Some non-singular and non-perturbative results induced by α' corrections in string theory

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Based on joint work with Peng Wang, Houwen Wu and Shuxuan Ying

arXiv:1906.09650 (PhysRevD.100.046016,2019), 1909.00830 (JHEP10(2019)263), 1910.05808 and works in preparation.

Motivation and Background

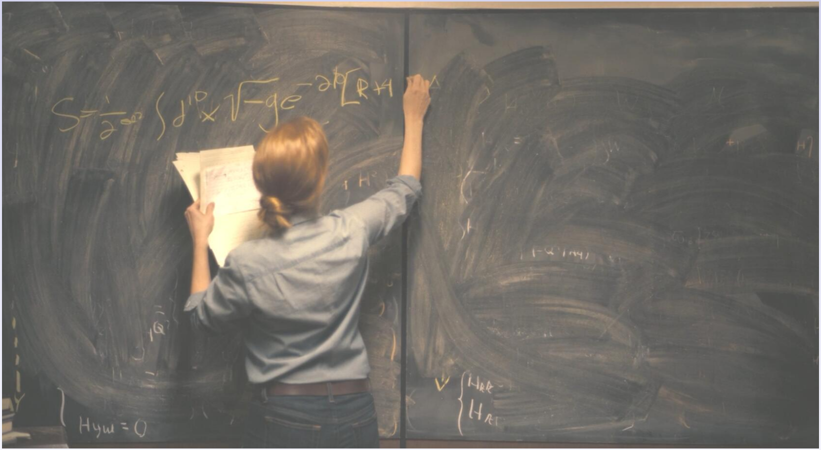

Figure: Interstellar: Murphy wrote down the gravity theory based on the information sent by Cooper from the black hole.

Motivation and Background

For bosonic closed string theory, the tree level string effective action of the massless sector is

$$I_0 = \int d^{d+1}x\sqrt{-g}e^{-2\phi}(R + 4(\partial\phi)^2 - \frac{1}{12}\mathcal{H}^2),$$

where ϕ is the dilaton and $\mathcal{H}_{\mu\nu\rho} = 3\partial_{[\mu}b_{\nu\rho]}$ is the field strength of the antisymmetric Kalb-Ramond field $b_{\mu\nu}$. For simplicity, we always set $b_{\mu\nu} = 0$ here. Many non-perturbative progresses in last two decades are based on this (SUSY-) action.

There are some long-standing unsolved problems:

- No de Sitter (dS) or anti-de Sitter (AdS) vacua (different from Einstein gravity).
- The big-bang singularity (same as Einstein gravity).
- $\cdots$

The first one leads to severe harms to the foundation of string theory itself and many popular applications such as (Bosonic) AdS/CFT, AdS/QCD, AdS/CMT, cosmology...

Motivation and Background

To see the problems clearer, we focus on the FLRW background

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j,$$

Define

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{d}{dt}\log a(t) \,(\text{Hubble parameter}), \quad e^{-\Phi} = \sqrt{-g}e^{-2\phi} \,(O(d,d) \text{ dilaton})$$

The EOM (Friedmann equations) are

$$\begin{aligned} \dot{H} - \dot{\Phi}H &= 0, \\ \dot{\Phi}^2 - dH^2 &= 0, \\ \ddot{\Phi} - dH^2 &= 0, \end{aligned}$$

There is a remarkable duality, the scale-factor duality:

$$\begin{aligned} a(t) &\rightarrow \frac{1}{a(t)} \Longleftrightarrow H(t) \rightarrow -H(t), \\ \Phi(t) &\rightarrow \Phi(t) \Longleftrightarrow \phi(t) \rightarrow \phi(t) - \log\sqrt{-g}, \end{aligned}$$

which turns out to be a special case of a more general symmetry: $O(d,d)$ symmetry.

$$ds^2 = -dt^2 + e^{2\int^t H(\tau)d\tau}\delta_{ij}\,dx^i dx^j,$$

The EOM

$$\begin{aligned} \dot{H} - \dot{\Phi}H &= 0, \\ \dot{\Phi}^2 - dH^2 &= 0, \\ \ddot{\Phi} - dH^2 &= 0, \end{aligned}$$

- To have dS vacua, there must be $H(t) = H_0 > 0$, which is not possible from the EOM.
- The evolutionary solutions are

$$H_\pm(t) = \frac{\dot{a}_\pm}{a_\pm} = \pm\frac{1}{\sqrt{d}\,|t|}, \qquad \Phi = -\ln\left|\frac{t}{t_0}\right|,$$

singular at $t = 0$.

Note the scale-factor duality combined with time reversal $t \to -t$ introduces a pre-bing-bang phase, which is different from the Einstein cosmology.

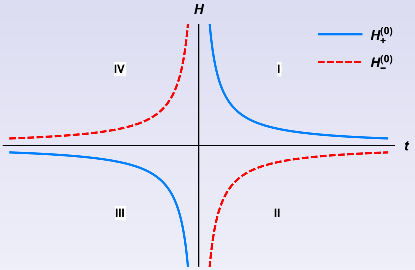

Figure: The evolutions of the Hubble parameters of four solutions (we set $d = 3$ in this plot).

