Fractional excitation, spin-charge separation, spin incoherent and Luttinger liquids

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Outlines

1. I. Fundamental concepts: Spin-charge separation
2. II. Yang-Gaudin model: Fractional quasiparticles and Luttinger liquids
3. III. Testing the theory of spin-charge separation
4. IV. Outlook

I. Fundamental concepts: Spin-charge separation

feature

The quasiparticle zoo

Quasiparticles are an extremely useful concept that provides a more intuitive understanding of complex phenomena in many-body physics. As such, they appear in various contexts, linking ideas across different fields and supplying a common language.

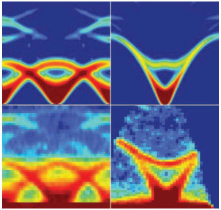
LANDAU QUASIPARTICLES

A concept emerges

One of the biggest surprises for experimental physicists of the early twentieth century was the discovery of the superfluid behaviour of liquid helium. And Moscow-based Pyotr Kapitza knew just the right theorist to explain it: Lev Landau[1]. It was unfortunate then that the brilliant and outspoken Landau had got himself imprisoned by the KGB in 1938, the same year that superfluidity was publicly announced. Undeterred, Kapitza wrote to Landau's captors and miraculously managed to persuade them that it would be a great loss to Soviet science if Landau was left to perish in prison. He was promptly released, with Kapitza appointed as his guarantor, and set to work without delay. Sure enough, two years later he published a paper on helium superfluidity that, along the way, boldly introduced the notion

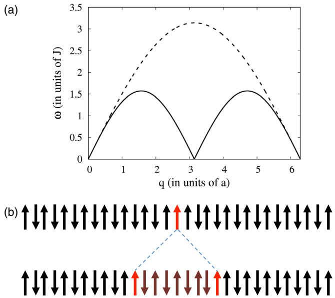
Even stranger quasiparticles showed up in the early 1980s, with the discovery of the fractional quantum Hall effect. Robert Laughlin proposed that the elementary quasiparticle excitations involved carry just a fraction of an electron charge — as if, remarkably, electrons are split up. Electronic transport measurements in 1997 detected quasiparticles carrying only a third of an electron charge and thereby proved — quite spectacularly — that Laughlin's quasiparticles were more than just mathematical entities.

Laughlin noted in his Nobel Prize lecture[3] that both superfluidity and the fractional quantum Hall effect are examples of emergent phenomena, which can only be understood by considering the system as a whole and are impossible to derive from microscopic principles. But one of the things an emergent phenomenon can do, Laughlin wrote, is create new particles.

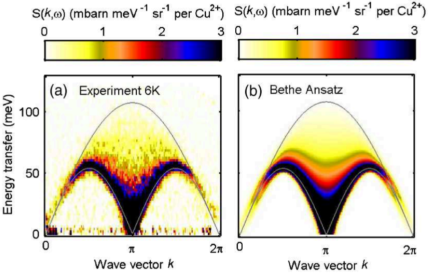


Calculated (top) and measured (bottom) dynamical structure factors for tin selenide, a highly thermoelectric material. Reproduced from ref. 30, NPG.

Nature Physics 12, 1085 (2016)



Two-Spinon continuum



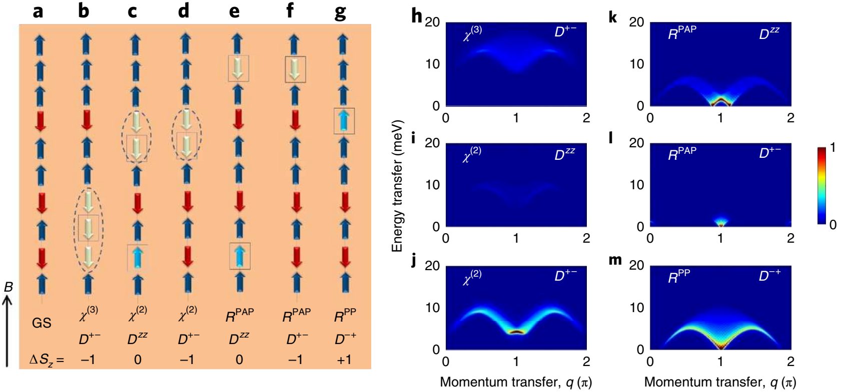
Two-Spinon continuum

Many neutron scattering experiments present novel measurements of such fractional excitations through measuring the dynamic structure factor:

$$S^{a\bar{a}}(q,\omega) = \frac{1}{N}\sum_{j,j'=1}^{N}\int_{-\infty}^{\infty} dt e^{-iq(j-j')+i\omega t} < S_j^a(t) S_{j'}^{\bar{a}}(0) >, \quad a = z, -, +$$

Faddeev & Takhtajian, Phys. Lett. A 85, 375 (1981)

Lake et al. PRL 111, 137205 (2013); Stone et al, PRL 91, 037205 (2003)



Spin strings and excitation spectra of spin-1/2 XXZ model $SrCo_2V_2O_8$

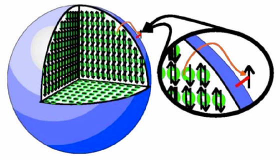
Wang, et al. Nature 554, 219 (2018); A K Bera, et al. Nat. Phys. 16, 625 (2020)

Wu, et al. PNAS, 111, 39 (2014), Wu, et al. Nature Comm. 10:698 (2019)

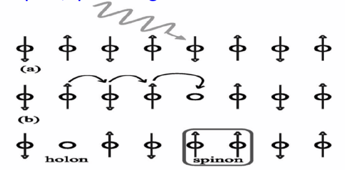
Spin-charge separation

Fermi Liquids vs **Luttinger liquid**

Electronic metal, Kondo impurities, Helium-3, heavy fermions etc.



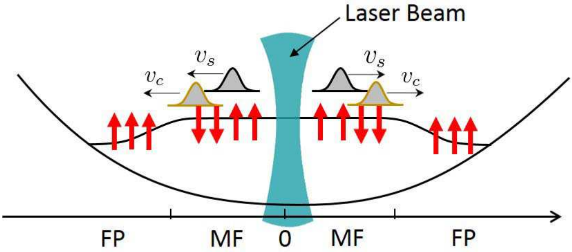
1D correlated electronic systems, spin liquids, quantum gases etc.



