# Saddle-points at holographic entanglement transitions: enhanced corrections

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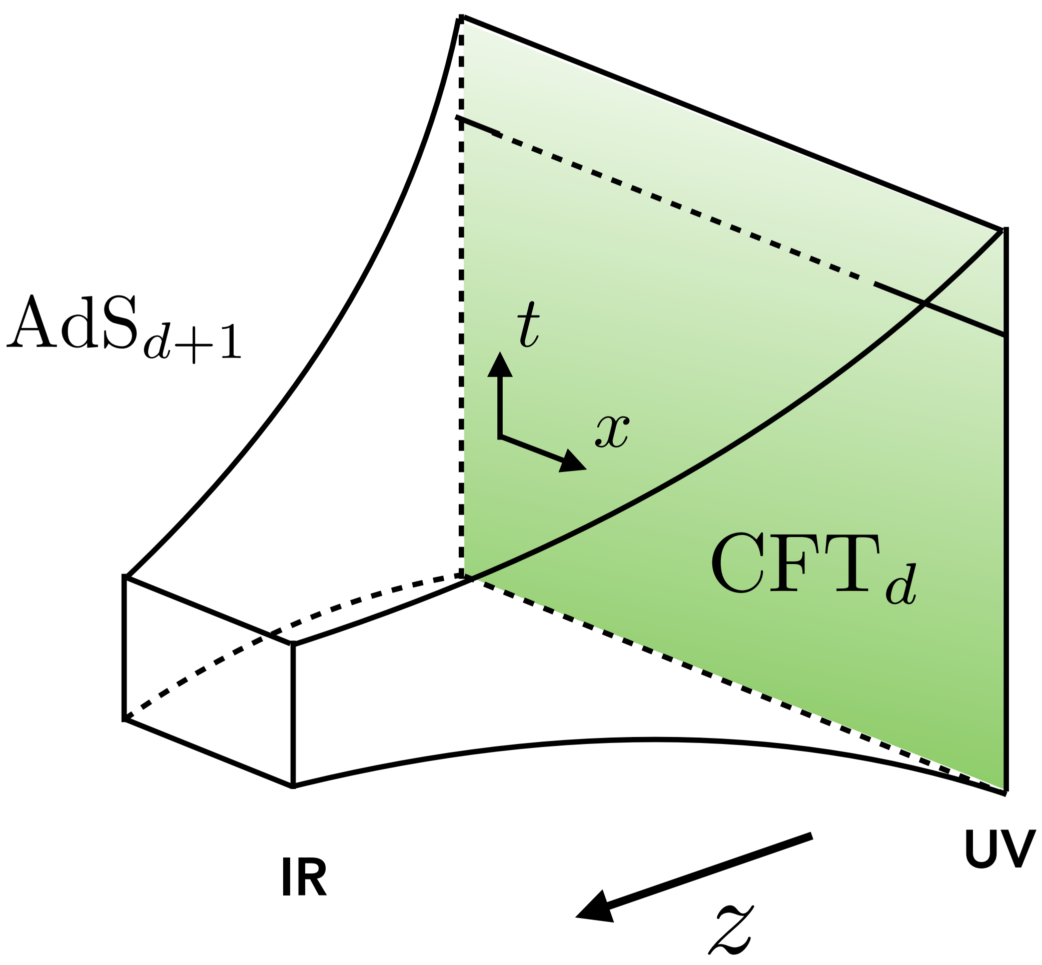
Huajia Wang

# Motivation




Holography: geometric manifestation of CFT ingredients in 1-higher dimensional bulk AdS spacetime

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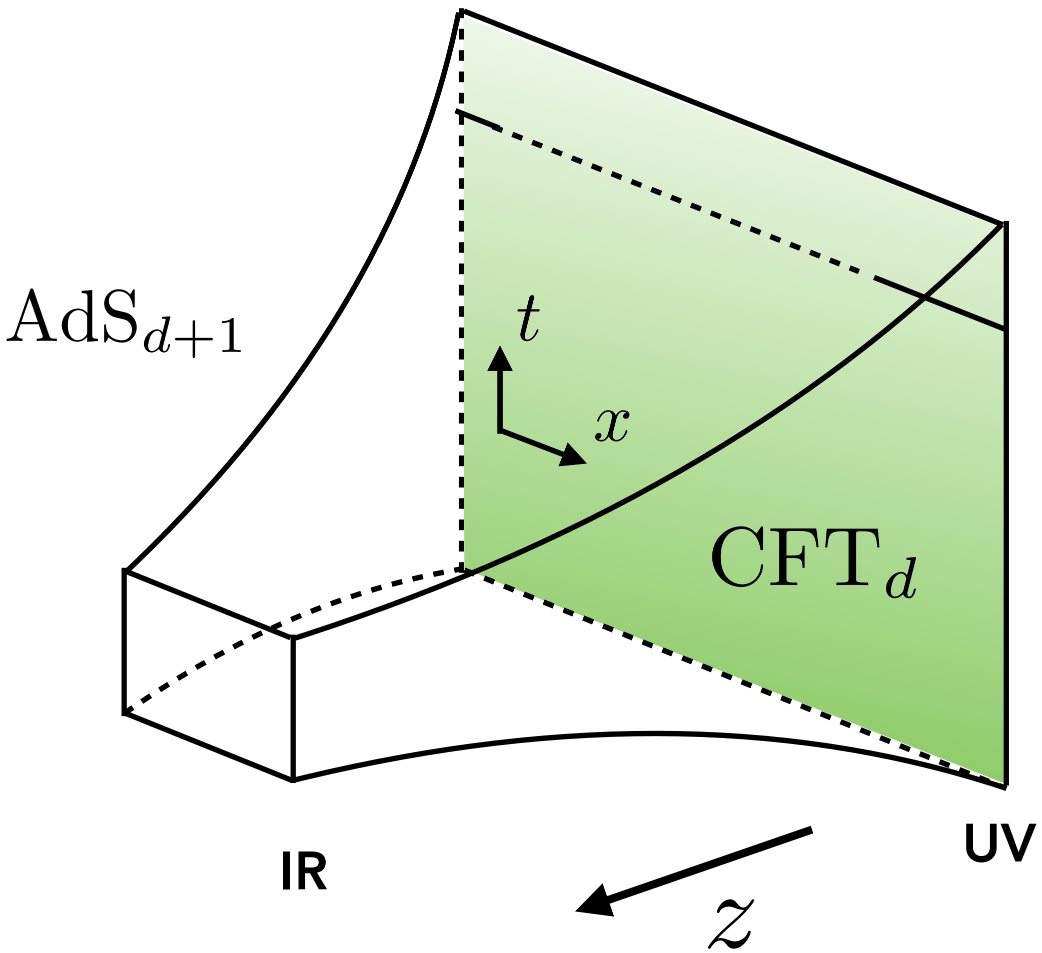


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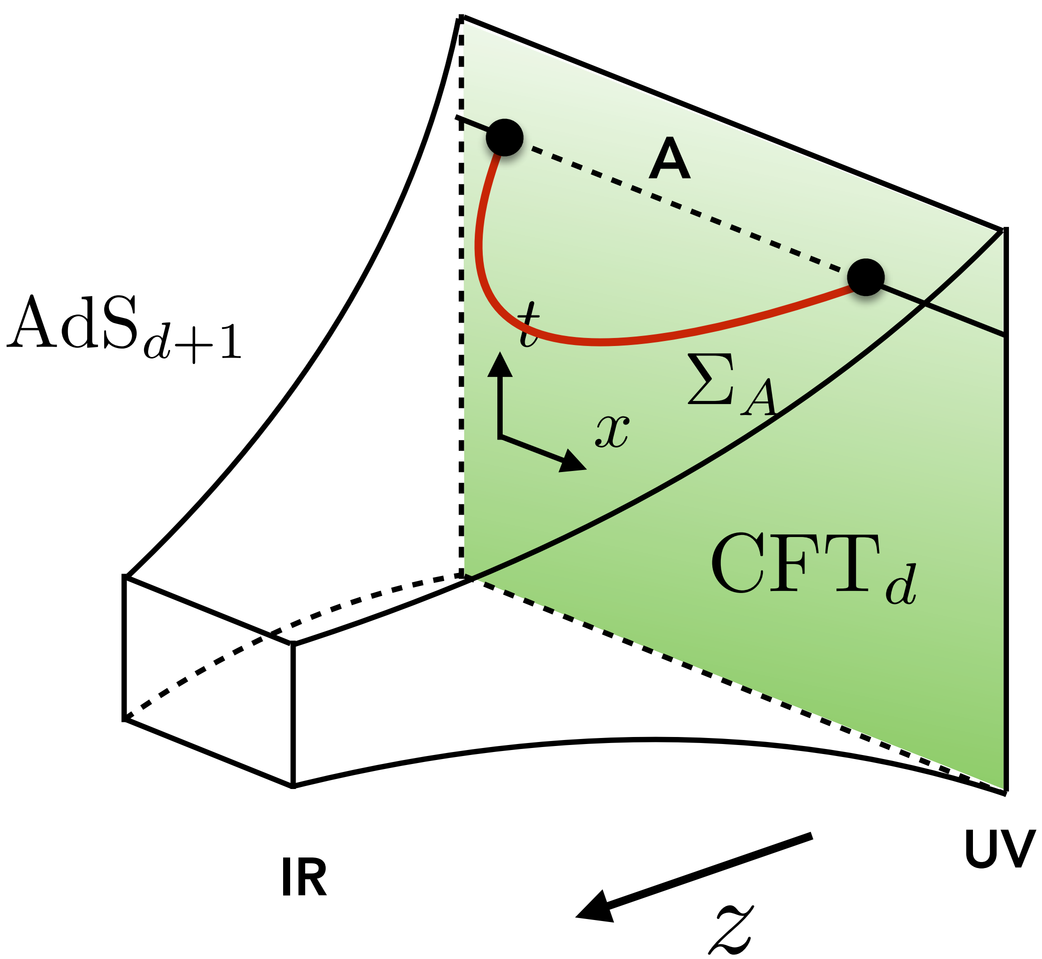


Holography: geometric manifestation of CFT ingredients in 1-higher dimensional bulk AdS spacetime

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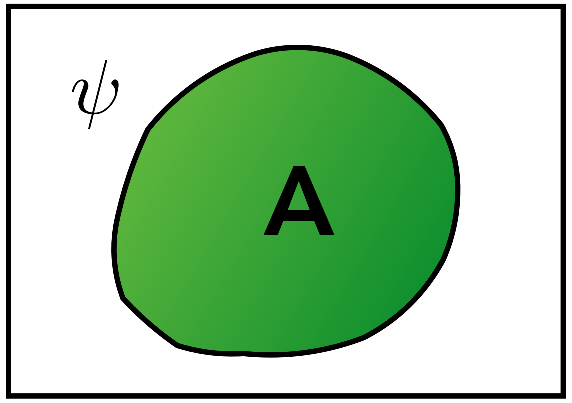
Simplest probe of geometry: entanglement entropy via the RT formula

$$S_A = \frac{\text{Area}(\Sigma_A)}{4G_N}$$

# Motivation

Entanglement entropy: $\rho_A = \mathrm{tr}_{\bar{A}} |\psi\rangle\langle\psi|$

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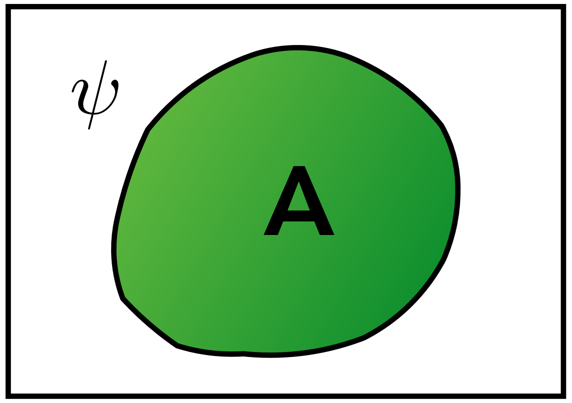


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Entanglement entropy via replica trick — Renyi entropies

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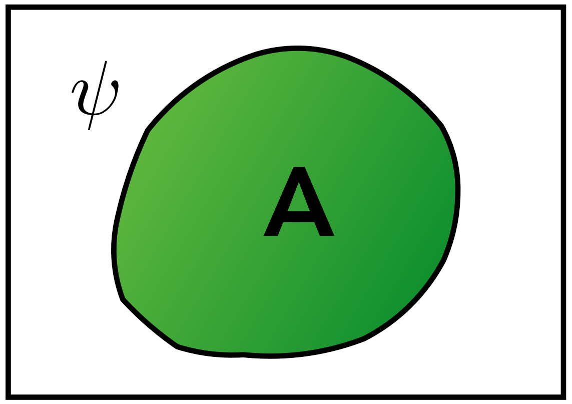


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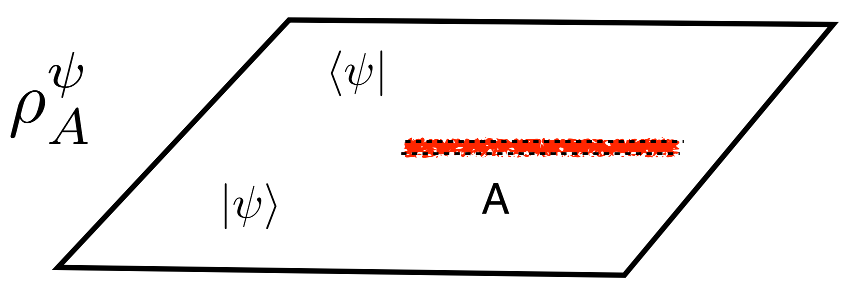


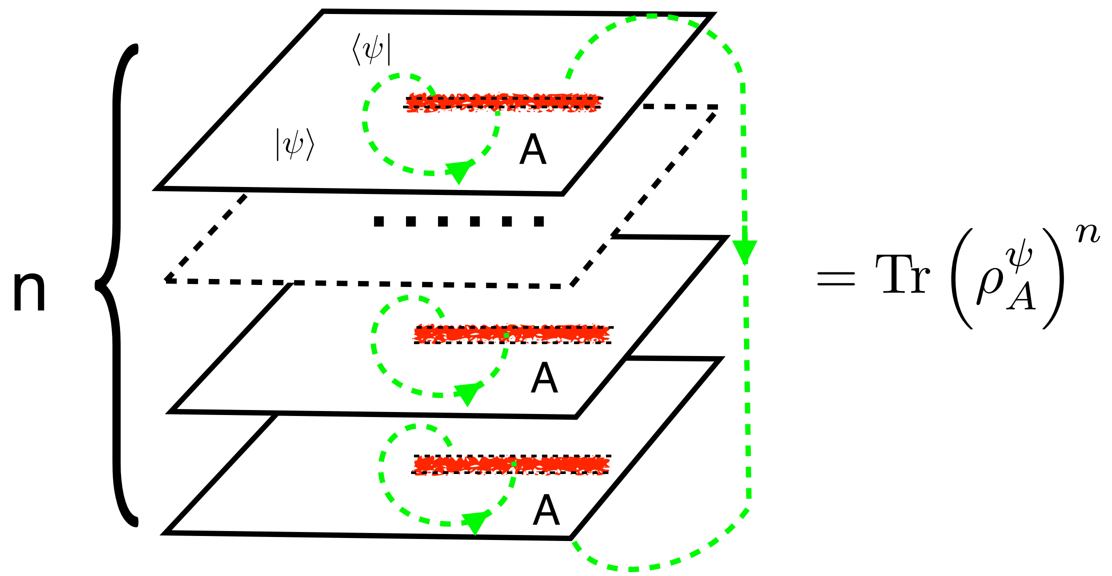

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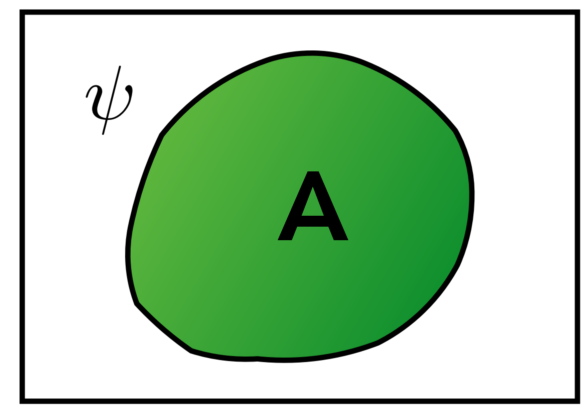


