New Positivity Bounds from Full Crossing Symmetry

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Andrew Tolley, Zi-Yue Wang & **SYZ**, 2011.02400

What are positivity bounds?

$$\mathscr{L} = \sum_i \Lambda^4 f_i \, \mathcal{O}_i \left(\frac{\text{boson}}{\Lambda}, \frac{\text{fermion}}{\Lambda^{3/2}}, \frac{\partial}{\Lambda} \right)$$

f_i : Wilson coefficients

Is every set of Wilson coefficients allowed?

Short answer: No!

UV completion satisfies: Lorentz invariance, causality/analyticity, unitarity, crossing symmetry, …

↓

Positivity bounds on Wilson coefficients

Some fundamental properties of S-matrix

Unitarity: conservation of probabilities $S^\dagger S = 1$

Lorentz invariance: amplitude $A(p_i^\mu p_{j\mu})$

Causality/Analyticity: $A(p_i^\mu p_{j\mu})$ as analytic function

Locality: $A(p_i^\mu p_{j\mu})$ is polynomially bounded at high energies

Crossing symmetry: $A(s, t) = A(u, t) = A(t, s)$ (for scalar)

Simplest example: P(X)

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{\Lambda^4}(\partial_\mu\phi\partial^\mu\phi)^2 + \cdots$$

$$A(s, t = 0) = \frac{4\lambda s^2}{\Lambda^4} + \cdots$$

Positivity bound: $\lambda > 0$

Theories with $\lambda < 0$ do not have a local and Lorentz invariant UV completion

$$\mathcal{L}_{\mathrm{DBI}} \sim -\sqrt{1 + (\partial\phi)^2}$$ ✓

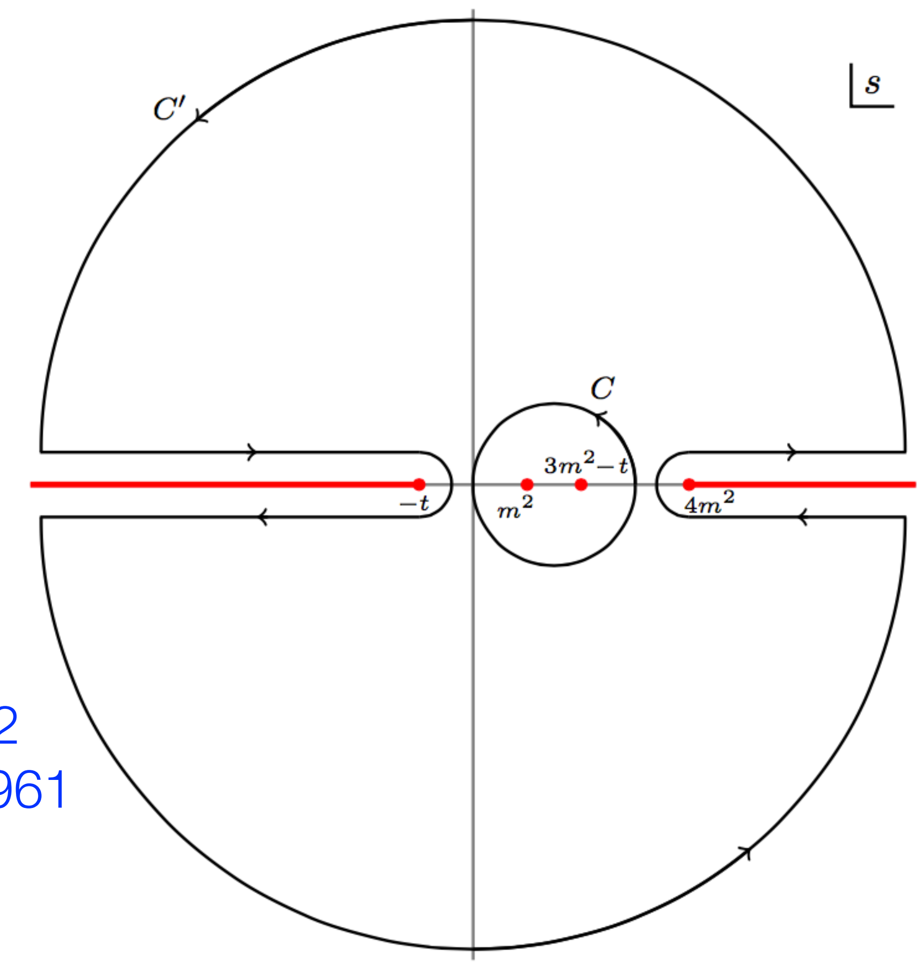
$$\mathcal{L}_{\overline{\mathrm{DBI}}} \sim -\sqrt{1 - (\partial\phi)^2}$$ ✗

Fixed t dispersion relation

$$A(s,t) = \frac{1}{2\pi i}\oint_{\mathcal{C}} ds' \, \frac{A(s',t)}{s'-s}$$

Analyticity

$$= \frac{\lambda}{m^2-s} + \frac{\lambda}{m^2-u} + \int_{\mathcal{C}^{\pm}_{\infty}} ds' \frac{A(s',t)}{s'-s}$$
$$+ \int_{4m^2}^{\infty} \frac{d\mu}{\pi}\left(\frac{\mathrm{Im}A(\mu,t)}{\mu-s} + \frac{\mathrm{Im}A(\mu,t)}{\mu-u}\right),$$



Froissart-Martin bound

$$\lim_{s\to\infty} |A(s,t)| < C s^{1+\epsilon(t)}, \qquad \epsilon(t) < 1, \quad 0 \le t < 4m^2.$$

Martin 1962
Froissart 1961

$s \leftrightarrow u$ crossing symmetry

$$B(s,t) = A(s,t) - \frac{\lambda}{m^2-s} - \frac{\lambda}{m^2-u} - \frac{\lambda}{m^2-t}$$

Twice subtracted dispersion relation

$$v = s + \frac{t}{2} - 2m^2$$
$$\bar{\mu} = \mu - \frac{4m^2}{3}$$

$$B(v,t) = a(t) + \int_{4m^2}^{\infty} \frac{d\mu}{\pi(\bar{\mu}+\bar{t}/2)} \frac{2v^2\,\mathrm{Im}A(\mu,t)}{(\bar{\mu}+\bar{t}/2)^2 - v^2}$$

What does the dispersion relation imply?

$$B^{(2N,M)}(t) = \frac{1}{M!}\partial_v^{2N}\partial_t^M B(v,t)\Big|_{v=0} = \sum_{k=0}^{M} \frac{(-1)^k}{k!2^k} I^{(2N+k,M-k)}$$

$$I^{(q,p)}(t) = \frac{q!}{p!}\frac{2}{\pi}\int_{4m^2}^{\infty} \frac{d\mu \;\; \partial_t^p \mathrm{Im}\, A(\mu,t)}{(\bar{\mu}+\bar{t}/2)^{q+1}}$$

partial wave unitary
positivity of Legendre polynomials
analyticity

$\Rightarrow$

$$\frac{\partial^n}{\partial t^n}\mathrm{Im}[A(s,t)] > 0$$

$$s \geq 4m^2 \,,\; 0 \leq t < 4m^2$$

$\Rightarrow$ $I^{(q,p)}(t) > 0$

Strategy: linearly combine different $\partial_s^{2N}\partial_t^M B(s,t)$ to overcome the $(-1)^k$

$$I^{(q,p)} < \frac{q}{\mathcal{M}^2} I^{(q-1,p)}$$

$$\mathcal{M}^2 = (t+4m^2)/2$$

The Y positivity bounds

Recurrence relation:

de Rham, Melville, Tolley & **SYZ**, 1702.06134

$$Y^{(2N,M)} = \sum_{r=0}^{M/2} c_r B^{(2N+2r,M-2r)} + \frac{1}{\mathcal{M}^2} \sum_{k \text{ even}}^{(M-1)/2} (2(N+k)+1)\beta_k Y^{(2(N+k),M-2k-1)} > 0$$

$$B^{(2N,M)}(t) = \frac{1}{M!} \partial_v^{2N} \partial_t^M \tilde{B}(v,t)\Big|_{v=0}$$

$$\text{sech}(x/2) = \sum_{k=0}^{\infty} c_k x^{2k} \quad \text{and} \quad \tan(x/2) = \sum_{k=0}^{\infty} \beta_k x^{2k+1}$$

$$\mathcal{M}^2 = (t+4m^2)/2$$

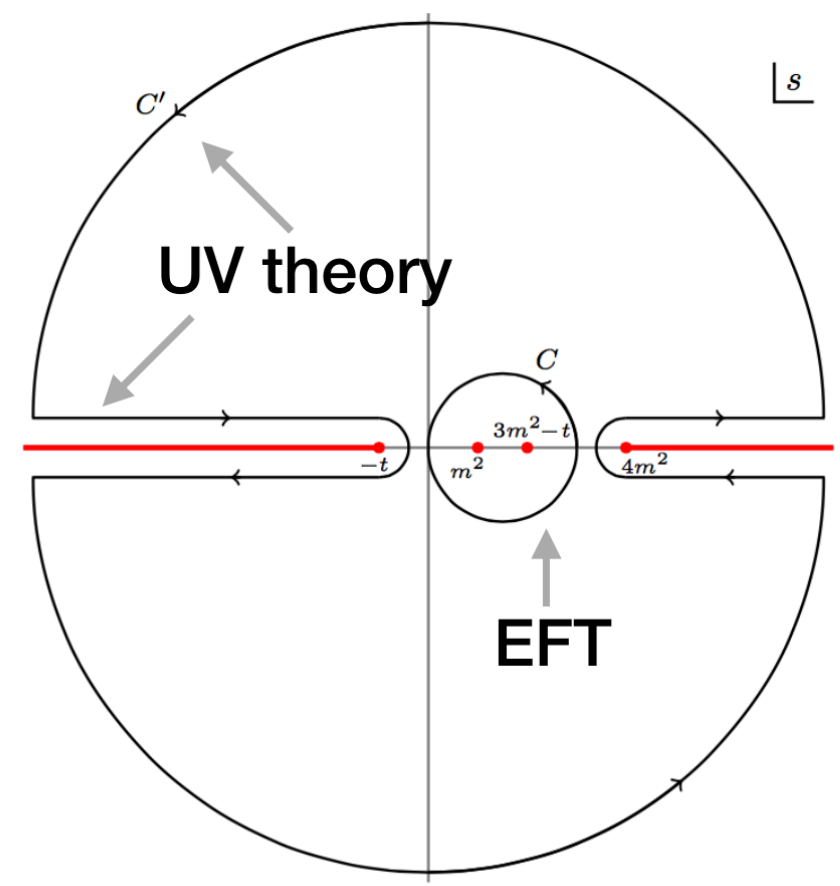
What role do the EFT play?

$$Y^{(2N,M)} \sim \sum_{n,m} c_{n,m}\, \partial_s^n \partial_t^m A(s,t) > 0$$

At low energies, EFT amplitude $\simeq$ full amplitude

$$Y_{\text{EFT}}^{(2N,M)} \sim \sum_{n,m} c_{n,m}\, \partial_s^n \partial_t^m A_{\text{EFT}}(s,t) > 0$$

contains Wilson coeff's



Inequalities on Wilson coefficients

Y bounds for spinning particles

Difficulties with nonzero spins

- nontrivial crossing
- kinematic singularities

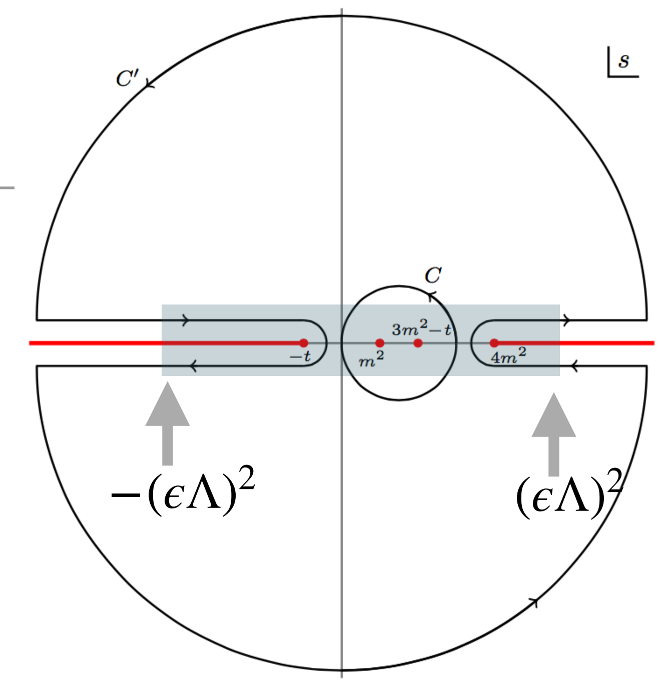
Making use of regularized transversely amplitude

Y bounds **formally the same** as scalar case:

$$Y^{(2N,M)}_{\tau_1\tau_2}(t) = \sum_{r=0}^{M/2} c_r B^{(2N+2r,M-2r)}_{\tau_1\tau_2}(t) + \frac{1}{\mathcal{M}^2} \sum_{\text{even } k=0}^{(M-1)/2} (2N+2k+1)\beta_k Y^{(2N+2k,M-2k-1)}_{\tau_1\tau_2}(t) > 0$$

de Rham, Melville, Tolley & **SYZ**,1706.02712

Improved positivity bounds



$A(s,t)$ calculable within EFT ($E < \epsilon\Lambda$, $\epsilon \lesssim 1$)

Low energy subtracted amplitude:

$$B_{\epsilon\Lambda}(v,t) \equiv B(v,t) - \int_{4m^2}^{(\epsilon\Lambda)^2} \frac{\mathrm{d}\mu}{(\ldots)} \mathrm{Im}A(\mu,t) = \int_{(\epsilon\Lambda)^2}^{\infty} \frac{\mathrm{d}\mu}{(\ldots)} \mathrm{Im}A(\mu,t)$$

→ Improved bounds $Y_{\epsilon\Lambda}^{(2N,M)}(t) > 0$

Tree amplitude positivity bounds (weak coupling $g \ll 1$)

$\epsilon\Lambda = \Lambda_{\mathrm{th}}$: UV heavy mass scale

de Rham, Melville, Tolley & **SYZ**,1702.08577

Positivity Bounds from Full Crossing Symmetry

Fixed t dispersion relation is only $s \leftrightarrow u$ symmetric.