Microscopic difference: **quasiparticles** **spin-charge separation**

Macroscopic similarity：**specific heat linearly depends on temperature *T***
susceptibility and compressibility are independent of *T*

For interacting fermions in higher dimensions, an excited electron is dressed by interaction and fluctuation and forms a quasiparticle with a dispersion $\omega(k) = E(k_F) + \frac{k_F}{m^*}(k - k_F)$. The spectral function has a Lorenzen peak. Whereas in 1D Luttinger liquid, the spectral function has two singular features corresponding to the spinon and holon.



Challenges: a definitive observation of the SC separation is still missing

1) Observation of the separate collective excitation spectra of charge and spin
2) Determination of independent spin and charge sound velocities and their Luttinger parameters
3) Confirmation of the power-law decay with distance in correlation functions or of the power law of dynamical response functions

Kim, et. al. PRL, 77(19), 4054 (1996); Nat. Phys., 2(6):397 (2006)

Auslaender, et. al. Science, 308(5718):88 (2005)

Jompol, et. al. Science, 325 597 (2009)

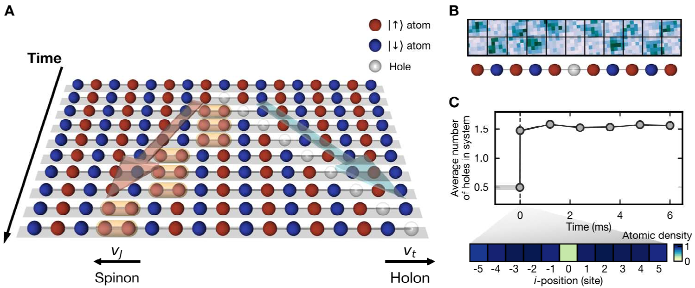
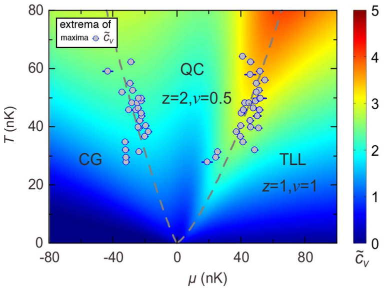
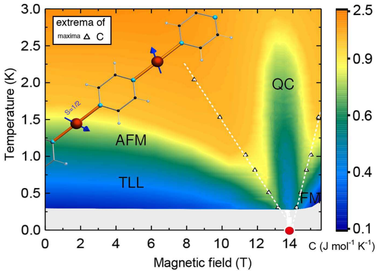


FIG. 1. **Probing spin-charge deconfinement with cold atoms**. **A,** Cartoon depicting fractionalization of a fermionic excitation into quasiparticles. The dynamics is initiated by removing a fermion from the Hubbard chain. This quench creates a spin (spinon) and a charge (holon) excitation, which propagate along the chain at different velocities v_J and v_t. **B,** Using quantum gas microscopy, we simultaneously detect the spin and density on every site of the chain after a variable time after the quench. **C,** Average number of holes in the chain as a function of time. Error bars denote 1 s.e.m. The quench, performed at 0 ms creates a hole with a probability of $\sim 78\%$ in the central site of the chain (bottom).

I. Bloch's group, Vijayan et. al., Science 367, 186-189 (2020)



Quantum criticality of bosons



Quantum criticality of spinons (Cooper pyrazine dinitrate (CuPzN))

Luttinger liquid : $R_W^\kappa = K = v_s/v_N = \hbar\pi n/(mv_s)$

momentum distribution $n(k) \sim 1/k^{(1-1/2K)}$

Scaling function : $c_v/T = c(\mu, T) + T^{d/z+1-2/\nu z}\mathcal{K}\left(\frac{\mu-\mu_c}{T^{1/\nu z}}\right)$

Critical exponents : $z = 2, \ \nu = 1/2, \ d = 1$

Correlation length : $\xi \sim |\mu-\mu_c|^{-\nu}, \ \Delta \sim \xi^{-z}$

Critical temperatures : $T_1 = -a_1|\mu-\mu_c|, \quad T_2 = a_2|\mu-\mu_c|^{\nu z}$

Yang, Chen, Zhang, Sun, Dai, Guan, Yuan, Pan, Phys. Rev. Lett. 119, 165701 (2017)

He, Jiang, Yu, Lin, Guan, PRB 96, 220401(R) (2017); Breunig, et al., Sci. Adv. 2017; 3:eaao3773

PHYSICAL REVIEW LETTERS **125,** 190401 (2020)

Emergence and Disruption of Spin-Charge Separation in One-Dimensional Repulsive Fermions

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At low temperature, collective excitations of one-dimensional (1D) interacting fermions exhibit spin-charge separation, a unique feature predicted by the Tomonaga-Luttinger liquid (TLL) theory, but a rigorous understanding remains challenging. Using the thermodynamic Bethe ansatz (TBA) formalism, we analytically derive universal properties of a 1D repulsive spin-1/2 Fermi gas with arbitrary interaction strength. We show how spin-charge separation emerges from the exact TBA formalism, and how it is disrupted by the interplay between the two degrees of freedom that brings us beyond the TLL paradigm. Based on the exact low-lying excitation spectra, we further evaluate the spin and charge dynamical structure factors (DSFs). The peaks of the DSFs exhibit distinguishable propagating velocities of spin and charge as functions of interaction strength, which can be observed by Bragg spectroscopy with ultracold atoms.