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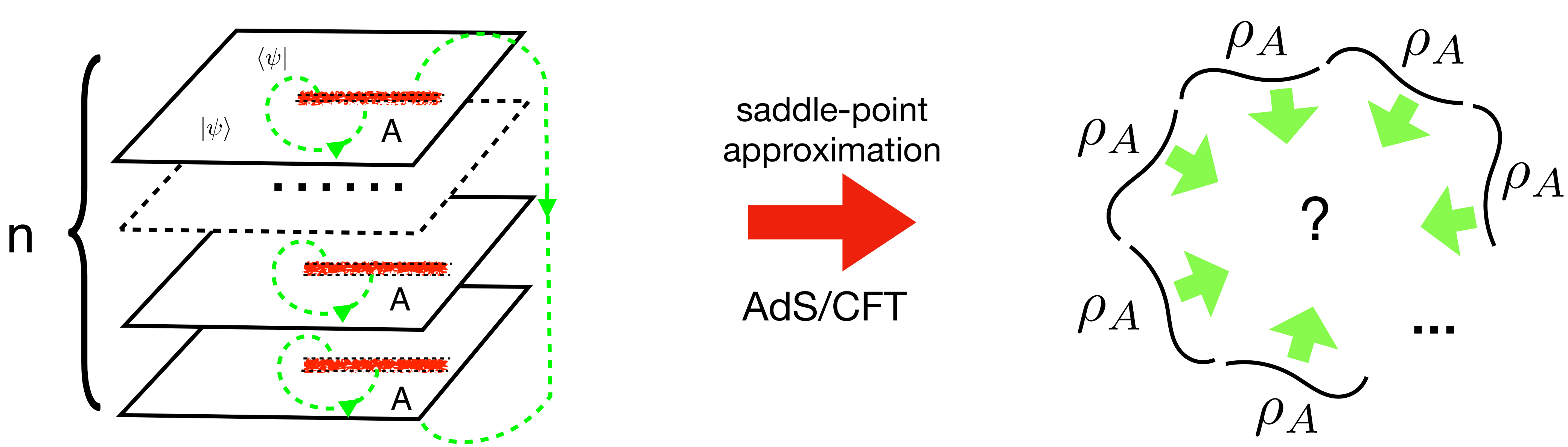

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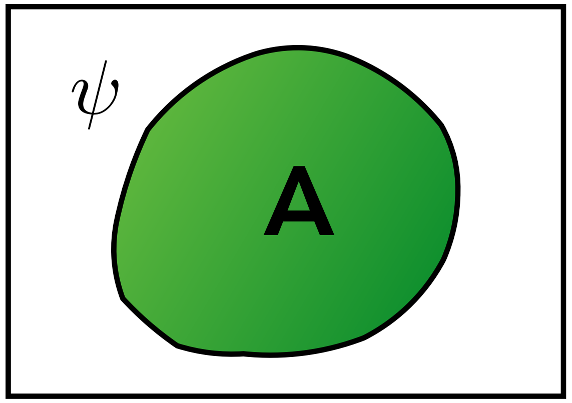


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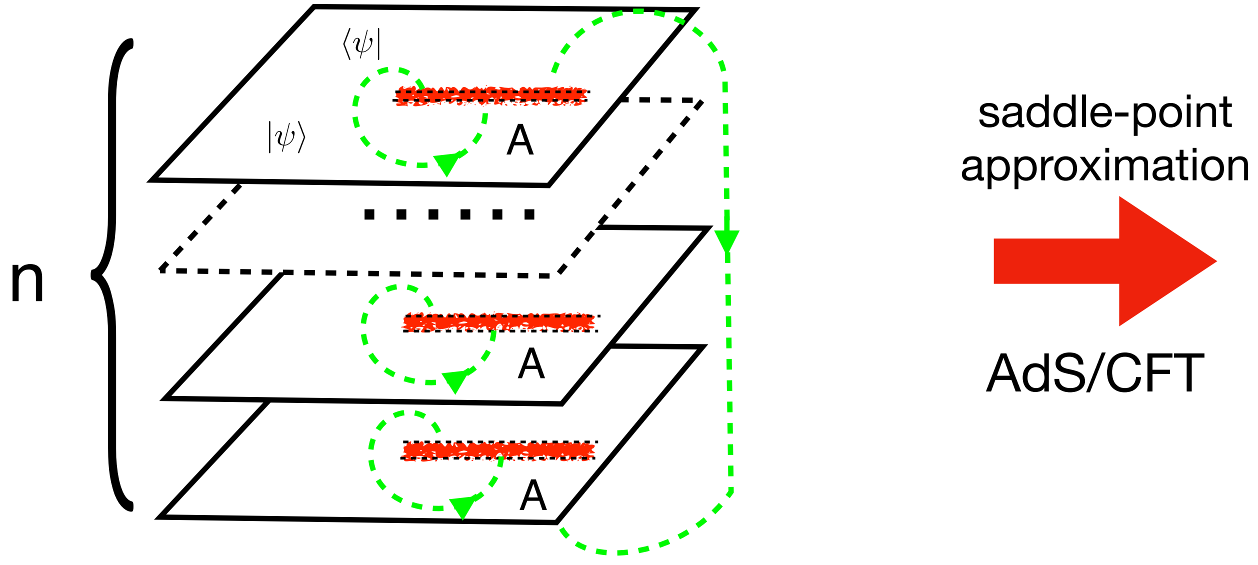


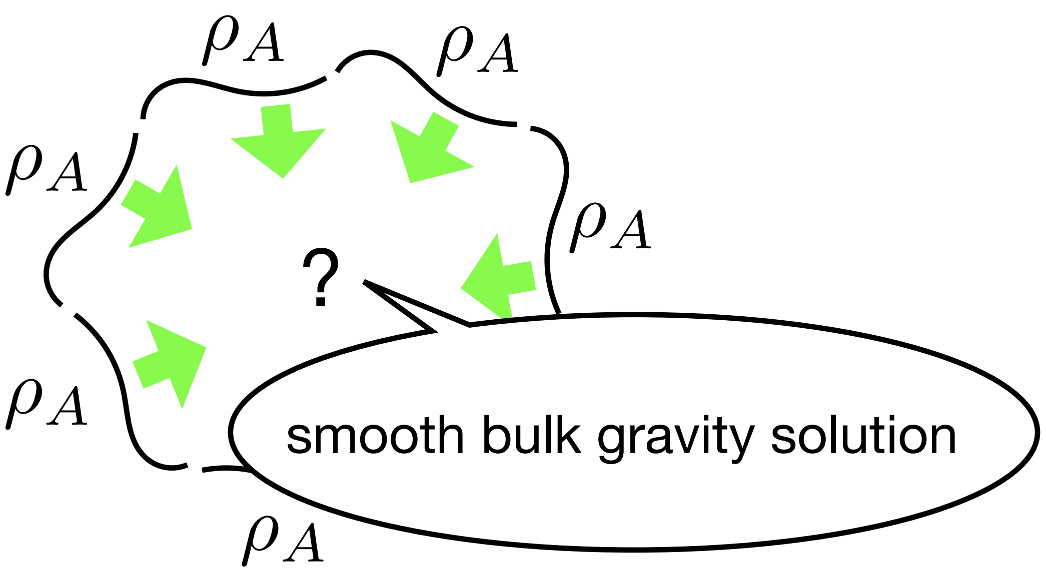

## Motivation

- "cosmic brane" prescription for constructing (quotient of) bulk    X. Dong, 16
- brane tension: $T = \frac{n-1}{4nG_N}$
- can analytically continue n to real values
- take replica limit, extract leading order in O(n-1)
- RT formula emerges: $S_A = \frac{\text{Area}(\Sigma_A)}{4G_N}$    Lewkowycz & Maldacena 14

# Motivation

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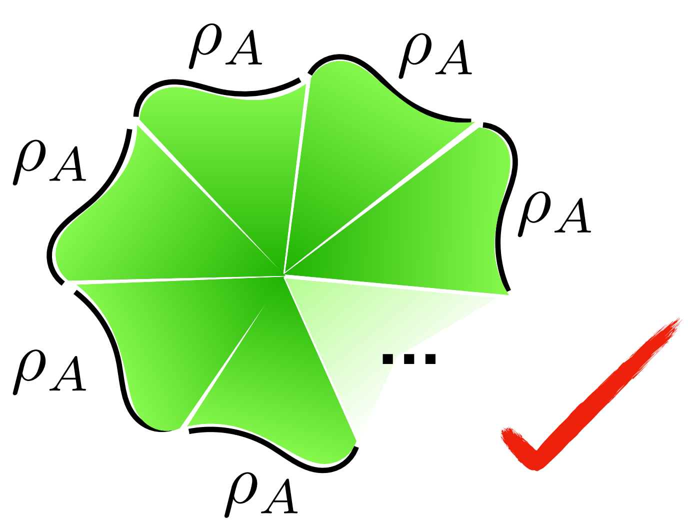
1. Only "replica symmetric" saddles are important

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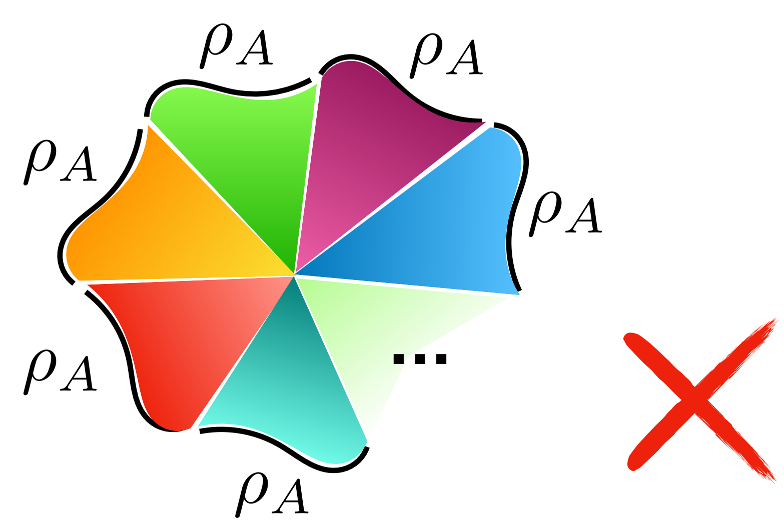

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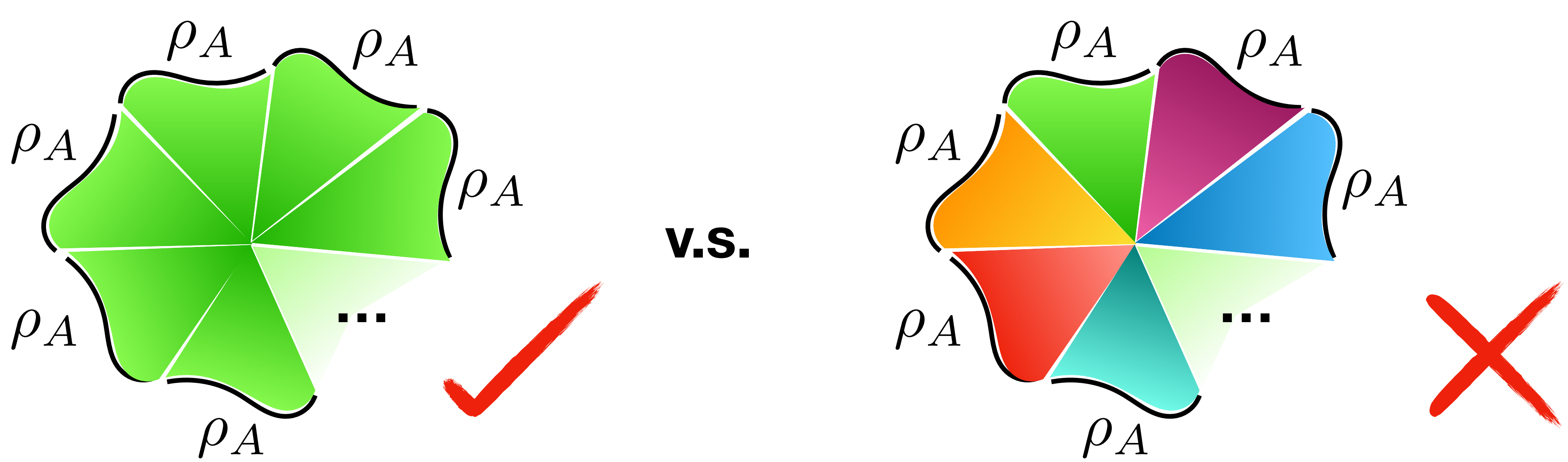


2. One particular saddle-point dominates the path-integral

$$S_A \sim \frac{\text{Area}\left(\Sigma_A\right)}{4G_N} + \mathcal{O}\left(e^{-\mathcal{O}(1)/4G_N}\right)$$

