Correlation functions in the TTbar deformed CFTs

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彭桓武高能基础理论研究中心

Outlines

- **Introduction to TTbar deformation**
- **Correlation functions in TTbar deformed theory （Plane, SUSY, Torus） [1st order deformation]**
- **TTbar flow effects on partition function [2nd order deformation]**
-
- **OTOC in Deformed theories [1st order deformation]**
- **Future Problems**

The $T\bar{T}$ operator

Consider the following bi-local operator in a 2d QFT.

$$T\bar{T}(z,z') = T_{zz}(z)T_{\bar{z}\bar{z}}(z') - T_{z\bar{z}}(z)T_{z\bar{z}}(z')$$

$z = x + it$

[Zamolodchikov]

The expectation value of this operator is given by the expectation values of the stress tensor itself and is a constant.

$$\langle T\bar{T}\rangle = \langle T_{zz}\rangle\langle T_{\bar{z}\bar{z}}\rangle - \langle T_{z\bar{z}}\rangle^2$$

This is true very generally in a reasonably well behaved 2d QFT which has a local conserved stress tensor.

The operator can also be defined (upto total derivatives) at coincident points.

$$T\bar{T} \equiv T_{zz}T_{\bar{z}\bar{z}} - T_{z\bar{z}}^2$$

Higher Dimensional Deformation

Cardy, 1801.06895; M. Taylor, 1805.10287

$$\mathcal{T} = T^{ij}T_{ij} - \frac{1}{(d-1)}T_i^iT_j^j\,.$$

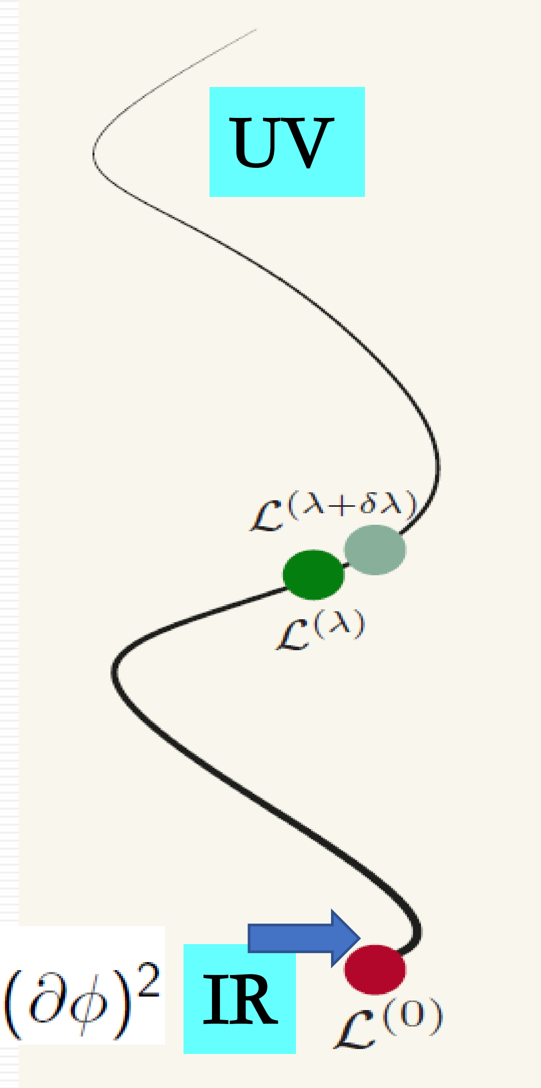
Deformation of TTbar

$$\mathcal{L}^{(\lambda+\delta\lambda)} = \mathcal{L}^{(\lambda)} + \delta\lambda\, T\bar{T}$$

$$\frac{dS(\lambda)}{d\lambda} = \int d^2x\, T\bar{T}(x).$$

Nambu-Goto

$$\mathcal{L}^{(\ell^2)} = \frac{1}{\ell^2}\left(\sqrt{2\ell^2\,\partial\phi\bar{\partial}\phi + 1} - 1\right)$$



free scalar $\mathcal{L} = (\partial\phi)^2$

- The spectrum of the deformed theory can be solved exactly and non-perturbatively.

[Smirnov-Zamolodchikov; Cavaglia-Negro-Szecsenyi-Tateo]

- Deforming an integrable QFT by this operator preserves integrability.

[F. A. Smirnov and A. B. Zamolodchikov,16]

- Deforming by $T\bar{T}$ = coupling the theory to Jackiw-Teitelbohm gravity

[Dubovsky-Gorbenko-…]

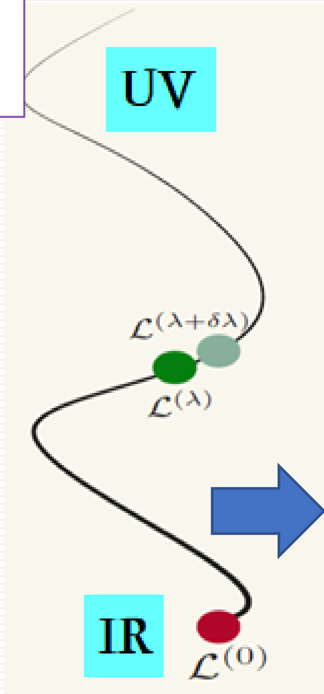
[Kentaroh Yoshida,⋯]