Full crossing symmetry: $B(u, t) = B(s, t) = B(t, s)$

Moment of positive distribution (without t derivatives)

Partial wave expansion: $A(s,t) \sim \sum_{\ell=0}^{\infty} (2\ell + 2\alpha) C_\ell^{(\alpha)}(\cos\theta) a_\ell(s)$

$C_\ell^{(\alpha)}(x)$: Gegenbauer polynomials

Partial wave unitary bounds:

$$\mathrm{Im}\, a_\ell(s) \geq |a_\ell(s)|^2$$

Positive "distribution": $\rho_\ell(\mu) \sim \mathrm{Im} a_\ell(\mu) > 0$

$$f^{(2N,0)} \sim \partial_s^{2N+2} B_{\epsilon\Lambda}(s,t)\Big|_{\substack{m^2 \to 0 \\ s,t \to 0}} = \sum_\ell \int \mathrm{d}\mu \rho_{\ell,\alpha}(\mu) \frac{1}{\mu^{2N}} > 0$$

Define "moment"

$$\left\langle\!\!\left\langle \frac{1}{\mu^{2N}} \right\rangle\!\!\right\rangle = \frac{\sum_\ell \int \mathrm{d}\mu \rho_{\ell,\alpha}(\mu) \frac{1}{\mu^{2N}}}{\sum_\ell \int \mathrm{d}\mu \rho_{\ell,\alpha}(\mu)} = \frac{f^{(2N,0)}}{f^{(0,0)}}$$

Nonlinear s derivative bounds

Cauchy-Schwarz inequality

$$\left\langle\!\!\left\langle \frac{1}{\mu^{2I}} \right\rangle\!\!\right\rangle \left\langle\!\!\left\langle \frac{1}{\mu^{2J}} \right\rangle\!\!\right\rangle \geq \left\langle\!\!\left\langle \frac{1}{\mu^{I+J}} \right\rangle\!\!\right\rangle^2 \quad \Rightarrow \quad f^{(2I,0)} f^{(2J,0)} \geq \left(f^{(I+J,0)}\right)^2$$

Low energy expansion

$$B_{\epsilon\Lambda}(s, t=0) = \frac{\tilde{a}_{1,0}}{\Lambda^4} s^2 + \frac{\tilde{a}_{2,0}}{\Lambda^8} s^4 + \frac{\tilde{a}_{3,0}}{\Lambda^{12}} s^6 + \cdots$$

Constraints on Wilson coeffs: $\tilde{a}_{2I,0}\, \tilde{a}_{2J,0} \geq \left(\tilde{a}_{I+J,0}\right)^2$

$$\Rightarrow \quad 0 < \tilde{a}_{I,0} < \tilde{a}_{1,0}$$

Constrained the s terms from both sides!

EFThedron

This can be generalized to “EFThedron”:

positivity of Hankel matrix of Wilson coefficients (s derivatives)

$$\det \begin{bmatrix} \tilde{a}_{1,0} & \tilde{a}_{2,0} & \cdots & \tilde{a}_{\frac{i}{2}+1,0} \\ a_{2,0} & \tilde{a}_{3,0} & \cdots & \tilde{a}_{\frac{i}{2}+2,0} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{\frac{i}{2}+1,0} & \tilde{a}_{\frac{i}{2}+2,0} & \cdots & \tilde{a}_{i+1,0} \end{bmatrix} \geq 0$$

Arkani-Hamed, Y.-t. Huang, T.-C. Huang, unpublished

What about t derivatives?

Add on $s \leftrightarrow t$ crossing!

Consider 1st t derivative $f^{(0,1)} \sim \partial_t B(s,t)$

$$\left(\frac{f^{(0,1)}}{f^{(0,0)}} + \left\langle\!\!\left\langle \frac{3}{2\mu} \right\rangle\!\!\right\rangle\right)^2 = \left\langle\!\!\left\langle \frac{2(D-3)\ell + 2\ell^2}{(D-2)\mu} \right\rangle\!\!\right\rangle^2 \leq \left\langle\!\!\left\langle \left(\frac{2(D-3)\ell + 2\ell^2}{(D-2)\mu}\right)^2 \right\rangle\!\!\right\rangle$$

Cauchy-Schwarz

Impose $s \leftrightarrow t$ on su dispersion relation

Tolley, Wang & **SYZ**, 2011.02400

$$B_{\rm tr}(s,t) = B_{\rm tr}(t,s)$$

$$\Rightarrow \quad \left\langle\!\!\left\langle \frac{\ell(\ell+D-3)\left[4-5D-2(3-D)\ell+2\ell^2\right]}{\mu^2} \right\rangle\!\!\right\rangle + \mathcal{O}\left(\frac{1}{\mu^3}\right) = 0$$

$$\Rightarrow \quad -\frac{3}{2\Lambda^2} f^{(0,0)} < f^{(0,1)} < \frac{5D-4}{(D-2)\Lambda^2} f^{(0,0)}$$

Constraining t dependent terms

Low energy expansion with $t \neq 0$

$$B_{\text{tr}}(s,t) = \frac{\tilde{a}_{1,0}}{\Lambda^4}x + \frac{\tilde{a}_{0,1}}{\Lambda^6}y + \frac{\tilde{a}_{2,0}}{\Lambda^8}x^2 + \cdots$$

$$x \equiv -(st+su+tu), \quad y \equiv -stu$$

Triple crossing bound

$$-\frac{3}{2}\tilde{a}_{1,0} < \tilde{a}_{0,1} < \frac{5D-4}{(D-2)}\tilde{a}_{1,0}$$

Now, the y coefficient is constrained from both sides!

Implications for soft theories

Weakly broken Galileon theory

$$\Lambda_3^{4-D}\mathcal{L}_{\rm mg} = -\frac{1}{2}\partial_\mu\pi\partial^\mu\pi - \frac{1}{2}m^2\pi^2 + \sum_{n=3}^{D+1}\frac{g_n}{\Lambda_3^{3n-3}}\pi\partial^{\mu_1}\partial_{[\mu_1}\pi\partial^{\mu_2}\partial_{\mu_2}\pi\cdots\partial^{\mu_n}\partial_{\mu_n]}\pi + \ldots$$

$$B_{\rm mg}(s,t) \sim \frac{1}{\Lambda_3^{D-4}}\left(\frac{m^2}{\Lambda_3^6}x + \frac{1}{\Lambda_3^6}y + \frac{1}{\Lambda_3^8}x^2 + \cdots\right)$$

$$\left.\begin{array}{c} \tilde{a}_{1,0} \sim g^2 \text{ with } g \ll 1 \\ \tilde{a}_{N+1,0}\,\tilde{a}_{1,0}^{N-1} \geq \tilde{a}_{2,0}^N. \\ -\frac{3}{2}\tilde{a}_{1,0} < \tilde{a}_{0,1} < \frac{5D-4}{(D-2)}\tilde{a}_{1,0} \end{array}\right\} \Rightarrow B(s,0) \sim \frac{g^2}{\Lambda^{D-4}}\left(\frac{x}{\Lambda^4} + \frac{y}{\Lambda^6} + \frac{x^2}{\Lambda^8} + \cdots\right)$$

$$\Rightarrow \frac{g^2}{\Lambda^D} \sim \frac{m^2}{\Lambda_3^{D+2}}, \quad \frac{g^2}{\Lambda^{D+2}} \sim \frac{1}{\Lambda_3^{D+2}}. \Rightarrow \Lambda \sim m,$$

No healthy hierarchy as EFT, so no standard UV completion!

