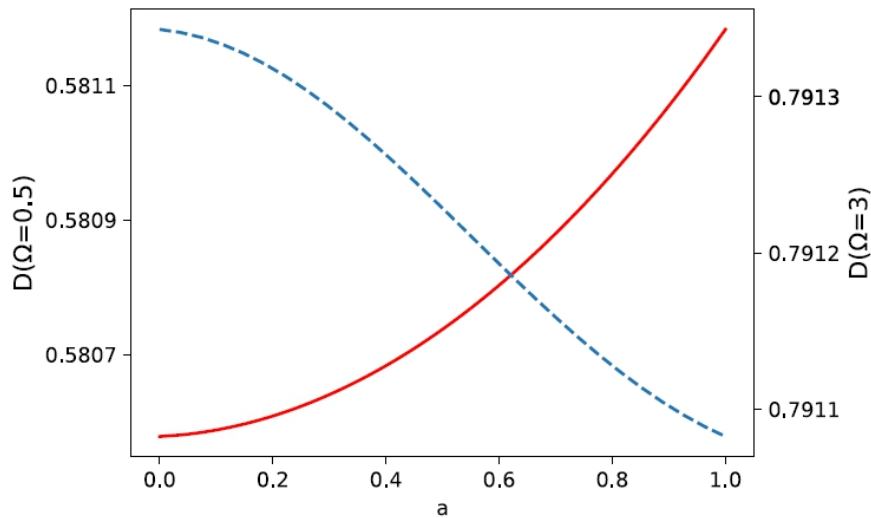
$$\eta_1 = \lambda \int dk I_{-,k}$$

$$I_{\pm,k} = \int_{-\infty}^{\infty} \chi(\tau/\sigma) \exp[\pm i\Omega\tau + i\omega t(\tau) - ikx(\tau)] d\tau / (\sqrt{4\pi\omega})$$

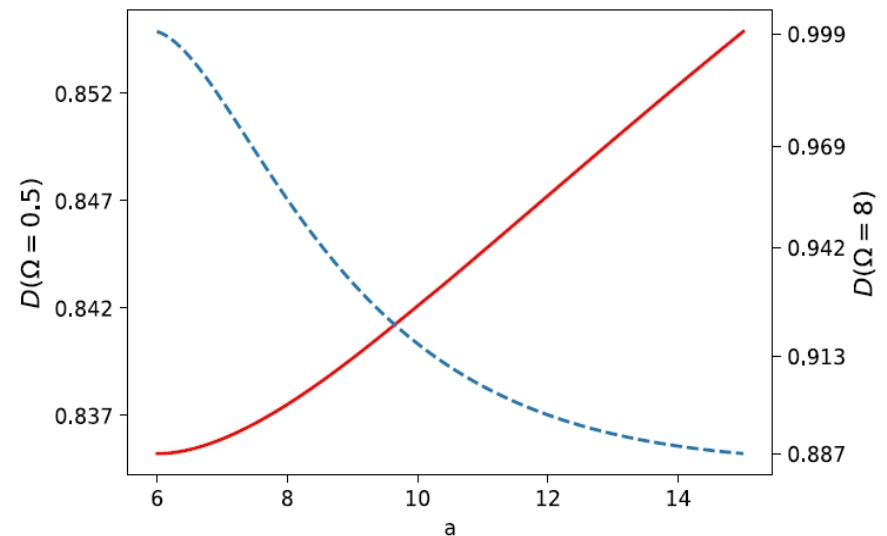
Anti-Unruh effect

The Anti-Unruh effect will lead to the increase of the coherence of the quantum state for accelerated atoms.

The decoherence factor $D = |\langle \psi_0 | \psi_1 \rangle|$

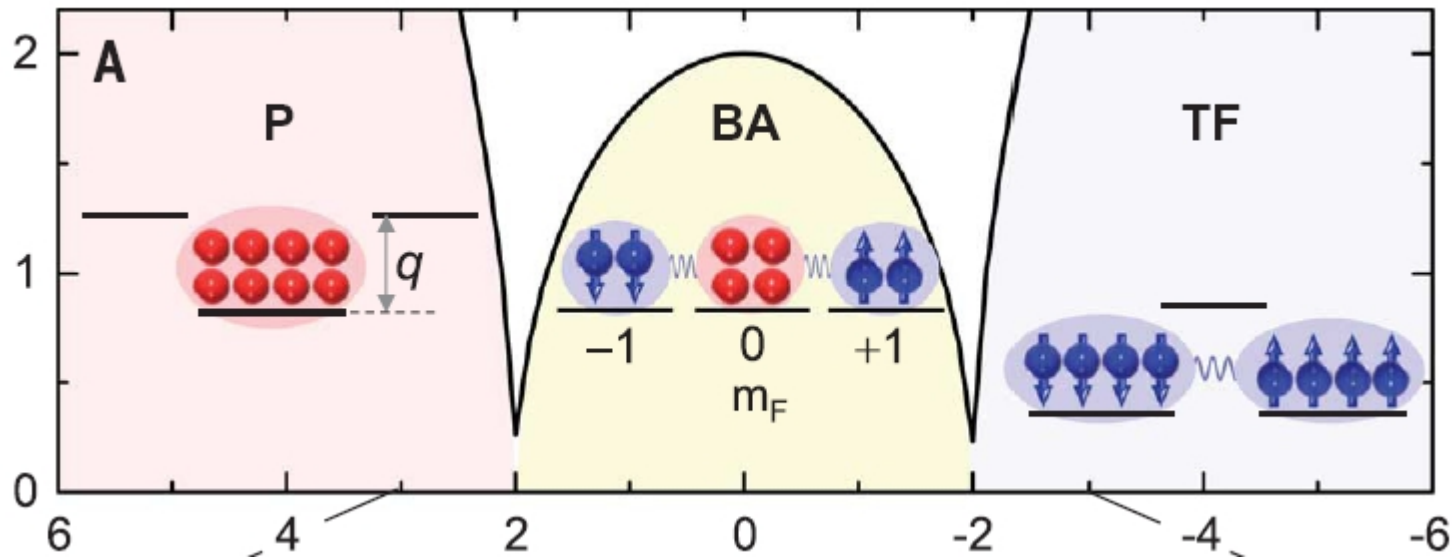


Finite interaction time



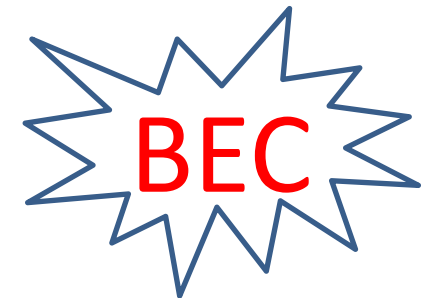
Infinite interaction time

Possibility of anti-Unruh effect



Luo *et al.*, *Science* **355**, 620–623 (2017)