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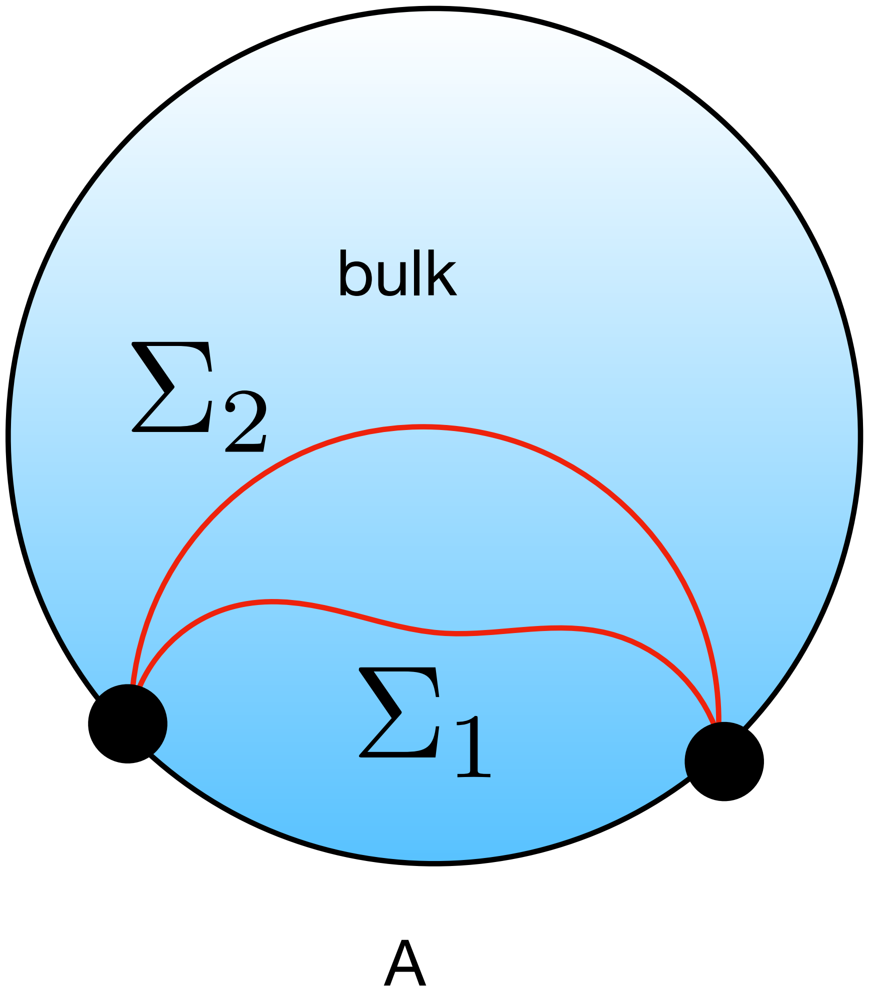
Two competing saddle points having comparable contribution

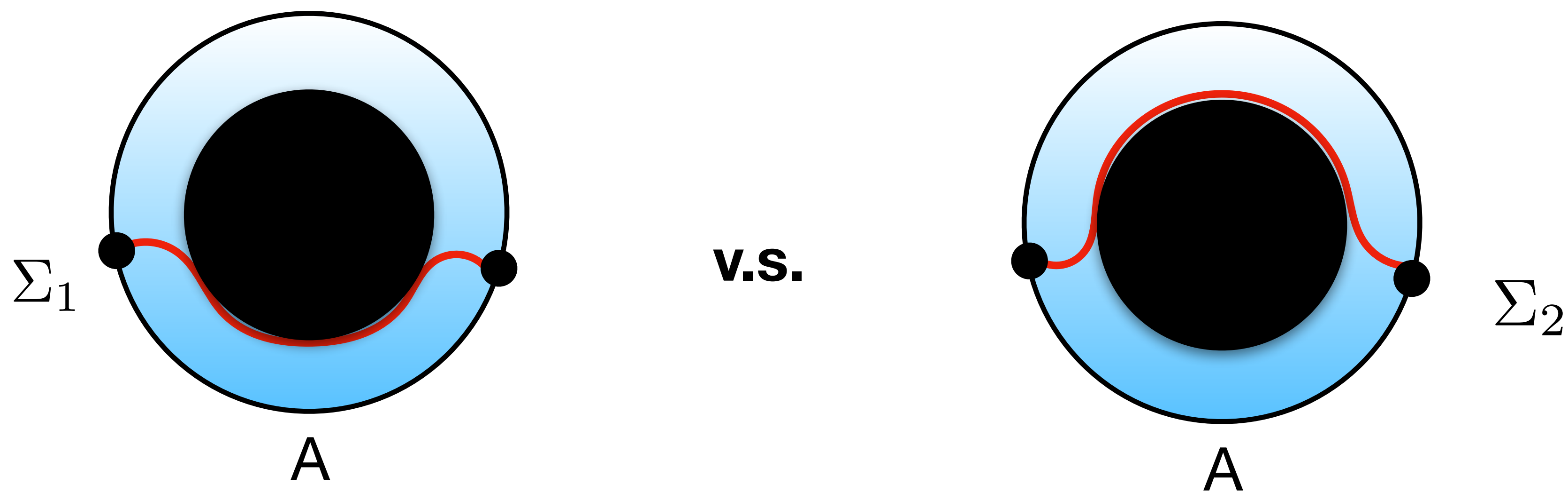



$$\text{Area}\,(\Sigma_2) \approx \text{Area}\,(\Sigma_2)$$

“entanglement phase transition”

# Motivation

**A prototype: entanglement entropy of a black hole micro-state (chaotic)**



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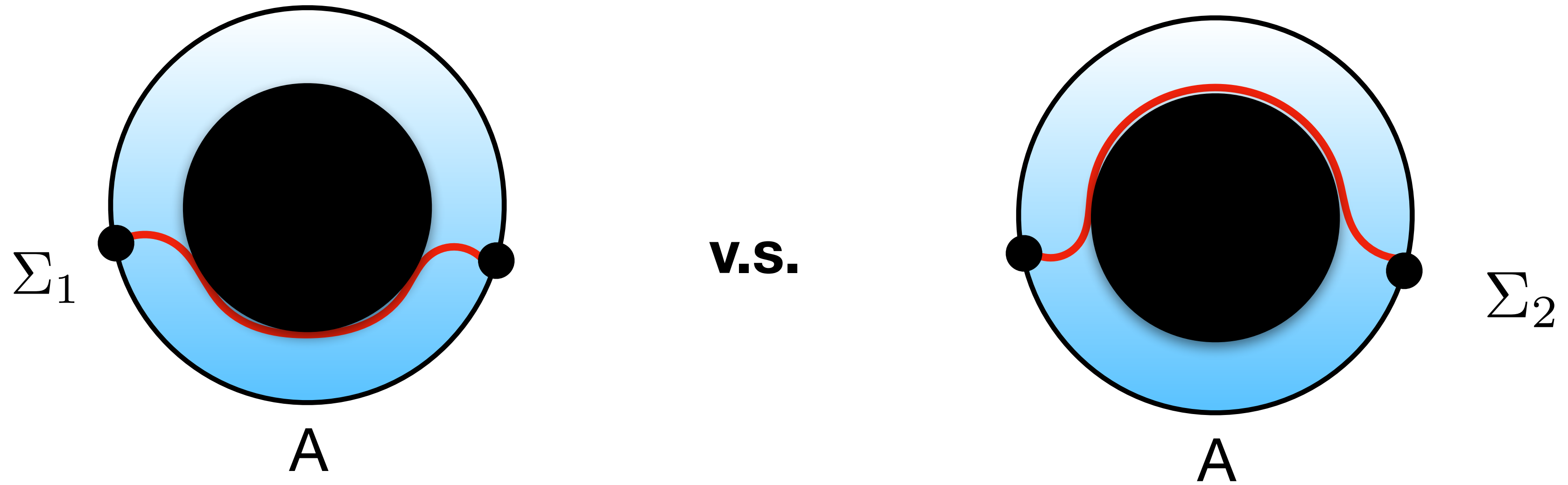


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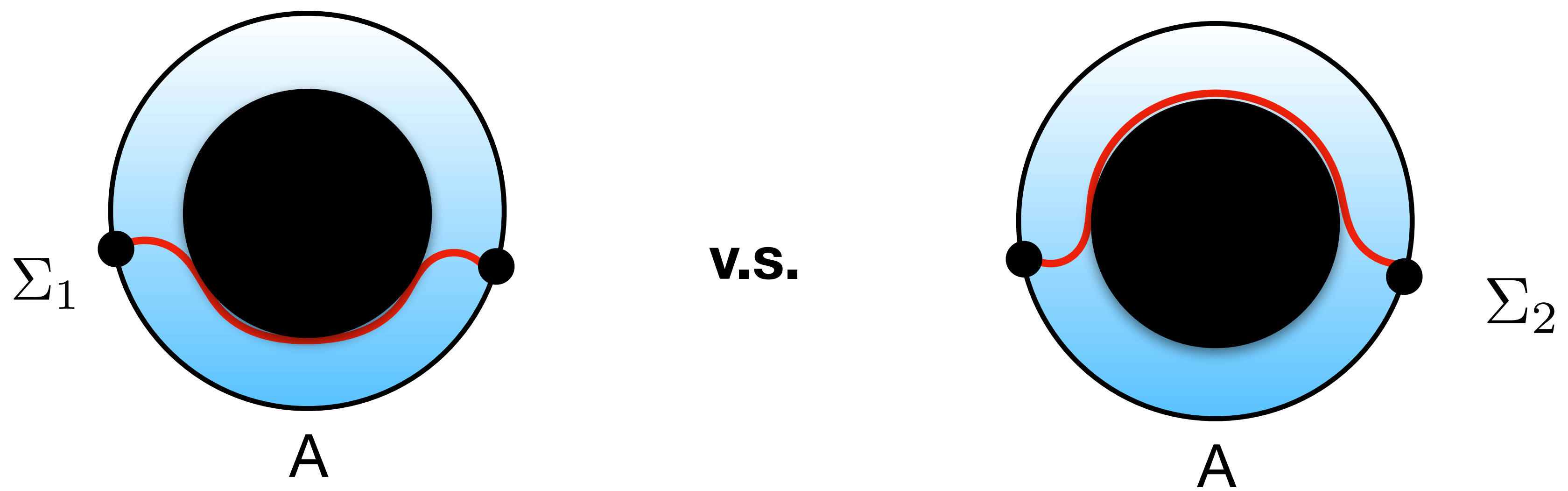


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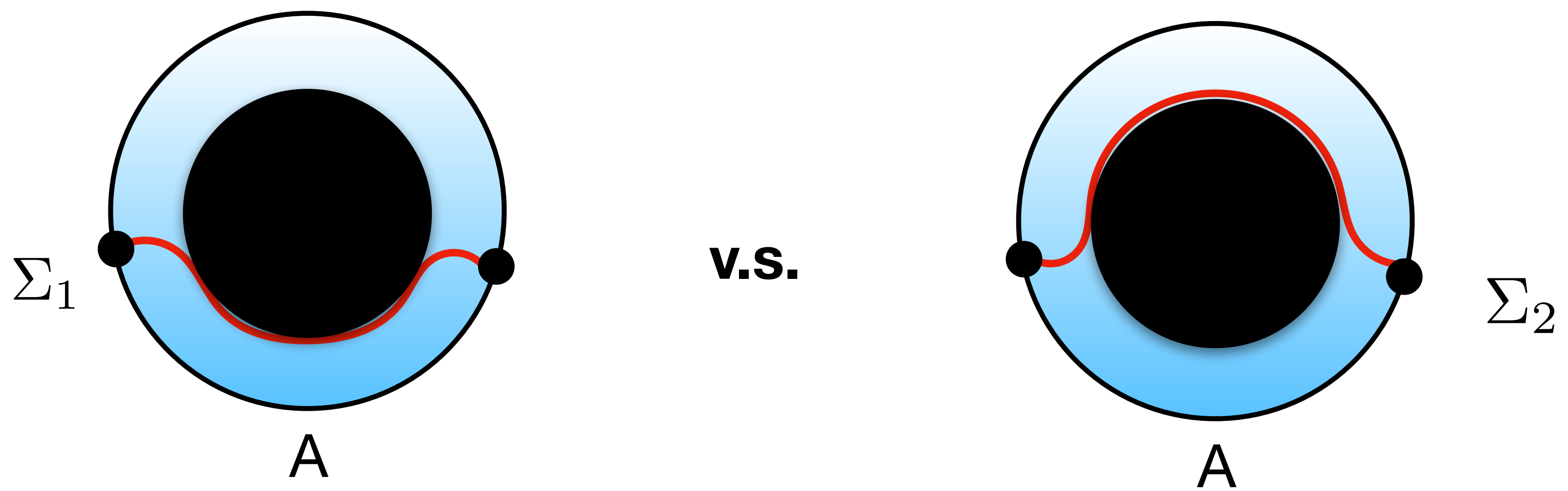


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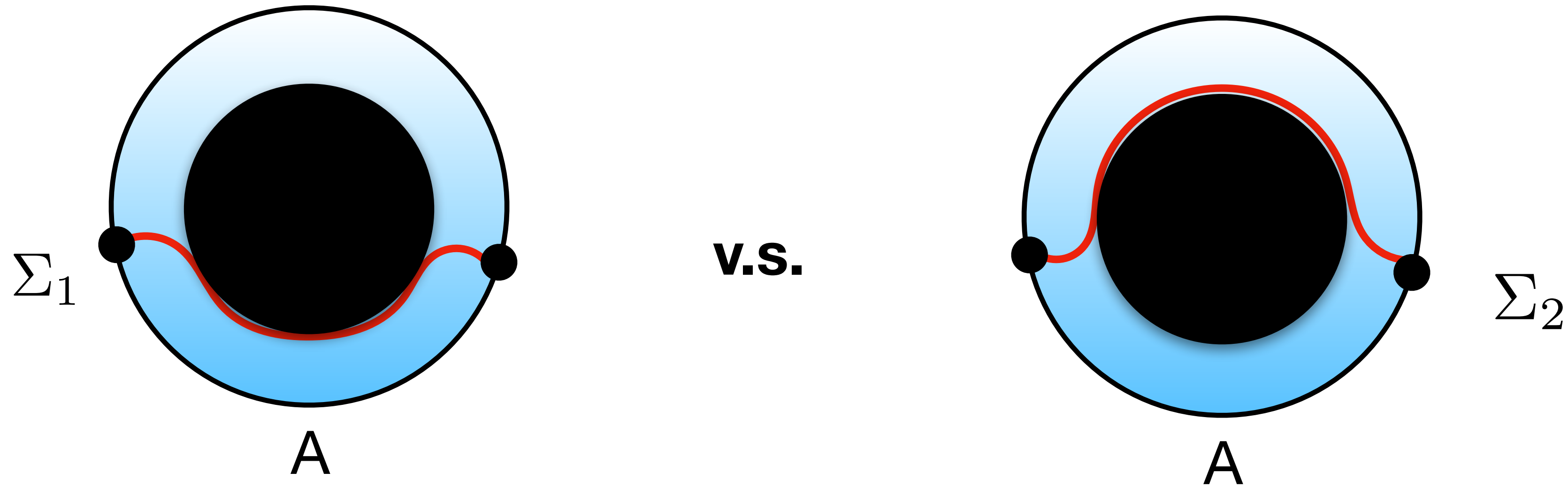


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- Effects of replica non-symmetric saddle-points?
- Focus on a particular aspect of the transition: enhanced correction

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Has been shown for *chaotic* high energy eigenstates

$$|E\rangle = \sum_{|E_i+E_J-E|<\Delta} c_{iJ} |E_i\rangle_A \otimes |E_J\rangle_{\bar{A}}$$

C. Murthy and M. Srednicki, 2019

$c_{iJ}$ are independent Gaussian random variables, an ansatz based on eigenstate thermalization hypothesis (ETH), i.e. small subsystems look thermal

