Constraints on Low-Energy Effective Field Theories from Weak Cosmic Censorship

Bo Ning

Sichuan University

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Motivation

Weak cosmic censorship conjecture (WCCC) : Penrose, 1969

Singularities are hidden behind event horizons.

- Proof via gedanken experiment:
  Throwing positive-energy matter into extremal or near-extremal black hole cannot destroy the horizon. Wald 1974; Sorce-Wald 2017

Beyond Einstein-Maxwell theory ?

- Quantum corrections could leave low-energy relics in the form of higher-order derivative terms beyond Einstein-Maxwell.
- Regard WCCC as a physical principle to constrain the higher-order EFTs.

Motivation

Consider most general quartic order corrections to Einstein-Maxwell theory

$$I = \int d^4x \, \sqrt{-g}\left(\frac{1}{2\kappa}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \Delta L\right),$$

$$\begin{aligned}\Delta L = \; & c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\ & + c_4 R F_{\mu\nu}F^{\mu\nu} + c_5 R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} + c_6 R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} \\ & + c_7 F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + c_8 F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu} \,.\end{aligned}$$

We obtain the following parameter bounds from WCCC by considering gedanken experiment for extremal black holes:

$$c_2 + 4c_3 + \frac{10c_4}{\kappa} + \frac{3c_5}{\kappa} + \frac{3c_6}{\kappa} \leq 0 \,.$$

Outline

- Motivation
- Charged black holes in higher derivative theory
- Gedanken experiments and WCCC
  - Test particle
  - Sorce-Wald method
- Discussion

Charged black holes in higher derivative theory

- Charged non-spinning solutions (*linear correction* to Reissner-Nordström):

Motl et al, 2007

$$A_t = -\frac{q}{r} + \frac{2q^3}{5r^5}\left[c_5\kappa + c_6\kappa\left(6 - \frac{5mr}{q^2}\right) + 8c_7 + 4c_8\right]$$

$$\begin{aligned}
-g_{tt} = & 1 - \frac{\kappa m}{r} + \frac{\kappa q^2}{2r^2} + c_2\left(\frac{\kappa^3 m q^2}{r^5} - \frac{\kappa^3 q^4}{5r^6} - \frac{2\kappa^2 q^2}{r^4}\right) \\
& + c_3\left(\frac{4\kappa^3 m q^2}{r^5} - \frac{4\kappa^3 q^4}{5r^6} - \frac{8\kappa^2 q^2}{r^4}\right) \\
& + c_4\left(-\frac{6\kappa^2 m q^2}{r^5} + \frac{4\kappa^2 q^4}{r^6} + \frac{4\kappa q^2}{r^4}\right) \\
& + c_5\left(\frac{4\kappa^2 q^4}{5r^6} - \frac{\kappa^2 m q^2}{r^5}\right) \\
& + c_6\left(\frac{\kappa^2 m q^2}{r^5} - \frac{\kappa^2 q^4}{5r^6} - \frac{2\kappa q^2}{r^4}\right) \\
& + c_7\left(-\frac{4\kappa q^4}{5r^6}\right) + c_8\left(-\frac{2\kappa q^4}{5r^6}\right) + O(c_i^2)\,.
\end{aligned}$$

Charged black holes in higher derivative theory

- Singularity will be hidden by a horizon if
  (absorbing the correction to metric function as mass shift)

$$m \geq \sqrt{\frac{2}{\kappa}}|q|\left(1 - \frac{4}{5q^2}c_0\right), \qquad c_0 \equiv c_2 + 4c_3 + \frac{c_5}{\kappa} + \frac{c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2}$$

- For extremal solution, location of degenerate horizon
  ($g_{tt} = 0$ and $dg_{tt}/dr = 0$)

$$r_H^c = \frac{m\kappa}{2} + \frac{4}{5m}\left(c_2 + 4c_3 + \frac{10c_4 + c_5 + c_6}{\kappa} - \frac{16c_7 + 8c_8}{\kappa^2}\right)$$

Electrostatic potential on extremal horizon

$$\Phi_H^c = -(\xi^a A_a)|_{\mathcal{H}} = \sqrt{\frac{2}{\kappa}}\left(1 + \frac{4c_0'}{5q^2}\right), \qquad c_0' = -\frac{10c_4}{\kappa} - \frac{2c_5}{\kappa} - \frac{2c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2}$$

Gedanken experiments and WCCC

Consider throwing matter into an *extremal* black hole.