Possibility { Low temperature
New phenomena

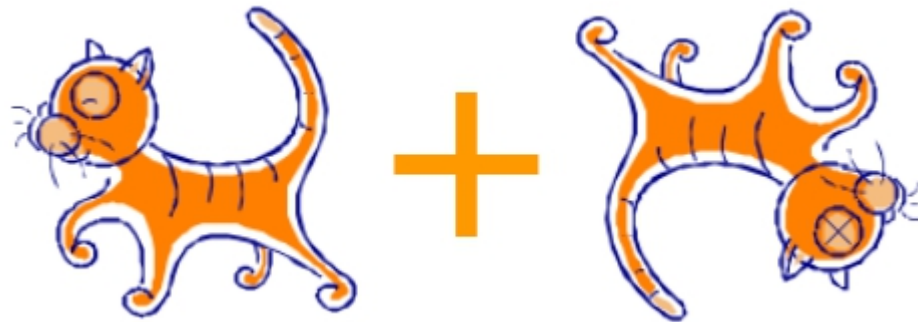


Outline

- Unruh and Anti-Unruh effects
- Quantum many-body states
- Influence of acceleration
- Conclusion and future

NoonN states

$$|\text{NoonN}\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle^{\otimes N} + |\downarrow\rangle^{\otimes N} e^{iN\varphi} \right)$$



Schrodinger's cat states

Two-mode squeezed states

$$S(z) = \exp\left[-\frac{1}{2}(za^\dagger b^\dagger - z^* ab)\right], \quad z = r * e^{i\theta}$$

$$|\text{SMSV}\rangle = S(z)|0,0\rangle$$

$$= \frac{1}{\cosh r} \sum_{n=0}^{\infty} (\tanh r)^n |n,n\rangle$$

$$\propto |0,0\rangle + (.)|1,1\rangle + (..) |2,2\rangle + \dots$$

Schwinger representation

Consider N two-level atoms, their collective spin operators

$$\mathbf{J} = (J_x, J_y, J_z)$$

$$J^2|j, m\rangle = j(j+1)|j, m\rangle, \quad J_z|j, m\rangle = m|j, m\rangle$$

$$j = N/2, N/2 - 1, \dots, \text{mod}(N, 2)/2, \quad m = j, j-1, \dots, -j$$

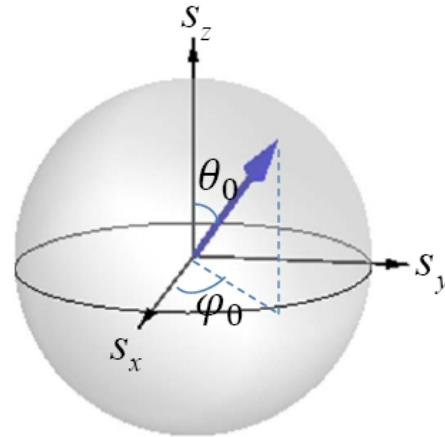
$$|j = \frac{N}{2}, m\rangle$$

$(j+m)$ atoms at the excited state, $(j-m)$ atoms in ground state

Coherent states

$$|\theta_0, \varphi_0\rangle = \bigotimes_{k=1}^N \left[\cos(\theta_0/2) |\uparrow\rangle_k + e^{i\varphi_0} \sin(\theta_0/2) |\downarrow\rangle_k \right]$$

$$(\Delta J_{\perp,1})^2 (\Delta J_{\perp,2})^2 = \frac{|\langle \mathbf{J} \rangle|^2}{4}$$



$$|\theta_0, \varphi_0\rangle = \sum_{m=-j}^j \sqrt{C_{2j}^{j+m}} \cos^{j+m}(\theta_0/2) \sin^{j-m}(\theta_0/2) e^{i(j-m)\varphi_0} |j, m\rangle$$

$$C_{2j}^{j+m} = \frac{(2j)!}{(j+m)!(j-m)!}$$

Dicke/Twin-Fock (TF) states

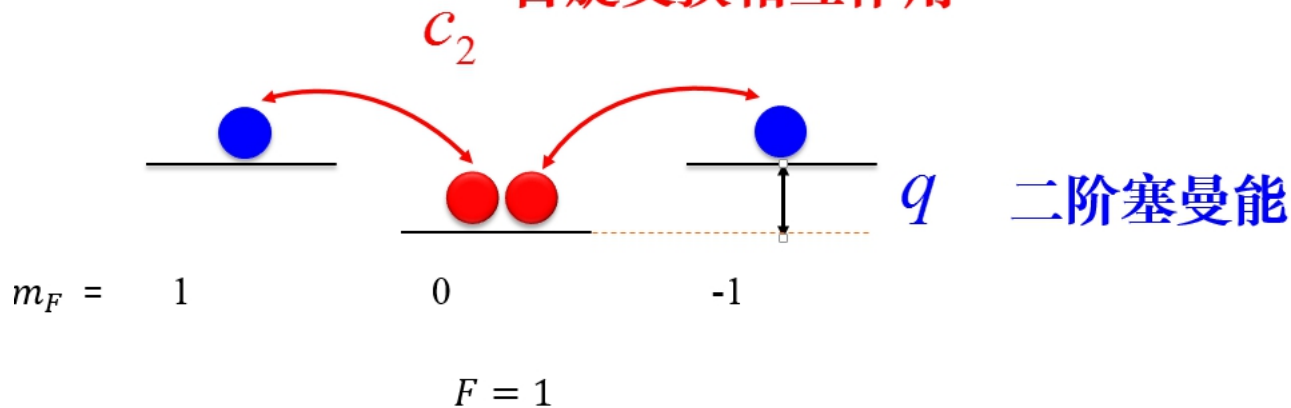
$$|\text{Two-mode Dicke}\rangle = |\uparrow\rangle^{\otimes\left(\frac{N}{2}+m\right)} \otimes |\downarrow\rangle^{\otimes\left(\frac{N}{2}-m\right)} = |j, m\rangle$$