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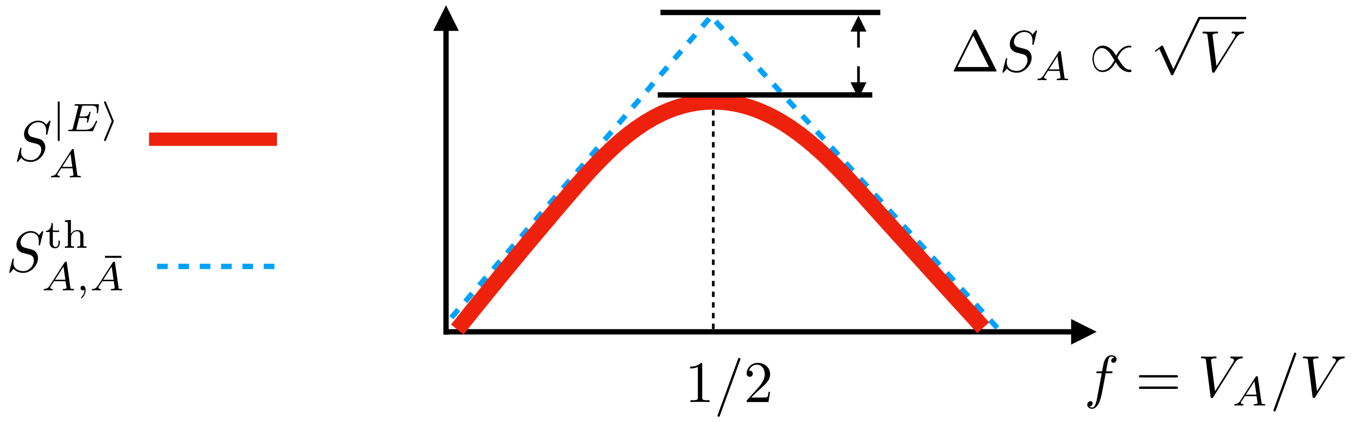

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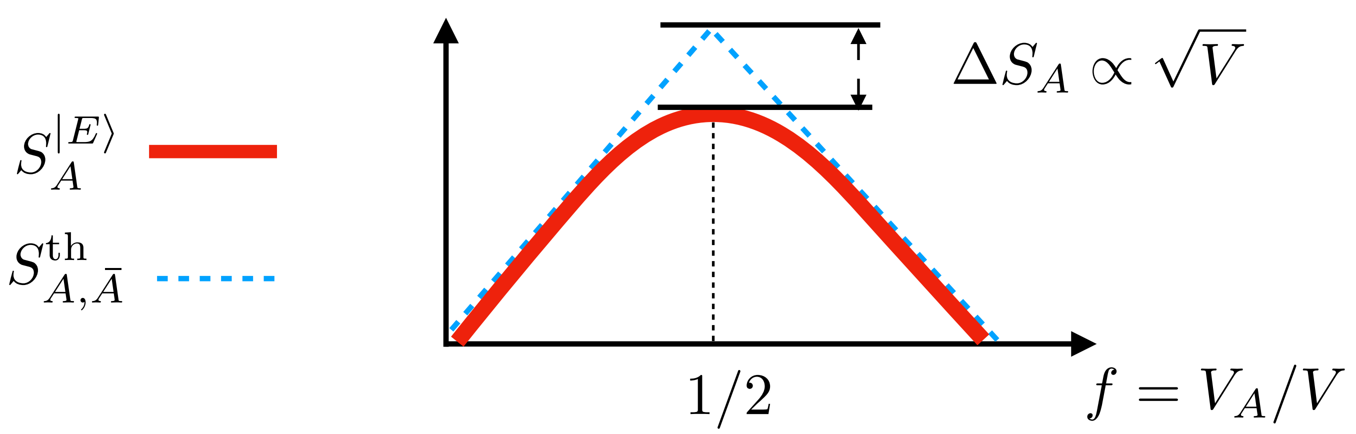


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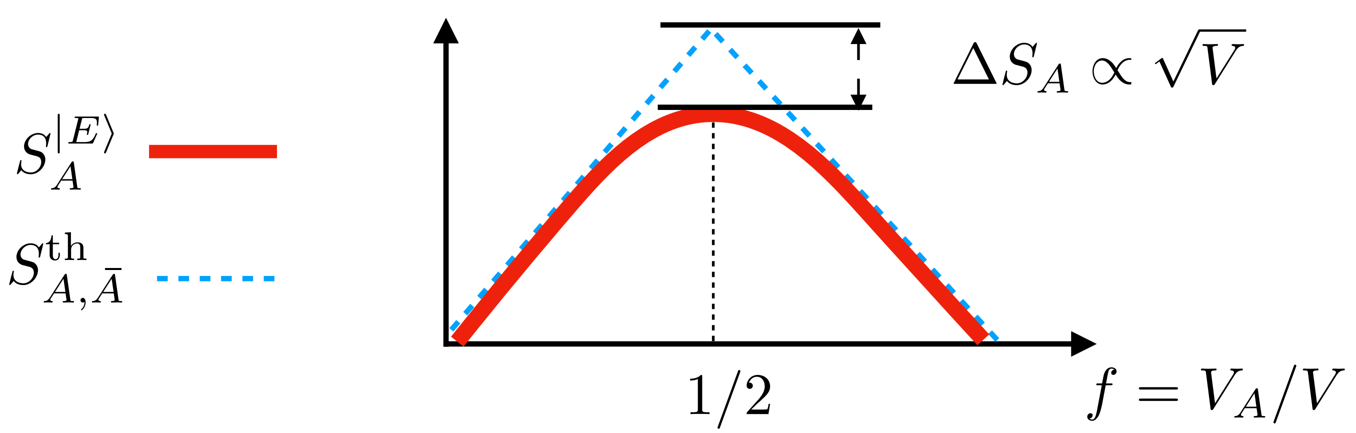


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Via replica-trick: revisit renyi entropy calculation in AdS/CFT

**Outline:**

- Renyi entropy for (chaotic) black hole micro-states
- Construct and re-sum bulk saddles: effective action
- Enhanced correction from effective action
- Discussions and outlooks

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*In terms of Euclidean path-integral, how does a chaotic energy eigenstate differ from a true canonical ensemble?*

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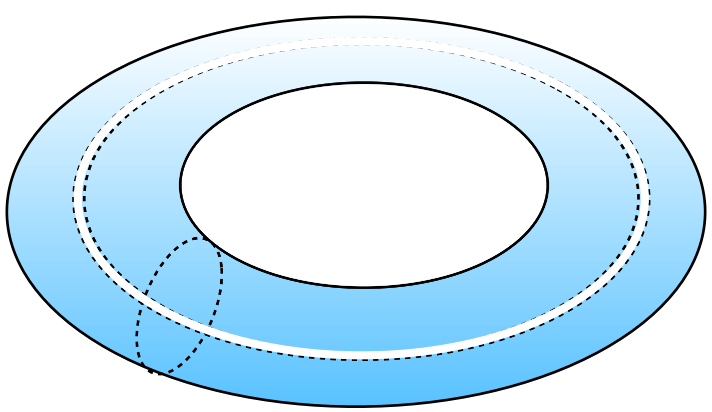

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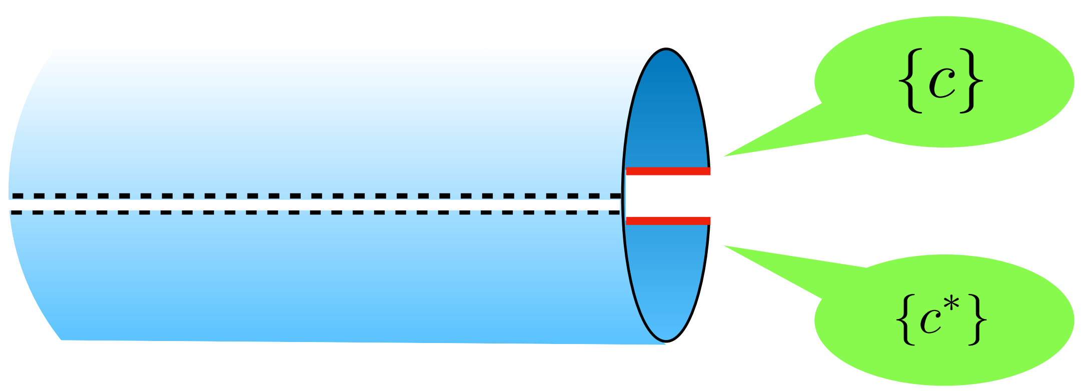
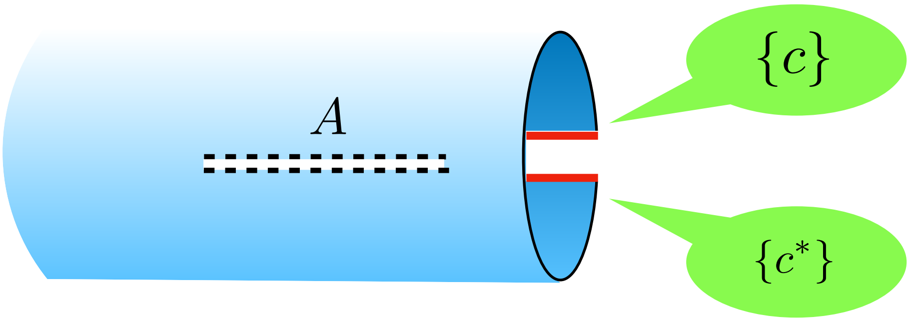



Black hole geometry = canonical ensemble

Black hole micro-state = $|E\rangle\langle E|$

## Renyi entropy for Chaotic Black hole micro-states

Reduced density matrix: $\rho_A = \mathrm{Tr}_{\bar{A}} \rho_E$



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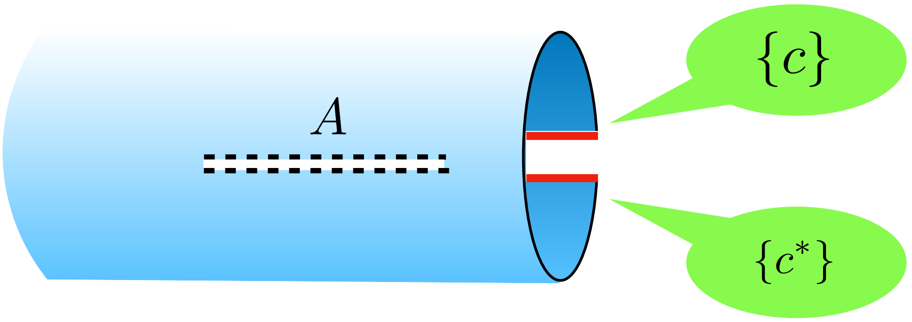


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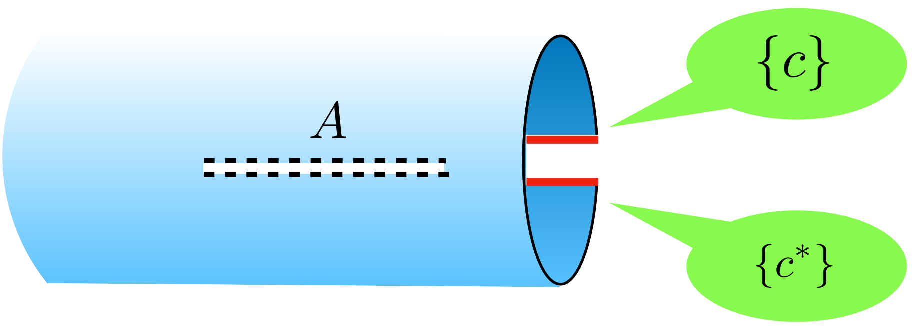


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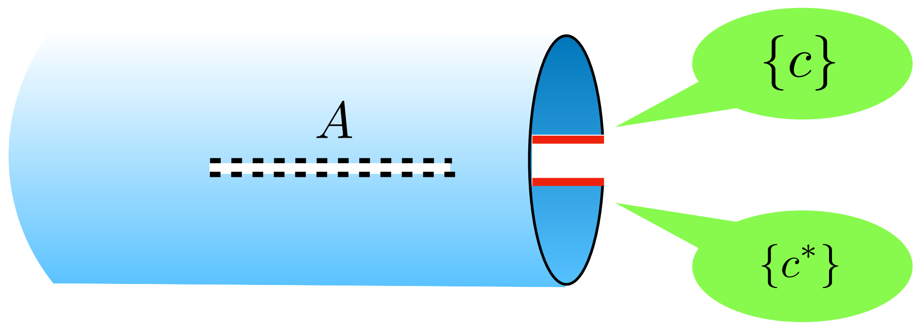


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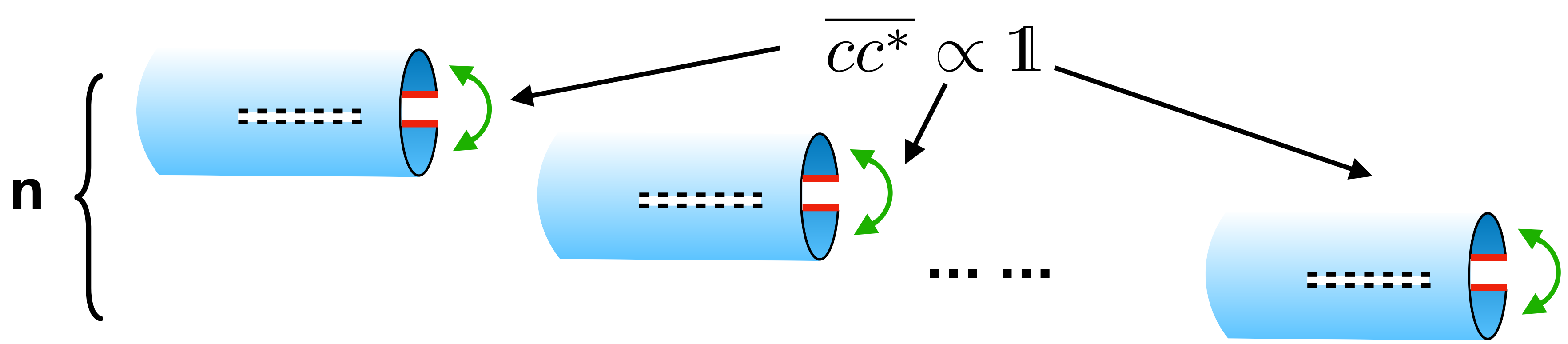

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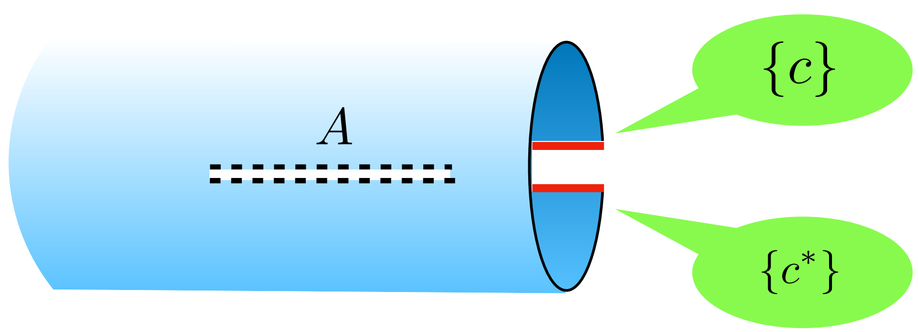


