QUERY COMPLEXITY AND CUTOFFS IN ADS_3/CFT_2

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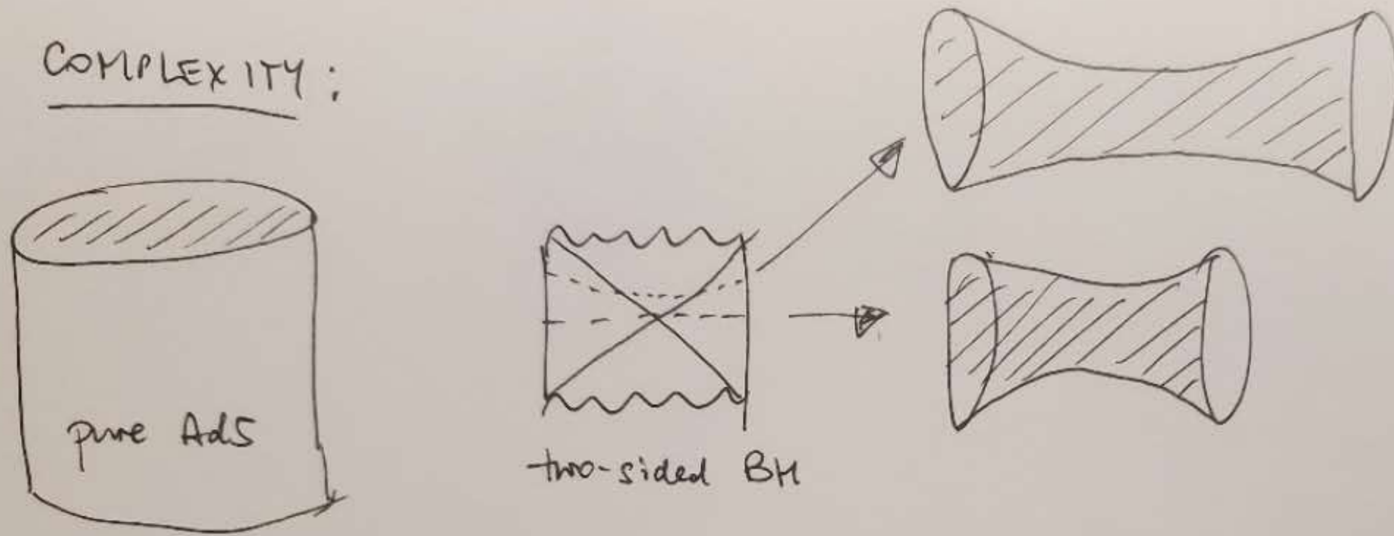
In progress,
see also 2004.11377

COMPLEXITY:

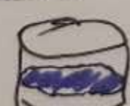


Susskind et al, ±2015:

$\boxed{\text{GROWING SPATIAL SIZE IN ADS} \longleftrightarrow \text{GROWING "STATE COMPLEXITY" IN CFT}}$

N.B. NEITHER SIDE IS A PRIORI WELL-DEFINED
THERE IS FLEXIBILITY AND NEED FOR CREATIVE INPUT

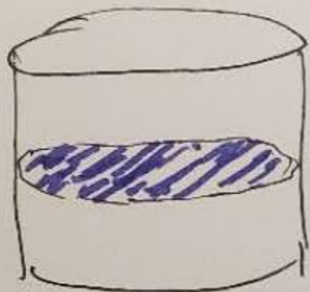
HOW TO QUANTIFY SPATIAL SIZE?

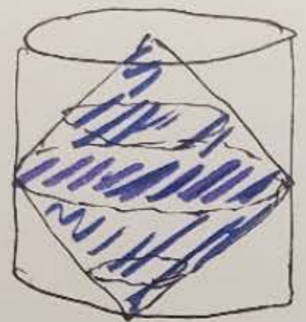
- MAX VOLUME SPATIAL SLICE 
- ACTION IN WDW PATCH 
- OTHERS...

WHAT IS STATE COMPLEXITY?

- CIRCUIT COMPLEXITY
- DISTANCE IN FUBINI-STUDY METRIC
- PATH INTEGRAL OPTIMIZATION
- ...

A PRELIMINARY: COMPLEXITY DEPENDS ON THE CUTOFF:





BOTH ARE STRICTLY SPEAKING INFINITE.

WHY?

THIS DISTANCE
IS INFINITE
BECAUSE IT REPRESENTS
ALL POSSIBLE SCALES

- WITHOUT A CUTOFF, STATE COMPLEXITY MUST BE INFINITE BECAUSE WE MUST PREPARE THE STATE OVER ALL SCALES

COMPLEXITY IS ONLY WELL-DEFINED AFTER WE SPECIFY A CUTOFF (CFT SCALE)

PLAN FOR TODAY:

A COMPLETELY NEW IDEA FOR:

- STATE
- CUTOFF
- STATE COMPLEXITY

LOGICAL STARTING POINTS:

- SUBREGION DUALITY
- MODULAR PARALLEL TRANSPORT
- QUERY COMPLEXITY

BIGGEST CONCEPTUAL CHANGE:

THINK OF THE STATE AS AN ALGORITHM, WHICH TAKES:

$$\underset{\text{INPUT}}{\text{OPERATORS } \mathcal{O}_1\, \mathcal{O}_2 \cdots \mathcal{O}_n} \xrightarrow[\text{SPECIFIED AS AN ALGORITHM}]{\text{COMPUTATION}} \underset{\text{OUTPUT}}{\mathbb{R} \text{ EXPECTATION VALUES}}$$

STATE AS A MAP **OPERATORS** $\longrightarrow$ **EXPECTATION VALUES**:

OPERATOR	EXP. VALUE
OP_1	#
OP_2	#
OP_3	#
OP_4	#
OP_5	#
$\vdots$	$\vdots$
OP_{100}	
OP_{101}	
OP_{102}	
OP_{103}	
$\vdots$	

SPECIFYING THESE NUMBERS CHARACTERIZES THE STATE

QUESTION: HOW EXACTLY DO WE SPECIFY THESE NUMBERS?