$$|\text{Twin-Fock}\rangle_N = |\uparrow\rangle^{\otimes\frac{N}{2}} \otimes |\downarrow\rangle^{\otimes\frac{N}{2}} = |j, 0\rangle$$

for $N=4$, $|2, 0\rangle$

$$|\uparrow\uparrow\downarrow\downarrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle$$

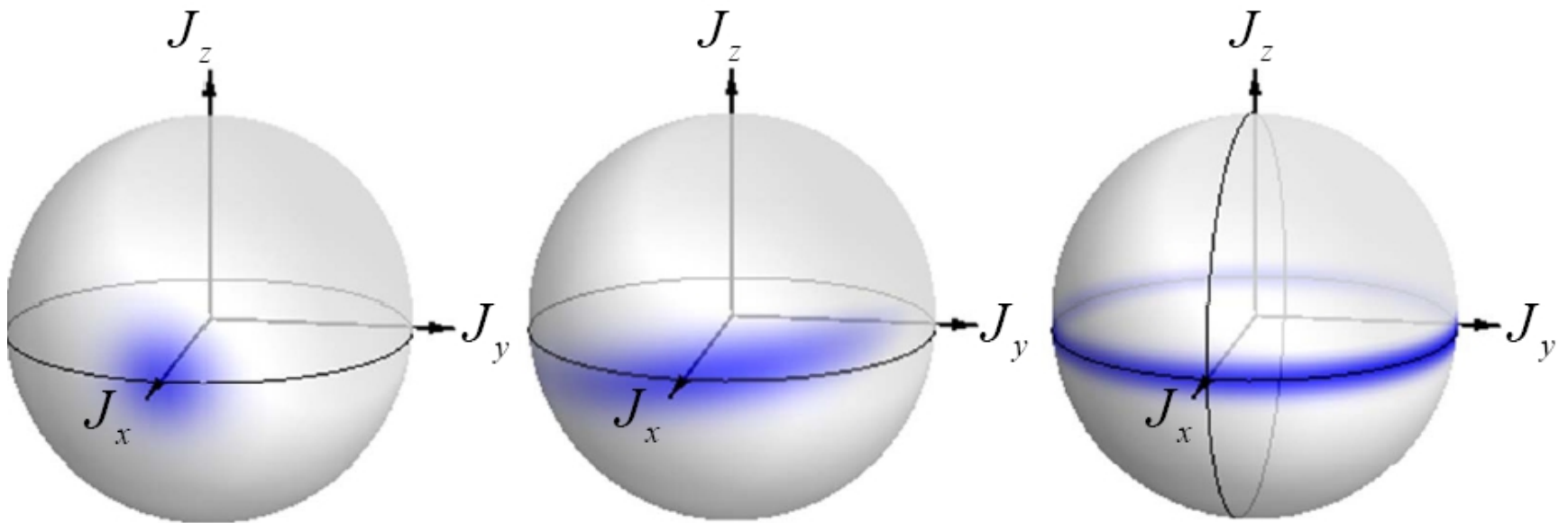
自旋交换相互作用



Squeezing parameter

➤ General definition

$$\xi^2 = \frac{(\Delta J_{\perp})_{\min}^2}{N/4} \quad \xi_R^2 = \frac{N (\Delta J_{\perp})_{\min}^2}{|\langle \mathbf{J} \rangle|^2}$$



Application of TF states

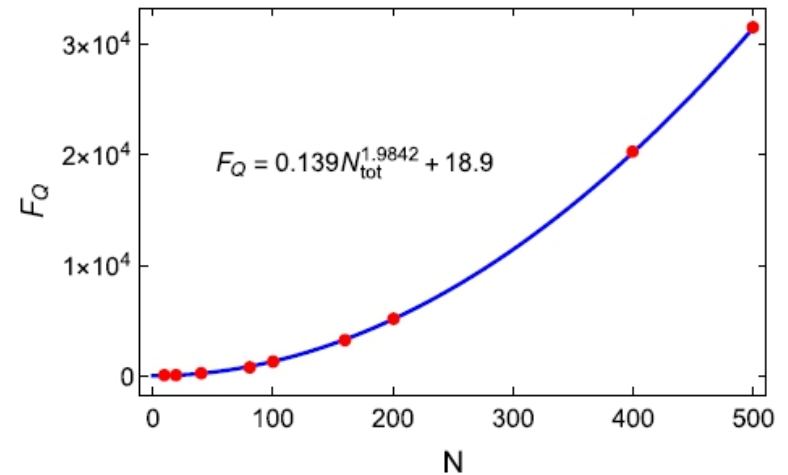
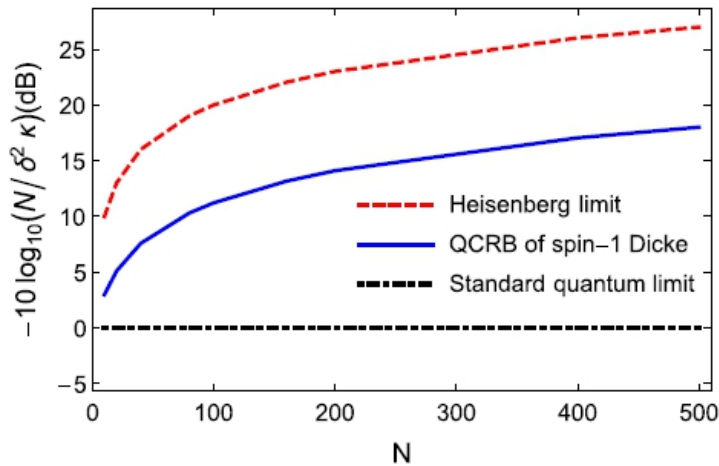
PHYSICAL REVIEW A **99**, 042118 (2019)