Positivity Bounds from Full Crossing Symmetry

——Generalizations

$s \leftrightarrow u$ symmetric expansion

Introduce a new variable $w = -su$

Expand the dispersion relation

$$B_{\epsilon\Lambda}(s,t) = b(t) + \sum_{\ell} \int \mathrm{d}\mu \rho_{\ell,\alpha}(\mu)\mu^2 \frac{2+\frac{t}{\mu}}{1+\frac{t}{\mu}-\frac{w}{\mu^2}} \frac{C_\ell^{(\alpha)}(1+\frac{2t}{\mu})}{2C_\ell^{(\alpha)}(1)}$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} c_{m,n} w^m t^n$$

by brute force

$$c_{m,n} \equiv \left\langle \frac{D_{m,n}(\eta)}{\mu^{2m+n-2}} \right\rangle$$

$\eta = \ell(\ell + 2\alpha)$

[in 4D $\eta = \ell(\ell + 1)$]

$D_{m,n}(\eta) = d_n \eta^n + d_{n-1}\eta^{n-1} + \cdots + d_0$ with $d_n > 0$

D^{su} bounds

Tolley, Wang & **SYZ**, 2011.02400

Find the minimum of $D_{m,n}(\eta)$ $\quad c_{m,n} = \left\langle \frac{D_{m,n}(\eta)}{\mu^{2m+n-2}} \right\rangle \geq \min_\eta(D_{m,n}(\eta)) \left\langle \frac{1}{\mu^{2m+n-2}} \right\rangle$

More generally, find minimum of $D^{\text{su}}_{m,n}(\eta, k) \equiv \sum_{i\geq 0} k_i D_{m+i,n-2i}(\eta),$ $\quad k_0 > 0$

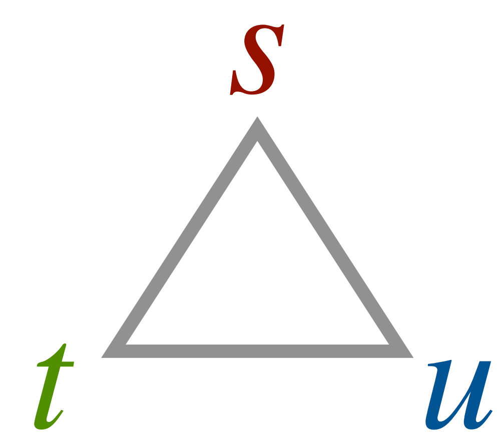
$$c_{m,2k} + \sum_{i\geq 1} k_i c_{m+i,2k-2i} > S_{m,2k}(k)\ c_{m+k,0},$$
$$c_{m,2k+1} + \sum_{i\geq 1} k_i c_{m+i,2k+1-2i} > S_{m,2k+1}(k)\sqrt{c_{m+k,0}c_{m+k+1,0}},$$

choose any set of k_i $\qquad S_{m,n}(k) = \min_\eta D^{\text{su}}_{m,n}(\eta, k).$

stu triple crossing symmetry

Impose $s \leftrightarrow t$ on su dispersion relation

$$B_{\text{tr}}(s,t) = B_{\text{tr}}(t,s)$$



$$B(s,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} c_{m,n} w^m t^n \qquad c_{m,n} \equiv \left\langle \frac{D_{m,n}(\eta)}{\mu^{2m+n-2}} \right\rangle \qquad B(s,t) = B(u,t) = B(t,s)$$

Triple crossing constraints

$$\left\langle \frac{\Gamma_{m,n}(\eta)}{\mu^{2m+n-2}} \right\rangle = 0$$

$$\Gamma_{m,m+1}(\eta) = D_{m,m+1}(\eta) - 2D_{m+1,m-1}(\eta),$$
$$\Gamma_{m,n}(\eta) = \Gamma_{m+1,n}(\eta) + \Gamma_{m,n-1}(\eta).$$

$\Gamma_{m,n}$ constraints

	η^0	η^2	η^4	η^6	η^8	η^{10}	η^{12}	$\Gamma_{m,n}$ constraints from $s \leftrightarrow t$ symmetry
μ^2	$c_{0,0}$							
1	$c_{1,0}$	$c_{0,2}$						
$\frac{1}{\mu^2}$	$c_{2,0}$	$c_{1,2}$	$c_{0,4}$					$c_{1,2} - 2c_{2,0} = 0$
$\frac{1}{\mu^4}$	$c_{3,0}$	$c_{2,2}$	$c_{1,4}$	$c_{0,6}$				$c_{1,4} - 3c_{3,0} = 0$
$\frac{1}{\mu^6}$	$c_{4,0}$	$c_{3,2}$	$c_{2,4}$	$c_{1,6}$	$c_{0,8}$			$c_{2,4} - c_{3,2} - 2c_{4,0} = 0$, $c_{1,6} - 2c_{2,4} - 2c_{3,2} - 8c_{4,0} = 0$
$\frac{1}{\mu^8}$	$c_{5,0}$	$c_{4,2}$	$c_{3,4}$	$c_{2,6}$	$c_{1,8}$	$c_{0,10}$		$c_{3,4} - 2c_{4,2} = 0$, $c_{2,6} + c_{3,4} - 3c_{4,2} - 5c_{5,0} = 0$, $c_{1,8} + 4c_{2,6} + 3c_{3,4} - 10c_{4,2} - 25c_{5,0} = 0$
$\frac{1}{\mu^{10}}$	$c_{6,0}$	$c_{5,2}$	$c_{4,4}$	$c_{3,6}$	$c_{2,8}$	$c_{1,10}$	$c_{0,12}$	$c_{3,6} - 3c_{5,2} - 2c_{6,0} = 0$, $c_{2,8} + 3c_{3,6} - 10c_{5,2} - 15c_{6,0} = 0$, $c_{1,10} + 6c_{2,8} + 12c_{3,6} - 42c_{5,2} - 84c_{6,0} = 0$

	η^1	η^3	η^5	η^7	η^9	η^{11}	$\Gamma_{m,n}$ constraints from $s \leftrightarrow t$ symmetry
μ	$c_{0,1}$						
$\frac{1}{\mu}$	$c_{1,1}$	$c_{0,3}$					
$\frac{1}{\mu^3}$	$c_{2,1}$	$c_{1,3}$	$c_{0,5}$				$c_{1,3} - c_{2,1} = 0$
$\frac{1}{\mu^5}$	$c_{3,1}$	$c_{2,3}$	$c_{1,5}$	$c_{0,7}$			$c_{2,3} - 2c_{3,1} = 0$, $c_{1,5} + c_{2,3} - 3c_{3,1} = 0$
$\frac{1}{\mu^7}$	$c_{4,1}$	$c_{3,3}$	$c_{2,5}$	$c_{1,7}$	$c_{0,9}$		$c_{2,5} - 3c_{4,1} = 0$, $c_{1,7} + 3c_{2,5} - 10c_{4,1} = 0$
$\frac{1}{\mu^9}$	$c_{5,1}$	$c_{4,3}$	$c_{3,5}$	$c_{2,7}$	$c_{1,9}$	$c_{0,11}$	$c_{3,5} - c_{4,3} - 2c_{5,1} = 0$, $c_{2,7} + 2c_{3,5} - 2c_{4,3} - 8c_{5,1} = 0$, $c_{1,9} + 5c_{2,7} + 7c_{3,5} - 7c_{4,3} - 35c_{5,1} = 0$

D^{stu} bounds

Tolley, Wang & **SYZ**, 2011.02400

Now, $\Gamma_{m,n}(\eta) = e_n \eta^n + e_{n-1}\eta^{n-1} + \cdots + e_0$

Define $D^{\rm stu}_{m,n}(\eta,k,\kappa) = D^{\rm su}_{m,n}(\eta,k) + \sum_{j\geq 0} \kappa_j \Gamma_{m'+j,n'-2j}(\eta),$ $n' < n$

$$D^{\rm stu}_{m,n}(\eta,k,\kappa) = d'_n \eta^n + d'_{n-1}\eta^{n-1} + \cdots + d'_0 \text{ with } \bar{d}_n > 0$$