- Assuming *stability* of our family of solutions, i.e., the in-falling matter finally turns original extremal BH into a one-parameter family solutions ($m(w)$, $q(w)$).

- At first order in w, WCCC holds only if

$$\delta m - \sqrt{\frac{2}{\kappa}}\left(1 + \frac{4c_0}{5q^2}\right)\delta q \geq 0$$

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Gedanken experiments: Test particle

For regular solution (m, q), consider a test particle with mass δm_0 and charge δq_0 falling in from infinity, with action

$$S_p = \int d\tau \, (\delta m_0 - \delta q_0 \vec{u} \cdot \vec{A})$$

canonical momentum $\vec{p} = \delta m_0 \, \vec{u} - \delta q_0 \, \vec{A}$ is conserved along trajectory, $\vec{\xi} \cdot \vec{p} = \text{const}$
applying to infinity and horizon,

$$\delta m_0 \left(\vec{u}^H \cdot \vec{\xi}\right) - \Phi_H^c \delta q_0 = \delta m_0 (\vec{u}^\infty \cdot \vec{\xi}) = -\delta E_\infty$$

Final space-time is parameterized by ($m + \delta m$, $q + \delta q$). Conservation of charge and ADM mass (gravitational radiation neglectable) gives $\delta q = \delta q_0$, $\delta m = \delta E_\infty$, hence

$$\delta m - \Phi_H^c \delta q = -\delta m_0 (\vec{u}^H \cdot \vec{\xi}) \geq 0$$

→ $$\delta m \geq \sqrt{\frac{2}{\kappa}} \left(1 + \frac{4c_0'}{5q^2}\right) \delta q$$

(different from ★)

Gedanken experiments: Sorce-Wald method

General method to derive the “law of energy conservation” for in-falling process:

- Following Iyer-Wald’s construction for Noether charge: Iyer-Wald 1994

$$\delta \boldsymbol{L} = \mathbf{E}(\phi)\delta\phi + d\boldsymbol{\Theta}(\phi, \delta\phi)$$

For any vector ξ^a one can construct the associated Noether current $\mathbf{J}_\xi = \boldsymbol{\Theta}(\phi, \mathcal{L}_\xi \phi) - i_\xi \boldsymbol{L}$ which is closed and can be written as $\mathbf{J}_\xi = d\mathbf{Q}_\xi + \xi_d \mathbf{C}^d$

Assuming ξ^a is Killing vector and on-shell, one find $\delta \mathbf{J}_\xi = d i_\xi \boldsymbol{\Theta}(\phi, \delta\phi)$ combined with $\delta \mathbf{J}_\xi = d\delta \mathbf{Q}_\xi + \xi^a \delta \mathbf{C}_a$ yields

$$\int_{\partial\Sigma} [\delta \mathbf{Q}_\xi - i_\xi \boldsymbol{\Theta}(\phi, \delta\phi)] = -\int_\Sigma \xi^a \delta \mathbf{C}_a$$

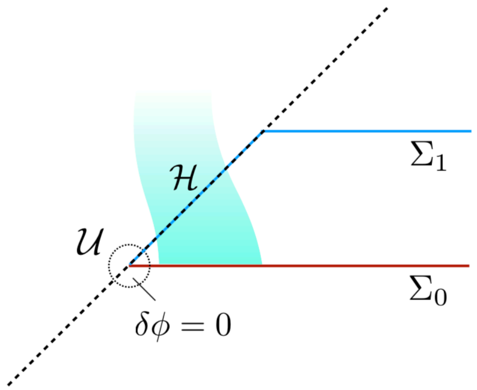
Choosing $\partial\Sigma = \infty$ and ξ^a time-like Killing vector, LHS is variation of ADM mass; identifying $(\delta \mathbf{C}_a)_{bcd} := \boldsymbol{\epsilon}_{ebcd} \left(\delta T^e{}_a + A_a \delta j^e\right)$, one arrive

$$\delta \mathcal{M} = -\int_\Sigma \boldsymbol{\epsilon}_{ebcd}\, \xi^a \left(\delta T^e{}_a + A_a \delta j^e\right)$$