How to resolve these problems?

Adding matter sources? compactification? quantization? ....?

(!!) Do not forget the tree level theory

$$I_0 = \int d^{d+1}x\sqrt{-g}e^{-2\phi}(R + 4(\partial\phi)^2 - \frac{1}{12}\mathcal{H}^2),$$

is valid only in the perturbative regime:

$$g_s = e^{2\phi} << 1 \quad \text{and} \quad |R|\alpha' << 1$$

- The first condition $g_s = e^{2\phi} << 1$ concerns quantum/loop/topology corrections.
- Since $\alpha' \sim \ell^2_{\text{string}}$, the second condition $|R|\alpha' << 1$ concerns the classical stringy correction. This means we have not really included "string" effects!

Motivation and Background

Beyond the perturbative regime, the tree level string effective action receives two kinds of corrections:

- *Classical* stringy effects, namely the higher-derivative expansion, controlled by α'.
- *Quantum* loop corrections, controlled by the string coupling $g_s = e^{2\phi}$.

Ignoring matter sources, the most general perturbative form of the string effective action has the following structure

$$\begin{aligned}
I &= \int d^{d+1}x\sqrt{-g}e^{-2\phi}\Big\{ \\
&\quad \Big[(R+4(\partial\phi)^2-\frac{1}{12}\mathcal{H}^2)+\frac{\alpha'}{4}(R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}+\cdots)+\mathcal{O}(\alpha'^2)\Big] \\
&+ e^{2\phi}\Big[(c^1_R R+c^1_\phi(\partial\phi)^2+c^1_{\mathcal{H}}\mathcal{H}^2)+\alpha'(c^1_{\alpha' R}R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}+\cdots)+\mathcal{O}(\alpha'^2)\Big] \\
&+ e^{4\phi}\Big[(c^2_R R+c^2_\phi(\partial\phi)^2+c^2_{\mathcal{H}}\mathcal{H}^2)+\alpha'(c^2_{\alpha' R}R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}+\cdots)+\mathcal{O}(\alpha'^2)\Big] \\
&+ \cdots\Big\},
\end{aligned}$$

with unknown $c^i_{[\cdots]}$.

The loop (quantum) corrections have no help on the vacuum (classical) problem, but indeed could smooth out the singularity, by implementing some non-local dilaton potentials.

However, there is not much progress with α' corrections,

$$I = \int d^{d+1}x\sqrt{-g}e^{-2\phi}\Big[(R + 4(\partial\phi)^2 - \frac{1}{12}\mathcal{H}^2) + \frac{\alpha'}{4}(R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} + \cdots) + \mathcal{O}(\alpha'^2)\Big]$$

The main reason is that, the higher-derivative α' corrections usually would change the order of the differential equations in the equations of motion (EOM). At the tree level, the EOM are second order differential equations; at the first order in α', the EOM become fourth order differential equations; and so on.

It seems hopeless...

The Hohm-Zwiebach action and dS/AdS vacua

It is well known that for FLRW metric, the tree level action can be recast in an $O(d,d)$ covariant form [Veneziano 1991]. To this end, it is convenient to choose the gauge $b_{0i}=0$ and write the fields in the form

$$g_{\mu\nu}=\left(\begin{array}{cc} -1 & 0 \\ 0 & G_{ij}(t) \end{array}\right), \qquad b_{\mu\nu}=\left(\begin{array}{cc} 0 & 0 \\ 0 & B_{ij}(t) \end{array}\right).$$

The action can be rewritten as

$$I_0=\int dte^{-\Phi}\left[-\dot{\Phi}^2-\frac{1}{8}\mathrm{Tr}\left(\dot{\mathcal{S}}^2\right)\right],$$

where $\mathcal{S}$ is the standard form of $O(d,d)$ matrix

$$\mathcal{S}=\left(\begin{array}{cc} BG^{-1} & G-BG^{-1}B \\ G^{-1} & -G^{-1}B \end{array}\right),$$

This action is manifestly invariant under the $O(d,d)$ transformations

$$\Phi\longrightarrow\Phi, \qquad \mathcal{S}\longrightarrow\tilde{\mathcal{S}}=\Omega^T\mathcal{S}\Omega,$$

where Ω is a constant matrix, satisfying

$$\Omega^T\eta\,\Omega=\eta, \qquad \eta=\left(\begin{array}{cc} 0 & I \\ I & 0 \end{array}\right).$$

η is the invariant metric of the $O(d,d)$ group.