DOI: 10.1103/PhysRevLett.125.190401

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1. I. Fundamental concepts: Spin-charge separation
2. II. Yang-Gaudin model: Fractional quasiparticles and Luttinger liquids
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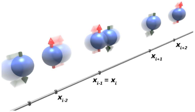
II. Yang-Gaudin model: Fractional quasiparticles and Luttigner liquids



$$\mathcal{H} = \sum_{j=\downarrow,\uparrow} \int_0^L \phi_j^\dagger(x)\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\phi_j(x)dx$$
$$+g_{1D}\int_0^L \phi_\downarrow^\dagger(x)\phi_\uparrow^\dagger(x)\phi_\uparrow(x)\phi_\downarrow(x)dx$$
$$-\frac{H}{2}\int_0^L \left(\phi_\uparrow^\dagger(x)\phi_\uparrow(x)-\phi_\downarrow^\dagger(x)\phi_\downarrow(x)\right)dx$$

Yang-Gaudin model(1967)

- $g_{1D} > 0$: antiferromagnetic
- $g_{1D} < 0$: ferromagnetic

- H: effective magnetic field
- $g_{1D} = -\frac{\hbar^2 c}{m}$, $c = -2/a_{1D}$, $a_{1D} = -\frac{a_\perp^2}{a_{3D}} + Aa_\perp$

Yang Phys. Rev. Lett. **19**, 1312 (1967)
Gaudin, Phys. Lett. **24**, 55 (1967)
Guan, Batchelor and Lee, Rev. Mod. Phys. 85, 1633 (2013)

- Bethe wave function for $(0 < x_{Q1} < \ldots < x_{Qi} < x_{Qj} \ldots < x_{QN} < L)$

$$\psi = \sum_P A_{\sigma_1 \ldots \sigma_N}(P_1, \ldots, P_N | Q_1, \ldots, Q_N) \exp \mathrm{i}(k_{P1} x_{Q1} + \ldots + k_{PN} x_{QN})$$

- Bethe Ansatz equations

$$\exp(\mathrm{i} k_j L) = \prod_{\ell=1}^{M} \frac{k_j - \lambda_\ell + \mathrm{i}\, c/2}{k_j - \lambda_\ell - \mathrm{i}\, c/2}, \quad j = 1, \ldots, N$$

$$\prod_{\ell=1}^{N} \frac{\lambda_\alpha - k_\ell + \mathrm{i}\, c/2}{\lambda_\alpha - k_\ell - \mathrm{i}\, c/2} = -\prod_{\beta=1}^{M} \frac{\lambda_\alpha - \lambda_\beta + \mathrm{i}\, c}{\lambda_\alpha - \lambda_\beta - \mathrm{i}\, c}, \quad \alpha = 1, \ldots, M$$

- Energy

$$E = \frac{\hbar^2}{2m} \sum_{j=1}^{N} k_j^2$$

CN Yang, Phys. Rev. Lett. **19**, 1312 (1967)

Spin charge separation: beyond the free fermion theory

Bethe ansatz equations

$$e^{\mathrm{i}k_iL}=\prod_{j=1}^{M}\frac{k_i-\Lambda_j+\frac{1}{2}\mathrm{i}c}{k_i-\Lambda_j-\frac{1}{2}\mathrm{i}c},\quad i=1,\ldots,N$$

$$\prod_{i=1}^{N}\frac{\Lambda_j-k_i+\frac{1}{2}\mathrm{i}c}{\Lambda_j-k_i-\frac{1}{2}\mathrm{i}c}=\prod_{\ell=1}^{N}\frac{\Lambda_j-\Lambda_\ell+\mathrm{i}c}{\Lambda_j-\Lambda_\ell-\mathrm{i}c},\quad j=1,\ldots,M$$

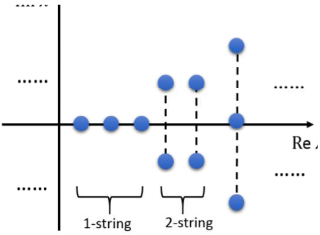
- An effective antiferromagnetic Heisenberg spin chain for $c>0$

$$H=\frac{J}{2}\sum_{i=1}^{N}\hat{S}_i\bullet\hat{S}_{i+1}-h\sum_{i}^{N}s_i^z,\quad J\approx\frac{4E_F}{c}$$

$$E\quad=\quad\frac{\pi^2}{3L^2}N\left(N^2-1\right)-\frac{nJ}{2}\sum_{i=1}^{M}\frac{1}{r_i^2+1/4}+O(c^{-2})$$

$$\left(\frac{r_i+\frac{i}{2}}{r_i-\frac{i}{2}}\right)^N=-\prod_{j=1}^{M}\frac{r_i-r_j+\mathrm{i}}{r_i-r_j-\mathrm{i}}$$

Oelkers, Bathchelor, Bortz and Guan, J. Phys. A 39, 1073 (2006)
Lee, Guan, Sakai and Batchelor, PRB 85, 085414 (2012)
Yang, Pu, Phys. Rev. A 95, 051602 (2017)



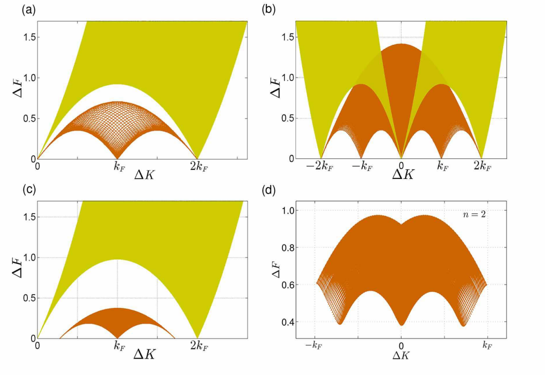
Spin strings : $\lambda_\alpha^{n,a} = \lambda_\alpha^n + \frac{1}{2}\mathrm{i}(n+1-2a) + O(e^{-\delta L}), \quad a = 1, \cdots, n$

$$k_j L = 2\pi I_j - \sum_{n=1}^{\infty} \sum_{\alpha=1}^{M_n} \theta\left(\frac{2(k_j - \lambda_\alpha^n)}{nc}\right), \qquad \theta_y(x) = 2\tan^{-1}(x/y)$$

$$\sum_{j=1}^{N} \theta\left(\frac{2(k_j - \lambda_\alpha^n)}{nc}\right) = 2\pi J_\alpha^n + \sum_{m=1}^{\infty} \sum_{\beta=1}^{M_m} \Theta_{mn}\left(\frac{2(\lambda_\alpha^n - \lambda_\beta^m)}{c}\right)$$

$$I_j \in \sum_{n=1}^{\infty} \frac{M_n}{2} + \mathbb{Z}, \quad J_\alpha^n \in \frac{N - M_n}{2} + \frac{1}{2} + \mathbb{Z}$$