Find the minimum of $D^{\rm stu}_{m,n}$

this improves the $D^{\rm su}$ bounds

$$c_{m,2k} + \sum_{i\geq 1} k_i c_{m+i,2k-2i} > U_{m,2k}(k) c_{m+k,0},$$

$$c_{m,2k+1} + \sum_{i\geq 1} k_i c_{m+i,2k+1-2i} > U_{m,2k+1}(k)\sqrt{c_{m+k,0}c_{m+k+1,0}},$$

$$U_{m,n}(k) = \max_\kappa \min_\eta D^{\rm stu}_{m,n}(\eta,k,\kappa),$$

$\bar{D}^{stu}$ bounds

Tolley, Wang & **SYZ**, 2011.02400

Define $\bar{D}^{\rm stu}_{m,n}(\eta,k,\kappa) = -D^{\rm su}_{m,n}(\eta,k) + \sum_{j\geq 0} \kappa_j \Gamma_{m'+j,n'-2j}(\eta),$ $\kappa_0 > 0,\ n' > n$

$$\bar{D}^{\rm stu}_{m,n}(\eta,k,\kappa) = \bar{d}_n \eta^n + \bar{d}_{n-1}\eta^{n-1} + \cdots + \bar{d}_0$$

Find the minimum of $\bar{D}^{\rm stu}_{m,n}$

$$c_{m,2k} + \sum_{i\geq 1} k_i c_{m+i,2k-2i} < -T_{m,2k}(k)\ c_{m+k,0},$$

$$c_{m,2k+1} + \sum_{i\geq 1} k_i c_{m+i,2k+1-2i} < -T_{m,2k+1}(k)\sqrt{c_{m+k,0}c_{m+k+1,0}},$$

$$T_{m,n}(k) = \max_\kappa \min_\eta \bar{D}^{\rm stu}_{m,n}(\eta,k,\kappa),$$

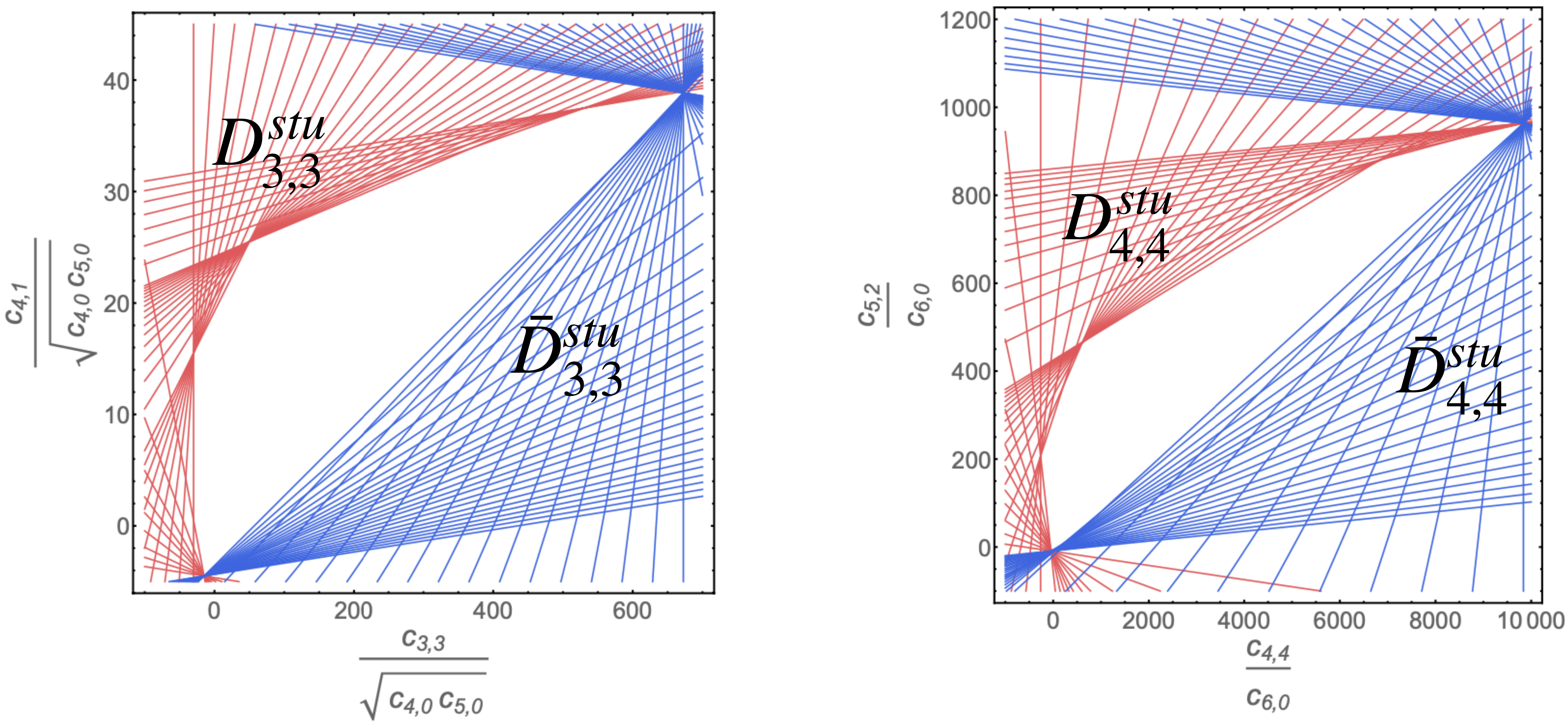
Bounding coefficients from the opposite side!