ANSWER: WE GIVE AN ALGORITHM FOR COMPUTING THEM

CUTOFF?

SOME OPERATORS WILL BE **UV** OPERATORS.

WE DON'T SPECIFY THEIR EXPECTATION VALUES IN A STATE AT A CUTOFF.

$\Longrightarrow$

CUTOFF RESTRICTS THE DOMAIN OF MAP

OPERATORS $\xrightarrow{\text{STATE}}$ **EXPECTATION VALUES**

STATE AS AN ALGORITHM TO EVALUATE MAP

OPERATORS $\xrightarrow[\text{EXHIBITED AS AN ALGORITHM}]{\text{STATE}}$ **EXPECTATION VALUES**

EXAMPLE 1: $|0\rangle_{CFT}$

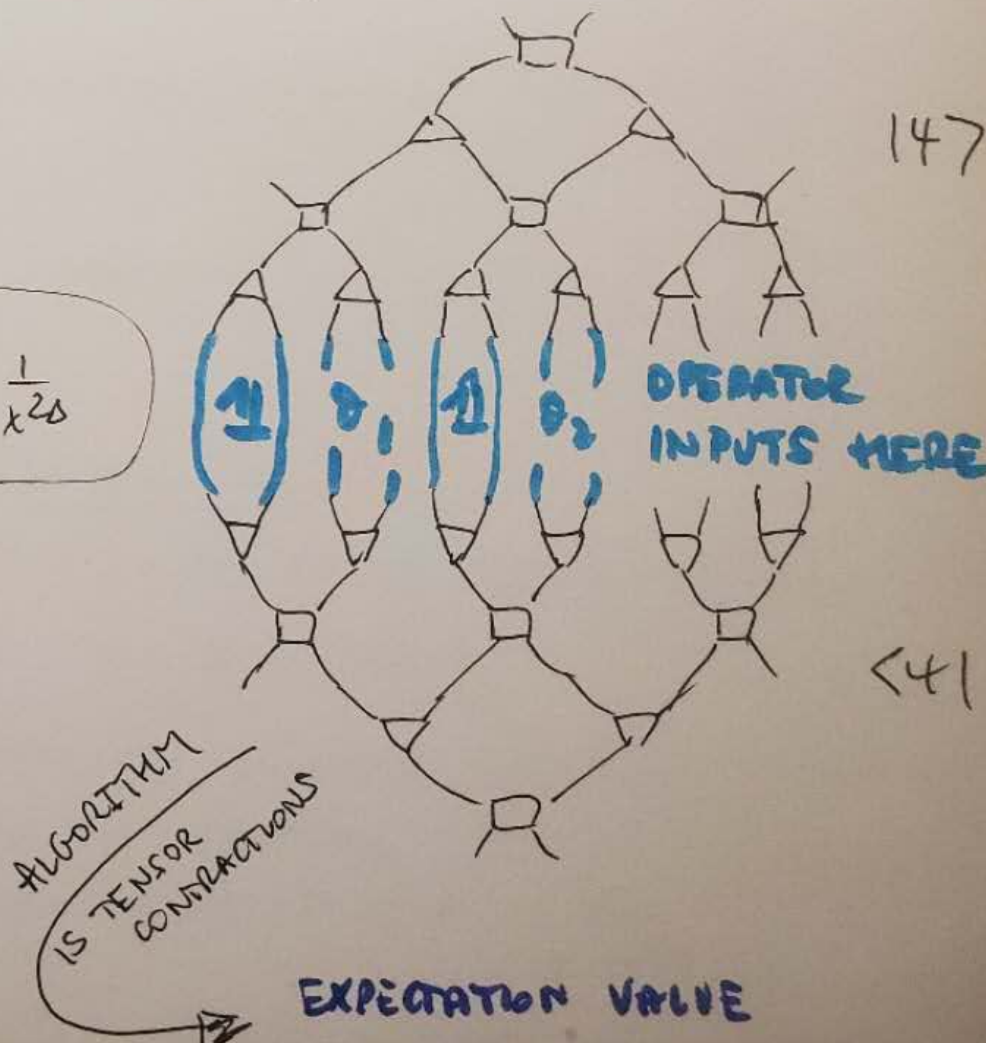
- SPECIFIED BY SYMMETRIES

$L_{-1}|0\rangle = L_0|0\rangle = L_{+1}|0\rangle = 0$

- USE THOSE SYMMETRIES TO EVALUATE MAP $\Rightarrow$ $\langle \mathcal{O}_\Delta(x)\,\mathcal{O}_\Delta(0)\rangle = \frac{1}{x^{2\Delta}}$ etc.