Gedanken experiments: Sorce-Wald method

δj^e , $\delta T^e{}_a$: associated current and stress tensor of in-falling matter passing through hypersurface Σ (chosen to be $H \cup \Sigma_1$)

Assuming all the matter fall into black hole far earlier than joint moment of H and Σ_1 , replace integral on Σ with H and obtain



$$\delta \mathcal{M} - \Phi_{\rm H} \int_{\mathcal{H}} \epsilon_{abcd}\, \delta j^a = - \int_{\mathcal{H}} \epsilon_{ebcd}\, \xi^a \delta T^e{}_a$$

- on horizon $\xi^a \propto n^a$ (null), RHS is non-negative due to *null energy condition*
- charge crossing the horizon: $\delta \mathcal{Q} \equiv \int_{\mathcal{H}} \epsilon_{abcd}\, \delta j^a$

→ $$\delta \mathcal{M} - \Phi_{\rm H} \delta \mathcal{Q} \geq 0.$$

Gedanken experiments: Sorce-Wald method

- Explicit form of Q_ξ :

$$(\mathbf{Q}_\xi)_{c_3 c_4} = \boldsymbol{\epsilon}_{abc_3c_4}\left(M^{abc}\,\xi_c - E^{abcd}\,\nabla_{[c}\,\xi_{d]}\right)$$

with

$$M^{abc} \equiv -2\nabla_d E^{abcd} + E_F^{ab} A^c$$

$$E^{abcd} \equiv \frac{\delta L}{\delta R_{abcd}}, \qquad E_F^{ab} \equiv \frac{\delta L}{\delta F_{ab}}\,.$$

Gedanken experiments: Sorce-Wald method

$\delta\mathcal{M}$ and $\delta\mathcal{Q}$ for black hole in the higher theory might not be the same as the ones for Reissner-Nordström black hole, i.e., δM and δQ .

- Corrections to Q_ξ due to the higher-dimension Lagrangian ΔL fall off too quickly to contribute to the ADM mass, hence $\delta\mathcal{M} = \delta M$
- Straightforward calculations using C_a give $\delta\mathcal{Q} \equiv \int_{\mathcal{H}} \epsilon_{abcd}\, \delta j^a = \delta Q + \mathcal{O}(c_i^2)$

Hence gives the same constraint as test particle case.

Gedanken experiments: Parameter bound

"Law of energy conservation" gives (with NEC)

$$\delta m - \sqrt{\frac{2}{\kappa}}\left(1 + \frac{4c_0'}{5q^2}\right)\delta q \geq 0$$

comparing with ★ which holds WCCC

$$\delta m - \sqrt{\frac{2}{\kappa}}\left(1 + \frac{4c_0}{5q^2}\right)\delta q \geq 0\,.$$

we must have $c_0' \geq c_0$, or

$$c_2 + 4c_3 + \frac{10c_4}{\kappa} + \frac{3c_5}{\kappa} + \frac{3c_6}{\kappa} \leq 0$$

Our key result !

Discussion

How does this bound work in the real world?

- Non-linear EM terms contribute to c_7 and c_8 which do not appear in the bound, implying that QED automatically bypass the WCCC constraint.

- Next leading order correction to Einstein-Maxwell background is given by g-γ-γ amplitudes with a scalar or spinor loop. For minimally-coupled case, 1-loop effective actions for Einstein-Maxwell background induced by spinor and scalar are given by Bastianelli et al, 2009, 2012

$$L_{\text{spinor}} \propto 5RF^2 - 26R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} + 2R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} ,$$
$$L_{\text{scalar}} \propto -\frac{5}{2}RF^2 - 2R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} - 2R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} ,$$

Our bound holds for both theories. WCCC not only holds for Einstein-Maxwell, but may also hold at one-loop level !

Discussion

Compare with the bound obtained from Weak gravity conjecture (WGC):

Motl et al, 2006; Cheung et al,2018

$$c_2 + 4c_3 + \frac{c_5}{\kappa} + \frac{c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2} \geq 0$$

- contains c_7 and c_8
- two bounds seem orthogonal to each other

Combing WCCC and WGC bounds together will be a useful tool to scrutinize the theory space of the higher order EFTs.

THANK YOU !