Enhancing test precision for local Lorentz-symmetry violation with entanglement

Lei Li,¹ Xinwei Li,² Baocheng Zhang,^{1,*} and Li You²

¹*School of Mathematics and Physics, China University of Geosciences, Wuhan 430074, China*

²*State Key Laboratory of Low Dimensional Quantum Physics, Department of Physics, Tsinghua University, Beijing 100084, China*



$$\delta H = -C_0^{(2)} \frac{(\mathbf{p}^2 - 3p_z^2)}{6m_e}$$

$$H_V = \kappa j_z^2$$

Squeezing parameter

$$\xi_E^2 = \frac{\min_{\vec{n}} [(N-1)(\Delta J_{\vec{n}})^2 + \langle J_{\vec{n}}^2 \rangle]}{\langle J^2 \rangle - N/2}$$

This is a spin-squeezing parameter related to entanglement

It derives from

$$(N-1)(\Delta J_{\vec{n}})^2 + \langle J_{\vec{n}}^2 \rangle \geq \langle J^2 \rangle - N/2$$

For our purpose,

$$\xi_E^2 = \frac{(N-1)(\Delta J_z)^2 + \langle J_z^2 \rangle}{\langle J^2 \rangle - N/2}$$

Outline

- Unruh and Anti-Unruh effects
- Quantum many-body states
- **Influence of acceleration**
- Conclusion and future

Two atoms

Consider two atoms with initial states

$$|\Psi_i\rangle = (\alpha|g\rangle_A|e\rangle_B + \beta|e\rangle_A|g\rangle_B)|0\rangle_A|0\rangle_B$$

When one or two of the two atoms are accelerated, how would the initial entangled state be changed?

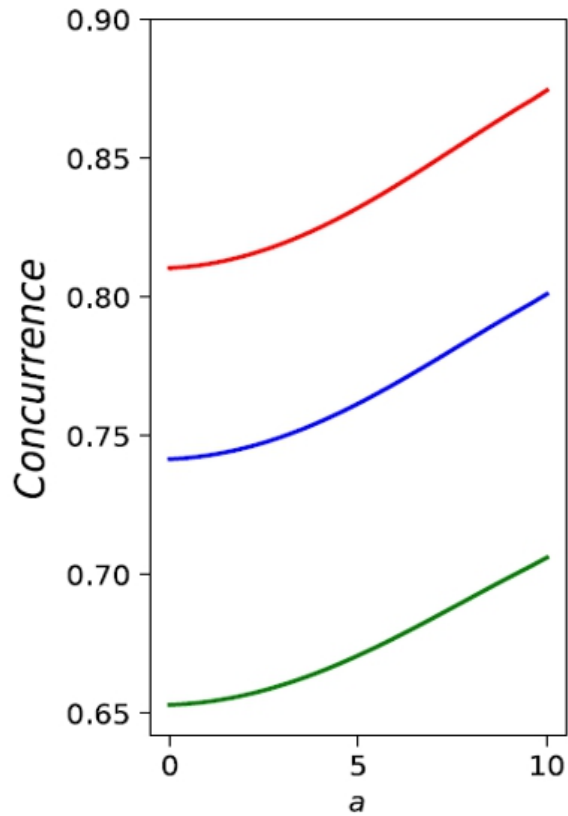
➤ **Concurrence:**

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

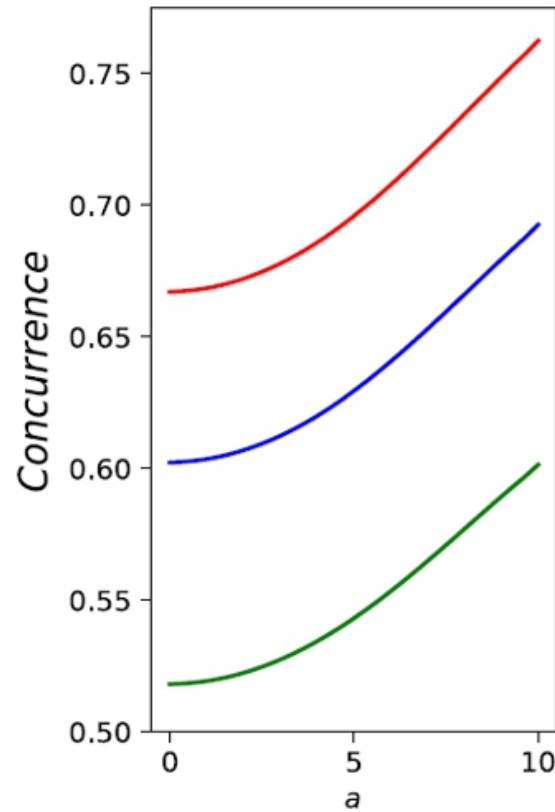
$\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are the eigenvalues of the Hermitian matrix $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$