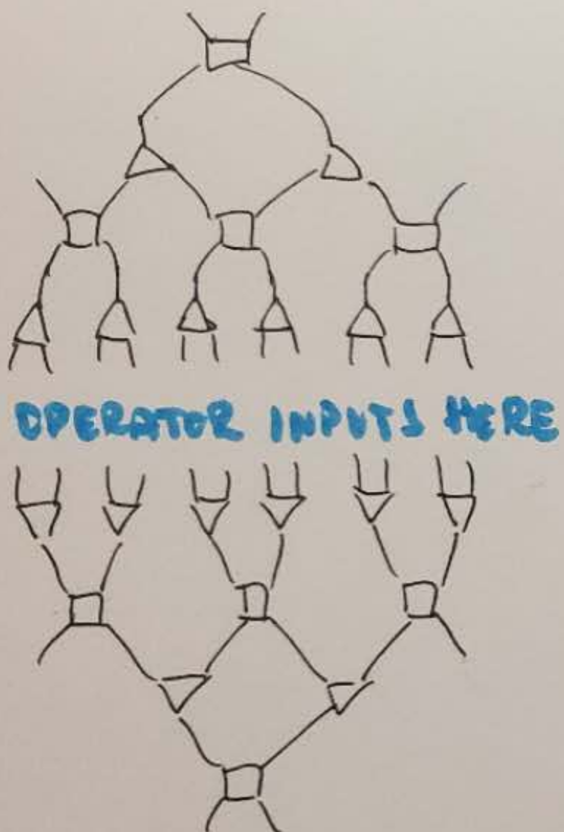
- IT'S OK HERE, BUT IT DOESN'T GENERALIZE TO STATES WITHOUT SPECIAL PROPERTIES / SYMMETRIES

EXAMPLE 2: TENSOR NETWORKS



QUERY COMPLEXITY OF AN ALGORITHM:

- HOW MANY TIMES ALGORITHM CALLS SOME KEY SUBROUTINE



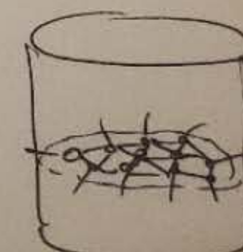
TENSOR NETWORK $\xrightarrow{\text{GENERALIZE}}$ ALGORITHM FOR COMPUTING CORRELATION FUNCTIONS

TENSOR CONTRACTION ⟶ SUBROUTINE OF ALGORITHM

#{TENSOR CONTRACTIONS} ⟶ QUERY COMPLEXITY

⇓ SPECIAL CASE:

- QUERY COMPLEXITY OF A TENSOR NETWORK PROPORTIONAL TO #{TENSORS}
- THIS RECOVERS THE ORIGINAL MOTIVATION FOR THE "VOLUME PROPOSAL": A HYPOTHETICAL SPACE-FILLING TENSOR NETWORK

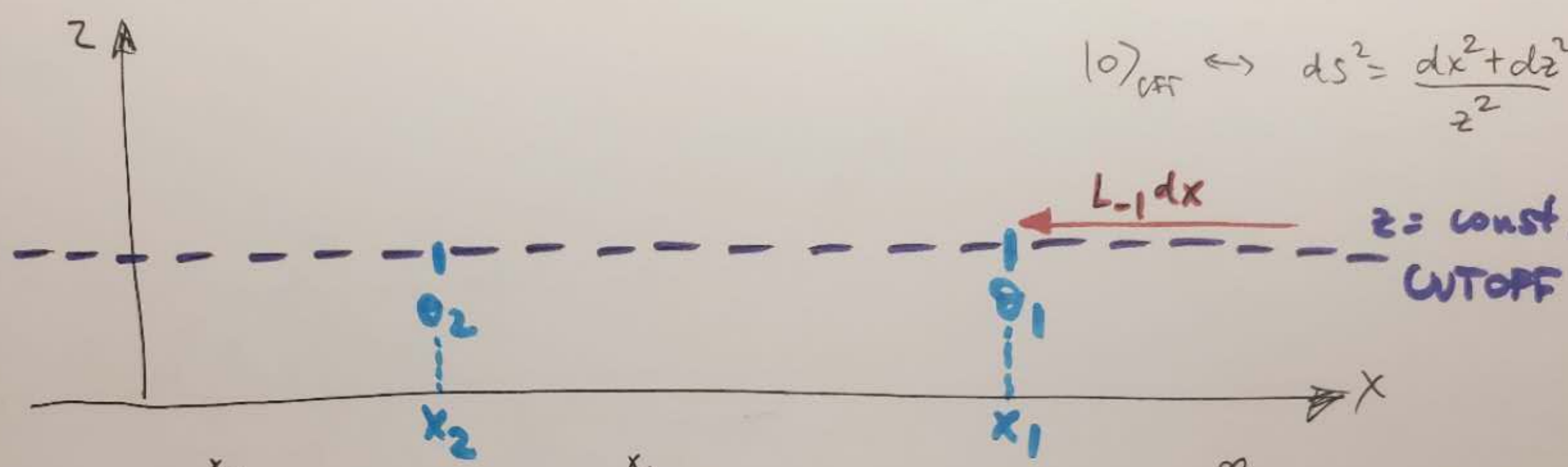


OUR PROPOSAL:

STATE COMPLEXITY IS QUERY COMPLEXITY OF THE OPTIMAL ALGORITHM TO EVALUATE OPERATORS $\xrightarrow{\text{STATE}}$ EXP VALUES

CONSTRUCTING THE ALGORITHM: EXAMPLE

$$|0\rangle_{CFT} \leftrightarrow ds^2 = \frac{dx^2+dz^2}{z^2}$$



$$\langle 0|\, e^{\int_{-\infty}^{x_2} dx\, L_1}\, \mathcal{O}_2\, e^{\int_{x_2}^{x_1} dx\, L_{-1}}\, \mathcal{O}_1\, e^{\int_{x_1}^{\infty} dx\, L_{-1}}\, |0\rangle =$$