Leading D^{stu} and $\bar{D}^{stu}$ bounds

(m,n)	$D^{\rm stu}_{m,n}$ bound	$\bar{D}^{\rm stu}_{m,n}$ bound
$(1,1)$	$c_{1,1} > -\frac{3}{2}\sqrt{c_{1,0}c_{2,0}}$	$c_{1,1} < 8\sqrt{c_{1,0}c_{2,0}}$
$(2,1)$	$c_{2,1} > -\frac{5}{2}\sqrt{c_{2,0}c_{3,0}}$	$c_{2,1} < \frac{465}{38}\sqrt{c_{2,0}c_{3,0}}$
$(2,2)$	$c_{2,2} > -\frac{9}{2}c_{3,0}$	$c_{2,2} < \frac{2961}{58}c_{3,0}$
$(3,1)$	$c_{3,1} > -\frac{7}{2}\sqrt{c_{3,0}c_{4,0}}$	$c_{3,1} < \frac{1097}{58}\sqrt{c_{3,0}c_{4,0}}$
$(3,2)$	$c_{3,2} > -7c_{4,0}$	$c_{3,2} < \frac{10027}{59}c_{4,0}$
$(3,3)$	$c_{3,3} + \frac{3}{4}c_{4,1} > -\frac{147}{8}\sqrt{c_{4,0}c_{5,0}}$, $c_{3,3} - 8c_{4,1} > -154\sqrt{c_{4,0}c_{5,0}}$, $c_{3,3} - \frac{481}{12}c_{4,1} > -\frac{7777}{8}\sqrt{c_{4,0}c_{5,0}}$, $c_{3,3} - 104c_{4,1} > -3369\sqrt{c_{4,0}c_{5,0}}$	$c_{3,3} - \frac{650}{41}c_{4,1} < -\frac{2310}{41}\sqrt{c_{4,0}c_{5,0}}$
$(4,2)$	$c_{4,2} > -\frac{17}{2}c_{5,0}$	$c_{4,2} < \frac{3923}{12}c_{5,0}$
$(4,3)$	$c_{4,3} + \frac{3}{4}c_{5,1} > -\frac{253}{8}\sqrt{c_{5,0}c_{6,0}}$, $c_{4,3} - \frac{180}{41}c_{5,1} > -\frac{8705}{82}\sqrt{c_{5,0}c_{6,0}}$, $c_{4,3} - \frac{325}{12}c_{5,1} > -\frac{16825}{24}\sqrt{c_{5,0}c_{6,0}}$, $c_{4,3} - \frac{169}{2}c_{5,1} > -\frac{11187}{4}\sqrt{c_{5,0}c_{6,0}}$ $c_{4,3} - \frac{743}{4}c_{5,1} > -\frac{63279}{8}\sqrt{c_{5,0}c_{6,0}}$	$c_{4,3} - \frac{73153}{1748}c_{5,1} < -\frac{708543}{3496}\sqrt{c_{5,0}c_{6,0}}$
$(4,4)$	$c_{4,4} + \frac{25}{24}c_{5,2} > -\frac{147}{8}c_{6,0}$, $c_{4,4} - \frac{125}{37}c_{5,2} > -\frac{71175}{74}c_{6,0}$, $c_{4,4} - \frac{785}{52}c_{5,2} > -\frac{83490}{13}c_{6,0}$, $c_{4,4} - \frac{2485}{69}c_{5,2} > -\frac{1144125}{46}c_{6,0}$	$c_{4,4} - 15c_{5,2} < -\frac{195}{2}c_{6,0}$, $c_{4,4} + \frac{368085}{36544}c_{5,2} < -\frac{2365845}{18272}c_{6,0}$

Tolley, Wang & **SYZ**, 2011.02400

Enclosed convex region from bounds



Constraining coefficients from both sides!

The PQ bounds: refinement of the Y bounds

Relaxing inequality: $\frac{1}{(\epsilon\Lambda)^2}\left\langle \frac{L_\ell^i}{\mu^{j-1}}\right\rangle > \left\langle \frac{L_\ell^i}{\mu^{j}}\right\rangle > (\epsilon\Lambda)^2\left\langle \frac{L_\ell^i}{\mu^{j+1}}\right\rangle$

$$P_{m,n} \equiv c_{m,n} + \frac{1}{(\epsilon\Lambda)^2}\sum_{i=1}^{\lfloor\frac{n+1}{2}\rfloor} M_m^{2i-1} P_{m+i-1,n+1-2i} - \sum_{j=1}^{\lfloor\frac{n}{2}\rfloor} M_m^{2j} Q_{m+j,n-2j},$$

$$Q_{m,n} \equiv c_{m,n} + (\epsilon\Lambda)^2\sum_{i=1}^{\lfloor\frac{n+1}{2}\rfloor} M_m^{2i-1} Q_{m+i,n+1-2i} - \sum_{j=1}^{\lfloor\frac{n}{2}\rfloor} M_m^{2j} P_{m+j,n-2j},$$

Tolley, Wang & **SYZ**, 2011.02400

Linear bounds $\boxed{P_{m,n} > (\epsilon\Lambda)^{4k} Q_{m+k,n}, \quad k = 0,1,2,...,}$

$P_{m,n} > 0$ is roughly $Y^{(2N,M)} > 0$

Nonlinear bounds $\boxed{P_{m,n}\, P_{m+2,n} > (Q_{m+1,n})^2.}$

Cauchy-Schwarz inequality

We are not alone in thinking this way!

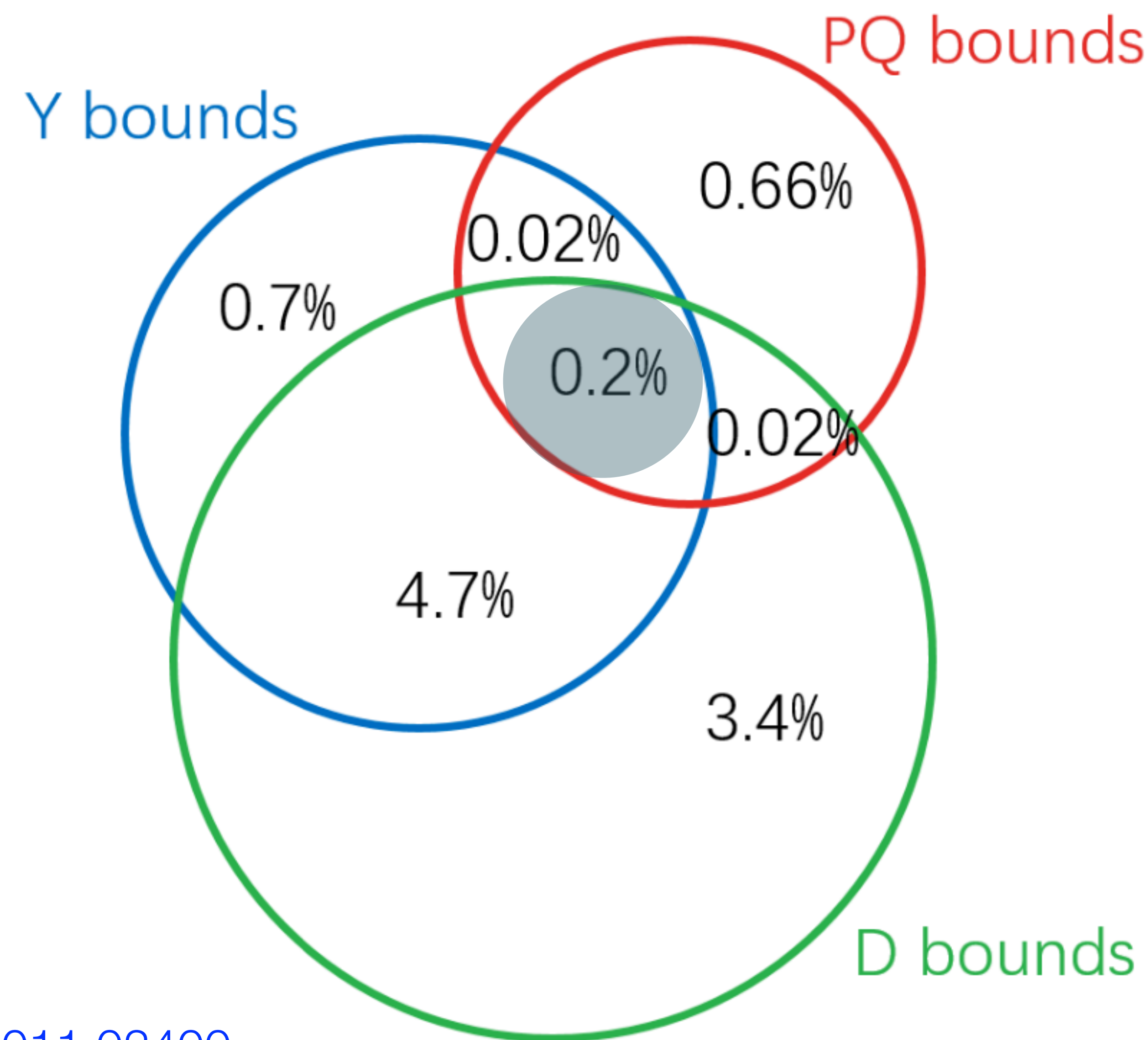
Caron-Huot & Van Duong, 2011.02957

Used powerful semi-definite programing

EFT coefficient	**Lower bound**	**Upper bound**
$\tilde{g}_3$	-10.346	3
$\tilde{g}_4$	0	0.5
$\tilde{g}_5$	-4.096	2.5
$\tilde{g}_6$	0	0.25
$\tilde{g}'_6$	-12.83	3
$\tilde{g}_7$	-1.548	1.75
$\tilde{g}_8$	0	0.125
$\tilde{g}'_8$	-10.03	4
$\tilde{g}_9$	-0.524	1.125
$\tilde{g}'_9$	-13.60	3
$\tilde{g}_{10}$	0	0.0625
$\tilde{g}'_{10}$	-6.32	3.75