the spin-flipped state $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$

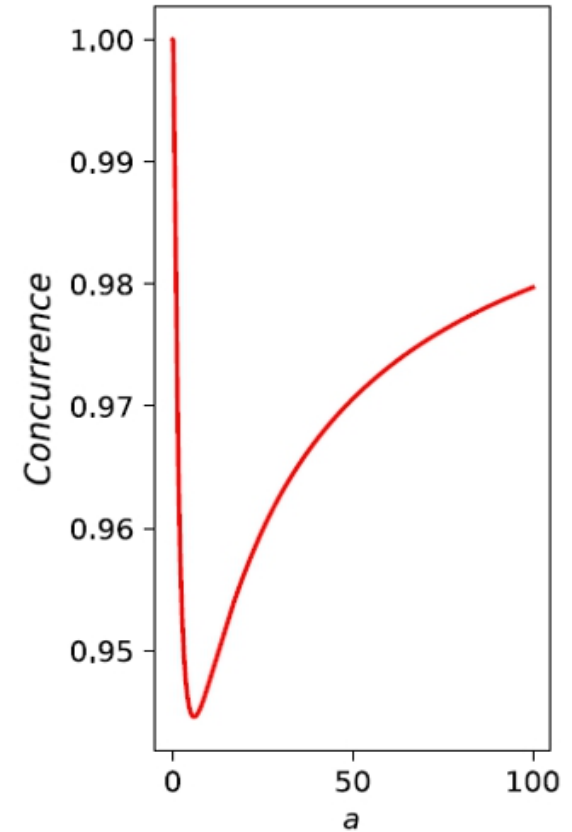
Results of Acceleration



One atom at acceleration



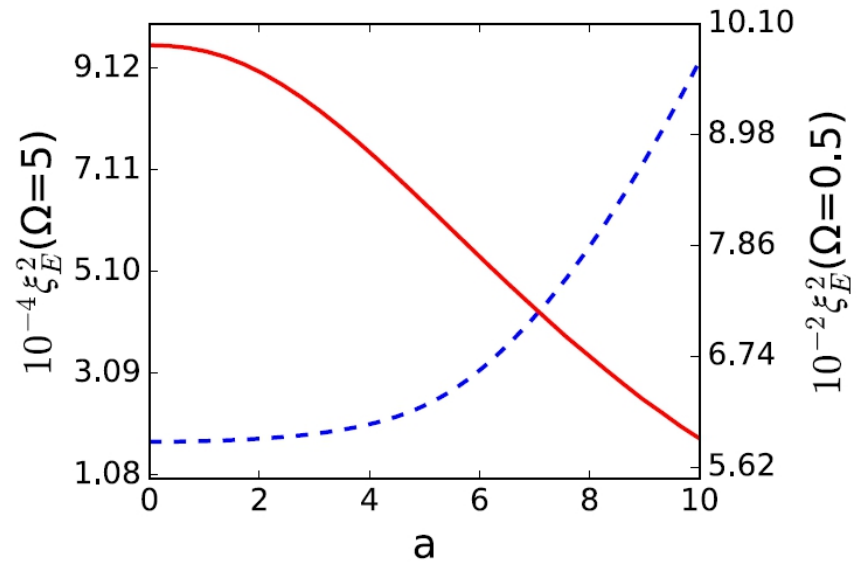
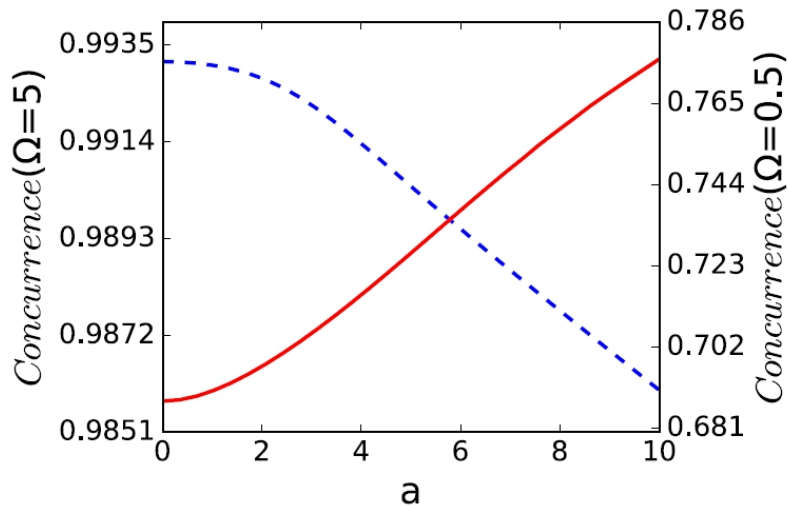
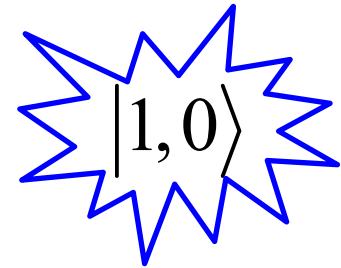
Two atoms at acceleration



Maximal entangled states

Minimal TF states

$$|\psi_i\rangle = \frac{1}{\sqrt{2}}(|g\rangle_A |e\rangle_B + |e\rangle_A |g\rangle_B)$$



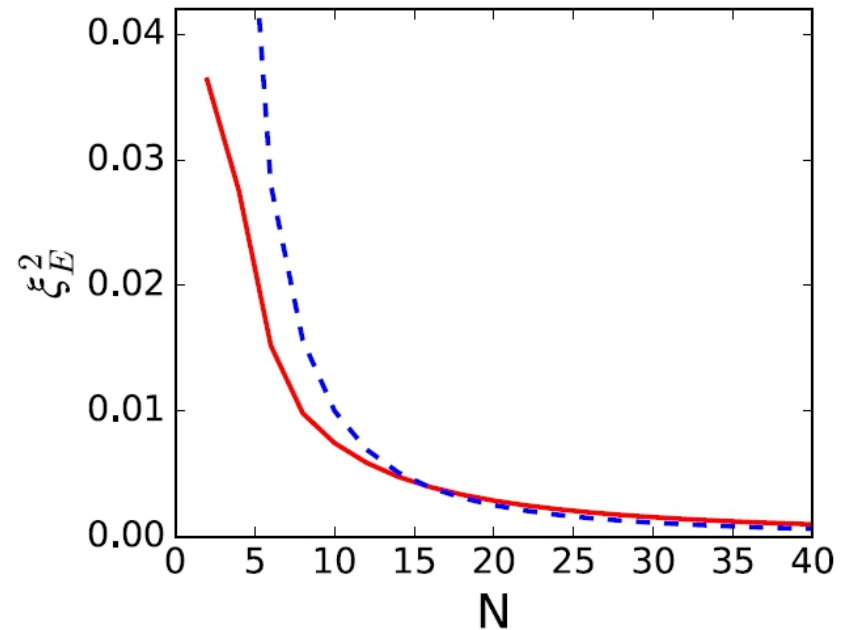
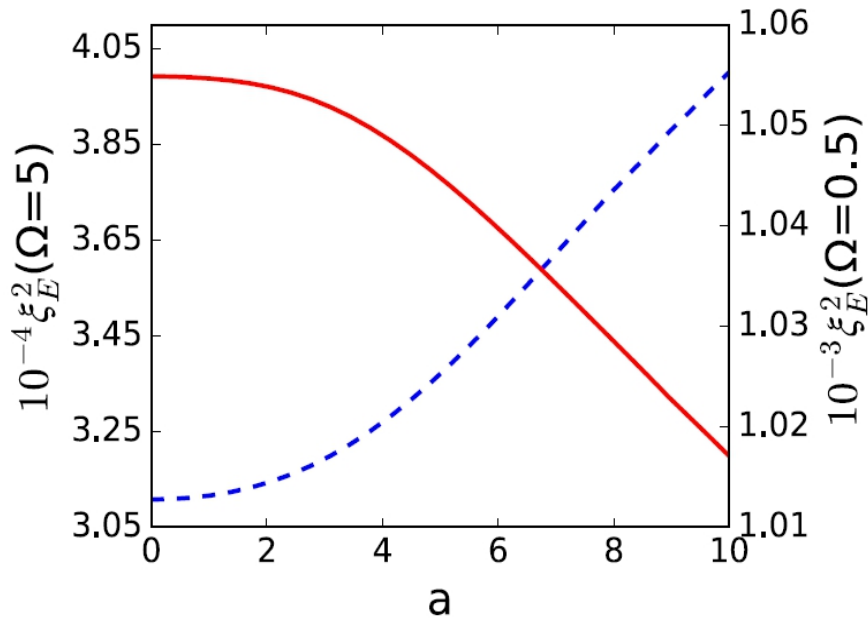
N-atom TF state