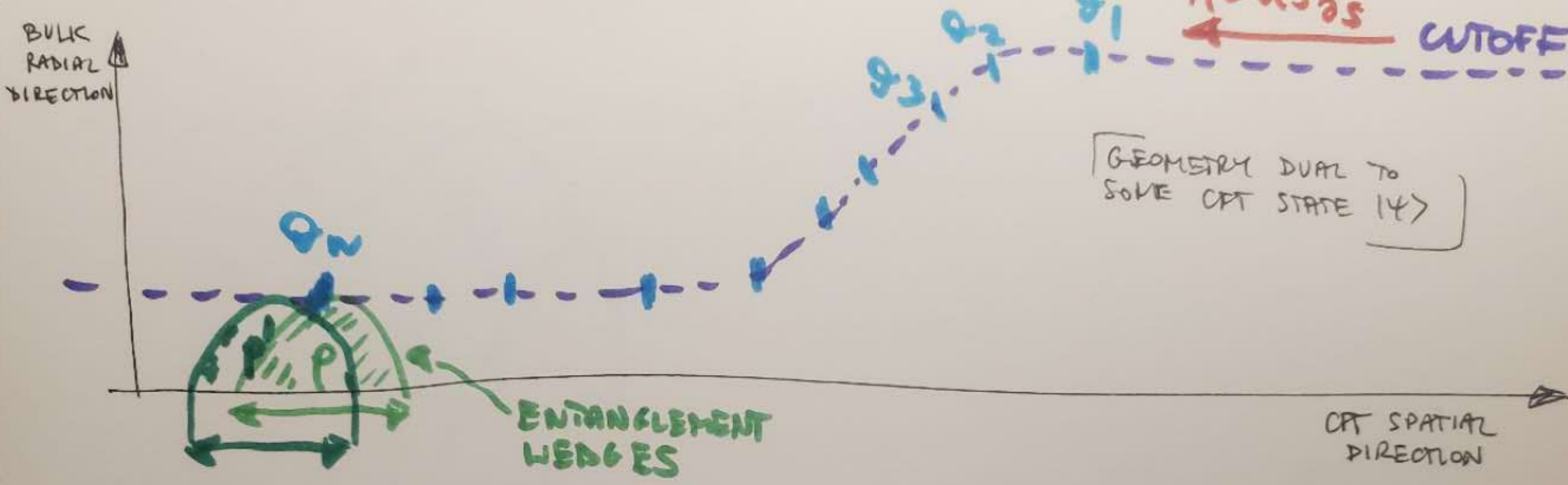
$$\langle 0|\, \mathcal{O}_2\, e^{L_1(x_1-x_2)}\, \mathcal{O}_1\, |0\rangle =$$

$$\langle 0|\, \mathcal{O}_2(0)\, \mathcal{O}_1(x_2-x_1)\, |0\rangle$$

- AN OPERATOR-VALUED ONE FORM, WHICH TRANSLATES ALONG CUTOFF SURFACE
- WE CAN WRITE IT AS $\boxed{ds \frac{\partial}{\partial s}}$

- ITS WILSON LINES TRANSLATE **INPUT OPERATORS** ALONG CUTOFF SURFACE

CONSTRUCTING THE ALGORITHM:



$$\langle\psi|\left(P\exp\int_{-\infty}^{s_N} A\right)\mathcal{O}_n(1+A)\ldots(1+A)\mathcal{O}_3(1+A)\mathcal{O}_2(1+A)\mathcal{O}_1\left(P\exp\int_{s_1}^{\infty} A\right)|\psi\rangle$$

- IF WE CAN FIND $A = ds\frac{\partial}{\partial s}$, WE WILL HAVE AN ALGORITHM
- IF A CAN BE CONSTRUCTED USING ONLY CFT INGREDIENTS, THIS WILL BE A CFT ALGORITHM, NOT A BULK COMPUTATION

COMMENT ON **OPERATOR INPUTS:**

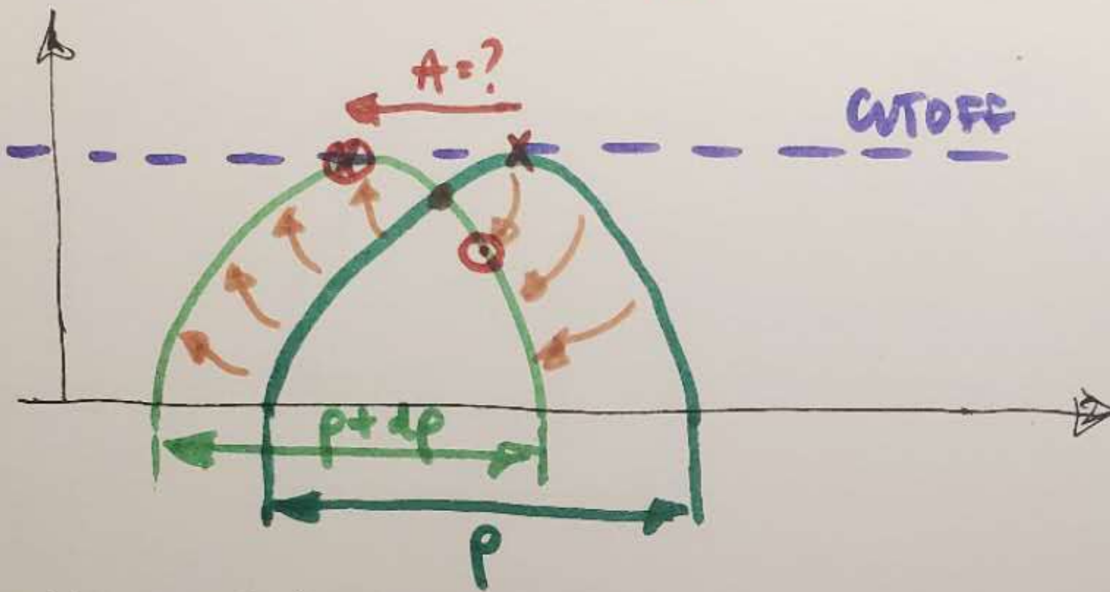
- THEY SHOULD LIVE ON THE **CUTOFF** SURFACE $\longleftrightarrow$