- **Correlation functions in deep UV of the deformed theory given by diffusion equation**[Cardy, 19]

$$\langle\prod_n \Phi_n(x_n)\rangle_\lambda = \int \prod_n G(x_n - y_n; \tilde{\lambda})\langle\prod_n \Phi_n(y_n)\rangle_0 \prod_n d^2 y_n$$

$$\varepsilon\mu \to 0\,, \quad \lambda\mu^2 \to 0 \quad \text{with } \tilde{\lambda}\mu^2 = -\lambda\mu^2 \log|\varepsilon\mu| \text{ fixed,}$$

$$G(x-y;\tilde{\lambda}) = (4\pi\tilde{\lambda})^{-1} e^{-(x-y)^2/4\tilde{\lambda}}$$



- **By canonical transformation in phase space, Flow equation of correlation functions** [Jorrit Kruthoff, Onkar Parrikar,20]

Correlation functions in deformed theory

Zero-pt Correlation function:[Aharony-Shouvik-Giveon-Jiang-Kutasov,18]； [Shouvik-Jiang,18][Cardy,19]….

$$\langle O_1 \ldots O_n \rangle_\lambda \sim \lambda \int d^2 z \langle T \bar{T} O_1 \ldots O_n \rangle_0$$

N<5 Point, S. He, Hongfei Shu [1907.12603]

The deformation of correlation function

$$\langle \mathcal{O}_1(z_1,\bar{z}_1)\mathcal{O}_2(z_2,\bar{z}_2)\cdots\mathcal{O}_n(z_n,\bar{z}_n)\rangle_\lambda = \lambda\int d^2z\langle T\bar{T}(z,\bar{z})\mathcal{O}_1(z_1,\bar{z}_1)\mathcal{O}_2(z_2,\bar{z}_2)\cdots\mathcal{O}_n(z_n,\bar{z}_n)\rangle$$

Energy-Momentum conservation

$$\begin{aligned}\langle \mathcal{O}_1(z_1,\bar{z}_1)\mathcal{O}_2(z_2,\bar{z}_2)\cdots\mathcal{O}_n(z_n,\bar{z}_n)\rangle_\lambda =&\lambda\int d^2z\Big(\sum_{i=1}^{n}\big(\frac{h_i}{(z-z_i)^2}+\frac{\partial_{z_i}}{z-z_i}\big)\Big)\Big(\sum_{i=1}^{n}\big(\frac{\bar{h}_i}{(\bar{z}-\bar{z}_i)^2}+\frac{\partial_{\bar{z}_i}}{\bar{z}-\bar{z}_i}\big)\Big)\\ &\times\langle \mathcal{O}_1(z_1,\bar{z}_1)\mathcal{O}_2(z_2,\bar{z}_2)\cdots\mathcal{O}_n(z_n,\bar{z}_n)\rangle,\end{aligned}$$

Conformal Ward Identity

The deformed Two-Point Function(Step 1)

$$\lim_{\ell\to 0}\langle T(z+\ell)\bar{T}(\bar{z}-\ell)\mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\rangle$$

$$=\lim_{\ell\to 0}\Big(\sum_{i=1}^{2}\Big(\frac{\bar{h}}{(\bar{z}-\bar{z}_i-\ell)^2}+\frac{\partial_{\bar{z}_i}}{\bar{z}-\bar{z}_i-\ell}\Big)\Big)\Big(\sum_{i=1}^{2}\Big(\frac{h}{(z-z_i+\ell)^2}+\frac{\partial_{z_i}}{z-z_i+\ell}\Big)\Big)\langle\mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\rangle$$

Regularization （Step 2）

Log term associated with boundary term of Non local deformation (TTar). Nonlocal divergence given by Peskin’s 4D QFT.

$$\langle T(z)\bar{T}(z)\mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\rangle_\lambda=\lambda h\bar{h}z_{12}^2\bar{z}_{12}^2\mathcal{I}_2(z_1,z_2)\langle\mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\rangle$$

$$=\lambda h\bar{h}\frac{8\pi}{|z_{12}|^2}\Big(\frac{4}{\epsilon}+2\log|z_{12}|^2+2\log\pi+2\gamma-5\Big)\langle\mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\rangle.$$

[P. Kraus, J. Liu and D. Marolf，18; Cardy, 19]

Four-Point in CFTs

$$\langle \mathcal{O}^\dagger(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\mathcal{O}^\dagger(z_3,\bar{z}_3)\mathcal{O}(z_4,\bar{z}_4)\rangle = \frac{G(\eta,\bar{\eta})}{z_{13}^{2h} z_{24}^{2h} \bar{z}_{13}^{2\bar{h}} \bar{z}_{24}^{2\bar{h}}},$$

with the cross ratios

$$\eta = \frac{z_{12}z_{34}}{z_{13}z_{24}}, \quad \bar{\eta} = \frac{\bar{z}_{12}\bar{z}_{34}}{\bar{z}_{13}\bar{z}_{24}}.$$