$$\rho_t = B_0^2 |j, 0\rangle \langle j, 0| + \sum_{m=-N/2}^{N/2} \sum_{m'=-N/2}^{N/2} B_m B_{m'}^* |j, m\rangle \langle j, m'|$$

$$B_0^2 = \sum_{k=0}^{N/2} [(C_{N/2}^k)^4 (D_0 D_1)^N (\eta_0 \eta_1)^{2k}],$$

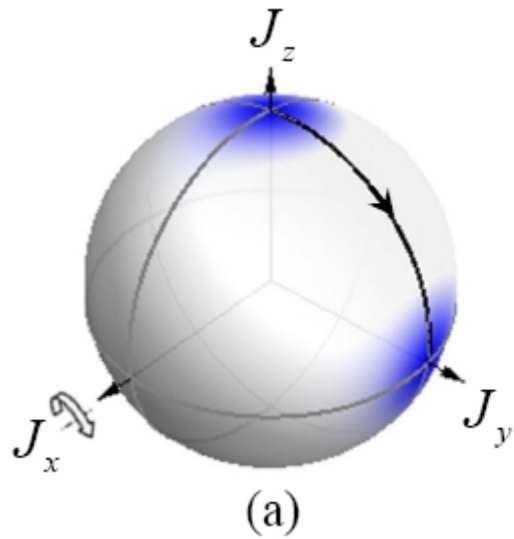
$$B_m = \left[\sum_{k=0}^{N/2-|m|} C_{N/2}^k C_{N/2}^{k+|m|} (D_0 D_1)^{N/2} (\eta_0 \eta_1)^k \right. \\ \left. \times (\theta(m)(-i\eta_0)^m + \theta(-m)(i\eta_1)^{|m|}) \right] \quad C_n^r = \frac{n!}{r!(n-r)!}$$

Results of Acceleration

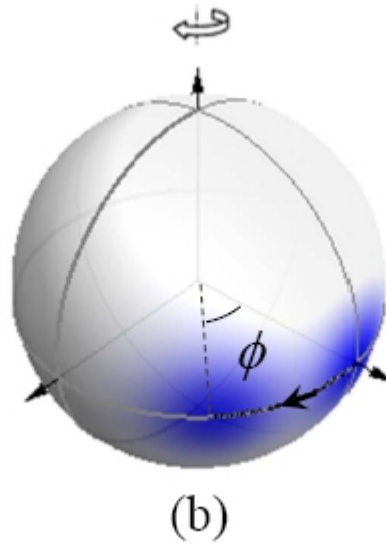


$$\xi_E^2 = \frac{(N-1)(\Delta J_z)^2 + \langle J_z^2 \rangle}{\langle J^2 \rangle - N/2}$$

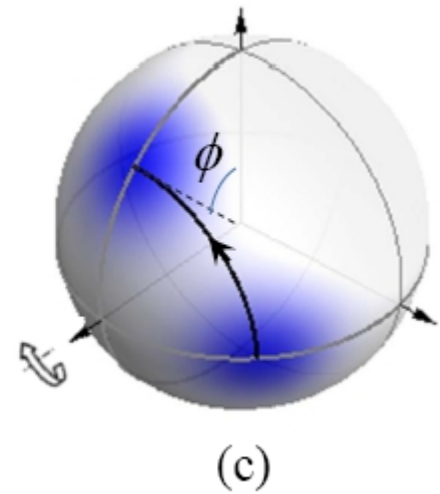
Ramsey Interferometer



$\pi/2$ Pulses



Evolves freely



$\pi/2$ Pulses

Phase sensitivity

$$(\Delta\theta)^2 = \frac{(\Delta J_z^2)_o^2}{|d\langle J_z^2 \rangle_o/d\theta|^2}$$

Evolution

$$\rho_o = U \rho_i U^\dagger \quad U = \exp(-i\theta J_y)$$

$$U J_z U^\dagger = J_z \cos \theta - J_x \sin \theta$$

$$\langle J_z^2 \rangle_o = \langle J_z^2 \rangle_i \cos^2 \theta + \langle J_x^2 \rangle_i \sin^2 \theta$$

Optimal phase sensitivity

$$(\Delta\theta)_P^2 = \frac{2(\Delta J_z^2)_i (\Delta J_x^2)_i + V_{xz}}{4(\langle J_x^2 \rangle_i - \langle J_z^2 \rangle_i)^2}$$

$$V_{xz} = \langle (J_x J_z + J_z J_x)^2 \rangle_i + \langle J_z^2 J_x^2 + J_x^2 J_z^2 \rangle_i - 2\langle J_z^2 \rangle_i \langle J_x^2 \rangle_i$$