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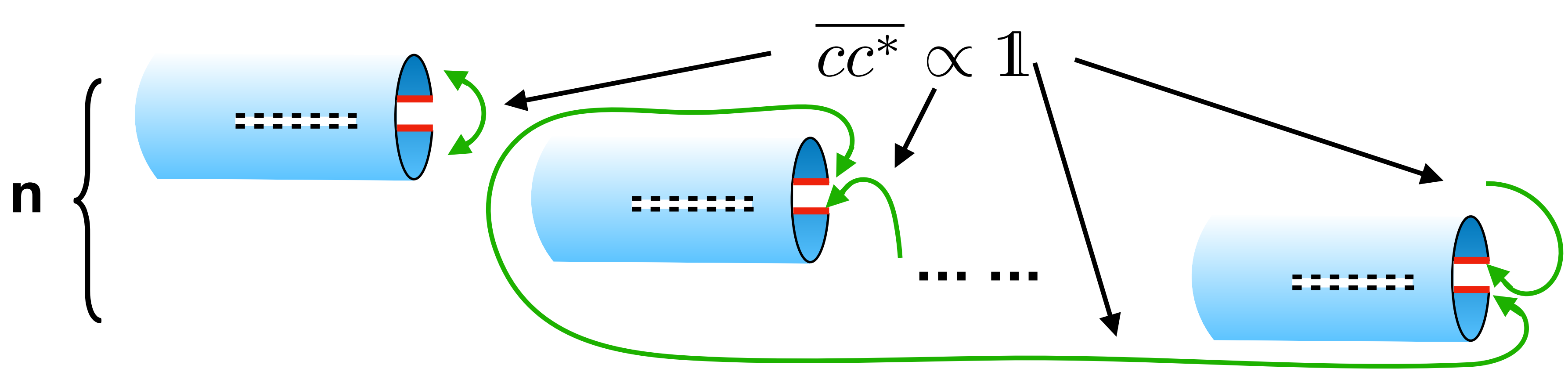

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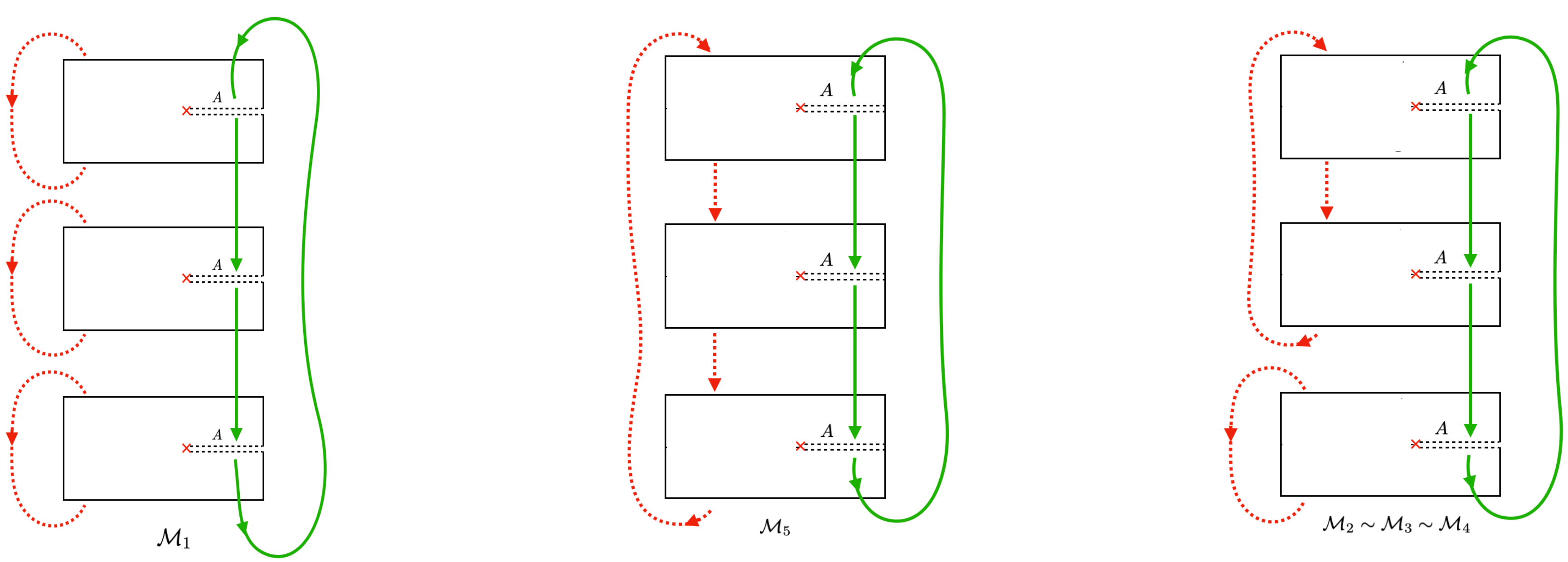


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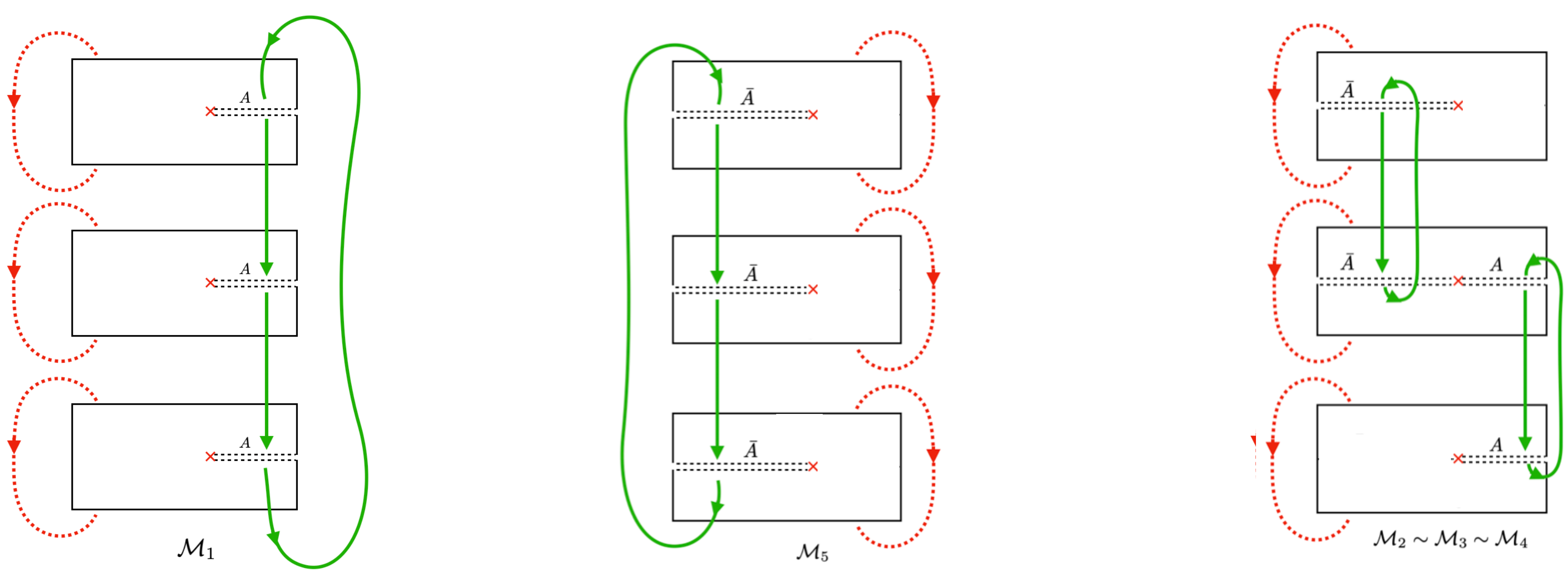


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- Different branch structures along A and A complement (to maintain topology)

## Renyi entropy for Chaotic Black hole micro-states

- Different (planar) wick-contractions give rise to different closed boundary branched manifolds $\{\mathcal{M}_i\}$ for computing $\overline{Z_n}$

- Need to fill them into bulk solutions $\{\mathcal{B}_i\}$, corresponding to different bulk saddles for computing $\overline{S_n(A)}$

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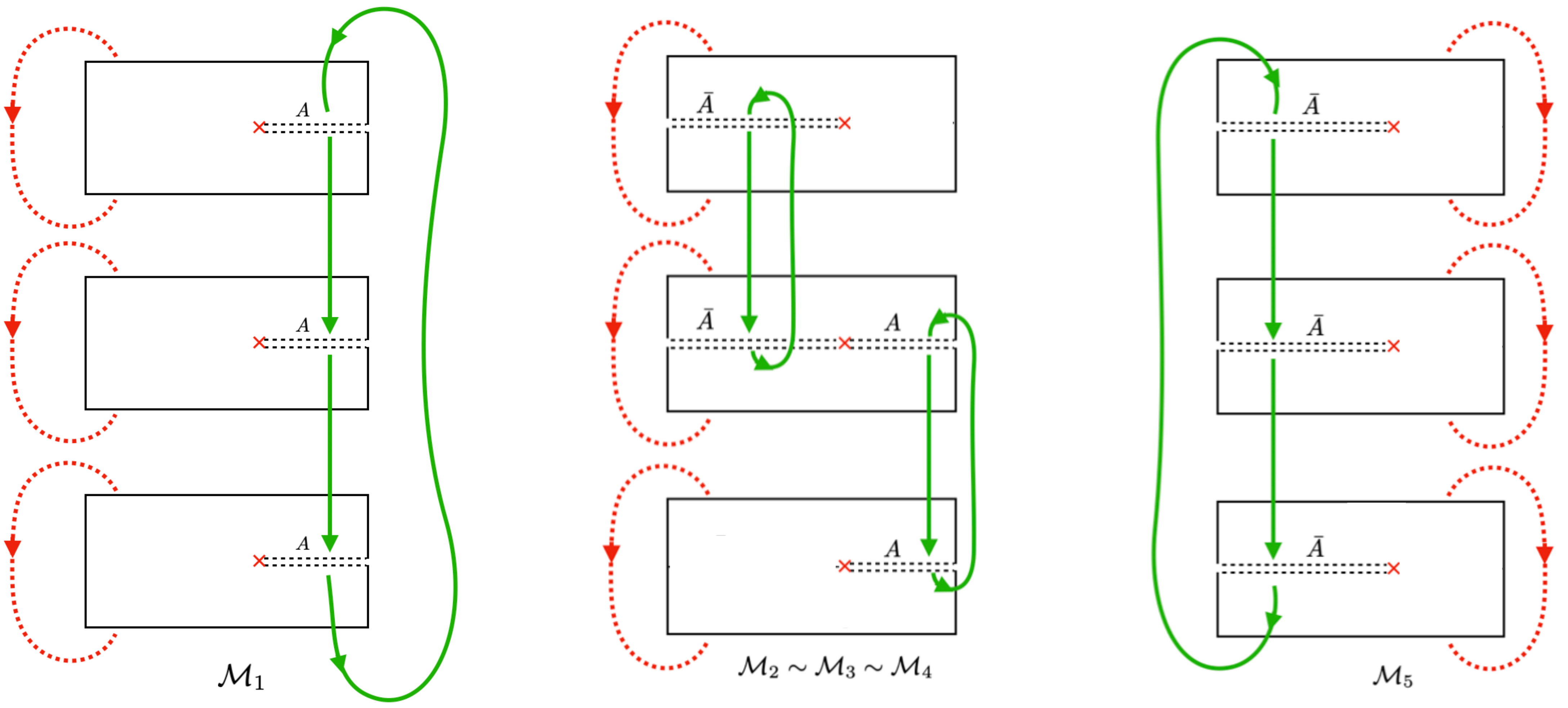
Example: n=3

In canonical representation, need to fill the following boundary manifolds:



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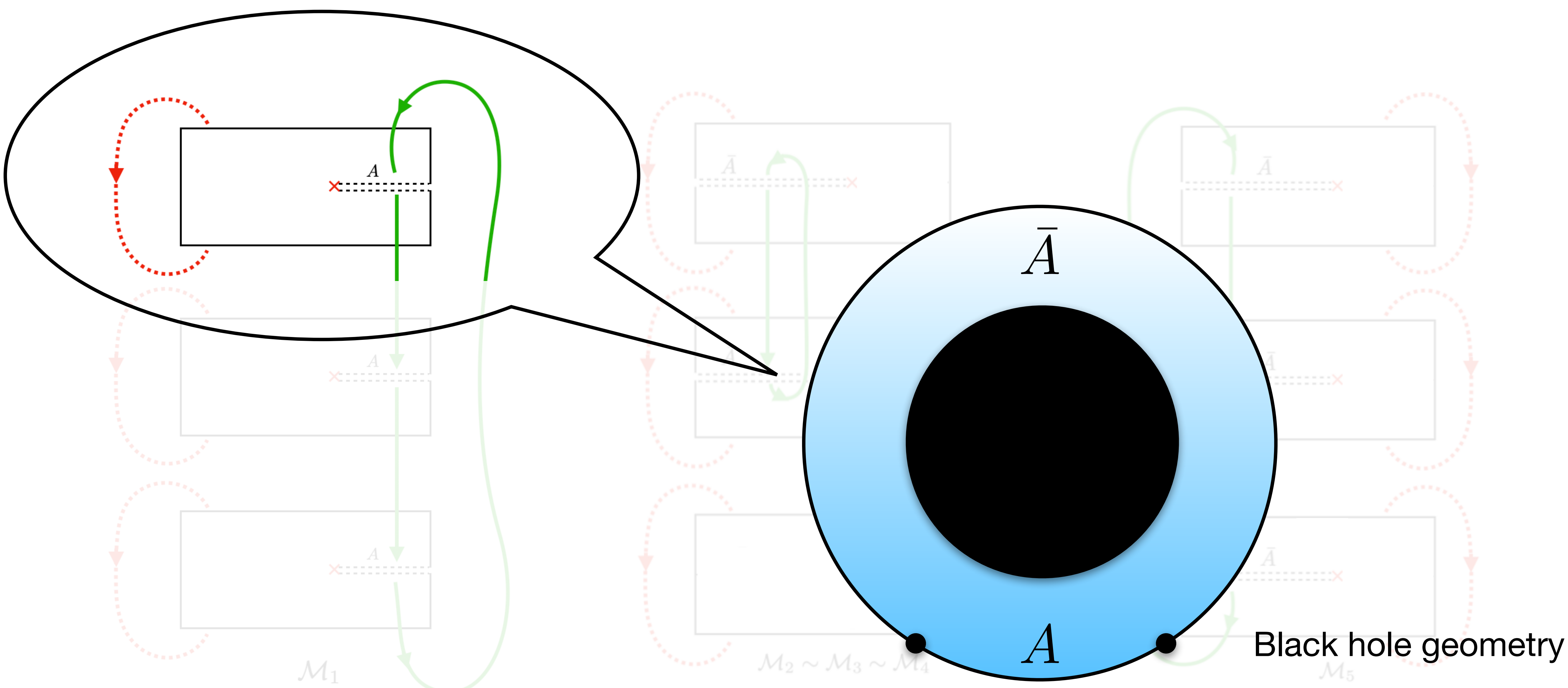