The Hohm-Zwiebach action and dS/AdS vacua

Sen [1991,1992] proved that, to all orders in α', for configurations independent of m coordinates, the action possesses an $O(m,m)$ symmetry. In particular for FLRW metric which depends on t only, the symmetry is $O(d,d)$. The standard form of $O(d,d)$ matrix receives higher order corrections.

$$\mathcal{S}_{corrected} = \begin{pmatrix} BG^{-1} & G - BG^{-1}B \\ G^{-1} & -G^{-1}B \end{pmatrix} + \alpha' \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} + \mathcal{O}(\alpha'^2),$$

Meissner [1996] demonstrated that to the first order in α', the $O(d,d)$ matrix can maintain the standard form in term of α' corrected fields,

$$I_1 = \int dt e^{-\Phi} \left\{ -\dot{\Phi}^2 - \frac{1}{8}\mathrm{Tr}\dot{\mathcal{S}}^2 + \alpha'\lambda_0 \left[\frac{1}{16}\mathrm{Tr}\dot{\mathcal{S}}^4 + \frac{1}{96}\left(\mathrm{Tr}\dot{\mathcal{S}}^2\right)^2 \right] \right\}.$$

The Hohm-Zwiebach action and dS/AdS vacua

Based on a reasonable assumption that, to all orders in α', the standard $O(d,d)$ matrix can be maintained by field redefinitions, Hohm and Zwiebach [Hohm:2019ccp, Hohm:2019jgu] proved that, for FLRW background

$$
\begin{aligned}
I &= \int d^D x\sqrt{-g}e^{-2\phi}\left(R+4\left(\partial\phi\right)^2+\frac{1}{4}\alpha'\left(R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}+\ldots\right)+\alpha'^2(\ldots)+\ldots\right),\\
&= \int dte^{-\Phi}\left(-\dot{\Phi}^2+\sum_{k=1}^{\infty}\left(\alpha'\right)^{k-1}c_k\mathrm{tr}\left(\dot{\mathcal{S}}^{2k}\right)\right)\\
&= \int dte^{-\Phi}\left(-\dot{\Phi}^2+g(H)-Hf(H)\right),
\end{aligned}
$$

where $b_{\mu\nu}=0$ and

$$
\begin{aligned}
f\left(H\right) &= d\sum_{k=1}^{\infty}\left(-\alpha'\right)^{k-1}2^{2(k+1)}kc_kH^{2k-1}=-2dH-2d\alpha' H^3+\mathcal{O}\left(\alpha'^2\right),\\
g\left(H\right) &= Hf(H)-\int_0^H dx f(x)=-dH^2-\frac{3}{2}d\alpha' H^4+\mathcal{O}\left(\alpha'^2\right).
\end{aligned}
$$

Hitherto, we only know $c_1=-\frac{1}{8}$ and $c_2=\frac{1}{64}$ for the bosonic string theory ($c_2=\frac{1}{128}$ for heterotic string and $c_2=0$ for type II strings) and $c_{k\geq 3}$ are undetermined.

The Hohm-Zwiebach action and dS/AdS vacua

$$
\begin{aligned}
f\left(H\right) &= d\sum_{k=1}^{\infty}\left(-\alpha'\right)^{k-1}2^{2(k+1)}kc_k H^{2k-1} = -2dH - 2d\alpha' H^3 + \mathcal{O}\left(\alpha'^2\right), \\
g\left(H\right) &= d\sum_{k=1}^{\infty}\left(-\alpha'\right)^{k-1}2^{2k+1}\left(2k-1\right)c_k H^{2k} = -dH^2 - \frac{3}{2}d\alpha' H^4 + \mathcal{O}\left(\alpha'^2\right).
\end{aligned}
$$

The EOM of the Hohm-Zwiebach action are given by

$$
\begin{aligned}
\ddot{\Phi} + \frac{1}{2}Hf\left(H\right) &= 0, \\
\dot{\Phi}^2 + g\left(H\right) &= 0, \\
\frac{d}{dt}\left(e^{-\Phi}f\left(H\right)\right) &= 0,
\end{aligned}
$$

Now, if $f(H_0) = g(H_0) = 0$ have solutions for some $H_0 > 0$, there are dS vacua,

$$
ds^2 = -dt^2 + e^{2H_0 t}dx_i^2, \qquad \Phi = \Phi_0.
$$

The Hohm-Zwiebach action and dS/AdS vacua

For instance, if

$$
\begin{aligned}
f(H) &= -\frac{2d}{\sqrt{\alpha'}}\sin(\sqrt{\alpha'}H), \\
g(H) &= Hf(H) - \int_0^H dx f(x) = -\frac{2dH}{\sqrt{\alpha'}}\sin(\sqrt{\alpha'}H) + \frac{2d}{\alpha'}(1-\cos(\sqrt{\alpha'}H))
\end{aligned}
$$

dS vacua are

$$
H_0 = \frac{1}{\sqrt{\alpha'}}2n\pi, \qquad n > 0,
$$

which is non-perturbative since $H_0 \sim \frac{1}{\sqrt{\alpha'}}$ and cannot be obtained unless all α' corrections are included.