For Dicke states

$$(\Delta\theta)_{PD}^2 = \frac{(4m^2 + 1)[j(j + 1) - m^2] - 4m^2}{2[j(j + 1) - 3m^2]^2}$$

When $m=j$, $(\Delta\theta)_{PD}^2 = \frac{1}{2j}$ Standard quantum limit

When $m=0$, $(\Delta\theta)_{PD}^2 = \frac{1}{2j(j + 1)}$

Approaching the Heisenberg limit: $\frac{2}{N(N + 2)}$

For accelerated states

Considering the approximation, $m, m' \ll j$

We obtained,

$$\langle J_z^2 \rangle = \sum_{m=-N/2}^{N/2} m^2 |B_m|^2 \quad \Delta J_z^2 = \sqrt{\sum_{m=-N/2}^{N/2} m^4 |B_m|^2 - \left(\sum_{m=-N/2}^{N/2} m^2 |B_m|^2 \right)^2}$$

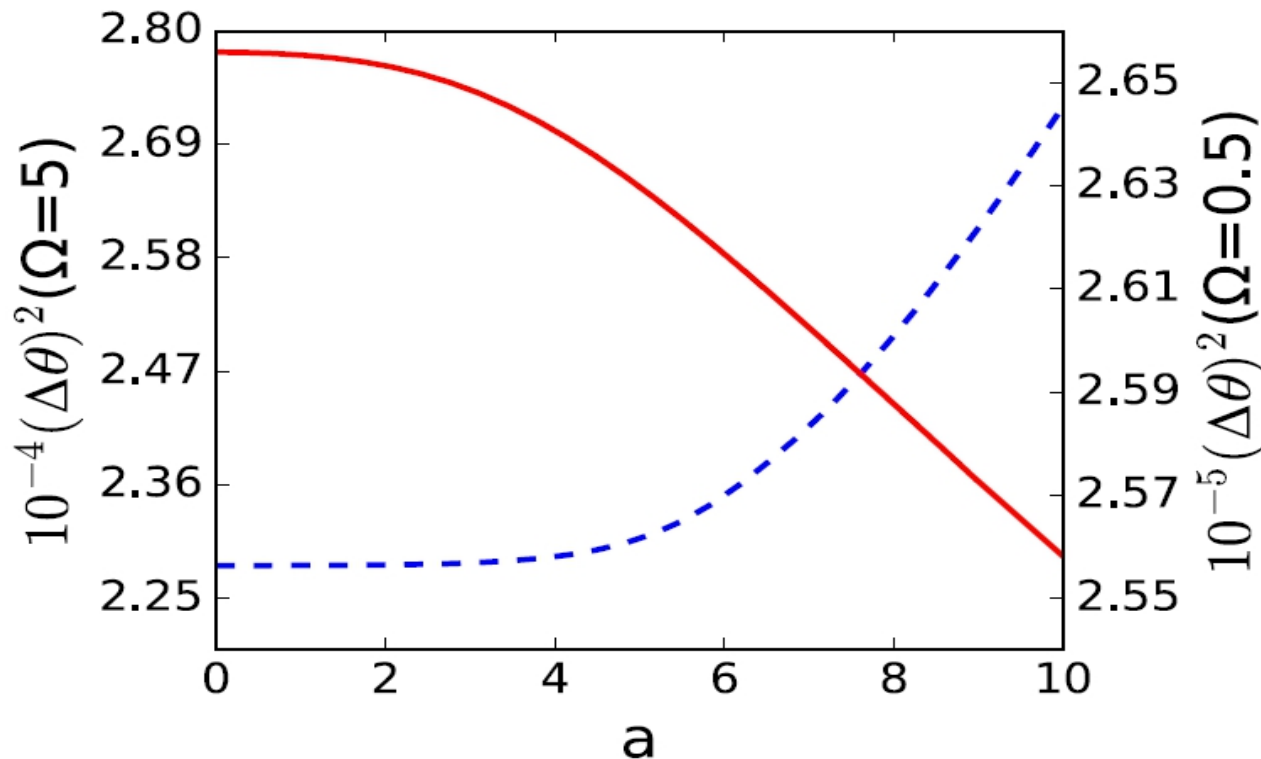
$$\langle J_x^2 \rangle \simeq \frac{1}{2} j(j+1) B_0^2 \quad \Delta J_x^2 \simeq \frac{B_0}{2\sqrt{2}} j(j+1)$$

$$V_{xz} \simeq \frac{1}{2} j(j+1) \left[1 + \sum_{m=-N/2}^{N/2} |B_m|^2 (4m^2 + 1) \right]$$