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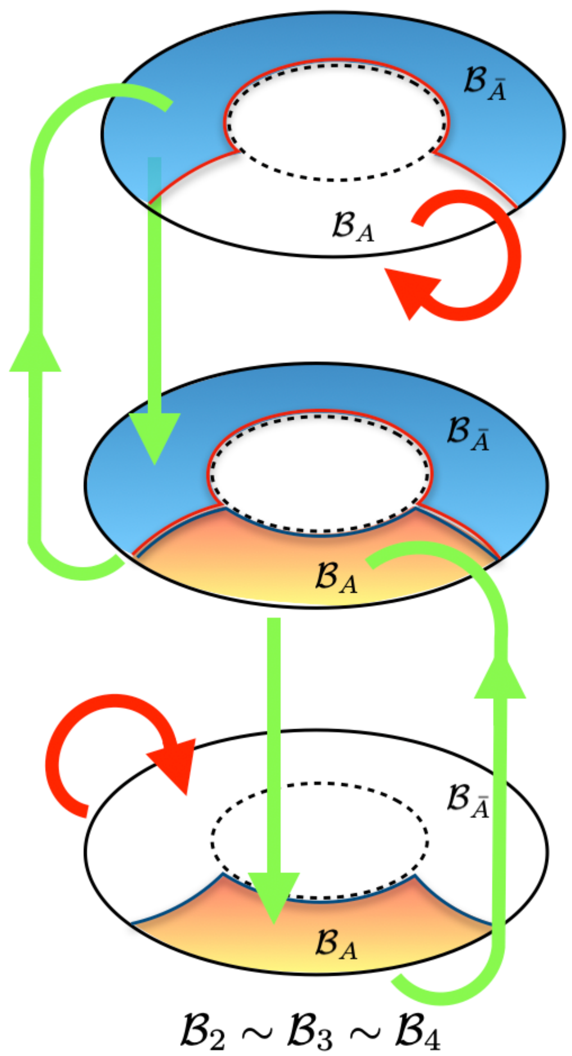


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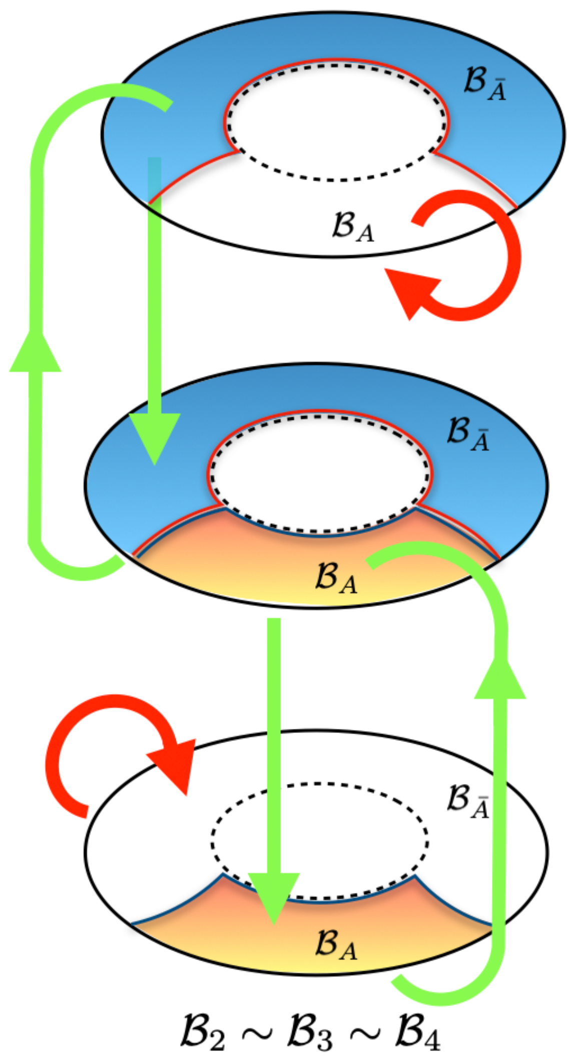


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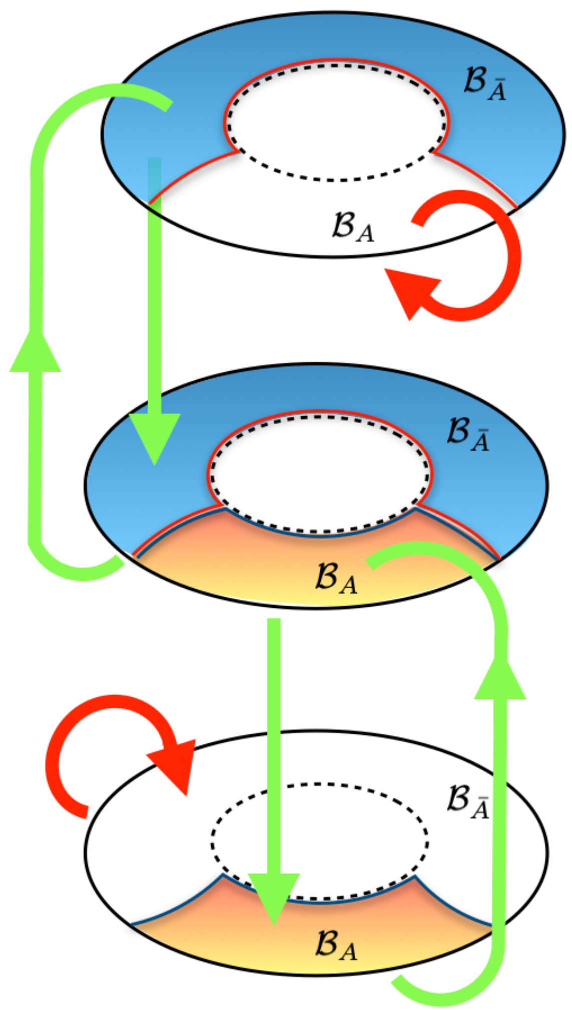


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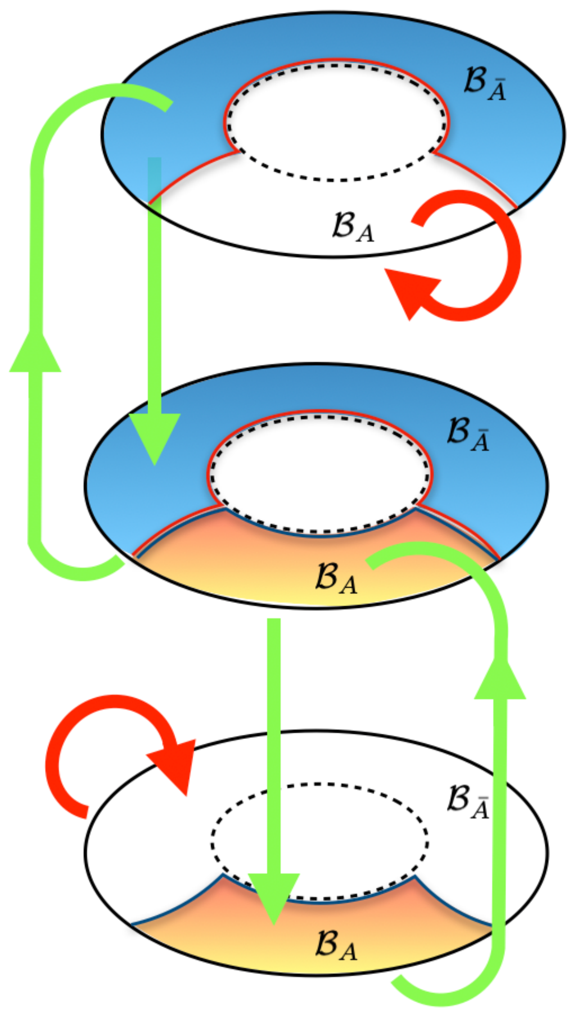
- We are interested in high temperature BH micro-states

# Constructing bulk saddle-points

Example: n=3

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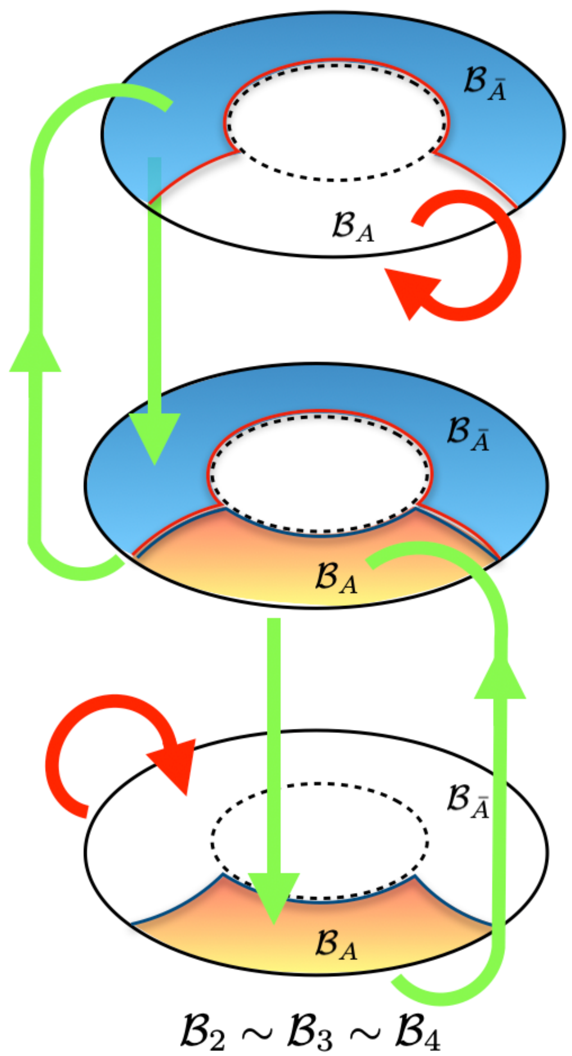
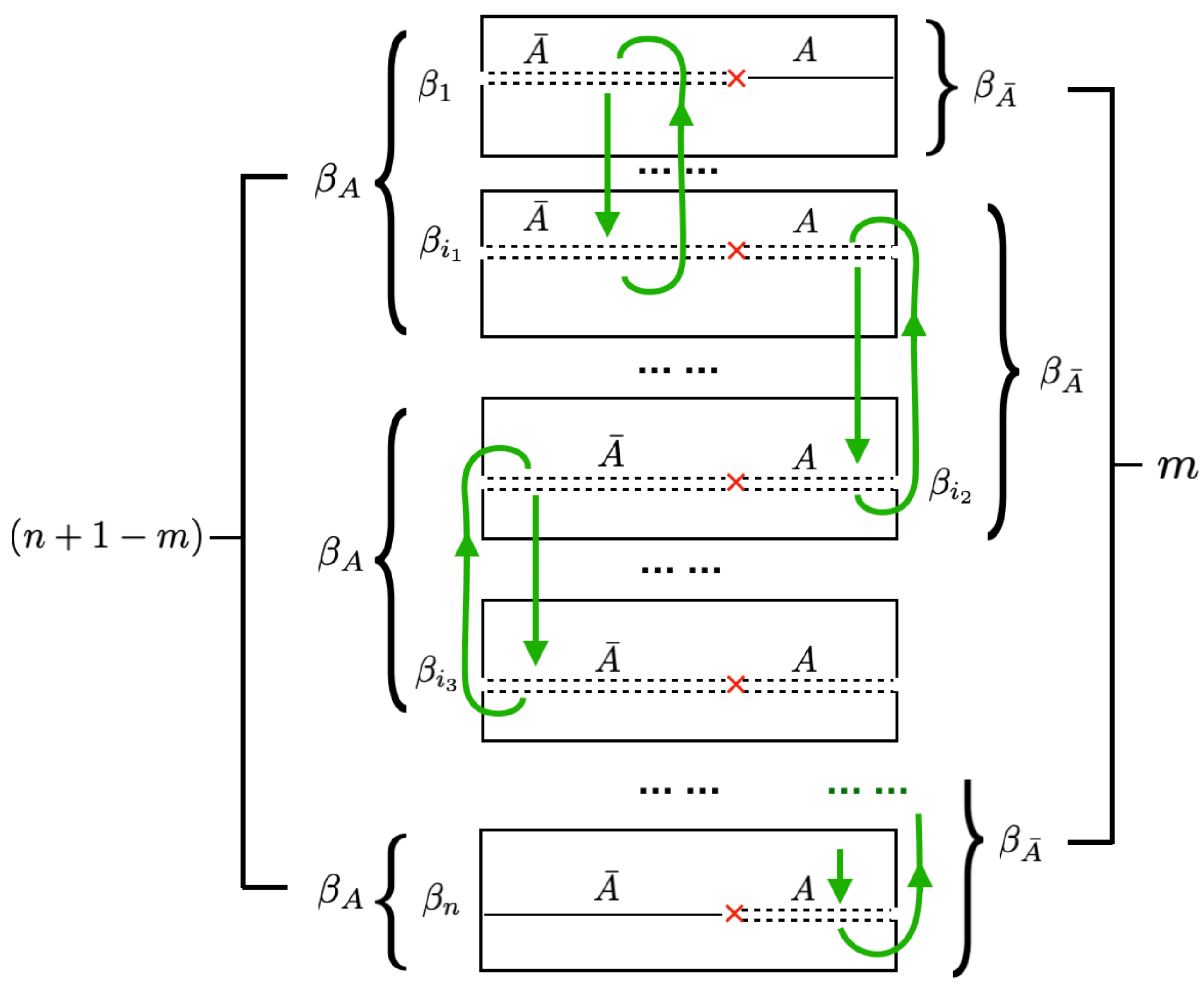



- In general, breaks replica symmetry
- Difficult to construct such bulk geometries
- We are interested in high temperature BH micro-states
- In this case, we can neglect features along the thermal circles and boundary phenomena near $\partial A$
- bulk solutions can be approximately constructed by simply gluing “segments” of black hole geometries, subject to matching conditions