BULK $\xleftrightarrow{\text{SUBREGION DUALITY}}$ BOUNDARY

- THEY SHOULD COMMUTE WITH ρ AND ρ':

$$[\mathcal{O}, \rho] = 0$$
$$[\mathcal{O}, \rho + d\rho] = 0$$

CONSTRUCTING $A = ds\frac{\partial}{\partial s}$



• WE KNOW A <u>CFT</u> OPERATION THAT DOES WHAT THE **BROWN** ARROWS DO:

$i\,dV$

MODULAR PARALLEL TRANSPORT

(1903.04493)

• IN THE BULK, THIS IS A ROTATION ABOUT THE COMMON POINT OF RT-SURFACES

• BUT THIS SENDS × to ⊙

$\boxed{\text{• HOW TO SEND ⊙ to ⊗ ?}}$

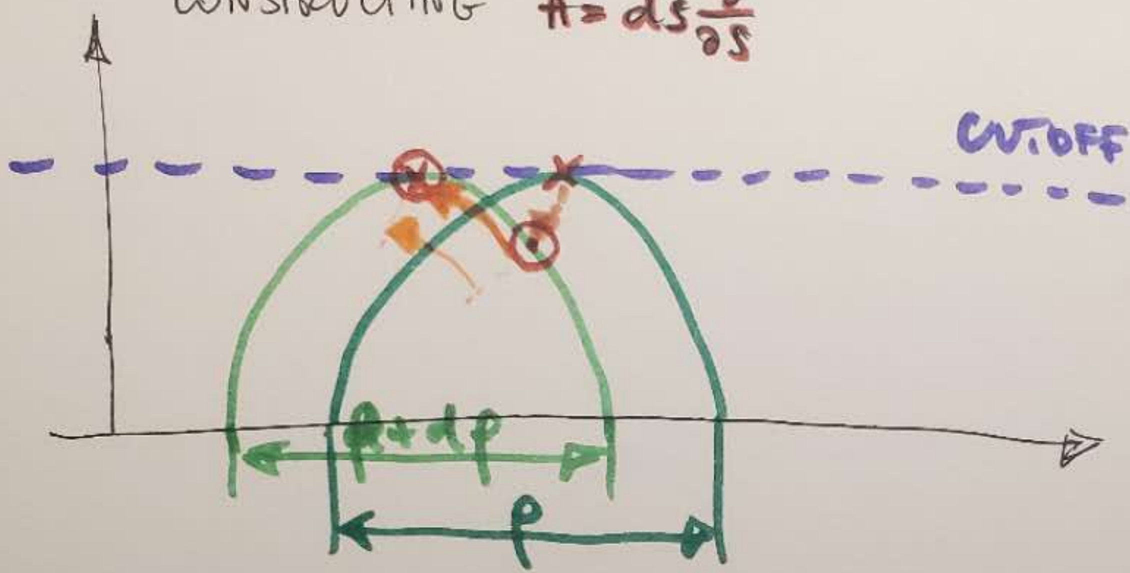
• THIS OPERATION ~~HAS~~ TRANSPORTS ENTANGLEMENT WEDGES TO ENTANGLEMENT WEDGES:

$$\boxed{i[dV, \rho] = d\rho}$$

• FOR NOW, WE WRITE:

$$\boxed{1 + A = (1 + i\,dV) \times (\ldots)}$$

CONSTRUCTING $A = ds\frac{\partial}{\partial s}$



- TAKING ⊙ TO ⊗ IS A MOTION **ALONG RT SURFACE**
- THEREFORE, IT IS A **ZERO MODE** $i\,dP$ OF $p+dp$:

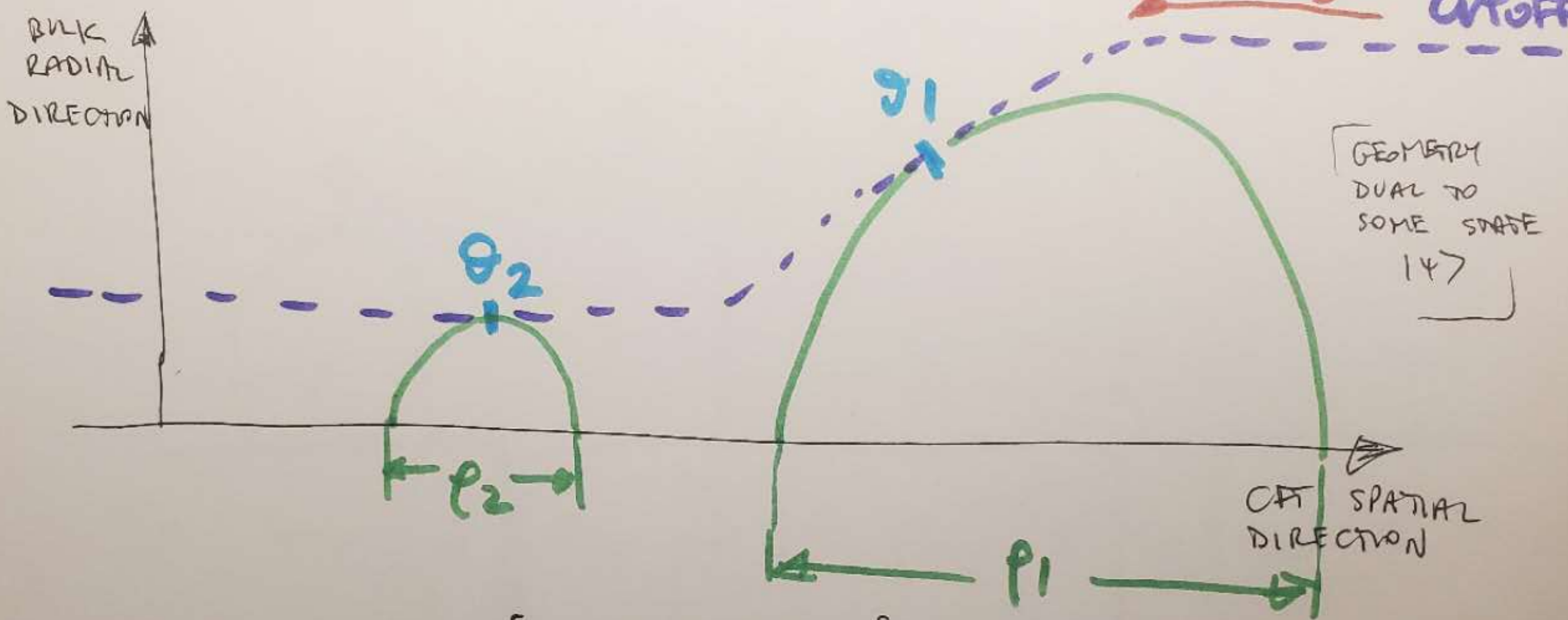
$$\boxed{i\,[dP,\, p+dp] = 0}$$

this is a CFT statement ⟷ IN BULK: dP PRESERVES TANGENT ENTANGLEMENT WEDGE

$$\Rightarrow \overbrace{\boxed{\begin{aligned} 1 + A &= (1 + i\,dV)(1 + i\,dP) \\ &= 1 + ds\frac{\partial}{\partial s}\end{aligned}}}^{\text{CFT CONSTRUCTS}}$$

- dP IS COMPLETELY DETERMINED FROM dV (THEREFORE, A CFT OBJECT)
- THERE IS A TECHNICAL CONSTRUCTION WHERE dV IS THE "VELOCITY" AND dP IS "ACCELERATION"

SIMPLIFY THE ALGORITHM:



$$\mathrm{Tr}\left(\mathrm{Pexp}\int_{-\infty}^{s_2} A\right)\theta_2\left(\mathrm{Pexp}\int_{s_2}^{s_1} A\right)\theta_1\left(\mathrm{Pexp}\int_{s_1}^{\infty} A\right)|\psi\rangle\langle\psi|$$

$\Downarrow$

BUT:

$$\boxed{\left(\mathrm{Pexp}\int_{s_1}^{\infty} A\right)\rho(\infty)\left(\mathrm{Pexp}\int_{-\infty}^{s_1} A\right) = \rho(s_1)}$$

BOILS DOWN TO:

$\mathrm{tr}\,\theta_1\rho(s_1)$ if $\theta_2 = \mathbb{1}$

OR

$\mathrm{tr}\,\theta_2\rho(s_2)$ if $\theta_1 = \mathbb{1}$

REMEMBER:

$$i[dV, \rho] = d\rho \qquad A = i(dV + d\rho)$$
$$i[d\rho, \rho] = 0$$

SIMPLIFY THE ALGORITHM:

THIS IS: ρ_1

$$\mathrm{Tr}\left(\mathcal{P}\exp\int_{-\infty}^{s_1} A\right)\underbrace{\left(\mathcal{P}\exp\int_{s_1}^{s_2} A\right)\theta_2\left(\mathcal{P}\exp\int_{s_2}^{s_1} A\right)}_{\text{"PRECURSOR OF } \theta_2 \text{ THAT ACTS IN } \rho_1\text{"}}\underbrace{\theta_1}_{\text{ALREADY ACTS IN } \rho_1}\left(\mathcal{P}\exp\int_{s_1}^{\infty} A\right)|\psi\rangle\langle\psi|$$

(HEISENBERG PICTURE VERSION OF θ_2 WRT A)

