On the consistency of a class of R-symmetry gauged $6D$ $\mathcal{N}=(1,0)$ supergravities

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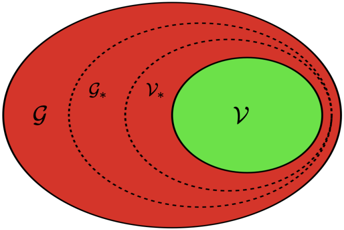
2. Structure of minimal 6d supergravity
3. New quantization condition on the anomaly coefficients
4. $U(1)_R$ symmetry gauged 6d minimal supergravities
5. Conclusion & discussions

Background & Motivation



[W. Taylor TASI 2010]

$\mathcal{G}$={ apparently consistent low-energy field theories coupled to gravity}
$\mathcal{G}*$={ fully consistent low-energy field theories coupled to gravity}
$\mathcal{V}*$={ complete set of low-energy theories from string constructions}
$\mathcal{V}$={ low-energy theories arising from known string constructions}

$$\boxed{\mathcal{G} \supseteq \mathcal{G}* \supseteq \mathcal{V}* \supseteq \mathcal{V}}$$

One interesting question is that

$$\mathcal{V}* = \mathcal{G}*?$$

In this talk, I will only discuss gravity models equipped with supersymmetry

By "apparently consistent", I mean at least the theory satisfies constraints such as anomaly-freedom, positivity of certain couplings $\cdots$

Supersymmetry+anomaly freedom can be quite restrictive

32 SUSY

- 11d supergravity $\longrightarrow$ M theory
- 10d (1,1) and (2,0) supergravity

16 SUSY Chiral theories

- 10d N=(1,0) supergravity with $E_8 \times E_8$ and SO(32)$\times$ SO(32) $\longrightarrow$ Heterotic string
- 6d (2,0) supergravity $\longrightarrow$ IIB on K3

16 SUSY Non Chiral theories

- No constraint from anomaly-freedom
- In principle, arbitrary gauge group is allowed
- String theory leads to upper bound on the rank of gauge group $r_G \leq 26 - d$

d	r_g	Compactifications
9	17	Heterotic on S^1
	9	CHL string
	1	M-theory on Klein bottle, Type IIB on DP bg
8	18	Heterotic on T^2
	10	CHL on S^1
	2	9d, $r_g = 1$ on S^1
7	19	Heterotic on T^3
	11	CHL on T^2
	7	F-theory on $K3 \times S^1/\mathbb{Z}_3$
	5	F-theory on $K3 \times S^1/\mathbb{Z}_4$
	3	F-theory on $K3 \times S^1/\mathbb{Z}_{5,6}$ or $T^4 \times S^1/\mathbb{Z}_{3,4,5}$

[Chaudhuri, Hockney, Lykken 95] [Dabholkar, Park 96], [de Boer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi 01], [Aharony, Komargodski, Patir 07], ...

[Hee-Choel Kim's talk at Oxford]

8 SUSY ($d \leq 6$)

- Too many models
- Not aiming at a complete analysis
- Focus on specific models that are phenomenologically interesting and do not have obvious string constructions

Structure of minimal 6d supergravity

multiplet	Field content
Gravity	$(g_{\mu\nu}\,,\psi^{+}_{\mu}\,,B^{+}_{\mu\nu})$
Tensor	$(B^{-}_{\mu\nu}\,,\chi^{-}\,,\phi)$
Vector	$(A_{\mu}\,,\lambda^{+})$
Hyper	$(\psi^{-}\,,4\varphi)$

- Fermions are chiral (symplectic Majorana-Weyl) and transform under certain irreps of the gauge and R-symmetry groups
- 2-from field strengths are (anti)self dual

They contribute to local and global anomalies

Local anomalies: gauge, gravitational & mixed anomalies

$I_8 = d_1 \mathrm{Tr}(R^4) + d_2(\mathrm{Tr}(R^2)^2) + d_3 \mathrm{Tr}(R^2)\mathrm{Tr}(F^2) + d_4 \mathrm{Tr}(F^4)$

- The anomaly cancellation is through Green-Schwarz mechanism (except for IIB) provided that

$$
\begin{aligned}
I_8 &= \tfrac{1}{2}\Omega_{\alpha\beta} Y^\alpha Y^\beta \\
Y^\alpha &= \frac{1}{16\pi^2}\left(\frac{1}{2}a^\alpha \,\mathrm{tr}\, R^2 + b_r^\alpha \left(\frac{2}{\lambda_r}\mathrm{tr}\, F_r^2\right) + 2\, c^\alpha\, F^2\right)
\end{aligned}
$$