# Constructing bulk saddle-points

**Recipe for a generic saddle-point for general $n$:**



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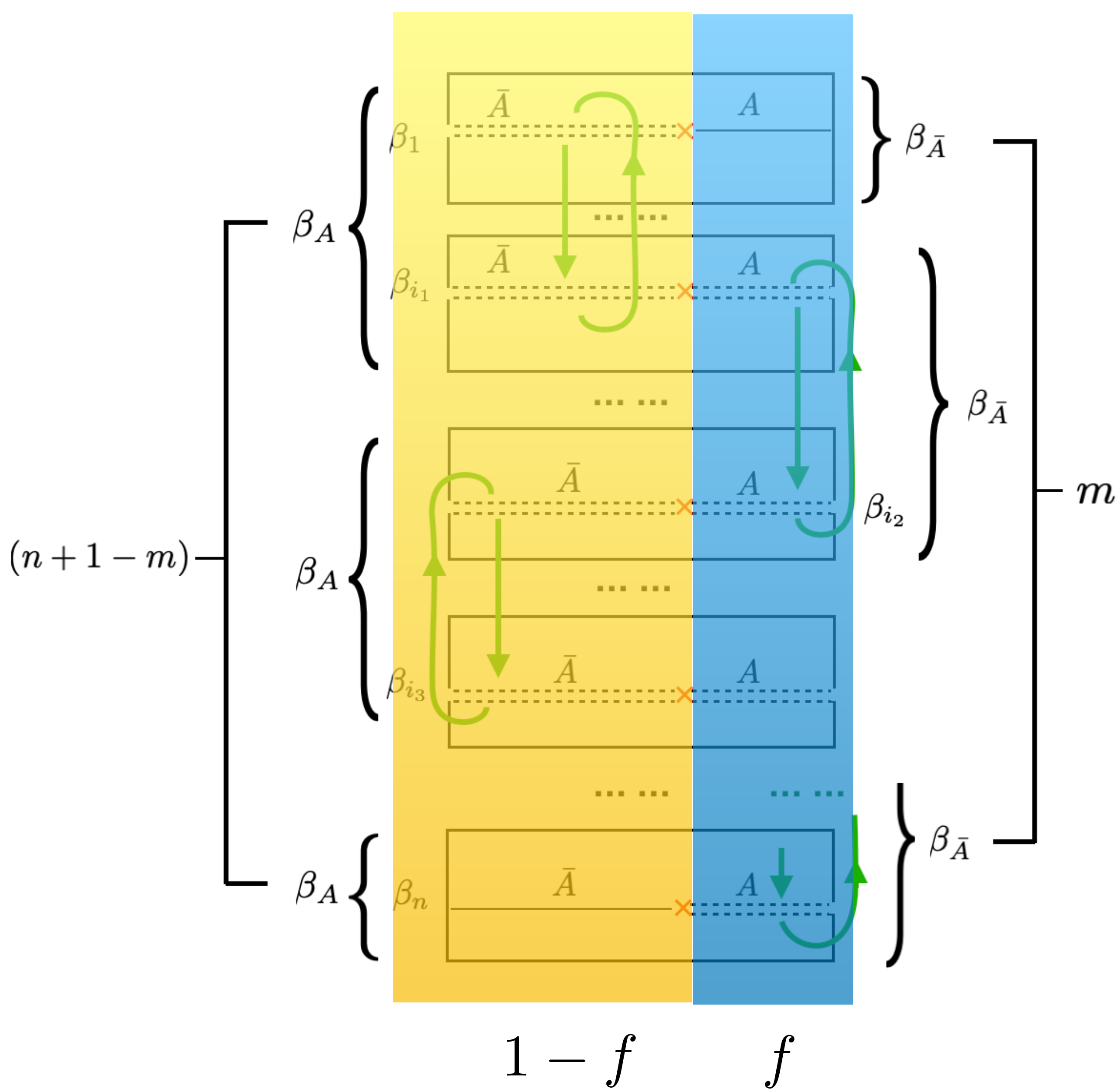



$\times(n+1-m)$

BH with temperature $\beta_A$

$\times m$

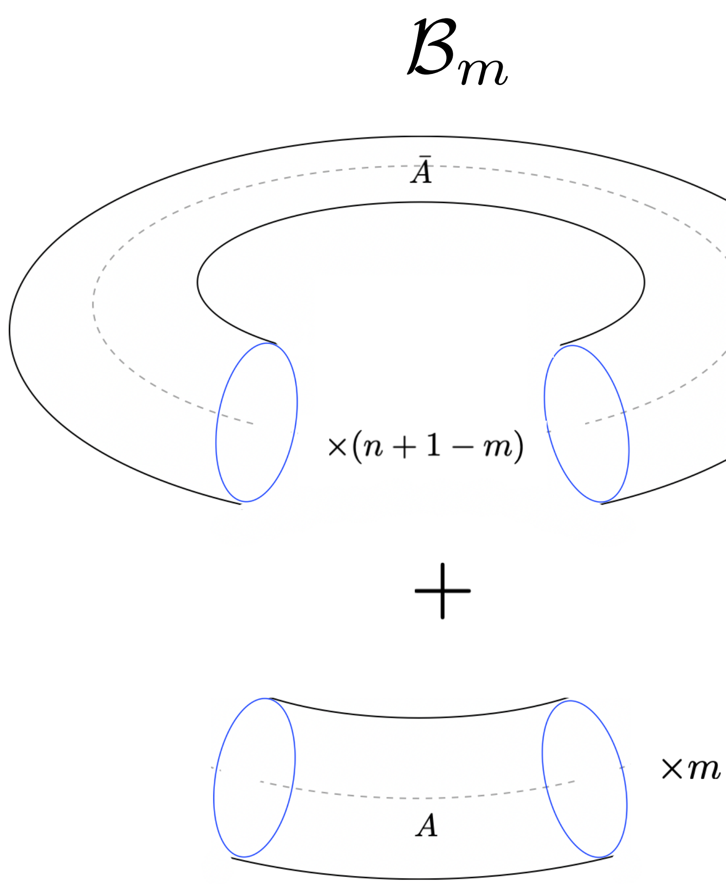
BH with temperature $\beta_{\bar{A}}$

$$f E_{\beta_A} + (1-f) E_{\beta_{\bar{A}}} = E$$

# Constructing bulk saddle-points

**Recipe for a generic saddle-point for general $n$ :**

Alternatively, the quotient of such a geometry can be obtained by ***double defect*** construction, similar to the cosmic brane construction

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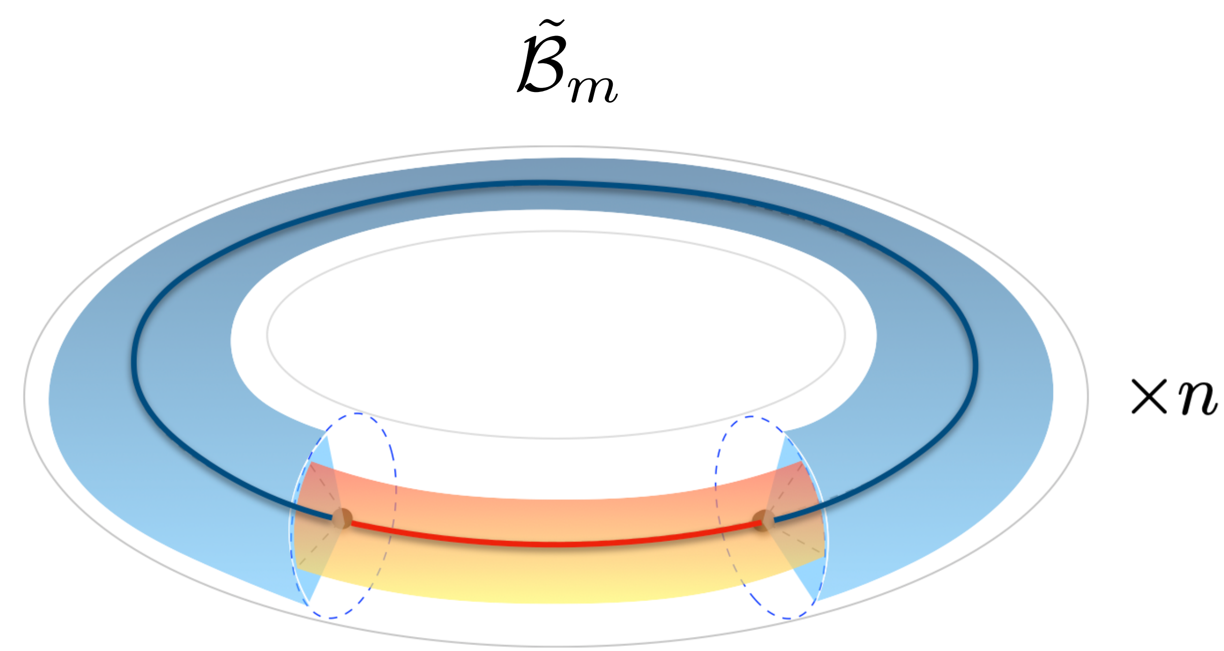



$$T_A = \frac{n-m}{4nG_N}, \quad T_{\bar{A}} = \frac{m-1}{4nG_M}$$

$\Sigma_A$ (red)
$\Sigma_{\bar{A}}$ (blue)

# Re-summing saddles: effective action for cosmic brane

$$Z_n = \sum_{m=1}^{n} d(m)\; Z_n\left(\mathcal{B}_m\right)$$

degeneracy from different gluing choices

$$Z_n(\tilde{\mathcal{B}}_m) = \int \mathcal{D}g\; \mathcal{D}\Sigma_{A,\bar{A}}\; e^{-S_{\rm bulk} - \sum_{i=A,\bar{A}} T_i S_{\rm brane}(\Sigma_i)}$$

$$S_{\rm brane}(\Sigma) = \int_\Sigma dy \sqrt{g}$$

"double defects"

$$T_A = \frac{n-m}{4nG_N}, \;\; T_{\bar{A}} = \frac{m-1}{4nG_M}$$

**Can re-sum over m explicitly:**

$$Z_n = \int \mathcal{D}_{g,\Sigma_{A,\bar{A}}} e^{-nS_{\rm bulk}} \sum_{m=1}^{n} d(m)\; e^{-\frac{n-m}{4G_N} S_{\rm brane}(\Sigma_A) - \frac{m-1}{4G_N} S_{\rm brane}(\Sigma_{\bar{A}})}$$

$$= \int \mathcal{D}_{g,\Sigma_{A,\bar{A}}} e^{-n\left(S_{\rm bulk} + S_{\rm eff}(\Sigma_{A,\bar{A}})\right)}$$

effective action for the cosmic branes

# Re-summing saddles: effective action for cosmic brane

**Effective action from re-summation:**

$$S_{\text{eff}} = \begin{cases} \frac{n-1}{4G_N} S_{\text{brane}}(\Sigma_A) - \ln\left({}_2\mathrm{F}_1\left[1-n, -n; 2; e^{\frac{\Delta I_{\text{brane}}}{4G_N}}\right]\right), & \Delta I_{\text{brane}} < 0 \\ \frac{n-1}{4G_N} S_{\text{brane}}(\Sigma_{\bar{A}}) - \ln\left({}_2\mathrm{F}_1\left[1-n, -n; 2; e^{-\frac{\Delta I_{\text{brane}}}{4G_N}}\right]\right), & \Delta I_{\text{brane}} > 0 \end{cases}$$

- "non-local" effective brane action from re-summing all saddles
- can analytically continue to non-integer n, and take replica limit.
- solve the dynamics of this effective brane action (in fixed-area basis)
- in high T limit, path-integral reduces to 1-dimensional:

$F_1(\mathcal{E})$

$$Z_n = \int d\mathcal{E}\, e^{nS_A(\mathcal{E}) + nS_{\bar{A}}(E-\mathcal{E}) - I_{\text{eff}}(S_A(\mathcal{E}), S_{\bar{A}}(E-\mathcal{E}), n)}$$

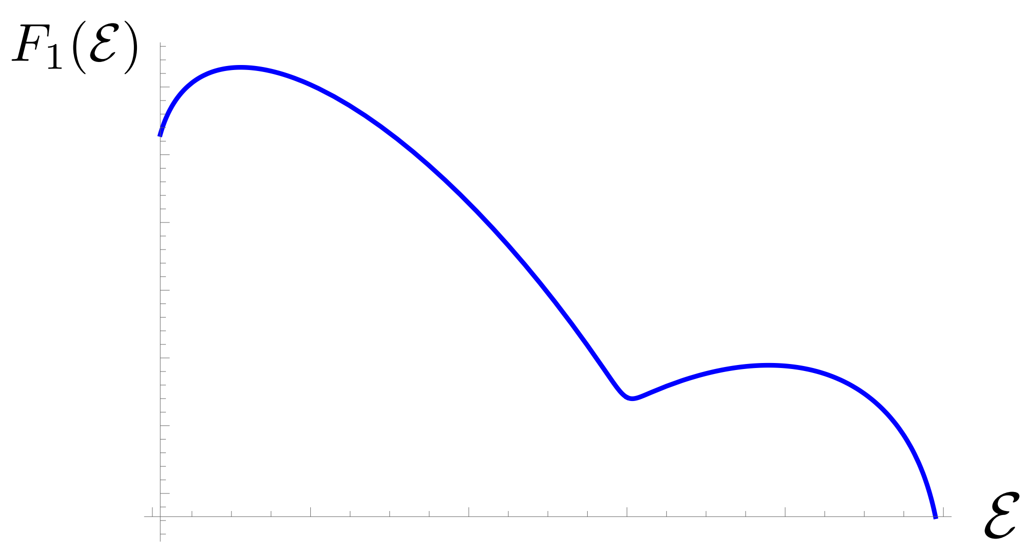
$e^{S_{A,\bar{A}}(x)}$ :subsystem density of states at subsystem energy x

## Outline:

- Renyi entropy for (chaotic) black hole micro-states
- Construct and re-sum bulk saddles: effective action
- Enhanced correction from effective action
- Discussions and outlooks

## Enhanced correction from effective action

- We can use compute the effective action approximately using stationary point




$$F_1 \propto VTc \gg 1$$

$$Z_n = \int d\mathcal{E}\; e^{F_1(\mathcal{E})} \propto \frac{e^{F_1(\mathcal{E}^*)}}{\sqrt{F_1''(\mathcal{E}^*)}},\; F_1'(\mathcal{E}^*) = 0$$

## Enhanced correction from effective action

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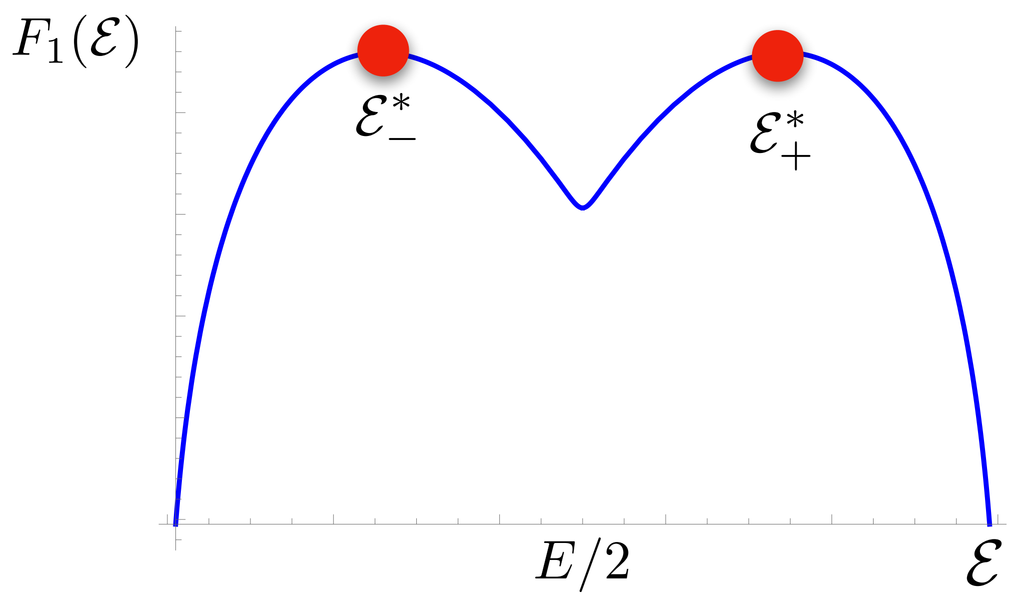
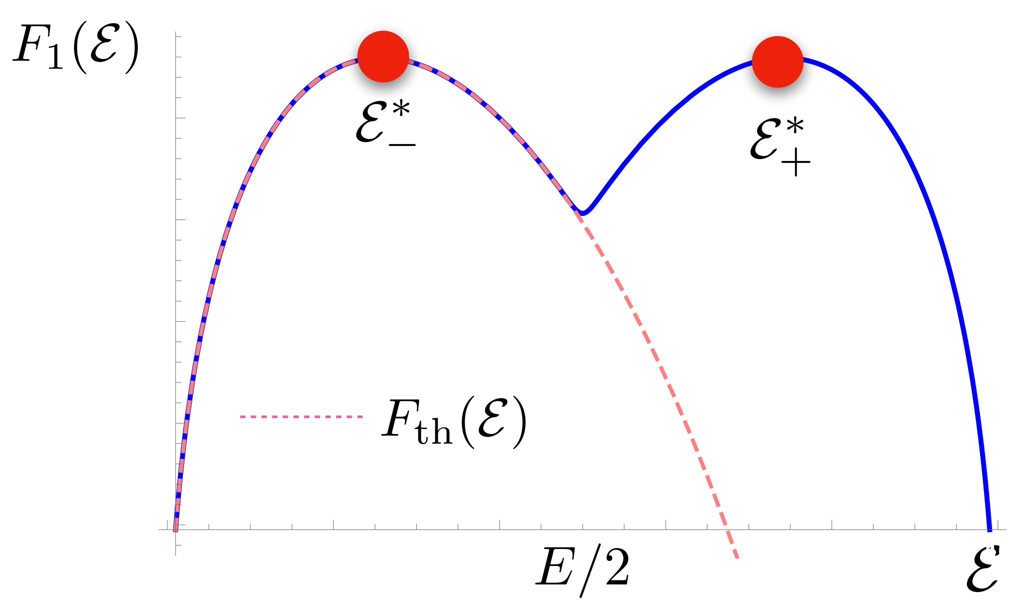
- At the transition $V_A = V_{\bar{A}}$ , $F_1$ is reflection symmetric about $E/2$