To investigate if nonperturbative AdS vacua are also allowed, an appropriate ansatz is crucial. The usual AdS metric form would make the derivation too complicated. So, We take the ansatz [arXiv:1906.09650]

$$ds^2 = dx^2 + a^2(x)\left(-dt^2 + dy^2 + dz^2 + \ldots\right).$$

We proved for this metric, Meissner's argument also applies and

$$\begin{aligned} I &= \int d^D x \sqrt{-g} e^{-2\phi}\left(R + 4(\partial\phi)^2 + \frac{1}{4}\alpha'\left(R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} + \ldots\right) + \alpha'^2(\ldots) + \ldots\right), \\ &= \int dx e^{-\Phi}\left(\Phi'^2 + \bar{g}(\bar{H}) - \bar{H}\bar{f}(\bar{H})\right), \end{aligned}$$

$$\begin{aligned} \bar{H}(x) &= \frac{a'(x)}{a(x)}, \\ \bar{f}(\bar{H}) &= d\sum_{k=1}^{\infty}\left(-\alpha'\right)^{k-1} 2^{2(k+1)} k\bar{c}_k \bar{H}^{2k-1}, \\ \bar{g}(\bar{H}) &= d\sum_{k=1}^{\infty}\left(-\alpha'\right)^{k-1} 2^{2k+1}(2k-1)\bar{c}_k \bar{H}^{2k}. \end{aligned}$$

It turns out that

$$\bar{c}_{2k-1} = c_{2k-1}, \qquad \bar{c}_{2k} = -c_{2k}, \qquad \text{for} \quad k = 1, 2, 3\ldots$$

The Hohm-Zwiebach action and dS/AdS vacua

The EOM are

$$\begin{aligned}\Phi'' + \frac{1}{2}\bar{H}\bar{f}\left(\bar{H}\right) &= 0,\\ \frac{d}{dx}\left(e^{-\Phi}\bar{f}\left(\bar{H}\right)\right) &= 0,\\ (\Phi')^2 + \bar{g}\left(\bar{H}\right) &= 0,\end{aligned}$$

So, if $\bar{f}(\bar{H}_0) = \bar{g}(\bar{H}_0) = 0$ have solutions for some $\bar{H}_0 > 0$, there are AdS vacua,

$$ds^2 = dx^2 + e^{2\bar{H}_0 x}\left(-dt^2 + dy^2 + dz^2 + \ldots\right).$$

To see this more clearly, we apply the transformation $x \to -\log[\bar{H}_0\xi]/\bar{H}_0$ and recover the familiar Poincare coordinate

$$ds^2 = \frac{1/\bar{H}_0^2}{\xi^2}\big(-dt^2 + d\xi^2 + dy^2 + dz^2 + \ldots\big).$$

An intriguing fact is that, since

$$\bar{c}_{2k-1} = c_{2k-1}, \qquad \bar{c}_{2k} = -c_{2k}, \qquad \text{for} \quad k = 1, 2, 3 \ldots$$

if in dS case $f(H) \sim \sin(\sqrt{\alpha'}H)$, there must be $\bar{f}(\bar{H}) \sim \sinh(\sqrt{\alpha'}\bar{H})$ in AdS case, or vice versa. But the $sinh$ function has no nontrivial zero. So, for this trial function, AdS or dS vacua cannot coexist and only one of them survives.
This looks like merely a coincidence. But we have some reasons to conjecture that by plugging the dS (AdS) metric into the yet unknown infinite α' expansion, one could sum the series into an expression including a factor that is very close to this trial function.
To see this explicitly, in [arXiv:1703.05217], we showed that, for the nonlinear sigma model of string theory

$$S = -\frac{1}{4\pi\alpha'} \int_{\Sigma} g_{ij}(X) \partial_\alpha X^i \partial^\alpha X^j,$$

we can expand X^i at some point $\bar{x}$, say, $X^i(\tau, \sigma) = \bar{x}^i + \sqrt{\alpha'}\mathbb{Y}^i(\tau, \sigma)$, where the $\mathbb{Y}^i$'s are dimensionless fluctuations. Locally around any point, one can always pick the Riemann Normal Coordinates (RNC)

$$\begin{aligned} g_{ij}(X) \quad &= \quad \eta_{ij} + \frac{\ell_s^2}{3} R_{iklj} \mathbb{Y}^k \mathbb{Y}^l + \frac{\ell_s^3}{6} D_k R_{ilmj} \mathbb{Y}^k \mathbb{Y}^l \mathbb{Y}^m \\ &\quad + \frac{\ell_s^4}{20} \left(D_k D_l R_{imnj} + \frac{8}{9} R_{iklp} R^p{}_{mnj} \right) \mathbb{Y}^k \mathbb{Y}^l \mathbb{Y}^m \mathbb{Y}^n + \ldots . \end{aligned}$$

Mutual exclusion of dS and AdS vacua?