$\Omega_{\alpha\beta}$ is the SO(1,n_T) invariant metric. $n_T = \#$ of tensor multiplets

$$
\begin{aligned}
H_{(3)} &= dB_{(2)} \Rightarrow H_{(3)} = dB_{(2)} + \sum_i \mathrm{CS}_i \\
\Delta S &= \tfrac{1}{2}\int \Omega_{\alpha\beta} B^\alpha \wedge Y^\beta
\end{aligned}
$$

Global anomaly (Witten anomaly)

- The R-symmetry of 6d minimal supergravity is SU(2),
- Fermions in pseudoreal irreps of gauge group
- Group with a non-trivial $\pi_6(G)$ [Vafa]

$$1 - 4\sum_R n_R m_R = 0$$

$n_R = \#$ of half-hyper, $\mathrm{tr}_R(F^4) = m_R \mathrm{tr}_R(\mathrm{tr}_R F^2)^2$

New quantization condition on the anomaly coefficients

$$d\star\left(G_{\alpha\beta}H^\beta\right)=16\pi^2\alpha'\,\Omega_{\alpha\beta}\,Y^\beta\ ,\quad dH^\alpha=16\pi^2\alpha'\,Y^\alpha$$

$$Y^\alpha=\frac{1}{4}a^\alpha p_1\quad-\quad b_r^\alpha\,c_2^r+\frac{1}{2}\,c^\alpha\,(c_1)^2$$

$$p_1=\frac{1}{8\pi^2}\mathrm{tr}\,R^2\ ,\quad c_2^r\quad=\quad-\frac{1}{8\pi^2}\left(\frac{1}{\lambda_r}\mathrm{tr}_r\,F^2\right)\ ,\quad c_1=\frac{F_1}{2\pi}$$

- A consistent supergravity theory may be put on an arbitrary spin manifold and that any smooth gauge field configurations should be allowed in the sugra "path-integral". In particular, taking $M_6=CP^3$

$$\int_{\Sigma_4}Y\in\Lambda_S\ (\text{string charge lattice})\,,\quad\Sigma_4\in M_4$$

  The background charges must be cancelled by background strings

- Dirac quantization $p_1q_2+p_2q_1\in\mathbb{Z}$ & completeness of gauge charge $\Rightarrow\Lambda_S$ is unimodular (self-dual) [Monnier, Moore and Park]

- More rigorously, the quantization condition on the anomaly coefficients can be obtained by requiring the Green-Schwarz term be well defined [Monnier and Moore]

$$e^{\frac{i}{2}\int_{M_6}\Omega_{\alpha\beta}B^{\alpha}\wedge Y^{\beta}} = e^{\frac{i}{2}\int_{X_7}\Omega_{\alpha\beta}H^{\alpha}\wedge Y^{\beta}}$$

- Independent of the choice of $X_7 \Rightarrow$ triviality of the partition of a TQFT on closed X_7 with spin structure

1. Λ_S is unimodular;
2. $\frac{1}{2}\mathfrak{b} \in H^4(BG;\Lambda_S)$; $\mathfrak{b} = \bigoplus_r b_r K_r \oplus c$; K_r is normalized Killing form

$$\frac{1}{2}\mathfrak{b}(x,x) \in \Lambda_S\,, \quad \text{and} \quad \mathfrak{b}(x,y) \in \Lambda_S \quad \text{for} \quad x \neq y\,.$$

3. $a \in \Lambda_S$ is a characteristic element $a \cdot x = x \cdot x \mod 2$;
4. $\Omega_7^{\mathrm{Spin}}(BG) = 0$,

U(1)$_R$ symmetry gauged 6d minimal supergravities

Salam Sezgin model

$$e^{-1}\mathscr{L} = (1/4\kappa^2)R - \tfrac{1}{4}(\partial_M\sigma)^2 - \tfrac{1}{12}e^{2\kappa\sigma}G_{MNP}G^{MNP} - \tfrac{1}{4}e^{\kappa\sigma}F_{MN}F^{MN} - \tfrac{1}{2}g^2\kappa^{-4}e^{-\kappa\sigma}$$

- $n_T = 1$
- The maximally symmetric vacuum of this model is Minkowski$_4 \times S^2$ instead of Minkowski$_6$
- Compactification on S^2 gives rise to chiral theory in 4d
- However, the model by itself is anomalous