Four-Point in Deformed-CFTs

$$
\begin{aligned}
&\langle \mathcal{O}(z_1,\bar z_1)\mathcal{O}(z_2,\bar z_2)\mathcal{O}^\dagger(z_3,\bar z_3)\mathcal{O}(z_4,\bar z_4)\rangle_\lambda \\
=&\lambda\Big\{h\bar h z_{13}^2\bar z_{13}^2\mathcal{I}_{2222}(z_1,z_3,\bar z_1,\bar z_3)+h\bar h z_{24}^2\bar z_{13}^2\mathcal{I}_{2222}(z_2,z_4,\bar z_1,\bar z_3) \\
&+h\bar h z_{13}^2\bar z_{24}^2\mathcal{I}_{2222}(z_1,z_3,\bar z_2,\bar z_4)+h\bar h z_{24}^2\bar z_{24}^2\mathcal{I}_{2222}(z_2,z_4,\bar z_2,\bar z_4) \\
&+\left(\bar z_{13}^2\mathcal{I}_{111122}(z_1,z_2,z_3,z_4,\bar z_1,\bar z_3)+\bar z_{24}^2\mathcal{I}_{111122}(z_1,z_2,z_3,z_4,\bar z_2,\bar z_4)\right)\bar h z_{23}z_{14}\frac{\eta\partial_\eta G(\eta,\bar\eta)}{G(\eta,\bar\eta)} \\
&+\left(z_{13}^2\mathcal{I}_{221111}(z_1,z_3,\bar z_1,\bar z_3,\bar z_2,\bar z_4)+z_{24}^2\mathcal{I}_{221111}(z_2,z_4,\bar z_2,\bar z_4,\bar z_1,\bar z_3)\right)h\bar z_{23}\bar z_{14}\frac{\bar\eta\partial_{\bar\eta}G(\eta,\bar\eta)}{G(\eta,\bar\eta)} \\
&+z_{23}z_{14}\bar z_{23}\bar z_{14}\mathcal{I}_{11111111}(z_1,z_2z_3,z_4,\bar z_1,\bar z_3,\bar z_2,\bar z_4)\eta\bar\eta\frac{\partial_\eta\partial_{\bar\eta}G(\eta,\bar\eta)}{G(\eta,\bar\eta)}\Big\} \\
&\langle \mathcal{O}^\dagger(z_1,\bar z_1)\mathcal{O}(z_2,\bar z_2)\mathcal{O}^\dagger(z_3,\bar z_3)\mathcal{O}(z_4,\bar z_4)\rangle.
\end{aligned}
$$

N-point Function, SUSY $\cdots$

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]; SH to appear

Correlation functions in deformed CFTs w/. SUSY (1,1) (2,2)

H. Jiang, A. Sfondrini and G. Tartaglino-Mazzucchelli,19; （0，2）
C. K. Chang, C. Ferko, S. Sethi, A. Sfondrini and G. Tartaglino-Mazzucchelli, 19;（1，1）
E. A. Coleman, J. Aguilera-Damia, D. Z. Freedman and R. M. Soni,19;（2，2）
H. Jiang and G. Tartaglino-Mazzucchelli, 19 （0，1）

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]

SUSY transformation

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]

coordinates on superspace

$$Z = (z, \theta)$$

$$\bar{Z} = (\bar{z}, \bar{\theta})$$

$$D = \partial_\theta + \theta\partial_z, \quad D^2 = \partial_z$$

$$\oint dZ = \frac{1}{2\pi i}\oint dz \int d\theta.$$

- The stress tensor superfield $J(Z) = \Theta(z) + \theta T(z)$,

$$T\bar{T}(z) = \int d\theta d\bar{\theta} J(Z)\bar{J}(\bar{Z})$$

Ward ID SUSY

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]

supercoordinates transformations

A superfield $\Phi(Z, \bar{Z})$

$$\delta_E \Phi(Z, \bar{Z}) = [J_E, \Phi(Z, \bar{Z})] = \oint dZ' E(Z') J(Z') \Phi(Z, \bar{Z})$$

$$\delta_E \Phi(Z, \bar{Z}) = E(Z) \partial_z \Phi(Z, \bar{Z}) + \frac{1}{2} D E(Z) D \Phi(Z, \bar{Z}) + \Delta \partial_z E(Z) \Phi(Z, \bar{Z}),$$

SUSY -Ward ID：

$$\langle J(Z_0) \Phi_1(Z_1, \bar{Z}_1) \ldots \Phi_n(Z_n, \bar{Z}_n) \rangle_0$$
$$= \sum_{i=1}^{n} \left(\frac{\theta_{0i}}{Z_{0i}} \partial_{z_i} + \frac{1}{2Z_{0i}} D_i + \Delta_i \frac{\theta_{0i}}{Z_{0i}^2} \right) \langle \Phi_1(Z_1, \bar{Z}_1) \ldots \Phi_n(Z_n, \bar{Z}_n) \rangle_0$$

Correlation function with （1,1） SUSY

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]

2-Pt

$$\langle\Phi_1(Z_1,\bar{Z}_1)\Phi_2(Z_2,\bar{Z}_2)\rangle = c_{12}\frac{1}{Z_{12}^{2\Delta}\bar{Z}_{12}^{2\bar{\Delta}}}, \quad \Delta\equiv\Delta_1=\Delta_2, \quad \bar{\Delta}\equiv\bar{\Delta}_1=\bar{\Delta}_2$$

3-Pt

$$\langle\Phi_1(Z_1,\bar{Z}_1)\Phi_2(Z_2,\bar{Z}_2)\Phi_3(Z_3,\bar{Z}_3)\rangle = \left(\prod_{i<j=1}^{3}\frac{1}{Z_{ij}^{\Delta_{ij}}\bar{Z}_{ij}^{\bar{\Delta}_{ij}}}\right)(c_{123}+c'_{123}\theta_{123}\bar{\theta}_{123}),$$

$$c_{123}+c'_{123}\theta_{123}\bar{\theta}_{123} = c_{123}e^{c'_{123}\theta_{123}\bar{\theta}_{123}/c_{123}}.$$

$$\theta_{ijk} = \frac{1}{\sqrt{Z_{ij}Z_{jk}Z_{kl}}}(\theta_i Z_{jk}+\theta_j Z_{ki}+\theta_k Z_{ij}+\theta_i\theta_j\theta_k),$$

Deformed Correlation function with （1,1）SUSY

2-Pt

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]