When the background is maximally symmetric, the expansion is greatly simplified and can be summed into a closed form. For dS, we have

$$S_{dS} = -\frac{1}{4\pi}\int_\Sigma \partial\mathbb{Y}^i\partial\mathbb{Y}^j \left[\frac{\sin^2\left(\frac{\sqrt{\alpha'}}{R_{dS}}\mathbb{W}\right)}{\left(\frac{\sqrt{\alpha'}}{R_{dS}}\mathbb{W}\right)^2}\right]^a{}_i\,\eta_{aj}\,, \qquad \left(\mathbb{W}^2\right)^a{}_b \equiv \delta^a_b\mathbb{Y}^2 - \mathbb{Y}^a\mathbb{Y}_b.$$

If the background is AdS, we get

$$S_{AdS} = -\frac{1}{4\pi}\int_\Sigma \partial\mathbb{Y}^i\partial\mathbb{Y}^j \left[\frac{\sinh^2\left(\frac{\sqrt{\alpha'}}{R_{AdS}}\mathbb{W}\right)}{\left(\frac{\sqrt{\alpha'}}{R_{AdS}}\mathbb{W}\right)^2}\right]^a{}_i\,\eta_{aj}\,, \qquad \left(\mathbb{W}^2\right)^a{}_b \equiv \delta^a_b\mathbb{Y}^2 - \mathbb{Y}^a\mathbb{Y}_b.$$

Noting that $H_0 \sim 1/R_{dS}$ and $\bar{H}_0 \sim 1/R_{AdS}$, the results strongly suggest that the beta functions or EOMs of these two actions S_{dS} and S_{AdS} may behave very similarly to $f\left(H\right) \sim \sin\left(\sqrt{\alpha'}H\right)$ and $\bar{f}\left(\bar{H}\right) \sim \sinh\left(\sqrt{\alpha'}\bar{H}\right)$, or, equivalently speaking, there are nonperturbative dS vacua but not nonperturbative AdS vacua, or vice versa.

Resolution of the singularity

The Hohm-Zwiebach action also sheds light on the resolution of the big-bang singularity. This is conceivable since, say, the divergence of the electron self-energy is basically caused by the pointlike model.
However, straightforward perturbative calculation does not work. In the perturbative regime $|t| \to \infty$ $(\alpha' \to 0)$, the EOM

$$\begin{aligned} \ddot{\Phi} + \frac{1}{2} H f(H) &= 0, \\ \dot{\Phi}^2 + g(H) &= 0, \\ \frac{d}{dt}\left(e^{-\Phi} f(H)\right) &= 0, \end{aligned}$$

$$\begin{aligned} f(H) &= d \sum_{k=1}^{\infty} \left(-\alpha'\right)^{k-1} 2^{2(k+1)} k c_k H^{2k-1} = -2dH - 2d\alpha' H^3 + \mathcal{O}\left(\alpha'^2\right), \\ g(H) &= d \sum_{k=1}^{\infty} \left(-\alpha'\right)^{k-1} 2^{2k+1} (2k-1) c_k H^{2k} = -dH^2 - \frac{3}{2} d\alpha' H^4 + \mathcal{O}\left(\alpha'^2\right). \end{aligned}$$

can be solved iteratively to arbitrary order in $\frac{\sqrt{\alpha'}}{t}$,

$$
\begin{aligned}
H(t) &= \frac{\sqrt{2}}{\sqrt{\alpha'}}\left[\frac{t_0}{t} - 160c_2\frac{t_0^3}{t^3} + \frac{256\left(770c_2^2+19c_3\right)}{3}\frac{t_0^5}{t^5}\right. \\
&\quad \left. - \frac{2048\left(88232c_2^3+4644c_3c_2+41c_4\right)}{5}\frac{t_0^7}{t^7} + \mathcal{O}\left(\frac{t_0^9}{t^9}\right)\right], \quad t_0 \equiv \frac{\sqrt{\alpha'}}{\sqrt{2d}} \\
\Phi(t) &= -\frac{1}{2}\log\left(\beta^2\frac{t^2}{t_0^2}\right) - 32c_2\frac{t_0^2}{t^2} + \frac{256\left(44c_2^2+c_3\right)}{3}\frac{t_0^4}{t^4} \\
&\quad - \frac{2048\left(6976c_2^3+352c_3c_2+3c_4\right)}{15}\frac{t_0^6}{t^6} + \mathcal{O}\left(\frac{t_0^8}{t^8}\right),
\end{aligned}
$$

where β is an integration constant, and we used the universal $c_1 = -\frac{1}{8}$. This solution is obviously singular around the big-bang region $t = 0$. But only $c_1 = -1/8$ and $c_2 = 1/64$ are known.

Resolution of the singularity

To construct non-singular solutions of the EOM

$$\begin{aligned}\ddot{\Phi} + \frac{1}{2} H f(H) &= 0, \\ \dot{\Phi}^2 + g(H) &= 0, \\ \frac{d}{dt}\left(e^{-\Phi} f(H)\right) &= 0,\end{aligned}$$

two constraints must be respected by such cosmological solutions:

a. As $\alpha' \to 0$ or $|t| \to \infty$, the solutions must exactly match the the perturbative solution.

b. The constructed solution is anticipated to be regular everywhere.