# Enhanced correction from effective action

- We can use compute the effective action approximately using stationary point
- At the transition $V_A = V_{\bar{A}}$ , $F_1$ is reflection symmetric about $E/2$
- For generic n>1, the integrand has two well-separated stationary points, can be approximated by two independent full Gaussian integrals

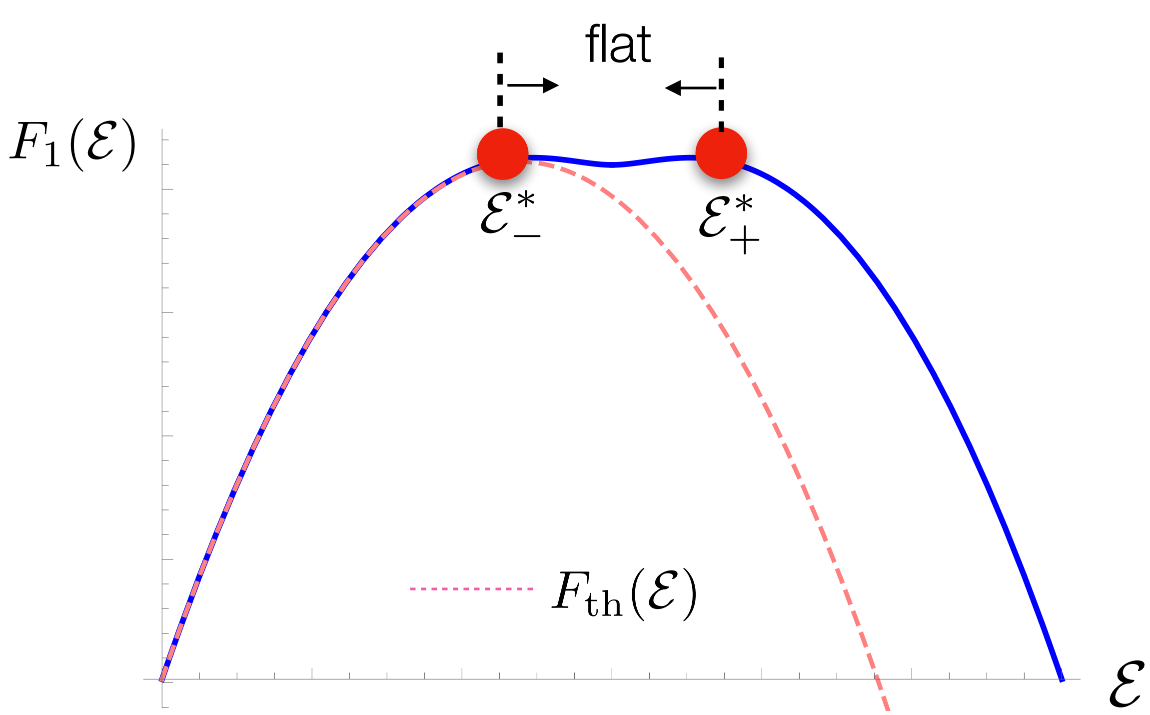



$$Z_n = \int d\mathcal{E}\; e^{F_1(\mathcal{E})} \approx 2 \times \frac{e^{F_1(\mathcal{E}^*_-)}}{\sqrt{F_1''(\mathcal{E}^*_-)}}$$

$$S_n^E(A) \approx S_n^{\rm th}(A) + \frac{\ln 2}{1-n}$$

# Enhanced correction from effective action

- As n->1, the two stationary points collide, a “flat direction” emerges in-between



## Enhanced correction from effective action

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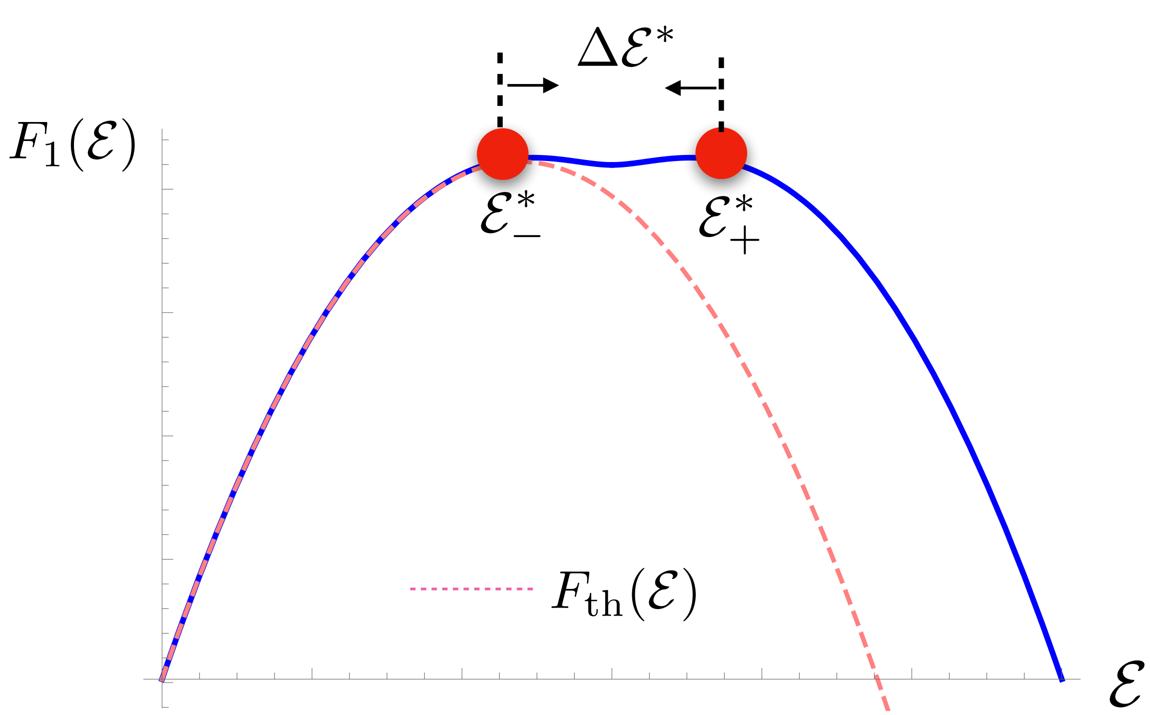


$$\Delta\mathcal{E}^* = \frac{V}{4}\frac{s'(E/V)}{s''(E/V)}\delta + \mathcal{O}(\delta^2)$$

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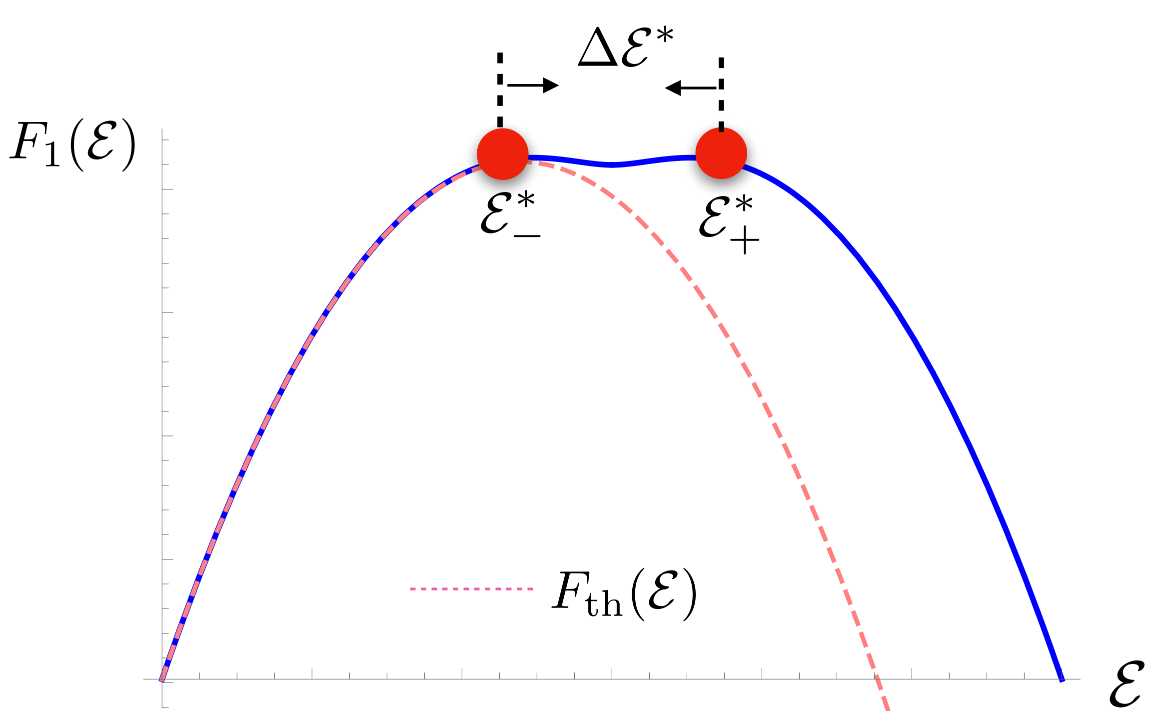


$$Z_n \approx \frac{e^{F_1(\mathcal{E}^*_-)}}{\sqrt{F_1''(\mathcal{E}^*_-)}}\left(1+\sqrt{F_1''(\mathcal{E}^*_-)}\Delta\mathcal{E}^*\right)$$

## Enhanced correction from effective action

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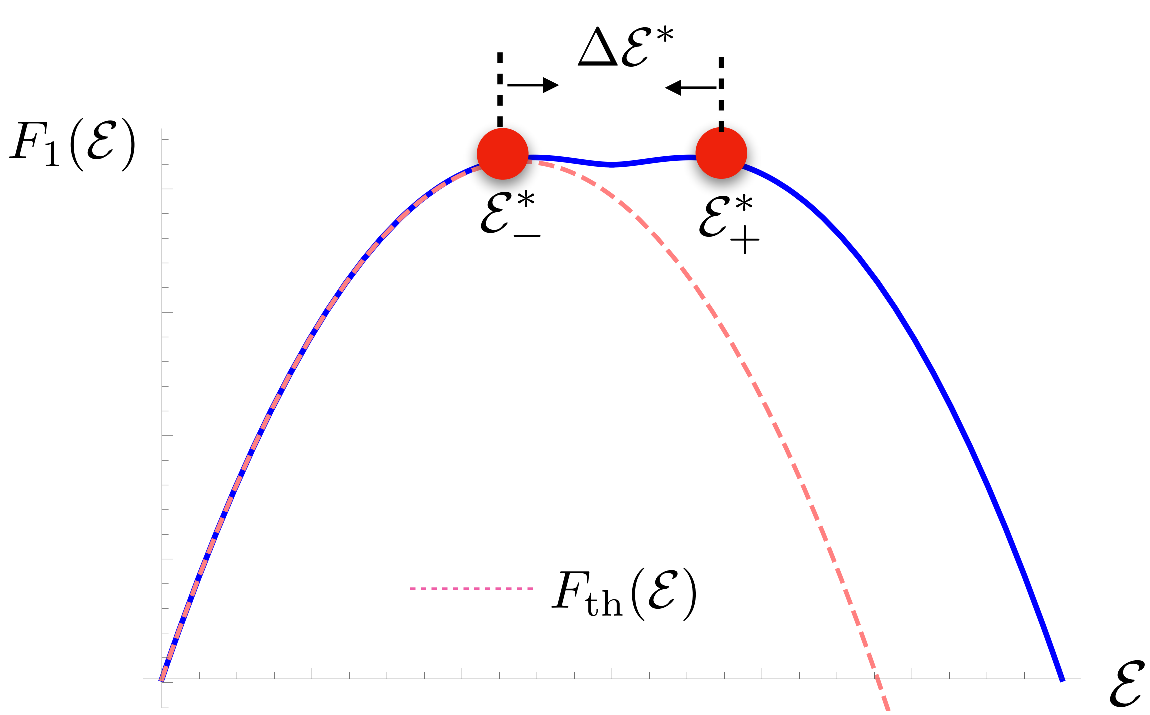
- The flat segment results in an enhancement compared to the Gaussian approximation
- In the replica limit n->1, $S_A = \lim_{n\to 1} \frac{\ln Z_n}{1-n}$ gives the enhanced correction to entanglement entropy




$$S_A^E = S_A^{\rm th} - \sqrt{\frac{C_V}{2\pi}}, \quad C_V \propto VTc$$

## Outline:

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## Discussions and Outlooks

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*could dynamically produce both sides of the transition, but would miss the important phenomena of enhanced correction at the transition.*

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- Ok, what exactly do they do at the transition?

*It is shown that one can get the enhanced correction if imposing by hand the ansatz (D. Marolf, et al, 2020):*

$$Z_n \approx \int d\mathcal{E}\ e^{-\mathrm{Min}\{S_E(\mathcal{B}_1,\mathcal{E}),S_E(\mathcal{B}_n,\mathcal{E})\}}$$

*which is different from* $\bigstar$*. Including the replica non-symmetric saddles dynamically implement this ansatz with small deviations.*

## Discussions and Outlooks

Questions for the future:

- More dynamics of the non-local effective action: interaction between brane defects?
- “flat segment”: exchanging soft modes between brane defects?
- Are cosmic branes more than auxiliary tools? Do they have intrinsic dynamics, including non-perturbative effects?
- How do these effects manifest without averaging over randomness?
- Enhanced corrections at other entanglement transitions, e.g. two-interval entanglements in AdS3/CFT2.

# Thank you!