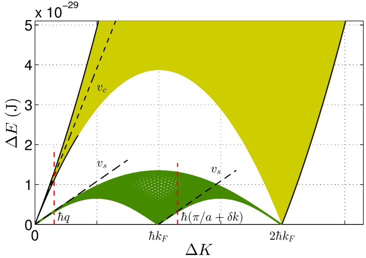
$$|J_\alpha^n| \le J_+^n = \frac{N}{2} - \sum_{m=1}^{n} m M_m - n \sum_{m=n+1}^{\infty} M_m + \frac{M_n}{2} - \frac{1}{2}$$



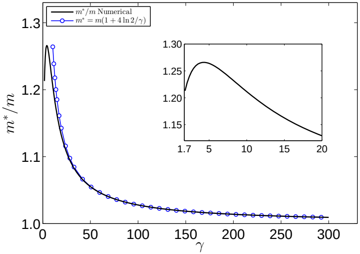
(a) Particle-hole and two-spinon excitations; (b) Four-spinon excitation; (c) Particle-hole and two-spinon excitations at finite magnetic field; (d) Two-spinon excitation with one length-2 string

$$\text{Momentum}: \; K = \sum_{j=1}^{N} k_j = \frac{2\pi}{L}\left(\sum_{j=1}^{N} I_j + \sum_{\alpha=1}^{M_n}\sum_{n=1}^{\infty} J_\alpha^n\right), \quad \text{Energy}: \; E = \sum_{j=1}^{N} k_j^2$$

$$\text{Two}-\text{spinon spectrum}: \; \Delta K = \pm n_c\pi + 2\pi\sum_{j=1}^{2}\int_0^{\lambda_j^{\mathrm{h}}} \rho_s^0(\lambda)\mathrm{d}\lambda, \qquad \Delta F = -\sum_{j=1}^{2}\phi_s^0(\lambda_j^{\mathrm{h}})$$



Particle-hole continuum spectra at $\gamma = 5.03$



Effective mass of quasiparticle

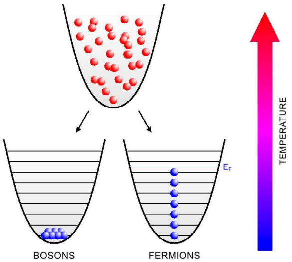
$$\text{Particle} - \text{hole spectrum}: \ \Delta E = v_c \hbar |\Delta K| \pm \frac{1}{2m^*}(\hbar \Delta K)^2$$

$$\text{Charge velocity}: \ v_c = \frac{\varepsilon_c'(k_0)}{2\pi\rho_c(k_0)}, \quad \text{Effective mass}: \ \frac{m}{m^*} = \frac{\varepsilon_c''(k_0)}{2(2\pi\rho_c(k_0))^2} - \frac{\pi\rho_c'(k_0)\varepsilon_c'(k_0)}{(2\pi\rho_c(k_0))^3}$$

$$\text{Strong coupling}: \ v_c \approx 2\pi n_c\left(1 - \frac{4\ln 2}{\gamma}\right), \quad m^* \approx m\left(1 + \frac{4\ln 2}{\gamma}\right)$$

He, Jiang, Lin, Hulet, Pu and Guan, Phys. Rev. Lett. 125, 190401 (2020)

Bosons and Fermions



Quantum statistics:

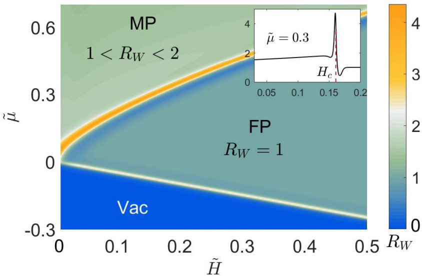
1. quantum man-body systems
2. microscopic state energy E_i
3. partition function $Z = \sum_{i=1}^{\infty} W_i e^{-E_i/(k_B T)}$
4. free energy $F = -k_B T \ln Z$
5. Energy of the quasiparticle: $\epsilon_{k,\sigma} = \epsilon_k + V^{-1} \sum_{k'} \sigma' f(k,k';\sigma,\sigma')\delta n_{k'\sigma'}$
6. challenge: finding new physics

$$\varepsilon(k) = k^2 - \mu - \frac{H}{2} - T\sum_{n=1}^{\infty} a_n * \ln\left(1 + e^{-\phi_n(k)/T}\right)$$

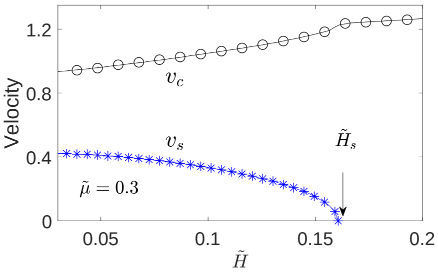
$$\phi_n(\lambda) = nH - Ta_n * \ln\left(1 + e^{-\varepsilon(\lambda)/T}\right) + T\sum_{m=1}^{\infty} T_{nm} * \ln\left(1 + e^{-\phi_m(\lambda)/T}\right)$$

$$p = \frac{T}{2\pi}\int_{-\infty}^{\infty} \ln[1 + \mathrm{e}^{-\varepsilon(k)/T}]\mathrm{d}k, \qquad n = 1, \ldots, \infty$$

Takahashi, Thermodynamics of One-Dimensional Solvable Models (Cambridge University Press)
Guan, Batchelor and Lee, Rev. Mod. Phys. 85, 1633 (2013)