Anomaly-free extension of Salam Sezgin model

- $n_T = 1$
- Gauge symmetry & irreps of hypers

$$\psi_{\mu+}^{A}\ ,\quad \chi_{-}^{A}\ ,\quad \lambda_{+}^{IA}\ ,\quad \psi_{-}^{aa'}$$

(A)	$(E_6/\mathbb{Z}_3)\times E_7\times U(1)_R$	$(1,912)_0$
(B)	$G_2\times E_7\times U(1)_R$	$(14,56)_0$
(C)	$F_4\times Sp(9)\times U(1)_R$	$(52,18)_0$

[Randjbar-Daemi, Salam, Sezgin and Strathdee];[Avramis, Kehagias and Randjbar-Daemi];[Avramis and Kehagias]

$$(A)\quad a=(2,-2)\ ,\quad b_6=(1,3)\ ,\quad b_7=(3,-9)\ ,\quad c=(2,18)$$

$$(B)\quad a=(2,2)\ ,\quad b_2=(3,15)\ ,\quad b_7=(3,1)\ ,\quad c=\left(2,-\frac{38}{3}\right)$$

$$(C)\quad a=(2,-2)\ ,\quad b_4=(2,-10)\ ,\quad b_9=\left(1,\frac{1}{2}\right)\ ,\quad c=(2,19)$$

Testing these models against Monnier& Moore's criteria

- Only model (C) obeys all of them
- $F_4 \times Sp(9) \times U(1)_R \quad (52, 18)_0$
- No obvious F-theory origin because of $\mathrm{U}(1)_R$ gauging

So far, the discussion utilized only the kinematic data, what about dynamics?

Setting hypers=0

$$
\begin{aligned}
S &= \int \Big\{ \frac{1}{4} R(\omega) \star 1\!\!1 - \frac{1}{4} \star d\phi \wedge d\phi - \frac{1}{2} G_{\alpha\beta} \star (H^\alpha) \wedge H^\beta + 16\pi^2 \alpha' \,\Omega_{\alpha\beta} B^\alpha \wedge Y^\beta \\
&\quad - \frac{1}{\sqrt{2}} \alpha' \, e \cdot \big(a \, \mathrm{tr} \star R(\omega) \wedge R(\omega) + b_i \, \mathrm{tr} \star F_i \wedge F_i \big) - \frac{1}{8\sqrt{2}\alpha'} \, (e \cdot c)^{-1} \star 1\!\!1 + \cdot
\end{aligned}
$$

$$
\begin{aligned}
G_{\alpha\beta} &= e_\alpha e_\beta + j_\alpha j_\beta \ , \qquad \Omega_{\alpha\beta} = -e_\alpha e_\beta + j_\alpha j_\beta \ , \\
e_\alpha &= \frac{1}{\sqrt{2}} \big(e^{-\phi}, \, -e^{\phi} \big) \ , \qquad j_\alpha = \frac{1}{\sqrt{2}} \big(e^{-\phi}, \, e^{\phi} \big)
\end{aligned}
$$

Tree level unitarity requires

$$
e \cdot a > 0 \,, \quad e \cdot b_i > 0
$$

$\frac{1}{4}$-BPS Dyonic string solutions [Gueven, Liu , Pope, Sezgin]

$$\begin{aligned} ds^2 &= c^2 dx^\mu dx_\mu + a^2 dr^2 + b^2 \left(\sigma_1^2 + \sigma_2^2 + \frac{4gP}{k}\sigma_3^2\right) \ , \\ G &= P\sigma_1 \wedge \sigma_2 \wedge \sigma_3 - u(r)\, d^2x \wedge dr \ , \\ F &= k\,\sigma_1 \wedge \sigma_2 \ , \qquad e^{2\phi} = \left(Q_0 + \frac{Q}{r^2}\right)\left(P_0 + \frac{P}{r^2}\right)^{-1} \end{aligned}$$

- Asymptotic to cone over (Minkowski)2×squashed S^3, dilaton blows up at infinity. The physical interpretation is unclear
- The near horizon limit is $\frac{1}{2}$-BPS squashed $AdS_3 \times S^3$

Conclusion & discussions

- We found one 6d $U(1)_R$ symmetry gauged minimal supergravity model passed all the known consistency criteria
- Study anomaly inflow on a probe string ala [Kim, Shiu, Vafa]
- Consistency of 4d supergravities

Thank you for listening