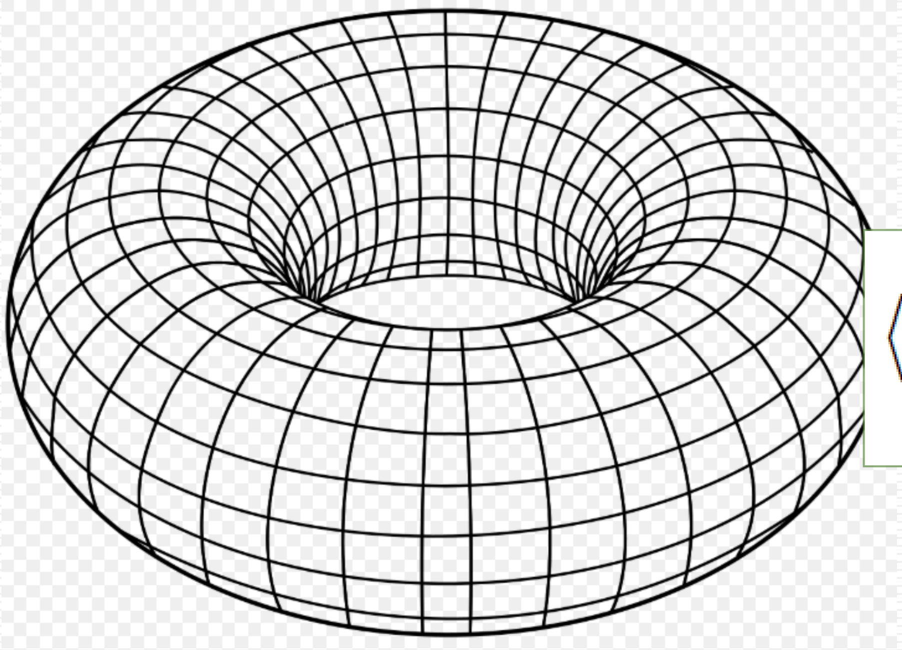
$$\int d^2z d\theta d\bar{\theta} \langle J(Z)\bar{J}(\bar{Z})\Phi_1(Z_1,\bar{Z}_1)\Phi_n(Z_2,\bar{Z}_2)\rangle / \langle \Phi_1(Z_1,\bar{Z}_1)\Phi_n(Z_2,\bar{Z}_2)\rangle$$
$$= \Delta\bar{\Delta}\int d^2z d\theta d\bar{\theta}\Big[\Big(-\frac{2}{Z_{12}}\Big(\frac{\theta_{01}}{z_{01}}-\frac{\theta_{02}}{z_{02}}\Big)-\frac{\theta_{12}}{Z_{12}}\Big(\frac{1}{Z_{01}}+\frac{1}{Z_{02}}\Big)+\Big(\frac{\theta_{01}}{z_{01}^2}+\frac{\theta_{02}}{z_{02}^2}\Big)\Big)$$
$$\times\Big(-\frac{2}{\bar{Z}_{12}}\Big(\frac{\bar{\theta}_{01}}{\bar{z}_{01}}-\frac{\bar{\theta}_{02}}{\bar{z}_{02}}\Big)-\frac{\bar{\theta}_{12}}{\bar{Z}_{12}}\Big(\frac{1}{\bar{Z}_{01}}+\frac{1}{\bar{Z}_{02}}\Big)+\Big(\frac{\bar{\theta}_{01}}{\bar{z}_{01}^2}+\frac{\bar{\theta}_{02}}{\bar{z}_{02}^2}\Big)\Big)\Big].$$

First order deformation

$$\frac{1}{\langle \Phi_1(Z_1,\bar{Z}_1)\Phi_2(Z_2,\bar{Z}_2)\rangle}\int d^2z d\theta d\bar{\theta}\langle J(Z)\bar{J}(\bar{Z})\Phi_1(Z_1,\bar{Z}_1)\Phi_2(Z_2,\bar{Z}_2)\rangle$$
$$=-\frac{4\pi\Delta^2}{Z_{12}\bar{Z}_{12}}\Big(-\frac{4}{\epsilon}+2\ln|z_{12}|^2+2\gamma+2\ln\pi-2\Big).$$

Similar structure as bosonic theory

Correlation functions in deformed CFTs on Torus

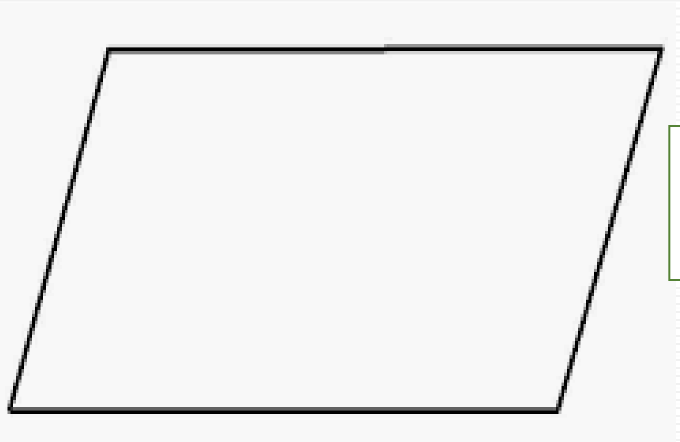


$$\langle\phi_1(z_1)\ldots\phi_n(z_n)\rangle_\lambda \sim \lambda \int_{T^2} d^2z \langle T\bar{T}(z)\phi_1(z_1)\ldots\phi_n(z_n)\rangle_0$$