Wilson ratio Phase diagram at $\tilde{T} = 0.005$



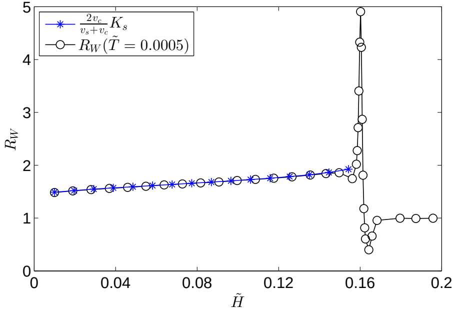
Spin and charge velocities at $\tilde{\mu} = 0.3$, $c = 1$

- **Wilson ratio–Essence of Luttinger liquid:** $R_W^\chi \propto \frac{\mathrm{Cov}(M,M)}{\mathrm{Cov}(E,E)}$ (or $R_W^\kappa \propto \frac{\mathrm{Cov}(N,N)}{\mathrm{Cov}(E,E)}$)

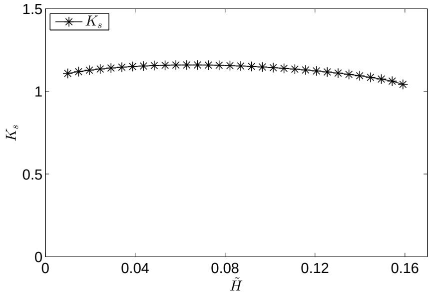
$$R_W^\chi = \frac{4}{3}\left(\frac{\pi k_B}{\mu_B g}\right)^2 \frac{\chi}{c_v/T} = \frac{2v_c}{v_s + v_c} K_s$$

$$R_W^\kappa = \frac{\pi^2}{3}\frac{\kappa}{c_V/T} = \frac{2v_s}{v_s + v_c} K_c$$

$$K_s = 2\pi v_s \chi_0, \qquad K_c = \frac{\pi v_c}{2}\kappa$$



Wilson ratio at $\tilde{\mu}=0.3$, $c=1$ and $\tilde{T}=0.005$



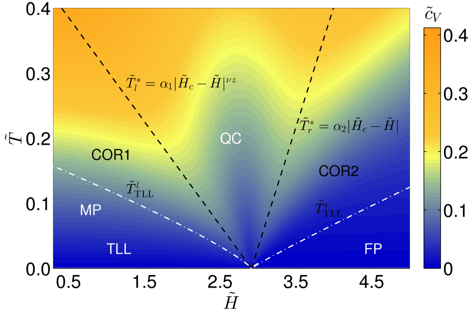
Spin Luttinger parameter at $\tilde{\mu}=0.3$, $c=1$

- **Wilson ratio–Essence of Luttinger liquid:** $R_W^{\chi} \propto \frac{\mathbf{Cov}(M,M)}{\mathbf{Cov}(E,E)}$ **(or** $R_W^{\kappa} \propto \frac{\mathbf{Cov}(N,N)}{\mathbf{Cov}(E,E)}$**)**

$$R_W = \frac{4}{3}\left(\frac{\pi k_B}{\mu_B g}\right)^2 \frac{\chi}{c_v/T} = \frac{2v_c}{v_s+v_c}K_s$$

$$R_W^{\kappa} = \frac{\pi^2}{3}\frac{\kappa}{c_V/T} = \frac{2v_s}{v_s+v_c}K_c$$

$$K_s = 2\pi v_s\chi_0, \qquad K_c = \frac{\pi v_c}{2}\kappa$$



- Tomonaga-Luttinger liquid
  $f = E_0 - \frac{\pi \mathbf{T}^2}{6}\left(\frac{1}{\mathbf{v_s}} + \frac{1}{\mathbf{v_c}}\right)$
- FG: Free gas $c_v \sim T^{1/2}$
- MP: mixed phase
  $1 < R_W < 2$
- FP: Ferromagnetic phase
  $R_W = 1$

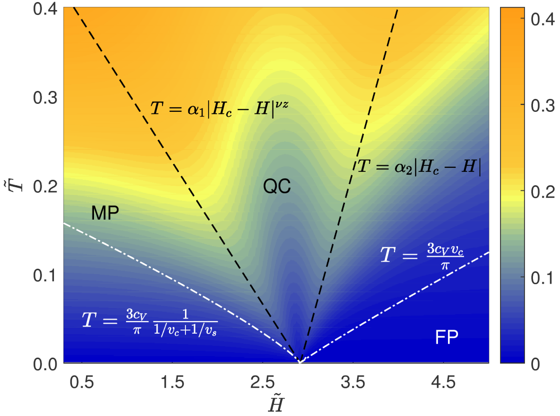
- Quantum criticality (QC) of spinons

$$c_v/T = c_v^0 + T^{d/z+1-2/\nu z}\mathcal{K}\left(\frac{\mu - \mu_c}{T^{1/\nu z}}\right), \quad z = 2,\, \nu = 1/2$$

- Coexistence of Liquid and gas near H_s (COR): $\epsilon_p = \sum_\alpha v_\alpha p + \frac{p^2}{2m^*_\alpha} + O(p^3)$

$$p = \mathbf{p_0^{\mathrm{Liquid}}} + \mathbf{p_0^{\mathrm{BG}}} + \mathbf{T^{\frac{1}{z}+1}}\mathcal{G}\left(\frac{s_0 \Delta H}{T^{1/\nu z}}\right)$$

$$p_0^{\mathrm{Liquid}} = \frac{\pi T^2}{6 v_c}, \quad p_0^{\mathrm{BG}} = \frac{2}{3\pi}\left(\mu_c + \frac{H}{2}\right)^{3/2}$$



Specific heat for $\tilde{\mu} = 2.5$, $c = 1$

- **Quantum phases at criticality**

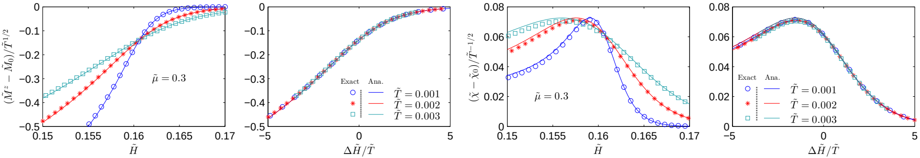
Critical exponents : $z = 2, \ \nu = 1/2, \ d = 1$