However, it is far from easy to look for such solutions. As an illustration, one can first make an ansatz for $f(H)$, whose first two terms of the expansion in α' agree with the perturbative results (easy). Then we have $g(H) = Hf(H) - \int_0^H f(x)\,dx$ (might be solvable). The insurmountable barrier is to solve $H(t)$ and $\Phi(t)$ by substituting $f(H)$ and $g(H)$ into the *nonlinear* EOM.

Resolution of the singularity

In recent works [arXiv:1909.00830, 1910.05808], we have constructed two non-perturbative non-singular solutions:

Resolution of the singularity

Solution A:

$$\Phi(t) = \frac{1}{2}\log\left(\sum_{k=1}^{\infty}\frac{\lambda_k}{1+\tau^{2k}}\right), \qquad \tau \equiv \frac{t}{t_0} = \frac{\sqrt{2d}}{\sqrt{\alpha'}}t,$$

$$H(t) = \frac{\left(\sum_{k=1}^{\infty}\frac{\lambda_k}{\tau^{2k}+1}\right)\sum_{k=1}^{\infty}\left(\frac{8k^2\lambda_k\tau^{4k-2}}{(\tau^{2k}+1)^3}-\frac{2k(2k-1)\lambda_k\tau^{2k-2}}{(\tau^{2k}+1)^2}\right)-\left(\sum_{k=1}^{\infty}\frac{2k\lambda_k\tau^{2k-1}}{(\tau^{2k}+1)^2}\right)^2}{\sqrt{2}\sqrt{\alpha'}\beta\left(\sum_{k=1}^{\infty}\frac{\lambda_k}{\tau^{2k}+1}\right)^{5/2}}$$

$$f(H(t)) = -\frac{2\sqrt{2}\beta d}{\sqrt{\alpha'}}\sqrt{\sum_{k=1}^{\infty}\frac{\lambda_k}{\tau^{2k}+1}}, \qquad g(H(t)) = -\frac{2d\left(\sum_{k=1}^{\infty}\frac{k\lambda_k\tau^{2k-1}}{(\tau^{2k}+1)^2}\right)^2}{\alpha'\left(\sum_{k=1}^{\infty}\frac{\lambda_k}{\tau^{2k}+1}\right)^2}.$$

One of the big advantages of this solution is that as long as $\Phi(t)$ is non-singular, $H(t)$ is guaranteed to be non-singular. We therefore only need to care about the singularity of $\Phi(t)$. Another advantage is that every individual term inside log is non-singular, in contrast to the perturbative solution where all terms are singular. Singularities appear if and only if

$$\sum_{k=1}^{\infty}\frac{\lambda_k}{1+\tau^{2k}} = 0,$$

has real roots.

Resolution of the singularity

In the perturbative regime $t \to \infty$ ($\alpha' \to 0$), $\Phi(t)$ is expanded as,

$$\begin{aligned}
\Phi(t) &= \frac{1}{2}\log\left(\sum_{k=1}^{\infty}\frac{\lambda_k}{1+\tau^{2k}}\right) = \frac{1}{2}\log\left(\frac{\lambda_1}{\tau^2}\right) + \frac{1}{2}\log\left(\sum_{k=1}^{\infty}\frac{1}{\tau^{2k-2}}\frac{\lambda_k/\lambda_1}{1+1/\tau^{2k}}\right) \\
&\to -\frac{1}{2}\log\left(\frac{\tau^2}{\lambda_1}\right) + \frac{\lambda_2-\lambda_1}{2\lambda_1}\frac{1}{\tau^2} + \frac{\lambda_1^2+2\left(\lambda_2+\lambda_3\right)\lambda_1-\lambda_2^2}{4\lambda_1^2}\frac{1}{\tau^4} + \cdots,
\end{aligned}$$

which has exactly the same pattern as the perturbative solution. Matching the coefficients of the perturbative solution fixes λ_i:

$$\lambda_1 = \frac{1}{\beta^2}, \quad \lambda_2 = 0, \quad \lambda_3 = \frac{4+512c_3}{3\beta^2}, \quad \lambda_4 = \frac{-4}{15\beta^2}(31+6272c_3+3072c_4), \quad \cdots$$

Using $H(t)$ produces the same λ_i. The solution is non-perturbative in the sense that it is defined in the whole regime $t \in (-\infty, \infty)$ and α' does not need to approach zero.

Up to any order n, though $\lambda_{k\leq n}$ are fixed by the (in the future) known $c_{k\leq n}$, one always has freedom to choose $\lambda_{k>n}$ as any real value to violate the singular condition.