TRACE OUT ρ_1-COMPLEMENT FIRST

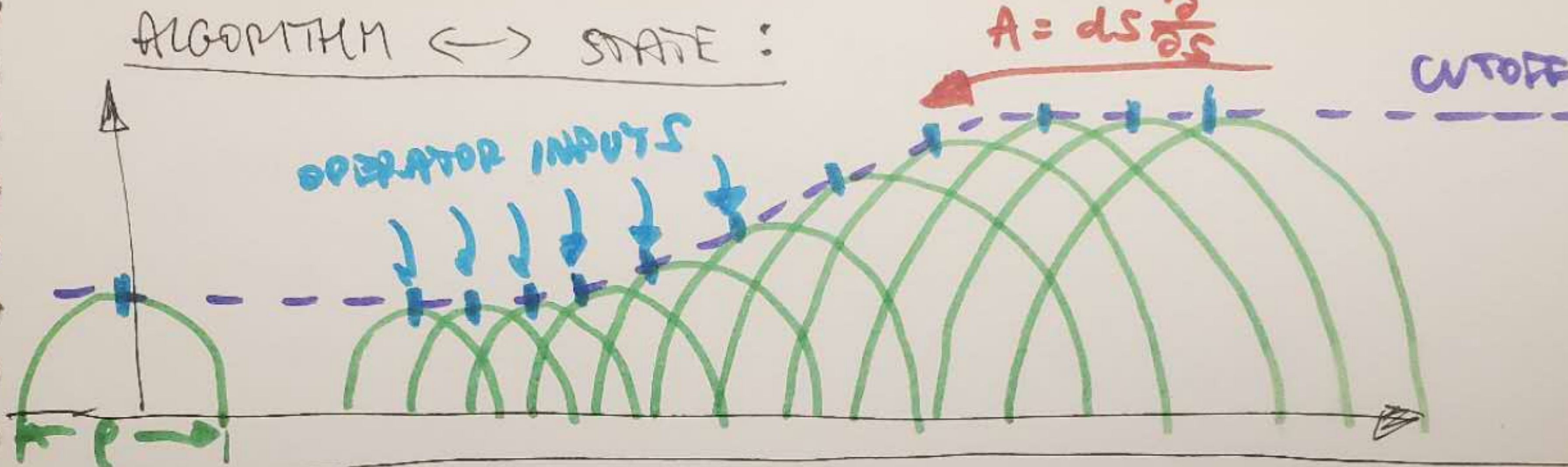
TRACE OUT $\rho(\infty)$ FIRST

using $\mathcal{P}\exp\int_{-\infty}^{\infty} A \equiv \mathcal{P}\exp\oint A = \mathbb{1}$

$$\Rightarrow \boxed{\mathrm{Tr}\left(\mathcal{P}\exp\int_{-\infty}^{s_2} A\right)\theta_2\left(\mathcal{P}\exp\int_{s_2}^{s_1} A\right)\theta_1\left(\mathcal{P}\exp\int_{s_1}^{\infty} A\right)\rho(\infty)}$$

FOR MORE $\Rightarrow$ INPUT OPERATORS ...

ALGORITHM $\longleftrightarrow$ STATE :



$$\boxed{\rho(\infty)\,(1+A)(\ldots)(1+A)(\ldots)(1+A)(\ldots)(1+A)(\ldots)\ \ldots\ (\ldots)(1+A)(\ldots)}$$

OPERATOR INPUTS

- MAP: OPERATORS $\longrightarrow$ EXPECTATION VALUES
- IN THE FORM OF AN ALGORITHM
- CONSTRUCTED ENTIRELY IN CFT LANGUAGE
- BUT IT "KNOWS" ABOUT THE CUTOFF
- DEPENDS CRUCIALLY ON THE MICROSCOPIC STATE THROUGH A

$$i\,[dV, \rho] = d\rho$$
$$i\,[dP, \rho] = 0$$

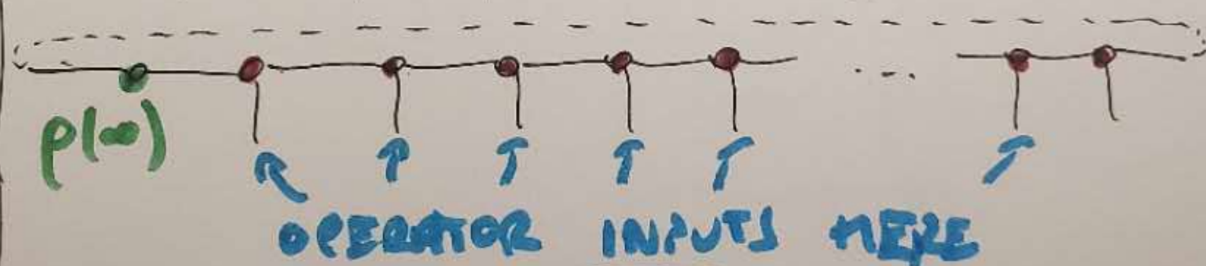
QUERY COMPLEXITY OF THIS ALGORITHM:

$$\mathrm{tr}\, \rho(\infty)\,(1+A)(\dots)(1+A)(\dots)(1+A)(\dots)\ \dots\ (\dots)(1+A)(\dots)$$

COUNTING SUBROUTINES...

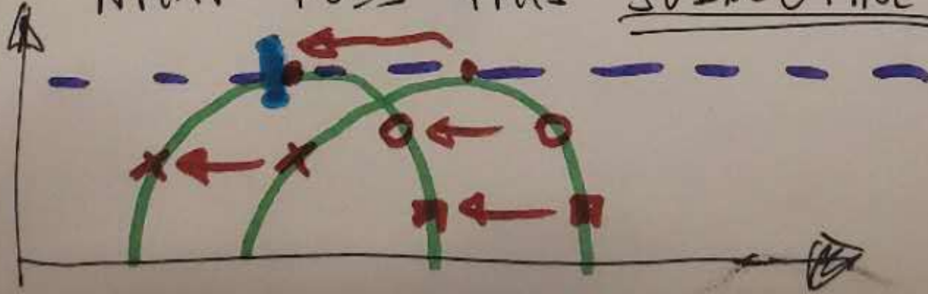
HERE THE SUBROUTINE IS: $\boxed{(1+A)(\dots)}$

BTW: YOU CAN REPRESENT THIS AS A MATRIX PRODUCT STATE:



THEN COUNTING $\boxed{(1+A)(\dots)}$ IS AGAIN COUNTING TENSORS IN A TENSOR NETWORK

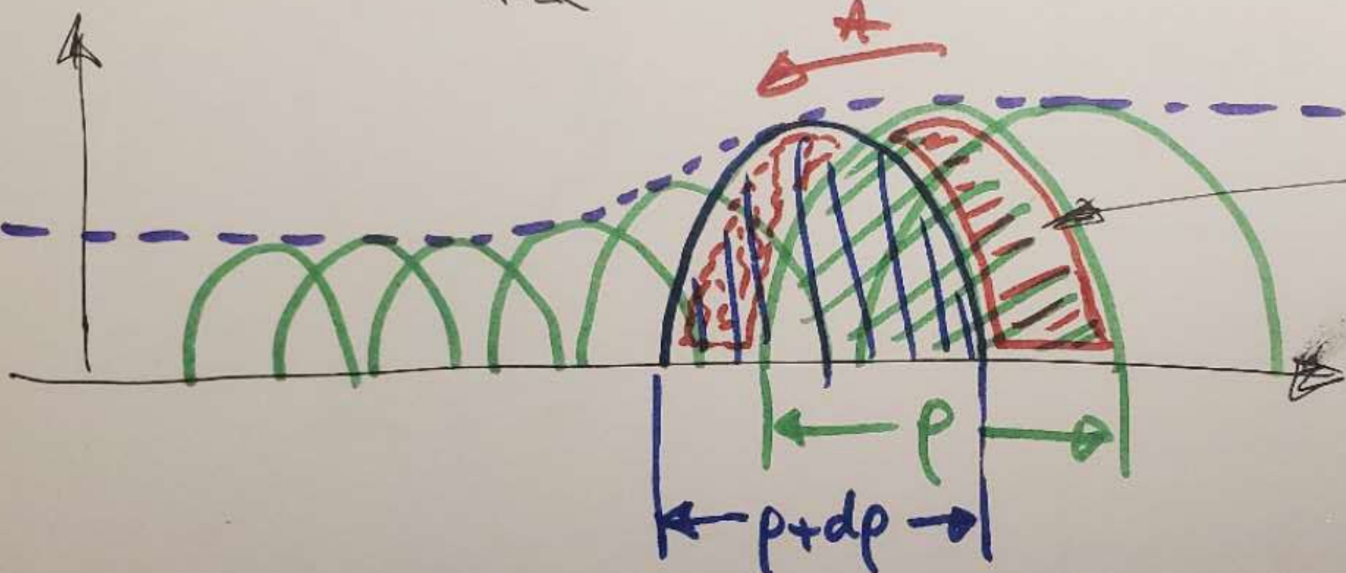
WHAT DOES THIS SUBROUTINE DO?



- IT MAPS ONE ENTANGLEMENT CUTOFF-SIZED WEDGE TO THE NEXT
- THEN READS OFF AN **INPUT OPERATOR**

QUERY COMPLEXITY OF OUR ALGORITHM:

- SHOULD BE A MEASURE OF "HOW DIFFICULT" IT IS TO MAP CONSECUTIVE **CUTOFF-SIZED** ENTANGLEMENT WEDGES TO ONE ANOTHER



"**DIFFICULTY**" PROPORTIONAL TO ANGULAR WIDTH OF THESE **CRESCENTS**

- THERE IS AN ESSENTIALLY UNIQUE METRIC IN THE SPACE OF DENSITY MATRICES, WHICH IS CONSISTENT WITH $\boxed{i[A,\rho] = d\rho}$

"THE KIRILLOV-KOSTANT-SOURIAU METRIC FOR THE SJØKVIST CONNECTION **A**"

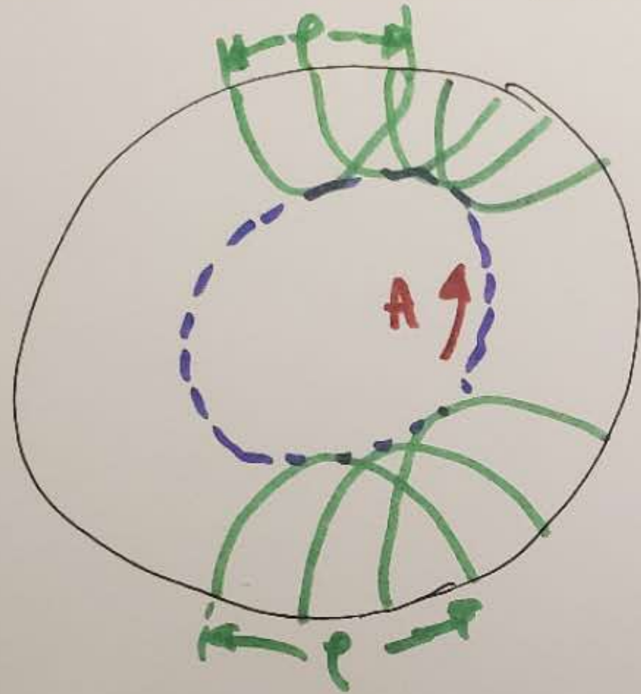
AGAIN: DEFINED PURELY IN CFT LANGUAGE

- IF THERE IS A BULK:

⇒ PROPORTIONAL TO: $\boxed{K\, ds_{BULK}}$

EXTRINSIC CURVATURE OF CUTOFF SURFACE

OUR QUERY COMPLEXITY IN THE BULK*:



QUERY COMPLEXITY

$$= \oint_{\text{CUTOFF}} K\, ds$$

* IF THERE IS A BULK.
IT IS DEFINED EVEN IN THE ABSCENCE OF A BULK

- IN PURE AdS_3: $QC = \oint K ds = \boxed{(-R)\int_{\text{inside}} dV + 2\pi}$

"VOLUME PROPOSAL"

- OTHERWISE, IT'S DIFFERENT

QUALITATIVELY:

- $QC = \oint K ds$ GIVES SOME MEASURE OF SPATIAL SIZE
- IT IS A COMPLEXITY OF A STATE AT A CUTOFF
- DEFINED PURELY IN CFT LANGUAGE

I DECLARE SUCCESS.

THANKS!