Correlation length : $\xi \sim |\mu - \mu_c|^{-\nu}, \ \Delta \sim \xi^{-z}$

Critical temperatures : $T_2 = \alpha_2 |\mu - \mu_c|, \quad T_1 = \alpha_1 |\mu - \mu_c|^{\nu z}$

TLL phase boundary : $T_{TLL1} = \frac{3}{\pi} \frac{c_V}{1/v_c + 1/v_s}, \quad T_{TLL2} = \frac{3}{\pi} c_V v_c$

Read off critical exponents from scaling functions: $\nu = 1/2,\ z = 2$



scaling of magnetization   scaling of susceptibility

$$M^z - M_0 = \frac{\left(1 - \frac{1}{\pi}\arctan\left(\frac{2}{c}k_0\right)\right)\arctan\left(\frac{2}{c}k_0\right) T^{1/2}}{\pi^{3/2}\sqrt{a}} Li_{\frac{1}{2}}\left(-e^{\frac{s_0 \Delta H}{T}}\right)$$

$$\chi - \chi_0 = -\frac{\left(1 - \frac{1}{\pi}\arctan\left(\frac{2}{c}k_0\right)\right)^2\arctan\left(\frac{2}{c}k_0\right) T^{-1/2}}{\pi^{3/2}\sqrt{a}} Li_{-\frac{1}{2}}\left(-e^{\frac{s_0 \Delta H}{T}}\right)$$

- Luttinger liquid theory: single-particle Green's function

$$G_\sigma(x,\tau) \sim \frac{1/\sqrt{v_s\tau - ix}}{(x^2+v_c^2\tau^2)^{\gamma_{K_c}}} \frac{e^{\mathrm{i}k_F x}}{\sqrt{v_c\tau - \mathrm{i}x}} + c.c. \qquad \gamma_{k_c} = (K_c + K_c^{-1} - 2)/8$$

- Incoherent spin liquid: single-particle Green's function ($E_{\mathrm{spin}} \ll K_B T \ll E_{\mathrm{charge}}$)

$$G_\sigma(x,\tau) \sim \frac{e^{-2k_F|x|(\ln 2/\pi)}}{(x^2+v_c^2\tau^2)^{\Delta_{K_c}}} \frac{e^{\mathrm{i}(2k_F x - \psi^+_{K_c})}}{\sqrt{v_c\tau - \mathrm{i}x}} + c.c.$$

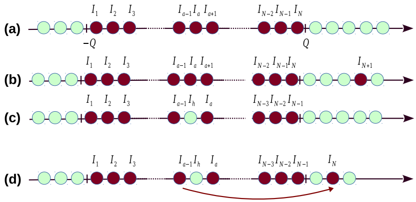
- Bosonization Hamiltonian:

$$H = \sum_{\alpha=\mathrm{c,s}} \sum_{q\neq 0} v_\alpha |q| \hat{b}^\dagger_{\alpha,q}\hat{b}_{\alpha,q} + \frac{\pi}{2L}\sum_{\alpha=\mathrm{c,s}}\left(v^\alpha_{\mathrm{N}}\,\Delta N^2_\alpha + v^\alpha_{\mathrm{J}}\,J^2_\alpha \right)$$
$$+\frac{2c}{L}\sum_{k_1,k_2,p}\sum_{r=\pm 1} \hat{\psi}^\dagger_{\uparrow,r,k_1}\hat{\psi}^\dagger_{\downarrow,-r,k_2}\hat{\psi}_{\downarrow,r,k_2+2rk_{\mathrm{F}}+p}\hat{\psi}_{\uparrow,-r,k_1-2rk_{\mathrm{F}}-p}$$

Near by the right/left Fermi point, $r=\pm 1$, $\hat{\psi}$ is the Fermi field operator.

G. A. Fiete, Rev. Mod. Phys., 79, 801 (2007)

Cheianov and Zvonarev, Physics Rev. Lett. 92, 176401 (2004)



Elementary excitations

- (a) the quantum numbers for the ground state in charge and spin degrees
- (b) adding a particle near right Fermi point (ΔN_α or backward scattering $2\Delta D_\alpha$).
- (c) a hole excitation. The total number of particles is $N-1$ in either spin or charge degrees.
- (d) a single particle-hole excitation ($N_\alpha^\pm$).

Calculate the total momentum and elementary excitation energies

Origin of spin charge separation

- Bethe ansatz result: $\Delta E = \frac{2\pi}{L}\sum_{\alpha=\mathrm{c,s}} v_\alpha (N_\alpha^- + N_\alpha^+) + \frac{\pi}{2L}\sum_{\alpha=\mathrm{c,s}} \left\{ v_\alpha (\hat{Z}^{-1}\Delta\vec{N})_\alpha^2 + v_\alpha [\hat{Z}^{\mathrm{t}}(2\vec{D})]_\alpha^2 \right\}$

  Dressed charge $\hat{Z} = \begin{pmatrix} Z_{\mathrm{cc}}(Q_{\mathrm{c}}) & Z_{\mathrm{cs}}(Q_{\mathrm{s}}) \\ Z_{\mathrm{sc}}(Q_{\mathrm{c}}) & Z_{\mathrm{ss}}(Q_{\mathrm{s}}) \end{pmatrix}$; Quantum number: $\Delta\vec{N} = \begin{pmatrix} \Delta N_{\mathrm{c}} \\ \Delta N_{\mathrm{s}} \end{pmatrix}$, $\vec{D} = \begin{pmatrix} D_{\mathrm{c}} \\ D_{\mathrm{s}} \end{pmatrix}$