Resolution of the singularity

Suppose the coefficients $c_{k\le n}$ are known and $N \ge n-1$, solution B is:

$$
\begin{aligned}
\Phi(t) &= -\frac{1}{2N}\log\left[\sum_{k=0}^{N}\rho_k\tau^{2k}\right], \qquad \tau \equiv \frac{t}{t_0} = \frac{\sqrt{2d}}{\sqrt{\alpha'}}t, \\
H(t) &= \frac{-\left(\sum_{k=0}^{N}\rho_k\tau^{2k}\right)\sum_{k=0}^{N}2k(2k-1)\rho_k\tau^{2k-2} + \left(\sum_{k=0}^{N}2k\rho_k\tau^{2k-1}\right)^2}{\sqrt{2}\sqrt{\alpha'}\beta N\left(\sum_{k=0}^{N}\rho_k\tau^{2k}\right)^{2-\frac{1}{2N}}}, \\
f(H(t)) &= -\frac{2\sqrt{2}\beta d}{\sqrt{\alpha'}}\left(\sum_{k=0}^{N}\rho_k\tau^{2k}\right)^{-\frac{1}{2N}}, \quad g(H(t)) = -\frac{d\left(\sum_{k=0}^{N}2k\rho_k\tau^{2k-1}\right)^2}{2\alpha' N^2\left(\sum_{k=0}^{N}\rho_k\tau^{2k}\right)^2}.
\end{aligned}
$$

Similar as solution A, matching the coefficients of the perturbative solution in the perturbative regime $t\to\infty$ $(\alpha'\to 0)$ fixes ρ_i:

$$
\rho_N = \beta^{2N}, \quad \rho_{N-1} = N\beta^{2N}, \quad \rho_{N-2} = \frac{\beta^{2N}}{12}\left(6N^2+11+4c_3\right), \quad \cdots
$$

It should be noted that only $\rho_N, \rho_{N-1}\cdots\rho_{N-n+1}$ are fixed by the known coefficients $c_1, c_2\cdots c_n$. Other parameters $\rho_0, \rho_1\cdots\rho_{N-n}$ can take any real numbers to violate the singular condition

$$
\sum_{k=0}^{N}\rho_k\tau^{2k} = 0,
$$

In particular, we should set $\rho_0 > 0$ to avoid $t=0$ becoming a singularity.

A map between α' corrected EOM and loop corrected EOM

Since

$$\dot{g}(H) = g'(H)\dot{H}(t) = Hf'(H)\dot{H}(t) = H\dot{f}(H),$$

The α' corrected EOM

$$\begin{aligned}
\ddot{\Phi} + \frac{1}{2}Hf(H) &= 0, \\
\dot{\Phi}^2 + g(H) &= 0, \\
\frac{d}{dt}\left(e^{-\Phi}f(H)\right) &= 0,
\end{aligned}$$

can be recast as

$$\begin{aligned}
2\ddot{\Phi} - 2df(H)^2 + \frac{d}{dt}\left[g(H) + df(H)^2\right]\frac{f(H)}{\dot{f}(H)} &= 0, \\
\dot{\Phi}^2 - df(H)^2 + \left[g(H) + df(H)^2\right] &= 0, \\
\dot{f}(H) - f(H)\dot{\Phi} &= 0. \qquad (1)
\end{aligned}$$

A map between α' corrected EOM and loop corrected EOM

It was discovered long time ago that the big-bang singularity could be regularized by loop corrections. In the context of discussing singularity resolution, it is sufficient to implement some effective dilaton potentials to stand for loop corrections. A phenomenological loop corrected effective theory then is

$$\begin{aligned} I_{\mathrm{Loop}} &= \int d^{d+1}x\sqrt{-g}e^{-2\phi}\left[R+4\left(\partial_\mu\phi\right)^2 - V\left(e^{-\Phi(x)}\right)\right], \\ &\overset{\text{FLRW}}{=\!=\!=} \int dte^{-\Phi}\left[-\dot{\Phi}+dH^2-V(e^{-\Phi})\right], \end{aligned}$$

where in the second line, we applied the FLRW background.

A map between α' corrected EOM and loop corrected EOM

The EOM is,

$$
\begin{aligned}
2\ddot{\Phi} - 2dH^2 - \frac{\partial V}{\partial \Phi} &= 0, \\
\dot{\Phi}^2 - dH^2 - V &= 0, \\
\dot{H} - H\dot{\Phi} &= 0.
\end{aligned}
$$

Using the third equation, we have

$$
\frac{\partial V}{\partial \Phi} = \frac{dV(\Phi)}{dt}\frac{1}{\dot{\Phi}} = \frac{dV}{dt}\frac{H(t)}{\dot{H}(t)}.
$$

Therefore, the EOM can be rewritten as

$$
\begin{aligned}
2\ddot{\Phi} - 2dH^2 - \frac{dV}{dt}\frac{H(t)}{\dot{H}(t)} &= 0, \\
\dot{\Phi}^2 - dH^2 - V &= 0, \\
\dot{H} - H\dot{\Phi} &= 0.
\end{aligned} \tag{2}
$$

A map between α' corrected EOM and loop corrected EOM

The loop corrected EOM

$$\begin{aligned} 2\ddot{\Phi} - 2dH^2 - \frac{dV}{dt}\frac{H(t)}{\dot{H}(t)} &= 0, \\ \dot{\Phi}^2 - dH^2 - V &= 0, \\ \dot{H} - H\dot{\Phi} &= 0. \end{aligned}$$

and the α' corrected EOM (1)

$$\begin{aligned} 2\ddot{\Phi} - 2df(H)^2 + \frac{d}{dt}\left[g(H) + df(H)^2\right]\frac{f(H)}{\dot{f}(H)} &= 0, \\ \dot{\Phi}^2 - df\left(H\right)^2 + \left[g\left(H\right) + df\left(H\right)^2\right] &= 0, \\ \dot{f}\left(H\right) - f\left(H\right)\dot{\Phi} &= 0. \end{aligned}$$