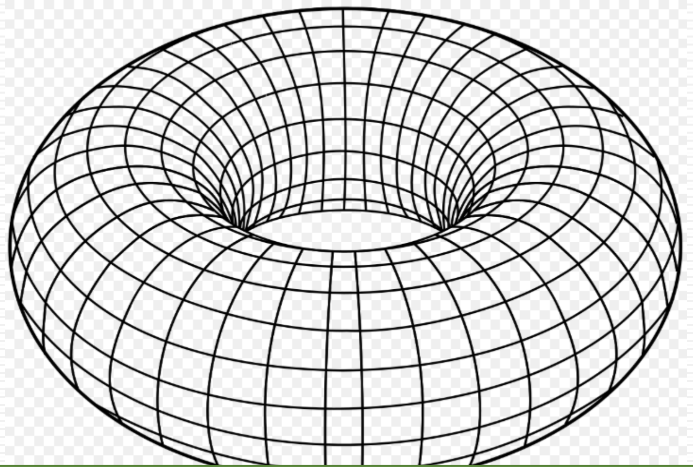
S. He, Yuan Sun [2004.07486]

Correlation function on Torus

S. He, Yuan Sun [2004.07486]



$$z = e^{2\pi i w}$$



$$\langle T(w)X\rangle - \langle T\rangle\langle X\rangle$$
$$= \sum_i \Big(h_i (P(w-w_i) + 2\eta_1) + (\zeta(w-w_i) + 2\eta_1 w_i)\partial_{w_i} \Big)\langle X\rangle + 2\pi i \partial_\tau \langle X\rangle,$$

$$X = \phi_1(w_1, \bar{w}_1)\phi_2(w_1, \bar{w}_2)\ldots\phi_n(w_n, \bar{w}_n)$$

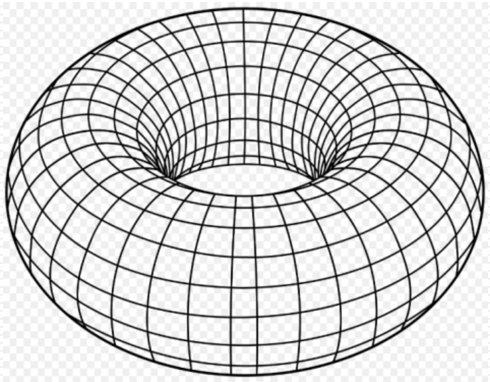
P. Di Francesco, P. Mathieu and D. Senechal, "Conformal FieldTheory,"

Weierstrass P-function and zeta function

$$P(w) \sim 1/w^2, \zeta(w) \sim 1/w$$

Ward Identity on Torus

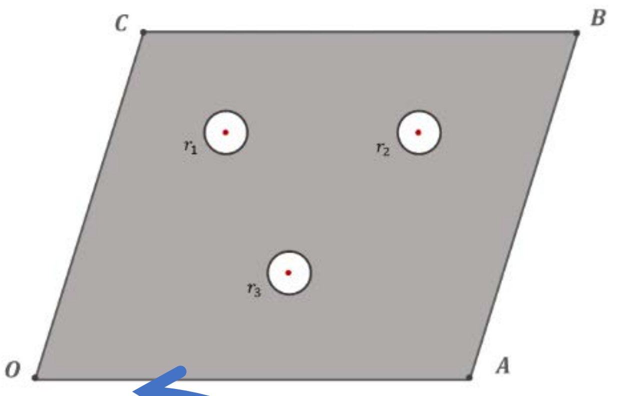
S. He, Yuan Sun [2004.07486]



$$T_{pl}(z)=\sum_{n\in\mathbb{Z}}\frac{L_n}{z^{n+2}}, \quad Z=\mathrm{tr}(q^{L_0-c/24}\bar{q}^{\bar{L}_0-c/24}), \quad q=e^{2\pi i\tau}$$

$$\langle X(\{w_i\})\rangle=\frac{1}{Z}\mathrm{tr}(X(\{w_i\})q^{L_0-c/24}\bar{q}^{\bar{L}_0-c/24})$$

replacement $\phi_1(w_1,\bar{w}_1)\rightarrow\bar{T}(w_1,\bar{w}_1)$



$$\begin{aligned}&\langle T(w)\bar{T}(\bar{v})X\rangle\\&=2\pi i\partial_\tau\langle\bar{T}(\bar{v})X\rangle+2\pi i(\partial_\tau\ln Z)\langle\bar{T}(\bar{v})X\rangle+\sum_i h_i\Big(-\zeta'(w_i-w)+2\eta_1\Big)\langle\bar{T}(\bar{v})X\rangle\\&\quad+\sum_i\Big(-\zeta(w_i-w)+2\eta_1 w_i-2\eta_1 w-\pi i\Big)\partial_{w_i}\langle\bar{T}(\bar{v})X\rangle\end{aligned}$$

Regularlization

$$\int_{T^2}d^2z\ldots\rightarrow\int_{T^2-D(z_i)}d^2z\ldots$$

$$\lambda\int_{T^2-\sum_i D(w_i)}d^2v\langle T(v)\bar{T}(\bar{v})X\rangle,$$

Further, Multiple T& Tbar on Torus

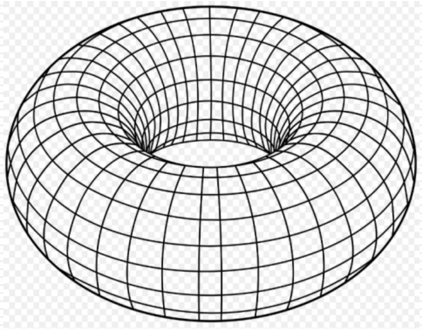
S. He, Yuan Sun [2004.07486]