- The leading order of correlations:

$$\langle \varphi(x,t)\varphi(0,0)\rangle = \frac{e^{-2iD_c(k_{F\uparrow}+k_{F\downarrow})x} e^{-2iD_s k_{F\downarrow} x}}{\prod_{a=c,s}(x - iv_a t)^{2\Delta_a^+}(x + iv_a t)^{2\Delta_a^-}}$$

$$\langle \varphi(x,t)\varphi(0,0)\rangle_T = \frac{(\pi T/v_a)^{2(\Delta_a^+ + \Delta_a^-)}\, e^{-2iD_c(k_{F\uparrow}+k_{F\downarrow})x} e^{-2iD_s k_{F\downarrow} x}}{\prod_{a=c,s}\sinh^{2\Delta_a^+}\left[\frac{\pi T}{v_a}(x - \mathrm{i}v_a t)\right]\sinh^{2\Delta_a^-}\left[\frac{\pi T}{v_a}(x + \mathrm{i}v_a t)\right]}$$

Giamarchi, T. *Quantum Physics in one dimension* (Oxford University Press, 2004)
Guan, Batchelor and Lee, Rev. Mod. Phys. 85, 1633 (2013)
Lee, Guan, Sakai, Batchelor, Phys. Rev. B 85, 085414 (2012)

Outlines

1. I. Fundamental concepts: Spin-charge separation
2. II. Yang-Gaudin model: Fractional quasiparticles and Luttinger liquids
3. III. Testing the theory of spin-charge separation
4. IV. Outlook

III. Testing the theory of spin charge separation

Linear response theory

$$\begin{aligned} H &= H_0 + f(t)B \\ \langle A(t)\rangle - A_0 &= \int_{-\infty}^{\infty} \chi(t-t')f(t')dt' \end{aligned}$$

Kubo formula gives

$$\begin{aligned} \chi(t-t') &= \frac{1}{\mathrm{i}\hbar}\langle[A(t),B(t')]\rangle\Theta(t-t') \\ \chi_{AB}(t) &= \frac{1}{\mathrm{i}\hbar}\frac{1}{Z}\sum_{mn} e^{-\beta E_m}\left(e^{\mathrm{i}E_{mn}t/\hbar}\langle m|A|n\rangle\langle n|B|m\rangle - e^{\mathrm{i}E_{nm}t/\hbar}\langle m|B|n\rangle\langle n|A|m\rangle\right) \\ \chi_{AB}(\omega) &= \frac{1}{Z}\sum_{mn} e^{-\beta E_m}\left(\frac{A_{mn}B_{nm}}{\omega-\omega_{mn}+i\eta} - \frac{B_{mn}A_{nm}}{\omega+\omega_{mn}+i\eta}\right) \end{aligned}$$

The dynamic structure factor relative to the operator F

$$\begin{aligned} S_F(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\langle F(t)F^{+}\rangle e^{\mathrm{i}\omega t}dt \\ &= \frac{1}{Z}\sum_{mn} e^{-\beta E_m}|\langle m|F|n\rangle|^2\delta(\omega - w_{nm}) \end{aligned}$$

A simple tuning of the frequency of both Bragg lasers, one can measure both the density and spin responses. The responses are the imaginary part of the charge and spin susceptibilities, respectively. That relates to the energy (momentum) transfer from the external field to the system. Here Δ_σ is the detuning.

$$\begin{aligned} P(k,\omega) &= 2\pi\Omega_{\mathrm{Br}}^2 S(k,\omega), \qquad \Omega_{\mathrm{Br}} = \hbar\Gamma^2/(4\Delta)\sqrt{I_A I_B}/I_{\mathrm{sat}} \\ \Delta P &= \propto \left(\frac{1}{\Delta_\uparrow^2}+\frac{1}{\Delta_\downarrow^2}\right) S_{\uparrow\uparrow}(k,\omega) + \frac{2}{\Delta_\downarrow\Delta_\uparrow} S_{\uparrow\downarrow}(k,\omega) \\ S_D(k,\omega) &= 2\left[S_{\uparrow\uparrow}(k,\omega)+S_{\uparrow\downarrow}(k,\omega)\right] \\ S_S(k,\omega) &= 2\left[S_{\uparrow\uparrow}(k,\omega)-S_{\uparrow\downarrow}(k,\omega)\right] \\ S_{\sigma,\sigma'}(q,\omega) &= \frac{1}{2\pi N}\int \mathrm{d}t e^{-\mathrm{i}\omega t}\langle\hat{\rho}_\sigma(q,t)\hat{\rho}^\dagger_{\sigma'}(q,0)\rangle \end{aligned}$$

Momentum transfer by Bragg perturbation

$$p(k,\omega) \propto \left(\frac{1}{\Delta_\uparrow^2}+\frac{1}{\Delta_\downarrow^2}\right) S_{\uparrow\uparrow}(k,\omega)+\frac{2}{\Delta_\uparrow\Delta_\downarrow}S_{\uparrow\downarrow}(k,\omega)$$

$$S_{\sigma,\sigma'}(q,\omega) = \frac{1}{2\pi N}\int \mathrm{d}t e^{-\mathrm{i}\omega t}\langle\hat\rho_\sigma(q,t)\hat\rho^\dagger_{\sigma'}(q,0)\rangle$$

charge and spin density sensitive measurement

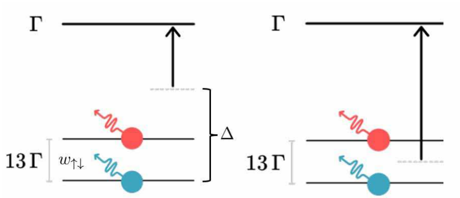
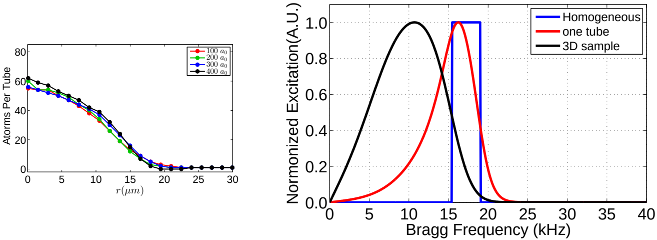


Figure: charge spin

charge channel: $\Delta_\uparrow \approx \Delta_\downarrow (\gg w_{\uparrow\downarrow})$ spin channel: $\Delta_\uparrow = -\Delta_\downarrow$