Comparison of different bounds

First few bounds (up to level μ^{-4})



Tolley, Wang & **SYZ**, 2011.02400

Application: Bounds on SU(2) ChPT

$$\mathscr{L}_{\rm chpt} = \frac{F^2}{4}\left\langle u_\mu u^\mu + \chi_+ \right\rangle + \frac{l_1}{4}\left\langle u^\mu u_\mu \right\rangle^2 + \frac{l_2}{4}\left\langle u_\mu u_\nu \right\rangle \langle u^\mu u^\nu \rangle + \dots$$

$$U = \sqrt{1 - \frac{\pi^a \pi^a}{F^2}}\mathbf{1} + i\frac{\pi^a \tau^a}{F}$$

$$u_\mu = i u^\dagger \partial_\mu U u^\dagger = u_\mu^\dagger \qquad u = \sqrt{U},$$

$$l_1^r = \frac{1}{96\pi^2}\left(\bar{l}_1 + \ln\frac{M_\pi^2}{\mu^2}\right), \quad l_2^r = \frac{1}{48\pi^2}\left(\bar{l}_2 + \ln\frac{M_\pi^2}{\mu^2}\right)$$

$Y^{(2,2)}(t = 4m^2)$ bounds

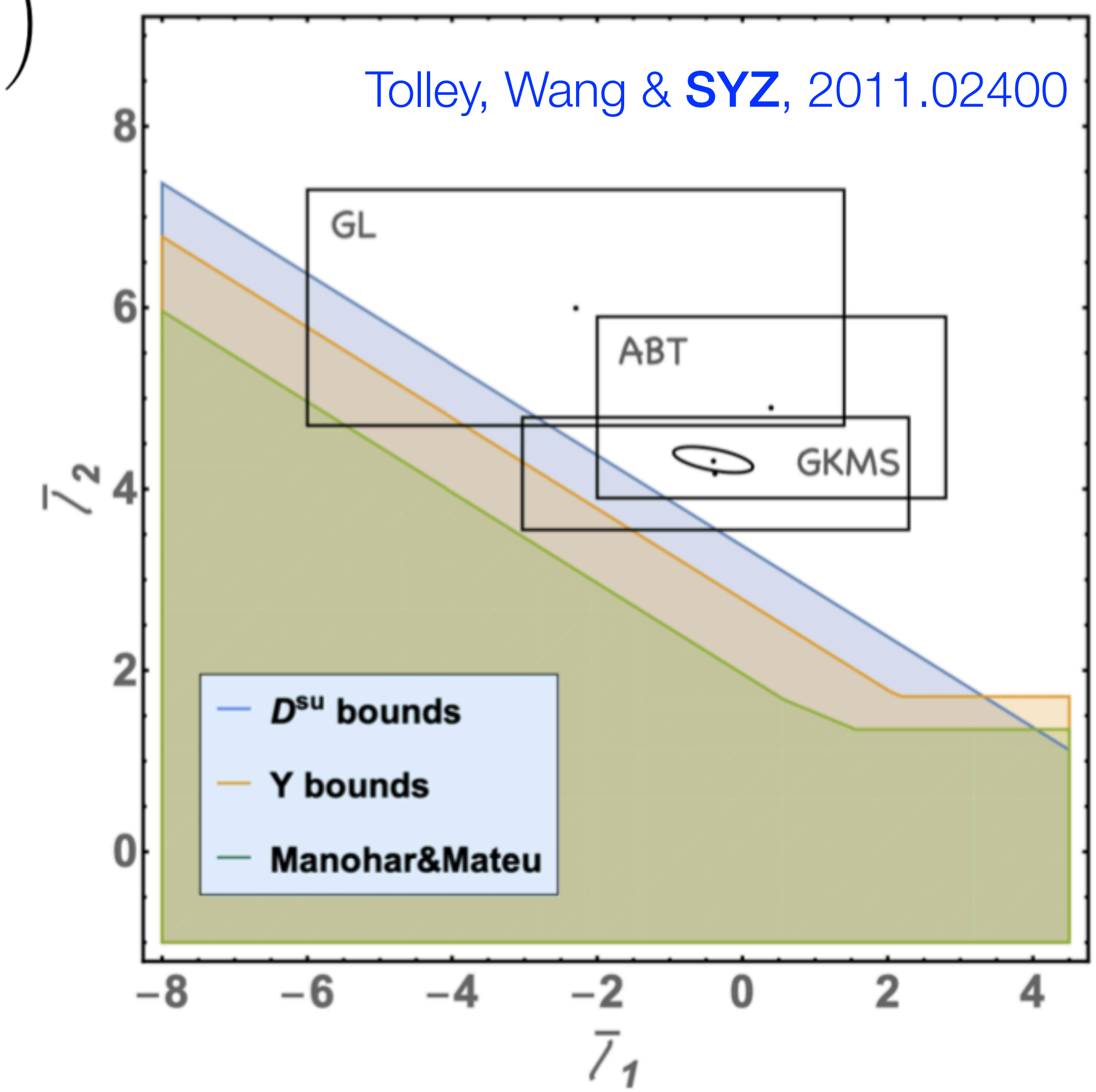
$$\bar{l}_1 + 2\bar{l}_2 > \frac{1559}{280}, \qquad \bar{l}_2 > \frac{719}{420},$$

1st nonlinear PQ bound

$$c_{1,0}\ c_{3,0} > c_{2,0}^2 \implies \bar{l}_1 + 2\bar{l}_2 > 3.85,$$

1st $D^{\rm su}$ bound

$$c_{1,1} + \frac{3}{2}\sqrt{c_{1,0}\ c_{2,0}} > 0 \implies \bar{l}_1 + 2\bar{l}_2 > 6.74.$$