$$
\begin{aligned}
&\mathrm{tr}(T(w)[T(u_1)\ldots T(u_n)\bar{T}(v_1)\ldots\bar{T}(v_m)]Xq^{L_0-c/12})\\
=&2\pi i\frac{\partial}{\partial\tau}\mathrm{tr}(T(u_1)\ldots T(u_n)\bar{T}(v_1)\ldots\bar{T}(v_m)Xq^{L_0-c/24})\\
&+\sum_i h_i\Big(-\zeta'(w_i-w)+2\eta_1\Big)\mathrm{tr}(T(u_1)\ldots T(u_n)\bar{T}(v_1)\ldots\bar{T}(v_m)Xq^{L_0-c/24})\\
&+\sum_i\Big(-\zeta(w_i-w)+2\eta_1 w_i-2\eta_1 w-\pi i\Big)\partial_{w_i}\mathrm{tr}(T(u_1)\ldots T(u_n)\bar{T}(v_1)\ldots\bar{T}(v_m)e^{L_0-c/24})\\
&+\frac{c}{12}\sum_j P''(u_j-w)\mathrm{tr}(T(u_1)\ldots\hat{T}(u_j)\ldots T(u_n)\bar{T}(v_1)\ldots\bar{T}(v_m)Xq^{L_0-c/24})\\
&+\sum_j 2\Big(P(w-u_j)+2\eta_1\Big)\mathrm{tr}(T(u_1)\ldots T(u_n)\bar{T}(v_1)\ldots\bar{T}(v_m)Xq^{L_0-c/24})\\
&+\sum_j\Big(\zeta(w-u_j)+2\eta_1 u_j-2\eta_1 w-\pi i\Big)\partial_{u_i}\mathrm{tr}(T(u_1)\ldots T(u_n)\bar{T}(v_1)\ldots\bar{T}(v_m)Xq^{L_0-c/24})
\end{aligned}
$$

- **Can be computable but very complicated.**
- **New recursion relation found!**
- **Algorithm for correlation function with Multiple T&Tbar is offered!**

First order deformation on Torus

S. He, Yuan Sun [2004.07486]



$$Z' = \int D\phi e^{-S+\lambda \int d^2 z T\bar{T}(z)}$$
$$= Z(1 + \lambda \int d^2 z \langle T\bar{T}\rangle(z)) + \frac{1}{2}\lambda^2 \int\int d^2u_1 d^2u_2 \langle T\bar{T}(u_1) T\bar{T}(u_2)\rangle + \ldots).$$

$$\lambda Z \int d^2 z \langle T\bar{T}\rangle(z) = \lambda (2\pi)^2 \tau_2 \partial_\tau \partial_{\bar{\tau}} Z.$$

$$Z = \mathrm{tr}(q^{L_0 - c/24} \bar{q}^{\bar{L}_0 - c/24}), \quad q = e^{2\pi i \tau}$$

- **The same as the first order in** 1806.07426.

Correlation function of Generic Operator may not be Modular Covariant.

TTbar flow effect on the partition function

S. He, Yuan Sun, Yu-Xuan Zhang [2011.02902]

Go beyond the first order?

- Conformal symmetry does not hold.

- Wald Identity breaks down due to higher order deformation of TTbar.

- Solve the effective action order by order and go further.

TTbar deformed action

$$S^\lambda = \int_{\mathcal{M}} \sqrt{g}\mathrm{d}^2x\mathcal{L}^\lambda(\phi, \nabla_a\phi, g_{ab}).$$

TTbar Flow equation

$$\frac{\mathrm{d}\mathcal{L}^\lambda}{\mathrm{d}\lambda} = \frac{1}{2}\epsilon^{\mu\nu}\epsilon^{\rho\sigma}T^\lambda_{\mu\rho}T^\lambda_{\nu\sigma};$$

$$\mathcal{L}^\lambda = \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\mathcal{L}^{(n)}, \quad T^\lambda_{\mu\nu} = \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}T^{(n)}_{\mu\nu}.$$

$$\mathcal{L}^{(n+1)} = \frac{1}{2}\sum_{i=0}^{n} C_n^i\left(T^{\mu}{}_{\mu}^{(i)}T^{\nu}{}_{\nu}^{(n-i)} - T^{\mu}{}_{\nu}^{(i)}T^{\nu}{}_{\mu}^{(n-i)}\right),$$

$$T^{(n)}_{\mu\nu} = 2\frac{\partial\mathcal{L}^{(n)}}{\partial g^{\mu\nu}} - g_{\mu\nu}\mathcal{L}^{(n)},$$

$$C_n^i \equiv \frac{n!}{i!(n-i)!}.$$

TTbar deformed Partition function

$$
\begin{aligned}
Z^{\lambda} &= \int \mathcal{D}\phi \; \mathrm{e}^{-\int_{\mathcal{M}} \mathcal{L}^{\lambda}[\phi]} \\
&= Z^{(0)} - \lambda Z^{(0)} \int_{\mathcal{M}} \langle \mathcal{L}^{(1)} \rangle + \frac{\lambda^2}{2} Z^{(0)} \Big(\int_{\mathcal{M}} \int_{\mathcal{M}'} \langle \mathcal{L}^{(1)}(x) \mathcal{L}^{(1)}(x') \rangle - \int_{\mathcal{M}} \langle \mathcal{L}^{(2)} \rangle \Big) + \mathcal{O}(\lambda^3) \\
&\equiv Z^{(0)} + \lambda Z^{(1)} + \frac{\lambda^2}{2} Z^{(2)} + \mathcal{O}(\lambda^3),
\end{aligned}
$$

Deformed Partition function up to second order

$$
\begin{aligned}
Z^{(0)} &= \int \mathcal{D}\phi \; \mathrm{e}^{-\int_{\mathcal{M}} \mathcal{L}^{(0)}[\phi]}, \\
Z^{(1)} &= -Z^{(0)} \int_{\mathcal{M}} \langle \mathcal{L}^{(1)} \rangle, \\
Z^{(2)} &= Z^{(0)} \Big(\int_{\mathcal{M}} \int_{\mathcal{M}'} \langle \mathcal{L}^{(1)}(x) \mathcal{L}^{(1)}(x') \rangle - \int_{\mathcal{M}} \langle \mathcal{L}^{(2)} \rangle \Big).
\end{aligned}
$$