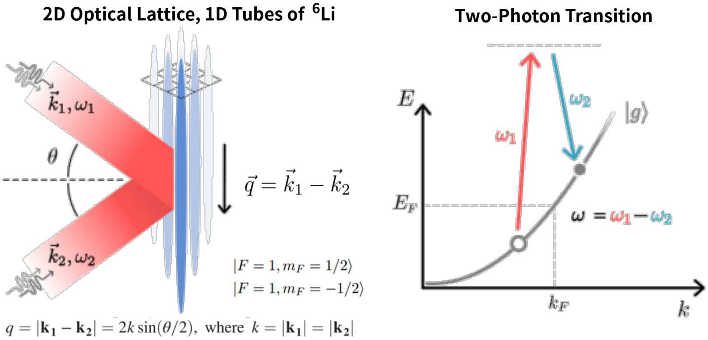
Charge DSF of homogeneous Fermi gas

$$S(q,\omega,k_F,T,N)=\frac{\mathrm{Im}\chi(q,\omega,k_F,T,N)}{\pi(1-\mathrm{e}^{-(\hbar\omega/k_BT)}}$$

Dynamic susceptibility

$$\begin{aligned}\mathrm{Im}\chi &= \frac{N\pi m}{2\hbar^2 qk_F}(n_{q_-}-n_{q_+})\\ &= \frac{Nm^*/(2\hbar^2qk_c)}{(1-\mathrm{e}^{-\beta\hbar\omega})}\left\{\frac{1}{\mathrm{e}^{\beta\left[\frac{m^*}{2q^2}\left(\omega-\frac{\hbar q^2}{2m^*}\right)^2-\frac{m^*}{2}v_c^2\right]}+1}-\frac{1}{\mathrm{e}^{\beta\left[\frac{m^*}{2q^2}\left(\omega+\frac{\hbar q^2}{2m^*}\right)^2-\frac{m^*}{2}v_c^2\right]}+1}\right\}\end{aligned}$$

He, Jiang, Lin, Hulet, Pu, Guan, Phys. Rev. Lett. 125, 190401 (2020)

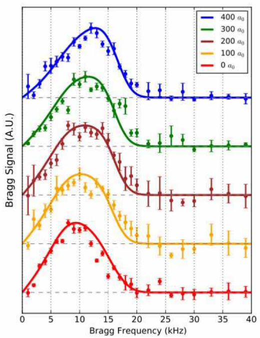


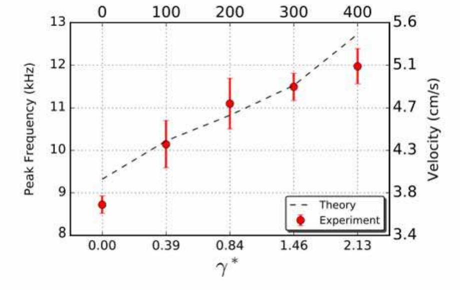
Bragg spectroscopy for the spin-1/2 Fermi gas

Bragg spectroscopy is used to measure a response of the gas to density (charge) mode excitations at a momentum q and frequency ω, as a function of the interaction strength.

$$S_{\sigma,\sigma'}(q,\omega) = \frac{1}{2\pi N}\int \mathrm{d}t e^{-\mathrm{i}\omega t}\langle \hat{\rho}_\sigma(q,t)\hat{\rho}^\dagger_{\sigma'}(q,0)\rangle$$

R. Hulet's group, Phys. Rev. Lett. 121, 103001 (2018)





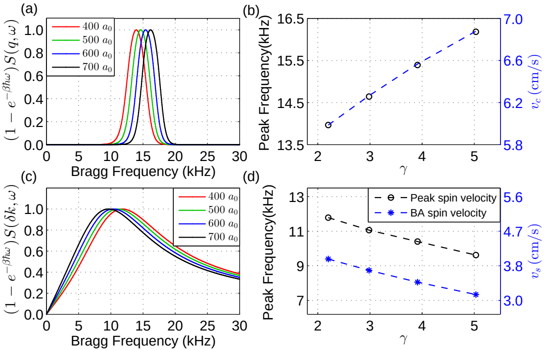
Charge DSF of homogeneous Fermi gas(imagine part of the response function is related to the energy transfer from the external field to the system)

$$S(q, \omega, k_F, T, N) = \frac{\mathrm{Im}\chi(q, \omega, k_F, T, N)}{\pi(1 - \mathrm{e}^{-(\hbar\omega/k_B T)})}$$

Dynamic susceptibility

$$\mathrm{Im}\chi = \frac{N\pi m}{2\hbar^2 q k_F}(n_{q_-} - n_{q_+}), \quad q_\pm = \frac{\omega m}{\hbar q} \pm \frac{q}{2}$$

T. L. Yang, Phys. Rev. Lett. 121 103001 (2018)



Spin DSF ($H=0$): Luttinger liquid(LL) theory gives the DSF around $k=\pi+\delta k$

$$S_{LL} = \frac{1}{1-\mathrm{e}^{-(\hbar w/k_B T)}}\frac{A_{LL}}{T}\mathrm{Im}\left[\rho\left(\frac{w+v_F\delta k}{4\pi T}\right)\rho\left(\frac{w-v_F\delta k}{4\pi T}\right)\right]$$

$$\rho(x) = \Gamma(1/4-ix)/\Gamma(4/3-ix), \quad A_{LL}=-c_\perp^2\alpha/2$$

He, Jiang, Lin, Hulet, Pu, Guan, Phys. Rev. Lett. 125, 190401 (2020)

IV. Outlook

- Fractional excitations (anyon, spinon, magnon, ...)
- Quasi-long range order (correlation function, M-body reduced density matrices...)
- Quantum liquid (Luttinger liquid, spin charge separation...)
- Quantum criticality (scaling laws, field theory, dynamical criticality...)
- Evolution dynamics (transport properties, hydrodynamic, thermalization...)
- Strongly correlated matter (topological matters, disorder, dynamical phase transition...)
- Quantum technology (quantum information, quantum metrology, quantum precision measurement, quantum battery, quantum refrigeration and quantum heat engine ...)
- Conformal Field Theory, gauge field, Yang-Mills theory, quantum gravity of black holes, ...

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