For accelerated states

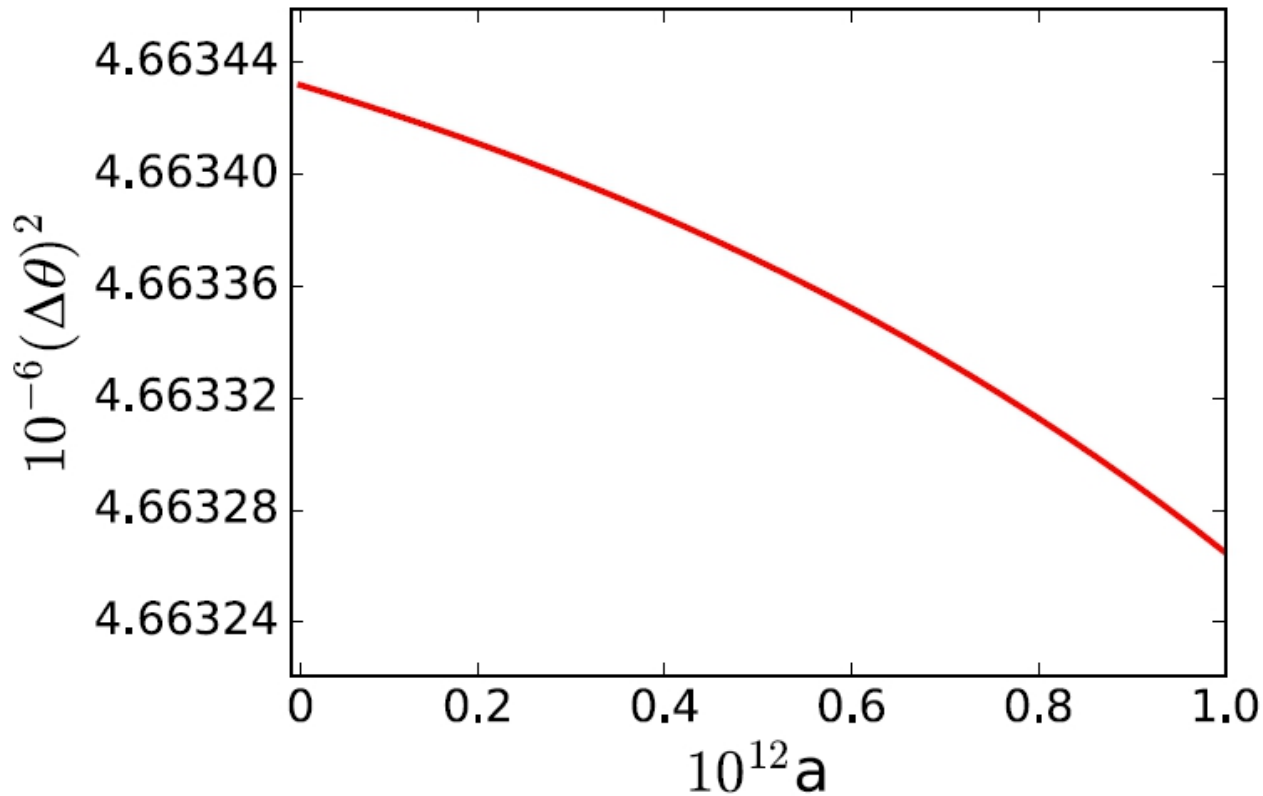
Phase sensitivity

$$(\Delta\theta)_{PA}^2 \simeq \frac{1}{2j(j+1)} + \frac{\sqrt{2}B_0\Delta J_z^2}{2j(j+1)} + \frac{\sum_{m=-N/2}^{N/2} |B_m|^2(4m^2+1)}{2j(j+1)}$$



For accelerated states

For the real experimental conditions



$$T \sim 10^{-9} \text{ K}$$

$$\Omega \sim 2\pi \text{ Hz}$$

$$N \sim 10\,000$$

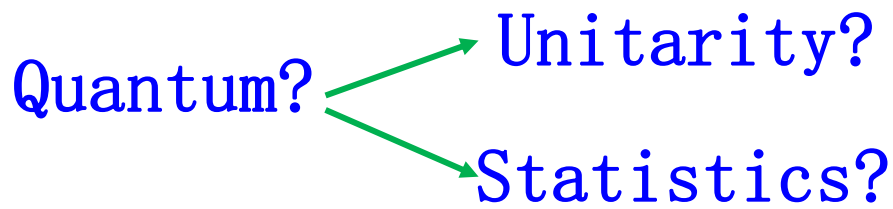
10^{10} m/s^2 **VS** 10^{17} m/s^2

Outline

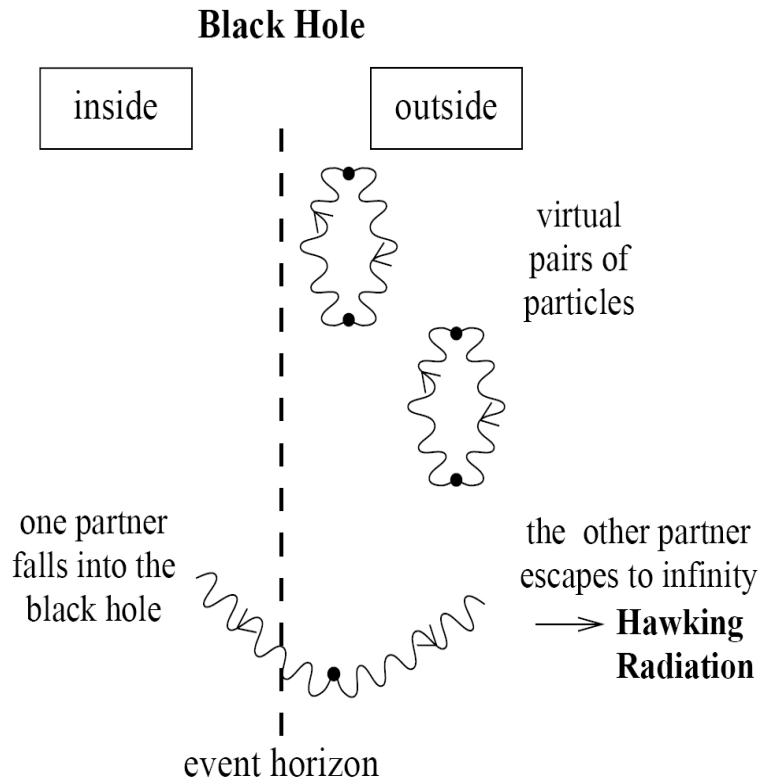
- Unruh and Anti-Unruh effects
- Quantum many-body states
- Influence of acceleration
- Conclusion and future

Summary and discussion

- ◆ Anti-Unruh effect can be used as experimental signals
- ◆ Anti-Unruh effect can increase the entanglement among many atoms.
- ◆ The acceleration is reduced for the actual experimental measurement using BEC.
- Is the evolution induced by acceleration unitary?



Information loss?



Irrespective of what initial state a black hole starts with before collapsing, it will evolve eventually into a thermal state after being completely exhausted into emitted radiations.

The radiations should be altered

[B. Zhang et al, IJMPD 22 \(2013\) 1341014](#)

The interior should be reinterpreted

[B. Zhang, PRD 92 \(2015\) 081501\(R\)](#)

[B. Zhang, et al, PLB 765 \(2017\) 226](#)

Is information loss a feature for semi-classical theory? **Unruh effect!**



Thank you!