TTbar deformed free boson on Torus

$$\mathcal{L}^{(0)} = 2g\partial\phi\bar{\partial}\phi.$$

$$\mathcal{L}^{(1)} = -4T^{(0)}\bar{T}^{(0)} = -4g^2(\partial\phi\bar{\partial}\phi)^2,$$

$$\mathcal{L}^{(2)} = -4(T^{(0)}\bar{T}^{(1)} + \bar{T}^{(0)}T^{(1)}) = 32g^3(\partial\phi\bar{\partial}\phi)^3,$$

Deformed Partition function up to second order

1st order

$$Z^{(1)} = 4\tau_2\partial_\tau\partial_{\bar{\tau}}Z^{(0)}.$$

1806.07426

2004.07486

Free Fermion : vanishing !!

2nd order

$$Z^{(2)} = 16\left(\tau_2^2\partial_\tau^2\partial_{\bar{\tau}}^2 + i\tau_2(\partial_\tau^2\partial_{\bar{\tau}} - \partial_{\bar{\tau}}^2\partial_\tau)\right)Z^{(0)} + \left(72\partial_\tau\partial_{\bar{\tau}} - 6\tau_2^{-2}\right)Z^{(0)}$$

$$Z^{(0)}\left(\int_{\mathcal{M}}\int_{\mathcal{M}'}\langle\mathcal{L}^{(1)}(x)\mathcal{L}^{(1)}(x')\rangle\right)$$

1806.07426

$$-\int_{\mathcal{M}}\langle\mathcal{L}^{(2)}\rangle$$

Comments

$$Z^{(2)} = 16\left(\tau_2^2\partial_\tau^2\partial_{\bar\tau}^2 + i\tau_2(\partial_\tau^2\partial_{\bar\tau} - \partial_{\bar\tau}^2\partial_\tau)\right)Z^{(0)} + \left(72\partial_\tau\partial_{\bar\tau} - 6\tau_2^{-2}\right)Z^{(0)}$$

$$Z^{(0)}\left(\int_{\mathcal{M}}\int_{\mathcal{M}'}\langle\mathcal{L}^{(1)}(x)\mathcal{L}^{(1)}(x')\rangle\right)$$

1806.07426

TTbar-flow effects;
Independent of renormalization scheme.

$$\mathcal{Z}(\omega_1,\omega_2,\ell) = \sum_n e^{i\tau_1|\omega_1|P_n - \tau_2|\omega_1|\mathcal{E}_n^{(\ell)}}.$$

$$Z^\lambda = \int \mathcal{D}\phi\, \mathrm{e}^{-\int_{\mathcal{M}}\mathcal{L}^\lambda[\phi]}$$

$$\partial_\lambda\langle P_s\rangle_n = -\pi^2\left(E_n\partial_L\langle P_s\rangle_n + P_n\frac{s\langle P_s\rangle_n}{L}\right).$$

Solving Bergurs' s equation to define the partition function in operator formalism.

Finally, the 2nd term in deformed free fermion on torus and plane is vanishing.

The presence of flow effect is at the second order in the deformed partition function.

Chaotic Behavior of deformed CFTs (OTOC)

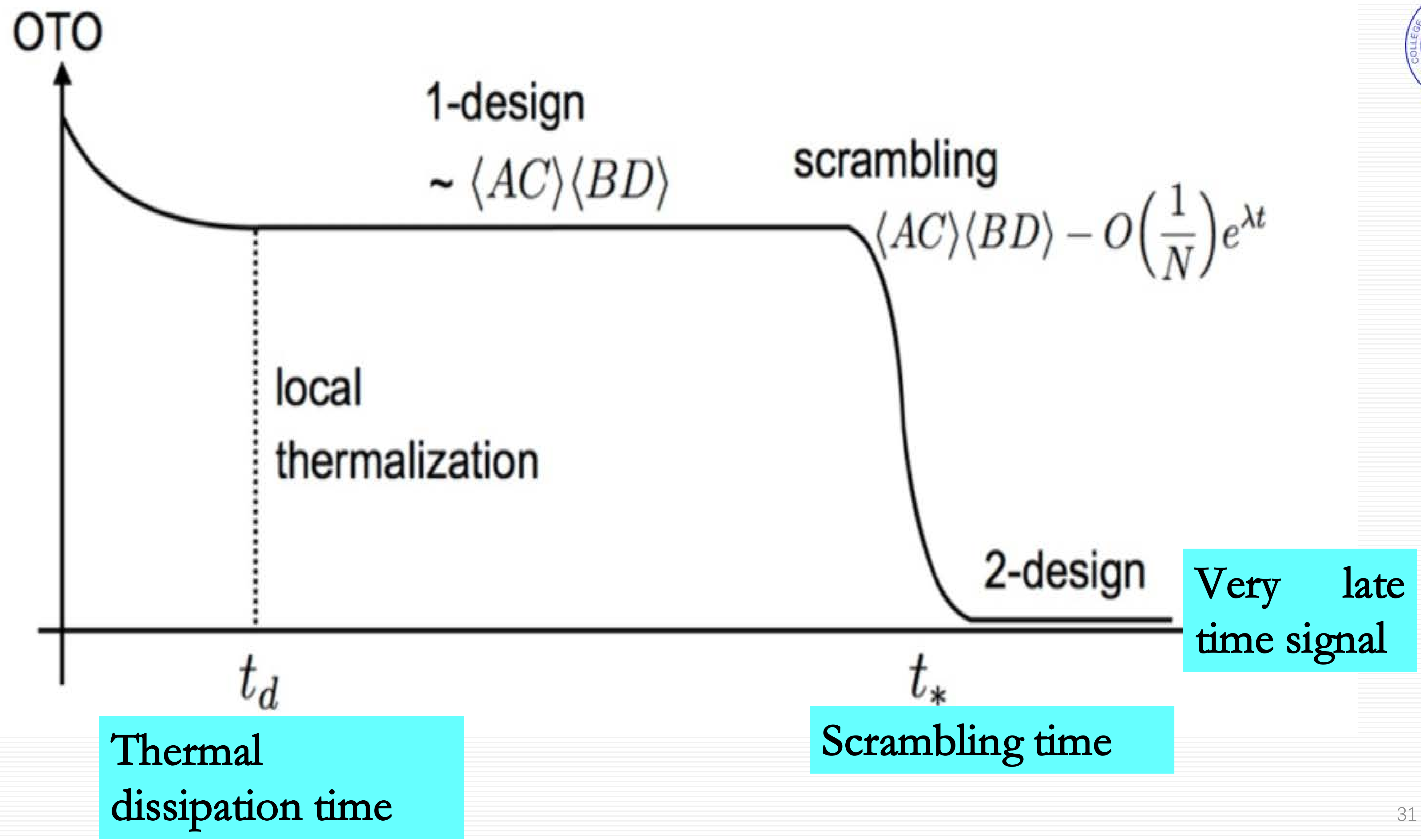
OTOC can diagnose the chaotic behavior of Many body system.