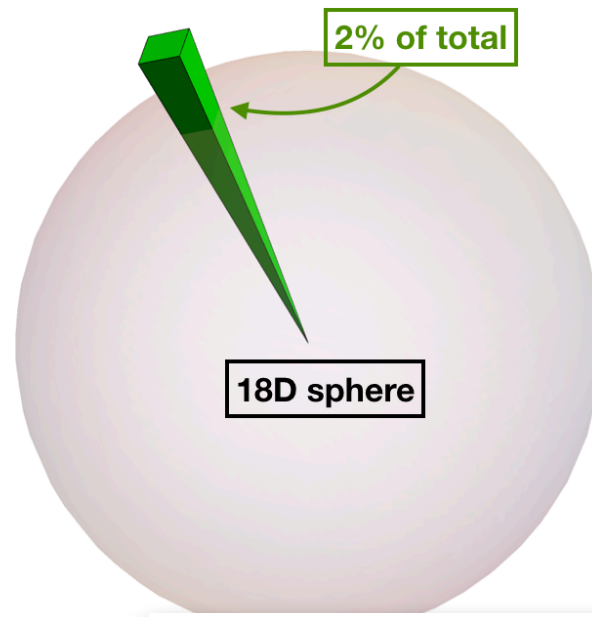
Can also bound other parameters

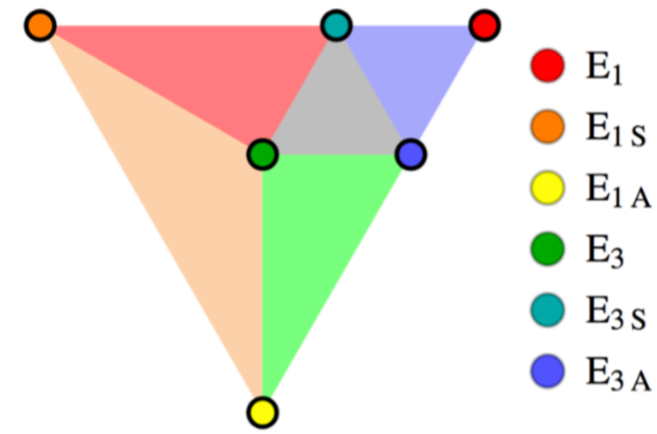
Wang, Feng, Zhang & **SYZ,** 2004.03992

Applications in other EFTs

- Standard Model EFT

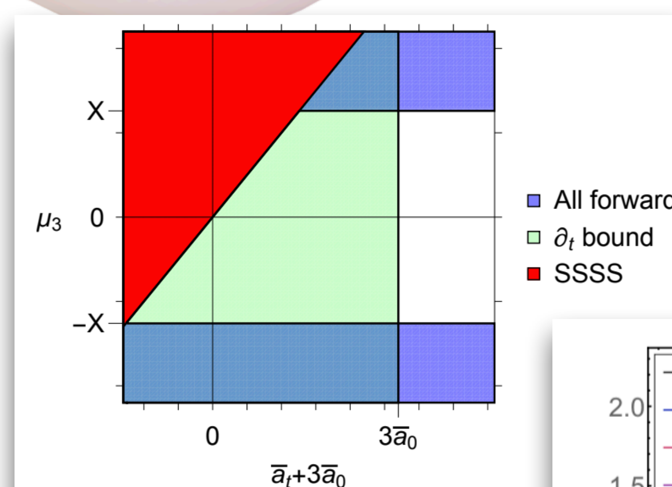
  Yamashita, Zhang & **SYZ**, 2009.04490
  Zhang & **SYZ**, 2005.03047
  Bi, Zhang & **SYZ**, 1902.08977
  Zhang & **SYZ**,1808.00010





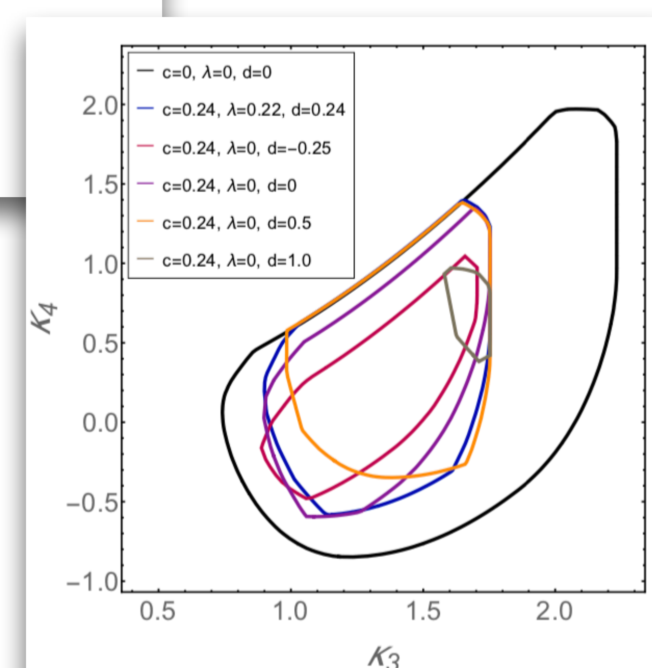
- Proca EFT

  de Rham, Melville, Tolley & **SYZ**, 1804.10624



- Spin-2 EFTs

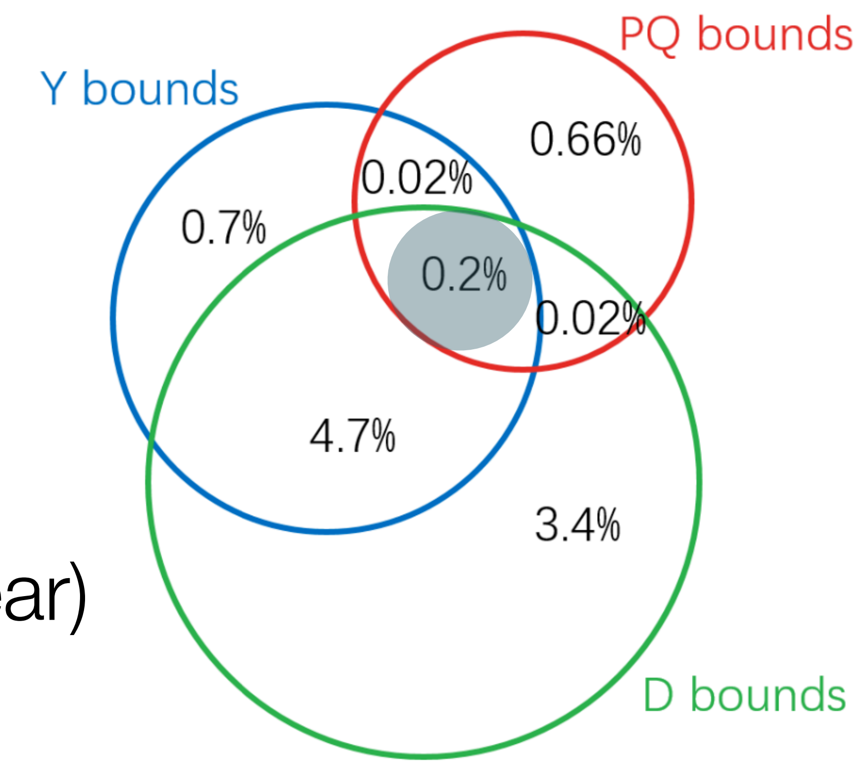
  de Rham, Melville, Tolley & **SYZ**, 1804.10624
  Wang, Zhang & **SYZ**, 2011.2011.05190



+ many other works by other authors

Summary(1)

- Utilizing 1) dispersion relation
  2) partial wave expansion
  3) full crossing symmetry



- Found new sets of bounds

  1. PQ-type bounds (linear and nonlinear)
  2. D^{su}-type bounds
  3. D^{stu} and $\bar{D}^{stu}$-type bounds

- Excluded some soft theories such as Galileon to have a Wilsonian UV completion

Summary (2)

D^{stu} and $\bar{D}^{stu}$-type bounds:

$$\mathscr{L} = \sum_i \Lambda^4 f_i \, \mathcal{O}_i \left(\frac{\text{boson}}{\Lambda}, \frac{\text{fermion}}{\Lambda^{3/2}}, \frac{\partial}{\Lambda} \right)$$

f_i : Wilson coefficients

Not only are f_i's bounded,

but also they are bounded from both sides

which loosely means that $f_i \sim \mathcal{O}(1)$

Thank you!