We immediately identify a map,

$$\begin{aligned} \alpha' \text{ EOM} & \qquad & \text{Loop EOM} \\ g\left(H_{\alpha'}\right) + df\left(H_{\alpha'}\right)^2 & \longleftrightarrow & -V_L, \\ f\left(H_{\alpha'}\right) & \longleftrightarrow & H_L, \\ \Phi_{\alpha'} & \longleftrightarrow & \Phi_L + \Phi_0, \end{aligned} \tag{3}$$

where Φ_0 is a constant.

A map between α' corrected EOM and loop corrected EOM

We want to stress that this does not mean there must exist such a map between the true complete loop corrections and complete α' corrections, since they might not share the same solution $\Phi(t)$ and the loop corrected action we used is a greatly simplified model.
However, this phenomenological but instructive map is still very useful to mutually generate new solutions for either of them. Especially the loop corrected solutions generated from the α' corrected solutions are more reasonable than those in literature. We gave examples in the paper [arXiv:1909.00830].

The most general *metric* theory of gravity leading to second order field equations in D-dimensions, is given by the Lovelock gravity (Lovelock theorem), which is constructed by the dimensionally extended Euler densities:

$$
\begin{aligned}
I_{Love} &= \int d^D x \sqrt{-\tilde{g}} \sum_{k=0}^{\left[\frac{D-1}{2}\right]} \alpha_k \lambda^{2k-2} \mathcal{L}_k, \\
&= \int d^D x \sqrt{-\tilde{g}} \Big(\alpha_0 + \alpha_1 \tilde{R} + \alpha_2 (\tilde{R}^2 + \tilde{R}_{\alpha\beta\mu\nu} \tilde{R}^{\alpha\beta\mu\nu} - 4 \tilde{R}_{\mu\nu} \tilde{R}^{\mu\nu}) + \mathcal{O}(\tilde{R}^3) \Big), \\
\mathcal{L}_k &\equiv \frac{1}{2^k} \delta^{\mu_1 \cdots \mu_k \nu_1 \cdots \nu_k}_{\rho_1 \cdots \rho_k \sigma_1 \cdots \sigma_k} \tilde{R}_{\mu_1 \nu_1}{}^{\rho_1 \sigma_1} \cdots \tilde{R}_{\mu_k \nu_k}{}^{\rho_k \sigma_k},
\end{aligned}
$$

where $[(D-1)/2]$ denotes the integer part of $(D-1)/2$. α_k are dimensionless and λ has a length scale. Notation "tilde" indicates the Einstein frame. The action (4) only has a finite number of terms for $k < D/2$. Terms for $k > D/2$ vanish identically, and the term $k = D/2$ is a topological invariant. To match the Einstein-Hilbert action, we have $\alpha_0 = -2\Lambda$ and $\alpha_1 = 1$.

The term of α_2 is the Gauss-Bonnet.

Derive Lovelock Gravity from String Theory

As early as the mid-1980s, it has been speculated that Lovelock theory might be derived from string theory. If string theory is as powerful as claimed, this should be true.

- This has been done up to the linear α' correction (Gauss-Bonnet).
- However, The higher order α' corrections include higher derivatives of the metric and cannot be rewritten as higher order Lovelock gravity.
- Moreover, a conceptual mismatch exists: for a particular dimension $D = d + 1$, Lovelock gravity has finite terms but α' corrections are infinitely many.

In a coming paper, we will show that in cosmological background, Lovelock Gravity indeed can be derived from string theory α' corrections, with the coefficients identified as

$$\alpha_k = \frac{2k-1}{(k+1)!} 2^{2k+1} c_k, \tag{4}$$

where c_k's are the coefficient of k-th α' corrections in the HZ action.

Derive Lovelock Gravity from String Theory

Summary and Discussions

- The α' corrections do permit dS/AdS vacua and provide possible resolutions for the singularities.
- A phenomenological map between the α' corrected EOM and loop corrected EOM is identified.

- We addressed vacuum scenario and set $b_{\mu\nu} = 0$. One might rotate time dependent $b_{\mu\nu}(t)$ into the evolution to get some new features. Particularly, the string coupling could be stabilized by some configurations of $b_{\mu\nu}(t)$ — work to appear very soon.
- With some subtleties, naked spatial singularity can also be resolved by α' corrections — work to appear very soon.
- With α' corrections, exotic matter is not necessary to support traversable Wormhole — work in progress.
- Matter sources in an $O(d, d)$ fashion are expected to lead to more realistic configurations.
- String cosmology can be reformulated in a solider manner!

Thank you!

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