- **In Chaotic system, the late time behavior of physical quantities are very sensitive to the early time input.**
- **There are many quantities to capture the behavior:**

  Lyapnov parameter, scrambling time scale and Rulle resonance. [A. Larkin and Y. Ovchinnikov,1969],[A. Kitaev,15]

- **For (non) integrable models do not show any chaotic signals, e.g OTOC (2nd REE).** [E. Perlmutter,16],[Y. Gu and X. L. Qi,16].[S. He, Feng-Li Lin, J.J. Zhang, JHEP 1708 (2017) 126; JHEP 1712 (2017) 073].
- **One can also look at the spectrum form factor to test the time evolution behavior.**
- **By empirism, Holographic CFTs should have chaotic signals.**

[E. Perlmutter,16],[J. L. Karczmarek, J. M. Maldacena and A.Strominger,16],[J. M. Maldacena, D. Stanford,16].



OTO
1-design
~ $\langle AC\rangle\langle BD\rangle$
scrambling
$\langle AC\rangle\langle BD\rangle - O\left(\frac{1}{N}\right)e^{\lambda t}$
local
thermalization
2-design
t_d
t_*
Very late time signal
Thermal dissipation time
Scrambling time

OTOC in TT-deformed CFTs

$$\frac{\langle W(t)VW(t)V\rangle_\beta}{\langle W(t)W(t)\rangle_\beta\langle VV\rangle_\beta}$$

Put the excitations on the thermal deformed CFTs (Cylinder)

$$\frac{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)\rangle_\beta\langle V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}$$
$$\times\Big(1-\lambda(\frac{2\pi}{\beta})^2\int d^2z_b|z_b|^2\frac{\langle\big(T(z_b)-\frac{c}{24z^2}\big)\big(\bar{T}(\bar{z}_b)-\frac{c}{24\bar{z}^2}\big)W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)\rangle}{\langle W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)\rangle}$$
$$-\lambda(\frac{2\pi}{\beta})^2\int d^2z_c|z_c|^2\frac{\langle\big(T(z_c)-\frac{c}{24z^2}\big)\big(\bar{T}(\bar{z}_c)-\frac{c}{24\bar{z}^2}\big)V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}{\langle V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}$$
$$+\lambda(\frac{2\pi}{\beta})^2\int d^2z_a|z_a|^2\frac{\langle\big(T(z_a)-\frac{c}{24z^2}\big)\big(\bar{T}(\bar{z}_a)-\frac{c}{24\bar{z}^2}\big)W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}{\langle W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}+\mathcal{O}(\lambda^2)\Big)$$

Late time of OTOC

S. He, Hongfei Shu [1907.12603]

$$\frac{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)\rangle_\beta\langle V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}$$

$$\xrightarrow{T\bar{T}} \frac{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)\rangle_\beta\langle V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}\left\{1-\lambda C_1(x)+\lambda C_2(x)e^{-\frac{2\pi}{\beta}t}+\cdots\right\},$$

The choices of the sign of λ do not affect the late time behavior exp[-2πβt] in above equation.

D. J. Gross, J. Kruthoff, A. Rolph and E. Shaghoulian,19

Summary

- **First order deformation to correlation functions in deformed theory on the plane and torus with/without SUSY.**

- **The second order correction contains the TTbar flow effect in path integral formalism.**

- **Entanglement entropy of local excited states, Quantum chaos in TTbar and JTbar CFTs (Maynot be mentioned in this talk, Refer to S. He, Hongfei Shu [1907.12603])**

Future Problems

- **Generic higher order correlation functions in deformed theory.**
- **Entanglement entropy, Renyi Entropy, Entanglement of Purification (EOP), Complexity, Psuedo Entanglement entropy etc. in deformed CFTs**
- **Correlation function in Deep UV [Cardy,19; 2006.03054](Non-local)**
- **Modular structure of higher point correlation functions on torus (Sub class).**
- **2D Bosonization of TTbar…**
- **Correlation functions in Lorentz-breaking theory [M. Guica, Cardy…], …**
- **……**

Thanks for your attention!